

Lab 4 - Series Expansions

```
In[168]:= Needs["PlotLegends`"]
dir = NotebookDirectory[];
SetDirectory[dir];
```

Series Sin and Cosine around x=0 out to n terms.

```
In[171]:= SerCos[x_, n_] := Normal[Series[Cos[a], {a, 0, n}]] /. a -> x
```

```
In[172]:= SerSin[x_, n_] := Normal[Series[Sin[a], {a, 0, n}]] /. a -> x
```

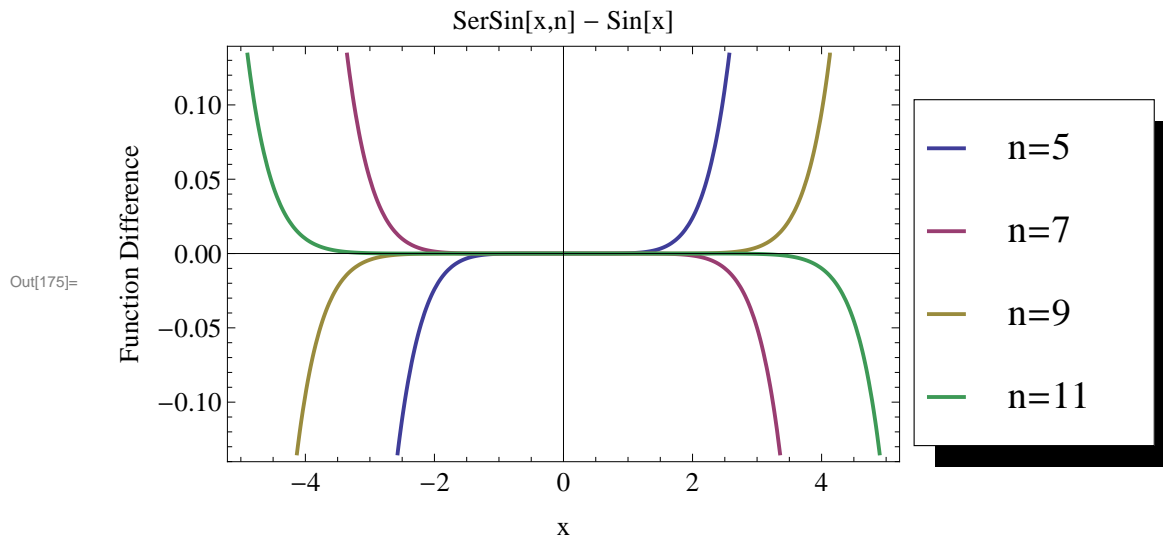
Evaluate the difference between SerSin and Sin.

```
In[173]:= serSinDiff[x_, n_] := SerSin[x, n] - Sin[x]
```

```
In[174]:= data = Table[serSinDiff[x, n], {n, 5, 11, 2}]
```

```
Out[174]:= {x - \frac{x^3}{6} + \frac{x^5}{120} - Sin[x], x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} - Sin[x],
x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} + \frac{x^9}{362880} - Sin[x], x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} + \frac{x^9}{362880} - \frac{x^{11}}{39916800} - Sin[x]}
```

```
In[175]:= graph = Plot[{data[[1]], data[[2]], data[[3]], data[[4]]}, {x, -5, 5}, Frame -> True,
FrameLabel -> {"Function Difference", ""}, {"x", "SerSin[x,n] - Sin[x]"},
PlotLegend -> {Style["n=5", 20], Style["n=7", 20], Style["n=9", 20], Style["n=11", 20]},
LegendPosition -> {.85, -0.4}, PlotStyle -> Thick, ImageSize -> Large, LabelStyle -> Larger]
```



```
In[176]:= Export["difference.png", graph]
```

```
Out[176]= difference.png
```

