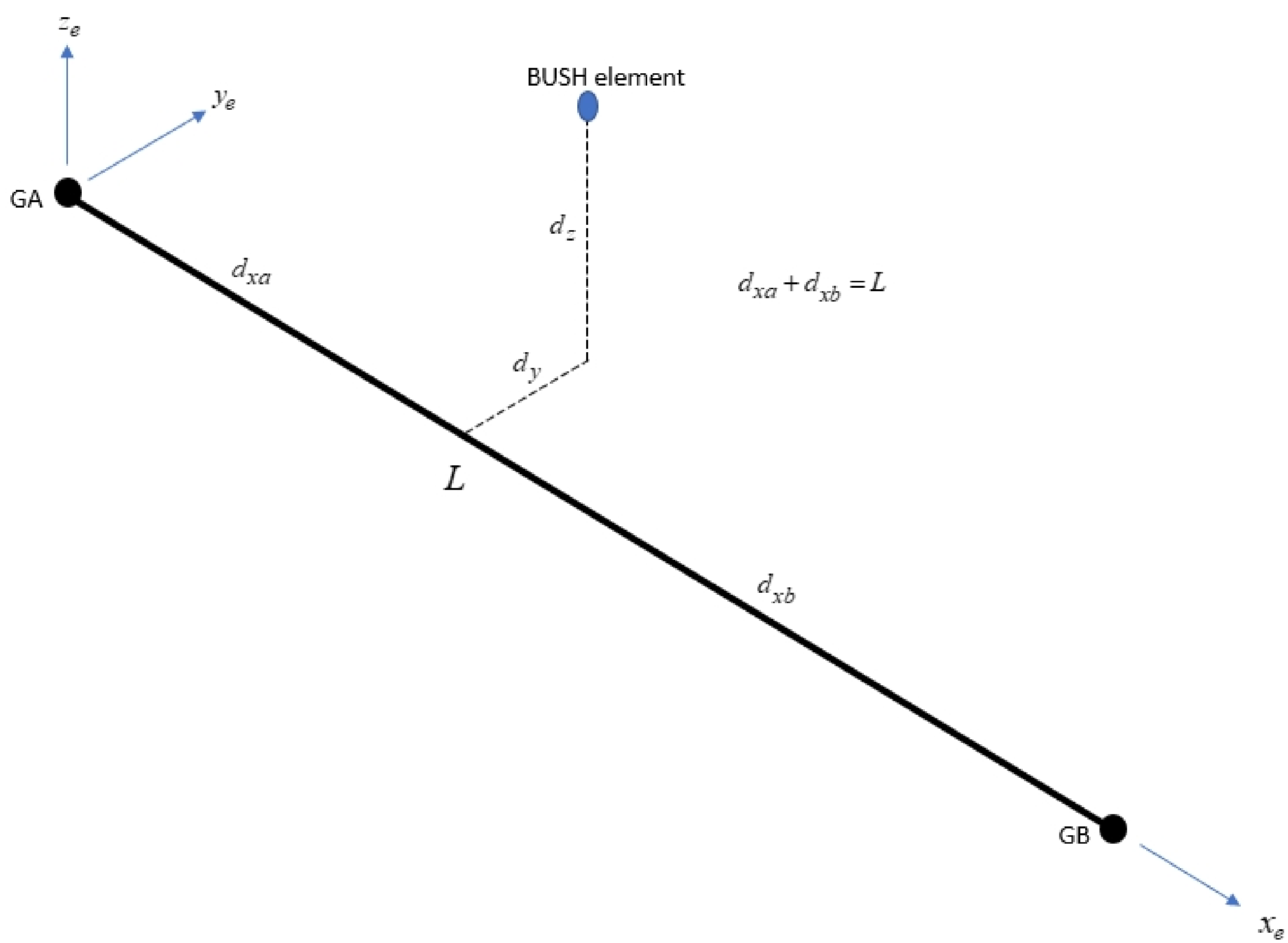
12 [illegible]

BUSH Element Geometry

(in local element coordinates)



The stiffness equations of the Shell-to-BUSH element can be expressed as

$$Ku \quad F$$

where K is a 12x12 matrix, u and F are the 12 degree of freedom displacement and force vectors (6 at each end) respectively. For clarity, rather than showing the whole 12x12 matrix, we express the above in partitioned form as:

$$\begin{bmatrix} K_{aa} & K_{ab} \\ K_{ab}^{T} & K_{bb} \end{bmatrix} \begin{Bmatrix} u_a \\ u_b \end{Bmatrix} \quad \begin{Bmatrix} F_a \\ F_b \end{Bmatrix}$$

