Lab 4 - Series Expansions

In[189]:=
```
Needs["PlotLegends`"]
dir = NotebookDirectory[];
SetDirectory[dir];
```

Series Sin and Cosine around x=0 out to n terms.

In[192]:=
```
SerCos[x_, n_] := Normal[Series[Cos[a], {a, 0, n}]] /. a → x
```

In[193]:=
```
SerSin[x_, n_] := Normal[Series[Sin[a], {a, 0, n}]] /. a → x
```

Evaluate the difference between SerSin and Sin.

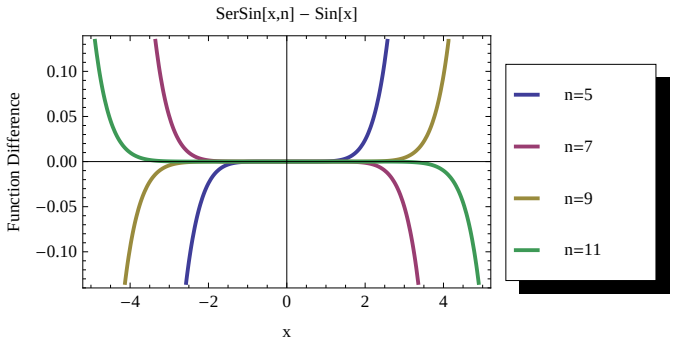
In[194]:=
```
serSinDiff[x_, n_] := SerSin[x, n] - Sin[x]
```

In[195]:=
```
data = Table[serSinDiff[x, n], {n, 5, 11, 2}]
```

Out[195]=
$$\left\{x-\frac{x^3}{6}+\frac{x^5}{120}-\text{Sin}[x],\ x-\frac{x^3}{6}+\frac{x^5}{120}-\frac{x^7}{5040}-\text{Sin}[x],\right.$$
$$\left.x-\frac{x^3}{6}+\frac{x^5}{120}-\frac{x^7}{5040}+\frac{x^9}{362\,880}-\text{Sin}[x],\ x-\frac{x^3}{6}+\frac{x^5}{120}-\frac{x^7}{5040}+\frac{x^9}{362\,880}-\frac{x^{11}}{39\,916\,800}-\text{Sin}[x]\right\}$$

In[196]:=
```
graph = Plot[{data[[1]], data[[2]], data[[3]], data[[4]]}, {x, -5, 5}, Frame → True,
  FrameLabel → {{"Function Difference", ""}, {"x", "SerSin[x,n] - Sin[x]"}},
  PlotLegend → {"n=5", "n=7", "n=9", "n=11"},
  LegendPosition → {.85, -0.4}, PlotStyle → Thick]
```

Out[196]=



In[197]:=
```
Export["difference.png", graph]
```

Out[197]=
```
difference.png
```

Define SerCos^2 + SerSin^2

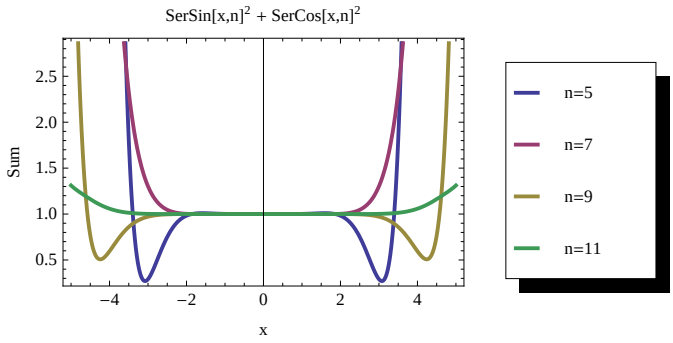
In[198]:=
```
sumSquareSer[x_, n_] := SerCos[x, n]^2 + SerSin[x, n]^2
```

In[199]:=
```
data = Table[sumSquareSer[x, n], {n, 5, 11, 2}]
```

Out[199]=
$$\left\{\left(1-\frac{x^2}{2}+\frac{x^4}{24}\right)^2+\left(x-\frac{x^3}{6}+\frac{x^5}{120}\right)^2,\ \left(1-\frac{x^2}{2}+\frac{x^4}{24}-\frac{x^6}{720}\right)^2+\left(x-\frac{x^3}{6}+\frac{x^5}{120}-\frac{x^7}{5040}\right)^2,\right.$$
$$\left(1-\frac{x^2}{2}+\frac{x^4}{24}-\frac{x^6}{720}+\frac{x^8}{40\,320}\right)^2+\left(x-\frac{x^3}{6}+\frac{x^5}{120}-\frac{x^7}{5040}+\frac{x^9}{362\,880}\right)^2,$$
$$\left.\left(1-\frac{x^2}{2}+\frac{x^4}{24}-\frac{x^6}{720}+\frac{x^8}{40\,320}-\frac{x^{10}}{3\,628\,800}\right)^2+\left(x-\frac{x^3}{6}+\frac{x^5}{120}-\frac{x^7}{5040}+\frac{x^9}{362\,880}-\frac{x^{11}}{39\,916\,800}\right)^2\right\}$$