Define SerCos^2 + SerSin^2

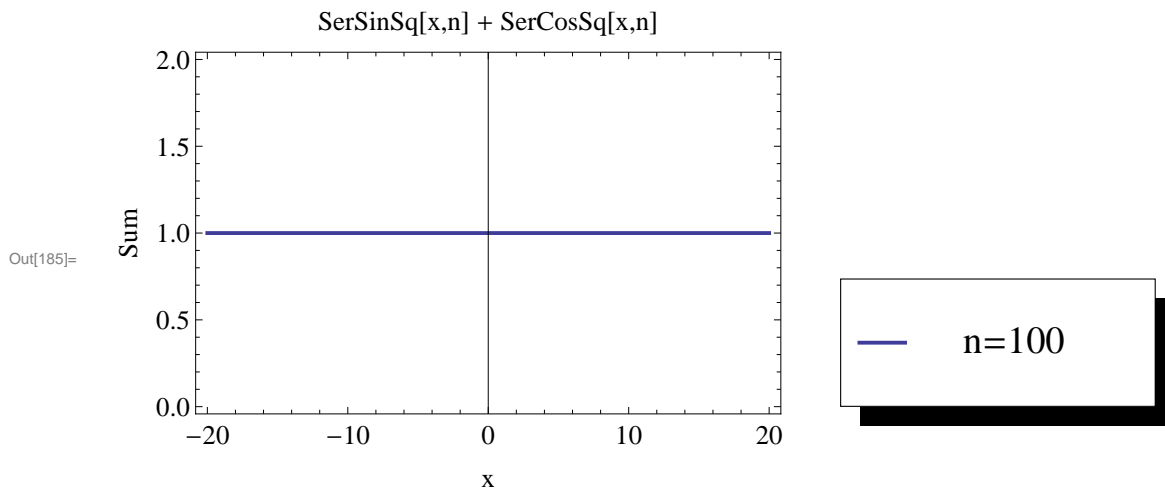
```
In[177]:= sumSquareSer[x_, n_] := SerCos[x, n]^2 + SerSin[x, n]^2
```

```
In[178]:= data = Table[sumSquareSer[x, n], {n, 5, 11, 2}]
```

$$\text{Out[178]= } \left\{ \left(1 - \frac{x^2}{2} + \frac{x^4}{24} \right)^2 + \left(x - \frac{x^3}{6} + \frac{x^5}{120} \right)^2, \left(1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} \right)^2 + \left(x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} \right)^2, \right. \\ \left(1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \frac{x^8}{40320} \right)^2 + \left(x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} + \frac{x^9}{362880} \right)^2, \\ \left. \left(1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \frac{x^8}{40320} - \frac{x^{10}}{3628800} \right)^2 + \left(x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} + \frac{x^9}{362880} - \frac{x^{11}}{39916800} \right)^2 \right\}$$

```
In[179]:= graph = Plot[{data[[1]], data[[2]], data[[3]], data[[4]]}, {x, -5, 5},
  Frame → True, FrameLabel → {{{"Sum", ""}, {"x", "SerSin[x,n]2 + SerCos[x,n]2"}}},
  PlotLegend → {Style["n=5", 20], Style["n=7", 20], Style["n=9", 20], Style["n=11", 20]},
  LegendPosition → {.85, -0.4}, PlotStyle → Thick, ImageSize → Large, LabelStyle → Larger]
```

```
In[185]:= graph = Plot[data[[Length[data]]], {x, -20, 20}, Frame → True,
  FrameLabel → {"Sum", ""}, {"x", "SerSinSq[x,n] + SerCosSq[x,n]"},
  PlotStyle → Thick, PlotLegend → {Style["n=100", 20]},
  LegendPosition → {.85, -0.4}, ImageSize → Large, LabelStyle → Larger]
```



```
In[186]:= Export["sumSerSq.png", graph]
```

Out[186]= sumSerSq.png

Define three rotation matrices

```
In[187]:= rX[th_] := {{1, 0, 0}, {0, Cos[th], Sin[th]}, {0, -Sin[th], Cos[th]}}
```

```
In[188]:= MatrixForm[rX[x]]
```

