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calculators okay

Suppose $F(x)$ is an antiderivative of $f(x)$.

$[F'(x) = f(x)]$ f is the rate of change of F

FTC: $$\int_a^b f(x)\,dx = \underbrace{F(b)-F(a)}_{\text{net change in } F}$$

$$\approx f(x_1)\Delta x + f(x_2)\Delta x + \dots + f(x_n)\Delta x$$

The approximate rate of change of the number of people in an ER (in people per hour) is recorded during several intervals. Use this information to approximate the net change in the number of people in the ER.

interval hours	0-2	2-4	4-6	6-8
rate of change	5	7	-1	-3

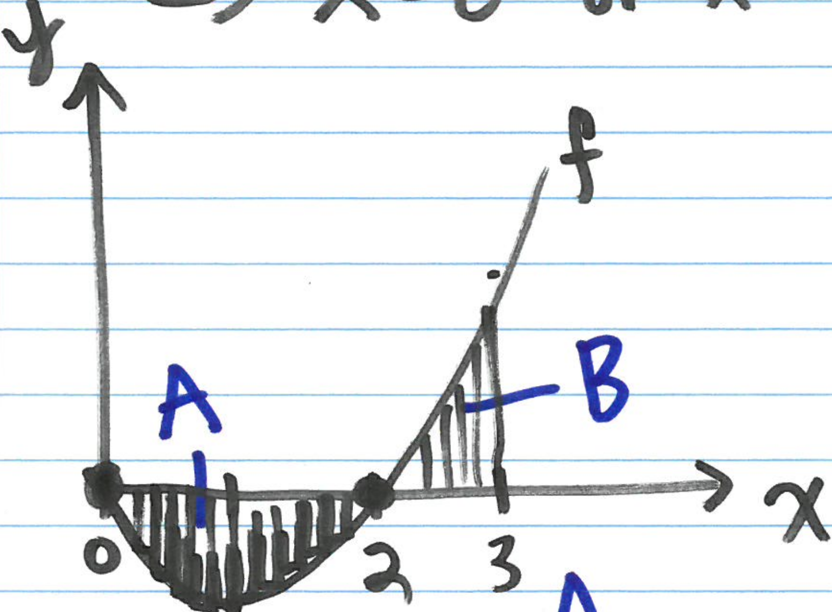
$$5 \text{ people/hour} \cdot 2 \text{ hours} + 7 \cdot 2 + -2 + -3 \cdot 2$$

$$10 + 14 - 2 - 6 = 16$$

The net change is 16 people.

Find the area bounded by $f(x) = x^2 - 2x$ between $x=0$ and $x=3$.

$$f(x) = x^2 - 2x = 0$$
$$= x(x-2) = 0$$
$$\Rightarrow x=0 \text{ or } x=2$$



$$\text{area} = \overbrace{\int_0^2 -f(x)\,dx}^{A} + \overbrace{\int_2^3 f(x)\,dx}^{B}$$
$$= \int_0^2 -x^2 + 2x\,dx + \int_2^3 x^2 - 2x\,dx$$

$$= -\frac{1}{3}x^3 + x^2 \Big|_0^2 + \frac{1}{3}x^3 - x^2 \Big|_2^3$$

$$= \left[-\frac{8}{3} + 4\right] - 0 + \cancel{\left[\frac{27}{3} - 9\right]} - \left[\frac{8}{3} - 4\right.$$

$$= -\frac{8}{3} + 4 - \frac{8}{3} + 4$$

$$= 8 - \frac{16}{3}$$

$$= \boxed{\frac{8}{3}}$$

A grocery store receives roses at a rate of $R(t) = 9t$ (in bouquets per week) and sells them at a rate of $S(t) = 3t^2 - 3t + 9$ where t is the time in weeks after the beginning of May. Assume that these rates account for all change in roses.

a) What quantity does the definite integral

$$\int_0^4 [R(t) - S(t)]\,dt \quad \text{represent?}$$

Explain.

$R(t) - S(t)$ is the total rate of change in the number of roses, so

$$\int_0^4 R(t) - S(t)\,dt \text{ is the}$$

net change in the number of roses at the store between $t = 0$ and $t = 4$.

b) If the store starts with 5 bouquets, how many will there be after 4 weeks?

Net change

$$= \int_0^4 9t - (3t^2 - 3t + 9)\, dt$$

$$= \int_0^4 -3t^2 + 12t - 9 \, dt$$

$$= -t^3 + 6t^2 - 9t \Big|_0^4$$

$$= -64 + 96 - 36$$

$$= -4$$

There will be $5 - 4 = 1$ bouquets after 4 weeks.

Rules of integration

$$\int e^{kx}\,dx = \frac{1}{k}e^{kx} + C$$

$$\int x^r\,dx = \frac{1}{r+1}x^{r+1} + C \quad (r \neq -1)$$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int c f(x) + g(x)\,dx$$

$$= c\int f(x)\,dx + \int g(x)\,dx$$

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx$$

$$+ \int_c^b f(x)\,dx$$

Find

$$\int \frac{e^{2t}}{1+e^{2t}}\,dt$$

Let $u = 1+e^{2t}$

$du = 2\underbrace{e^{2t}\,dt}$ $\quad e^{2t}dt = \frac{1}{2}du$

$$\int \frac{1}{u}\,\frac{1}{2}\,du$$

$$\frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C$$

$$= \boxed{\frac{1}{2}\ln|1+e^{2t}| + C}$$

What do we replace $e^{2t}dt$ by?

Find $\int_2^6 \frac{1}{\sqrt{4x+1}}\,dx$

Let $u = 4x+1$

$du = 4\,dx$

By substitution

$$\int_2^6 \frac{1}{\sqrt{4x+1}}\,dx = \frac{1}{4}\int_2^6 \frac{1}{\sqrt{4x+1}} \cdot 4\,dx$$

$$= \frac{1}{4}\int_9^{25} \frac{1}{\sqrt{u}}\,du$$

Note: When changing the variable in the integral, using substitution, we also substitute the limits for the u variable that correspond to $x=2$ and $x=6$:

$$9 = 4(2)+1$$

$$25 = 4(6)+1$$

$$\frac{1}{4}\int_9^{25} \frac{1}{\sqrt{u}}\,du$$

$$= \frac{1}{4}\int_9^{25} u^{-1/2}\,du$$

$$= \frac{1}{4}\,2u^{1/2}\Big|_9^{25}$$

$$= \frac{1}{2}5 - \frac{1}{2}3$$

$$= \boxed{1.}$$