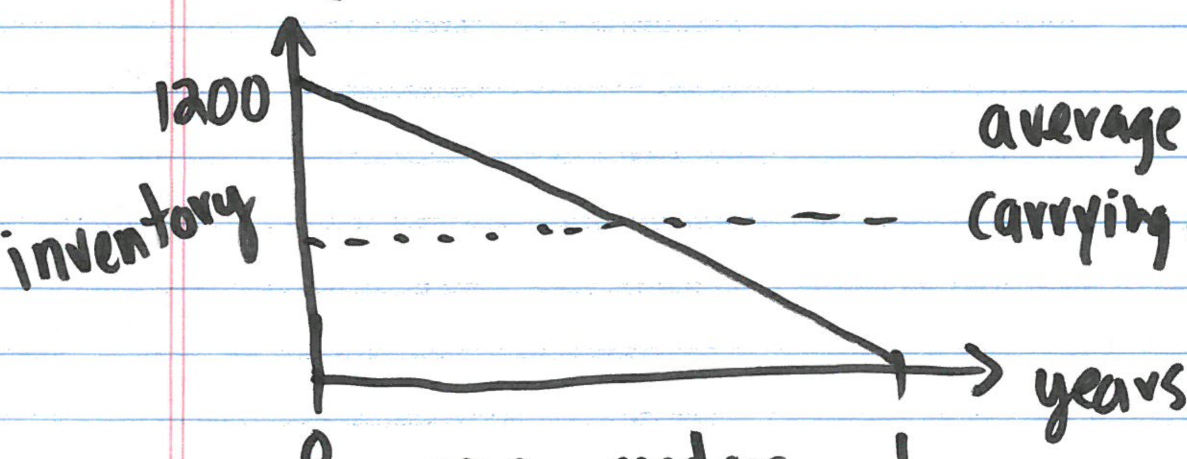
2/26

WebAssign – Optimization Wed
2 more Fri

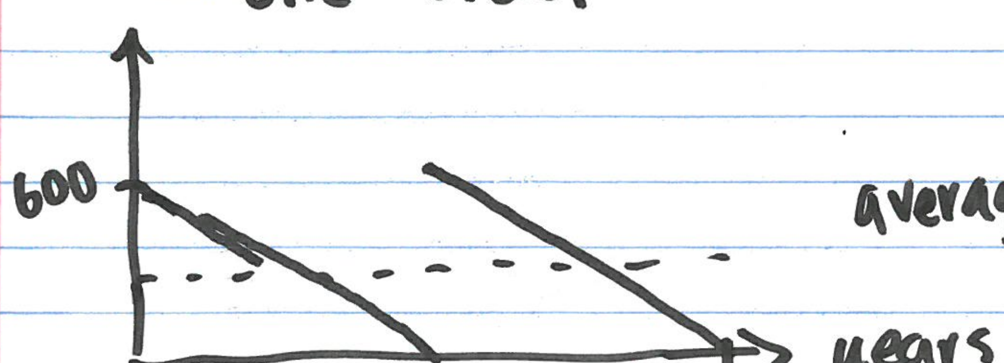
A supermarket manager anticipates ~~oran~~ selling 1200 cases of frozen orange juice at a steady rate during the next year. It costs \$8 per year to carry one case of orange juice. If the manager places orders of equal size at equally-spaced intervals the carrying cost can be determined by plotting the inventory over time



average # cases = 600

carrying cost = $8 \cdot 600$
= 4800 dollars

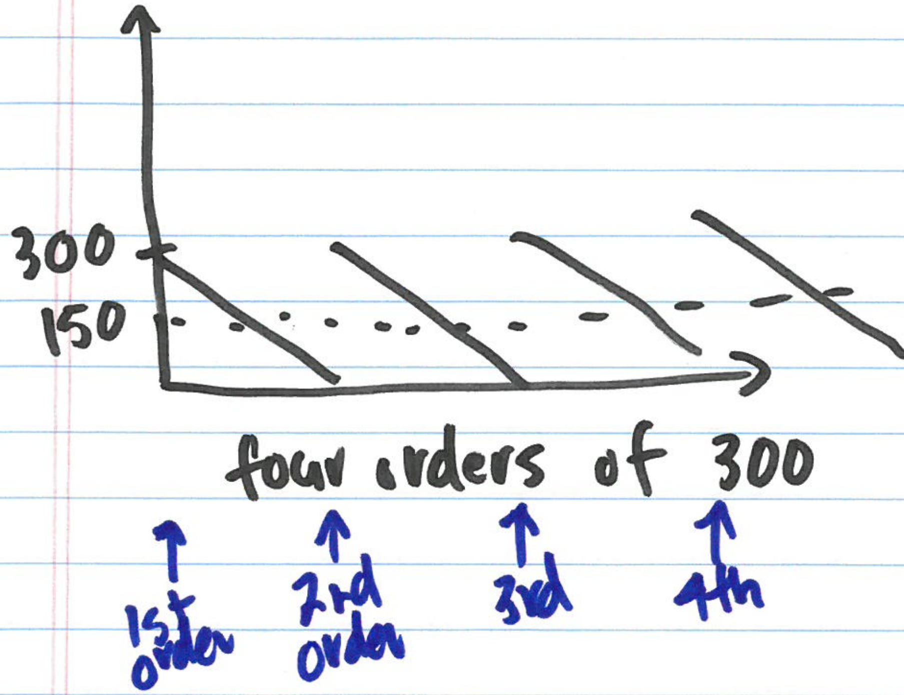
one order



average cases = $\frac{600}{2}$
= 300

two orders of 600

carrying cost = 2400 dollars



avg cases = 150
carrying cost = 1200

Manager reduces carrying cost by increasing number of orders but each order cost money

Inventory Control

$$\begin{bmatrix}\text{inventory}\\ \text{cost}\end{bmatrix} = \begin{bmatrix}\text{carrying}\\ \text{cost}\end{bmatrix} + \begin{bmatrix}\text{ordering}\\ \text{cost}\end{bmatrix}$$

Problem: If delivery of each order cost \$75 how much should the manager order to minimize inventory cost?

