test_ordre_schema

October 15, 2022

[1]:
```
import sys
import os
import matplotlib.pyplot as plt
from scipy.stats import linregress

lib_path = os.path.realpath(os.path.join(os.getcwd(), ".."))
if lib_path not in sys.path:
    sys.path = [lib_path] + sys.path
```

[2]:
```
from src.main import *
from matplotlib import rc
from src.interpolation_methods import grad, grad_center4

%matplotlib widget
rc("text", usetex=True)
rc("font", family="serif")
rc("figure", figsize=(10, 5))
rc("figure", dpi=100)
rc("font", size=18)
rc("legend", fontsize=16)
rc("text.latex", preamble=r"\usepackage{siunitx}")
```

[3]:
```
Delta = 1.0
```

[4]:
```
list_n = [10, 100, 500, 1000, 5000]  # , 10000, 100000, 500000]
list_x = [
    np.linspace(
        0.5 * Delta / (k * int(Delta)),
        Delta - 0.5 * Delta / (k * int(Delta)),
        k * int(Delta),
    )
    for k in list_n
]
list_xf = [np.linspace(0.0, Delta, k * int(Delta) + 1) for k in list_n]

x = list_x[0]
xf = list_xf[0]
```

```python
# print('I : ', bulle.indicatrice_liquide(x))
# print('If : ', bulle.indicatrice_liquide(xf))
```

[5]:
```python
def test_cvgce_order(function):
    np.seterr(all="ignore")
    list_scheme = ["upwind", "center", "center_h", "quick", "weno"]
    list_labels = ["Upwind", "Center", "Center\_h", "QUICK", "WENO"]
    fig, ax = plt.subplots(1)
    for i_scheme, scheme in enumerate(list_scheme):
        print("=================")
        print("| Interpolation |")
        print("=================")
        print(scheme)
        err_schema = []
        dx_list = []
        # figT, axT = plt.subplots(1)
        for i in range(len(list_x)):
            x = list_x[i]
            xf = list_xf[i]
            dx = x[1] - x[0]
            dx_list.append(dx)
            T = function(x)
            T_f = function(xf)
            T_int = interpolate(T, schema=scheme)
            err_schema.append(np.sum(np.abs(T_f - T_int) * dx))
            # axT.plot(xf, T_int, label=i)
        # axT.plot(xf, T_f, '+', label='Ref')
        # axT.legend()
        # axT.set_title(scheme)
        slope, intercept, r_value, p_value, std_err = linregress(
            np.log(dx_list), np.log(err_schema)
        )
        print("Ordre : ", slope)
        print(r"$R^2$ : ", r_value**2)
        ax.loglog(
            dx_list,
            err_schema,
            "-+",
            label=list_labels[i_scheme]
            + r", $\mathcal{O}(\Delta x^{%.3g})$" % (slope,)
            + r", $R^2 = %g$" % (r_value**2,),
        )
        ax.set_xlabel(r"$\Delta x$")
        ax.set_ylabel(r"$\int_{L}{\left| T_{interp} - T_f \right| dx}$")
        ax.legend()
        ax.minorticks_on()
        ax.grid(visible=True, which="major")
```

```
        ax.grid(visible=True, which="minor", alpha=0.2)
        fig.tight_layout()
    return fig, ax
```

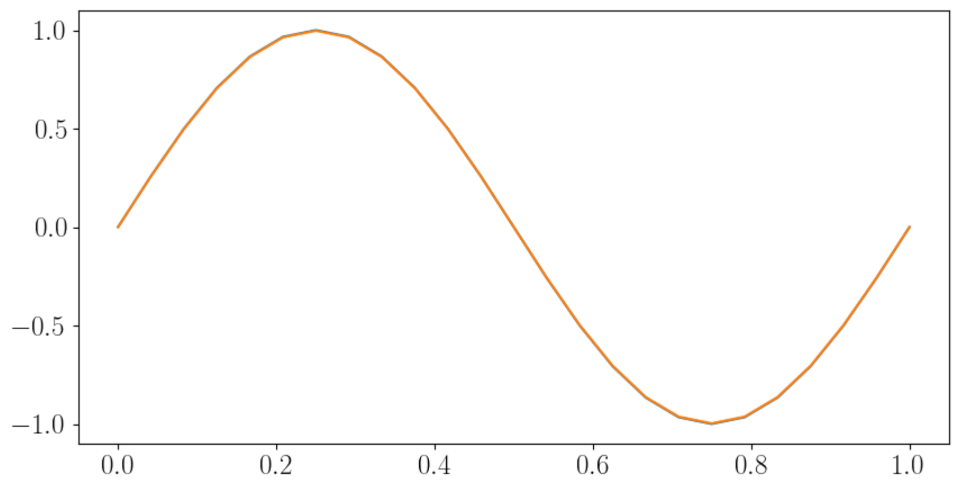
1 Ordre de convergence de l'interpolation

1.1 Avec un sinus

[6]:
```
def sinus(x):
    return np.sin(x * 2 * np.pi / Delta)


# returns the sine integral between x-dx/2. and x+dx/2.
def int_sinus(x):
    dx = x[1] - x[0]
    res = (
        -Delta
        / (2 * np.pi)
        * (
            np.cos((x + dx / 2.0) * 2 * np.pi / Delta)
            - np.cos((x - dx / 2.0) * 2.0 * np.pi / Delta)
        )
        / dx
    )
    return res
    # return get_T_creneau(x, phy_prop=phy_prop, markers=bulle)
```

[7]:
```
x = np.linspace(0.0, 1.0, 25)
y = sinus(x)
y1 = int_sinus(x)
plt.figure()
plt.plot(x, y)
plt.plot(x, y1)
```

