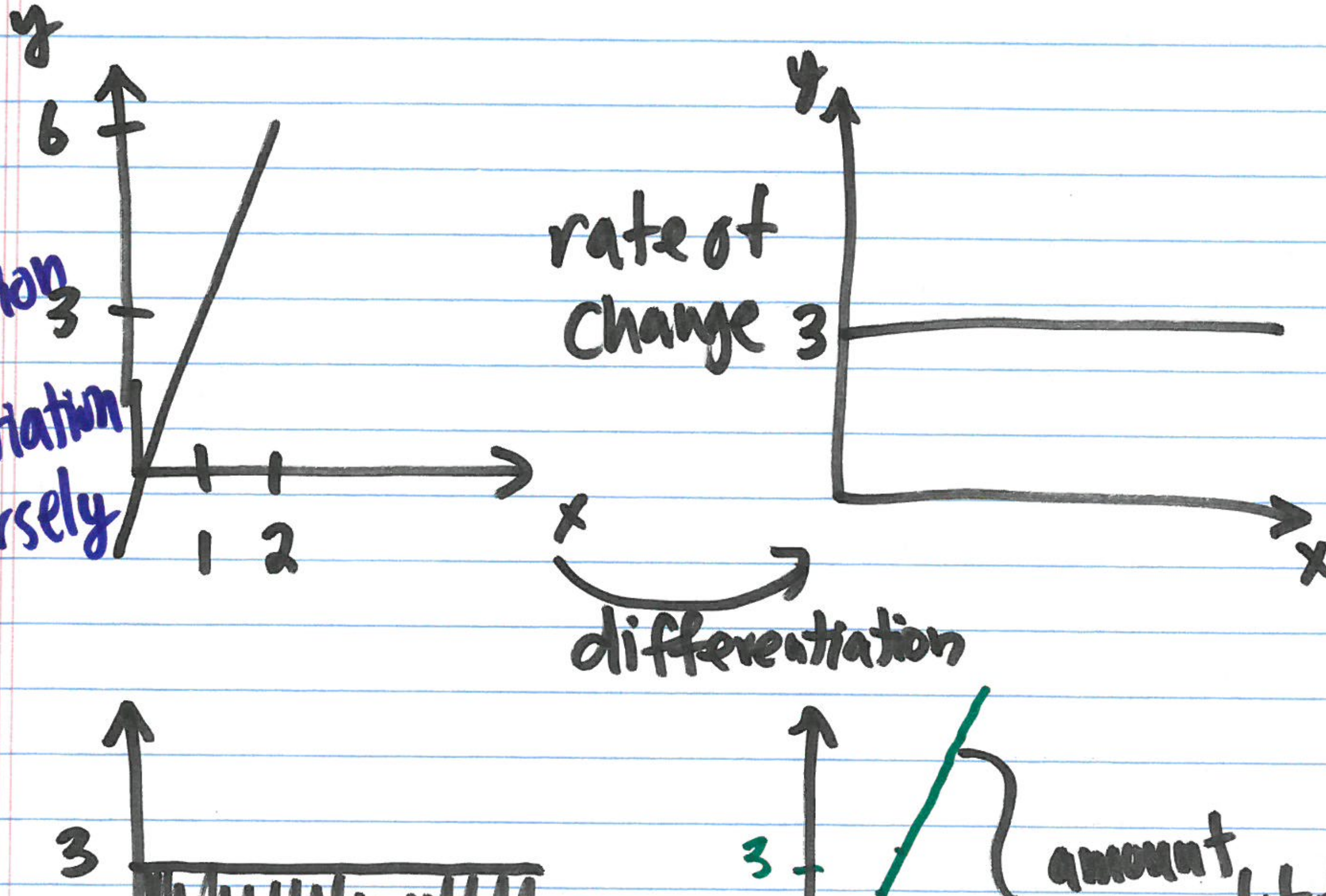
3/28 - Integration (6.1)

Minihomework: Antidifferentiation
due Friday

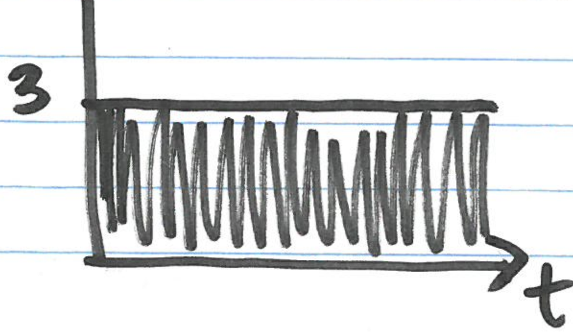
Test 4: April 15

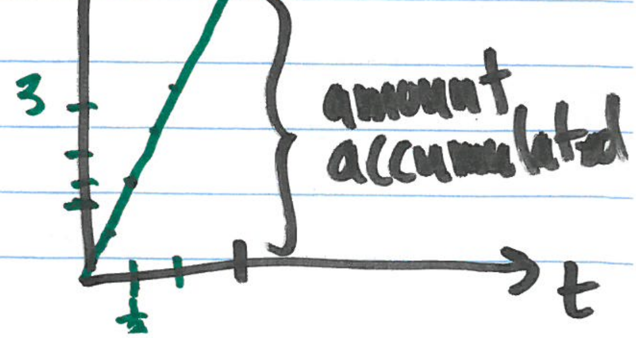
What is integration?

To find a number or a formula that represents an accumulating quantity.

