Subject Code	PHY1	**Physics 1**
Module Code	4.0	**Vectors**
Lesson Code	4.6	**Analytical Method of Adding Vectors**
Time Frame		30 minutes

Components	Tasks	TA[1] (min)	ATA[2] (min)
TARGET 	By the end of this learning guide, the student should be able to: • employ the component method of adding vectors.	1	
HOOK 	In the previous lessons, you have learned how to add vectors using graphical methods. Your ruler and protractor were your main tools in applying such methods. Accuracy and precision while drawing all those arrows are very important as they affect your answer. But what if you do not have your ruler and protractor, and you are asked to determine resultant vectors? What if you are not good at drawing arrows? What if you wanted to verify if your answer using the graphical method is correct? All of these questions or problems could be answered by considering another method.	2	
IGNITE 	Another method of adding vectors is through the analytical method. As compared to the graphical method, you will not use your ruler and protractor anymore to get the resultant vector. Instead, you will be applying your knowledge and skills in geometry and simple trigonometry like what you did in the previous lesson. You will be using your skills in getting the components of a vector in the analytical method, and hence it is also called the **component method**. So how is the component method done? All you have to do is to follow these three simple steps. 1. Get the x- and y-components of each of the given vectors, and add all x- and y-components, respectively. 2. Get the magnitude of the resultant vector by applying the Pythagorean theorem, that is: $R = \sqrt{{R_x}^2 + {R_y}^2}$ [Equation 1]	14	

[1] Time allocation suggested by the teacher.
[2] Actual time allocation spent by the student (for information purposes only).

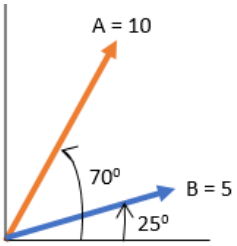
3. Determine the direction of the resultant vector by using the following equation:

$$\theta = \tan^{-1}(R_y/R_x) \qquad \text{[Equation 2]}$$

Let us study the following examples.

Example 1

Determine the resultant vector of **A** and **B** as shown below.



Following the three steps listed above, our first step is to get the x- and y-components of each vector.

For a more systematic solution, you may tabulate your results this way.

Vector	x-component	y-component
A	$A_x = 10 \cos 70^0 = 3.42$	$A_y = 10 \sin 70^0 = 9.40$
B	$B_x = 5 \cos 25^0 = 4.53$	$B_y = 5 \sin 25^0 = 2.11$
Total	$R_x = 3.42 + 4.53 = 7.95$	$R_y = 9.40 + 2.11 = 11.51$

Note: *As the x- and y-components are vectors themselves, their direction must be considered in adding these components. Make sure to reflect this by including a negative sign if the component is along the negative x-axis or along the negative y-axis.*

Our second step is to get the magnitude of the resultant vector by using equation 1.

$$R = \sqrt{R_x^2 + R_y^2}$$

$$= \sqrt{7.95^2 + 11.51^2}$$

$$= 13.99$$

The magnitude of the resultant vector is 14 units.

Our last step under the component method is to determine the direction and so we will be using equation 2.

$$\theta = \tan^{-1}(R_y/R_x)$$

$$= \tan^{-1}(11.51/7.95)$$

$$= 55.37^0$$

The direction is 55^0 counterclockwise from the + x-axis.

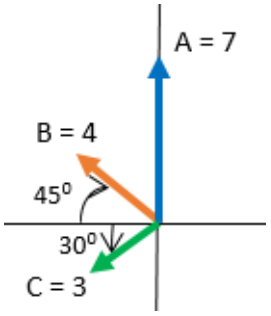
Final Answer: **A** + **B** = 14, 55^0 counterclockwise from the + x-axis.

