3/19 · 5.1

Goals: 1) Represent exponential growth and decay

2) Understand the kind of information we need to determine the initial value C and the growth constant k in an exponential growth model.

Idea: ~~To determine C~~

Let $f(t) = Ce^{kt}$ general form of exponential growth

be a function representing exponential growth.

To determine C, we can use f(0).

In some cases we do not need to have f(0) given in order to find C.

$$\cancel{1.5 = Ce^{15\cdot 14}}$$

$$\cancel{\Rightarrow C = \frac{1.5}{e^{15\cdot 14}}}$$

Consider the population of fruit flies (from last class)
Suppose in the colony of fruit flies, the initial rate of growth is 7.7 fruit flies per day. What was the initial population size?

$k=\frac{\ln 2}{9}$

$$P(t) = Ce^{kt} \qquad \underbrace{k=\frac{\ln 2}{9}}_{\text{last time}}$$

$$P(t) = Ce^{\frac{\ln 2}{9}t}$$

$$P'(t) = Ce^{\frac{\ln 2}{9}t}\cdot\frac{\ln 2}{9}$$

Since initial rate of growth is 7.7

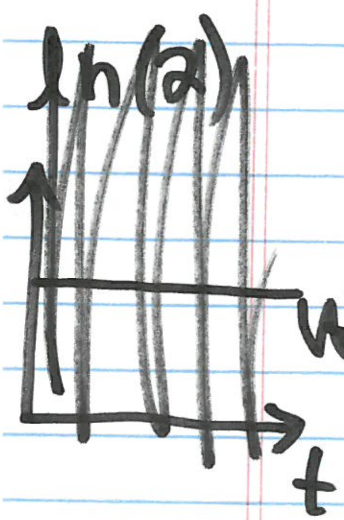
$$P'(0) = 7.7 = C\cdot\frac{\ln 2}{9}$$

$$C = \frac{7.7 \cdot 9}{\ln(2)} \approx 100$$

How large will the colony be after 41 days?

$$P(t) = 100e^{\frac{\ln 2}{9}t}$$

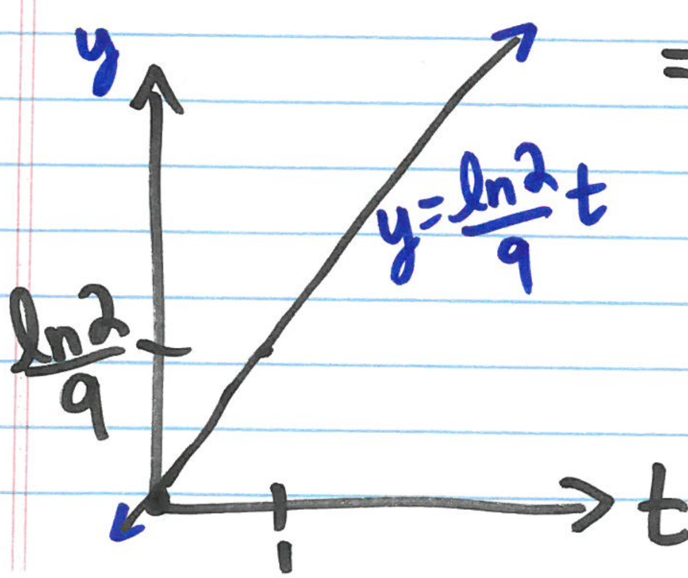
$$P(41) = 100e^{\frac{\ln 2}{9} \cdot 41} \approx \boxed{2,350 \text{ fruit flies}}$$



~~What will rate of~~

$$\frac{d}{dt} C \cdot e^{\frac{\ln 2}{9}t} = Ce^{\frac{\ln 2}{9}t} \cdot \frac{d}{dt}\frac{\ln 2}{9}t$$

$$= Ce^{\frac{\ln 2}{9}t} \cdot \frac{\ln 2}{9}$$



Note: $\frac{\ln 2}{9}t$ is a constant $\left(\frac{\ln 2}{9}\right)$ times t

What will the rate of growth be at $t = 41$ days?

$$P'(41) = 100e^{\frac{\ln 2}{9} \cdot t} \cdot \frac{\ln 2}{9} \Big|_{t=41}$$

$$= \underbrace{100e^{\frac{\ln 2}{9} \cdot 41}}_{\overset{\shortparallel}{P(41)}} \cdot \frac{\ln 2}{9}$$

$$= 2{,}350 \cdot \frac{\ln 2}{9}$$

$$= P(41) \cdot k$$

The population of a state grows exponentially. If the population in 2016 is 8 million people and grows at a rate of 0.2 million per year, what will the population be in 2050?

Let $P(t)$ be the population (in millions). where t is the number of years after 2016.

$P(t) = Ce^{k \cdot t}$

We need to determine C, k, and then find $P(34)$.

$P(0) = 8$

$P'(0) = 0.2$

$P(0) = C = 8$

$P(t) = 8e^{kt}$

Differentiate P and evaluate at $t=0$ to find k.

$$P'(t) = 8e^{kt} \cdot k$$

$$P'(0) = 8e^{0} \cdot k$$

$$= 8k = 0.2$$

$$k = \frac{0.2}{8}$$

$$= 0.025$$

$$P(t) = 8e^{0.025t}$$

$$P(34) = 8e^{0.025 \cdot 34}$$

$$\cong 18.717$$

The population will be about 18.7 million people in 2050