4/6 due

WebAssign: Integrals and Areas under Curves 4/15

Further integration after 4/15

problems on substitution included

Ex: (1) The rate of health expenditure grew exponentially starting in 2000 ($t=0$) so that the rate of expenditure (in billions of dollars per year) t year after 2000 was

$$R(t) = 380e^{0.12t}$$

Find the total amount of health expenditure between 2000 and 2010.

Let's call the total expenditure $T(t)$. Then $R(t) = T'(t)$.

Net change in T between $t=0$ and $t=10$ is the total amount spent between 2000 and 2010.

We want to know $T(10) - T(0)$.

$$T(10) - T(0) = \int_0^{10} R(t)\,dt$$

The net change is the integral of the rate of change (FTC).

$$T(10) - T(0) = \int_0^{10} 380e^{0.12t}\,dt$$

$$= \frac{380}{0.12} e^{0.12t} \Big|_0^{10}$$

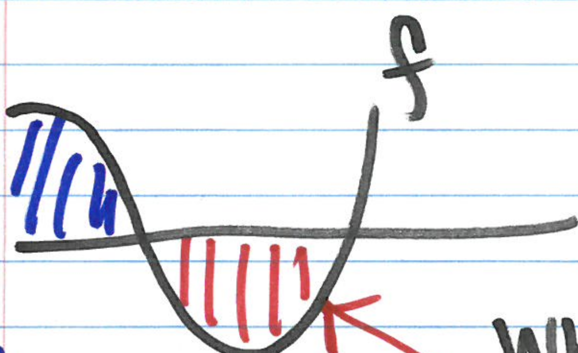
$$= \frac{380}{0.12} e^{1.2} - \frac{380}{0.12}$$

$$\approx 7347$$

$$\boxed{\$7347 \text{ billion}}$$

Fact 1

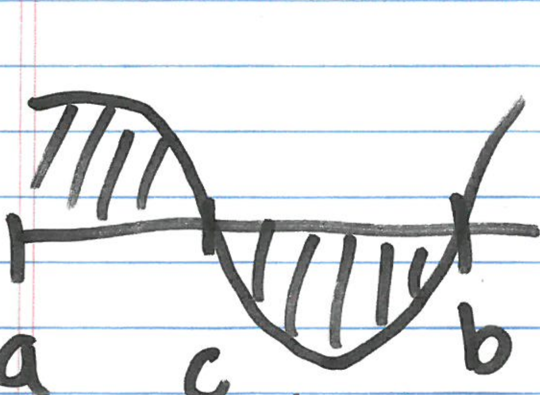
How negative values contribute to an integral



When we integrate f, these negative value decrease the value of the integral

Fact 2

One way to deal w/ negative values

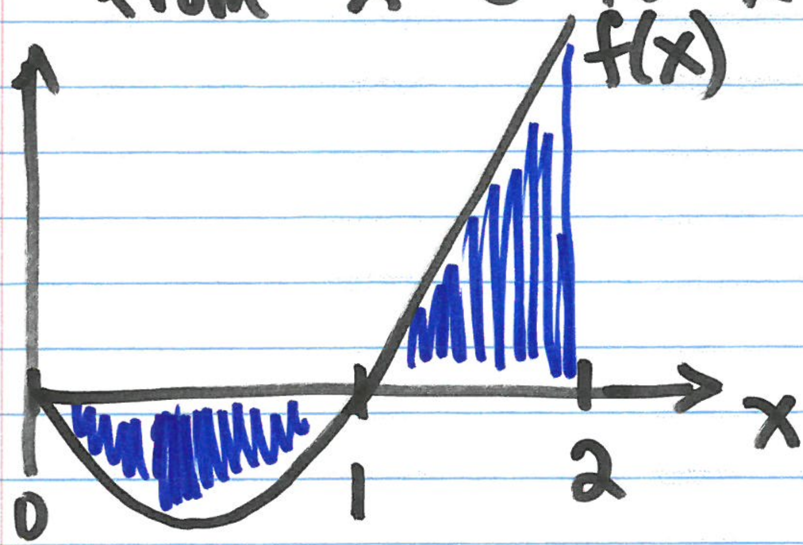


$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

* Need to consider what we want to compute

In some applications, we may want to integrate instead $-f(x)$ on the interval $[c,b]$.

