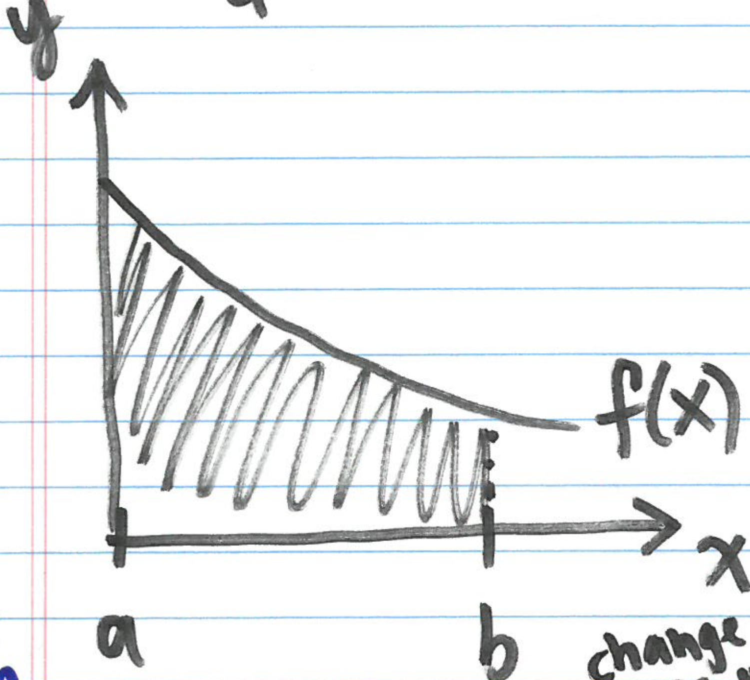
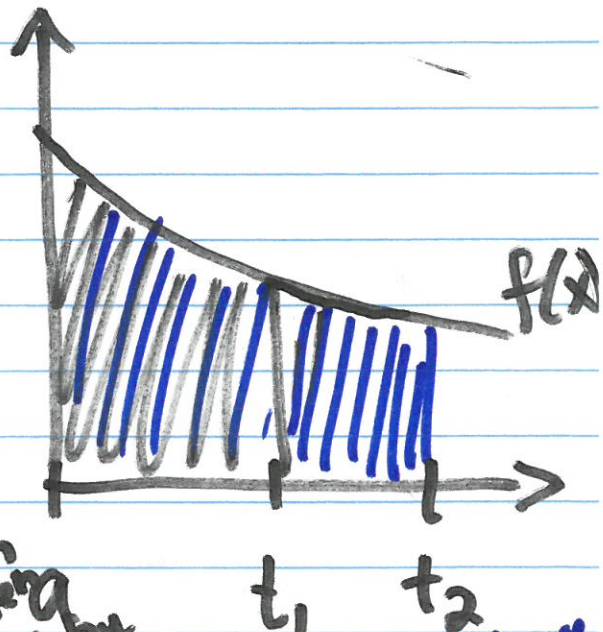
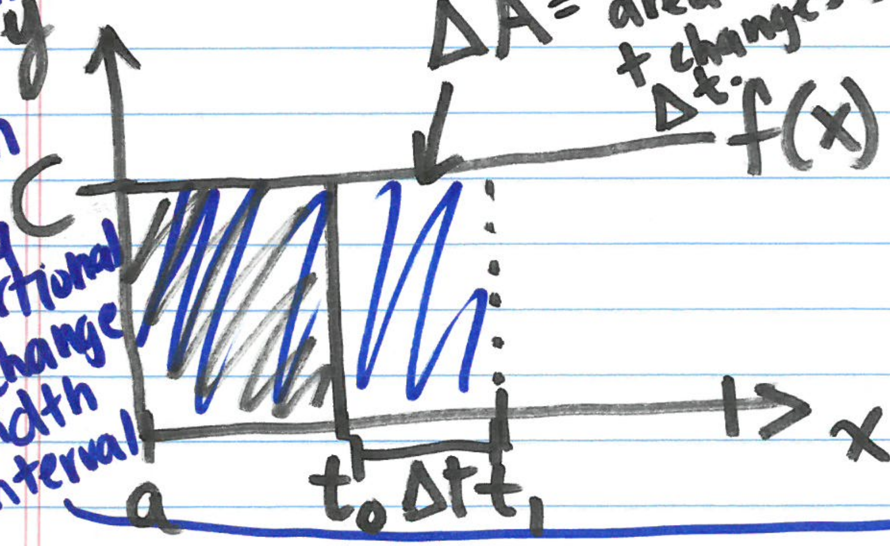
4/20

