Exponential decay and half-life

The half-life of a radioactive element is the amount of time required for half of the mass to decay.

Finding the growth constant when given the half-life is similar to finding the growth constant if you are given how long it takes a population to double. ~~It is common~~ Since the growth constant will be negative (it represents a decreasing function), it is common to write the form for exponential decay as

$$P(t) = Ce^{-\lambda t}$$

λ is then called the decay const.

As in example 5 of section 5.1, if the half-life is 5730 years then we have:

$$P(5730) = \frac{1}{2}P(0)$$

amount present after 5730 years is half initial amount

$$Ce^{-\lambda 5730} = \frac{1}{2}C$$

Properties II and III of logarithms

$$-5730\lambda = \ln\left(\tfrac{1}{2}\right) \overset{\downarrow}{=} -\ln(2)$$

$$\lambda = \frac{-\ln 2}{-5730} \approx 0.00012$$

If we know the decay constant λ, we can find the half-life:

5.1 Exercise 17

The decay constant for cesium 137 is .023 when time is measured in years. Find its half-life.

Let $t_{1/2}$ denote the half-life, and let $P(t)$ be the amount of cesium left given that we started with an initial amount C.

$$P(t) = Ce^{-0.023t}$$

$$P(t_{1/2}) = \tfrac{1}{2}P(0)$$

$$= \tfrac{1}{2}\cdot C$$

$$Ce^{-0.023 \cdot t_{1/2}} = \frac{1}{2}C$$

$$e^{-0.023 \cdot t_{1/2}} = \frac{1}{2}$$

$$-0.023 t_{1/2} = \ln\left(\frac{1}{2}\right)$$

$$t_{1/2} = \frac{\ln\left(\frac{1}{2}\right)}{-0.023}$$

The half life is ~30.1 years