If we denote k_i ($i=1,\dots 6$) as the 6 stiffness from the PBUSH Bulk Data entry, the above partitions are:

$$K_{aa} \quad \begin{bmatrix} k_1 & 0 & 0 & 0 & d_z k_1 & d_y k_1 \\ 0 & k_2 & 0 & d_z k_2 & 0 & d_{xa} k_2 \\ 0 & 0 & k_3 & d_y k_3 & d_{xa} k_3 & 0 \\ 0 & d_z k_2 & d_y k_3 & k_4 \; d_y^2 k_3 \; d_z^2 k_2 & d_{xa} d_y k_3 & d_{xa} d_z k_2 \\ d_z k_1 & 0 & d_{xa} k_3 & d_{xa} d_y k_3 & k_5 \; d_{xa}^2 k_3 \; d_z^2 k_1 & d_y d_z k_1 \\ d_y k_1 & d_{xa} k_2 & 0 & d_{xa} d_z k_2 & d_y d_z k_1 & k_6 \; d_{xa}^2 k_2 \; d_y^2 k_1 \end{bmatrix}$$

$$K_{ab} \quad \begin{bmatrix} k_1 & 0 & 0 & 0 & d_z k_1 & d_y k_1 \\ 0 & k_2 & 0 & d_z k_2 & 0 & d_{xb} k_2 \\ 0 & 0 & k_3 & d_y k_3 & d_{xb} k_3 & 0 \\ 0 & d_z k_2 & d_y k_3 & (k_4 \; d_y^2 k_3 \; d_z^2 k_2) & d_{xb} d_y k_3 & d_{xb} d_z k_2 \\ d_z k_1 & 0 & d_{xa} k_3 & d_{xa} d_y k_3 & k_5 \; d_{xa} d_{xb} k_3 \; d_z^2 k_1 & d_y d_z k_1 \\ d_y k_1 & d_{xa} k_2 & 0 & d_{xa} d_z k_2 & d_y d_z k_1 & k_6 \; d_{xa} d_{xb} k_2 \; d_y^2 k_1 \end{bmatrix}$$

