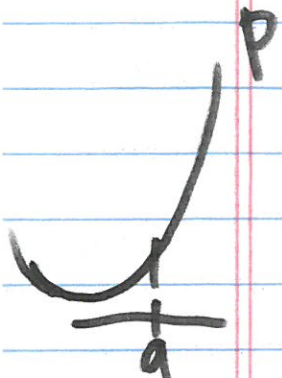
2/10 Q: What is a limit?

A limit is a number that all y values of a function approach as x approaches a given number.

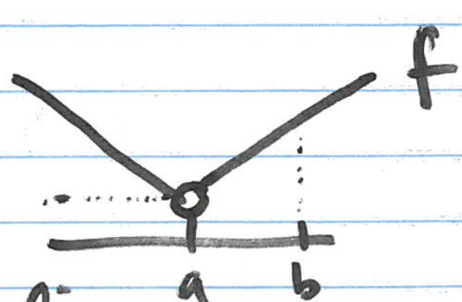
Q: What are some examples where a limit exists or does not exist?



limit at a exists and equals p(a)

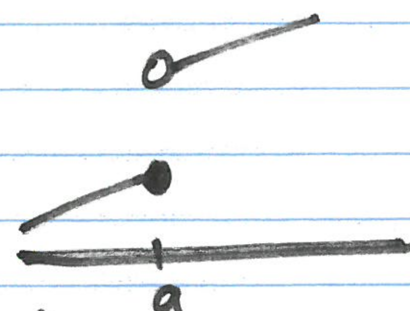
continuity

continuous



$\lim_{x \to a} f(x)$ exists (1,2,3)

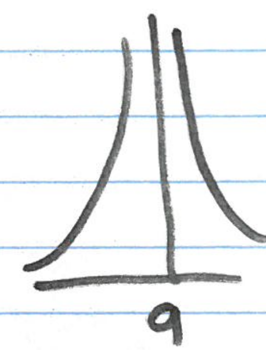
not continuous



$\lim_{x \to a} f(x)$ does not exist

1,2,3

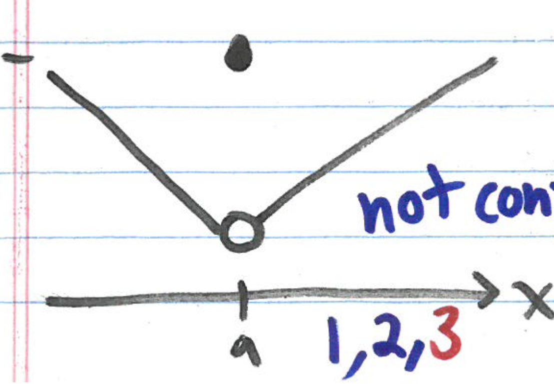
not continuous



limit at a does not exist

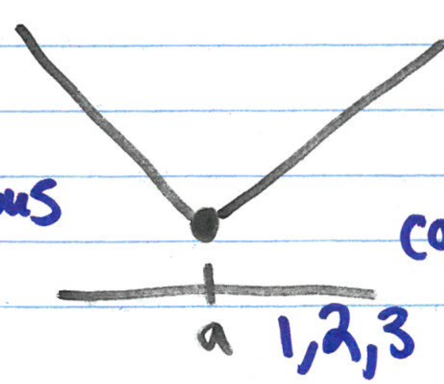
not continuous

1,2,3



not continuous

1,2,3



continuous

1,2,3

f is continuous at $x=a$ if

$$\lim_{x \to a} f(x) = f(a).$$

Three parts

① limit exists at a

② f defined at $x=a$

③ the limit equals $f(a)$.

f is continuous on a bounded interval if you can the graph without lifting your pencil.

The value of f changes gradually.

Differentiability

The derivative of a function at a point is:

① the slope of the tangent line

② the instaneous rate of change of the function

$$\text{slope} = \frac{\Delta y}{\Delta x}$$

= how much a graph rises over how much it runs

= how much a graph rises per unit change in the independent variable

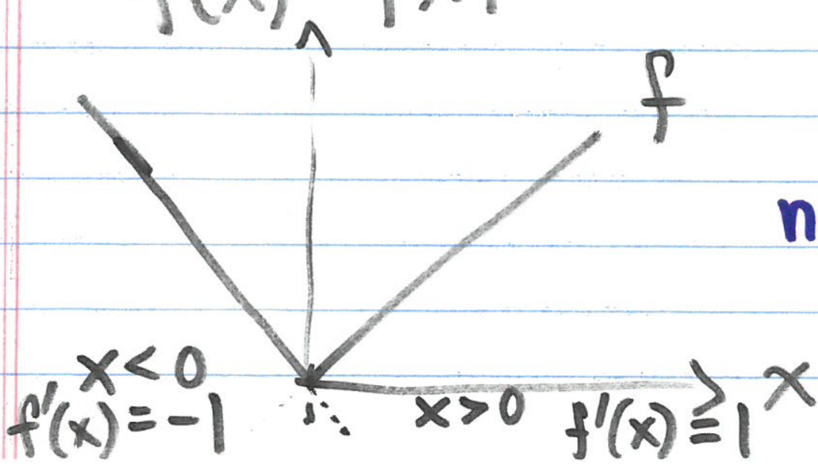
Differentiability

f is continuous if the value changes gradually

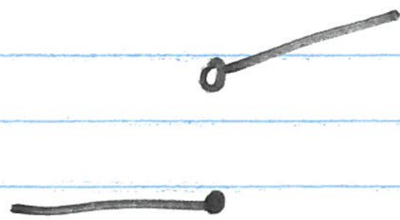
f is differentiable if the slope changes gradually.

f is non differentiable if the slope changes abruptly.

$f(x) = |x|$

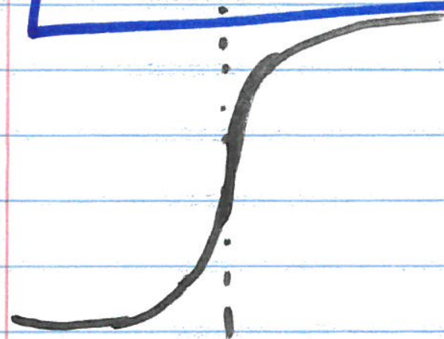


nondifferentiable



non differentiable

- If f is not continuous, then f is not differentiable.



non differentiable

Ex: $f(x) = \begin{cases} \dfrac{x^3 - 2x^2}{x-2} & x \neq 2 \\ 1 & x = 2 \end{cases}$

Is f continuous, differentiable at $x = 2$?

① $\lim\limits_{x \to 2} f(x) = \lim\limits_{x \to 2} \dfrac{x^3 - 2x^2}{x-2}$

$\lim\limits_{x \to 2} x - 2 = 2 - 2 = 0$

$\lim\limits_{x \to 2} x^3 - 2x^2 = 8 - 8 = 0$

$\dfrac{x^3 - 2x^2}{x-2} = \dfrac{x^2(x-2)}{(x-2)} = x^2$ for $x \neq 2$

$\lim\limits_{x \to 2} \dfrac{x^3 - 2x^2}{x-2} = \lim\limits_{x \to 2} x^2 = 2^2 = 4$

② $f(2) = 1$, so f is defined at 2.

③ $\lim_{x \to 2} f(x) = 4$, but $f(2) = 1$ does not equal 4, so ~~lim~~ f is not continuous. Since f is not continuous, it is not differentiable.

Limits at ∞

What value (if any) does the function approach as $x \to \infty$.
goes to

Ex: $\lim_{x \to \infty} \frac{1}{x} = 0.$

$x > 10$, $x > 1000$ ...

$f(x) < \frac{1}{10}$, $f(x) < \frac{1}{1000}$ ...

$$\lim_{x \to \infty} \frac{1}{x^2} = 0.$$

Ex: $\lim_{x \to \infty} \frac{3x^2 + 5x + 1}{4x^2 + 2x + 1}$

$$\frac{(3x^2+5x+1)/x^2}{(4x^2+2x+1)/x^2}$$

distribute $\frac{1}{x^2}$

$$\frac{\frac{3x^2}{x^2}+\frac{5x}{x^2}+\frac{1}{x^2}}{\frac{4x^2}{x^2}+\frac{2x}{x^2}+\frac{1}{x^2}} = \frac{3}{4}$$ (see below)

$$= \frac{3+\frac{5}{x}+\frac{1}{x^2}}{4+\frac{2}{x}+\frac{1}{x^2}}.$$

$$\lim_{x\to\infty} 3+\frac{5}{x}+\frac{1}{x^2} = 3+0+0=3$$

$$\lim_{x\to\infty} 4+\frac{2}{x}+\frac{1}{x^2} = 4+0+0=4$$

So $\displaystyle\lim_{x\to\infty} \frac{3x^2+5x+1}{4x^2+2x+1} = \frac{3}{4}$