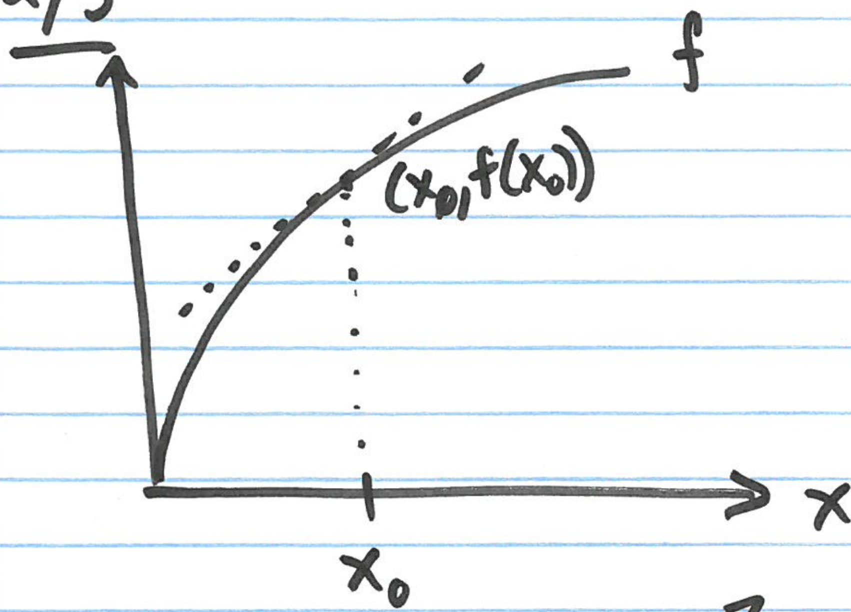
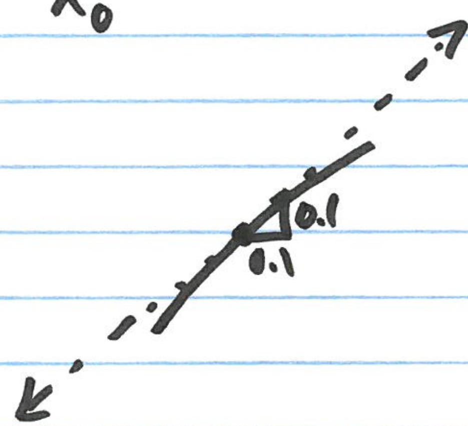
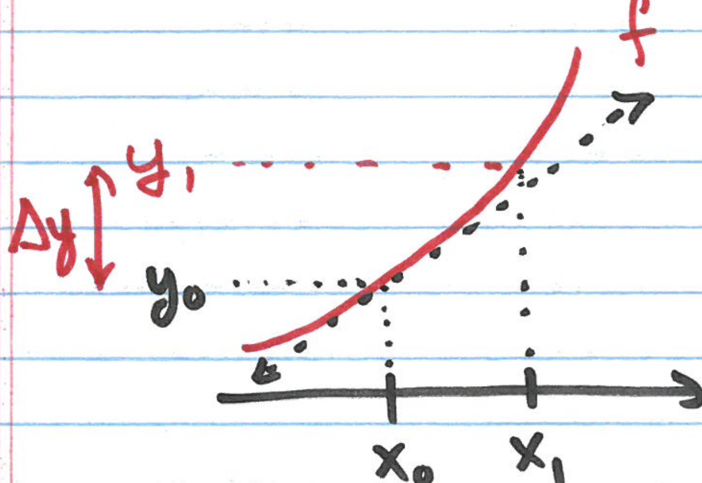
2/3



zoom in



Tangent line approximation



unknown

$f, \Delta y$

known

$x_0, y_0, f'(x_0)$

x_1

Want to know y_1

Ex: Suppose $x_0 = 1$, $f(x_0) = f(1) = y_0 = 2$ and $f'(1) = 0.5$.

