ESTIMATE OF CEASELESS SIZE NEEDED FOR TOTAL FACEBOOK COVERAGE

$C =$ Ceaseless users
$F =$ Facebook users

Assume each Facebook user has α friends which are randomly distributed and that $C \subset F$.

$$
\begin{aligned}
|F| &= N \\
|C| &= n
\end{aligned}
$$

For a node x, the probability P(x has no friends in C) is,

$$
\begin{aligned}
P &= (1 - n/N)^{\alpha} = (1 - n/N)^{N(\alpha/N)} \quad \text{for N} \gg 0, \\
P &\approx e^{-\alpha n/N} \text{ because } (1 + \mathrm{x}/\mathrm{N})^{\mathrm{N}} \to \mathrm{e}^{\mathrm{x}} \text{ as N} \to \infty
\end{aligned}
$$

Then, $1 - P =$ probability(x has a friend in C)
E(nodes with a friend in C) $= \mathrm{N}(1 - \mathrm{P}) = \mathrm{N}(1 - \mathrm{e}^{-\alpha\mathrm{n}/\mathrm{N}})$

For each user added to C, the expected prayer coverage gain of Facebook is,

$$E'(n) = \alpha e^{-\alpha n/N}$$

We may expect that all Facebook users are covered with a 50% certainty when,

$$
\begin{aligned}
(1 - P)^N &= 1/2 \\
(1 - e^{-\alpha n/N}) &= (1/2)^{1/N} \\
e^{-\alpha n/N} &= 1 - (1/2)^{1/N} \\
&\approx \frac{\ln 2}{N} \\
\frac{\alpha n}{N} &\approx \ln\left(\frac{N}{\ln 2}\right) \\
n &\approx \frac{N}{\alpha} \ln\left(\frac{N}{\ln 2}\right)
\end{aligned}
$$

How many users are needed to get r coverage of F?

$$\begin{aligned}
N(1-e^{-\alpha n/N}) =& rN \\
1-e^{-\alpha n/N} =& r \\
e^{-\alpha n/N} =& 1-r \\
-\alpha n/N =& \ln(1-r) \\
n =& \frac{N}{\alpha}\ln(1-r)^{-1} \\
n =& \frac{N}{\alpha}\ln\left(\frac{1}{1-r}\right)
\end{aligned}$$

With these assumptions, we can estimate how many people need to use Ceaseless to cover Facebook users in personal prayer.
Let $N = 1.3$ billion Facebook users and $\alpha = 130$ friends per user on average.
Then to have 99% coverage of F we need to have,

$$n = \frac{1.3\times 10^9}{130}\ln\left(\frac{1}{1-.99}\right) \approx 4.605\times 10^7 \approx 47 \text{ million}$$

To have 90% coverage of F we need to have,

$$n = \frac{1.3\times 10^9}{130}\ln\left(\frac{1}{1-.90}\right) \approx 2.303\times 10^7 \approx 24 \text{ million}$$