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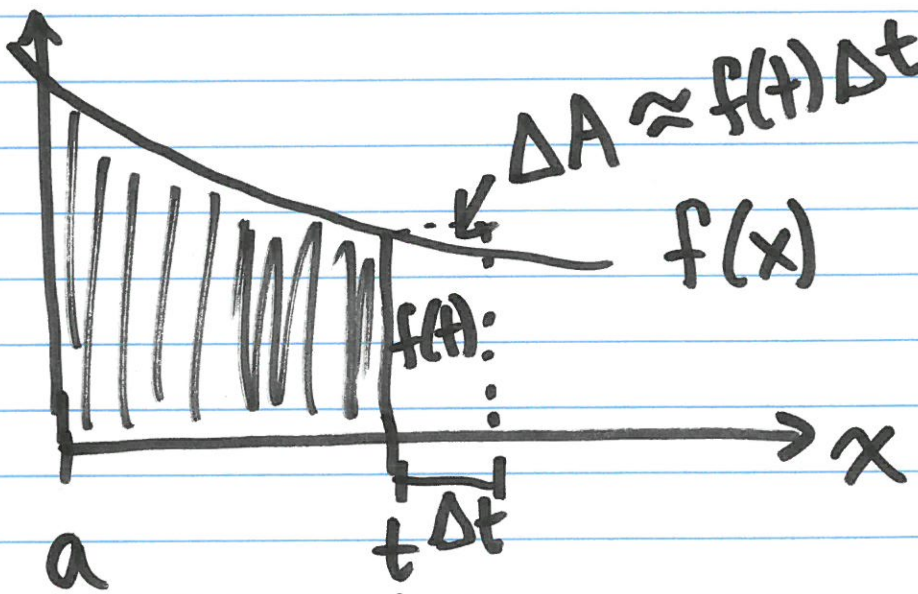
Volumes of solids of revolution

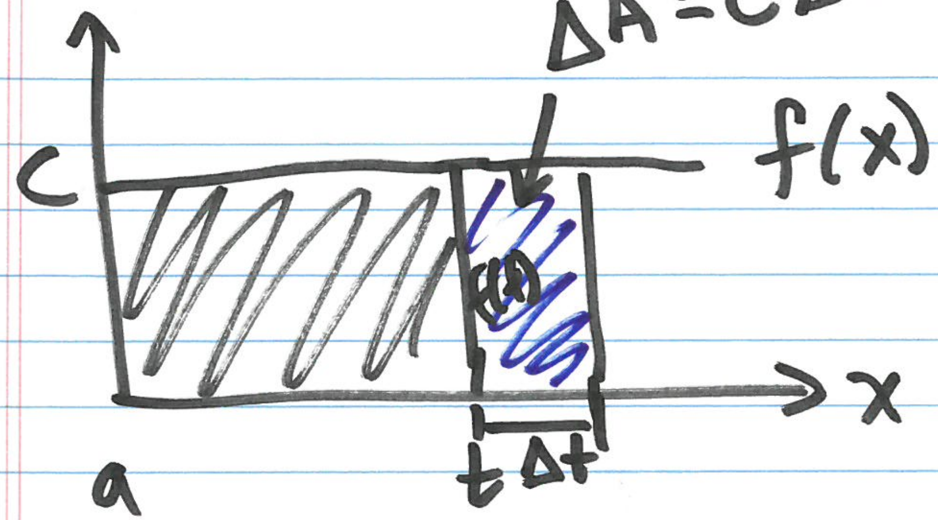
Let $A(t)$ be the area under the graph of $f(x)$ between $x=a$ and $x=t$.



