Suggested exercises

1.6 Exercises 1-37 odd

21-37 odd

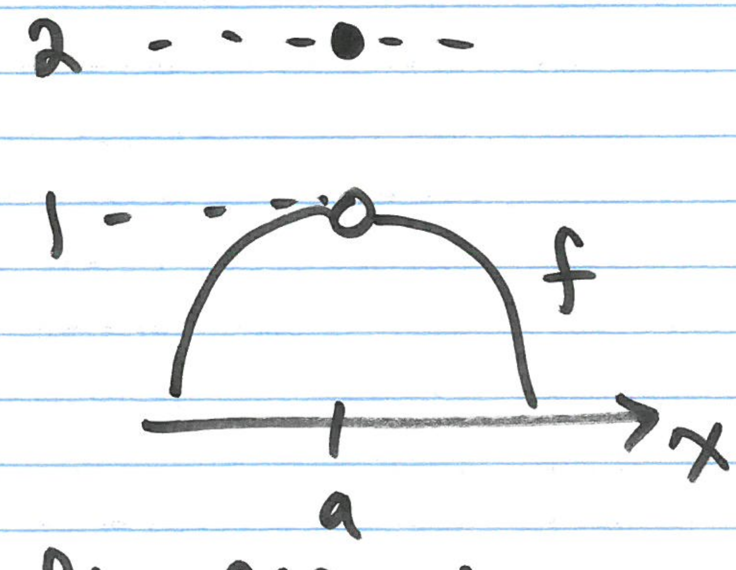
more challenging ones

2/8 Limits

The number L is the limit of f(x) as x approaches a provided f(x) can be made arbitrarily close to L for all x sufficiently close (but not equal to) a.

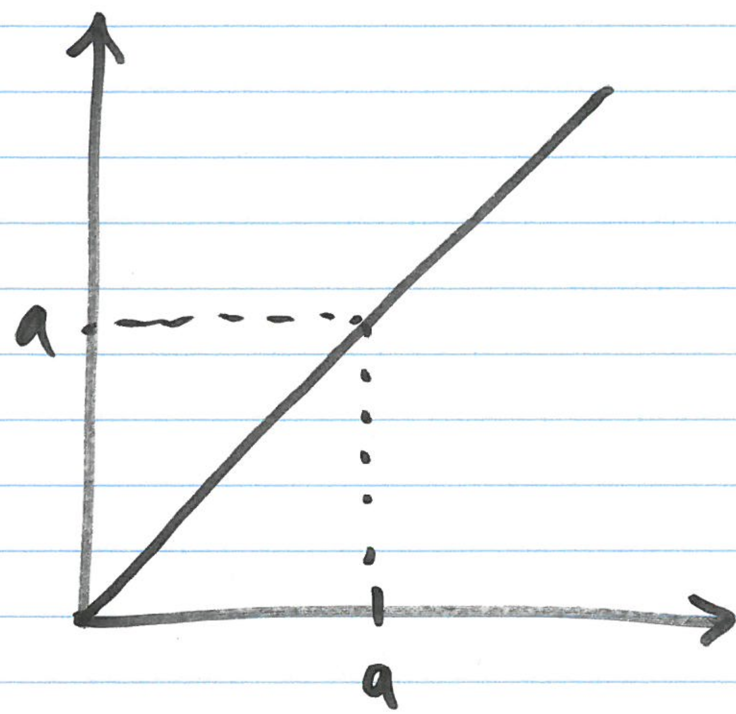
f(a), the value of f at a, does not affect $\lim_{x \to a} f(x)$.

Ex:



$\lim_{x \to a} f(x) = 1$

Last time we saw

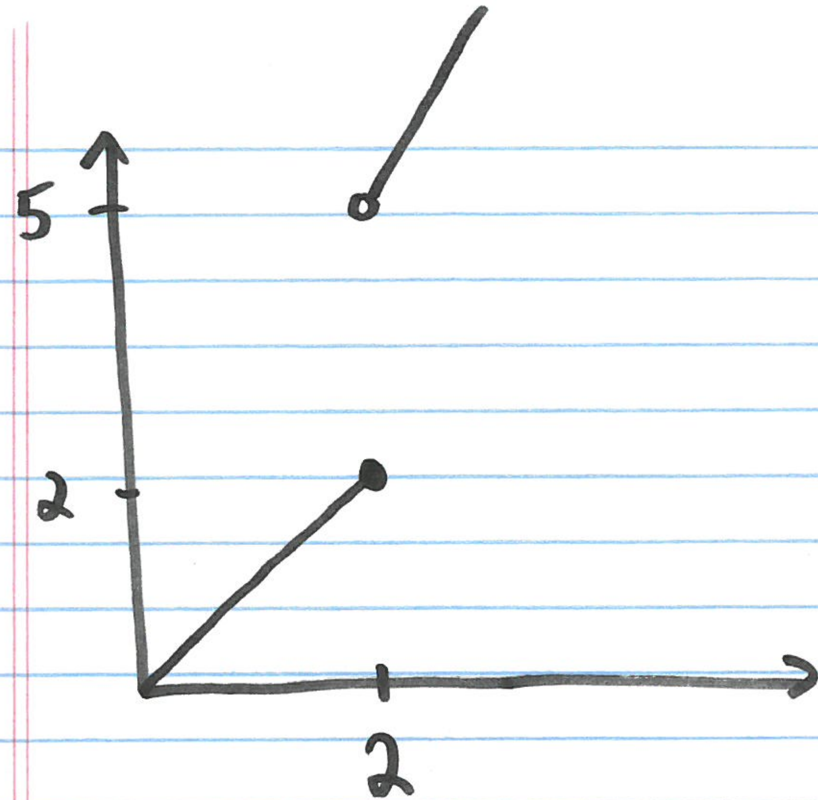


$$\lim_{x \to a} x = a$$

It ~~als~~ is also possible that $\lim_{x \to a} f(x)$ does not exist

<u>Ex</u>: Consider the piecewise-defined function

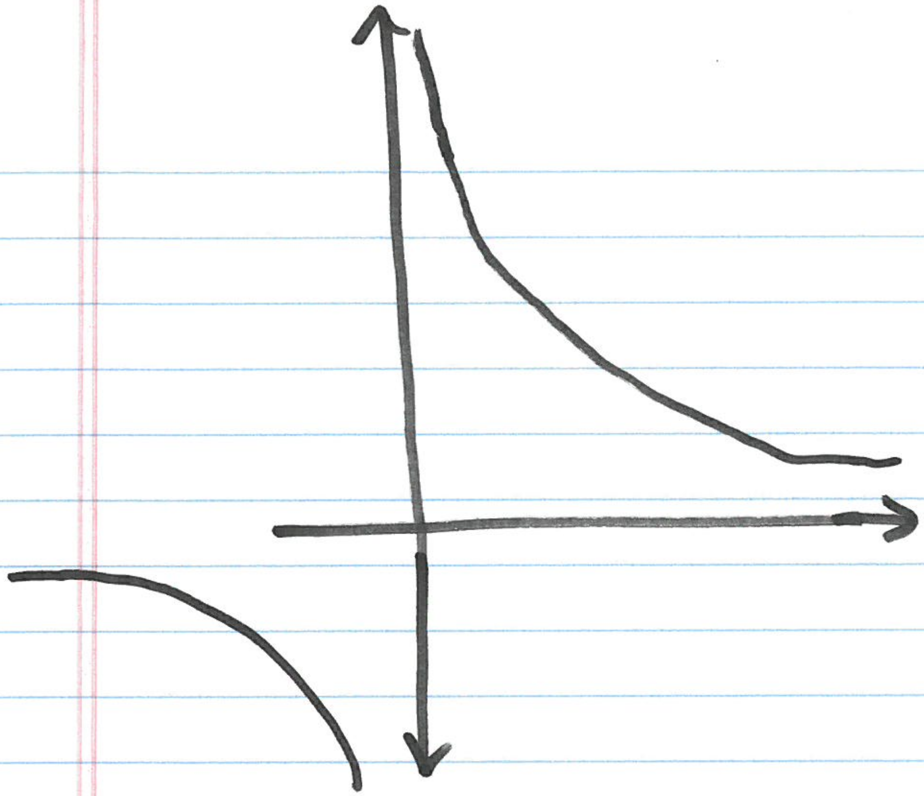
$$f(x) = \begin{cases} x & x \leq 2 \\ 2x+1 & x > 2 \end{cases}$$



$\lim_{x \to 2} f(x)$ does not exist because the function goes to one value (2) as x approaches 2 from the left and it goes to a different value (5) when x approaches 2 from the right.

Ex: $f(x) = \frac{1}{x}$

$\lim_{x \to 0} \frac{1}{x}$



$\lim_{x\to 0} \frac{1}{x}$ does not exist because the y-values on every interval around 0 are unbounded.