Note: *You have to consider the sign of the angle when interpreting for the direction. When you get a positive angle, it means that the direction of measuring from the x-axis is counterclockwise and when you get negative, it means clockwise. As to whether the reference is from the positive or negative x-axis, all you have to do is to check on the sign of the total x-component of the given vectors. If it is positive then that means that the measured angle is from the positive x-axis, otherwise the measurement is from the negative x-axis.*

Tip: *To check the direction, it is best to sketch the components of the resultant vector and identify which quadrant does the resultant vector lie. Keep in mind that the angle you get using equation 2 is opposite to the y-component of the resultant vector.*

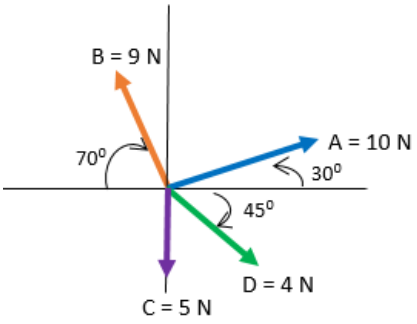
Example 2

What is the resultant of the three vectors shown below?



Step 1:

Vector	x-component	y-component
A	$A_x = 0$	$A_y = 7$
B	$B_x = -4 \cos 45^0 = -2.83$	$B_y = 4 \sin 45^0 = 2.83$
C	$C_x = -3 \cos 30^0 = -2.60$	$C_y = -3 \sin 30^0 = -1.5$
Total	$R_x = (-2.83) + (-2.60)$ $= -5.43$	$R_y = 2.83 + (-1.5)$ $= 8.33$

	Step 2: $R = \sqrt{R_x{}^2 + R_y{}^2}$ $= \sqrt{(-5.43)^2 + (8.33)^2}$ $= 9.94$ Step 3: $\theta = \tan^{-1}(R_y/R_x)$ $= \tan^{-1}(8.33/-5.43)$ $= -56.9^0$ Final Answer: **R** = 10, 57^0 clockwise from the –x-axis.		
NAVIGATE 	It is now your turn to apply the component method in vector addition by answering the following. I have provided the answers below the item so that you can check your answers. Given the vectors on the right, solve the following: a. **A** + **B** b. **C** + **D** c. **A** + **B** + **C** + **D**  Answers: a. **A** + **B** = 15 N, 67^0, counterclockwise from the +x-axis b. **C** + **D** = 8 N, 70^0, clockwise from the +x-axis c. **A** + **B** + **C** + **D** = 10 N, 34^0, counterclockwise from the +x-axis	6	
KNOT 	In summary, I would like to reiterate the steps to follow when using the component method in vector addition. 1. Get the x- and y-components of each of the given vectors, and add all x- and y-components, respectively. 2. Get the magnitude of the resultant vector by applying the Pythagorean theorem, that is:	7	

$$R = \sqrt{R_x{}^2 + R_y{}^2}$$

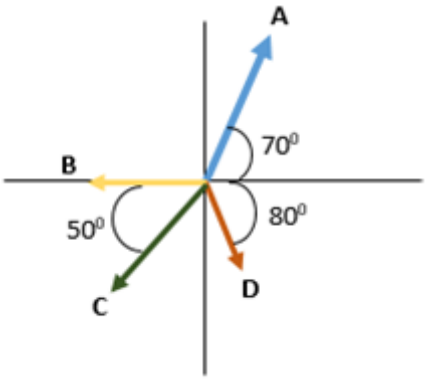
3. Determine the direction of the resultant by using the following equation:

$$\theta = \tan^{-1}(R_y/R_x).$$

Now it is time to assess what you have learned.

Write your answers (with complete solutions) on a clean sheet of paper. Follow your teacher's instructions regarding submission. All items will be graded.

Given the vectors shown below where A = 8 m, B = 5 m, C = 7 m and D = 4 m, solve the following:



1. **A + C**
2. **C + D**
3. **A + B + C + D**

References:

1. Lumen Learning. (n.d.). Vector addition and subtraction: analytical methods. https://courses.lumenlearning.com/physics/chapter/test-vector-addition-and-subtraction-analytical-methods/
2. Giancoli, D. C. (2007). *Physics: Principles with Applications.* (6th edition). Pearson Education, Inc.
3. Henderson, T. (1996-2020). The Physics Classroom. Component Method of Vector Addition. https://www.physicsclassroom.com/class/vectors/Lesson-1/Component-Addition

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