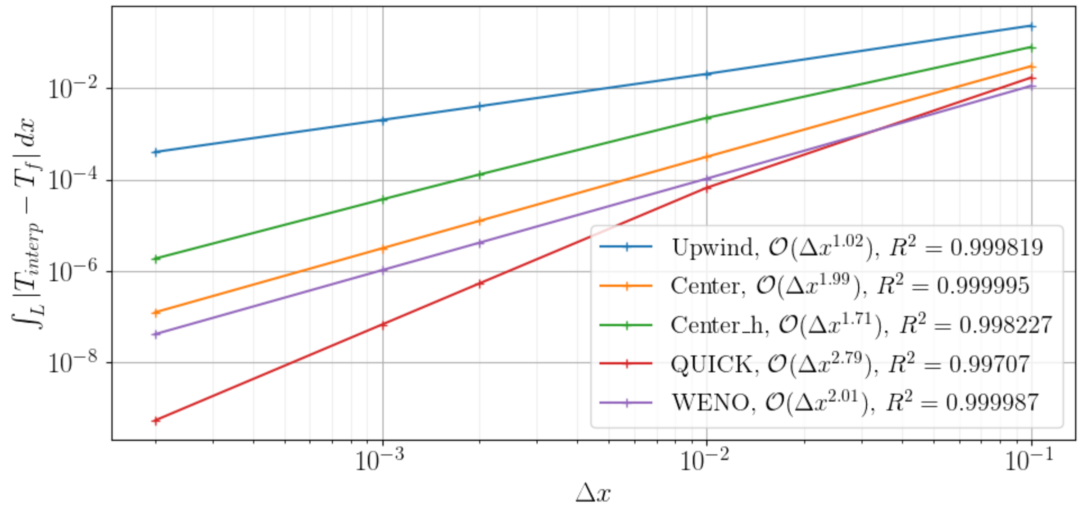
[7]:
```
[<matplotlib.lines.Line2D at 0x7f89acfeeda0>]
```



[8]:
```
fig, _ = test_cvgce_order(sinus)
fig.savefig(os.path.join(lib_path, "figures", "convergence_operateurs_base",␣
 ↪"sin.pdf"))
```

```
=================
| Interpolation |
=================
upwind
Ordre :  1.0224253418357305
$R^2$ :  0.9998191800788256
=================
| Interpolation |
=================
center
Ordre :  1.9936064146722758
$R^2$ :  0.999994582628658
=================
| Interpolation |
=================
center_h
Ordre :  1.7138306824963354
$R^2$ :  0.9982272252574688
=================
| Interpolation |
=================
quick
Ordre :  2.7870505032370794
$R^2$ :  0.997069987289377
=================
```

```
| Interpolation |
=================
weno
Ordre :   2.0099139511946933
$R^2$ :   0.9999866323120541
```



On constate de manière attendue que le schéma upwind est d'ordre 1, le schéma centré d'ordre 2, le schéma centré harmonique est d'ordre un peu inférieur (1.86).

Comme prévu le schéma QUICK est d'ordre 3, en revanche de manière surprenante le WENO est d'ordre 2.

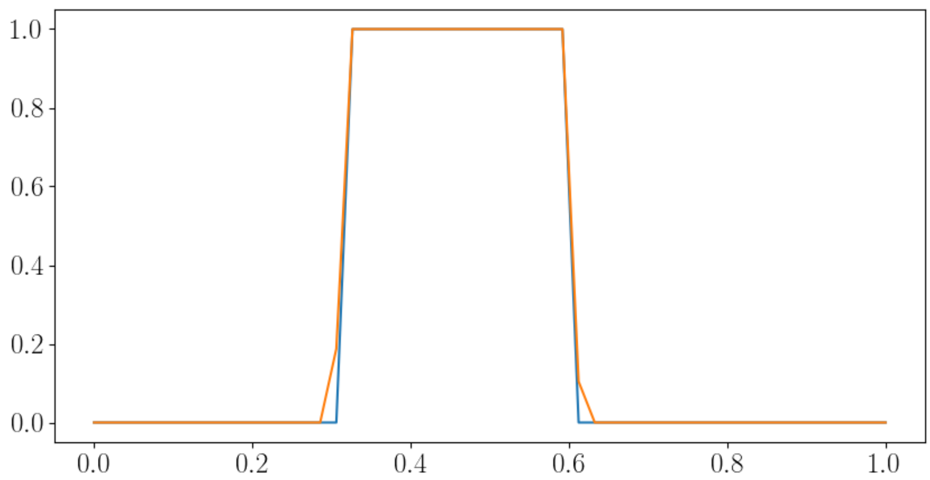
1.2 Avec un creneau

[9]:
```python
x0 = 3.75 / 12.0
x1 = 7.25 / 12.0



def creneau(x):
    res = np.where(np.bitwise_and(x0 < x, x < x1), 1.0, 0.0)
    return res



def int_creneau(x):
    dx = x[1] - x[0]
    res = np.where(np.bitwise_and(x0 < x, x < x1), 1.0, 0.0)
    ind_0 = np.abs(x - x0) < dx / 2.0
    res[ind_0] = (x[ind_0] + dx / 2.0 - x0) / dx
    ind_1 = np.abs(x - x1) < dx / 2.0
```

```
    res[ind_1] = 1.0 - (x[ind_1] + dx / 2.0 - x1) / dx
    return res
```

[10]:
```
x = np.linspace(0.0, 1, 50)
y = creneau(x)
y1 = int_creneau(x)
plt.figure()
plt.plot(x, y)
plt.plot(x, y1)
```

