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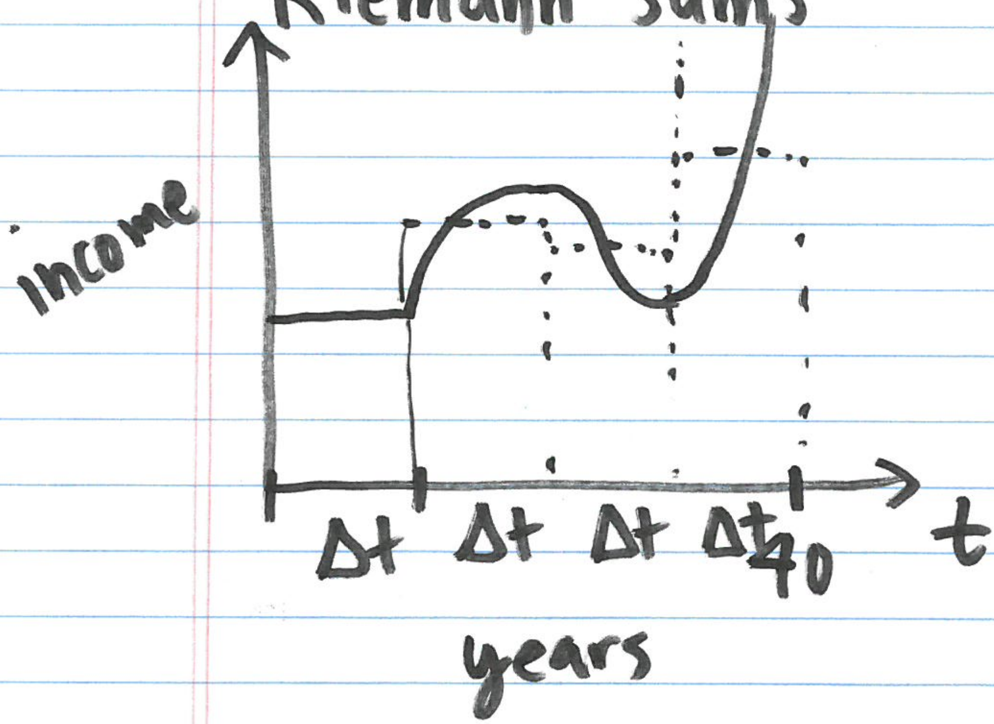
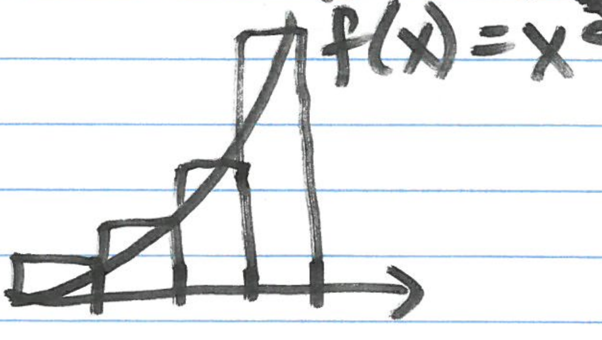
WebAssign: Integration
due 4/15

Riemann sums



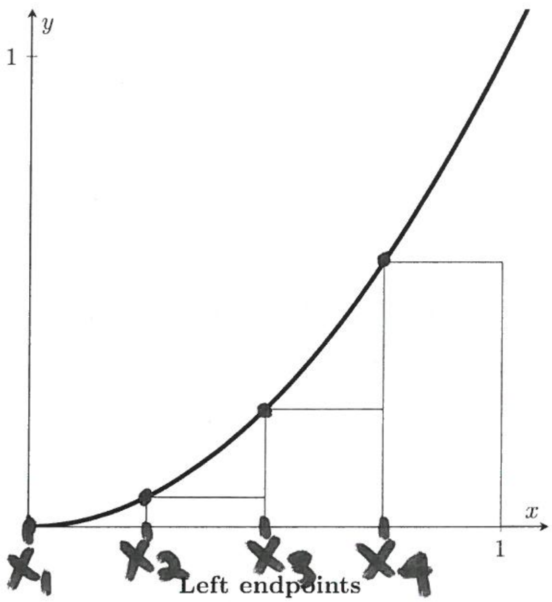
The total change was computed by finding an average ~~change~~ rate of change on each interval and multiplying by the width of each interval Δt.