Ex (6): Find the area bounded by the x-axis and the graph of $f(x) = x^2 - x$ from $x=0$ to $x=2$.



As shown f takes on negative values. These values should contribute to the area

We want $\int_0^2 |f(x)|\,dx$

$$\int_0^2 |x^2 - x|\,dx$$

Since we always want our integrand to be positive (in this case)

We need to split up the interval so that we can just change the sign in front of x^2-x based on whether x^2-x is positive or negative.

$$x^2-x=0$$

$$\Rightarrow x(x-1)=0$$

$$\Rightarrow x=0 \text{ or } x=1$$

$$|x^2-x| = \begin{cases} x^2-x & x \geq 1 \\ -(x^2-x) & 0 \leq x \leq 1 \end{cases}$$

So we can split up the integral as

$$\int_0^2 f(x)\,dx = \int_0^1 -(x^2-x)\,dx + \int_1^2 (x^2-x)\,dx$$

$$-\left[\int_0^1 (x^2 - x)\,dx\right] + \int_1^2 (x^2 - x)\,dx$$

$$= -\left[\frac{1}{3}x^3 - \frac{1}{2}x^2 \Big|_0^1\right]$$

$$+ \left[\frac{1}{3}x^3 - \frac{1}{2}x^2 \Big|_1^2\right]$$

$$= -\left[\left(\frac{1}{3} - \frac{1}{2}\right) - 0\right] + \left[\left(\frac{8}{3} - \frac{4}{2}\right) - \left(\frac{1}{3} - \frac{1}{2}\right)\right]$$

$$= -\frac{1}{3} + \frac{1}{2} + \frac{8}{3} - 2 - \frac{1}{3} + \frac{1}{2}$$

$$= \frac{1}{6} + \frac{7}{3} - \frac{3}{2}$$

$$= \frac{1}{6} + \frac{14}{6} - \frac{9}{6}$$

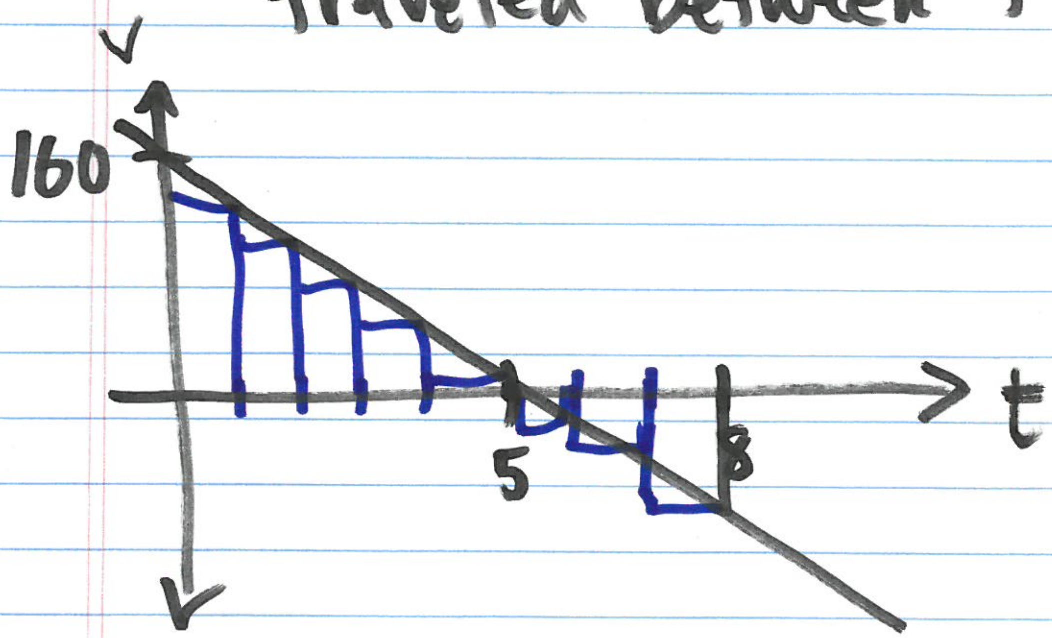
$$= 1$$

Displacement versus distance traveled

<u>Ex</u> A rocket is fired vertically into the air. Its velocity t seconds after liftoff is $v(t) = -32t + 160$ feet per second.

a) Find the rocket's displacement between $t=0$ and $t=8$.

b) Find the total distance traveled between $t=0$ and $t=8$.



