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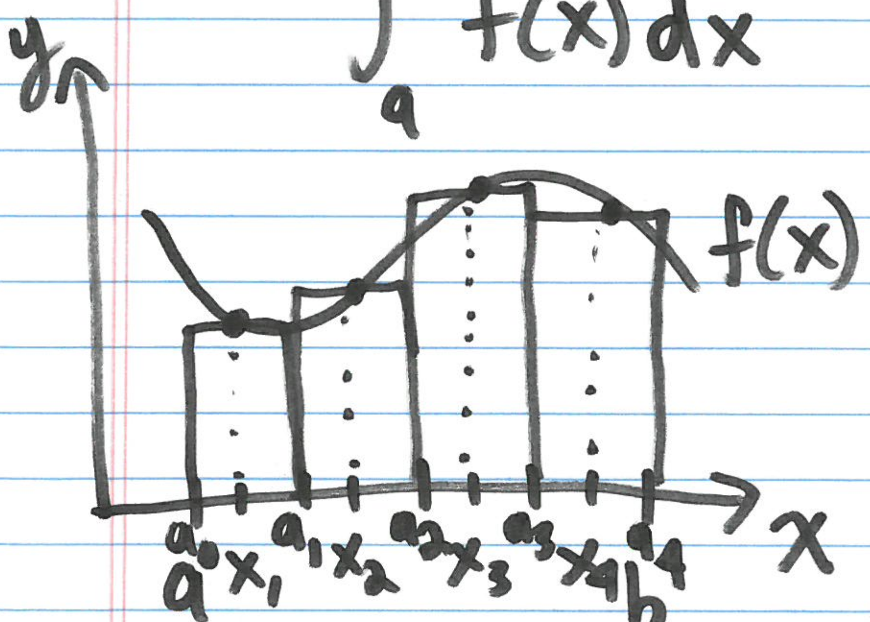
WebAssign — Friday / Monday

9.4 Approximation of Definite Integrals

Suppose we want to find

$$\int_a^b f(x)\,dx$$



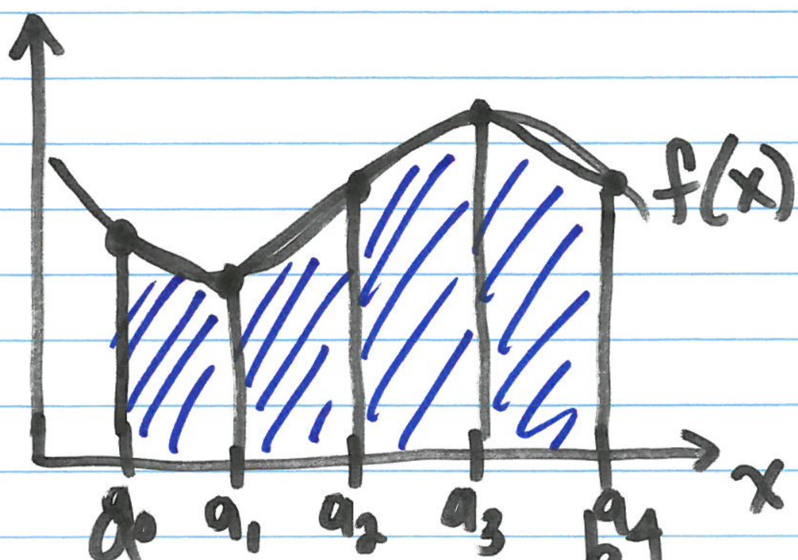
$n=4$

Let $x_1, x_2, \dots x_n$ be midpoints of each subinterval, then the Riemann sum approximation

$$\boxed{\int_a^b f(x)\,dx \approx f(x_1)\Delta x + f(x_2)\Delta x + \dots + f(x_n)\Delta x} = M$$

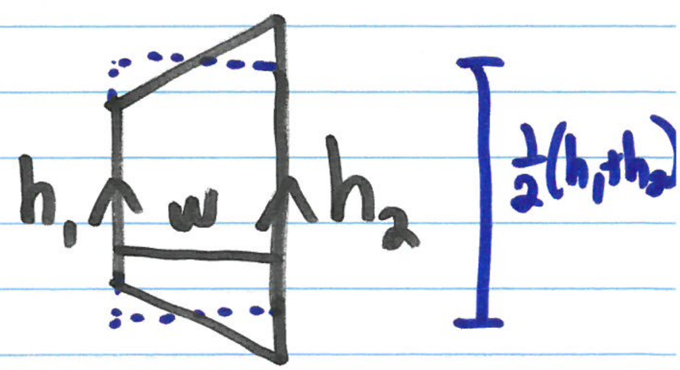
is called the midpoint rule.