In[200]:=
```
graph = Plot[{data[[1]], data[[2]], data[[3]], data[[4]]}, {x, -5, 5},
  Frame → True, FrameLabel → {{"Sum", ""}, {"x", "SerSin[x,n]^2 + SerCos[x,n]^2"}},
  PlotLegend → {"n=5", "n=7", "n=9", "n=11"},
  LegendPosition → {.85, -0.4}, PlotStyle → Thick]
```

Out[200]=



In[201]:=
```
Export["sumSer.png", graph]
```

Out[201]= sumSer.png

Define SerCosSq and SerSinSq.

In[202]:=
```
SerCosSq[x_, n_] := Normal[Series[Cos[a]^2, {a, 0, n}]] /. a → x
```

In[203]:=
```
SerSinSq[x_, n_] := Normal[Series[Sin[a]^2, {a, 0, n}]] /. a → x
```

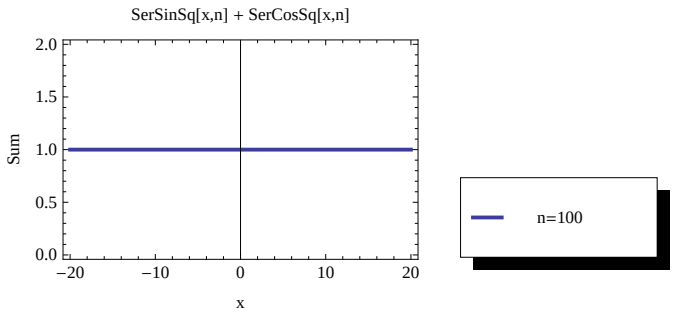
In[204]:=
```
sumSerSquare[x_, n_] := SerCosSq[x, n] + SerSinSq[x, n]
```

In[205]:=
```
data = Table[sumSerSquare[x, n], {n, 5, 100, 1}]
```

Out[205]=
```
{1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}
```

In[206]:=
```
graph = Plot[data[[Length[data]]], {x, -20, 20}, Frame → True,
  FrameLabel → {{"Sum", ""}, {"x", "SerSinSq[x,n] + SerCosSq[x,n]"}},
  PlotStyle → Thick, PlotLegend → {"n=100"}, LegendPosition → {.85, -0.4}]
```

Out[206]=



In[207]:=
```
Export["sumSerSq.png", graph]
```

Out[207]= sumSerSq.png

Define three rotation matrices

In[208]:=
```
rX[th_] := {{1, 0, 0}, {0, Cos[th], Sin[th]}, {0, -Sin[th], Cos[th]}}
```

In[209]:=
```
MatrixForm[rX[x]]
```

Out[209]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \text{Cos}[x] & \text{Sin}[x] \\ 0 & -\text{Sin}[x] & \text{Cos}[x] \end{pmatrix}$$

In[210]:=
```
rY[ski_] := {{Cos[ski], 0, Sin[ski]}, {0, 1, 0}, {-Sin[ski], 0, Cos[ski]}}
```

In[211]:=
```
MatrixForm[rY[x]]
```

Out[211]//MatrixForm=

$$\begin{pmatrix} \text{Cos}[x] & 0 & \text{Sin}[x] \\ 0 & 1 & 0 \\ -\text{Sin}[x] & 0 & \text{Cos}[x] \end{pmatrix}$$

In[212]:=
```
rZ[phi_] := {{Cos[phi], Sin[phi], 0}, {-Sin[phi], Cos[phi], 0}, {0, -0, 1}}
```

In[213]:=
```
MatrixForm[rZ[x]]
```

Out[213]//MatrixForm=

$$\begin{pmatrix} \text{Cos}[x] & \text{Sin}[x] & 0 \\ -\text{Sin}[x] & \text{Cos}[x] & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Full Rotation

