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WebAssign:

| | |
|---|---|
| optimization | ? |
| prod., quotient,... | 3/4 |
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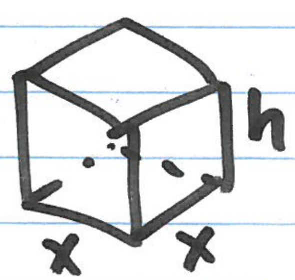
Manufacturer designs open top box w/ square base, surface area 144 $in^2$

What dimensions will maximize volume.



$x$ is width of base

$h$ is the height

objective function

$$V = \underline{x^2 \cdot h}$$

constraint

$$SA = \underline{x^2 + 4xh} = 144$$

$$h = \frac{144 - x^2}{4x}$$

$$= \frac{36}{x} - \frac{x}{4}$$

$$V = x^2\left(\frac{36}{x} - \frac{x}{4}\right)$$

$$= 36x - \frac{x^3}{4}$$

$$V' = 36 - \frac{1}{4}\frac{d}{dx}x^3$$

$$= 36 - \frac{3}{4}x^2 = 0$$

$$x^2 = \frac{4}{3} \cdot 36$$

$$x = \sqrt{48}$$

This is a critical value.
Is it a global min?
Check concavity, etc.

Suppose the width of a rectangle is 10 cm and is increasing at a rate of 5cm/s. Suppose the height is 3cm and increases at 2cm/s. What is the change of the area?

<u>Aside</u>: (Notation):

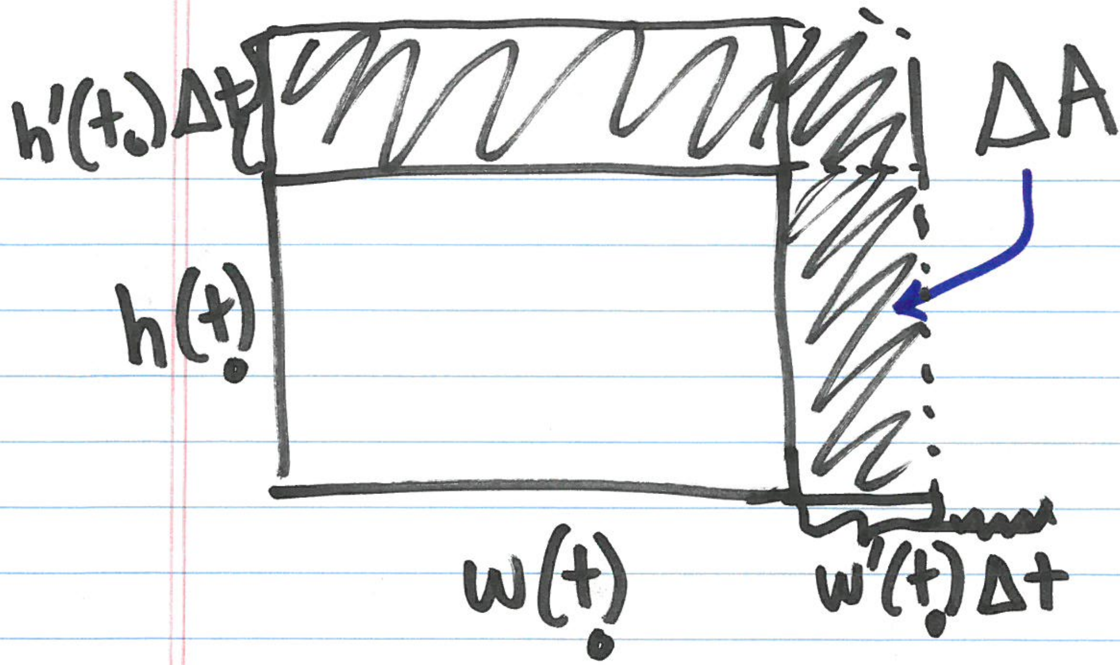
$$\frac{d}{dx} f(x)\Big|_{x=x_0} = f'(x_0)$$

$$\cancel{\frac{d}{dx} f(x_0) = 0}$$

This means evaluated at $x = x_0$.

$$f(x) = x^2$$

$$f'(1) = \frac{d}{dx}(x^2)\Big|_{x=1}$$




Can use geometry to find $\frac{\Delta A}{\Delta t}$ (I did not show this method)

This requires using the definition of the derivative $\lim\limits_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$.

$$A(t) = h(t) \cdot w(t)$$

rate of change of A

$$A'(t) = \frac{d}{dt}\big(h(t)\, w(t)\big)$$

$$= h(t)\,w'(t) + w(t)\,h'(t)$$

Let $t_0$ be the time when the width is 10 and height is 3, etc.

$$A'(t_0) = h(t_0)\underbrace{w'(t_0)}_{= \frac{d}{dt} w(t)\big|_{t=t_0}} + w(t_0)\,h'(t_0)$$

$$= 3_{cm} \cdot 5\,{}^{cm}/_{s} + 10_{cm} \cdot 2\,{}^{cm}/_{s}$$

$$= (15 + 20)\ {}^{cm^2}/_{s}$$

$$= 35\ {}^{cm^2}/_{s}$$

# Allometry

The weight in grams of a certain species of pike can be estimated from its length $x$ (in cm) using the equation $w(x) = 0.008x^3$.

If the length of a fish is increasing at a rate of 10 cm per year, how fast is the weight increasing? ~~when~~ if its length is 50 cm

Clarification: We need to think of weight as a function of time, so we need to think of length as a function of time.

Unknown: rate of change of weight with respect to time.

So we need to think of length as a function of time:

$$x(t)$$

Then the weight as function of time is the composition

$$w(x(t))$$

$$\frac{d}{dt}\left[w(x(t))\right] = w'(x(t)) \cdot x'(t)$$

Again we only know $x$ at one point in time $t_0$

$$\frac{d}{dt}\left[w(x(t))\right]\Big|_{t=t_0} = w'(x(t))\cdot x'(t)\Big|_{t=t_0}$$

$$\begin{aligned}
(w(x(t)))' &= w'(x(t_0))\cdot x'(t_0) \\
&= w'(50)\cdot \boxed{\overset{\shortparallel}{10\ \text{cm}/\text{yr}}}
\end{aligned}$$

$$\boxed{\begin{aligned} w'(x) &= 3\cdot 0.008x^2 \\ &= 0.024x^2 \end{aligned}}$$

$$\begin{aligned}
&= 0.024(50)^2\cdot 10 \\
&= 0.24\cdot 2500 \\
&= 600\ \text{grams}/\text{cm}\cdot \text{cm}/\text{year} \\
&= 600\ \text{grams}/\text{year}
\end{aligned}$$

units

$$w'(x) = \text{gram}/\text{cm}$$

# Exponential functions

$$f(x) = b^x$$

## Laws of exponents

$$b^x \cdot b^y = b^{x+y}$$

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$$\frac{b^x}{b^y} = b^x b^{-y} = b^{x-y}$$

$$a^x b^x = (ab)^x$$

$$b^{-x} = \frac{1}{b^x}$$

$$(b^y)^x = b^{xy}$$

$$\left(\frac{a^x}{b^x}\right) = \left(\frac{a}{b}\right)^x$$