We consider an integral in which the upper limit is a variable (Use t to represent time)

$$\int_a^b f(x)\,dx \qquad \int_a^t f(x)\,dx$$







ΔA = change in area when t changes by Δt.

for a constant function the change in the area is proportional to the change in the width of the interval

rate of change of A

$$\frac{\Delta A}{\Delta t} = C = f(x)$$

$$\boxed{\Delta A = C\Delta t}$$

Fundamental theorem of calculus version 2

$$\frac{d}{dt}\int_a^t f(x)\,dx = f(t)$$

Ex: $$\int_1^t \frac{1}{x^3}\,dx = \int_1^t x^{-3}\,dx$$

$$= -\frac{1}{2}x^{-2}\Big|_1^t$$

$$= -\frac{1}{2}t^{-2} - \left(-\frac{1}{2}(1)\right)$$

$$= \frac{1}{2} - \frac{1}{2t^2}$$

Take derivative $$\frac{d}{dt}\left(\frac{1}{2} - \frac{1}{2t^2}\right) = \frac{d}{dt}\left(\frac{1}{2} - \frac{1}{2}t^{-2}\right)$$

$$= t^{-3} = \frac{1}{t^3}.$$

We get back the integrand evaluated at t.

Improper integrals

Let $f(x)$ is nonnegative for $x \geq a$ then we define

$$\int_a^{\infty} f(x)\,dx = \lim_{t\to\infty}\left(\int_a^t f(x)\,dx\right)$$

improper integral (upper limit is ∞)

Ex: $$\int_1^{\infty} \frac{1}{x^3}\,dx = \lim_{t\to\infty}\left(\int_1^t \frac{1}{x^3}\,dx\right)$$

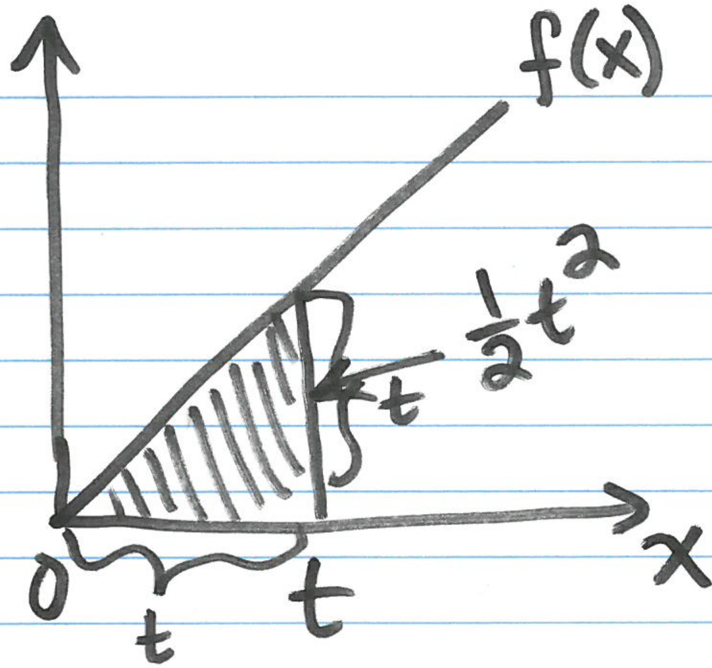
$$= \lim_{t\to\infty}\left(\frac{1}{2} - \frac{1}{2t^2}\right)$$