Key idea: displacement is the net change of the position

a) Let $s(t)$ be the position at time t.

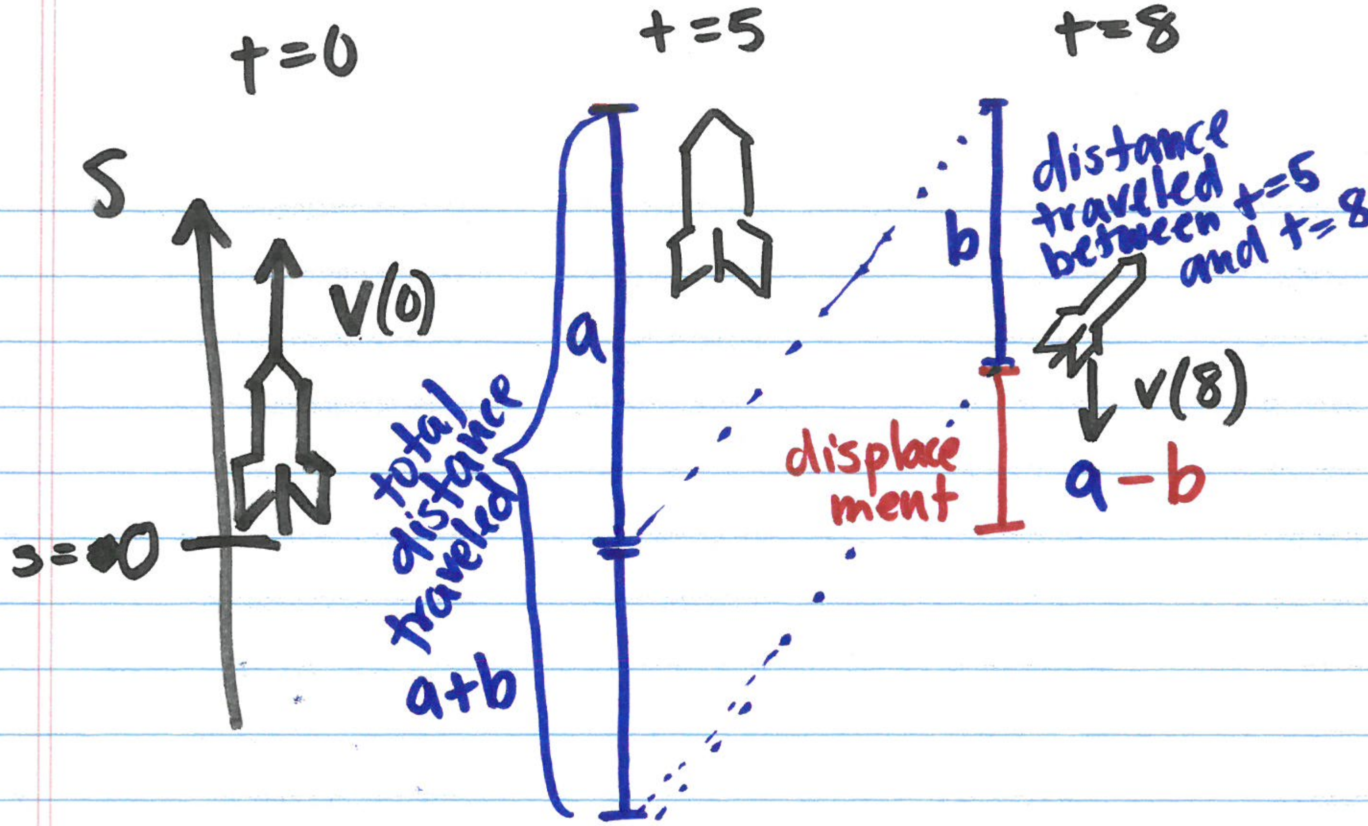
Then the displacement is the net change of the position.

We want to find

FTC: $s(8)-s(0)=\int_0^8 s'(t)\,dt$

$$=\int_0^8 v(t)\,dt$$

6.4 #5, 11, 28, 39, 40


t=0
t=5
t=8
s
v(0)
s=0
a
total distance traveled
a+b
b
distance traveled between t=5 and t=8
v(8)
displacement
a-b