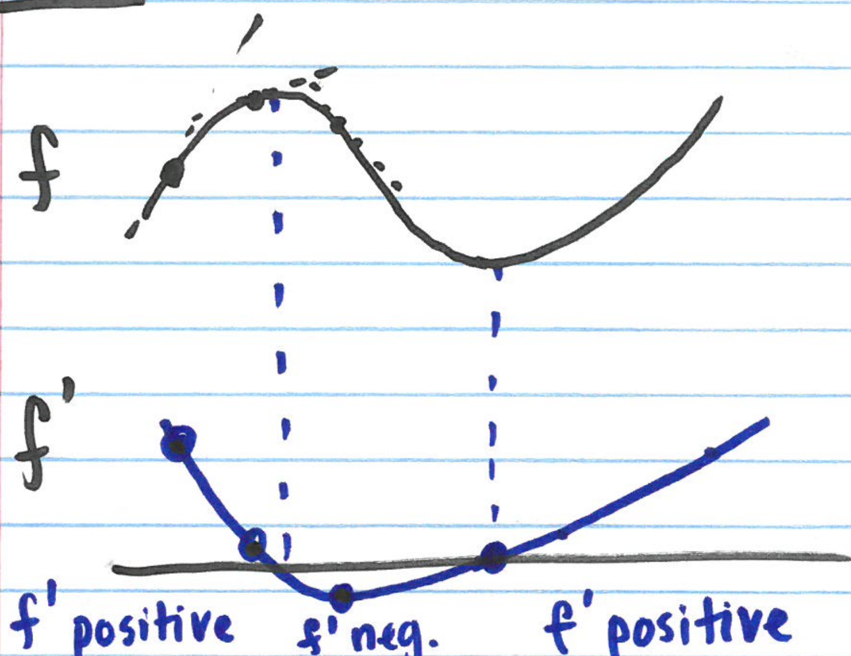
2/5



If the graph (function) is increasing then the derivative is positive.

— " — decreasing — " —

— " — negative

The general power rule

r is any number

$$\frac{d}{dx}[g(x)]^r = r\,[g(x)]^{r-1} \cdot \frac{d}{dx}[g(x)]$$

~~Simplistic example~~

~~the~~ we need to multiply by this correction factor because of the way that rates of change are composed.

Case: $g(x) = x$

$$\frac{d}{dx} x^r = r x^{r-1} \cdot \frac{d}{dx} x$$

$$= r x^{r-1} \cdot 1$$

$$= r x^{r-1}$$

When $g(x) = x$ the general power rule is the same as the power rule.

Ex: Differentiate $y = \dfrac{1}{x^3 + 4x}$

$$y = (x^3 + 4x)^{-1}$$

must use general power rule because the function inside paretheses is not just x.

$r = -1$

$r - 1 = -2$

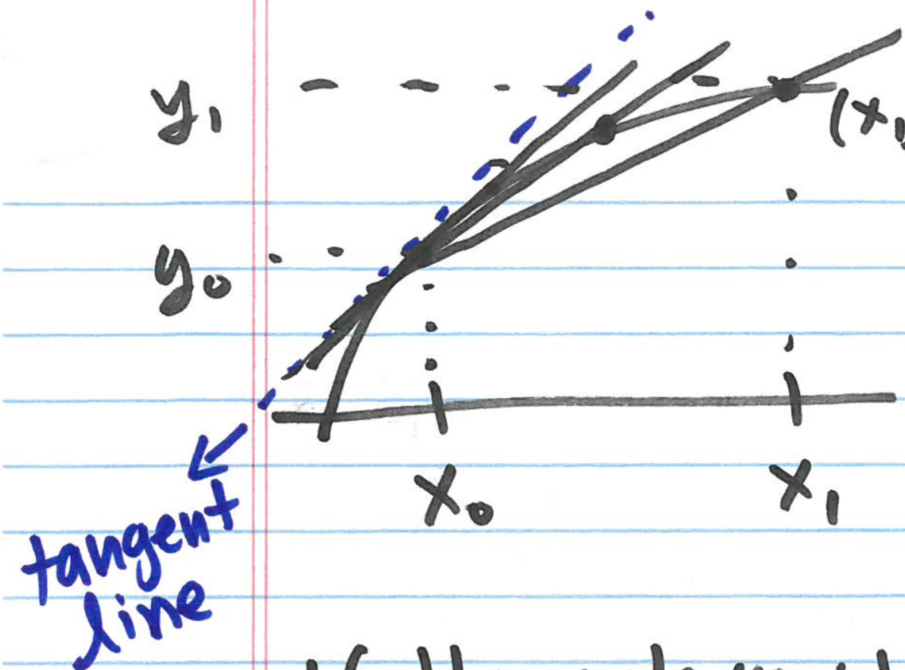
By the general power rule

$$\frac{d}{dx} \frac{1}{x^3 + 4x} = \frac{d}{dx} (x^3 + 4x)^{-1}$$

$$= -1 (x^3 + 4x)^{-2} \cdot \frac{d}{dx}(x^3 + 4x)$$

$$= -1 (x^3 + 4x)^{-2} (3x^2 + 4)$$

$$= -\frac{3x^2 + 4}{(x^3 + 4x)^2}$$



Idea:
If the point (x_1, y_1) approaches and ultimately reaches (x_0, y_0) then the angle that the secant line makes with the tangent line at (x_0, y_0) will vanish.