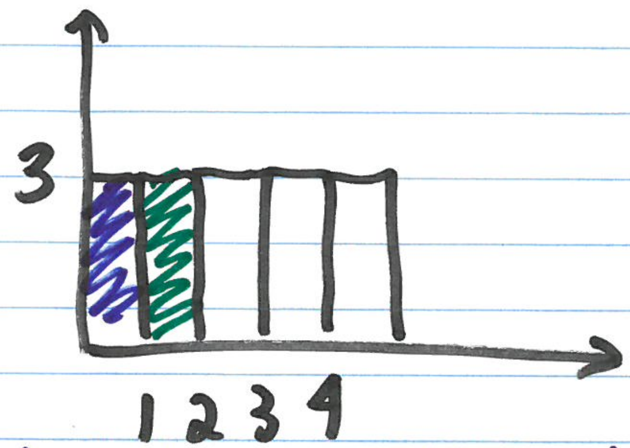


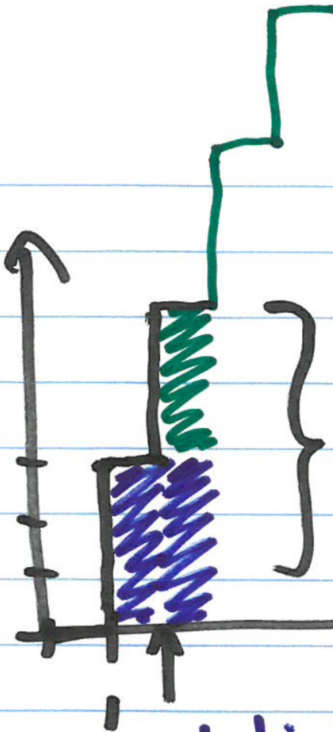
Integration and differentiation are inversely related





Summation





the value of the integral is total amount accumulated

In contrast to summation, With integration, the accumulation is continuous.

6.1 - Antidifferentiation

Ex A rocket is fired vertically into the air. Its velocity at t seconds after liftoff is $v(t) = 6t + 0.5$ m/s. Before launch the top of the rocket is 8m above the launch pad. Find the height of the rocket at time t.

Let $s(t)$ be the height of the rocket at time t.
Then $v(t) = s'(t)$. (velocity is rate of change of position).

$$s'(t) = 6t + 0.5$$

The function that we want to find $s(t)$ is called an antiderivative of $s'(t) = v(t)$

Definition: Suppose $f(x)$ is a given function. If $F(x)$ is a function having $f(x)$ as its derivative, that is

$$F'(x) = f(x)$$

then we call $F(x)$ an antiderivative of $f(x)$.

Ex: Find an antiderivative of $f(x) = x^2$.

Educated guess: $F_1(x) = x^3$

$F_1'(x) = 3x^2$. (Not quite an antiderivative of x^2)

$F_2(x) = \frac{1}{3}x^3$.

Check whether F_2 is an antiderivative of f:

$$F_2'(x) = \frac{1}{3}\frac{d}{dx}x^3$$
$$= \frac{1}{3}\cdot 3x^2$$
$$= x^2. \checkmark$$

So $F_2(x) = \frac{1}{3}x^3$ is an antiderivative of $f(x) = x^2$.

$$F_3(x) = \frac{1}{3}x^3 + 1$$
$$F_3'(x) = x^2$$

$F_3(x) = \frac{1}{3}x^3 + 1$ is also an antiderivative of x^2.

Find an antiderivative of $f(x) = e^{-2x}$.

$$F_1(x) = e^{-2x}.$$
$$F_1'(x) = (-2)e^{-2x}$$

$$F_2(x) = \frac{1}{(-2)} \cdot e^{-2x}$$

$$F_2'(x) = \frac{1}{(-2)}(-2)e^{-2x}$$

$$= e^{-2x}$$

$F_2(x)$ is an antiderivative of e^{-2x}.

$$F_3(x) = \frac{1}{(-2)}e^{-2x} + 5$$

$$F_3'(x) = e^{-2x}$$

$F_3(x)$ is also an antiderivative of e^{-2x}.

Theorems: (1) If $F(x)$ is an antiderivative of $f(x)$ then $F(x) + C$ is also an antiderivative of $f(x)$.

(2) If $F_1(x), F_2(x)$ are antiderivatives of $f(x)$ then $F_2(x) = F_1(x) + C$.

★ We can find all antiderivatives once we know one of them.

indefinite integral

<u>Def</u>: If $F(x)$ is an antiderivative of $f(x)$ then all antiderivative have the form $F(x) + C$ and the standard way to express this fact is

integrand

$$\int \underbrace{f(x)\,dx}_{\text{indefinite integral}} = \underbrace{F(x)}_{\text{an antiderivative}} + C$$

integral symbol

constant of integration

Since $\frac{1}{3}x^3$ is an antiderivative of x^2 (see previous example)

~~Find~~ $\int x^2 \, dx = \underbrace{\frac{1}{3}x^3 + C}_{\text{all antiderivatives of } x^2}$

Find $\int x^r \, dx$ $\quad (r \neq -1)$

$$\boxed{\int x^r \, dx = \frac{1}{r+1} x^{r+1} + C}$$

Let's check that $\frac{1}{r+1}x^{r+1} + C$ is an antiderivative of x^r

$$\frac{d}{dx}\left(\frac{1}{r+1}x^{r+1}\right) = \frac{1}{r+1}(r+1)x^r$$

$$= x^r.$$