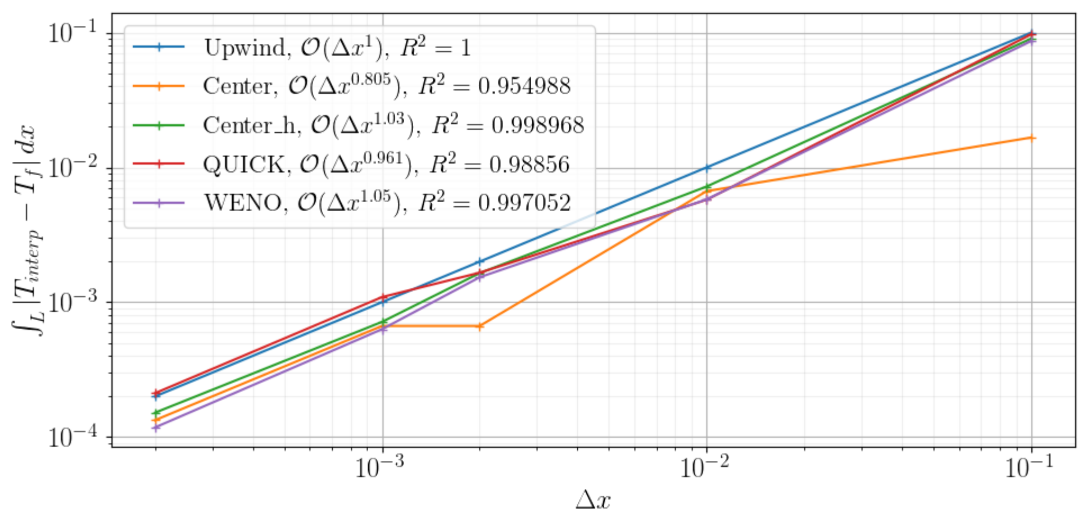
[10]:
```
[<matplotlib.lines.Line2D at 0x7f89911100b8>]
```



[11]:
```
fig, _ = test_cvgce_order(int_creneau)
# fig.savefig(os.path.join(lib_path, 'figures', 'convergence_operateurs_base',␣
 ↪'creneau.pdf'))
```

```
=================
| Interpolation |
=================
upwind
Ordre :  0.9999999999999415
$R^2$ :  0.9999999999999996
=================
| Interpolation |
=================
center
Ordre :  0.8047921026590918
$R^2$ :  0.9549881387586561
=================
```

```
| Interpolation |
=================
center_h
Ordre :   1.025640551765483
$R^2$ :   0.9989683836203196
=================
| Interpolation |
=================
quick
Ordre :   0.9610880181109256
$R^2$ :   0.9885599443470903
=================
| Interpolation |
=================
weno
Ordre :   1.048394887219534
$R^2$ :   0.9970522528290264
```



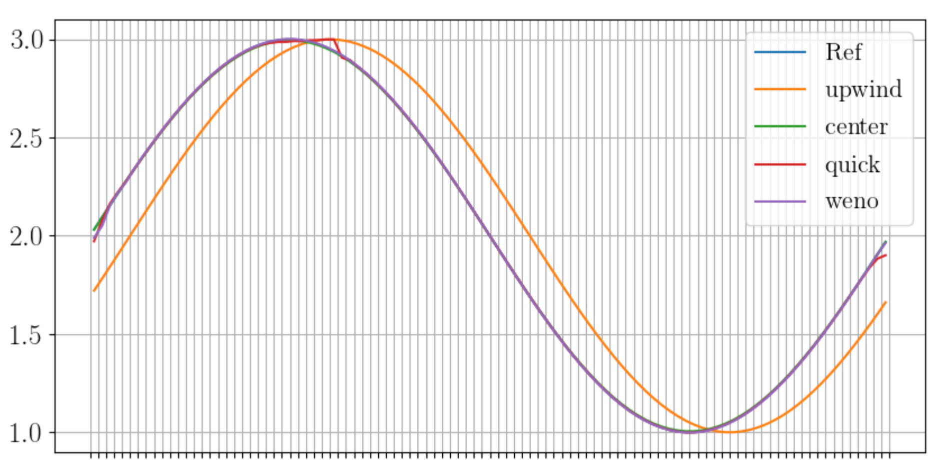
On en conclut que tous les schémas d'interpolation de créneaux étudiés ici sont d'ordre 1. En effet, la seule erreur d'interpolation est faite ici à proximité des interfaces et l'intégrale de l'erreur donne une convergence à l'ordre 1.

On en conclut que tous les schémas d'interpolation de créneaux étudiés ici sont d'ordre 1. En effet, la seule erreur d'interpolation est faite ici à proximité des interfaces et l'intégrale de l'erreur donne une convergence à l'ordre 1.

2 Test avec plusieurs itérations en interpolation et réinterpolation successives

[12]:
```
def eval_interp_iterative(function, n=100, step=10):
    dx = 1.0 / n
    x = np.linspace(dx / 2.0, 1.0 - dx / 2, n)
    y = function(x)
    plt.figure()
    plt.plot(x, y, label="Ref")
    list_scheme = ["upwind", "center", "quick", "weno"]
    for schema in list_scheme:
        y_int = y[:]
        for i in range(step):
            if i % 2 == 0:
                y_int = interpolate(y_int, schema=schema, cl=1)
            else:
                y_int = interpolate(y_int, schema=schema, cl=1)
                y_int = y_int[1:-1]

        plt.plot(x, y_int, label=schema)
    xf = np.empty(x.size + 1)
    xf[:-1] = x - dx / 2.0
    xf[-1] = x[-1] + dx / 2
    plt.legend()
    plt.grid()
    ti = plt.gca().set_xticks(xf)
    la = plt.gca().set_xticklabels([])
```