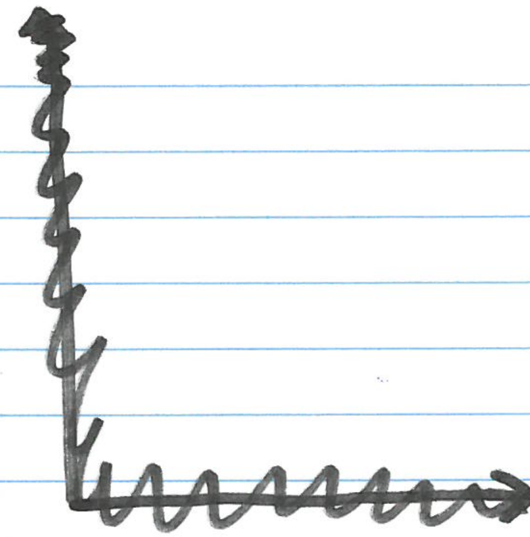
$$= \frac{1}{2} - 0 = \boxed{\frac{1}{2}}.$$

since $\lim_{t\to\infty} \frac{1}{t^2} = 0$

Ex: Find $$\int_0^t x\,dx = \frac{1}{2}x^2\Big|_0^t$$

$$= \frac{1}{2}t^2$$





$$\frac{d}{dt}\int_0^t x\,dx = \frac{d}{dt}\left(\frac{1}{2}t^2\right)$$

$$= t$$

$$\int_0^\infty x\,dx = \lim_{t\to\infty}\left(\int_0^t x\,dx\right)$$

$$= \lim_{t\to\infty} \frac{1}{2}t^2$$

$$= \infty \text{ or DNE.}$$

Find $\int_0^t -x^2+6x-5\,dt$

$= -\frac{1}{3}x^3 + 3x^2 - 5x \Big|_0^t$

$= -\frac{1}{3}t^3 + 3t^2 - 5t.$

Graph $g(t) = -\frac{1}{3}t^3 + 3t^2 - 5t$

~~critical values:~~ $g'(t) = -t^2 + 6t - 5$

$g''(t) = -2t + 6$

critical values: $g'(t) = -t^2 + 6t - 5 = 0$

$\Rightarrow t^2 - 6t + 5 = 0$

$\Rightarrow (t-5)(t-1) = 0$

$t = 1$ or $t = 5$

