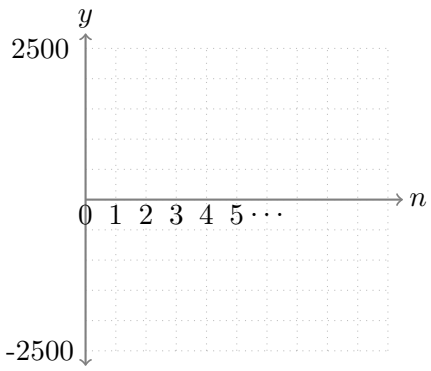
1. (9 points) A student takes out a \$12000 loan that charges an annual interest rate of 8%, compounded monthly, and makes monthly payments of \$100.
   (a) Write a difference equation that describes how to compute the balance each month based on the balance of the previous month.
   (b) How much will the student owe after 12 years?

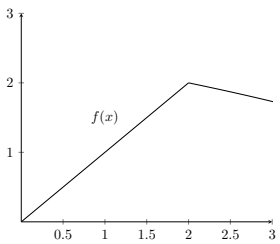
2. (5 points) On the coordinate grid to the right, sketch the graph of the difference equation $y_{n+1} = 1.05y_n - 100$, with initial value $y_0 = 500$. The points do not need to be exact, but your graph should correctly show the characteristics of the solution to the difference equation.



3. (25 points) Find the first and second derivatives of the following functions:
   (a) $f(x) = e^{-3x}$
   (b) $f(x) = \frac{1}{x}$
   (c) $f(x) = (4x+1)^{\frac{3}{2}}$
   (d) $f(x) = \pi + 2x$
   (e) $f(x) = x \ln x$

4. (20 points) Find the following integrals: (Hint: use substitution, if necessary)
   (a) $\int_0^3 e^{\frac{x}{4}} dx$
   (b) $\int (\frac{2}{x} + \sqrt{x} + \frac{1}{x^3}) dx$
   (c) $\int_1^2 2xe^{x^2} dx$
   (d) $\int x^2 \frac{1}{x^3+1} dx$

5. (5 points) For each of the following functions, find $\lim_{x \to 2} f(x)$, and say whether or not the function is continuous at $x = 2$.
   (a) $f(x) = \begin{cases} \frac{2x^2-8}{x^2-2x} & x \neq 2 \\ 3 & x = 2 \end{cases}$
   (b)



6. (10 points) During a heavy downpour, a room in a building becomes flooded with water. Suppose $f(t)$ represents the height of the water line (in inches) above the floor after t hours. Suppose $f(1) = 3$ and $f'(1) = 0.5$.

   (a) Estimate $f(1.2)$.

   (b) Suppose that $f(1) = 3$, $f'(1) = 0.5$, and $f''(1) < 0$. Then which of the following must be true? Circle all that apply.

   A. f is increasing at $t = 1$.
   B. f is decreasing at $t = 1$.
   C. f is concave down at $t = 1$.
   D. f is concave up at $t = 1$.
   E. f' is increasing $t = 1$.
   F. f' is decreasing $t = 1$.

7. (5 points) A biochemical reaction is set up to break down x grams of starch molecules into simple sugars. The rate at which sugar is produced can be described by the function

$$v(x) = \frac{0.003x}{57 + x},$$

   where x is the amount of starch, in grams. Find $v'(5)$.

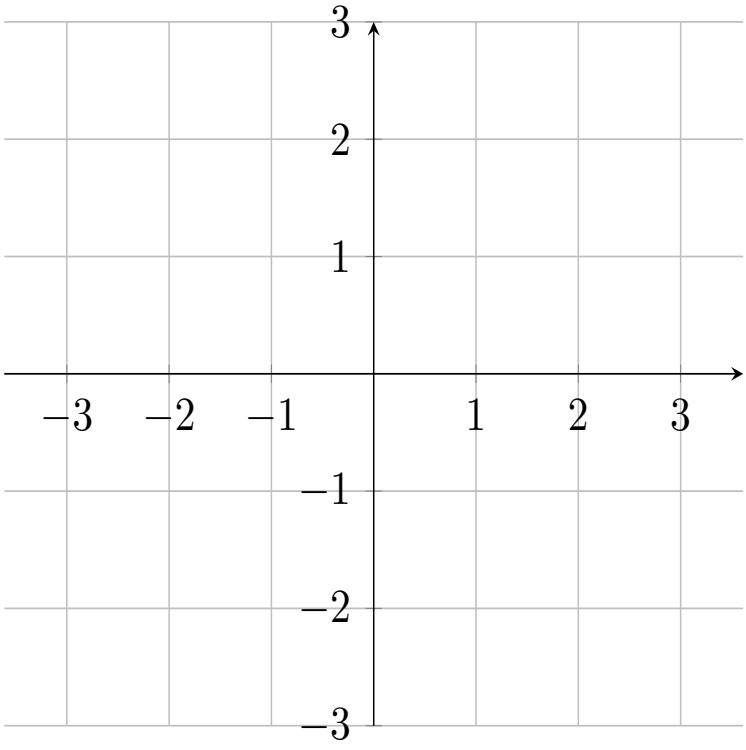
8. (10 points) The population of a bacterial culture grows exponentially.

   (a) Represent the population at time t by a function using the general form for exponential growth or decay.

   (b) If the initial population was $1,000$, and the initial rate of change was 200 per hour, what will the population be after 10 hours?

   (c) At what rate will the population be increasing at that time?

9. (10 points) Sketch the graph of $f(x) = 2x^3 + 3x^2 + 1$ in the cartesian coordinate grid below. Use information about the derivatives to find the locations of any relative minima/maxima and inflection points, and state their coordinates. You do not have to find the x-intercepts.



10. (10 points) Find the area bounded by the graph of $f(x) = 3x^3 + 3x^2 - 6x$ and the x-axis, between $x = -2$ and $x = 1$.

11. (10 points) You would like to build a wooden crate with 4 sides, a square base, and no top, with the minimum amount of wood possible. The volume of the box is to be 4 cubic feet. What dimensions will minimize the amount of wood you need? (Hint: surface area)

12. (10 points) Find the volume of the solid of revolution obtained by rotating the region under the graph of $f(x) = 2 + x$ about the x-axis from $x = 0$ to $x = 2$.

13. (10 points) Use the midpoint or trapezoidal rule with $n = 4$ to approximate the value of the following integral:

$$\int_0^2 xe^{-x}dx.$$