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Ex (rocket):

b) Note that $v(t) \le 0$ when $t \ge 5$.
(The rocket changes direction at $t=5$).

To compute distance traveled, we add how far it travels before $t=5$ and how far it travels after $t=5$.

Let D be the distance traveled

$$D = \int_0^5 v(t)\,dt + \int_5^8 |v(t)|\,dt$$

$$= \int_0^5 (-32t+160)\,dt$$

$$+ \int_5^8 (32t-160)\,dt$$

$$= (-16t^2 + 160t)\Big|_0^5 + (16t^2 - 160t)\Big|_5^8$$

$$\left[-16(5)^2+160(5)-0\right]$$
$$+\left[16(8)^2-160(8)-\left(16(5)^2-160\cdot 5\right)\right]$$

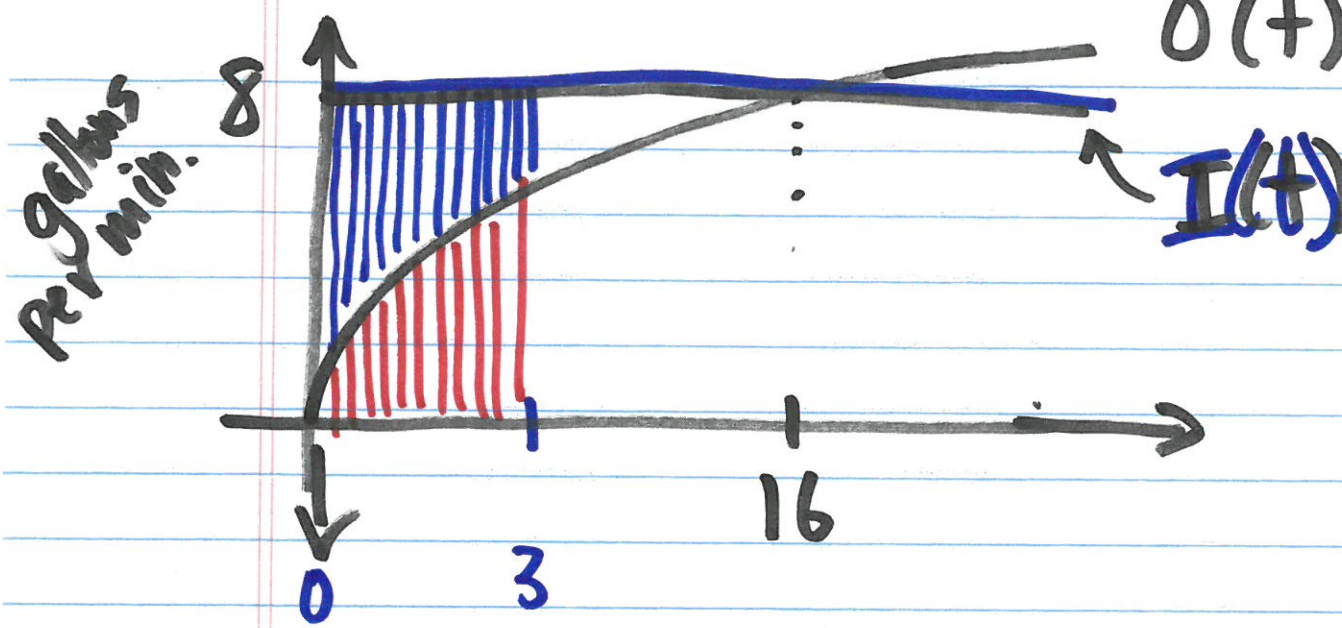
$= 544$

The distance traveled is 544 feet

This is the area bounded by the curve $v(t)$ and the x-axis between 0 and 8.

<u>Ex</u>: Water is pumped into an underground tank at a constant rate of 8 gal/min. Water leaks out the tank at a rate of $2\sqrt{t}$ gal/min. At time $t=0$, the tank contains 30 gallons of water.

How many gallons of water are in the tank after 3 minutes?



Let A(t) be the amount of water in the tank at time t.

total rate of change of A	= rate of inflow	− rate of outflow	
left of 16	total rate positive	A increasing	
right of t=16	total rate negative	A decreasing	

The integral of the rate of change is the net change.

net change between $t=0$ and $t=3$ $= A(3) - A(0)$

$$= \int_0^3 r(t)\,dt$$

$$\boxed{= \int_0^3 I(t) - O(t)\,dt}$$

$$= \int_0^3 (8 - 2\sqrt{t})\,dt$$

$$= \int_0^3 8 - 2t^{1/2}\,dt$$

$$= 8t - \frac{4}{3}t^{3/2}\Big|_0^3$$

$$= 8\cdot 3 - \frac{4}{3}3^{3/2} - 0$$

$$= 24 - \frac{4}{3}27^{1/2}$$

We want the amount after 3 minutes

$A(3)$

6.4 Exercises 39, 40

9.1 Integration by substitution

"u-substitution"

~~Ex:~~ Let $F(x)$ be an antiderivative of $f(x)$.
The rule of integration/antidifferentiation which reverses the chain rule is

$$(1) \quad \int f(g(x))g'(x)\,dx = F(g(x)) + C$$

We will use this to develop a technique for integration

Ex: $\int (x^2+1)^3 \cdot 2x\,dx$

It is possible to write the integrand as $f(g(x)) \cdot g'(x)$

$f(x) = x^3$

$g(x) = x^2+1$

We can use formula (1).

An antiderivative of $f(x) = x^3$ is

$$F(x) = \frac{1}{4}x^4$$

So by (1)

$$\int (x^2+1)^3 \cdot 2x\,dx = \frac{1}{4}(x^2+1)^4 + C$$

Check this:

$$\frac{d}{dx}\frac{1}{4}(x^2+1)^4 + C$$

$$= (x^2+1)^3 \cdot 2x \quad ✓$$

We use a mnemonic to aid this process. We

1. $\boxed{\text{determine an appropropriate } g(x)}$
2. replace $g(x)$ by u (variable)
3. replace $g'(x)dx$ by du

$$\int f(g(x))\,g'(x)\,dx = \int f(u)\,du$$

no x variable!

$$\int f(u)\,du = F(u) + C = F(g(x)) + C$$

substitute back $g(x)$ for u

Second solution to finding

$$\int (x^2+1)^3 \cdot 2x\,dx$$

① $g(x) = (x^2+1) \xrightarrow{\text{replace by}} u$

② $g'(x)\,dx = 2x\,dx \longrightarrow du$

$$\int (x^2+1)^3 \cdot 2x\,dx = \int u^3\,du$$

$$= \frac{1}{4}u^4 + C$$

$$= \frac{1}{4}(x^2+1)^4 + C$$

Exercises $^{9.1}$ #1