Suppose

## If (A,B) is not a winning position for player 1, it must be a losing position for player 1.

Proof: for every choice of player 1, because (A,B) is not a winning position, there must be a choice for player 2 that the remaining position (A1,B1) is not a winning position for player 1. Because Ai+Bi keeps decreasing after every round. Either, there is some way that player 1 loses or Ai + Bi <= 2 and (Ai, Bi) is not a winning position. It must be (1,1). So (A,B) is losing position for player 1.

## If , (A,B) is a winning number.

Proof: B=k\*A+C, k>=2, 0<=C<A. if (A,C) is a winning position, player can choose to make the position to be (A, A+C), then player 2 has only one choice to make it (A,C).

If (A,C) is a losing position, then p1 can choose to make it (A,C) and then win.

## If A==B, (A,B) is a losing number

If A<B<2A

Round 1, 1 make it (B-A,A), if A>=2(B-A) <-> A >=2/3B, then p1 loses else round2

Round 2, 2 make it (2A-B, B), if B>=2(2A-B) <-> A <= ¾ B, then p1 wins, else

Round 3, 1 makes it(2B-2A, 2A-B), if 2B-2A <= ¾(2A-B) <->, then 1 loses, else

Round 4,

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If A <= ½ B, 1 wins

if A >= 2/3B, 1 loses