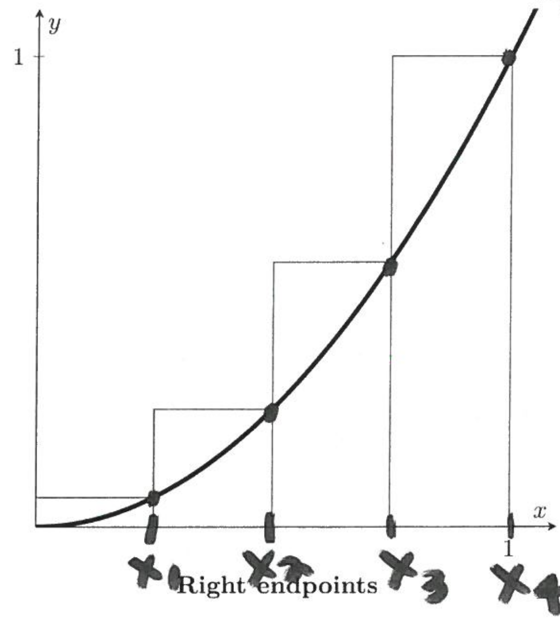
$f(x) = x^2$

n=4

Left endpoints

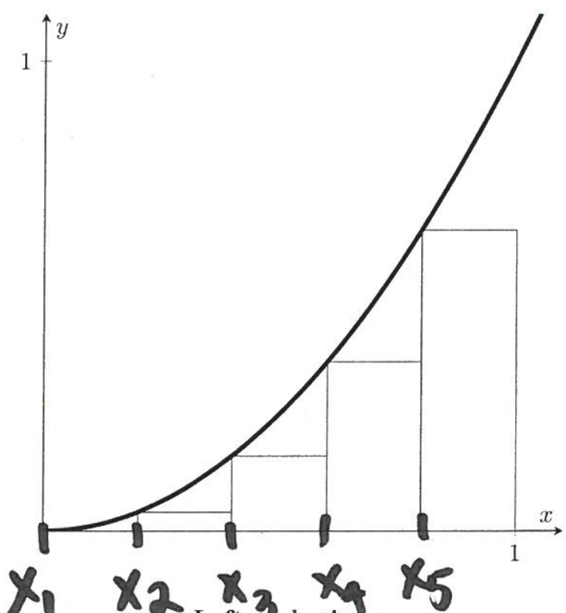


Right endpoints

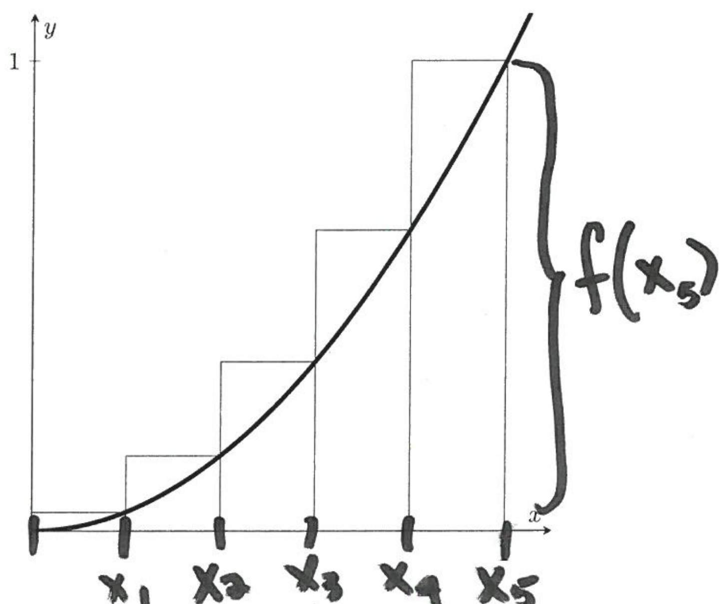


n=5

Left endpoints

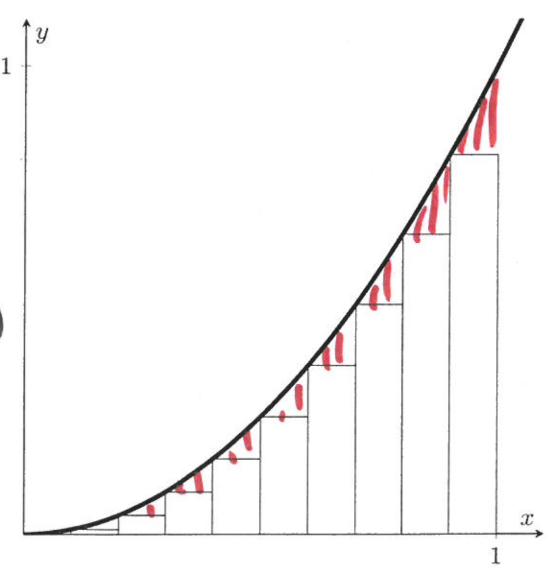


Right endpoints

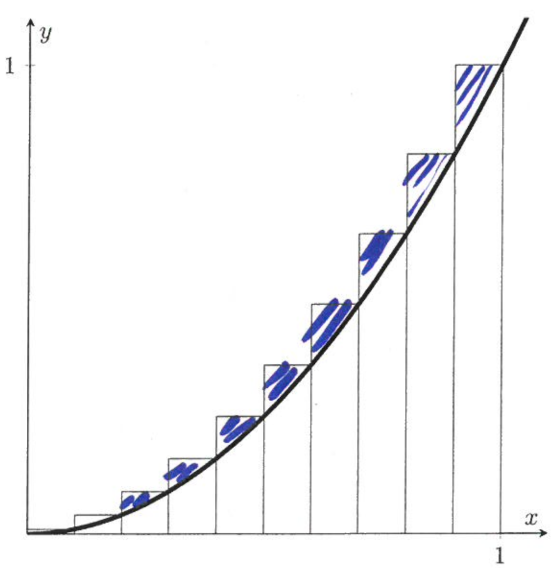


n=10

Left endpoints

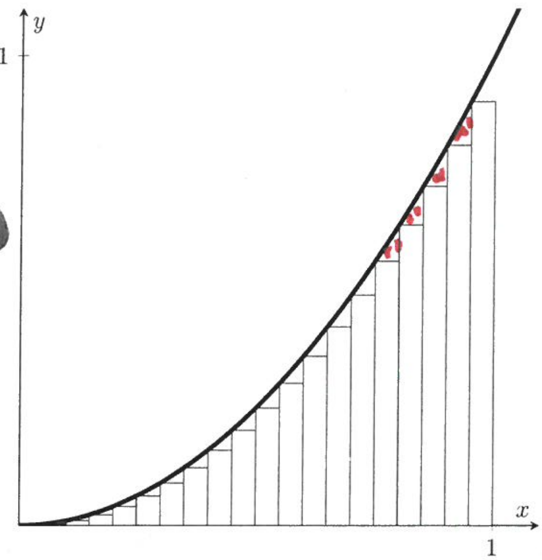


Right endpoints

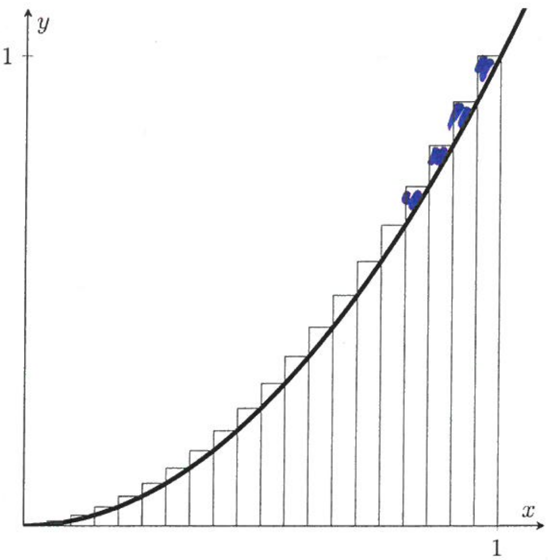


n=20

Left endpoints

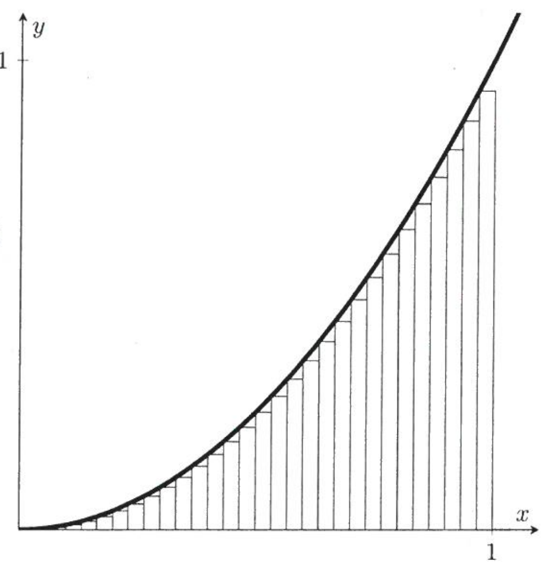


Right endpoints

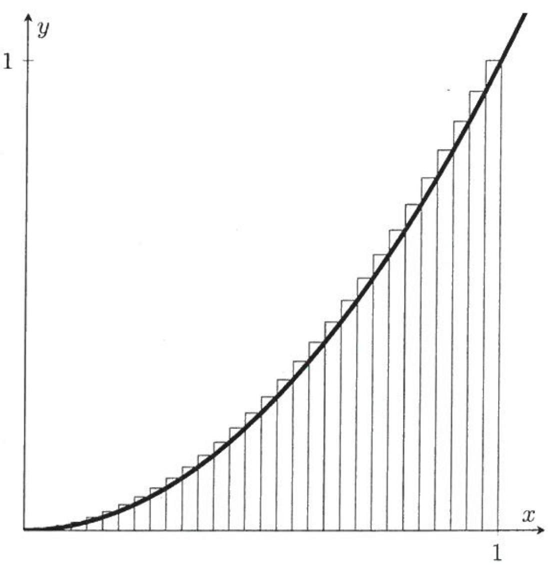


n=30

Left endpoints

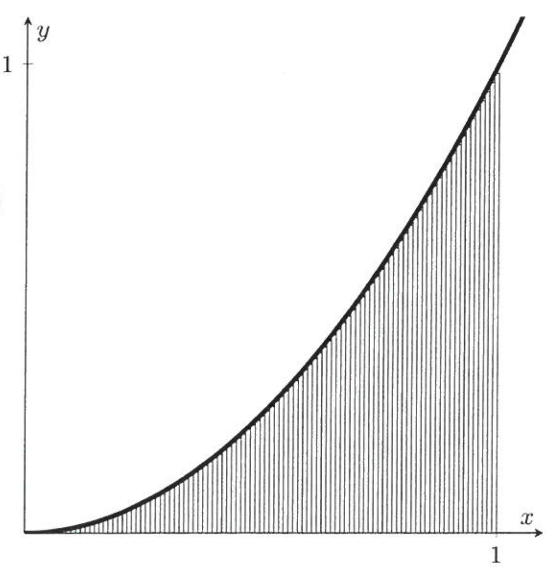


Right endpoints