[13]:
```
eval_interp_iterative(lambda x: sinus(x) + 2.0)
```



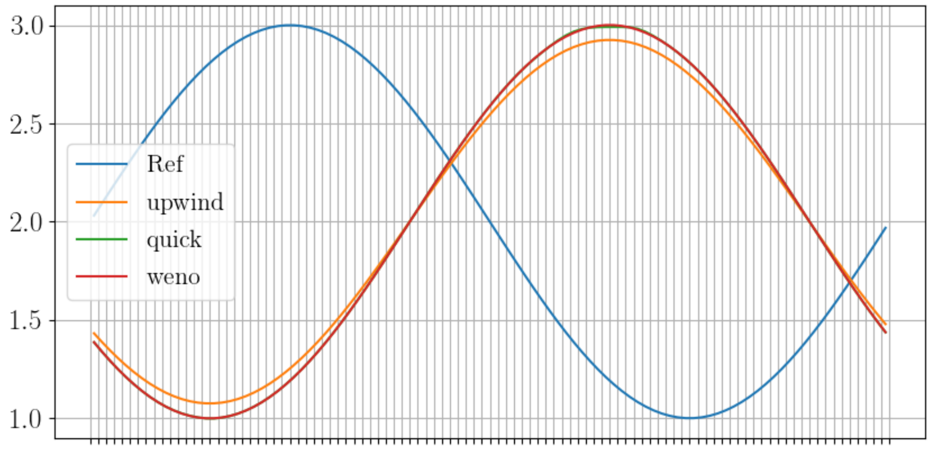
3 Test avec plusieurs itérations en Euler explicit

3.1 Profil

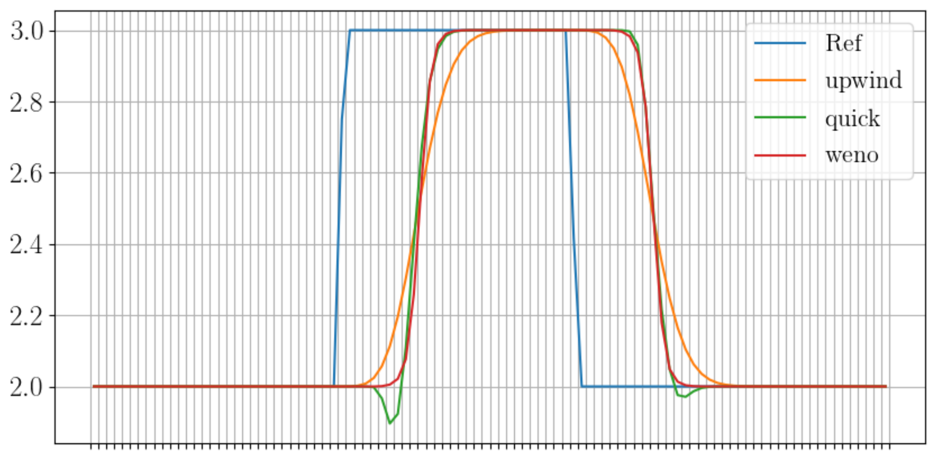
[14]:
```
def plot_profil(ax, x, y, lab=None):
    ax.plot(x, y, label=lab)
    ax.legend()
    ax.grid(visible=True)
    ax.set_xticks(xf)
    ax.set_xticklabels([])
```

[15]:
```
def eval_interp_euler_conv(function, n=200, step=10, cfl=0.5):
    dx = 1.0 / n
    dt = dx * cfl
    x = np.linspace(dx / 2.0, 1.0 - dx / 2, n)
    y = function(x)
    plt.figure()
    plt.plot(x, y, label="Ref")
    list_scheme = ["upwind", "quick", "weno"]
    for schema in list_scheme:
        y_int = y.copy()
        for i in range(step):
            flux = interpolate(y_int, schema=schema, cl=1)
            y_int += dt * (flux[:-1] - flux[1:]) / dx
        plt.plot(x, y_int, label=schema)
    xf = np.empty(x.size + 1)
    xf[:-1] = x - dx / 2.0
    xf[-1] = x[-1] + dx / 2
    plt.legend()
    plt.grid()
    ti = plt.gca().set_xticks(xf)
    la = plt.gca().set_xticklabels([])
```

[16]:
```
eval_interp_euler_conv(lambda x: sinus(x) + 2.0, n=100, step=4000, cfl=0.01)
```



[17]:
```
eval_interp_euler_conv(lambda x: int_creneau(x) + 2.0, n=100, step=1000, cfl=0.
↪01)
```



3.2 Ordre de convergence

[18]:
```
def eval_convergence_euler_conv(
    function, n_list=[50, 100], n_tour_domaine=0.03, cfl=0.0001
):
```

```python
already_plotted_ref = False
v = 1.0
D = 1.0
n_array = np.array(n_list)
dx_array = D / n_array
dt = np.min(dx_array / v * cfl)
print("dt : ", dt)
step = int(n_tour_domaine * D / (v * dt))
fig, ax = plt.subplots(1)
fig_prof, (ax_prof, ax_err) = plt.subplots(nrows=2, sharex=True)
list_scheme = ["upwind", "quick", "weno"]
for schema in list_scheme:
    err_pour_schema = []
    for k, n in enumerate(n_array):
        dx = dx_array[k]
        x = np.linspace(dx / 2.0, D - dx / 2, n)
        y = function(x)
        y_int = y.copy()
        for _ in range(step):
            flux = interpolate(y_int * v, schema=schema, cl=1)
            y_int += dt * (flux[:-1] - flux[1:]) / dx
        y_ref = function(x - step * v * dt)
        err_pour_schema.append(np.sqrt(np.sum((y_ref - y_int) ** 2.0)) / n)
        if k == len(n_array) - 1:
            if not already_plotted_ref:
                plot_profil(ax_prof, x, y_ref, "Ref")
                plot_profil(ax_err, [], [], "Ref")
                already_plotted_ref = True
            plot_profil(ax_prof, x, y_int, schema)
            ax_err.semilogy(x, np.abs(y_ref - y_int), label=schema)
            ax_err.legend()
    err_pour_schema_arr = np.array(err_pour_schema)
    slope, intercept, r_value, p_value, std_err = linregress(
        np.log(dx_array), np.log(err_pour_schema_arr)
    )
    ax.loglog(
        dx_array,
        err_pour_schema_arr,
        "-+",
        label=schema
        + r", $\mathcal{O}(\Delta x^{%.3g})$" % (slope,)
        + r", $R^2 = %g$" % (r_value**2,),
    )
    print()
    print(schema, slope)
    print(r"R^2", r_value**2.0)
ax_err.set_ylabel("Erreur")
```

