Phys 20 Lab 4 - Mathematica

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1 Part 1 - Mathematica

I am familiar with Mathematica and have included a sample notebook from another class that finds a polynomial expression for given data.

2 Part 1 - Series Expansion

2.1 SerCos and SerSin

I have defined two functions SerCos[x, n] and SerSin[x, n] that are the series expansion of Cos and Sin of x respectively out to n terms. Below is a plot of SerSin[x, n] - Sin[x] for various values of n.

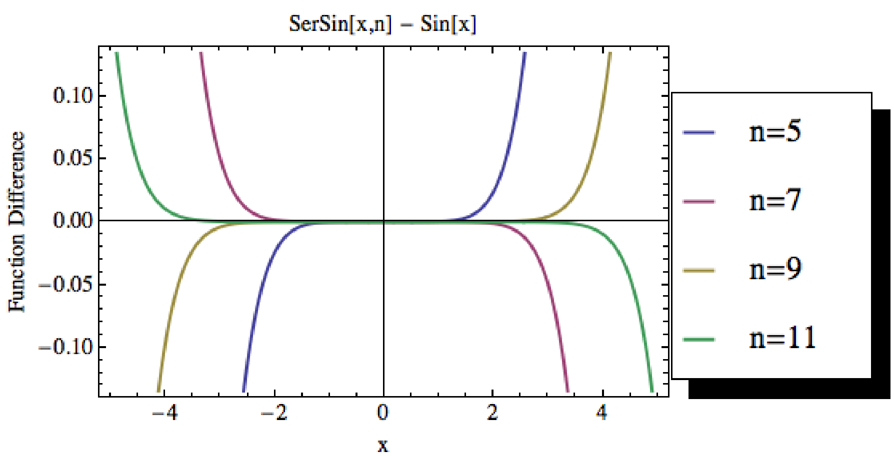


Figure 1: Difference between SerSin and Sin for various values of n

We can see that as n increases, the range of values for which the difference is 0 increases. If $n = 2a + 1$, then for even values of a, SerSin underestimates $x < 0$ and overestimates $x > 0$. The opposite is true for odd values of a. This happens because when a is even, the last expansion term is positive meaning the next term would subtract value form it since the sign alternate. The errors get larger and larger the further from 0 we go.

2.2 SerCosSq and SerSinSq

$SerCos^2 + SerSin^2$ does not uniformly equal one. Since they have different powers, the two expansions do not perfectly cancel out one another. Cos is even, while Sin is odd. As n increases, the range of values for which the sum is 1 (around 0) increases but it is not uniformly correct. The error eventually diverges to infinity.

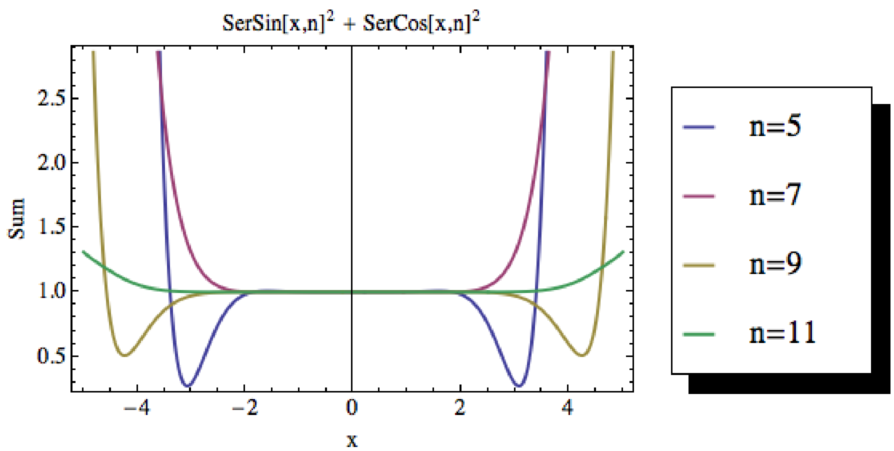


Figure 2: Sum of $SerSin^2 + SerCos^2$ for various values of n

$SerCosSq + SerSinSq$, where these are the expansions of Sin^2 and Cos^2 respectively, always sum to 1. Both of these are even functions that vary only by a single 1 in the Cosine expansion and the sign of each term. Therefore, the sum of the two will always be equal to 1 for any value of n.