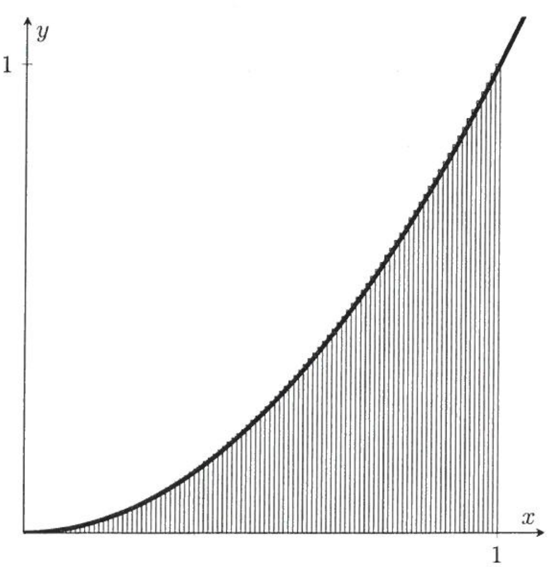


n=100

Left endpoints



Right endpoints



Each Riemann sum has the form

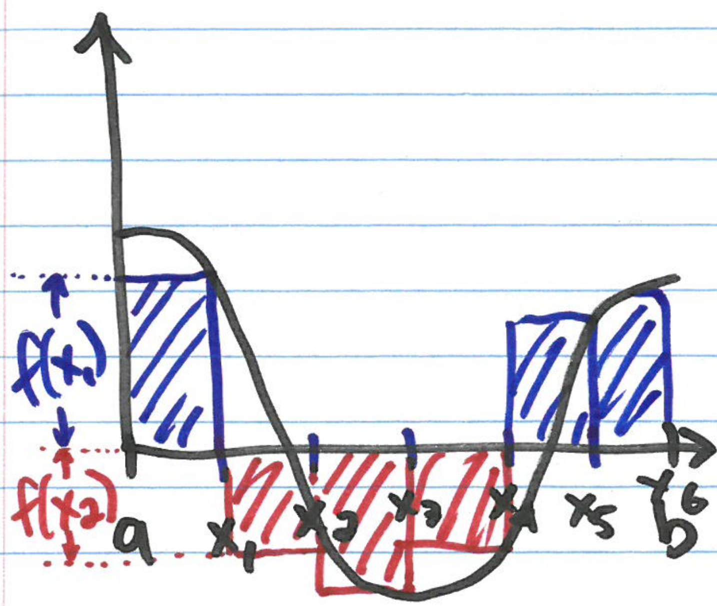
$$f(x_1)\Delta x + f(x_2)\Delta x + \dots + f(x_n)\Delta x$$

$$\lim_{n\to\infty} f(x_1)\Delta x + f(x_2)\Delta x + \dots + f(x_n)\Delta x$$

= [area under the curve of f* ~~from~~ on [a,b]]

* assuming f is positive on [a,b]

Suppose f is negative on part of [a,b]



n=6
right endpoints

$$\lim_{n\to\infty} f(x_1)\Delta x + f(x_2)\Delta x + \dots + f(x_n)\Delta x$$

$$= \int_a^b f(x)\,dx$$

negative values of f reduce the value of the integral

Note: The notation