Out[188]/MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos[x] & \sin[x] \\ 0 & -\sin[x] & \cos[x] \end{pmatrix}$$

```
In[189]:= rY[ski_] := {{Cos[ski], 0, Sin[ski]}, {0, 1, 0}, {-Sin[ski], 0, Cos[ski]}}
```

```
In[190]:= MatrixForm[rY[x]]
```

Out[190]/MatrixForm=

$$\begin{pmatrix} \cos[x] & 0 & \sin[x] \\ 0 & 1 & 0 \\ -\sin[x] & 0 & \cos[x] \end{pmatrix}$$

```
In[191]:= rZ[phi_] := {{Cos[phi], Sin[phi], 0}, {-Sin[phi], Cos[phi], 0}, {0, -0, 1}}
```

```
In[192]:= MatrixForm[rZ[x]]
```

Out[192]/MatrixForm=

$$\begin{pmatrix} \cos[x] & \sin[x] & 0 \\ -\sin[x] & \cos[x] & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Full Rotation

```
In[193]:= Rot3[a1_, a2_, a3_] := rZ[a1].rX[a2].rZ[a3]
```

```
In[194]:= MatrixForm[Rot3[x, y, z]]
```

Out[194]/MatrixForm=

$$\begin{pmatrix} \cos[x] \cos[z] - \cos[y] \sin[x] \sin[z] & \cos[y] \cos[z] \sin[x] + \cos[x] \sin[z] & \sin[x] \sin[y] \\ -\cos[z] \sin[x] - \cos[x] \cos[y] \sin[z] & \cos[x] \cos[y] \cos[z] - \sin[x] \sin[z] & \cos[x] \sin[y] \\ \sin[y] \sin[z] & -\cos[z] \sin[y] & \cos[y] \end{pmatrix}$$

```

In[195]:= MatrixForm[Simplify[Rot3[x, y, z]]]
Out[195]//MatrixForm=

$$\begin{pmatrix} \cos[x] \cos[z] - \cos[y] \sin[x] \sin[z] & \cos[y] \cos[z] \sin[x] + \cos[x] \sin[z] & \sin[x] \sin[y] \\ -\cos[z] \sin[x] - \cos[x] \cos[y] \sin[z] & \cos[x] \cos[y] \cos[z] - \sin[x] \sin[z] & \cos[x] \sin[y] \\ \sin[y] \sin[z] & -\cos[z] \sin[y] & \cos[y] \end{pmatrix}$$

Rotational inverse with negative angles
In[196]:= Rot3Inverse[a1_, a2_, a3_] := Rot3[-a3, -a2, -a1]
ReverseAngles times Regular is the identity.
In[197]:= revAngle = MatrixForm[Rot3Inverse[x, y, z].Rot3[x, y, z]]
Out[197]//MatrixForm=

$$\begin{pmatrix} \sin[y]^2 \sin[z]^2 + (-\cos[z] \sin[x] - \cos[x] \cos[y] \sin[z]) & & \\ -\cos[z] \sin[y]^2 \sin[z] + (-\cos[z] \sin[x] - \cos[x] \cos[y] \sin[z]) & (\cos[x] \cos[y] \cos[z] - \sin[x] \sin[z]) & \\ & \cos[y] \sin[y] \sin[z] + \cos[x] \sin[y] & (-\cos[z] \sin[x] - \cos[x] \cos[y] \sin[z]) \end{pmatrix}$$

In[198]:= Simplify[revAngle]
Out[198]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Inverse Matrix times Matrix is the Identity
In[199]:= invFunc = MatrixForm[Inverse[Rot3[x, y, z]].Rot3[x, y, z]]
Out[199]//MatrixForm=