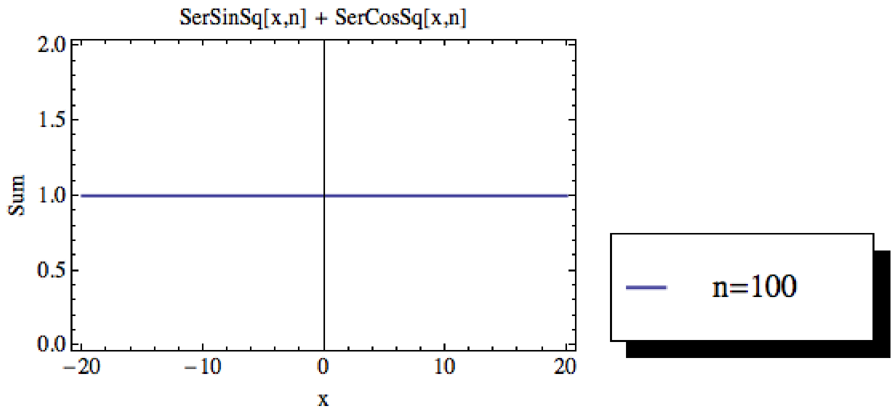


Figure 3: Sum of $SerSinSq + SerCosSq$ for n=100

3 Part 3 - Euler Expansion

Using the definitions provided, I made functions for Rx[th], Ry[ski], and Rz[phi].
Rot3 uses Euler's Expression to make another function:

$$Rot3[a1, a2, a3] := Rz[a1].Rx[a2].Rz[a3]$$

This expression does not simplify.
Rot3Inverse using negative angles is defined in terms of Rot3:

$$Rot3Inverse[a1, a2, a3] := Rot3[-a3, -a2, -a1]$$

The product is a very complicated expression of the three initial matrices, but luckily it simplifies to the identity matrix as expected.

Using Mathematica's Inverse function, we can get another complicated expression for Inverse[Rot3[x,y,z]] . Rot3[x,y,z], but again it simplifies to the identity matrix as expected.

Finally, by computing Rot3Inverse[x,y,z] - Inverse[Rot3[x,y,z]], we get our most complicated expression yet, but once we simplify it, we get the zero matrix meaning the two expressions are indeed equivalent.

4 Code and Info

4.1 Code

Code for this week's set is appended at the end of the file as a pdf version of the Mathematica code.

Lab 4 - Series Expansions

In[168]:=
```
Needs["PlotLegends`"]
dir = NotebookDirectory[];
SetDirectory[dir];
```

Series Sin and Cosine around x=0 out to n terms.

In[171]:=
```
SerCos[x_, n_] := Normal[Series[Cos[a], {a, 0, n}]] /. a → x
```

In[172]:=
```
SerSin[x_, n_] := Normal[Series[Sin[a], {a, 0, n}]] /. a → x
```

Evaluate the difference between SerSin and Sin.

In[173]:=
```
serSinDiff[x_, n_] := SerSin[x, n] - Sin[x]
```

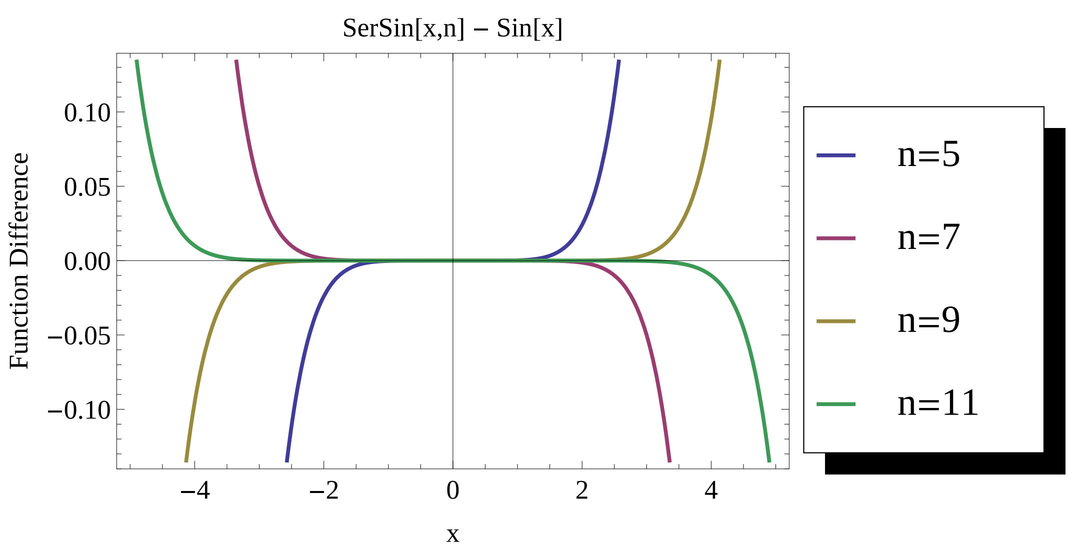
In[174]:=
```
data = Table[serSinDiff[x, n], {n, 5, 11, 2}]
```