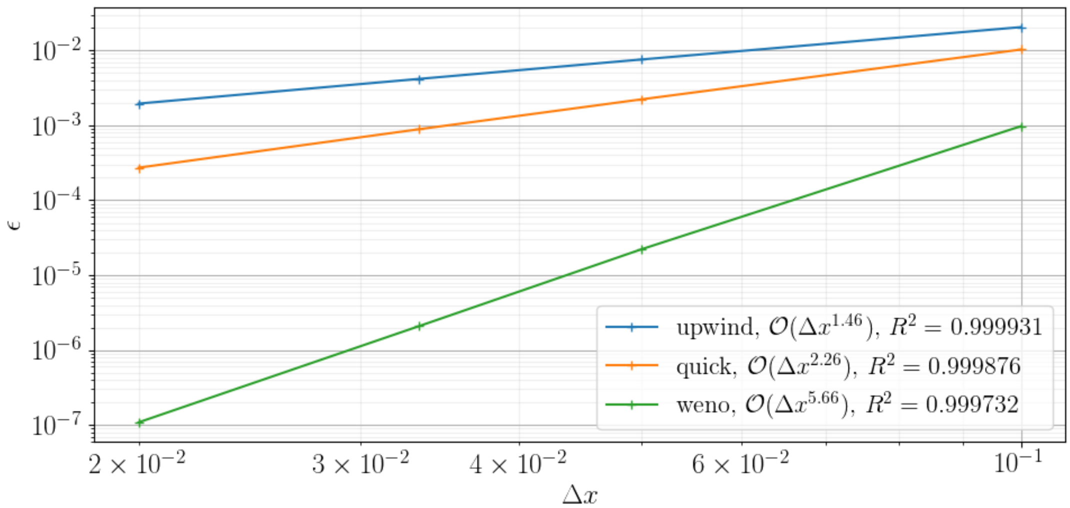
```python
    ax_prof.set_ylabel("Solution")
    ax_err.set_xlabel(r"$x$")
    ax_err.set_xlim(0.0, 1.0)
    ticks = np.linspace(0.0, 1.0, 10)
    ax_err.set_xticks(ticks, minor=False)
    ax_prof.set_xticks(ticks, minor=False)
    ax_err.set_xticklabels(
        [
            "0",
        ]
        + [
            "",
        ]
        * (len(ticks) - 2)
        + [
            "1",
        ],
        minor=False,
    )
    fig_prof.tight_layout()
    ax.grid(visible=True, which="major")
    ax.grid(visible=True, which="minor", alpha=0.2)
    ax.set_xlabel(r"$\Delta x$")
    ax.set_ylabel(r"$\epsilon$")
    ax.legend()
    fig.tight_layout()
    return fig, fig_prof
```

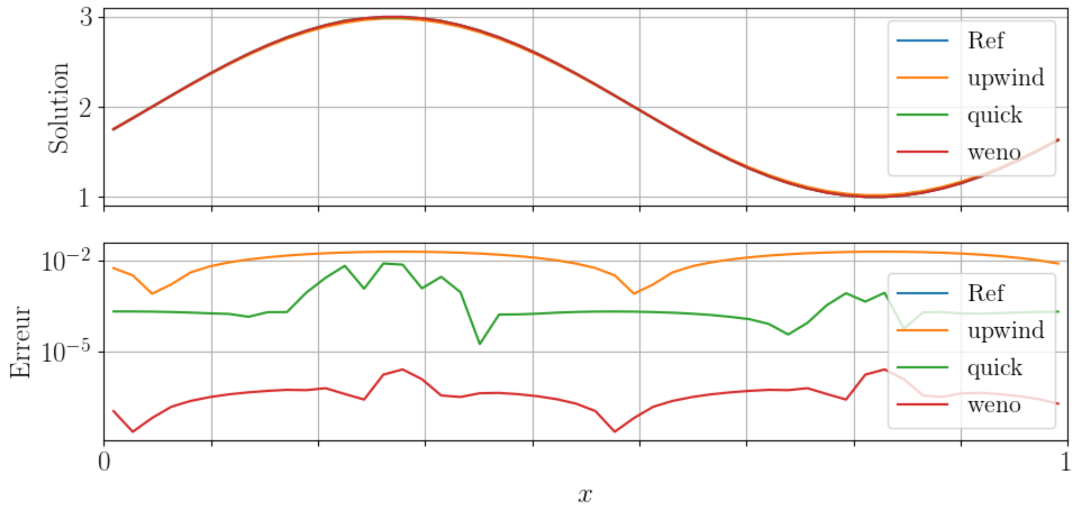
[19]:

```python
fig, fig_prof = eval_convergence_euler_conv(
    lambda x: int_sinus(x) + 2.0,
    n_list=[10, 20, 30, 50],
    cfl=0.0001,
    n_tour_domaine=0.05,
)
fig_prof.savefig(
    os.path.join(
        lib_path,
        "figures",
        "convergence_operateurs_base",
        "euler_sinus_profil.pdf",
    )
)
fig.savefig(
    os.path.join(
        lib_path,
        "figures",
        "convergence_operateurs_base",
```

```
        "euler_sinus_erreur.pdf",
    )
)
```

```
dt :  2.0000000000000003e-06

upwind 1.460035174009346
R^2 0.9999307735967031

quick 2.2550350972647655
R^2 0.9998760324477028

weno 5.660296635157494
R^2 0.99973191365036
```

