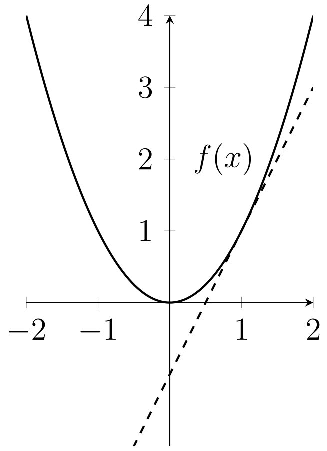
Learning objective 5 Explain what the value of the derivative at a given value of the independent variable says about the graph of the function.
Example: Consider the function f defined by $f(x) = x^2$. It is known that the derivative of f, evaluated at 1, is 2. What does the fact that $f'(1) = 2$ imply about the graph of f? (The graph of f is provided below if you want to refer to it, or use it to illustrate your answer.)



Solution: By the definition of the derivative, the value of the derivative function at a given value of the independent variable x is the slope of the tangent line at $(x, f(x))$. Here we are evaluating the derivative at $x = 1$. This gives the slope of the tangent line at $(1, f(1))$, which is $(1, 1)$. So $f'(1) = 2$ is the slope of the tangent line to the graph of f at the point $(1, 1)$.
Note: Evaluating the derivative at another x value would give the slope of the tangent line at a different point on the graph $(x, f(x))$.

Learning objective 6 Find the derivative of a power function $f(x) = x^r$ for any number r (using the power rule).
Power rule: Let r be any number, and let $f(x) = x^r$. Then $f'(x) = rx^{r-1}$.

1. $f(x) = x^5$

   Solution: Applying the power rule gives, $f'(x) = 5x^4$.

2. $g(x) = x^{\frac{1}{2}}$

   Solution: Applying the power rule gives $g'(x) = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-\frac{1}{2}}$.

3. $g(x) = x^{\frac{1}{4}}$

   Solution: Applying the power rule gives $g'(x) = \frac{1}{4}x^{\frac{1}{4}-1} = \frac{1}{4}x^{-\frac{3}{4}}$.

4. $h(x) = \sqrt[3]{x}$

   Solution: $\sqrt[3]{x}$ can be rewritten as $x^{\frac{1}{3}}$. Applying the power rule gives $h'(x) = \frac{1}{3}x^{\frac{1}{3}-1} = \frac{1}{3}x^{-\frac{2}{3}}$.

5. $f(x) = \frac{1}{\sqrt{x}}$

   Solution: $\frac{1}{\sqrt{x}}$ can be rewritten as $\frac{1}{x^{1/2}}$, which can be rewritten as $x^{-1/2}$. Applying the power rule gives $f'(x) = -\frac{1}{2}x^{-1/2-1} = -\frac{1}{2}x^{-3/2}$.