$g(1) = -\frac{1}{3} + 3 - 5 = -2 - \frac{1}{3}$

$= -\frac{7}{3}$

$$g(5) = -\frac{1}{3}(125) + 3 \cdot 25 - 25$$

$$= -\frac{125}{3} + 50$$

$$= \frac{150}{3} - \frac{125}{3}$$

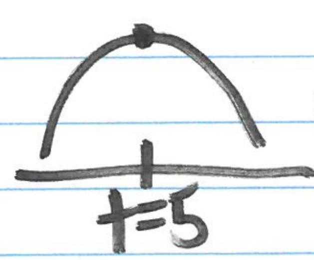
$$= \frac{25}{3}$$

Using the second derivative test, we have

$g''(1) = 4 > 0 \Rightarrow g$ concave up at $t=1$

$\Rightarrow t=1$ is a local min

$g''(5) = -4 < 0 \Rightarrow g$ concave down at $t=5$



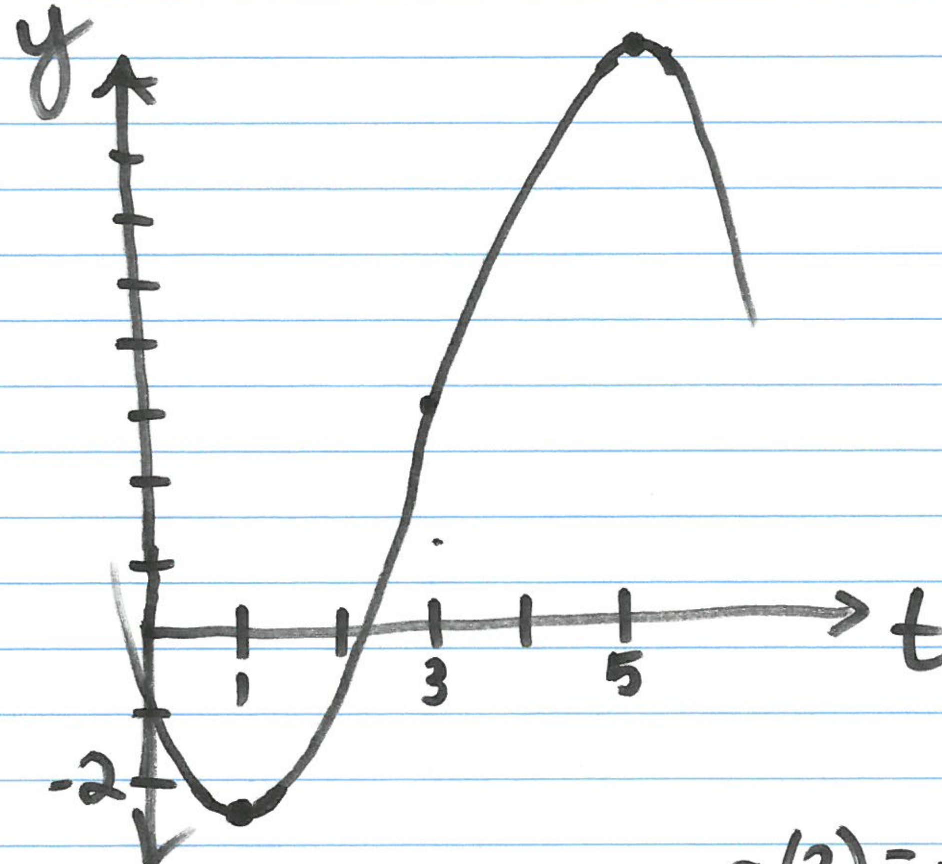
$\Rightarrow t=5$ is a local max

inflection points: $g''(x) = -2t + 6 = 0$

$\Rightarrow t = 3$

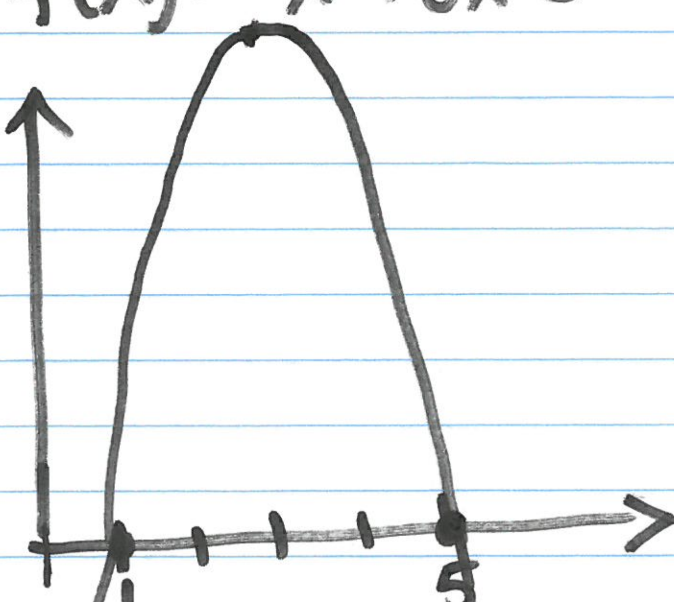
g changes concavity at $t=3$

$\Rightarrow$ $t=3$ is an inflection pt.



$$g(3) = -\frac{1}{3}3^3 + 3\cdot 3^2 - 15$$
$$= -9 + 27 - 15$$
$$= 3$$

graph: $f(x) = -x^2 + 6x - 5$



9.6 #21, 23, 25