If the angle vanishes then the slopes will ultimately be equal.

Note: We cannot set x_1 to x_0 because our ratio $\frac{\Delta y}{\Delta x}$ would be

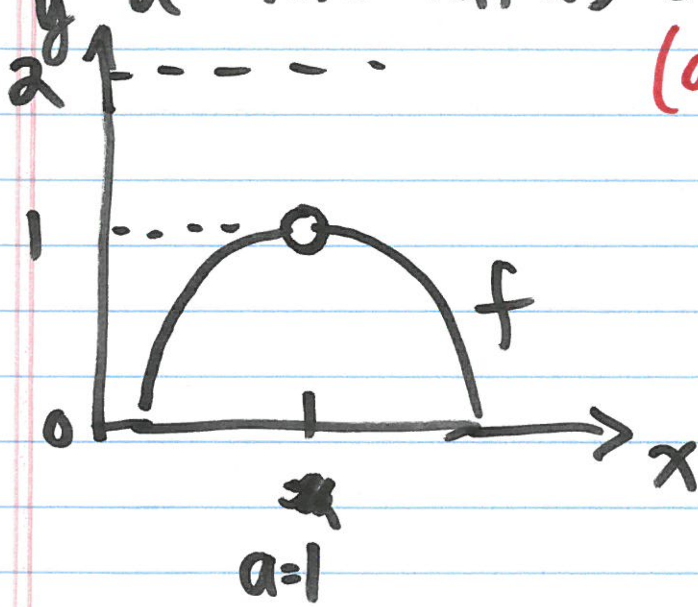
$$\frac{y_1 - y_0}{x_1 - x_0} = \frac{y_0 - y_0}{x_0 - x_0} = \frac{0}{0}$$

The concept that a quantity will ~~ultima~~ ultimately have some value as x approuch a number is the concept of a limit.

Limit

We say $L = \lim_{x \to a} f(x)$.

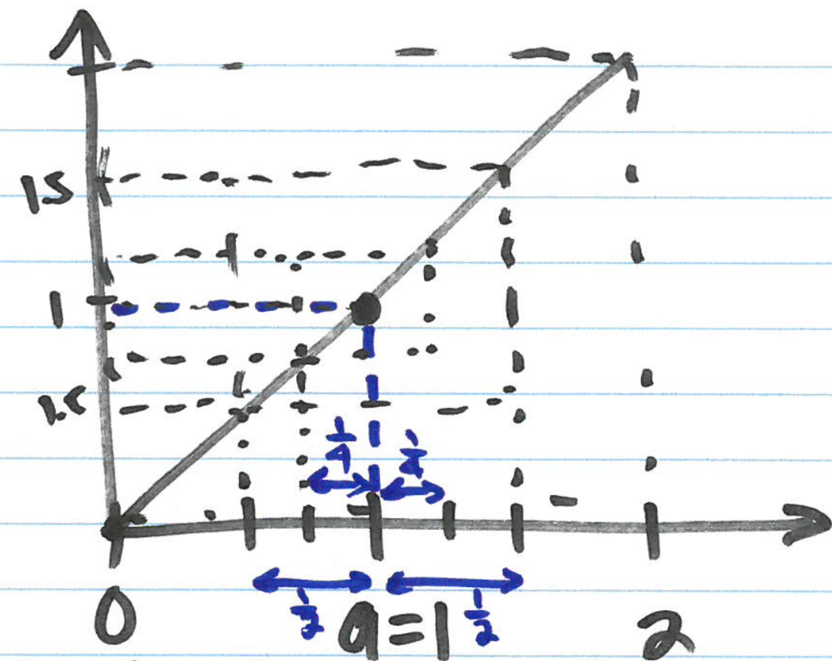
This means: as the values of x go to a the values of y (go to) L.
(approach and ultimately reach)



$L = 1$

"ultimately reach": does not mean that the y-value is ever equal to L

Let's clarify what this ultimately reach language mean?



Look at the y values on smaller & smaller intervals

Size of the interval around a: 1 $\quad \frac{1}{2} \quad \frac{1}{4}$ …

y-values in that interval 1 …

y values all eventually get close to 1.

~~The limit L is the a number~~

The number L is the limit of f(x) as x approaches a provided f(x) can be made arbitrarily close to L for all x sufficiently close to a