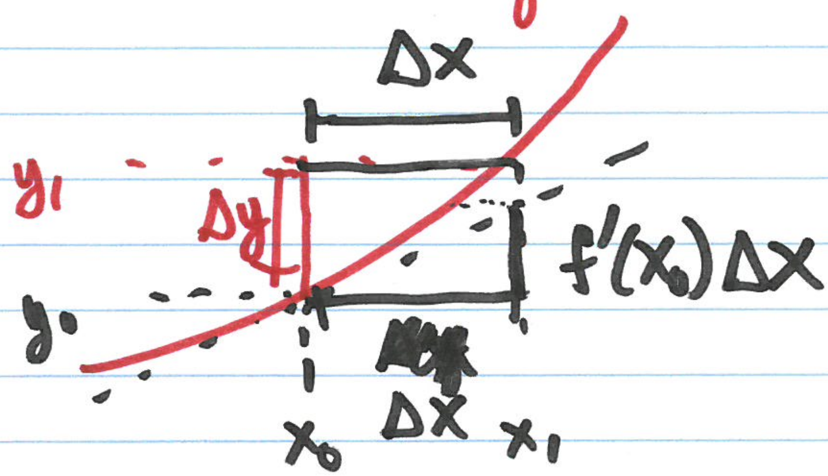
Estimate $f(1.1)$. $(x_1 = 1.1)$

Idea: Find y_1 by adding Δy to y_0

Find Δy by:

$$\frac{\Delta y}{\Delta x} \approx f'(x_0)$$

$$\Delta y \approx f'(x_0)\Delta x$$



Back to example:

$$\Delta x = x_1 - x_0 = 1.1 - 1 = 0.1$$

$$\frac{\Delta y}{\Delta x} \approx f'(1) = 0.5$$

$$\Delta y \approx 0.5 \cdot \Delta x$$
$$= 0.5 \cdot 0.1$$
$$= 0.05$$

$$y_1 - y_0 = 0.05$$

$$y_1 = y_0 + 0.05$$
$$= 2 + 0.05$$
$$= 2.05$$

The tangent line approximation does not give the true value if the slope increases between x_0 and x_1.

[redacted]

1) $y=\frac{1}{x}$. Find ~~the y-value~~

the slope of the curve when $x=0.5$.

slope of the tangent line at $x=0.5$

$f(x)=\frac{1}{x}=x^{-1}$ $\quad f'(x)=-1x^{-2}$

$f'(0.5)=-(0.5)^{-2}=-4$

2) Find the equation of the tangent line.

$$\boxed{\begin{array}{l}\text{point-slope formula}\\ y-y_0=m(x-x_0)\end{array}}$$