Out[174]=

$$\left\{x - \frac{x^3}{6} + \frac{x^5}{120} - \mathrm{Sin}[x],\ x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} - \mathrm{Sin}[x],\ x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} + \frac{x^9}{362880} - \mathrm{Sin}[x],\ x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} + \frac{x^9}{362880} - \frac{x^{11}}{39916800} - \mathrm{Sin}[x]\right\}$$

In[175]:=
```
graph = Plot[{data[[1]], data[[2]], data[[3]], data[[4]]}, {x, -5, 5}, Frame → True,
  FrameLabel → {{"Function Difference", ""}, {"x", "SerSin[x,n] - Sin[x]"}},
  PlotLegend → {Style["n=5", 20], Style["n=7", 20], Style["n=9", 20], Style["n=11", 20]},
  LegendPosition → {.85, -0.4}, PlotStyle → Thick, ImageSize → Large, LabelStyle → Larger]
```

Out[175]=



In[176]:=
```
Export["difference.png", graph]
```

Out[176]= difference.png

Define SerCos^2 + SerSin^2

In[177]:=
```
sumSquareSer[x_, n_] := SerCos[x, n]^2 + SerSin[x, n]^2
```

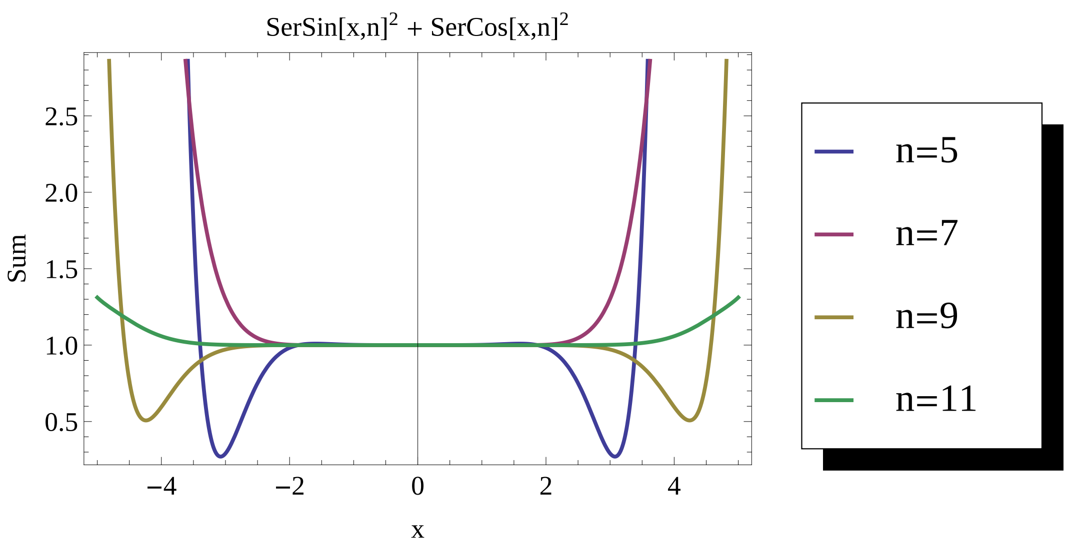
In[178]:=
```
data = Table[sumSquareSer[x, n], {n, 5, 11, 2}]
```