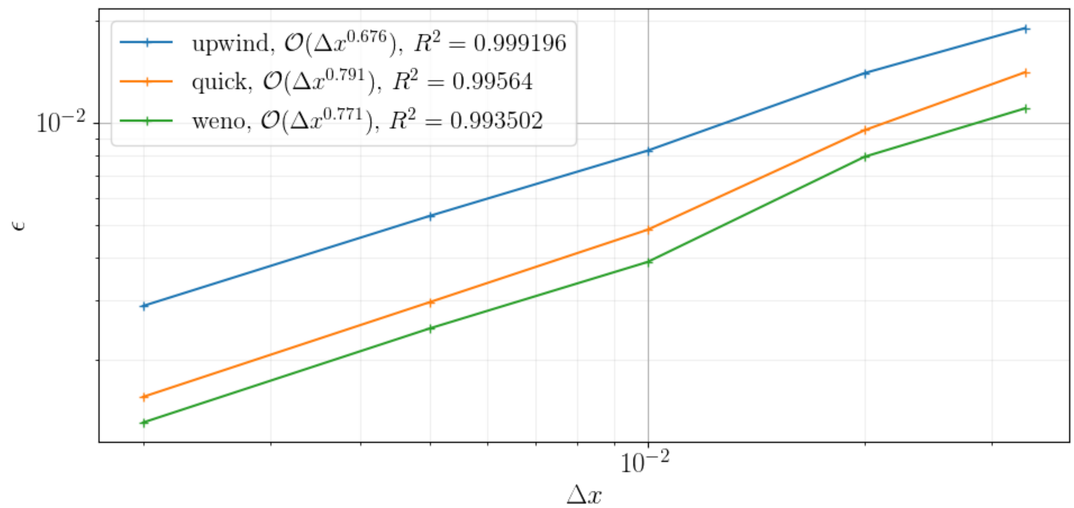


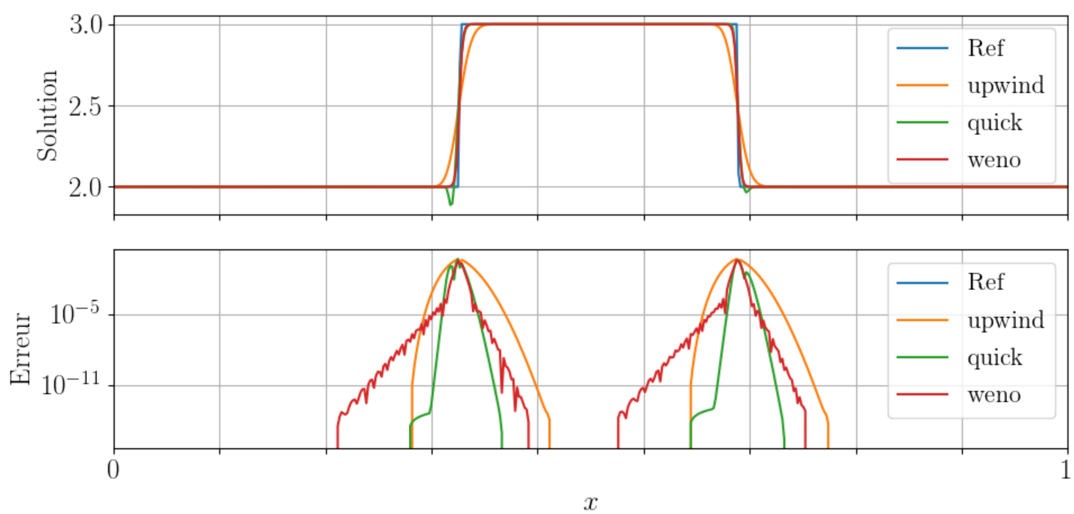


[20]:
```
fig, fig_prof = eval_convergence_euler_conv(
    lambda x: int_creneau(x) + 2.0,
    n_list=[30, 50, 100, 200, 500],
    cfl=0.01,
    n_tour_domaine=0.05,
)
fig_prof.savefig(
    os.path.join(
        lib_path,
        "figures",
        "convergence_operateurs_base",
        "euler_creneau_profil.pdf",
    )
)
fig.savefig(
    os.path.join(
        lib_path,
        "figures",
        "convergence_operateurs_base",
        "euler_creneau_erreur.pdf",
    )
)
```

```
dt :  2e-05

upwind 0.6755619842014863
R^2 0.9991955124110475

quick 0.7907206797437892
```

```
R^2 0.9956403976864822

weno 0.7711732487463624
R^2 0.9935023829325312
```





4 Diffusion avec plusieurs itérations en Euler explicit

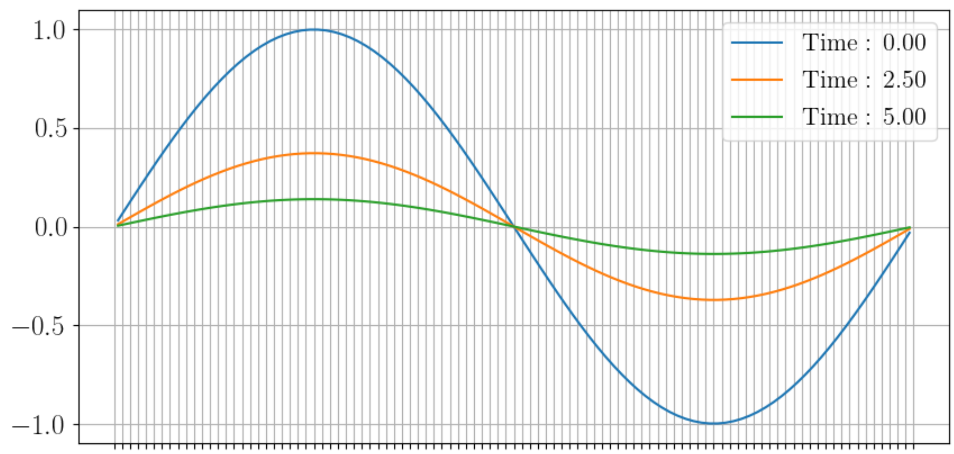
4.1 Profil

[21]:
```
def plot_profil(ax, x, y, lab=None):
    ax.plot(x, y, label=lab)
    # ax.legend()
    ax.grid(visible=True)
    ticks = ax.get_xticks(minor=False)
    ax.set_xticks(ticks, minor=False)
    ax.set_xticklabels(
        [
            "0",
        ]
        + [
            "",
        ]
        * (len(ticks) - 2)
        + [
            "L",
        ],
        minor=False,
    )
    # ax.set_xticklabels([], minor=True)
```

