1. a) $P'(0) = 0.5\,P(0)$

$P'(0)$: given; $P(0)$: want

$18 = 0.5\,P(0)$

$P(0) = 36$

b) $P(t) = P(0)\,e^{kt}$

$= 36e^{kt}$ $\quad k = 0.5$

$P(t) = Ce^{kt}$

$P(0) = 36$

$P(t) = 36e^{kt}$

$P'(t) = 36e^{kt} \cdot k$

$P'(0) = 36k$

$= 18$

$\Rightarrow k = 0.5$

c) $P(t) = 36e^{0.5t} = 72$

$\Rightarrow e^{0.5t} = 2$

$0.5t = \ln 2$

$t = 2\ln 2$

d) $P'(2\ln 2) = 0.5\,P(2\ln 2)$

$= 0.5 \cdot 72$

$= 36$ cells/hour

5. Integrate $\int \frac{2e^x}{e^x+1}\,dx$

Let $u = e^x + 1$

$du = e^x\,dx$

$\int \frac{2}{u}\,du = 2\int \frac{1}{u}\,du$

$= 2\ln|u| + C$

$$2\ln|e^x+1|+C$$

6. (Test 4 #3)

$A(t) = -t^2 + 12t$ ← absorption rate (g/hour)

$E(t) = 6$ ← emission rate (g/hour)

Let $C(t)$ stand for the amount of carbon t hours after sunrise.

$C(0) = 700{,}004$

Amount lost during first 12 hours

$$6\ \text{g/hour} \cdot 12\ \text{hours} = 72\ \text{g}$$

Also need the amount gained/absorbed

$$\text{amount gained over 12 hours} = \int_0^{12} -t^2 + 12t\,dt$$

$$= \frac{}{} - \frac{1}{3}t^3 + 6t^2 \Big|_0^{12}$$

$$= -\frac{12^3}{3} + 6 \cdot 12^2$$

$$= 288 \text{ grams}$$

$$C(12) = C(0) + 288 - 72$$

$$= 700{,}004 + 288 - 72$$

$$= 700{,}220 \text{ g}$$

Shorter version

$$C(12) = C(0) + \int_0^{12} [A(t) - R(t)]\,dt$$

3. $y_{n+1} = 0.95y_n + 1000$

$y_0 = 5000$

If y_n is a solution to

$$y_{n+1} = ay_n + b$$

then $$y_n = \frac{b}{1-a} + \left(y_0 - \frac{b}{1-a}\right)a^n$$

$a = 0.95$

$b = 1000$

$\frac{b}{1-a} = \frac{1000}{0.05} = 20000$

$y_0 \neq \frac{b}{1-a} \Rightarrow$ not constant

$|a| < 1 \Rightarrow$ asymptotic to $y = \frac{b}{1-a}$

$a > 0 \Rightarrow$ monotonic (not oscillating)

4. annual interest rate: $3\% = 0.03$

interest rate per period: $\frac{0.03}{12}$

$i = 0.0025$

Let y_n be the balance after n months.

change in balance = interest − withdrawal

$$y_{n+1} - y_n = 0.0025\, y_n - 2000$$

$$y_{n+1} = 1.0025\, y_n - 2000$$

2. $V(r) = 4\pi r^3 = (\pi r^2 h$ where $h = 4r)$

$r(t_0) = 10$

$r'(t_0) = 0.5$

$$\frac{d}{dt} V(r(t)) = V'(r(t)) \cdot r'(t)$$

The rate of change of $V(r(t))$ when the radius $r(t)$ is 10 inches and is increasing at 0.5 inches/s is:

[redacted]

$$V'(10) \cdot 0.5$$

$$= \frac{d}{dt} V(r) \Big|_{r=10} \cdot 0.5$$

[redacted] $= 12\pi r^2 \Big|_{r=10} \cdot 0.5$

$$= 1200\pi \cdot 0.5$$

$$= 600\pi \text{ inches}^3/\text{s}.$$