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Test 2 – Chapter 1 and Chapter 2
sections 2.1 – 2.3

Today: Given a function, determine where it is increasing, decreasing, where local maxima and minima occur, where it is concave up, concave down, and where inflection points occur.

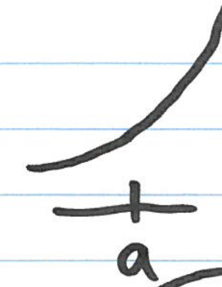
Assume that the function is not too badly behaved, ~~th~~ all derivatives change gradually wherever f is differentiable.

First derivative rule

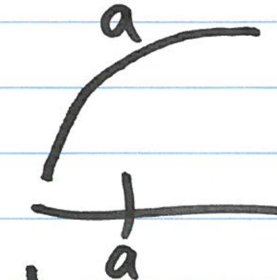
If $f'(a) > 0$ then f is increasing at a.
If $f'(a) < 0$ then f is decreasing at a.

Second derivative rule
If $f''(a) > 0$ then f is concave up at a.
If $f''(a) < 0$ then f is concave down at a.

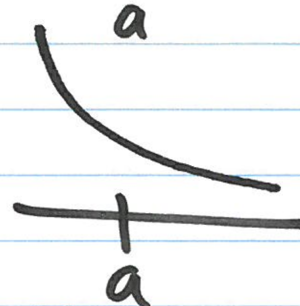
$f'(a) > 0 \Rightarrow f$ increasing
$f''(a) > 0 \Rightarrow f$ concave up



$f'(a) > 0$ f increasing
$f''(a) < 0$ concave down

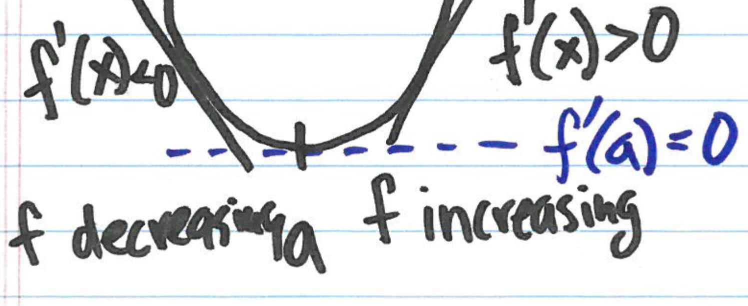


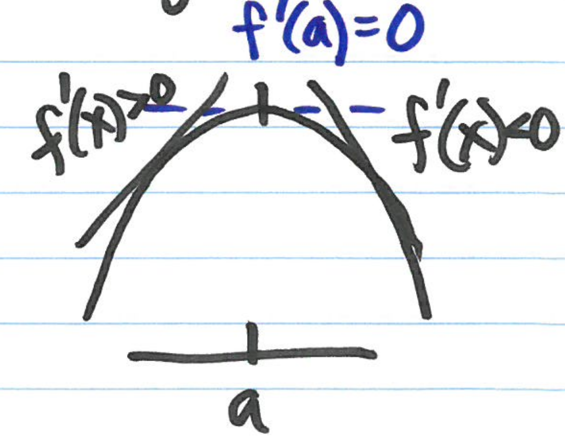
$f'(a) < 0$ f decreasing
$f''(a) > 0$ concave up