Limit theorems (Assume $\lim_{x\to a} f(x)$, $\lim_{x\to a} g(x)$ exists)

Limit theorems	notes
$\lim_{x\to a} k f(x) = k \lim_{x\to a} f(x)$	k constant
$\lim_{x\to a} [f(x)]^r = [\lim_{x\to a} f(x)]^r$	$r>0$; $f(x)^r$ must be defined
$\lim_{x\to a} [f(x)+g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$	

$$\lim_{x\to a} f(x)\cdot g(x) = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$$

Together with the fact that

$\lim_{x\to a} x = a$, we can

find the limit of any polynomial $p(x)$ using these limit theorems.

<u>Ex</u>: $\lim_{x\to 2} 3x^2+5x-1$

$p(x) = 3x^2+5x-1$

$$\lim_{x\to 2} p(x) = 3\lim_{x\to 2} x^2 + 5\lim_{x\to 2} x - \lim_{x\to 2} 1$$

$$= 3(2)^2 + 5(2) - 1$$

$$= 12 + 10 - 1$$

$$= 21$$

In general, for any polynomial

$$\lim_{x \to a} p(x) = p(a).$$ continuity of polynomials

Finding limits of rational functions

Recall: A rational function is a ratio of two polynomials

$$\frac{p(x)}{q(x)}$$

case 1: If $\lim\limits_{x \to a} q(x) \neq 0$

then $$\lim_{x \to a} \frac{p(x)}{q(x)} = \frac{\lim\limits_{x \to a} p(x)}{\lim\limits_{x \to a} q(x)}$$

$$= \frac{p(a)}{q(a)}.$$

case 2: If $\lim\limits_{x \to a} q(x) = 0$ and

$\lim\limits_{x \to a} p(x) \neq 0$ then

$$\lim_{x \to a} \frac{p(x)}{q(x)} \text{ does not exist}$$

case 3: If $\lim_{x \to a} p(x) = 0$

and $\lim_{x \to a} q(x) = 0$ then

we need to understand how the ratio $\frac{p(x)}{q(x)}$ behaves as $x \to 0$.

<u>Ex</u>: Find $\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$

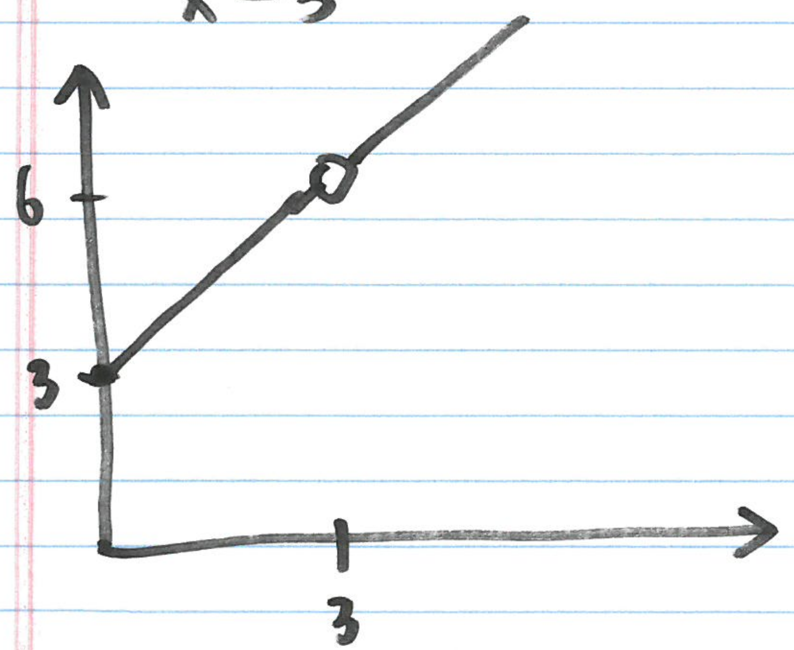
~~By~~ $\lim_{x \to 3} x^2 - 9 = 0$ by continuity of polynomials.

Similarly $\lim_{x \to 3} x - 3 = 0$

$$\frac{x^2 - 9}{x - 3} = \frac{(x+3)(x-3)}{(x-3)}$$

When dividing by $x-3$ we need to consider that $x-3$ can be zero for some x.

$$\frac{x^2-9}{x-3} = x+3 \qquad (x \neq 3)$$



$$\lim_{x \to 3} \frac{x^2-9}{x-3} = \lim_{x \to 3} x+3 = 6$$

(the fact that our function is not defined at $x=3$ does not affect the limit)