Out[178]=

$$\left\{\left(1-\frac{x^2}{2}+\frac{x^4}{24}\right)^2+\left(x-\frac{x^3}{6}+\frac{x^5}{120}\right)^2,\ \left(1-\frac{x^2}{2}+\frac{x^4}{24}-\frac{x^6}{720}\right)^2+\left(x-\frac{x^3}{6}+\frac{x^5}{120}-\frac{x^7}{5040}\right)^2,\right.$$
$$\left(1-\frac{x^2}{2}+\frac{x^4}{24}-\frac{x^6}{720}+\frac{x^8}{40320}\right)^2+\left(x-\frac{x^3}{6}+\frac{x^5}{120}-\frac{x^7}{5040}+\frac{x^9}{362880}\right)^2,$$
$$\left.\left(1-\frac{x^2}{2}+\frac{x^4}{24}-\frac{x^6}{720}+\frac{x^8}{40320}-\frac{x^{10}}{3628800}\right)^2+\left(x-\frac{x^3}{6}+\frac{x^5}{120}-\frac{x^7}{5040}+\frac{x^9}{362880}-\frac{x^{11}}{39916800}\right)^2\right\}$$

In[179]:=
```
graph = Plot[{data[[1]], data[[2]], data[[3]], data[[4]]}, {x, -5, 5},
  Frame → True, FrameLabel → {{"Sum", ""}, {"x", "SerSin[x,n]^2 + SerCos[x,n]^2"}},
  PlotLegend → {Style["n=5", 20], Style["n=7", 20], Style["n=9", 20], Style["n=11", 20]},
  LegendPosition → {.85, -0.4}, PlotStyle → Thick, ImageSize → Large, LabelStyle → Larger]
```

Out[179]=



In[180]:=
```
Export["sumSer.png", graph]
```

Out[180]= sumSer.png

Define SerCosSq and SerSinSq.

In[181]:=
```
SerCosSq[x_, n_] := Normal[Series[Cos[a]^2, {a, 0, n}]] /. a → x
```

In[182]:=
```
SerSinSq[x_, n_] := Normal[Series[Sin[a]^2, {a, 0, n}]] /. a → x
```

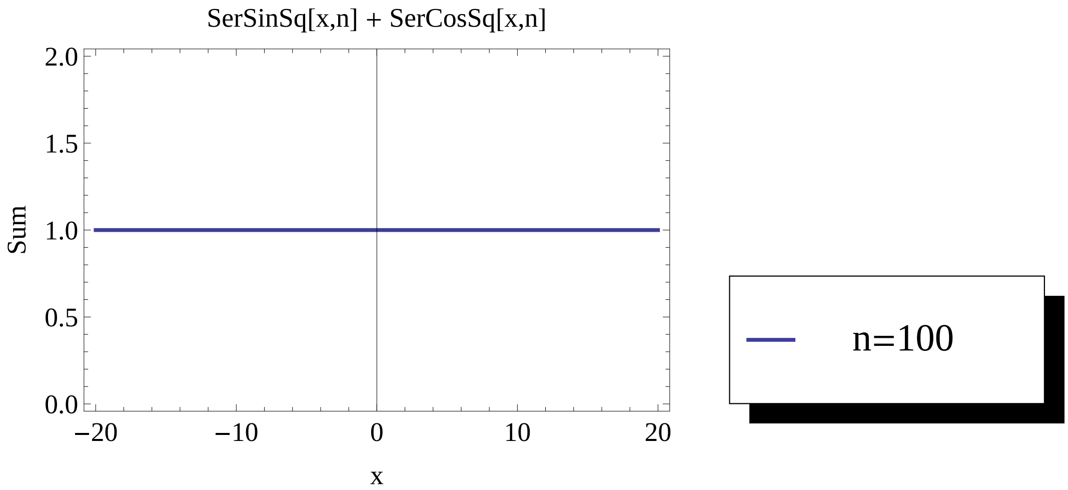
In[183]:=
```
sumSerSquare[x_, n_] := SerCosSq[x, n] + SerSinSq[x, n]
```

In[184]:=
```
data = Table[sumSerSquare[x, n], {n, 5, 100, 1}]
```

Out[184]=
```
{1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}
```

In[185]:=
```
graph = Plot[data[[Length[data]]], {x, -20, 20}, Frame → True,
  FrameLabel → {{"Sum", ""}, {"x", "SerSinSq[x,n] + SerCosSq[x,n]"}},
  PlotStyle → Thick, PlotLegend → {Style["n=100", 20]},
  LegendPosition → {.85, -0.4}, ImageSize → Large, LabelStyle → Larger]
```

