4/1

WebAssign: Riemann Sums
due Wednesday

Review

indefinite integral — antiderivative

rules for antidifferentiation

Ex: $\int (3e^{2x} + x^{-2})\,dx$

$= 3\int e^{2x}\,dx + \int x^{-2}\,dx$

$= 3\frac{1}{2}e^{2x} + \frac{1}{-1}x^{-1} + C$

$= \frac{3}{2}e^{2x} - \frac{1}{x} + C$

definite integral

Ex: $\int_1^2 \left(3e^{2x} + x^{-2}\right) dx$

$$= \frac{3}{2}e^{2x} - \frac{1}{x}\Big|_1^2$$

$$= \left[\frac{3}{2}e^{2\cdot 2} - \frac{1}{2}\right] - \left[\frac{3}{2}e^{2\cdot 1} - 1\right]$$

$$= \frac{3}{2}e^4 - \frac{1}{2} - \frac{3}{2}e^2 + 1$$

$$= \frac{3}{2}\left(e^4 - e^2\right) + \frac{1}{2}$$

Note: We do not need to include the constant when evaluati[ng] definite integrals

6.2 - Definite integral and net change

Problem: Given a function f that represents a rate of change of a quantity y on an interval [a,b] find the change in y.

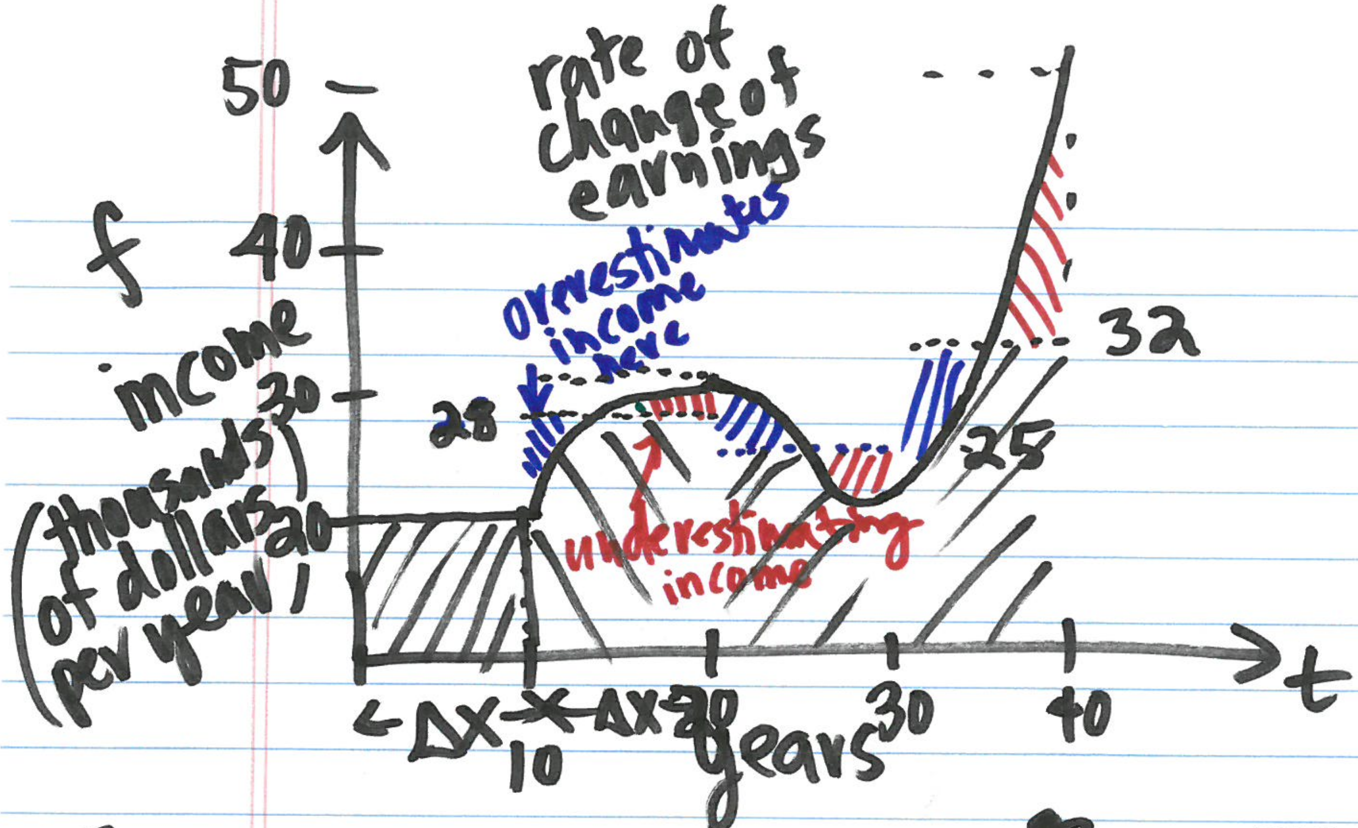