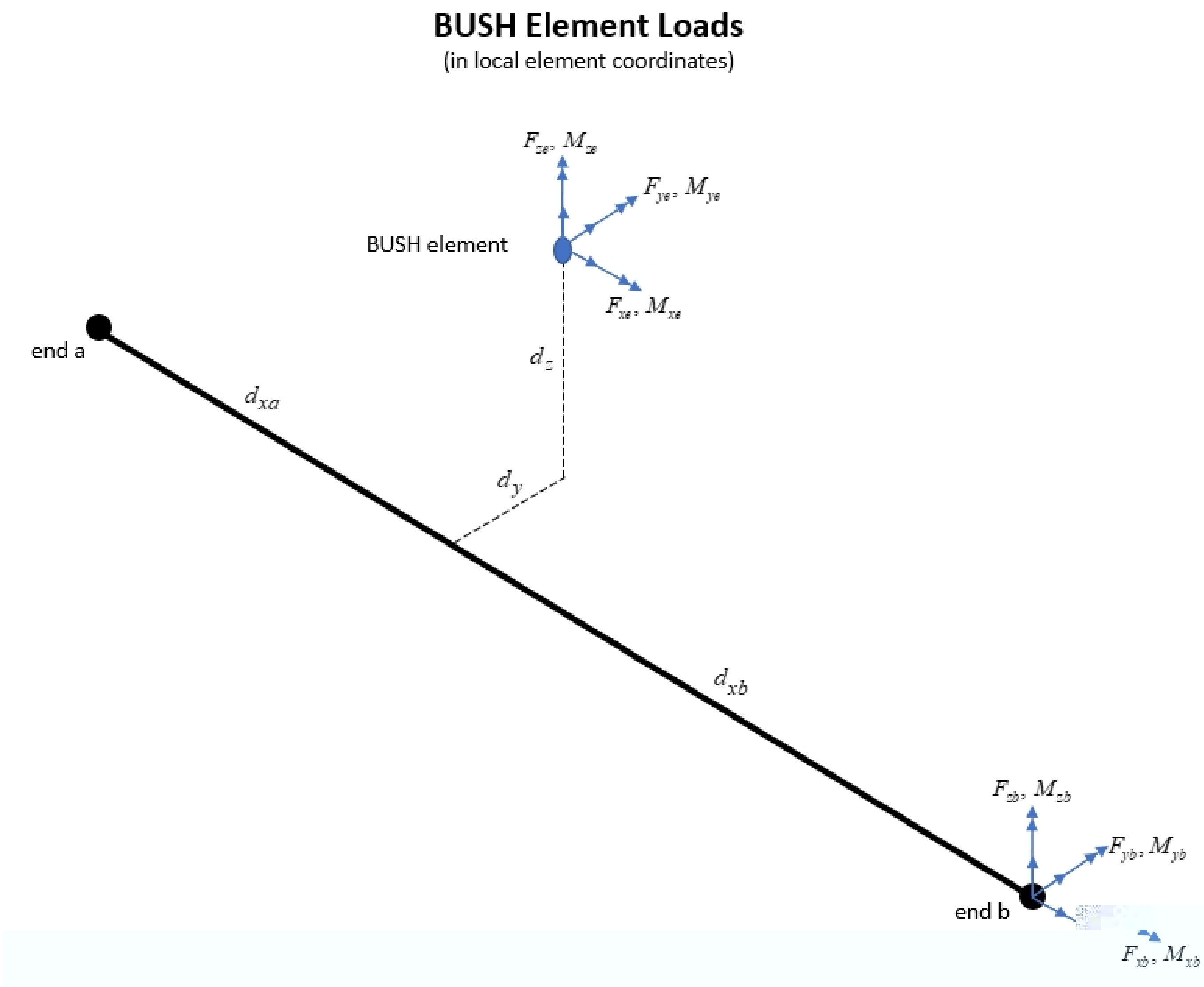
$$
K_{bb} \quad \begin{array}{cccccc}
{}_{1} & 0 & 0 & 0 & d_{z}\ {}_{1} & d_{y}\ {}_{1} \\
0 & {}_{2} & 0 & d_{z}\ {}_{2} & 0 & d_{xb}\ {}_{2} \\
0 & 0 & {}_{3} & d_{y}\ {}_{3} & d_{xb}\ {}_{3} & 0 \\
0 & d_{z}\ {}_{2} & d_{y}\ {}_{3} & {}_{4}\ d_{y}^{2}\ {}_{3}\ \ d_{z}^{2}\ {}_{2} & d_{xa}d_{y}\ {}_{3} & d_{xb}d_{z}\ {}_{2} \\
d_{z}\ {}_{1} & 0 & d_{xb}\ {}_{3} & d_{xb}d_{y}\ {}_{3} & {}_{5}\ d_{xb}^{2}\ {}_{3}\ \ d_{z}^{2}\ {}_{1} & d_{y}d_{z}\ {}_{1} \\
d_{y}\ {}_{1} & d_{xb}\ {}_{2} & 0 & d_{xb}d_{z}\ {}_{2} & d_{y}d_{z}\ {}_{1} & {}_{6}\ d_{xb}^{2}\ {}_{2}\ \ d_{y}^{2}\ {}_{1}
\end{array}
$$

An image of the full 12x12 matrix is shown below:

$$
K = \left[\begin{array}{cccccc|cccccc}
K_1 & 0 & 0 & 0 & d_zK_1 & -d_yK_1 & -K_1 & 0 & 0 & 0 & -d_zK_1 & d_yK_1 \\
0 & K_2 & 0 & -d_zK_2 & 0 & d_{xa}K_2 & 0 & -K_2 & 0 & d_zK_2 & 0 & d_{xb}K_2 \\
0 & 0 & K_3 & d_yK_3 & -d_{xa}K_3 & 0 & 0 & 0 & -K_3 & -d_yK_3 & -d_{xb}K_3 & 0 \\
0 & -d_zK_2 & d_yK_3 & K_4+d_y^2K_3+d_z^2K_2 & -d_{xa}d_yK_3 & -d_{xa}d_zK_2 & 0 & d_zK_2 & -d_yK_3 & -(K_4+d_y^2K_3+d_z^2K_2) & -d_{xb}d_yK_3 & -d_{xb}d_zK_2 \\
d_zK_1 & 0 & -d_{xa}K_3 & -d_{xa}d_yK_3 & K_5+d_{xa}^2K_3+d_z^2K_1 & -d_yd_zK_1 & -d_zK_1 & 0 & d_{xa}K_3 & d_{xa}d_yK_3 & -K_5+d_{xa}d_{xb}K_3-d_z^2K_1 & d_yd_zK_1 \\
-d_yK_1 & d_{xa}K_2 & 0 & -d_{xa}d_zK_2 & -d_yd_zK_1 & K_6+d_{xa}^2K_2+d_y^2K_1 & d_yK_1 & -d_{xa}K_2 & 0 & d_{xa}d_zK_2 & d_yd_zK_1 & -K_6+d_{xa}d_{xb}K_2-d_y^2K_1 \\
\hline
-K_1 & 0 & 0 & 0 & -d_zK_1 & d_yK_1 & K_1 & 0 & 0 & 0 & d_zK_1 & -d_yK_1 \\
0 & -K_2 & 0 & d_zK_2 & 0 & -d_{xa}K_2 & 0 & K_2 & 0 & -d_zK_2 & 0 & -d_{xb}K_2 \\
0 & 0 & -K_3 & -d_yK_3 & d_{xa}K_3 & 0 & 0 & 0 & K_3 & d_yK_3 & d_{xb}K_3 & 0 \\
0 & d_zK_2 & -d_yK_3 & -(K_4+d_y^2K_3+d_z^2K_2) & d_{xa}d_yK_3 & d_{xa}d_zK_2 & 0 & -d_zK_2 & d_yK_3 & K_4+d_y^2K_3+d_z^2K_2 & d_{xb}d_yK_3 & d_{xb}d_zK_2 \\
-d_zK_1 & 0 & -d_{xb}K_3 & -d_{xb}d_yK_3 & -K_5+d_{xa}d_{xb}K_3-d_z^2K_1 & d_yd_zK_1 & d_zK_1 & 0 & d_{xb}K_3 & d_{xb}d_yK_3 & K_5+d_{xb}^2K_3+d_z^2K_1 & -d_yd_zK_1 \\
d_yK_1 & d_{xb}K_2 & 0 & -d_{xb}d_zK_2 & d_yd_zK_1 & -K_6+d_{xa}d_{xb}K_2-d_y^2K_1 & -d_yK_1 & -d_{xb}K_2 & 0 & d_{xb}d_zK_2 & -d_yd_zK_1 & K_6+d_{xb}^2K_2+d_y^2K_1
\end{array}\right]
$$