Out[185]=



In[186]:=
```
Export["sumSerSq.png", graph]
```

Out[186]= sumSerSq.png

Define three rotation matrices

In[187]:=
```
rX[th_] := {{1, 0, 0}, {0, Cos[th], Sin[th]}, {0, -Sin[th], Cos[th]}}
```

In[188]:=
```
MatrixForm[rX[x]]
```

Out[188]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \mathrm{Cos}[x] & \mathrm{Sin}[x] \\ 0 & -\mathrm{Sin}[x] & \mathrm{Cos}[x] \end{pmatrix}$$

In[189]:=
```
rY[ski_] := {{Cos[ski], 0, Sin[ski]}, {0, 1, 0}, {-Sin[ski], 0, Cos[ski]}}
```

In[190]:=
```
MatrixForm[rY[x]]
```

Out[190]//MatrixForm=

$$\begin{pmatrix} \mathrm{Cos}[x] & 0 & \mathrm{Sin}[x] \\ 0 & 1 & 0 \\ -\mathrm{Sin}[x] & 0 & \mathrm{Cos}[x] \end{pmatrix}$$

In[191]:=
```
rZ[phi_] := {{Cos[phi], Sin[phi], 0}, {-Sin[phi], Cos[phi], 0}, {0, -0, 1}}
```

In[192]:=
```
MatrixForm[rZ[x]]
```

Out[192]//MatrixForm=

$$\begin{pmatrix} \mathrm{Cos}[x] & \mathrm{Sin}[x] & 0 \\ -\mathrm{Sin}[x] & \mathrm{Cos}[x] & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Full Rotation

In[193]:=
```
Rot3[a1_, a2_, a3_] := rZ[a1].rX[a2].rZ[a3]
```

In[194]:=
```
MatrixForm[Rot3[x, y, z]]
```

Out[194]//MatrixForm=

$$\begin{pmatrix} \mathrm{Cos}[x]\,\mathrm{Cos}[z] - \mathrm{Cos}[y]\,\mathrm{Sin}[x]\,\mathrm{Sin}[z] & \mathrm{Cos}[y]\,\mathrm{Cos}[z]\,\mathrm{Sin}[x] + \mathrm{Cos}[x]\,\mathrm{Sin}[z] & \mathrm{Sin}[x]\,\mathrm{Sin}[y] \\ -\mathrm{Cos}[z]\,\mathrm{Sin}[x] - \mathrm{Cos}[x]\,\mathrm{Cos}[y]\,\mathrm{Sin}[z] & \mathrm{Cos}[x]\,\mathrm{Cos}[y]\,\mathrm{Cos}[z] - \mathrm{Sin}[x]\,\mathrm{Sin}[z] & \mathrm{Cos}[x]\,\mathrm{Sin}[y] \\ \mathrm{Sin}[y]\,\mathrm{Sin}[z] & -\mathrm{Cos}[z]\,\mathrm{Sin}[y] & \mathrm{Cos}[y] \end{pmatrix}$$

```
In[195]:= MatrixForm[Simplify[Rot3[x, y, z]]]

Out[195]//MatrixForm=
( Cos[x] Cos[z] - Cos[y] Sin[x] Sin[z]     Cos[y] Cos[z] Sin[x] + Cos[x] Sin[z]    Sin[x] Sin[y] )
( -Cos[z] Sin[x] - Cos[x] Cos[y] Sin[z]    Cos[x] Cos[y] Cos[z] - Sin[x] Sin[z]    Cos[x] Sin[y] )
( Sin[y] Sin[z]                             -Cos[z] Sin[y]                          Cos[y]        )
```

Rotational inverse with negative angles

```
In[196]:= Rot3Inverse[a1_, a2_, a3_] := Rot3[-a3, -a2, -a1]
```

ReverseAngles times Regular is the identity.

```
In[197]:= revAngle = MatrixForm[Rot3Inverse[x, y, z].Rot3[x, y, z]]

Out[197]//MatrixForm=
(                                  Sin[y]^2 Sin[z]^2 + (-Cos[z] Sin[x] - Cos[x] Cos[y] Sin[z]
( -Cos[z] Sin[y]^2 Sin[z] + (-Cos[z] Sin[x] - Cos[x] Cos[y] Sin[z]) (Cos[x] Cos[y] Cos[z] - Sin[x] S
(                    Cos[y] Sin[y] Sin[z] + Cos[x] Sin[y] (-Cos[z] Sin[x] - Cos[x] Cos[y] Si

In[198]:= Simplify[revAngle]

Out[198]//MatrixForm=
( 1 0 0 )
( 0 1 0 )
( 0 0 1 )
```