$$A(t)=\int_a^t f(x)\,dx$$

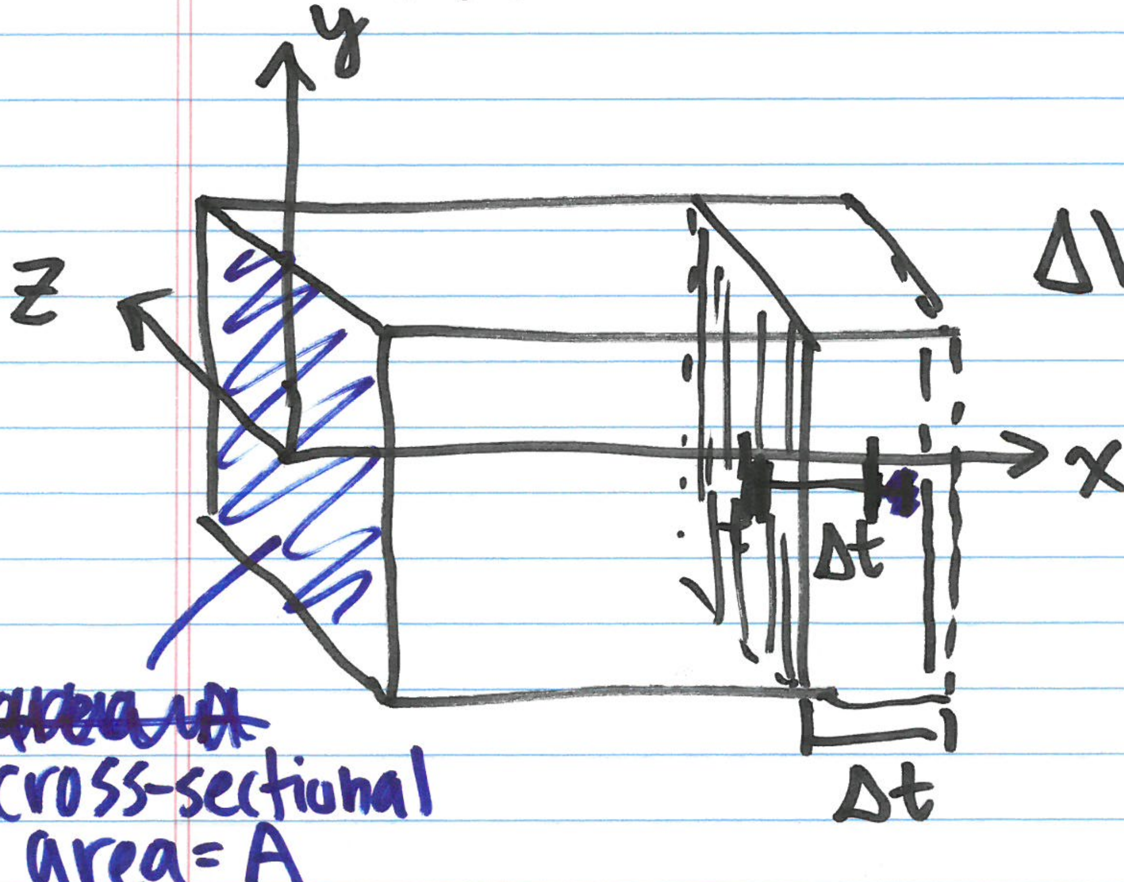
$$\frac{d}{dt}A(t)=f(t)$$

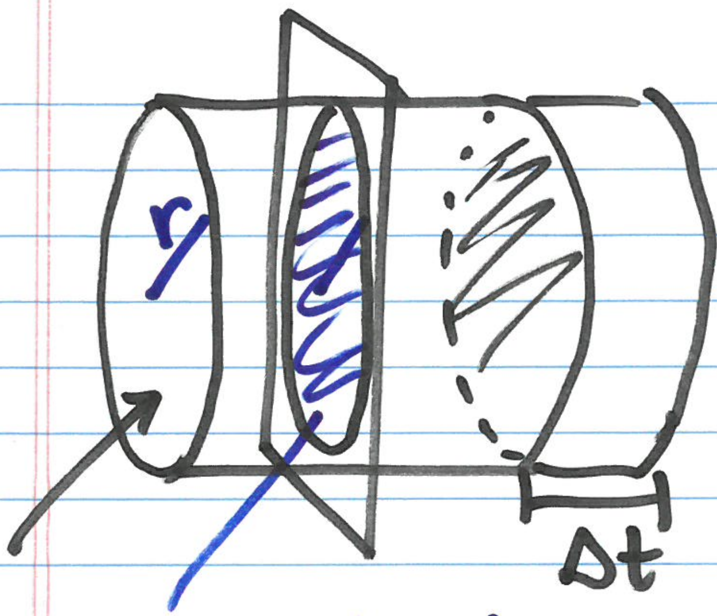
$$\Delta A = C\Delta t = f(t)\Delta t$$



Let $V(t)$ be the volume of a cuboid with cross-sectional area A.