$$\begin{pmatrix} \frac{(-\cos[z] \sin[x] - \cos[x] \cos[y] \sin[z]) (-\cos[y]^2 \cos[z] \sin[x] - \cos[z] \sin[x] \sin[y]^2 - \cos[x]^2 \cos[y]^2 \cos[z]^2 \sin[x]^2 + \cos[y]^2 \cos[z]^2 \sin[x]^2 + \cos[x]^2 \cos[z]^2 \sin[y]^2 + \cos[z]^2 \sin[x]^2 \sin[y]^2 + \cos[x]^2 \cos[y]^2 \sin[z]^2 + \cos[y]^2 \sin[x]^2 \sin[y] (-\cos[x]^2 \cos[z] \sin[y] - \cos[z] \sin[x]^2 \sin[y]) \sin[z]}{\cos[x]^2 \cos[y]^2 \cos[z]^2 + \cos[y]^2 \cos[z]^2 \sin[x]^2 + \cos[x]^2 \cos[z]^2 \sin[y]^2 + \cos[z]^2 \sin[x]^2 \sin[y]^2 + \cos[x]^2 \cos[y]^2 \sin[z]^2 + \cos[y]^2 \sin[x]^2 \sin[y] \sin[z] (\cos[x]^2 \cos[y] \cos[z]^2 + \cos[y] \cos[z]^2 \sin[x]^2 + \cos[x]^2 \cos[y] \sin[z]^2 + \cos[y] \sin[x]^2 \cos[z]^2 \sin[y]^2 + \cos[x]^2 \cos[z]^2 \sin[y]^2 + \cos[z]^2 \sin[x]^2 \sin[y]^2 + \cos[x]^2 \cos[y]^2 \sin[z]^2 + \cos[y]^2 \sin[x]^2 \sin[y]^2} \end{pmatrix}$$

In[200]:= Simplify[invFunc]
Out[200]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In[201]:= invDiff[x_, y_, z_] := Inverse[Rot3[x, y, z]] - Rot3Inverse[x, y, z]
In[202]:= diff = MatrixForm[invDiff[x, y, z]]
Out[202]//MatrixForm=

$$\begin{pmatrix} -\cos[x] \cos[z] + \cos[y] \sin[x] \sin[z] + \frac{\cos[x] \cos[y]^2 \cos[z]^2 \sin[x]^2 + \cos[y]^2 \cos[z]^2 \sin[x]^2 + \cos[x]^2 \cos[z]^2 \sin[y]^2 + \cos[z]^2 \sin[x]^2 \sin[y]^2 + \cos[x]^2 \cos[y]^2 \sin[z]^2 + \cos[y]^2 \sin[x]^2 \sin[y] \sin[z] (\cos[x]^2 \cos[y] \cos[z]^2 + \cos[y] \cos[z]^2 \sin[x]^2 + \cos[x]^2 \cos[y] \sin[z]^2 + \cos[y] \sin[x]^2 \cos[z]^2 \sin[y]^2 + \cos[x]^2 \cos[z]^2 \sin[y]^2 + \cos[z]^2 \sin[x]^2 \sin[y]^2 + \cos[x]^2 \cos[y]^2 \sin[z]^2 + \cos[y]^2 \sin[x]^2 \sin[y]^2}}{\cos[x]^2 \cos[y]^2 \cos[z]^2 + \cos[y]^2 \cos[z]^2 \sin[x]^2 + \cos[x]^2 \cos[z]^2 \sin[y]^2 + \cos[z]^2 \sin[x]^2 \sin[y]^2 + \cos[x]^2 \cos[y]^2 \sin[z]^2 + \cos[y]^2 \sin[x]^2 \sin[y] \sin[z] (\cos[x]^2 \cos[y] \cos[z]^2 + \cos[y] \cos[z]^2 \sin[x]^2 + \cos[x]^2 \cos[y] \sin[z]^2 + \cos[y] \sin[x]^2 \cos[z]^2 \sin[y]^2 + \cos[x]^2 \cos[z]^2 \sin[y]^2 + \cos[z]^2 \sin[x]^2 \sin[y]^2 + \cos[x]^2 \cos[y]^2 \sin[z]^2 + \cos[y]^2 \sin[x]^2 \sin[y]^2} \end{pmatrix}$$

In[203]:= Simplify[diff]
Out[203]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

In[204]:= Export["series.pdf", EvaluationNotebook]]]
Out[157]= series.pdf

```