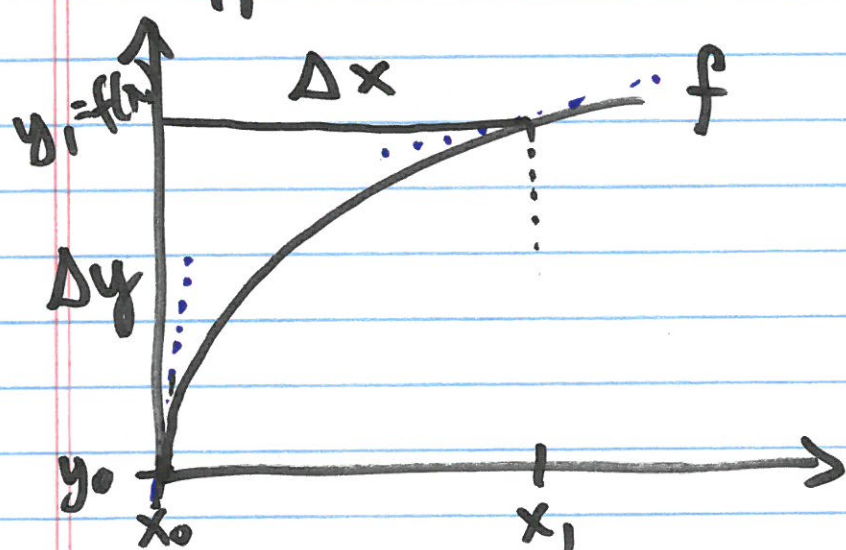
Use derivative to describe graphs

vocab: increasing, decreasing, ..
unbounded, asymptotic, ...
concave up, concave down, linear
nonlinear, continuous, differentiable

Rates of change

Suppose we have a function f



Notation:

$$y = f(x)$$
$$y_0 = f(x_0)$$
$$y_1 = f(x_1)$$
$$\vdots$$
$$\Delta x_1 = x_1 - x_0$$
$$\Delta x_2 = x_2 - x_0$$
$$\Delta y = y_1 - y_0$$
$$= f(x_1) - f(x_0)$$

The average rate of change of f between x_0 and x_1 is

$$\frac{\Delta y}{\Delta x} = \frac{y_1 - y_0}{x_1 - x_0} = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

<u>Ex1</u>: If $f(x) = x^2$, find the average rate of change of f over the interval

(a) $1 \le x \le 2$

(b) $1 \le x \le 1.1$

(c) $1 \le x \le 1.01$

(a) $x_1 = 2,\ x_0 = 1$

Math:

$$\frac{\Delta y}{\Delta x} = \frac{2^2 - 1^2}{2 - 1} = \frac{4-1}{1} = 3$$

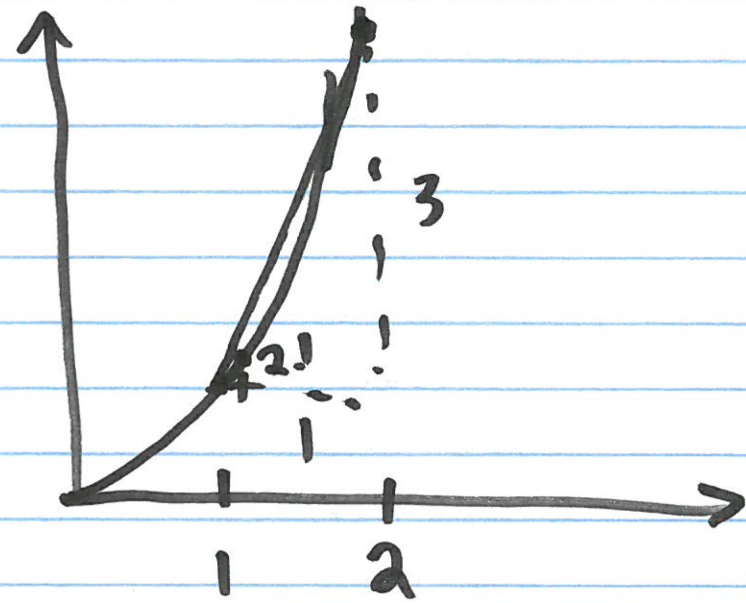
(b) $$\frac{\Delta y}{\Delta x} = \frac{(1.1)^2 - 1^2}{1.1 - 1} = \frac{1.21 - 1}{0.1} = \frac{.21}{0.1}$$