$f'(a) < 0$ f decreasing
$f''(a) < 0$ concave down

Relative minimum occurs where f changes from decreasing to increasing

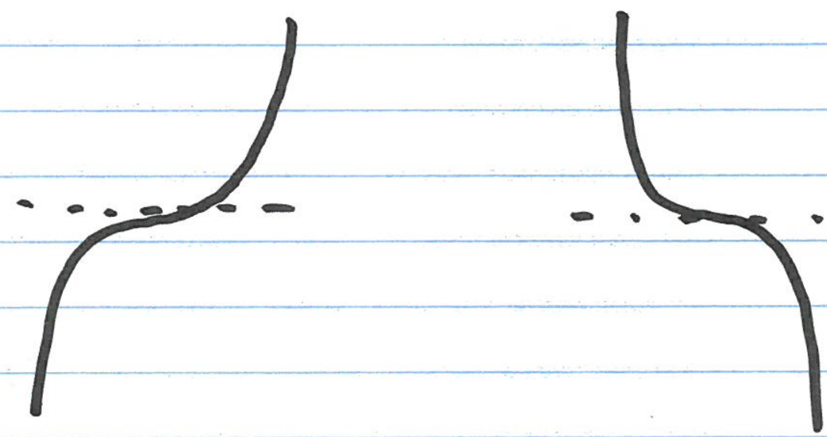




If f changes from increasing to decreasing (local max) or decreasing to increasing (local min) at $x=a$ then $f'(a)$ must be 0.

A number a such that $f'(a)=0$ is called a ~~critic~~ critical value.

A critical value is not necessarily an extremum. ⟺ Note: $f'(a)=0$ does not necessarily mean that f has a relative max or min.



To determine whether ~~if~~ a critical value is a local max or min, we need to determine whether f changes from increasing to decreasing or vice versa, using 1st or 2nd derivative test.

First derivative test:

If f' changes sign from positive to negative at $x=a$ then f has a local max at a

If f' changes from negative to positive at $x=a$ then f has a local min at $x=a$.

If f' does not change sign at $x=a$ then a is neither.

critical value is not a local max or min.

<u>Ex</u> Find the local maxima and minima of $f(x)=\frac{1}{3}x^3-2x^2+3x+1$.

Find critical values (where f could have min or max).

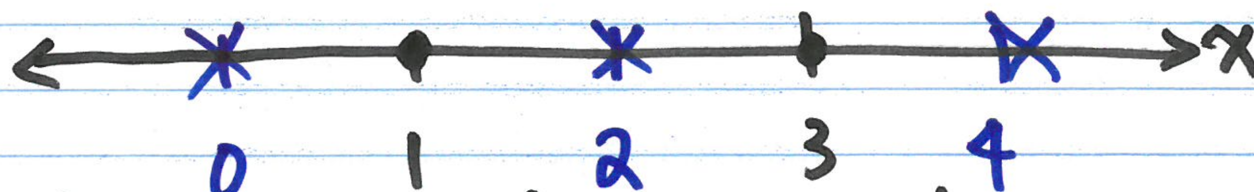
When is f' equal to 0?

$$f'(x)=x^2-4x+3=0$$

$$(x-3)(x-1)=0$$

$x=3$ or $x=1$ (critical values)

To see whether f' changes sign (condition of 1st derivative test) at the critical values, we evaluate f' at any points between 3 and 1 and on points to the left of 1 and right of 3.



f' positive $\quad$ f' negative $\quad$ f' positive.

$f'(0) = 3 > 0$

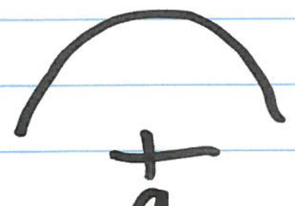
$f'(2) = 2^2 - 4(2) + 3 = -1 < 0$

$f'(4) = 4^2 - 4(4) + 3 = 3 > 0$

By the first derivative test f has local max at $x = 1$

and f has a local min at $x = 3$.

Another way that sometimes works to determine if f has a local max/min at $x=a$ is to determine the second derivative at a.



Second derivative test

If $f'(a)=0$ and $f''(a)<0$ then a is a local max.

If $f'(a)=0$ and $f''(a)>0$ then a is a local min.

Ex: Find local max/min

$$f(x)=\frac{1}{3}x^3-2x^2+3x+1$$

$f'(x) = x^2 - 4x + 3$, $f''(x) = 2x - 4$

Critical values

$f'(x) = 0$

$\Longrightarrow x = 3,\ x = 1$ (see last example)

Using the second derivative test,

$f''(3) = 2 > 0$ f is concave up at $x=3$. So 3 is a local min.

$f''(1) = -2 < 0$, so f is concave down at $x = 1$, so f has a local max at $x=1$.