Integration

3/30

6.1 - Antidifferentiation

Review

power rule
for integration/
antidifferentiation

power rule

$$\boxed{\int x^r \, dx = \frac{1}{r+1} x^{r+1} + C} \qquad (r \neq -1)$$

Always check that your antiderivative is correct (by differentiation)

$$\frac{d}{dx} \frac{1}{r+1} x^{r+1} = x^r \quad ✓$$

Always remember that an indefinite integral has a constant of integration.

Last time, we also saw that

If $F(x)$ is an antiderivative of f of $f(x)$, then every antiderivative has the form $F(x)+C$.

$$\int f(x)\,dx = F(x)+C$$

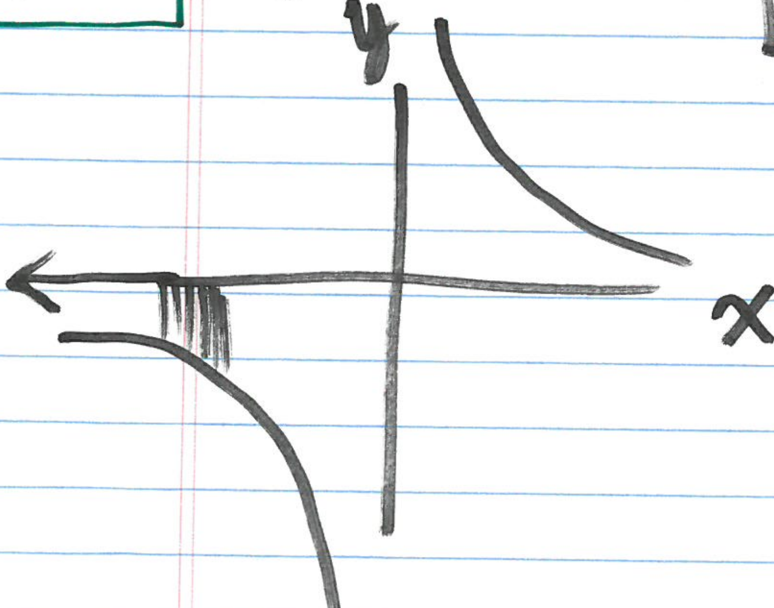
Start new material: For every rule of differentiation, there is a rule for antidifferentiation

$\boxed{\text{log rule}}$ $\int x^{-1}\,dx$ or $\boxed{\int \frac{1}{x}\,dx = \ln|x|+C}$

How to enter into WebAssign:

ln(abs(x)) + c

lower case



exponential rule for integration $\boxed{\int e^{kx}\,dx = \frac{1}{k}e^{kx} + C}$

Linearity of integration

$$\boxed{\int [f(x)+g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx} \quad (1)$$

$$\boxed{\int k f(x)\,dx = k\int f(x)\,dx} \quad (2)$$

k a constant.

Returning to the rocket example, we want an antiderivative of

velocity function

$6t+0.5 =$ v(t), which we called s(t) with s(0) = 8

$$s(t) = \int v(t)\,dt$$

$$= \int (6t + 0.5)\,dt$$

Using (1) linearity

$$= \int 6t\,dt + \int 0.5\,dt$$

Using (2) linearity

$$= 6\int t\,dt + \int 0.5\,dt$$

$$= 6\left(\tfrac{1}{2}t^2 + C_2\right) + 0.5t + C_1$$

We can ~~lump all cons~~ combine the constants into one constant.

$$= 3t^2 + 0.5t + C$$

We could have also used the power rule to find $\int 0.5dt$:

$$\int 0.5\,dt = 0.5\int 1\,dt$$

$$= 0.5\int t^0\,dt \qquad t^0 = 1$$

$$= 0.5t + C$$

Antiderivative of a constant

$$\boxed{\int C\,dt = Ct + D}$$

Since $s(0) = 8$

$$C = 8$$

So $s(t) = 3t^2 + 0.5t + 8$

Ex: Find $\int \sqrt{x}\,dx$

$$\int \sqrt{x}\,dx = \int x^{\frac{1}{2}}\,dx$$

Apply power rule with $r = \frac{1}{2}$

$$= \frac{1}{\frac{3}{2}} x^{\frac{3}{2}} + C$$

$r+1 = \frac{3}{2}$

$\frac{1}{r+1} = \frac{2}{3}$

$$= \frac{2}{3} x^{3/2} + C$$

Ex: $\int \frac{1}{x^2}\,dx = \int x^{-2}\,dx$

Using power rule with $r = -2$

$$= -x^{-1} + C$$

$$= -\frac{1}{x} + C$$

Ex: $$\int \left(x^{-3} + 7e^{5x} + \frac{4}{x}\right) dx$$

$$\int x^{-3}\,dx + 7\int e^{5x}\,dx + 4\int \frac{1}{x}\,dx$$

$$-\frac{1}{2}x^{-2} + 7\,\frac{1}{5}e^{5x} + 4\ln|x| + C$$

Check: $$\frac{d}{dx}\left(-\frac{1}{2}x^{-2} + 7\cdot\frac{1}{5}e^{5x} + 4\ln|x| + C\right)$$

$$= \underbrace{x^{-3} + 7e^{5x} + 4\,\frac{1}{x}}_{\text{We get back the integrand when we differentiate our antiderivative } \checkmark}$$

6.2 - The definite integral

In the rocket example:

$$\int v(t)\,dt = s(t) + C$$

Example of a definite integral

$$\int_a^b v(t)\,dt = \underbrace{s(b) - s(a)}_{\text{net change in position between } t=a \text{ and } t=b}$$

Definition: Suppose f is continuous function on an interval $[a,b]$. The definite integral of f from a to b is

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

The number $F(b) - F(a)$ is the net change of the function F as x varies from a to b.

a, b are called limits of integration.

$F(b) - F(a)$ is also written as $F(x)\Big|_a^b$

Ex: Evaluate $\int_1^2 x\,dx$

Need an antiderivative of x.

$$\int x\,dx = \underbrace{\frac{1}{2}x^2 + C}_{F(x)}$$

$$\int_1^2 x\,dx = F(2) - F(1)$$

$$= F(x)\Big|_1^2$$

$$= \left(\frac{1}{2}x^2 + C\right)\Big|_1^2$$

$$= \left[\frac{1}{2}(2)^2 + C\right] - \left[\frac{1}{2} + C\right]$$

$$= 2 + C - \frac{1}{2} - C$$

$$= \frac{3}{2}$$