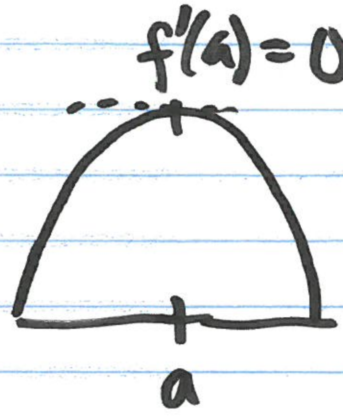
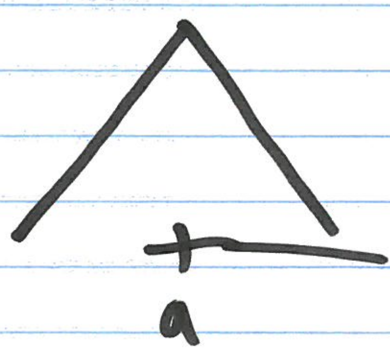
2/24

Test 3 2.5, 3.1-3.2, 4, 5.1



$f'(a)=0$

both have local maxima at a



f is not differentiable at $x=a$

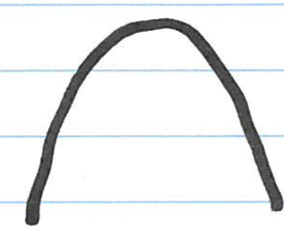
$f'(a)$ does not exist

To account for this, we consider a point where $f'(a)$ does not exist to be a critical value

<u>Definition:</u> $x=a$ is a critical value if $f'(a)=0$ or $f'(a)$ does not exist.

A global maximum (minimum) of f is a point where f has the largest (smallest) value on the entire domain.

<u>Note</u>: A local max can be a global max



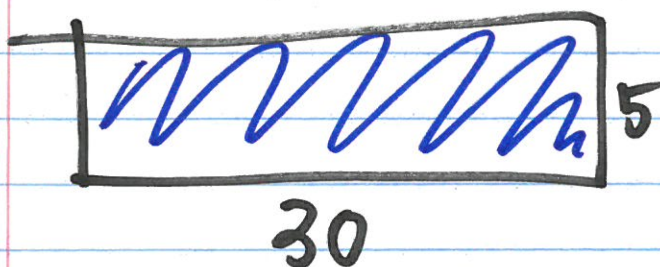
Note: In addition a global max can occur at an endpoint of the domain

<u>Note</u>: Global max might not exist

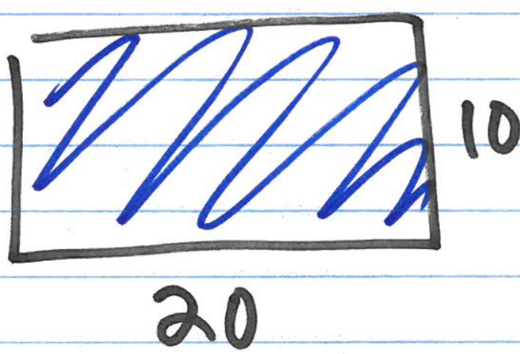
f unbounded

Optimization

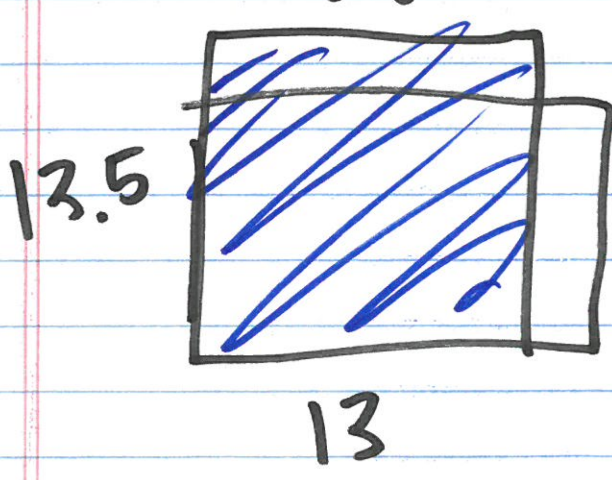
Ex: Enclose three sides of a rectangle with the largest possible area using a string of length 40 cm.