In[214]:=
```
Rot3[a1_, a2_, a3_] := rZ[a1].rX[a2].rZ[a3]
```

In[215]:=
```
MatrixForm[Rot3[x, y, z]]
```

Out[215]//MatrixForm=

$$\begin{pmatrix} \text{Cos}[x]\,\text{Cos}[z]-\text{Cos}[y]\,\text{Sin}[x]\,\text{Sin}[z] & \text{Cos}[y]\,\text{Cos}[z]\,\text{Sin}[x]+\text{Cos}[x]\,\text{Sin}[z] & \text{Sin}[x]\,\text{Sin}[y] \\ -\text{Cos}[z]\,\text{Sin}[x]-\text{Cos}[x]\,\text{Cos}[y]\,\text{Sin}[z] & \text{Cos}[x]\,\text{Cos}[y]\,\text{Cos}[z]-\text{Sin}[x]\,\text{Sin}[z] & \text{Cos}[x]\,\text{Sin}[y] \\ \text{Sin}[y]\,\text{Sin}[z] & -\text{Cos}[z]\,\text{Sin}[y] & \text{Cos}[y] \end{pmatrix}$$

In[216]:=
```
MatrixForm[Simplify[Rot3[x, y, z]]]
```

Out[216]//MatrixForm=

$$\begin{pmatrix} \text{Cos}[x]\,\text{Cos}[z]-\text{Cos}[y]\,\text{Sin}[x]\,\text{Sin}[z] & \text{Cos}[y]\,\text{Cos}[z]\,\text{Sin}[x]+\text{Cos}[x]\,\text{Sin}[z] & \text{Sin}[x]\,\text{Sin}[y] \\ -\text{Cos}[z]\,\text{Sin}[x]-\text{Cos}[x]\,\text{Cos}[y]\,\text{Sin}[z] & \text{Cos}[x]\,\text{Cos}[y]\,\text{Cos}[z]-\text{Sin}[x]\,\text{Sin}[z] & \text{Cos}[x]\,\text{Sin}[y] \\ \text{Sin}[y]\,\text{Sin}[z] & -\text{Cos}[z]\,\text{Sin}[y] & \text{Cos}[y] \end{pmatrix}$$

Rotational inverse with negative angles

In[217]:=
```
Rot3Inverse[a1_, a2_, a3_] := Rot3[-a3, -a2, -a1]
```

ReverseAngles times Regular is the identity.

In[218]:=
```
revAngle = MatrixForm[Rot3Inverse[x, y, z].Rot3[x, y, z]]
```

Out[218]//MatrixForm=

$$\begin{pmatrix} \text{Sin}[y]^2\,\text{Sin}[z]^2+(-\text{Cos}[z]\,\text{Sin}[x]-\text{Cos}[x]\,\text{Cos}[y]\,\text{Sin}[z \\ -\text{Cos}[z]\,\text{Sin}[y]^2\,\text{Sin}[z]+(-\text{Cos}[z]\,\text{Sin}[x]-\text{Cos}[x]\,\text{Cos}[y]\,\text{Sin}[z])\,(\text{Cos}[x]\,\text{Cos}[y]\,\text{Cos}[z]-\text{Sin}[x]\,\text{S} \\ \text{Cos}[y]\,\text{Sin}[y]\,\text{Sin}[z]+\text{Cos}[x]\,\text{Sin}[y]\,(-\text{Cos}[z]\,\text{Sin}[x]-\text{Cos}[x]\,\text{Cos}[y]\,\text{Si} \end{pmatrix}$$

In[219]:=
```
Simplify[revAngle]
```

Out[219]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Inverse Matrix times Matrix is the Identity