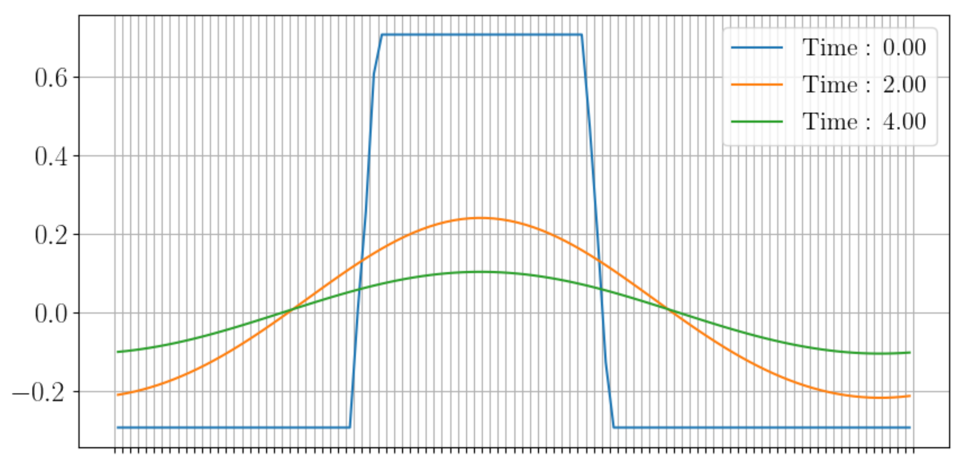
[22]:
```
def eval_interp_euler_diff(function, n=200, step=10, fo=0.5):
    dx = 1.0 / n
    dt = dx**2 * fo
    x = np.linspace(dx / 2.0, 1.0 - dx / 2, n)
    y = function(x)
    plt.figure()
    # plt.plot(x, y, label="Ref")
    list_scheme = [grad]
    for schema in list_scheme:
        y_int = y.copy()
        for i in range(step):
            flux = schema(y_int, cl=1)
            y_int += dt * (flux[1:] - flux[:-1]) / dx
            if i % (step // 2) == 0:
                plt.plot(x, y_int, label=r"Time : %.2f" % (i * dt,))
        plt.plot(x, y_int, label=r"Time : %.2f" % (i * dt,))
    xf = np.empty(x.size + 1)
    xf[:-1] = x - dx / 2.0
    xf[-1] = x[-1] + dx / 2
    plt.legend()
    plt.grid()
    ti = plt.gca().set_xticks(xf)
```

```
    la = plt.gca().set_xticklabels([])
```

[23]:
```
eval_interp_euler_diff(lambda x: sinus(x), n=100, step=1000, fo=50.0)
```



[24]:
```
eval_interp_euler_diff(lambda x: int_creneau(x) - (x1 - x0), n=100, step=1000,␣
↪fo=40.0)
```



4.2 Ordre de convergence

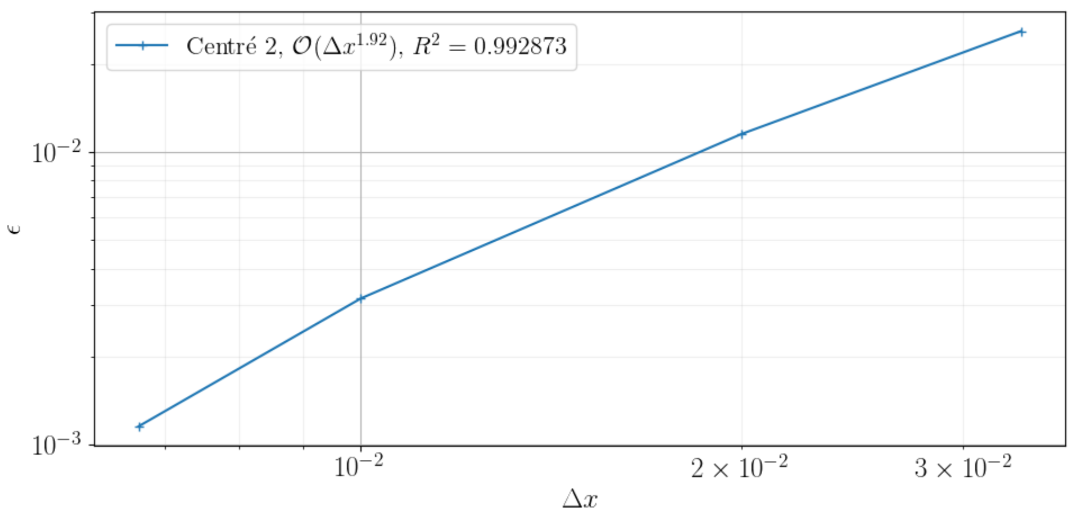
[25]:
```python
def eval_convergence_euler_diff(
    function,
    fonction_ref,
    n_list=[50, 100],
    fo=0.0001,
    n_tour_domaine=1.0,
    ref_label="Ref",
):
    already_plotted_ref = False
    v = 1.0
    D = 1.0
    n_array = np.array(n_list)
    dx_array = D / n_array
    dt = np.min(dx_array**2 * fo)
    print("dt : ", dt)
    step = int(n_tour_domaine * D / (v * dt))
    fig, ax = plt.subplots(1)
    fig_prof, (ax_prof, ax_err) = plt.subplots(nrows=2, sharex="col")
    list_scheme = [grad]
    for schema in list_scheme:
        err_pour_schema = []
        for k, n in enumerate(n_array):
            dx = dx_array[k]
            x = np.linspace(dx / 2.0, D - dx / 2, n)
            y = function(x)
            y_int = y.copy()
            for _ in range(step):
                flux = schema(y_int, cl=1)
                y_int += -dt * (flux[:-1] - flux[1:]) / dx
            y_ref = fonction_ref(x, step * dt)
            err_pour_schema.append(np.sqrt(np.sum((y_ref - y_int) ** 2.0)) / n)
            if k == len(n_array) - 1:
                if not already_plotted_ref:
                    plot_profil(ax_prof, x, y_ref, ref_label)
                    plot_profil(ax_err, [], [])  # , "Ref, $t = %.2f$" %␣
 ↪(step*dt,))
                    already_plotted_ref = True
                plot_profil(ax_prof, x, y_int, "Centré 2, $t = %.2f$" % (step *␣
 ↪dt,))
                ax_err.semilogy(x, np.abs(y_ref - y_int))  # , label='Centré 2')
                # ax_err.legend()
                ax_prof.legend()
        err_pour_schema_arr = np.array(err_pour_schema)
        slope, intercept, r_value, p_value, std_err = linregress(
            np.log(dx_array), np.log(err_pour_schema_arr)
```