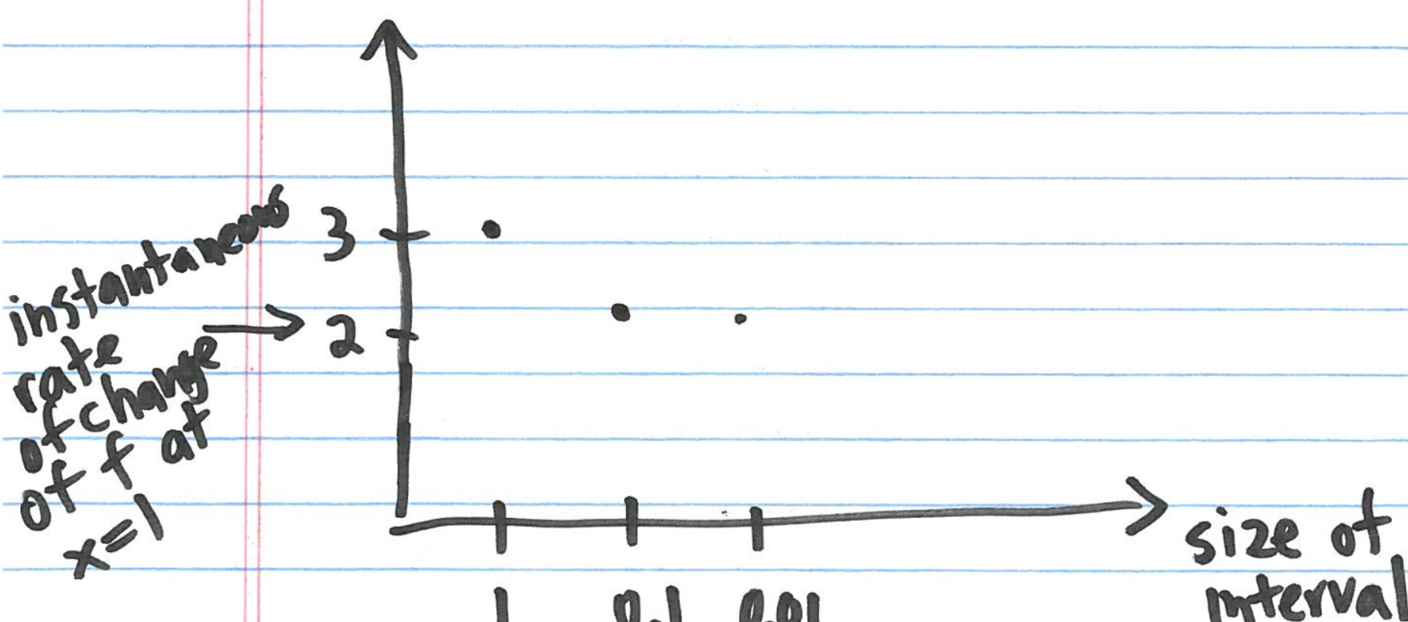
$$= 2.1$$

(c) $\dfrac{\Delta y}{\Delta x} = \dfrac{(1.01)^2 - 1^2}{1.01 - 1} = \dfrac{1.0201 - 1}{0.01}$

$= \dfrac{0.0201}{0.01}$

$= 2.01$





The Instantaneous rate of change of f at x_0 is $f'(x_0)$.
(the derivative evaluated at x_0).

Ex: Compare the average rates of change we computed to this instantaneous rate of change at $x=1$.

instantaneous rate of change at $x=1$ $= f'(1)$

$f(x)=x^2$ power function $r=2$

By the power rule

$$f'(x) = 2x^1 = 2x$$

So $f'(1) = 2\cdot 1$
$= 2$

Let $s(t)$ be the position of a body (e.g. the height of a ball above the ground).

$$\text{average velocity} = \frac{\Delta s}{\Delta t} = \frac{\text{change in position}}{\text{chang in time}}$$

$$\text{instantaneous velocity} = v(t) \equiv s'(t)$$

$\equiv$ is defined as

instantaneous velocity = instantaneous rate of change of position

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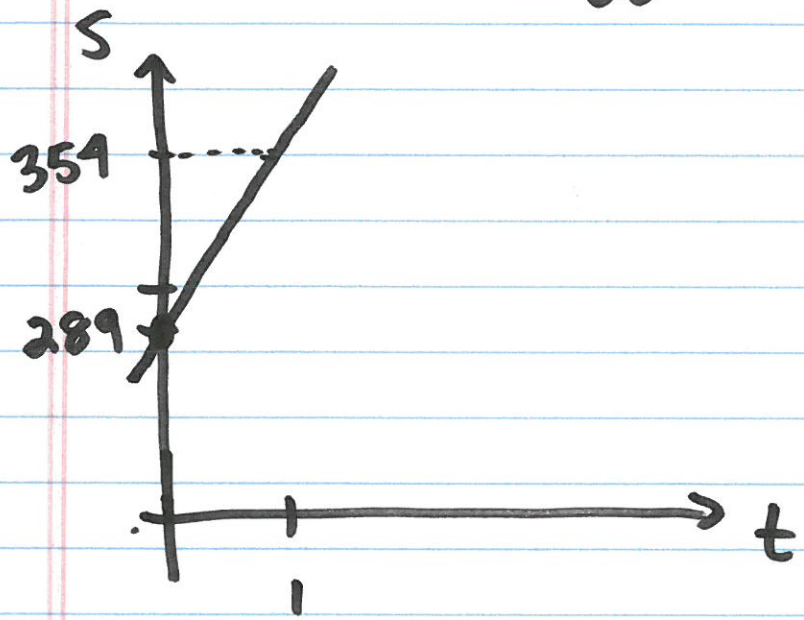
Let $s(t)$ is the position in miles ~~from~~ of a car that starts in Raleigh traveling ~~at~~ ~~65~~ mph southeast on I-40.

Then $s(t) = 65t + 289$

What is the average velocity during the first hour?

$$\frac{\Delta s}{\Delta t} = \frac{65 \text{ miles}}{1 \text{ hour}} = \frac{s(1) - s(0)}{1 - 0}$$

$$s(1) - s(0) = [65(1) + 289] - [65(0) + 289]$$
$$= 65$$



$$s'(0) = \text{slope of the tangent line at } (0, s(0))$$
$$= 65$$