1. (14 points) The population of a culture of cells is represented by a function $P(t)$ which satisfies the following differential equation:

$$P'(t) = 0.5P(t).$$

The initial *rate of growth* of the population is 18 cells per hour.

(a) What is the initial size of the population?

(b) Determine the function $P(t)$.

(c) After how long will the population reach 72?

(d) What will the rate of growth of the population be at that time?

2. (10 points) The volume of a cylinder, with a length equal to four times the radius, is $V(r) = 4\pi r^3$. If the radius is 10 inches and is increasing at rate of 0.5 inches/s, at what rate is the volume increasing?

3. (9 points) Identify the words below which correctly describe the graph of the difference equation $y_{n+1} = 0.95y_n + 1000$ with initial value $y_0 = 5000$. Write a brief justification for each word that correctly describes the graph.

monotonic	constant	oscillating
asymptotic	unbounded	non-constant

4. (5 points) Write a difference equation to represent an account that earns 3% interest, compounded monthly, if the initial balance is \$100,000 and \$2,000 is withdrawn every month.

5. (5 points) Integrate $\int \frac{2e^x}{e^x+1}\, dx$

6. (10 points) Suppose that over the course of one day, a tree absorbs carbon from the air at a rate of $A(t) = -t^2 + 12t$ grams per hour where t is the time (in hours) after sunrise. In addition, suppose the tree breaks down its stored carbon and emits it back into the air at a constant rate of $E(t) = 6$ grams per hour. Assume that these two rates account for all changes in the amount of carbon in the tree. If the tree contains 700,004 grams of carbon at the beginning of the day ($t = 0$), how many grams of carbon will it contain 12 hours later?