$$\int_a^b f(x)\,dx$$ is similar to

$$\sum_{i=1}^{n} f(x_i)\Delta x = f(x_1)\Delta x + f(x_2)\Delta x + \dots + f(x_n)\Delta x$$

Fundamental theorem of calculus

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

where $F(x)$ is an antiderivative of f.

Setting up integrals for problems involving integration

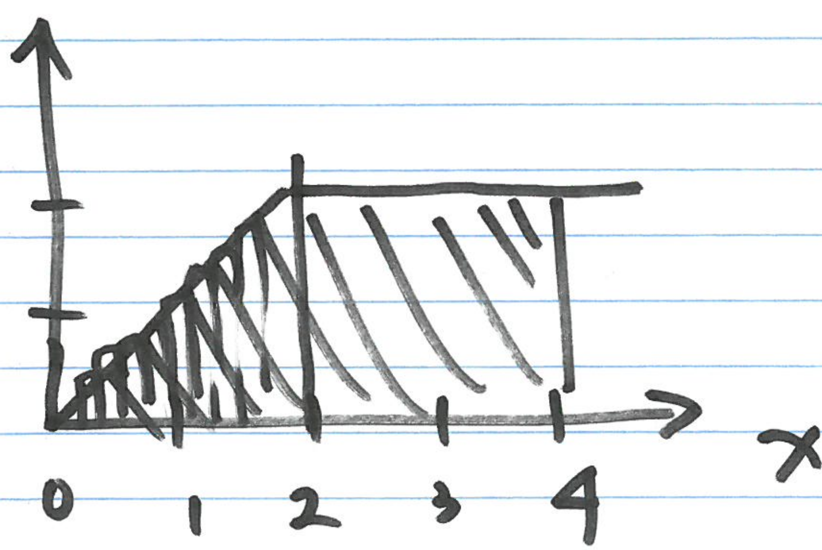
a) What is the quantity being integrated?

b) Do I want negative values to be added or subtracted from the quantity I am calculating?

Ex: Find the area under the graph of

$$h(x) = \begin{cases} x & \text{if } 0 \leq x \leq 2 \\ 2 & \text{if } x \geq 2 \end{cases}$$

from $x=0$ to $x=4$



We want $\int_0^4 h(x)\,dx$

We can split the integral into parts corresponding to the piecewise definition

$$\int_0^4 h(x)\,dx = \int_0^2 h(x)\,dx + \int_2^4 h(x)\,dx$$

$$\boxed{\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx}$$

$$= \int_0^2 x\,dx + \int_2^4 2\,dx$$

$$= \frac{1}{2}x^2\Big|_0^2 + 2x\Big|_2^4$$

$$= \left[\frac{1}{2}4 - 0\right] + \left[2\cdot 4 - 2\cdot 2\right]$$

$$= 2 + 4$$

$$= 6$$

Find the area under the graph of $4x^2 + e^{-\frac{1}{2}x}$ between $x = -1$ and $x = 1$. (see next page for estimate)

$$\int_{-1}^{1} \left(4x^2 + e^{-\frac{1}{2}x}\right) dx \qquad \int e^{kx} = \frac{1}{k}e^{kx} + C$$