In[220]:=
```
invFunc = MatrixForm[Inverse[Rot3[x, y, z]].Rot3[x, y, z]]
```

Out[220]//MatrixForm=

$$\begin{pmatrix} \frac{(-\text{Cos}[z]\,\text{Sin}[x]-\text{Cos}[x]\,\text{Cos}[y]\,\text{Sin}[z])\,(-\text{Cos}[y]^2\,\text{Cos}[z]\,\text{Sin}[x]-\text{Cos}[z]\,\text{Sin}[x]\,\text{Sin}[y]^2-\text{Cos}[x]}{\text{Cos}[x]^2\,\text{Cos}[y]^2\,\text{Cos}[z]^2+\text{Cos}[y]^2\,\text{Cos}[z]^2\,\text{Sin}[x]^2+\text{Cos}[x]^2\,\text{Cos}[z]^2\,\text{Sin}[y]^2+\text{Cos}[z]^2\,\text{Sin}[x]^2\,\text{Sin}[y]^2+\text{Cos}[x]^2\,\text{Cos}[y]^2\,\text{Sin}[z]^2+\text{Cos}[y]^2\,\text{Sin}[x]} \\ \frac{\text{Sin}[y]\,(-\text{Cos}[x]^2\,\text{Cos}[z]\,\text{Sin}[y]-\text{Cos}[z]\,\text{Sin}[x]^2\,\text{Sin}[y])\,\text{Sin}[z]}{\text{Cos}[x]^2\,\text{Cos}[y]^2\,\text{Cos}[z]^2+\text{Cos}[y]^2\,\text{Cos}[z]^2\,\text{Sin}[x]^2+\text{Cos}[x]^2\,\text{Cos}[z]^2\,\text{Sin}[y]^2+\text{Cos}[z]^2\,\text{Sin}[x]^2\,\text{Sin}[y]^2+\text{Cos}[x]^2\,\text{Cos}[y]^2\,\text{Sin}[z]^2+\text{Cos}[y]^2\,\text{Sin}[x]} \\ \frac{\text{Sin}[y]\,\text{Sin}[z]\,(\text{Cos}[x]^2\,\text{Cos}[y]\,\text{Cos}[z]^2+\text{Cos}[y]\,\text{Cos}[z]^2\,\text{Sin}[x]^2+\text{Cos}[x]^2\,\text{Cos}[y]\,\text{Sin}[z]^2+\text{Cos}[y]\,\text{S}}{\text{Cos}[x]^2\,\text{Cos}[y]^2\,\text{Cos}[z]^2+\text{Cos}[y]^2\,\text{Cos}[z]^2\,\text{Sin}[x]^2+\text{Cos}[x]^2\,\text{Cos}[z]^2\,\text{Sin}[y]^2+\text{Cos}[z]^2\,\text{Sin}[x]^2\,\text{Sin}[y]^2+\text{Cos}[x]^2\,\text{Cos}[y]^2\,\text{Sin}[z]^2+\text{Cos}[y]^2\,\text{Sin}[x]} \end{pmatrix}$$

In[221]:=
```
Simplify[invFunc]
```

Out[221]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In[222]:=
```
invDiff[x_, y_, z_] := Inverse[Rot3[x, y, z]] - Rot3Inverse[x, y, z]
```

In[223]:=
```
diff = MatrixForm[invDiff[x, y, z]]
```

Out[223]//MatrixForm=

$$\begin{pmatrix} -\text{Cos}[x]\,\text{Cos}[z] + \text{Cos}[y]\,\text{Sin}[x]\,\text{Sin}[z] + \frac{\text{Cos}[x]\,\text{Cos}[y]^2\,\text{C}}{\text{Cos}[x]^2\,\text{Cos}[y]^2\,\text{Cos}[z]^2+\text{Cos}[y]^2\,\text{Cos}[z]^2\,\text{Sin}[x]^2+\text{Cos}[x]^2\,\text{Cos}[z]^2\,\text{Sin}[y]^2+\text{Cos}[z]^2} \\ -\text{Cos}[y]\,\text{Cos}[z]\,\text{Sin}[x] - \text{Cos}[x]\,\text{Sin}[z] + \frac{\text{Cos}[y]\,\text{Cos}[z]\,\text{Si}}{\text{Cos}[x]^2\,\text{Cos}[y]^2\,\text{Cos}[z]^2+\text{Cos}[y]^2\,\text{Cos}[z]^2\,\text{Sin}[x]^2+\text{Cos}[x]^2\,\text{Cos}[z]^2\,\text{Sin}[y]^2+\text{Cos}[z]^2} \\ -\text{Sin}[x]\,\text{Sin}[y] + \frac{\text{Cos}[z]^2\,\text{Sin}[x]\,\text{Sin}[y]}{\text{Cos}[x]^2\,\text{Cos}[y]^2\,\text{Cos}[z]^2+\text{Cos}[y]^2\,\text{Cos}[z]^2\,\text{Sin}[x]^2+\text{Cos}[x]^2\,\text{Cos}[z]^2\,\text{Sin}[y]^2+\text{Cos}[z]^2\,\text{Sin}[x]^2\,\text{Sin}[y]^2} \end{pmatrix}$$

In[224]:=
```
Simplify[diff]
```

Out[224]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

In[225]:=
```
Export["series.pdf", EvaluationNotebook[]]
```

Out[188]= series.pdf