Let r be the number of orders.
Let x be the number of cases per order.

We have seen that the average number of cases per order is $\frac{x}{2}$.

<u>objective</u>

minimize cost

objective function: $C = \begin{bmatrix}\text{inventory}\\ \text{cost}\end{bmatrix} = \begin{bmatrix}\text{carrying}\\ \text{cost}\end{bmatrix} + \begin{bmatrix}\text{ordering}\\ \text{cost}\end{bmatrix}$

$$= 8 \cdot \frac{x}{2} + 75r$$

<u>constraint:</u>

$$\frac{1200}{r} = x$$

$$1200 = x \cdot r$$

$$r = \frac{1200}{x}$$

$$C = 4x + 75\left(\frac{1200}{x}\right)$$

$$= 4x + \frac{90000}{x} = 4x + 90000x^{-1}$$

find global minimum of C

$$C'(x) = 4 + (-1)\,90000\,x^{-2}$$

$$= 4 - \frac{90000}{x^2} = 0$$

$$4 = \frac{90000}{x^2}$$

$$x^2 = \frac{90000}{4}$$

$$= 22500$$

$$x = 150 \quad (x > 0)$$

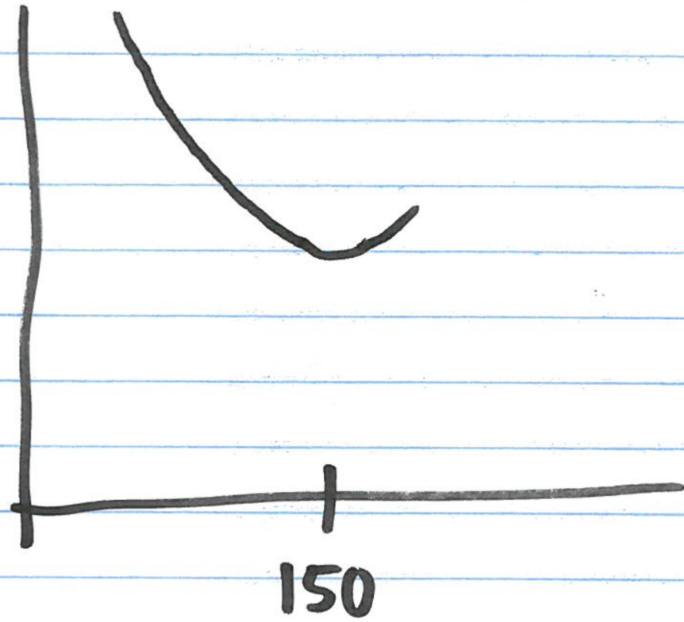
$$C''(x) = 2 \cdot \frac{90000}{x^3}$$

$$C''(150) = 2 \cdot \frac{90000}{150^3} > 0$$

150 is a local min.

$$C''(x) = 0 = \frac{2 \cdot 90000}{x^3}$$

is never 0. no inflection points



150 is the global min.

$x = 150$

$$r = \frac{1200}{x} = \frac{1200}{150} = 8$$

The manager should make 8 orders.

Goal: Apply product rule
quotient rule

Product rule

$$\boxed{\frac{d}{dx}\left[f(x)g(x)\right] = f(x)g'(x) + g(x)f'(x)}$$

Differentiate $y = (2x^3-5x)(3x+1)$

Let $f(x) = (2x^3-5x$

$g(x) = 3x+1$

$$\frac{d}{dx}\left[(2x^3-5x)(3x+1)\right]$$

$$= (2x^3-5x)\frac{d}{dx}(3x+1) + (3x+1)\cdot\frac{d}{dx}(2x^3-5x)$$

$$= (2x^3-5x)\cdot 3 + (3x+1)(6x^2-5)$$

$$= 6x^3-15x + 18x^3-15x+6x^2-5$$

$$= 24x^3+6x^2-30x-5$$

ex Find $\frac{dy}{dx}$ where

$$y = (x^2-1)^4(x^2+1)^5$$

Let $f(x) = (x^2-1)^4$

$$g(x) = (x^2+1)^5$$

$$\frac{d}{dx}\left[(x^2-1)^4(x^2+1)^5\right]$$

$$= (x^2-1)^4 \cdot \frac{d}{dx}(x^2+1)^5 + (x^2+1)^5 \cdot \frac{d}{dx}(x^2-1)^4$$

$$= (x^2-1)^4 \cdot 5(x^2+1)^4 \cdot 2x$$

$$+ (x^2+1)^5\, 4(x^2-1)^3 \cdot 2x$$

$$= 2x\,(x^2-1)^3(x^2+1)^4\left[(x^2-1)\cdot 5 + (x^2+1)\cdot 4\right]$$

$$= 2x(x^2-1)^3(x^2+1)^4\left[5x^2-5+4x^2+4\right]$$

$$= 2x(x^2-1)^3(x^2+1)^4(9x^2-1)$$

ex: $$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

Note: We must remember the order of the terms b/c of the minus sign in the numerator

$$\frac{d}{dx}\left[\frac{N}{D}\right] = \frac{D\frac{d}{dx}N - N\frac{d}{dx}D}{D^2}$$