$$f = y' \qquad F = y$$

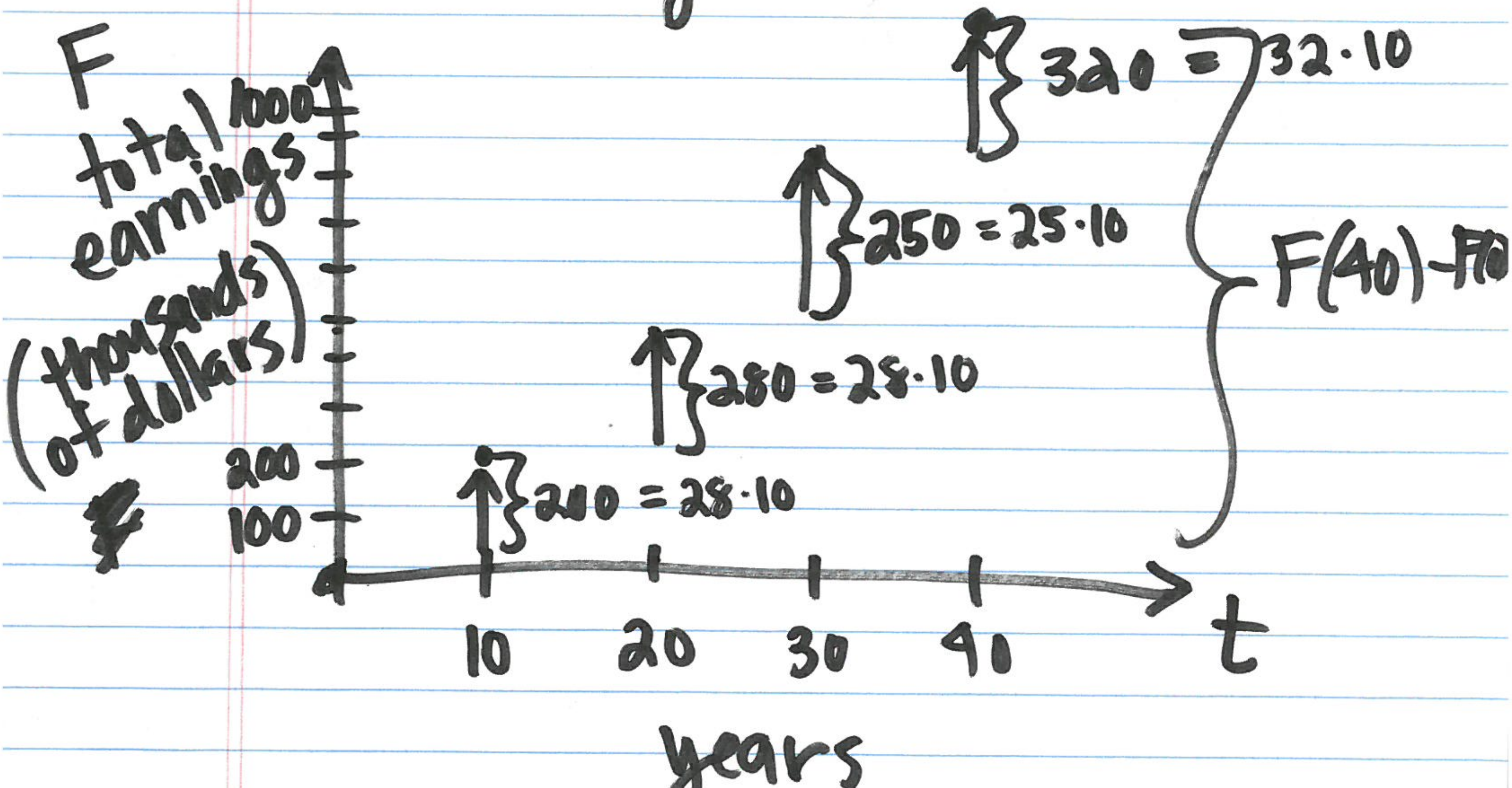
y is the antiderivative of the given function

$$\int_a^b \underset{f(x)}{\cancel{f(x)}}\, dx = F(b) - F(a)$$

End review

Suppose we are given a function f, Let's say it represents income and we want to know the total earnings after some time, say ≈ 40 years.





change in
~~t~~ total earnings =$1050 thousand

Riemann Sums

Given a function f, and an interval $[a,b]$ we form the Riemann sum with n terms by dividing the interval into n and adding the area of rectangles that "fit under the graph"

A sum of the form

$$f(x_1)\Delta x + f(x_2)\Delta x + \ldots + f(x_n)\Delta x$$

is called a Riemann sum for f on the interval $[a,b]$ if $\Delta x = \frac{b-a}{n}$ and the values $x_1, x_2, \ldots, x_n$ are from each of the distinct subintervals of the partition of $[a,b]$ into n parts.