$$\Delta V = A\,\Delta t$$

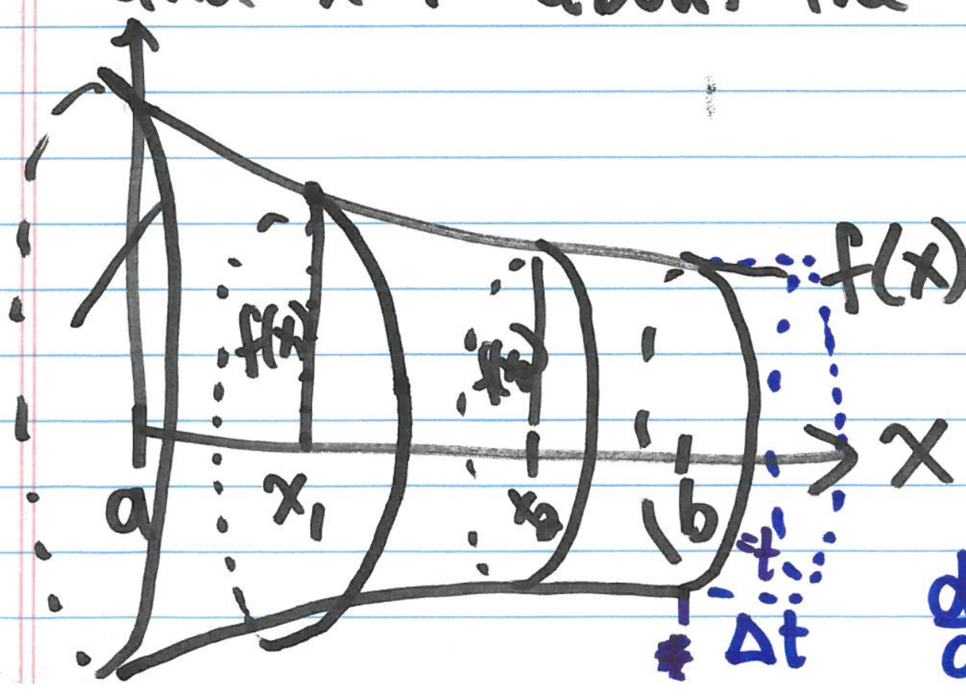




$\Delta V = A\Delta t$

cross-sectional area $A = \pi r^2$

Suppose we have a volume obtained by revolving the ~~fun~~ region bounded by a function $f(x)$ between $x=a$ and $x=b$ about the x-axis



$\Delta V \approx A(t)\Delta t$

$\frac{dV}{dt} = A(t)$

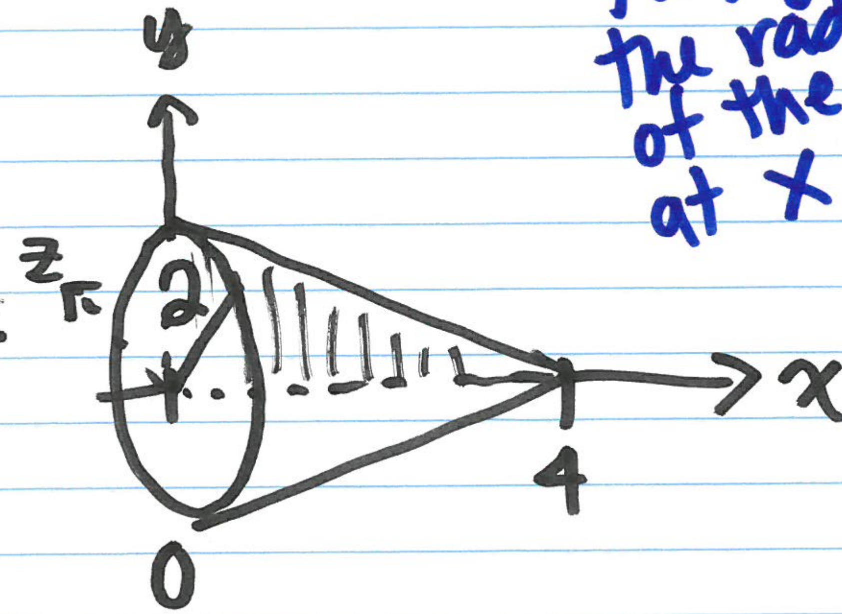
We will find the volume.

The area of a solid of revolution obtained by revolving the region below $f(x)$ about the x-axis between $x=a$ and $x=b$ is

$$\boxed{V = \int_a^b A(x)\,dx = \int_a^b \pi[f(x)]^2\,dx}$$

f(x) gives the radius of the cross-section at x

Ex:



Find the volume of the solid of revolution obtained by rotating the region below the graph of $f(x) = 2 - \frac{1}{2}x$ about the x-axis from $x=0$ to $x=4$.

$$\int_0^4 \pi \left(2 - \frac{1}{2}x\right)^2 dx$$

Use substitution

Let $u = 2 - \frac{1}{2}x$

$du = -\frac{1}{2}dx \Rightarrow -2du = dx$

u limits

$x = 0 \Rightarrow u = 2$

$x = 4 \Rightarrow u = 0$

$$\int_0^4 \pi \left(2 - \frac{1}{2}x\right)^2 dx = \int_2^0 \pi (u^2)(-2)du$$

$$= -2\pi \int_2^0 u^2 du$$

$$= -2\pi \left(\frac{1}{3}u^3\right)\Big|_2^0$$

$$= 0 - \left(-2\pi\left(\frac{1}{3}\cdot 2^3\right)\right)$$

$$= \frac{16\pi}{3}.$$

$$V = \frac{\pi r^2 h}{3}$$

$$= \frac{\pi \cdot 2^2 \cdot 4}{3}$$

$$= \frac{16\pi}{3}. \checkmark$$

Ex: Find VOSOR obtained by revolving $y = e^{-2x}$ from $x=0$ to $x=1$ about the x-axis.

$f(x) = e^{-2x}$