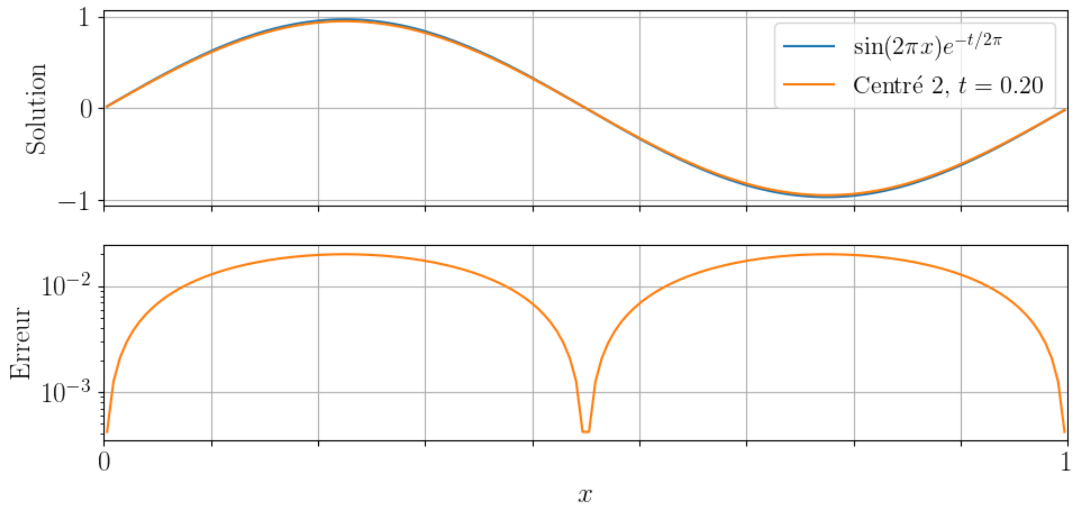
```
        )
        ax.loglog(
            dx_array,
            err_pour_schema_arr,
            "-+",
            label="Centré 2"  # schema
            + r", $\mathcal{O}(\Delta x^{%.3g})$" % (slope,)
            + r", $R^2 = %g$" % (r_value**2,),
        )
        print()
        print(schema, slope)
        print(r"R^2", r_value**2.0)
    ax_err.set_ylabel("Erreur")
    ax_prof.set_ylabel("Solution")
    ax_err.set_xlabel(r"$x$")
    ax_err.set_xlim(0.0, 1.0)
    ticks = np.linspace(0.0, 1.0, 10)
    ax_err.set_xticks(ticks, minor=False)
    ax_prof.set_xticks(ticks, minor=False)
    ax_err.set_xticklabels(
        [
            "0",
        ]
        + [
            "",
        ]
        * (len(ticks) - 2)
        + [
            "1",
        ],
        minor=False,
    )
    fig_prof.tight_layout()
    ax.grid(visible=True, which="major")
    ax.grid(visible=True, which="minor", alpha=0.2)
    ax.set_xlabel(r"$\Delta x$")
    ax.set_ylabel(r"$\epsilon$")
    ax.legend()
    fig.tight_layout()
    return fig, fig_prof
```

[26]:
```
fig, fig_prof = eval_convergence_euler_diff(
    lambda x: int_sinus(x),
    lambda x, t: np.sin(2 * np.pi * x) * np.exp(-t / (2 * np.pi)),
    ref_label=r"$\sin(2\pi x) e^{-t / 2\pi }$",
    n_list=[30, 50, 100, 150],
    fo=0.01,
```

```
    n_tour_domaine=0.2,
)
fig.savefig(
    os.path.join(
        lib_path,
        "figures",
        "convergence_operateurs_base",
        "diffusion_euler_sinus_erreur.pdf",
    )
)
fig_prof.savefig(
    os.path.join(
        lib_path,
        "figures",
        "convergence_operateurs_base",
        "diffusion_euler_sinus_profil.pdf",
    )
)
```

```
dt :  4.444444444444454e-07

<function grad at 0x7f893058f9d8> 1.915185500049257
R^2 0.9928734658112804
```




1
0
−1
Solution
$\sin(2\pi x)e^{-t/2\pi}$
Centré 2, $t = 0.20$
10^{-2}
10^{-3}
Erreur
0
1
x

[ ]: