MA131-002 Spring 2016 Test 4 Practice Problems

1. Find the following integrals:

   (a) $\int(6x^3 + 4x^2 + 4x + 3)dx$

   Solution:

   $$\int(6x^3 + 4x^2 + 4x + 3)dx = \frac{6}{4}x^4 + \frac{4}{3}x^3 + 2x^2 + 3x + C$$

   (b) $\int 2e^{-0.4x}dx$

   Solution:

   $$\begin{aligned}\int 2e^{-0.4x}dx &= -\frac{2}{0.4}e^{-0.4x} + C \\ &= -5e^{-0.4x} + C\end{aligned}$$

   (c) $\int \frac{1}{2x}dx$

   Solution:

   $$\begin{aligned}\int \frac{1}{2x}dx &= \frac{1}{2}\int \frac{1}{x}dx \\ &= \frac{1}{2}\ln|x| + C\end{aligned}$$

2. Find the following integrals:

   (a) $\int \frac{5x^4}{x^5+1}dx$

   Solution: Let $u = x^5 + 1$. Then $du = 5x^4dx$. By substitution, we get:

   $$\begin{aligned}\int \frac{5x^4}{x^5+1}dx &= \int \frac{1}{u}du \\ &= \ln(u) + C \\ &= \ln(x^5 + 1) + C.\end{aligned}$$

   (b) $\int(-x^{-2})\left(\frac{1}{x} + 2\right)^5 dx$

Solution: Let $u = \frac{1}{x} + 2$. Then $du = -x^{-2}dx$. By substitution, we get:

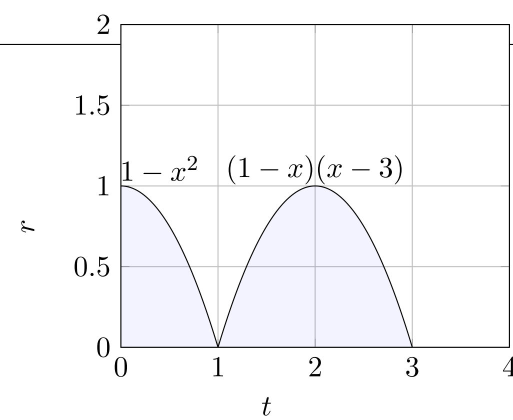
$$\begin{aligned} \int -x^{-2}(\frac{1}{x} + 2)^5 dx &= \int u^5 du \\ &= \frac{1}{6}u^6 + C \\ &= \frac{1}{6}(\frac{1}{x} + 2)^5 + C. \end{aligned}$$

(c) $\int \frac{2}{x+4} dx$

Solution: Let $u = x + 4$. Then $du = dx$, so $2dx = 2du$. By substitution, we get:

$$\begin{aligned} \int \frac{2}{x+4} dx &= \int \frac{1}{u} 2du \\ &= 2\ln|u| + C \\ &= 2\ln|x+4| + C. \end{aligned}$$

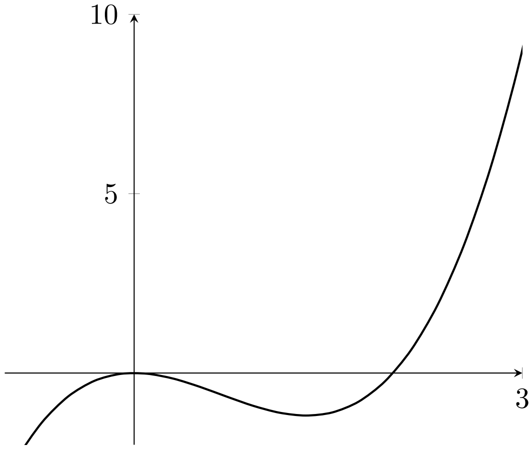
3. Find $\int_0^3 f(x)dx$, where $f(x)$ is shown:



Solution: We divide the interval from 0 to 3 into two subintervals where we can find the antiderivative of the function on each subinterval:

$$\begin{aligned} \int_0^3 f(x)dx &= \int_0^1 1 - x^2 dx + \int_1^3 (1-x)(x-3)dx \\ &= \int_0^1 1 - x^2 dx + \int_1^3 (-x^2 + 4x - 3)dx \\ &= x - \frac{1}{3}x^3 \Big|_0^1 + -\frac{1}{3}x^3 + 2x^2 - 3x \Big|_1^3 \\ &= \left(\left[1 - \frac{1}{3}\right] - [0]\right) + \left(\left[-\frac{1}{3}3^3 + 2(3)^2 - 3(3)\right] - \left[-\frac{1}{3} + 2 - 3\right]\right) \\ &= \frac{2}{3} + 0 + \frac{4}{3} \\ &= 2 \end{aligned}$$