Trapezoid rule



$n=4$

trapezoid:



area of trapezoid:

$$A = \frac{h_1+h_2}{2}\cdot w$$

Recall: $\Delta x = \dfrac{b-a}{n}$

$$\int_a^b f(x)\,dx \cong \frac{f(a_0)+f(a_1)}{2}\cdot\Delta x$$

area of 1st trapezoid

$$+ \frac{f(a_1)+f(a_2)}{2}\Delta x$$

area of 2nd trapezoid

$$+ \cdots$$

$$+ \frac{f(a_{n-1})+f(a_n)}{2}\Delta x$$

area of nth trapezoid

$$= \left[f(a_0)+2f(a_1)+\ldots 2f(a_{n-1}) + f(a_n)\right]\frac{\Delta x}{2}$$

$$= T$$

This approximation is called the trapezoid rule.

Simpson's rule:

$$\int_a^b f(x)\,dx \approx \frac{2}{3}M + \frac{1}{3}T$$

approximation using midpoint rule

approx. using trapezoid rule

Use ~~the~~ Simpson's rule with $n=2$ to approximate the

$$\int_{-2}^{2} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}\,dx$$

midpoint rule approximation:

$n=2$

$a=-2$

$b=2$

$$\Delta x = \frac{b-a}{n} = \frac{4}{2} = 2$$

Left endpoints	Right endpoints	Midpoints
$a_0 = -2$	$a_1 = 0$	-1
$a_1 = 0$	$a_2 = 2$	1

$$M = f(-1)\Delta x + f(1)\Delta x$$

$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}} \cdot 2 + \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}} \cdot 2$$

$$= \frac{4}{\sqrt{2\pi}} e^{-\frac{1}{2}}$$

$$T = \left[f(-2) + 2f(0) + f(2)\right] \cdot \frac{\Delta x}{2}$$

$$= \left[\frac{1}{\sqrt{2\pi}} e^{-2} + \frac{2}{\sqrt{2\pi}} \cdot 1 + \frac{1}{\sqrt{2\pi}} e^{-2}\right] \cdot 1$$

$$= \frac{2}{\sqrt{2\pi}} e^{-2} + \frac{2}{\sqrt{2\pi}}$$

$$S = \frac{2}{3} M + \frac{1}{3} T$$

$$= \frac{8}{3\sqrt{2\pi}} e^{-\frac{1}{2}} + \frac{2}{3\sqrt{2\pi}} e^{-2} + \frac{2}{3\sqrt{2\pi}}$$

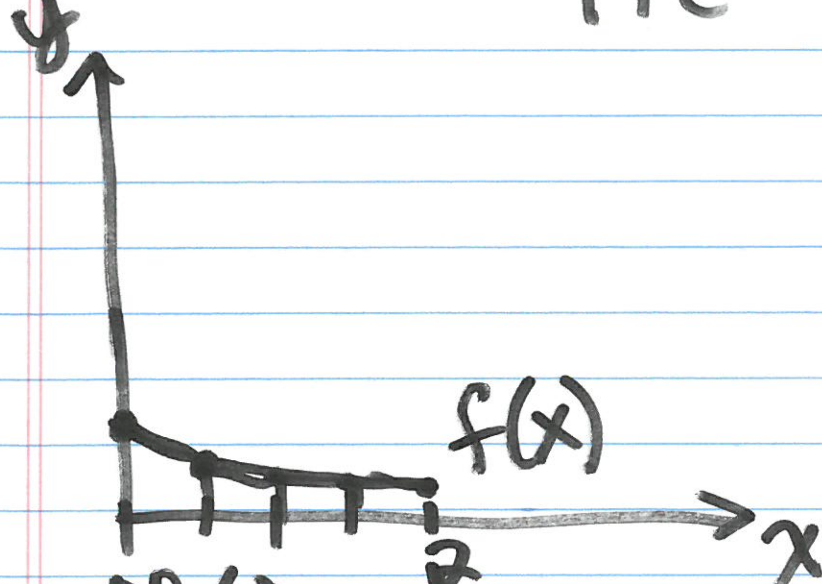
≈ 0.9472

Ex: Use the trapezoid rule with $n=4$ to approximate

$$\int_0^2 \frac{1}{1+e^x}\,dx$$

$$\Delta x = \frac{2}{4} = \frac{1}{2}$$

Let's draw $\frac{1}{1+e^x}$. $= f(x) = (1+e^x)^{-1}$



$f(0) = \frac{1}{2}$

$$f'(x) = -1(1+e^x)^{-2} \cdot e^x = \frac{-1 \cdot e^x}{(1+e^x)^2}$$

left endpoints

$a_0 = 0$

$a_1 = \frac{1}{2}$

$a_2 = 1$

$a_3 = \frac{3}{2}$

right endpt

$a_1 = \frac{1}{2}$

$a_2 = 1$

$a_3 = \frac{3}{2}$

$a_4 = 2$

$$T = \left[f(a_0) + 2f(a_1) + 2f(a_2) + 2f(a_3) + f(a_4)\right] \cdot \frac{\Delta x}{2}$$

$$= \left[\frac{1}{1+e^0} + 2\frac{1}{1+e^{\frac{1}{2}}} + 2\frac{1}{1+e^1} + 2\cdot\frac{1}{1+e^{3/2}} + \frac{1}{1+e^2}\right] \cdot \frac{1}{4}$$

≈ 0.569

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