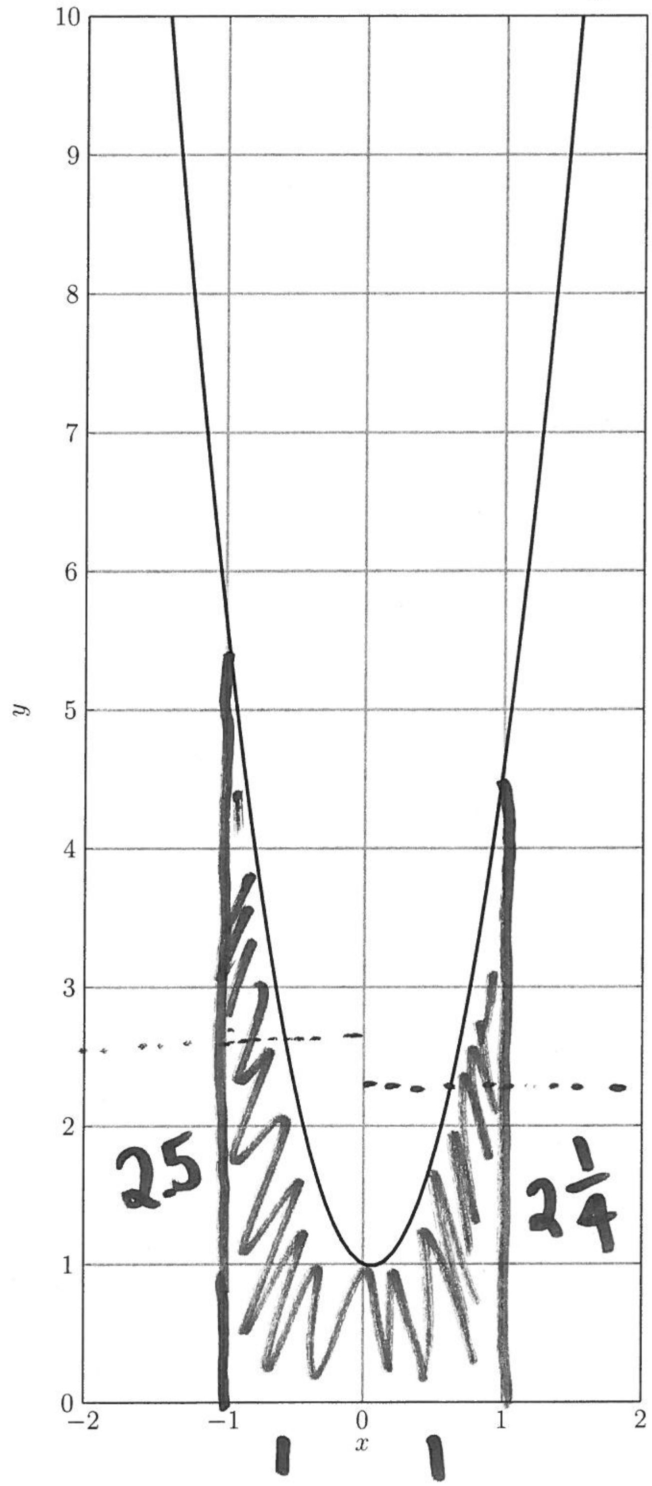
$$= 4\frac{1}{3}x^3 + (-2)e^{-\frac{1}{2}x}\Big|_{-1}^{1}$$

$$= \overbrace{\left[\frac{4}{3} + (-2)e^{-\frac{1}{2}}\right]}^{\text{evaluated at upper limit (1)}} - \overbrace{\left[-\frac{4}{3} + (-2)e^{\frac{1}{2}}\right]}^{\text{evaluated at lower limit (-1)}}$$

$$= \frac{4}{3} + -2e^{-\frac{1}{2}} + \frac{4}{3} + 2e^{\frac{1}{2}}$$

$$= \frac{8}{3} + 2e^{\frac{1}{2}} - 2e^{-\frac{1}{2}} \approx 4.75105$$

$f(x) = 4x^2 + e^{-\frac{1}{2}x}$



$A \cong 4.75$