4. Consider the functions $f(x) = x^3 - 2x^2$. Find the area bounded by the graph of $f(x)$ and the x-axis between $x = 0$ and $x = 3$. You may use the following graph as a reference, but you must show all of the work needed to determine any values associated with the function and its graph.



Solution:

We need to determine where the graph is positive and where it is negative. To do this we find when $f(x) = 0$:

$$\begin{aligned} &x^3 - 2x^2 = 0 \\ \Rightarrow &x^2(x-2) = 0 \\ \Rightarrow &x = 0 \text{ or } x = 2 \end{aligned}$$

We can see that $f(x)$ is negative between $x = 0$ and $x = 2$. To find the area we need to integrate the absolute value of the f between 0 and 2. The area of the first part is

$$\begin{aligned} \int_0^2 -(x^3 - 2x^2)dx &= \int_0^2 (2x^2 - x^3)dx \\ &= \frac{2}{3}x^3 - \frac{1}{4}x^4 \Big|_0^2 \\ &= \left[\frac{2}{3}(2)^3 - \frac{1}{4}(2)^4\right] - [0] \\ &= \frac{16}{3} - 4 \\ &= \frac{4}{3}. \end{aligned}$$

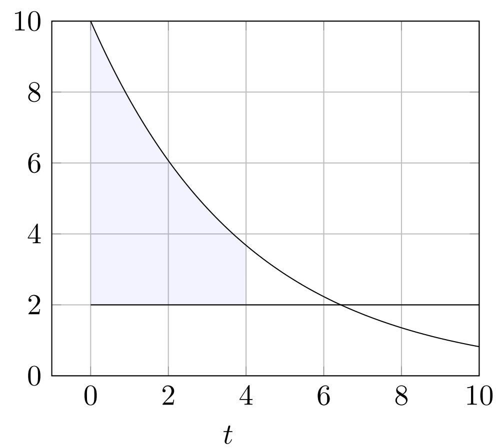
The area of the second part (between $x = 2$ and $x = 3$) is

$$\begin{aligned}\int_2^3 (x^3 - 2x^2)dx &= \frac{1}{4}x^4 - \frac{2}{3}x^3\Big|_2^3 \\ &= \left[\frac{1}{4}(3)^4 - \frac{2}{3}(3)^3\right] - \left[\frac{1}{4}(2)^4 - \frac{2}{3}(2)^3\right] \\ &= \left[\frac{81}{4} - 18\right] - \left[4 - \frac{16}{3}\right] \\ &= \frac{9}{4} + \frac{4}{3} \\ &= \frac{43}{12}\end{aligned}$$

The total area between the graphs of $f(x)$ and the x-axis is

$$\frac{43}{12} + \frac{4}{3} = \frac{59}{12}$$

5. Water is pumped out of a reservoir to provide water for a community at a constant rate of $2,000$ gallons per day. In addition, after a heavy rain, rainwater flows into the reservoir. The rate (in thousands of gallons per day) at which the water flows into the reservoir t days after the rain stops is given by $I(t) = 10e^{-0.25t}$. The graph of $I(t)$ and the constant function 2 are shown below. What quantity does the area between these curves between $t = 0$ and $t = 4$ represent?



Solution: The area between the curves between $t = 0$ and $t = 4$ represents the net change in the amount of water in the reservoir that is due to both rainwater flowing into the reservoir and water being pumped out of the reservoir during the four days after the heavy rainfall.

This area is

$$\begin{aligned}\int_0^4 10e^{-0.25t} - 2\ dt &= -\frac{10}{0.25}e^{-0.25t} - 2t\Big|_0^4 \\ &= \left[-\frac{10}{0.25}e^{-1} - 8\right] - \left[-\frac{10}{0.25}\right] \\ &\approx 17.28482\end{aligned}$$

Thus, the net change in the amount of water in the reservoir during the four days is about $17,284$ gallons.