A = 150

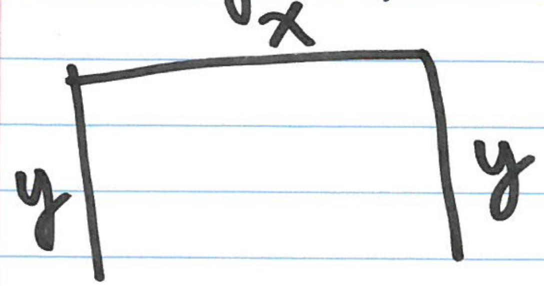


A = 200



A = 175.5

Using calculus solve this problem



draw diagram, label quantities

<u>constraint</u>

the length of the string is 40

$$y+y+x=40$$
$$2y+x=40$$

<u>objective</u>

maximize area

$$A=x\cdot y$$

Use constraint to write the objective as a function of only one variable

solve for y and plug in to A

$$y=\frac{40-x}{2}$$

$$A = x\left(\frac{40-x}{2}\right) = 20x - \frac{x^2}{2}$$

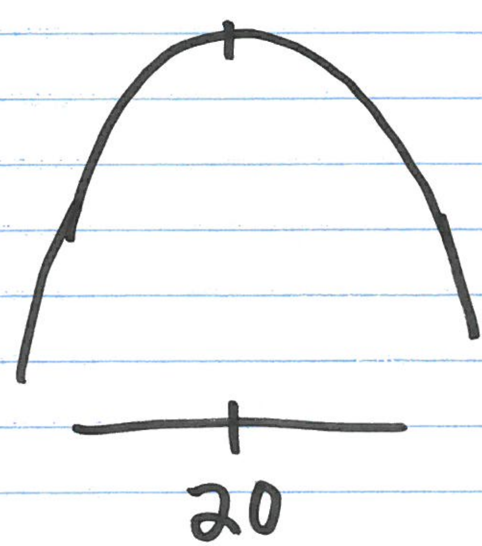
Now find the global max, ~~by~~ using the information provided by ~~its~~ the derivatives.

local maxima occur at critical values

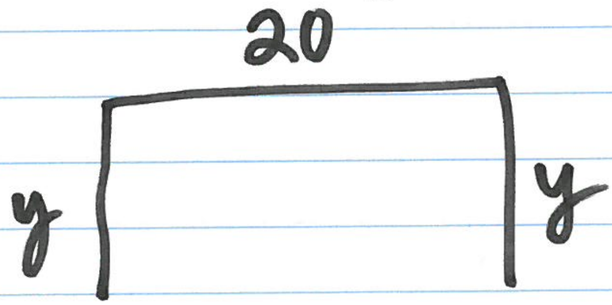
$$A'(x) = 20 - x$$

$$A'(x) = 0 \Rightarrow x = 20$$

$$A''(x) = -1 \Rightarrow A \text{ is concave down } \underline{\text{everywhere}}$$



A has a global max when $x = 20$.

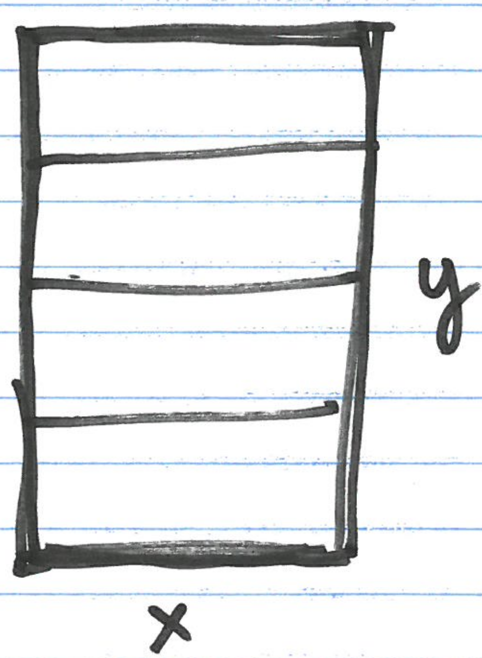


$$y = \frac{40 - x}{2} = \frac{20}{2} = 10$$

We enclose the rectangle using two sides of length 10 cm and one side of length 20 cm.

w/ 3 shelves

ex Suppose you build a bookshelf using two kinds of board. For the rectangular frame, you use thick wood that costs \$3 per foot. For the 3 shelves you use thinner wood that costs \$2 per foot. If the total area needed to hold the books is $18\ ft^2$ what dimensions of the bookshelf will minimize cost.

① 

Let x = width
y = height

objective

minimize cost

$$C = (2x+2y)\cdot 3$$

$$3x\cdot 2$$

<u>constraint</u>

$$18\ ft^2 = xy \;:\; A$$

$$y = \frac{18}{x}$$

$$\Rightarrow C = 12x + 6y$$
$$= 12x + 6\left(\frac{18}{x}\right)$$
$$= 12x + \frac{108}{x}$$

domain?