Inverse Matrix times Matrix is the Identity

```
In[199]:= invFunc = MatrixForm[Inverse[Rot3[x, y, z]].Rot3[x, y, z]]

Out[199]//MatrixForm=
(                 (-Cos[z] Sin[x]-Cos[x] Cos[y] Sin[z]) (-Cos[y]^2 Cos[z] Sin[x]-Cos[z] Sin[x] Sin[y]^2-Cos[x]
(  ---------------------------------------------------------------------------------------------------------------
(  Cos[x]^2 Cos[y]^2 Cos[z]^2+Cos[y]^2 Cos[z]^2 Sin[x]^2+Cos[x]^2 Cos[z]^2 Sin[y]^2+Cos[z]^2 Sin[x]^2 Sin[y]^2+Cos[x]^2 Cos[y]^2 Sin[z]^2+Cos[y]^2 Sin[x]
(                 Sin[y] (-Cos[x]^2 Cos[z] Sin[y]-Cos[z] Sin[x]^2 Sin[y]) Sin[z]
(  ---------------------------------------------------------------------------------------------------------------
(  Cos[x]^2 Cos[y]^2 Cos[z]^2+Cos[y]^2 Cos[z]^2 Sin[x]^2+Cos[x]^2 Cos[z]^2 Sin[y]^2+Cos[z]^2 Sin[x]^2 Sin[y]^2+Cos[x]^2 Cos[y]^2 Sin[z]^2+Cos[y]^2 Sin[x]
(                 Sin[y] Sin[z] (Cos[x]^2 Cos[y] Cos[z]^2+Cos[y] Cos[z]^2 Sin[x]^2+Cos[x]^2 Cos[y] Sin[z]^2+Cos[y] S
(  ---------------------------------------------------------------------------------------------------------------
(  Cos[x]^2 Cos[y]^2 Cos[z]^2+Cos[y]^2 Cos[z]^2 Sin[x]^2+Cos[x]^2 Cos[z]^2 Sin[y]^2+Cos[z]^2 Sin[x]^2 Sin[y]^2+Cos[x]^2 Cos[y]^2 Sin[z]^2+Cos[y]^2 Sin[x]

In[200]:= Simplify[invFunc]

Out[200]//MatrixForm=
( 1 0 0 )
( 0 1 0 )
( 0 0 1 )

In[201]:= invDiff[x_, y_, z_] := Inverse[Rot3[x, y, z]] - Rot3Inverse[x, y, z]

In[202]:= diff = MatrixForm[invDiff[x, y, z]]

Out[202]//MatrixForm=
(                                                                                          Cos[x] Cos[y]^2 C
( -Cos[x] Cos[z] + Cos[y] Sin[x] Sin[z] + --------------------------------------------------------------------------
(                                         Cos[x]^2 Cos[y]^2 Cos[z]^2+Cos[y]^2 Cos[z]^2 Sin[x]^2+Cos[x]^2 Cos[z]^2 Sin[y]^2+Cos[z]^2
(                                                                                          Cos[y] Cos[z] Si
( -Cos[y] Cos[z] Sin[x] - Cos[x] Sin[z] + --------------------------------------------------------------------------
(                                         Cos[x]^2 Cos[y]^2 Cos[z]^2+Cos[y]^2 Cos[z]^2 Sin[x]^2+Cos[x]^2 Cos[z]^2 Sin[y]^2+Cos[z]^2
(                                                                                          Cos[z]^2 Sin[x] Sin[y]
(          -Sin[x] Sin[y] + ------------------------------------------------------------------------------------------------
(                           Cos[x]^2 Cos[y]^2 Cos[z]^2+Cos[y]^2 Cos[z]^2 Sin[x]^2+Cos[x]^2 Cos[z]^2 Sin[y]^2+Cos[z]^2 Sin[x]^2 Sin[y]^2

In[203]:= Simplify[diff]

Out[203]//MatrixForm=
( 0 0 0 )
( 0 0 0 )
( 0 0 0 )

In[204]:= Export["series.pdf", EvaluationNotebook[]]

Out[157]= series.pdf
```