Ex: Let $f(x) = x^2$. Find the Riemann sum of f on the interval $[0, 1]$ with $n = 4$ terms, using the right endpoints of each subinterval as x_1, x_2, x_3, x_4

Procedure for computing Riemann sum

1. compute Δx
2. partition interval / find the endpoints of subintervals
3. choose x values $x_1, \ldots, x_n$
4. evaluate $f(x_i)$ and form the Riemann sum

1. Here $a = 0$, $b = 1$, $n = 4$

$$\Delta x = \frac{b-a}{n} = \frac{1-0}{4} = \frac{1}{4}$$

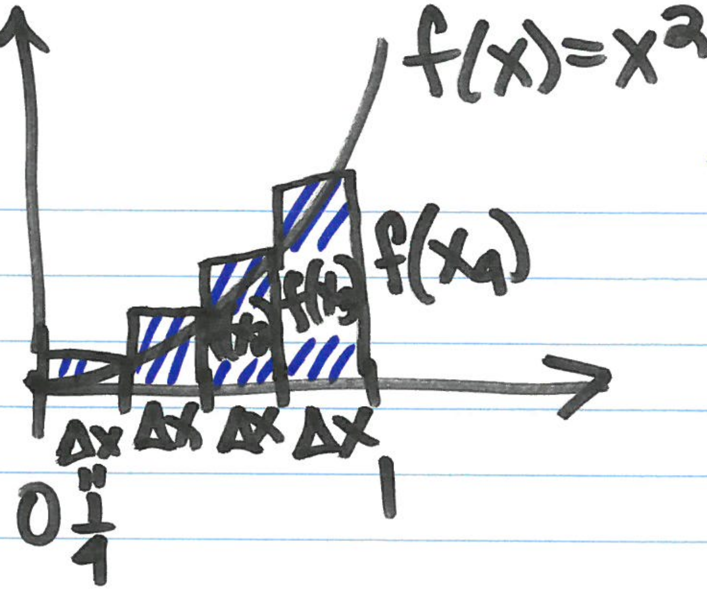
If $F(t)$ is our total earnings

$F'(t) = f(t)$ income is rate of change of total earnings

$F(40) - F(0) \approx$ area under the graph of f

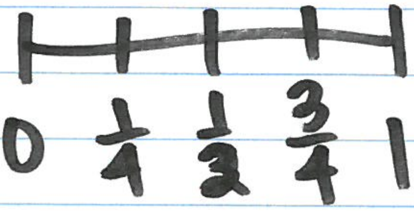
The fundamental theorem of calculus:

The area under the graph of f on the interval $[a,b]$ = ~~Dif~~ Change in the anti-derivative of f between a and b



The Riemann sum gives the area of these 4 rectangles

2.



	left endpoints	right endpoints
1st interval	0	$\frac{1}{4}$
2nd	$\frac{1}{4}$	$\frac{1}{2}$
3rd	$\frac{1}{2}$	$\frac{3}{4}$
4th	$\frac{3}{4}$	1

3.

$$\left.\begin{aligned} x_1 &= \tfrac{1}{4} \\ x_2 &= \tfrac{1}{2} \\ x_3 &= \tfrac{3}{4} \\ x_4 &= 1 \end{aligned}\right\}$$

these are the x-values of the right endpoints

4. $f(x_1) = \left(\frac{1}{4}\right)^2$

$f(x_2) = \left(\frac{1}{2}\right)^2$

$f(x_3) = \left(\frac{3}{4}\right)^2$

$f(x_4) = (1)^2$

Riemann sum:

$$R = f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + f(x_4)\Delta x$$

$$= \left[f(x_1) + f(x_2) + f(x_3) + f(x_4)\right] \cdot \Delta x$$

$$= \left[\frac{1}{16} + \frac{1}{4} + \frac{9}{16} + 1\right] \cdot \frac{1}{4}$$

$$= 0.46875$$

$$\approx \int_0^1 x^2 \, dx = \frac{1}{3}x^3\Big|_0^1$$

$$= \frac{1}{3} - 0 = \frac{1}{3}$$