# 1 Summary of useful facts about exponential functions and their applications

1. Every exponential function $f(x) = b^x$ has the following properties:

   (a) $b^0 = 1$

   (b) $b^x$ is positive for all $x$

   (c) $b^x$ is increasing if ____________ and decreasing if ____________.

2. Any exponential function $f(x) = b^x$ can be written in the form $f(x) = e^{kx}$ for some constant $k$ ($k = \ln b$). $e^{kx}$ is increasing if ____________ and decreasing if ____________.

3. The derivative of the exponential function is itself:

$$\frac{d}{dx}e^x = e^x.$$

4. The derivative of composite functions $\frac{d}{dx}e^{g(x)}$ can be computed using the chain rule, as long as we can differentiate the inner function $g(x)$. An important example is:

$$\frac{d}{dx}e^{kx} = ke^{kx}$$

5. A quantity whose instantaneous rate of change, at every instant, is proportional to the quantity at that instant, is said to grow exponentially. This fact can be expressed mathematically by the differential equation:

$$y' = ky \tag{1}$$

6. The general solution to differential equation (1) is $y = Ce^{kt}$ (check this by differentiating this expression). Thus, a quantity that grows or decays exponentially can be represented by a function $f(x) = Ce^{kx}$ for some constant $k$, called the growth (or decay) constant, and some constant $C$, often called the initial value $C = f(0)$.

7. The distinction between saying that a quantity or function *grows or decays exponentially* and saying that it *is an exponential function* is that (according to item 1 above) an exponential function $f$ satisfies $f(0) = 1$, while a quantity that grows or decays exponentially can have $f(0) = C$ for any constant $C$. In applications, the value of $f(0)$ is called the initial value.

8. Item 6 above provides a formula for a quantity that grows exponentially and a formula for the rate of change of such a quantity. These formulas provide a basis for relating the values of an exponentially growing quantity, or its rate of change, at any time (i.e. the data) to the values of $C$ and $k$ which uniquely determine the model.