Find x_0, y_0 and m

$m=-4$

$x_0=0.5$

$y_0=\frac{1}{0.5}=2$

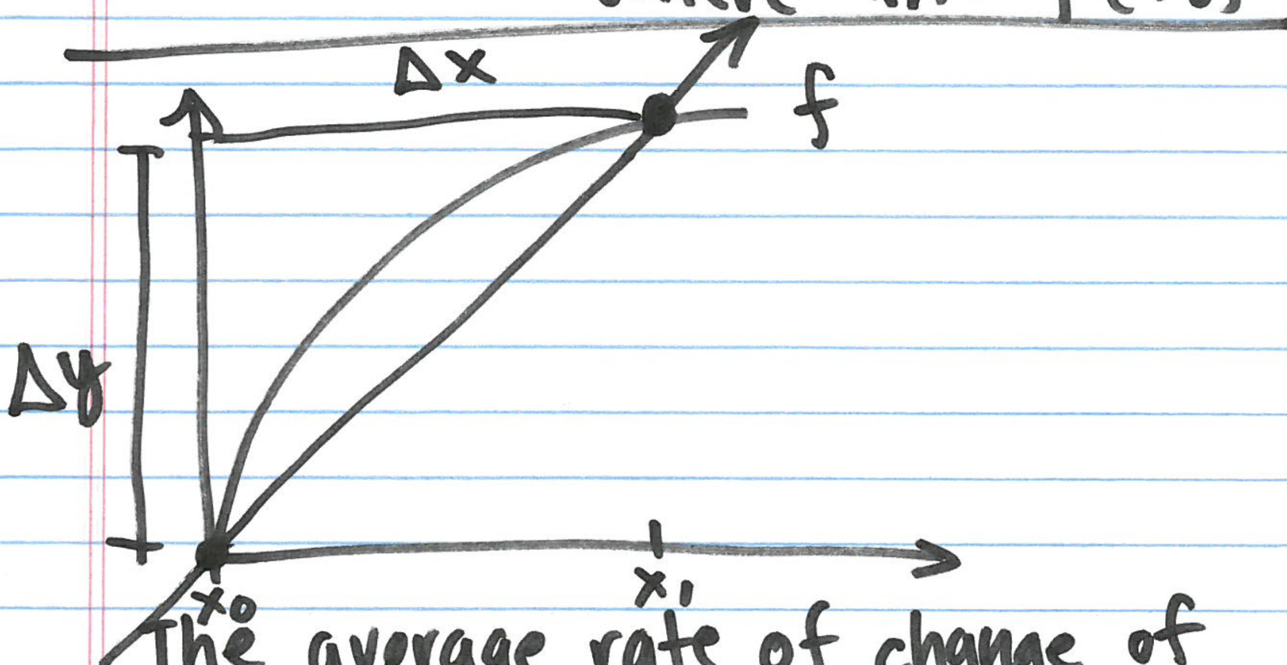
$y-2=-4(x-0.5)$

$$\Delta y = f'(x_0)\,\Delta x$$

$$y_1 - y_0 = f'(x_0)(x_1 - x_0)$$

$$= m\,(x_1 - x_0)$$

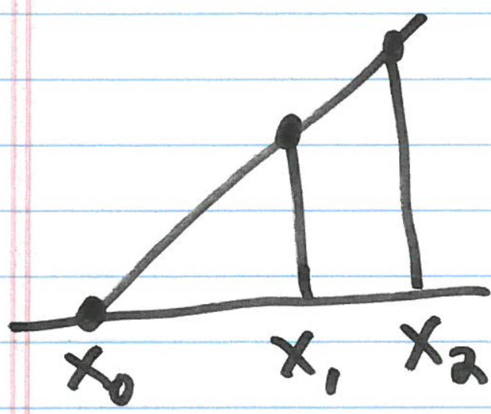
where $m = f'(x_0)$



The average rate of change of f on (x_0, x_1) is the slope of the line through (x_0, y_0) and (x_1, y_1).

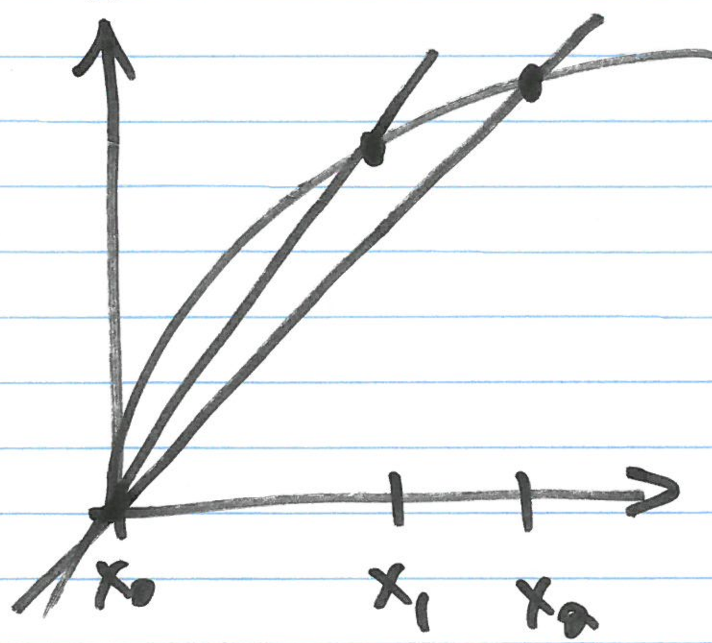
This line is called the secant line through (x_0, y_0) and (x_1, y_1).

If f is linear, all secant lines have the same slope.



For linear functions, the rate of change is constant.

For nonlinear functions the slope
ROC
is not constant.



The second derivative is the derivative of the first derivative.

$$f''(x) = \frac{d}{dx}\left[f'(x)\right]$$

"f double prime"

second derivative

The derivative of the velocity is the acceleration

$$a(t) = v'(t) = \frac{d}{dt}(v(t)) = \frac{d}{dt}s'(t)$$

$$= s''(t)$$

Rules of differentiation:

$$\frac{d}{dx}kf(x) = k\frac{d}{dx}f(x)$$

$$\frac{d}{dx}\left[f(x)+g(x)\right] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

$$\frac{d}{dx}\left[af(x)+bg(x)+ch(x)\right]$$
$$= a\frac{d}{dx}f(x)+b\frac{d}{dx}g(x)+c\frac{d}{dx}h(x)$$

Ex: A ball is thrown vertically and its height after t seconds is

$$s(t) = -16t^2+128t+5.$$

Find the velocity and acceleration functions.

$$v(t) = s'(t)$$
$$= -16\frac{d}{dt}t^2 + 128\cdot\frac{d}{dt}t + \frac{d}{dt}5$$
$$= -16\cdot 2t + 128\cdot 1 + 0$$
$$= -32t + 128$$

$$a(t) = v'(t)$$
$$= -32.$$