Note that the K_{aa} and K_{bb} are symmetric

The element engineering forces can be derived using the figure below:



The engineering forces in the BUSH element are:

$$
\begin{array}{l}
F_{xe} \quad F_{xb} \\
F_{ye} \quad F_{yb} \\
F_{ze} \quad F_{zb} \\
M_{xe} \quad F_{yb} d_z \quad F_{zb} d_y \quad M_{xb} \\
M_{ye} \quad F_{xb} d_z \quad F_{zb} d_{xb} \quad M_{yb} \\
M_{ze} \quad F_{xb} d_y \quad F_{yb} d_{xb} \quad M_{zb}
\end{array}
$$

This can be put into a form which includes all nodal forces as:

$$
\begin{array}{c}
F_{xe} \\ F_{ye} \\ F_{ze} \\ M_{xe} \\ M_{ye} \\ M_{ze}
\end{array}
\quad
\left.\begin{array}{cccccc|cccccc}
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & d_z & d_y & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & d_z & 0 & d_{xb} & 0 & . & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & d_y & d_{xb} & 0 & 0 & 0 & .
\end{array}\right.
\quad
\begin{array}{c}
F_{xa} \\ F_{ya} \\ F_{za} \\ M_{xa} \\ M_{ya} \\ M_{za} \\ \hline F_{xb} \\ F_{yb} \\ F_{zb} \\ M_{xb} \\ M_{yb} \\ M_{zb}
\end{array}
$$

The 6x12 transformation matrix in the above equation is used in MYSTRAN code to transform the nodal forces to engineering forces

The engineering forces in the BUSH element are:

$$
\begin{array}{llll}
F_{xe} & F_{xa} & & \\
F_{ye} & F_{ya} & & \\
F_{ze} & F_{za} & & \\
M_{xe} & F_{ya}d_z & F_{za}d_y & M_{xa} \\
M_{ye} & F_{xa}d_z & F_{za}d_{xa} & M_{ya} \\
M_{ze} & F_{xa}d_y & F_{yb}d_{xa} & M_{za}
\end{array}
$$

This can be put into a form which includes all nodal forces as:

$$
\begin{array}{cccccccc}
F_{xe} & 1 & 0 & 0 & 0 & 0 & 0 & F_{xa} \\
F_{ye} & 0 & 1 & 0 & 0 & 0 & 0 & F_{ya} \\
F_{ze} & 0 & 0 & 1 & 0 & 0 & 0 & F_{za} \\
M_{xe} & 0 & d_z & d_y & 1 & 0 & 0 & M_{xa} \\
M_{ye} & d_z & 0 & d_{xa} & 0 & 1 & 0 & M_{ya} \\
M_{ze} & d_y & d_{xa} & 0 & 0 & 0 & 1 & M_{za}
\end{array}
$$

The 6x transformation matrix in the above equation is used in MYSTRAN code to transform the element engineering forces to nodal forces