# Lecture #2(a): Magnetically Coupled Networks

*Theory*

ECE 20200: Linear Circuit Analysis II
Steve Naumov (Instructor)

# Lecture Overview

- This set of slides presented the following
  - Mutual Inductance
    - Defining Mutual Conductance
    - Dotted Sign Convention
    - Energy Considerations
    - Analysis of Magnetically Coupled Networks
  - The Linear Transformer
    - SSS Analysis
    - Reflected Impedances
  - The Ideal Transformer
    - Element Constraints
    - Dot Convention
    - SSS Analysis
    - Reflecting Elements

# Lecture #2(a): Magnetically Coupled Networks
## *Theory*

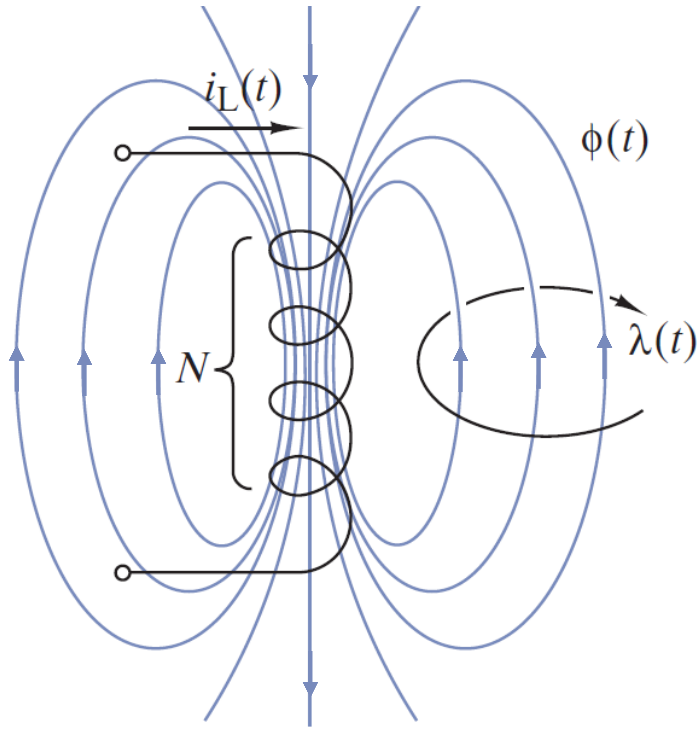
*Mutual Inductance*

# Inductance Review

- When a current $i_L(t)$ passes through a coiled wire having $N$ turns, a magnetic flux $\phi(t)$ is produced in a direction determined by the RHL
- The current $i_L(t)$ is related to the magnetic flux $\phi(t)$ as follows

$$\phi(t) = \mathcal{P} N i_L(t)$$

  - $i_L(t)$: Current through coil measured in Amperes
  - $\phi(t)$: Magnetic flux measured in Webbers
  - $N$: Number of turns of the coil
  - $\mathcal{P}$: Permeance of space occupied by $\phi$. Depends on magnetic properties of space.




- The flux $\phi(t)$ intercepts/links the $N$ turns. This effect is represented by flux linkage $\lambda(t)$ as

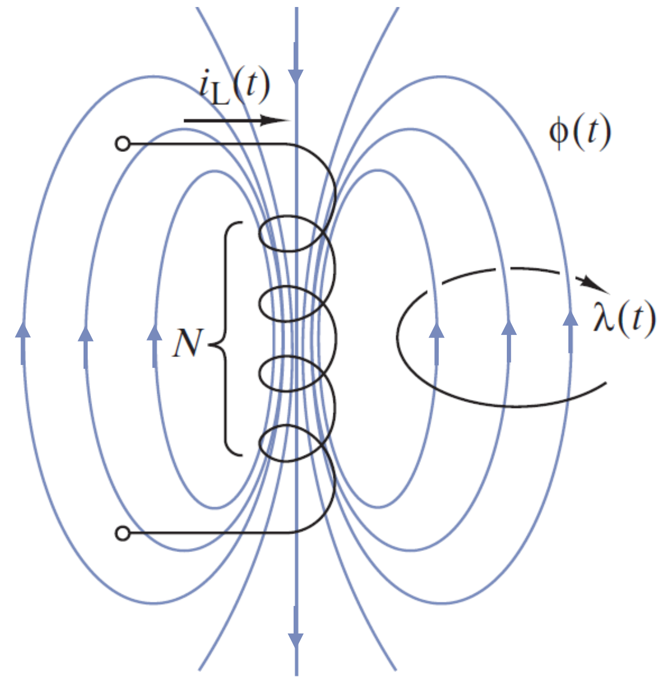
$$\lambda(t) = N\phi(t) \text{ (Webber-Turns)}$$

- Substituting $\phi(t)$ in the above relation yields

$$\lambda(t) = N\phi(t) = N\mathcal{P}Ni_L(t) = \mathcal{P}N^2 i_L(t)$$

# Inductance Review (cont'd)

- The flux linkage $\lambda(t)$ is proportional to the current $i_L(t)$ through the coil
$$\lambda(t) = \mathcal{P}N^2 i_L(t)$$
- The proportionality constant $L = \mathcal{P}N^2$ is known as the as the **self-inductance *L*** of the coil (measured in Henry's). Therefore,
$$\lambda(t) = L i_L(t)$$
- By Faraday's Law, the voltage across the $N$ turns of the coil can be expressed as follows
$$v_L(t) = \frac{d\lambda(t)}{dt}$$
- Therefore, the $v$-$i$ relation of a coil having a constant inductance $L = \mathcal{P}N^2$-is given by
$$v_L(t) = \frac{d\lambda(t)}{dt} = L\frac{di_L(t)}{dt}$$



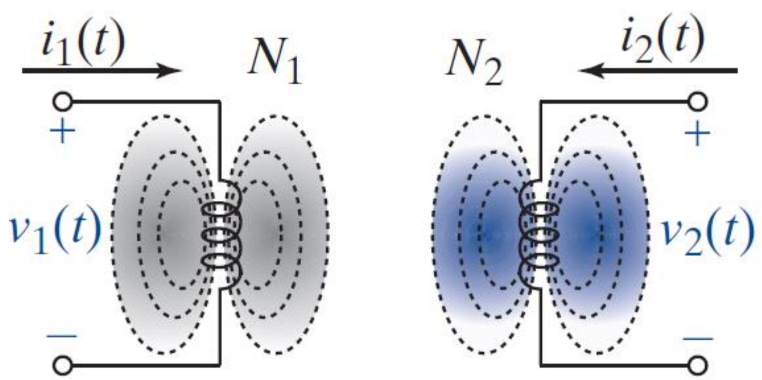
# Decoupled (Non-Neighboring) Coils

- Coil with $N_1$ Turns
  - Current $i_1(t)$ passes through the coil having $N_1$ turns and produces a magnetic flux $\phi_1(t)$

    $$\phi_1(t) = \mathcal{P}_1 N_1 i_1(t)$$
  - Flux $\phi_1(t)$ intercepts or links the $N_1$. The flux linkage $\lambda_1(t)$ is given by

    $$\lambda_1(t) = N_1 \phi_1(t)$$
  - By Faraday's Law, voltage $v_1(t)$ and flux linkage $\lambda_1(t)$ relate as follows

    $$v_1(t) = \frac{d\lambda_1(t)}{dt} = N_1 \frac{d\phi_1(t)}{dt} = [\mathcal{P}_1 N_1 N_1] \frac{di_1(t)}{dt} = [\mathcal{P}_1 N_1^2] \frac{di_1(t)}{dt} = L_1 \frac{di_1(t)}{dt}$$
- Coil with $N_2$ Turns
  - Similar equations can be developed
- Since the two coils are not in close proximity, their fluxes do not interact



# Magnetically Coupled (Neighboring) Coils

- The total magnetic flux for each for each coupled coil is given below

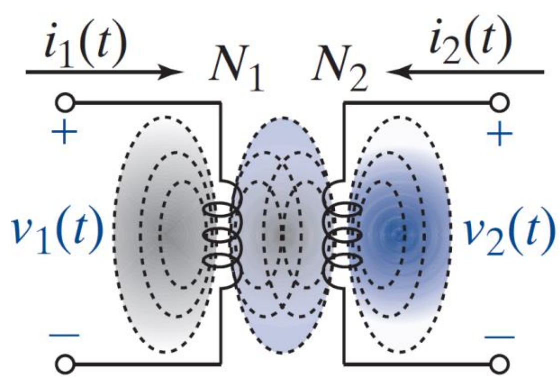
**Coil With Inductance $L_1$**

$$\phi_1 = \phi_{1\to1} + \phi_{2\to1} = \mathcal{P}_1 N_1 i_1 \text{ (total flux)}$$
$$\phi_{1\to1} = \mathcal{P}_{1\to1} N_1 i_1 \text{ (self term)}$$
$$\phi_{2\to1} = \mathcal{P}_{2\to1} N_2 i_2 \text{ (coupling term)}$$

**Coil With Inductance $L_2$**

$$\phi_2 = \phi_{2\to2} + \phi_{1\to2} = \mathcal{P}_2 N_2 i_2 \text{ (total flux)}$$
$$\phi_{2\to2} = \mathcal{P}_{2\to2} N_2 i_2 \text{ (self term)}$$
$$\phi_{1\to2} = \mathcal{P}_{1\to2} N_1 i_1 \text{ (coupling term)}$$

  - $\phi_{1\to1}$: Portion of total flux $\phi_1$ caused by $i_1$ linking only the $N_1$ turns of $L_1$
  - $\phi_{2\to1}$: Portion of total flux $\phi_1$ caused by $i_2$ linking $N_2$ turns of $L_2$ to $N_1$ turns of $L_1$
  - $\phi_{2\to2}$: Portion of total flux $\phi_2$ caused by $i_2$ linking only the $N_2$ turns of $L_2$
  - $\phi_{1\to2}$: Portion of total flux $\phi_2$ caused by $i_1$ linking $N_1$ turns of $L_1$ to $N_2$ turns of $L_2$



# Magnetically Coupled (Neighboring) Coils (cont'd)

- The flux linkage for each coupled coil is given below

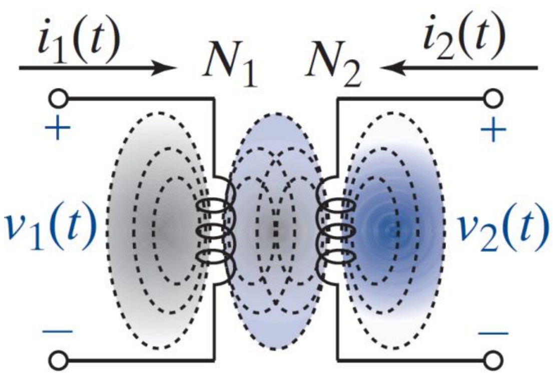
**Coil With Inductance $L_1$**

$\lambda_1 = \lambda_{1\to1} + \lambda_{2\to1} = N_1\phi_1$ (total link)
$\lambda_{1\to1} = N_1\phi_{1\to1}$ (self term)
$\lambda_{2\to1} = N_1\phi_{2\to1}$ (coupling term)

**Coil With Inductance $L_2$**

$\lambda_2 = \lambda_{2\to2} + \lambda_{1\to2} = N_2\phi_2$(total link)
$\lambda_{2\to2} = N_2\phi_{2\to2}$ (self term)
$\lambda_{1\to2} = N_2\phi_{1\to2}$ (coupling term)

- $\lambda_{1\to1}$: Portion of total linkage $\lambda_1$ caused by $\phi_{1\to1}$ linking only the $N_1$ turns of $L_1$
- $\lambda_{2\to1}$: Portion of total linkage $\lambda_1$ caused by $\phi_{2\to1}$ linking the $N_1$ turns of $L_1$
- $\lambda_{2\to2}$: Portion of total linkage $\lambda_2$ caused by $\phi_{2\to2}$ linking only the $N_2$ turns of $L_2$
- $\lambda_{1\to2}$: Portion of total linkage $\lambda_2$ caused by $\phi_{1\to2}$ linking the $N_2$ turns of $L_2$



# Magnetically Coupled (Neighboring) Coils (cont'd)

- By Faraday's Law, the voltage across each coil is as follows

**Coil With Inductance $L_1$**

$v_1 = v_{1\to1} + v_{2\to1}$ (total voltage)

$v_{1\to1} = N_1\phi'_{1\to1} = [\mathcal{P}_{1\to1}N_1^2]i'_1$

$v_{2\to1} = N_1\phi'_{2\to1} = [N_1\mathcal{P}_{2\to1}N_2]i'_2$

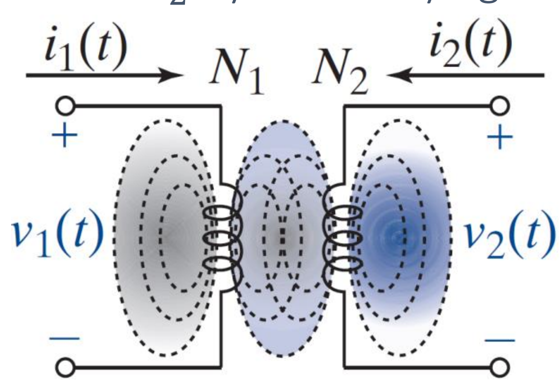
**Coil With Inductance $L_2$**

$v_2 = v_{2\to2} + v_{1\to2}$ (total voltage)

$v_{2\to2} = N_2\phi'_{2\to2} = [\mathcal{P}_{2\to2}N_2^2]i'_2$

$v_{1\to2} = N_2\phi'_{1\to2} = [N_2\mathcal{P}_{1\to2}N_1]i'_1$

  - $v_{1\to1}$: Self-induced voltage across $L_1$ by time-varying current $i_1$ in $L_1$
  - $v_{2\to1}$: Voltage induced across $L_1$ by time-varying $i_2$ in $L_2$
  - $v_{2\to2}$: Self-induced voltage across $L_2$ by time-varying current $i_2$ in $L_2$
  - $v_{1\to2}$: Voltage induced across $L_2$ by time-varying current $i_1$ in $L_1$



# Magnetically Coupled (Neighboring) Coils (cont'd)

- A total of four inductance parameters exist

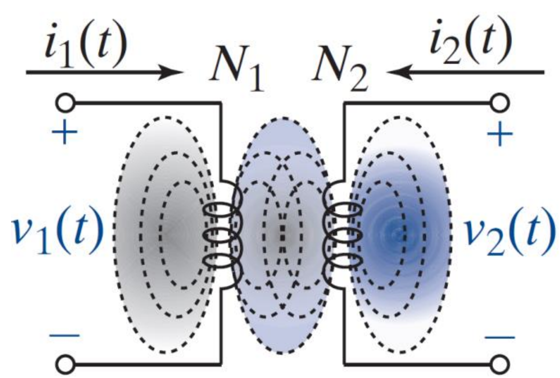
**Coil With Inductance $L_1$**

$$v_1 = v_{1\to1} + v_{2\to1} \text{ (total voltage)}$$
$$v_{1\to1} = \left[\mathcal{P}_{1\to1} N_1^2\right] i'_1 = L_1 i'_1$$
$$v_{2\to1} = \left[\mathcal{P}_{2\to1} N_2 N_1\right] i'_2 = M_{2\to1} i'_2$$

**Coil With Inductance $L_2$**

$$v_2 = v_{2\to2} + v_{1\to2} \text{ (total voltage)}$$
$$v_{2\to2} = \left[\mathcal{P}_{2\to2} N_2^2\right] i'_2 = L_2 i'_2$$
$$v_{1\to2} = \left[\mathcal{P}_{1\to2} N_2 N_1\right] i'_1 = M_{1\to2} i'_1$$

- $L_1$: Self-inductance of the first coil measured in Henry's
- $M_{2\to1}$: Mutual Inductance of coil #1 due to coil #2 measured in Henry's
- $L_2$: Self-inductance of the second coil measured in Henry's
- $M_{1\to2}$: Mutual Inductance of coil #2 due to coil #2 measured in Henry's



# Magnetically Coupled (Neighboring) Coils (cont'd)

- To summary, the total voltage across each coil is given by

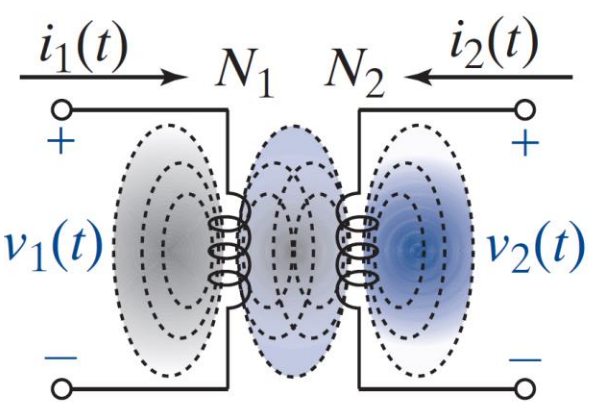
$$v_1(t) = v_{1\to1}(t) + v_{2\to1}(t) = L_1 \frac{di_1(t)}{dt} + M_{2\to1} \frac{di_2(t)}{dt}$$
$$v_2(t) = v_{2\to2}(t) + v_{1\to2}(t) = L_2 \frac{di_2(t)}{dt} + M_{1\to2} \frac{di_1(t)}{dt}$$

- If the magnetic properties of the space surrounding the flux are linear,

$$\mathcal{P}_{2\to1} = \mathcal{P}_{1\to2} = \mathcal{P}_M \qquad M = M_{2\to1} = M_{1\to2} = N_1 N_2 \mathcal{P}_M$$

- Incorporating the above equalities simplifies the voltage equations as follows

$$v_1(t) = v_{1\to1}(t) + v_{2\to1}(t) = L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt}$$
$$v_2(t) = v_{2\to2}(t) + v_{1\to2}(t) = L_2 \frac{di_2(t)}{dt} + M \frac{di_1(t)}{dt}$$



# Magnetically Coupled (Neighboring) Coils (cont'd)

- Although $M, L_1, L_2 \geq 0\mathrm{H}$, voltages $Mi_1'(t)$ and $Mi_2'(t)$ may be positive or negative depending on the rate of change of the respective current
- To account for the signs, we must adjust the voltage equations as follows

$$v_1(t) = v_{1\to1}(t) \pm v_{2\to1}(t) = L_1 \frac{di_1(t)}{dt} \pm M \frac{di_2(t)}{dt}$$

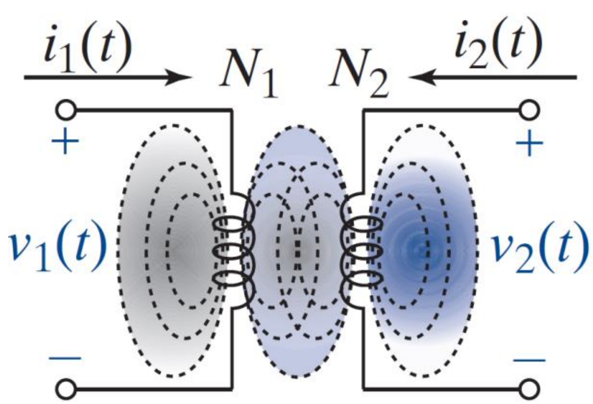
$$v_2(t) = v_{2\to2}(t) \pm v_{1\to2}(t) = L_2 \frac{di_2(t)}{dt} \pm M \frac{di_1(t)}{dt}$$

- How do we find the sign of the self-induced voltages $v_{1\to1}(t)$ and $v_{2\to2}(t)$?

**Passive Sign Convention (PSC)**

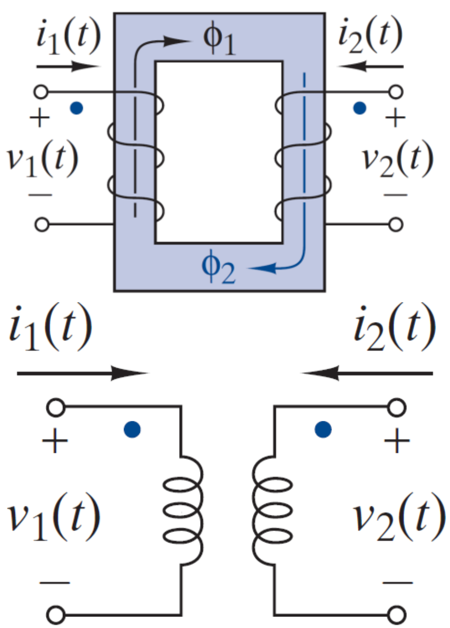
- How do we find the sign of the mutually-induced voltages $v_{2\to1}(t)$ and $v_{1\to2}(t)$?

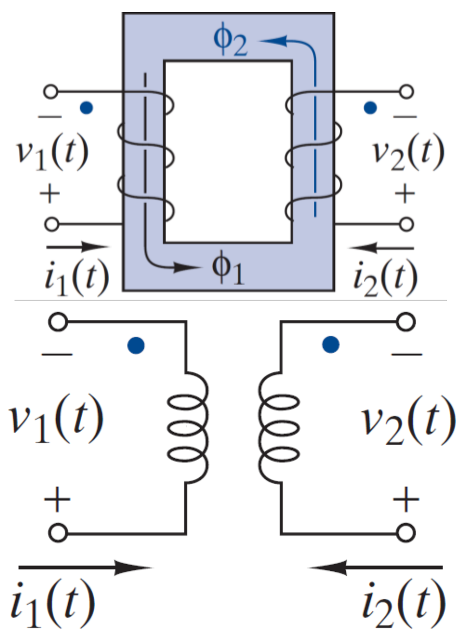
**Dotted Sign Convention (DSC)**



# Dotted Sign Convention (DSC)

- The sign of voltages $Mi_1'(t)$ and $Mi_2'(t)$ depends on two factors:
  - Spatial orientation of the windings (i.e. how the coils are wound)
  - The voltage/current references assigned at the terminals of each coil
- Additive Coupling: Current reference directions are such that flux created by each current is oriented in the **same direction**. For this scenario, the signs of $Mi_1'(t)$ and $Mi_2'(t)$ are both **positive**.

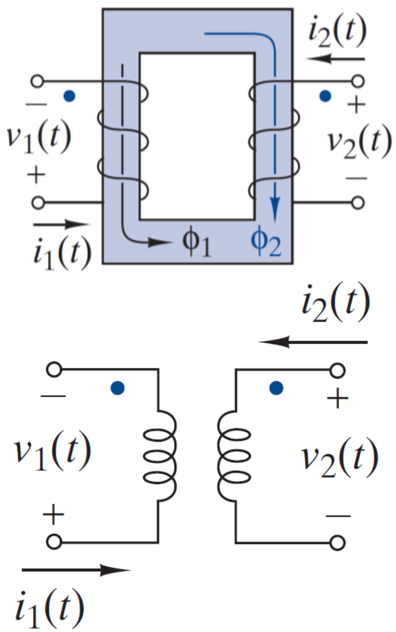


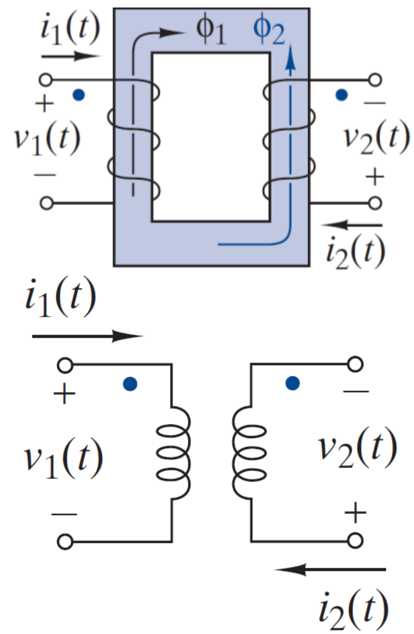




$$v_1(t) = L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt}$$
$$v_2(t) = L_2 \frac{di_2(t)}{dt} + M \frac{di_1(t)}{dt}$$

# Dotted Sign Convention (DSC) (cont'd)

- The sign of voltages $Mi_1'(t)$ and $Mi_2'(t)$ depends on two factors:
  - Spatial orientation of the windings (i.e. how the coils are wound)
  - The voltage/current references assigned at the terminals of each coil
- **Subtractive Coupling**: Current reference directions are such that the flux created by each current is oriented in the **opposite direction**. For this scenario, the signs of $Mi_1'(t)$ and $Mi_2'(t)$ are both **negative**.



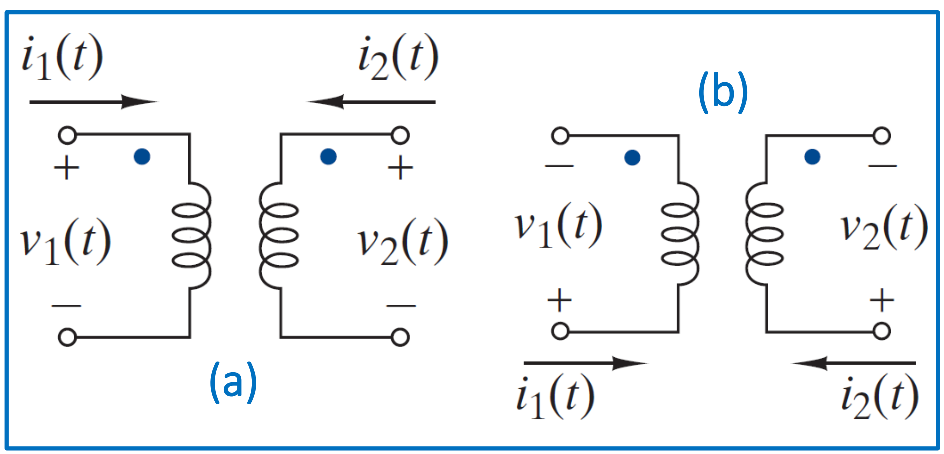




$$v_1(t) = L_1 \frac{di_1(t)}{dt} - M \frac{di_2(t)}{dt}$$
$$v_2(t) = L_2 \frac{di_2(t)}{dt} - M \frac{di_1(t)}{dt}$$

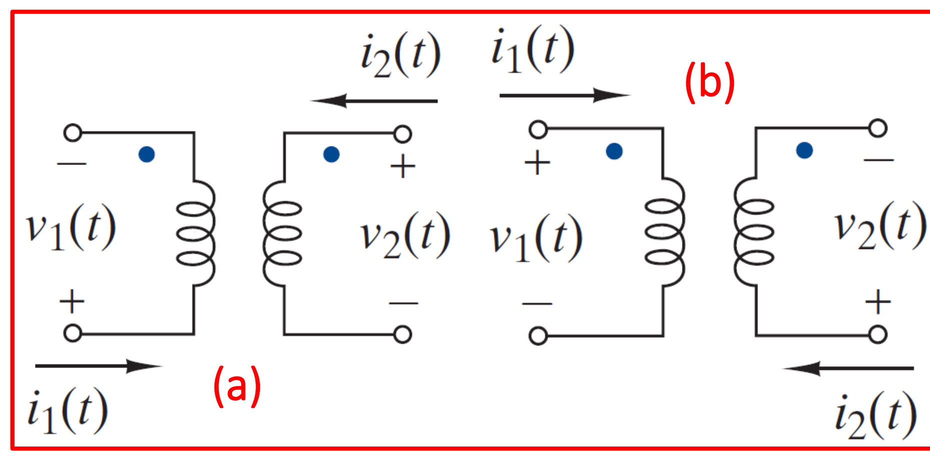
# Dotted Sign Convention (DSC) – Summary

- Current reference directions through the coils and reference polarities of voltages across the coils is chosen according to the passive sign convention.
- The dotted terminal of each coil indicates physical attributes (i.e. the spatial orientation) of the coil windings and **are fixed**.
- The Dot Convention Summarized

> **Mutual inductance is additive** when the **current <u>reference</u> directions both enter or both leave the dotted nodes**, otherwise, the **mutual inductance is subtractive**.




Additive




Subtractive

# Phasor Representation of Voltages

- The total voltage across each coil in the time domain was found to be

$$v_1(t) = v_{1\to1}(t) \pm v_{2\to1}(t) = L_1 \frac{di_1(t)}{dt} \pm M \frac{di_2(t)}{dt}$$
$$v_2(t) = v_{2\to2}(t) \pm v_{1\to2}(t) = L_2 \frac{di_2(t)}{dt} \pm M \frac{di_1(t)}{dt}$$

- If the mutually coupled coils operate in the sinusoidal steady state (SSS) at a radian frequency $\boldsymbol{\omega}$, the total voltage across each coil in the phasor (frequency) domain is as follows

$$\boldsymbol{V_1} = \boldsymbol{V}_{1\to1} \pm \boldsymbol{V_{2\to1}} = j\omega L_1 \boldsymbol{I_1} \pm j\omega M \boldsymbol{I_2}$$

$$\boldsymbol{V_2} = \boldsymbol{V}_{2\to2} \pm \boldsymbol{V_{1\to2}} = j\omega L_2 \boldsymbol{I_2} \pm j\omega M \boldsymbol{I_1}$$

# Lecture #2(a): Magnetically Coupled Networks *Theory*

*Power, Energy, and Coupling Coefficient*

# Power for Magnetically Coupled Coils

- The instantaneous power absorbed by each coil is given as

$$p_1(t) = v_1(t)i_1(t) = L_1 i_1(t)\frac{di_1(t)}{dt} \pm M i_1(t)\frac{di_2(t)}{dt}$$

$$p_2(t) = v_2(t)i_2(t) = L_2 i_2(t)\frac{di_2(t)}{dt} \pm M i_2(t)\frac{di_1(t)}{dt}$$

- The total instantaneous power $p(t)$ absorbed by both coils is

$$p(t) = p_1(t) + p_2(t)$$

$$p(t) = L_1\left[i_1(t)\frac{di_1(t)}{dt}\right] + L_2\left[i_2(t)\frac{di_2(t)}{dt}\right] \pm M\left[i_1(t)\frac{di_2(t)}{dt} + i_2(t)\frac{di_1(t)}{dt}\right]$$

- Each bracketed term above is a perfect derivative. So, $p(t)$ can be written as

$$p(t) = L_1\left[\frac{1}{2}\frac{di_1^2(t)}{dt}\right] + L_2\left[\frac{1}{2}\frac{di_2^2(t)}{dt}\right] \pm M\left[\frac{d[i_1(t)i_2(t)]}{dt}\right]$$

$$p(t) = \frac{d}{dt}\left[\frac{1}{2}L_1 i_1^2(t) + \frac{1}{2}L_2 i_2^2(t) \pm M i_1(t)i_2(t)\right]$$

# Energy for Magnetically Coupled Coils

- The total instantaneous power nd energy can be related as follows

$$p(t) = \frac{d}{dt}w(t) = \frac{d}{dt}\left[\frac{1}{2}L_1 i_1^2(t) + \frac{1}{2}L_2 i_2^2(t) \pm M i_1(t) i_2(t)\right] \Rightarrow$$

$$\boxed{w(t) = 0.5L_1 i_1^2(t) + 0.5L_2 i_2^2(t) \pm M i_1(t) i_2(t)}$$

- Since each coil is a passive element, $w(t) \geq 0$ for all $t$. The energy $w(t)$ may become negative only when the mutual term is negative.

$$w(t) = 0.5L_1 i_1^2(t) + 0.5L_2 i_2^2(t) - M i_1(t) i_2(t) \geq 0$$

- Completing the square gives

$$w(t) = 0.5L_1 i_1^2 - \sqrt{L_1 L_2} i_1 i_2 + 0.5L_2 i_2^2 + \sqrt{L_1 L_2} i_1 i_2 - M i_1 i_2 \geq 0$$

$$w(t) = 0.5\left(\sqrt{L_1} i_1 - \sqrt{L_2} i_2\right)^2 + i_1 i_2\left(\sqrt{L_1 L_2} - M\right) \geq 0$$

- From above, when will $w(t) \geq 0$?

# Mutual Coupling Coefficient $\boldsymbol{\kappa}$

$$w(t) = \frac{1}{2}\left(\sqrt{L_1}i_1(t) - \sqrt{L_2}i_2(t)\right)^2 + i_1(t)i_2(t)\left(\sqrt{L_1L_2} - M\right) \geq 0$$

- From the above inequality, the energy is non-negative provided

$$\boxed{0\ \mathrm{H} \leq M \leq \sqrt{L_1L_2}\ \mathrm{H}}$$

- The above establishes that the mutual inductance magnitude cannot be larger than the geometric mean of the self-inductances of the two coils.
- **Mutual Coupling Coefficient $\boldsymbol{k}$**: Indicates the degree in which $M \to \sqrt{L_1L_2}$. For larger $\boldsymbol{k}$, more flux from one coil couples with the other coil, and vice versa.

$$\boxed{k = M/\sqrt{L_1L_2} \to 0 \leq k \leq 1}$$

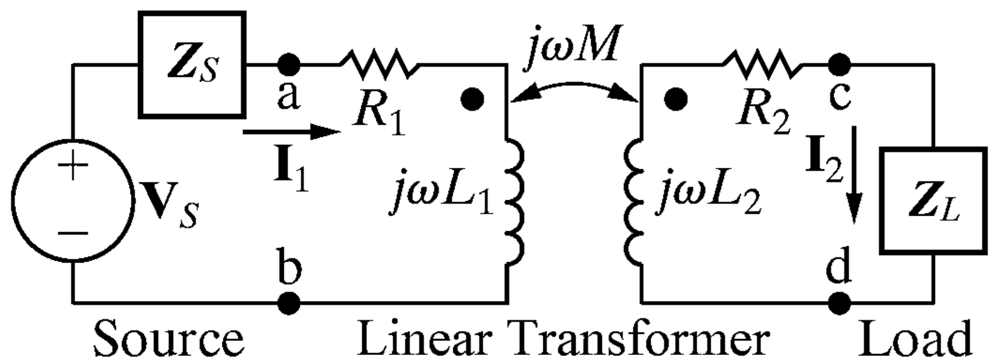
  - Coils are <u>decoupled</u> when $\boldsymbol{k = 0}$
  - Coils are <u>tightly coupled</u> when $\boldsymbol{k > 0.5}$
  - Coils are <u>loosely coupled</u> when $\boldsymbol{k \leq 0.5}$
  - Coils are <u>perfectly coupled</u> when $\boldsymbol{k = 1}$ (physically impossible)

# Lecture #2(a): Magnetically Coupled Networks
*Theory*

*Linear Transformer*

# Linear Transformer

- **Transformer**: A four-terminal (i.e. 2-port) circuit element that models a magnetic device that leverages the mutual inductance phenomenon of two coils in close spatial proximity.
  - **Primary Coil/Winding**: Coil connected to the source side having resistance $R_1$ and self-inductance $L_1$ Henry's
  - **Secondary Coil/Winding**: Coil connected to the load side having resistance $R_2$ and self-inductance $L_2$ Henry's
- **Linear Transformer**: A transformer whose coils are wound around a magnetically linear material and whose coupling coefficient $k \neq 1$
  - e.g. air, plastic, wood, etc.
  - Usually $k$ is a few tenths, but not always.
  - Uses include radios, TVs, and other high frequency communication applications.



# Linear Transformer – SSS Analysis

- Often, transformers operate in the SSS at a particular frequency $\omega$
- Mesh analysis of the "Typical Linear Transformer Network" in SSS gives
  - Mesh @ Primary Network

$$\boldsymbol{V_S} = \boldsymbol{I_1}(\boldsymbol{Z_s} + R_1 + j\omega L_1) - j\omega M\boldsymbol{I_2}$$
$$\boldsymbol{V_S} = \boldsymbol{I_1}\boldsymbol{Z_{11}} - j\omega M\boldsymbol{I_2}$$

$\boldsymbol{Z_{11}}$: Total self-impedance of the mesh network coupled to the primary coil

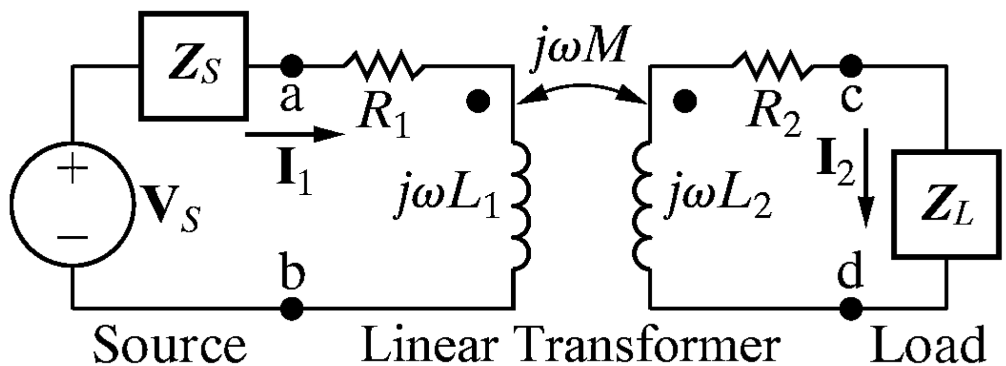
  - Mesh @ Secondary Network

$$\boldsymbol{I_2}(R_2 + j\omega L_2 + \boldsymbol{Z_L}) - j\omega M\boldsymbol{I_1} = 0\text{V}$$
$$-j\omega M\boldsymbol{I_1} + \boldsymbol{I_2}\boldsymbol{Z_{22}} = 0\text{V}$$

$\boldsymbol{Z_{22}}$: Total self-impedance of the mesh network coupled to the secondary coil

- Matrix equation governing the electrical behavior of the linear transformer network is given as

$$\begin{bmatrix} \boldsymbol{Z_{11}} & -j\omega M \\ -j\omega M & \boldsymbol{Z_{22}} \end{bmatrix} \begin{bmatrix} \boldsymbol{I_1} \\ \boldsymbol{I_2} \end{bmatrix} = \begin{bmatrix} \boldsymbol{V_s} \\ 0 \end{bmatrix}$$



# Linear Transformer – Equivalent Primary Network

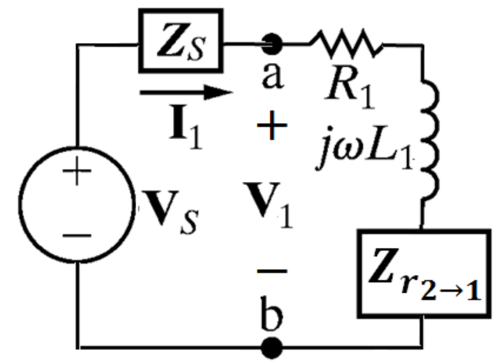
- Equivalent network on primary side can be established for analysis purposes
  - Solve secondary mesh equation for $\boldsymbol{I_2}$

    $\boldsymbol{I_2} = (j\omega M/\boldsymbol{Z_{22}})\boldsymbol{I_1}$
  - Substitute $\boldsymbol{I_2}$ into primary mesh equation

    $\boldsymbol{V_S} = \boldsymbol{I_1}\boldsymbol{Z_{11}} - j\omega M(j\omega M/\boldsymbol{Z_{22}})\boldsymbol{I_1}$
  - Simplify primary mesh equation

    $\boxed{\boldsymbol{V_S} = \left(\boldsymbol{Z_{11}} + [(\omega M)^2/\boldsymbol{Z_{22}}]\right)\boldsymbol{I_1}}$

Equivalent primary network




- Input Impedance

$$\boxed{\boldsymbol{Z_{IN}} = \boldsymbol{V_S}/\boldsymbol{I_1} = \boldsymbol{Z_{11}} + [(\omega M)^2/\boldsymbol{Z_{22}}] = \boldsymbol{Z_{11}} + [(\omega M/|\boldsymbol{Z_{22}}|)^2]\boldsymbol{Z_{22}^*} = \boldsymbol{Z_{11}} + \boldsymbol{Z_{r_{2\to1}}}}$$

  - Reflected Impedance $\boldsymbol{Z_{r_{2\to1}}}$: Additional impedance "seen" by $\boldsymbol{V_S}$ due to non-zero magnetic coupling among coils (as $M, \omega \to 0,\ \boldsymbol{Z_{r_{2\to1}}} \to 0$)
    - Transformer reverses sign of $\mathbf{Im}(\boldsymbol{Z_{22}})$ and scales $\boldsymbol{Z_{22}}$ by a factor $(\omega M/|\boldsymbol{Z_{22}}|)^2$
    - Sign is independent of dot markings on the coils
- Current through Primary

  $\boxed{\boldsymbol{I_1} = \boldsymbol{V_S}/\boldsymbol{Z_{IN}}}$

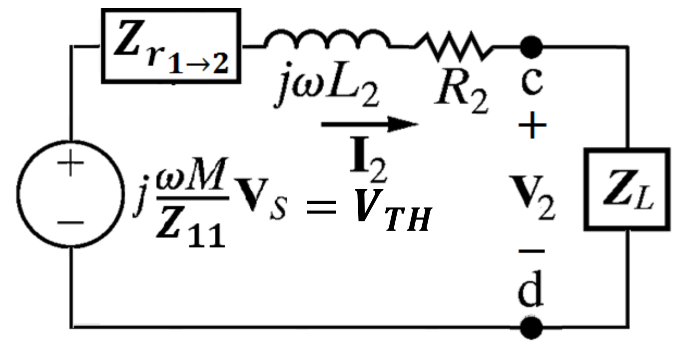
  Voltage across Primary

  $\boxed{\boldsymbol{V_1} = (\boldsymbol{Z_{IN}} - \boldsymbol{Z_S})\,\boldsymbol{I_1}}$

# Linear Transformer – Equivalent Secondary Network

- Equivalent network on secondary side can be established for analysis purposes
  - Solve primary mesh equation for $\boldsymbol{I_1}$
    $\boldsymbol{I_1} = (\boldsymbol{V_S} + j\omega M\boldsymbol{I_2})/\boldsymbol{Z_{11}}$
  - Substitute $\boldsymbol{I_1}$ into secondary mesh equation
    $-j\omega M[(\boldsymbol{V_S} + j\omega M\boldsymbol{I_2})/\boldsymbol{Z_{11}}] + \boldsymbol{I_2}\boldsymbol{Z_{22}} = 0\text{V}$
  - Simplify secondary mesh equation
    $\boxed{(j\omega M/\boldsymbol{Z_{11}})\boldsymbol{V_S} = \left(\boldsymbol{Z_{22}} + [(\omega M)^2/\boldsymbol{Z_{11}}]\right)\boldsymbol{I_2}}$

Equivalent Secondary Network




- Impedance Seen by Load

$$\boxed{\boldsymbol{Z_{TH}} = \boldsymbol{Z_{22}} - \boldsymbol{Z}_L + [(\omega M)^2/\boldsymbol{Z_{11}}] = \boldsymbol{Z_{22}} - \boldsymbol{Z}_L + [(\omega M/|\boldsymbol{Z_{11}}|)^2]\boldsymbol{Z}^*_{\boldsymbol{11}} = \boldsymbol{Z_{22}} - \boldsymbol{Z}_L + \boldsymbol{Z}_{r_{1\to 2}}}$$

  - Reflected Impedance $\boldsymbol{Z_{r_{1\to 2}}}$: Additional impedance "seen" by $\boldsymbol{Z_L}$ due to non-zero magnetic coupling among coils (as $M, \omega \to 0,\ \boldsymbol{Z_{r_{1\to 2}}} \to 0$)
    - Transformer reverses sign of $\mathbf{Im}(\boldsymbol{Z_{11}})$ and scales $\boldsymbol{Z_{11}}$ by a factor $(\omega M/|\boldsymbol{Z_{11}}|)^2$
    - Sign is independent of dot markings on the coils

- Current through Secondary
  $\boxed{\boldsymbol{I_2} = \boldsymbol{V_{TH}}/(\boldsymbol{Z_{TH}} + \boldsymbol{Z_L})}$

Voltage across Secondary
$\boxed{\boldsymbol{V_2} = \boldsymbol{Z_L}\boldsymbol{I_2}}$

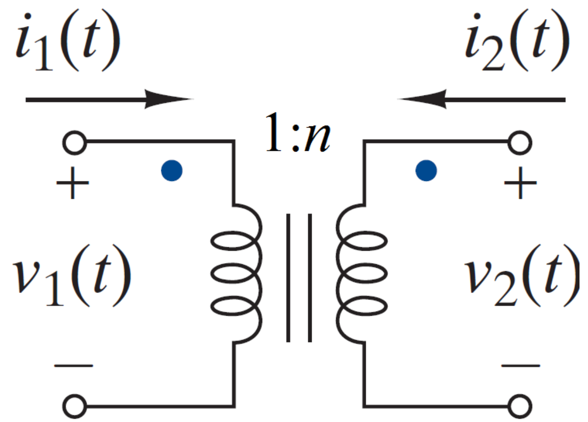
OC Voltage at Port c-d
$\boxed{\boldsymbol{V_{OC}} = (j\omega M/\boldsymbol{Z_{11}})\boldsymbol{V_S}}$

# Lecture #2(a): Magnetically Coupled Networks *Theory*

*Ideal Transformer*

# Ideal Transformer

- **Ideal Transformer**: A transformer that is designed to come as close as possible to the following ideal properties
  - Coils are wound on a linear magnetic material of high permeability (e.g. iron-based material), so coupling coefficient $\kappa = 1$ (i.e. perfect coupling)
  - Coils have "large" number of turns ($N_1, N_2 \to \infty$)
  - Coils have "large" self inductance ($L_1, L_2, M \to \infty$)
  - Each coil exhibits zero (average) power losses (i.e. $R_1 = R_2 = 0$)
- Ideal transformers are
  - Typically referred to as iron-core transformers
  - Used primarily in power-based applications, including power supply design




$1: N_2/N_1$

# Ideal Transformers – Perfect Coupling

- Perfect magnetic coupling (i.e. $k = 1$) means all flux from coil #1 links coil #2.
- Recall, the total flux in each winding is

Flux for Coil #1

$$\phi_1 = \phi_{1\to1} + \phi_{2\to1} = \mathcal{P}_1 N_1 i_1 \text{ (total flux)}$$
$$\phi_{1\to1} = \mathcal{P}_{1\to1} N_1 i_1 \text{ (self term)}$$
$$\phi_{2\to1} = \mathcal{P}_{2\to1} N_2 i_2 \text{ (coupling term)}$$

Flux for Coil #2

$$\phi_2 = \phi_{2\to2} + \phi_{1\to2} = \mathcal{P}_2 N_2 i_2 \text{ (total flux)}$$
$$\phi_{2\to2} = \mathcal{P}_{2\to2} N_2 i_2 \text{ (self term)}$$
$$\phi_{1\to2} = \mathcal{P}_{1\to2} N_1 i_1 \text{ (coupling term)}$$

- When the windings are perfectly coupled

Flux for Coil #1

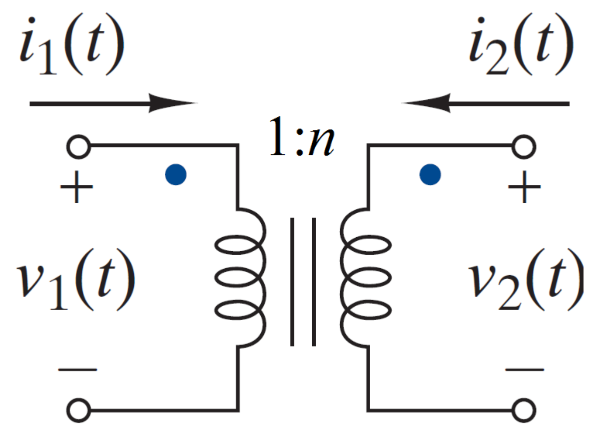
$$\phi_{2\to1} = \phi_2 = \mathcal{P}_{2\to1} N_2 i_2 = \mathcal{P}_2 N_2 i_2$$

Flux for Coil #2

$$\phi_{1\to2} = \phi_2 = \mathcal{P}_{2\to1} N_1 i_1 = \mathcal{P}_1 N_1 i_1$$

- The above equalities imply that

$$\mathcal{P}_{2\to1} = \mathcal{P}_2 = \mathcal{P}_{1\to2} = \mathcal{P}_1 = \mathcal{P}_M$$




$1: N_2/N_1$

# Ideal Transformers – Perfect Coupling (cont'd)

- Substitute a single $\mathcal{P}_M$ into the voltage equations gives

Voltage Across Coil #1

$$v_1 = [\mathcal{P}_M N_1^2] i_1' \pm [\mathcal{P}_M N_1 N_2] i_2'$$

Voltage Across Coil #2

$$v_2 = [\mathcal{P}_M N_2^2] i_2' \pm [\mathcal{P}_M N_1 N_2] i_1'$$

- Factor an $N_1$ out of $v_1$ and an $\pm N_2$ out of $v_2$ yields

Voltage Across Coil #1

$$v_1 = N_1([\mathcal{P}_M N_1] i_1' \pm [\mathcal{P}_M N_2] i_2')$$

Voltage Across Coil #2

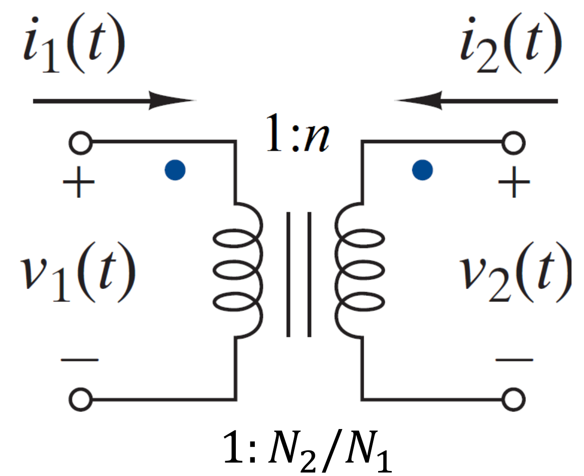
$$v_2 = \pm N_2 \left(\pm[\mathcal{P}_M N_2] i_2' + [\mathcal{P}_M N_1] i_1'\right)$$

- Divide $v_2$ by $v_1$ to yield

$$\frac{v_2}{v_1} = \frac{\pm N_2 \left(\pm[\mathcal{P}_M N_2] i_2' + [\mathcal{P}_M N_1] i_1'\right)}{N_1\left([\mathcal{P}_M N_1] i_1' \pm [\mathcal{P}_M N_2] i_2'\right)}$$

- Therefore, perfect magnetic coupling ($k = 1$) implies

$$\boxed{\frac{v_2(t)}{v_1(t)} = \pm\frac{N_2}{N_1}} \rightarrow \boxed{\frac{v_1}{v_2} = \pm\frac{N_1}{N_2}}$$



# Ideal Transformers – Zero Power Losses

- Ideal transformers consume zero instantaneous power. According to PSC, instantaneous power of both coils must be zero.

$$p(t) = p_1(t) + p_2(t) = v_1(t)i_1(t) + v_2(t)i_2(t) = 0\text{W}$$

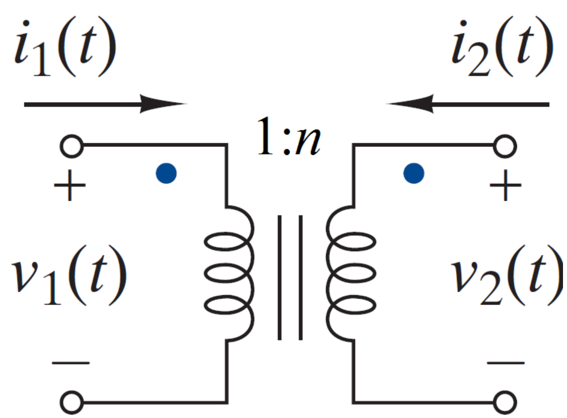
- Above implies coils neither consume nor store energy.
- Rearranging above yields

$$-v_2(t)i_2(t) = v_1(t)i_1(t) \Rightarrow \boxed{\frac{v_2(t)}{v_1(t)} = -\frac{i_1(t)}{i_2(t)}} \rightarrow \boxed{\frac{v_1(t)}{v_2(t)} = -\frac{i_2(t)}{i_1(t)}}$$

- Combine the zero power loss assumption and the perfect coupling assumption to yield

$$\boxed{\frac{v_2(t)}{v_1(t)} = \pm\frac{N_2}{N_1} = -\frac{i_1(t)}{i_2(t)}}$$

$$\boxed{\frac{v_1(t)}{v_2(t)} = \pm\frac{N_1}{N_2} = -\frac{i_2(t)}{i_1(t)}}$$



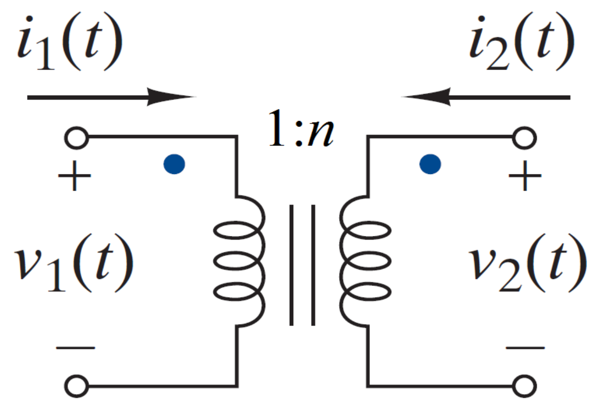

$1: N_2/N_1$

# Ideal Transformers: $i$–$v$ Characteristics

- Ideal Transformer $i$–$v$ Constraint (additive)

$$\boxed{\frac{v_2(t)}{v_1(t)} = \pm\frac{N_2}{N_1} = n}$$

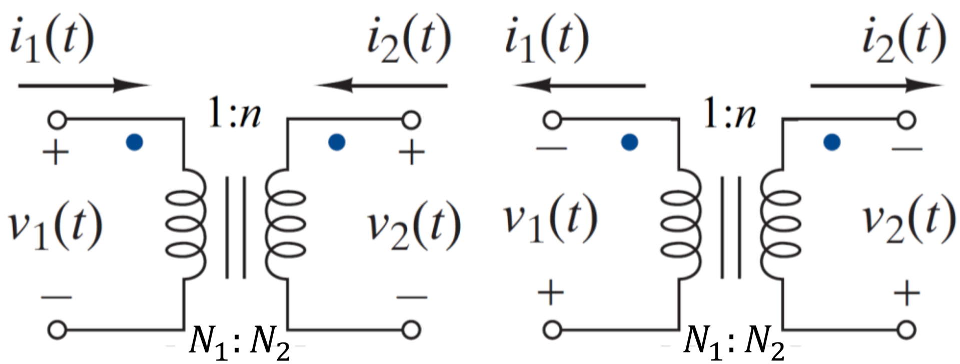
$$\boxed{\frac{i_1(t)}{i_2(t)} = -\frac{N_2}{N_1} = -n}$$




- Some Transformer Terminology
  - **Turns Ratio** $n = N_2/N_1$: The ratio of the number of turns in the secondary coil to the number of turns in the primary coil
  - **Step-up Transformer**: Ideal transformer with $n > 1$ such that it provides a secondary voltage $|v_2(t)|$ greater than the primary voltage $|v_1(t)|$
  - **Step-down Transformer**: Ideal transformer with $n < 1$ such that it provides a secondary voltage $|v_2(t)|$ less than the primary voltage $|v_1(t)|$
  - **Isolation Transformer**: Ideal transformer with $n = 1$ such that it provides a secondary voltage $|v_2(t)|$ that is the same as the primary voltage $|v_1(t)|$

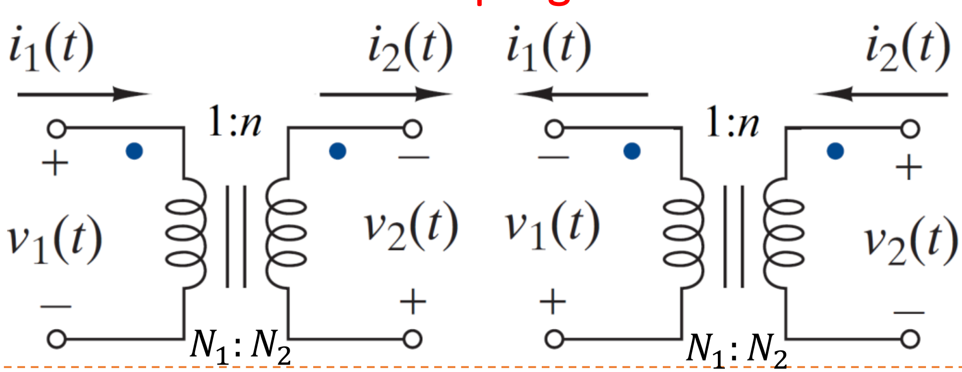
# Ideal Transformers – Dot Convention (Viewpoint #1)

- The preceding characteristics were found assuming additive coupling. Generally, the sign of the constraints are found by using the dot convention.
- **Dot Convention for Ideal Transformer** (Assuming Passive Sign Convention)
  - Additive Mutual Coupling




$$\frac{v_2(t)}{v_1(t)} = +\frac{N_2}{N_1} = +n$$
$$\frac{i_1(t)}{i_2(t)} = -\frac{N_2}{N_1} = -n$$

  - Subtractive Mutual Coupling

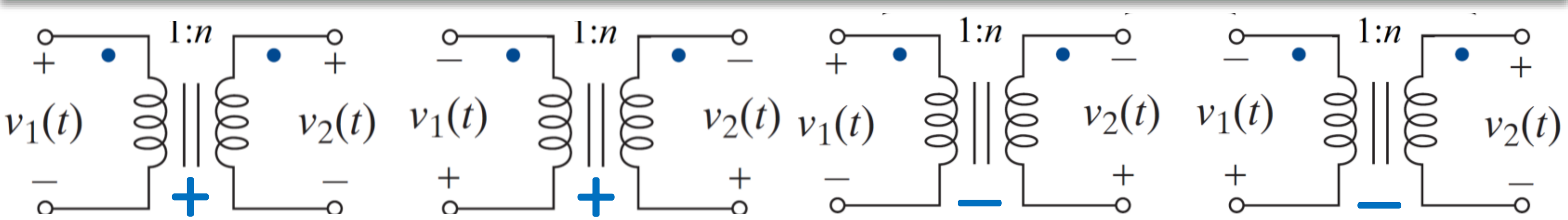



$$\frac{v_2(t)}{v_1(t)} = -\frac{N_2}{N_1} = -n$$
$$\frac{i_1(t)}{i_2(t)} = +\frac{N_2}{N_1} = +n$$

# Ideal Transformers – Dot Convention (Viewpoint #2)

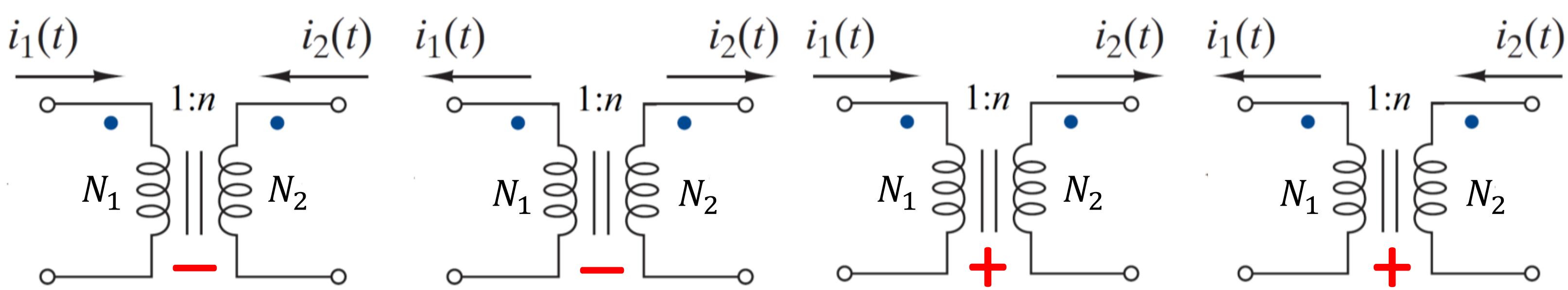
- Dotted Sign Convention for Ideal Transformer (Independent of PSC)
  - Voltage References

Use $+n = +N_2/N_1$ if the ideal transformer voltages are referenced as both positive or both negative at the dotted terminals, otherwise, use $-n = -N_2/N_1$




  - Current References

Use $-n = -N_2/N_1$ if the ideal transformer currents are referenced as both entering or both leaving the dotted terminals, otherwise, use $+n = N_2/N_1$



# Ideal Transformer – SSS Analysis

- KCL analysis of "Typical Ideal Transformer Network" in the SSS
  - KCL @ Node (a) (Primary)

    $$-\boldsymbol{I}_{s1} + (\boldsymbol{V}_1 - \boldsymbol{V}_{S1})/\boldsymbol{Z}_1 + \boldsymbol{I}_1 = 0\text{A}$$

    KCL @ Node (c) (Secondary)

    $$-\boldsymbol{I}_{s2} + (\boldsymbol{V}_2 - \boldsymbol{V}_{S2})/\boldsymbol{Z}_2 + \boldsymbol{I}_2 = 0\text{A}$$

  - Ideal Transformer Voltage Constraint

    $$0\boldsymbol{I}_1 + 0\boldsymbol{I}_2 - (N_2/N_1)\boldsymbol{V}_1 + \boldsymbol{V}_2 = 0\text{V}$$

    Ideal Transformer Current Constraint

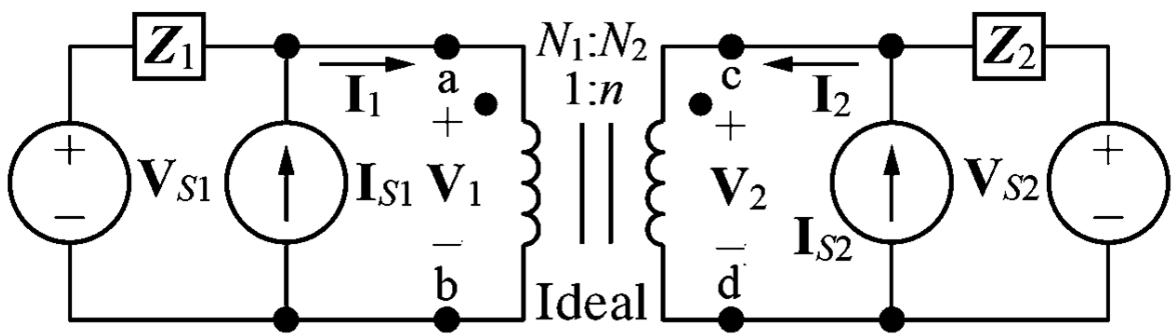
    $$\boldsymbol{I}_1 + (N_2/N_1)\boldsymbol{I}_2 + 0\boldsymbol{V}_1 + 0\boldsymbol{V}_2 = 0\text{A}$$

- Matrix equation governing behavior of typical ideal transformer network

$$\begin{bmatrix} 1 & 0 & \boldsymbol{Z}_1^{-1} & 0 \\ 0 & 1 & 0 & \boldsymbol{Z}_2^{-1} \\ 0 & 0 & -N_2/N_1 & 1 \\ 1 & N_2/N_1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{I}_1 \\ \boldsymbol{I}_2 \\ \boldsymbol{V}_1 \\ \boldsymbol{V}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{I}_{s1} + \boldsymbol{V}_{s1}/\boldsymbol{Z}_1 \\ \boldsymbol{I}_{s2} + \boldsymbol{V}_{s2}/\boldsymbol{Z}_2 \\ 0 \\ 0 \end{bmatrix}$$

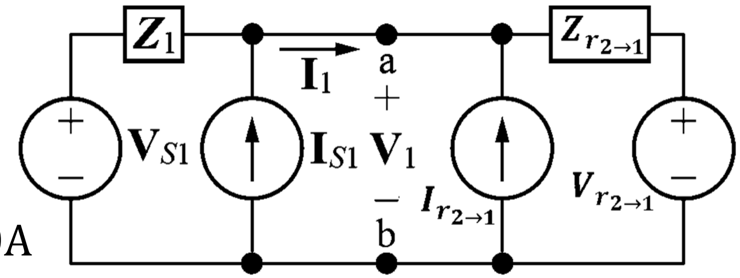
$$\boldsymbol{V}_2/\boldsymbol{V}_1 = N_2/N_1$$

$$\boldsymbol{I}_1/\boldsymbol{I}_2 = -N_2/N_1$$



# Ideal Transformer – Equivalent Primary Network

- Equivalent network on primary side can be established for analysis purposes
  - Sub. for $\boldsymbol{I_2}$ and $\boldsymbol{V_2}$ into secondary EQ.
    $-\boldsymbol{I_{s2}} + [(N_2/N_1)\boldsymbol{V_1} - \boldsymbol{V_{S2}}]/\boldsymbol{Z_2} - (N_1/N_2)\boldsymbol{I_1} = 0\text{A}$
  - Solve above for $\boldsymbol{I_1}$
    $-(N_2/N_1)\boldsymbol{I_{s2}} + (N_2/N_1)[(N_2/N_1)\boldsymbol{V_1} - \boldsymbol{V_{S2}}]/\boldsymbol{Z_2} = \boldsymbol{I_1}$
  - Substitute $\boldsymbol{I_1}$ expression into primary equation
    $-\boldsymbol{I_{s1}} + (\boldsymbol{V_1} - \boldsymbol{V_{S1}})/\boldsymbol{Z_1} - (N_2/N_1)\boldsymbol{I_{s2}} + (N_2/N_1)[(N_2/N_1)\boldsymbol{V_1} - \boldsymbol{V_{S2}}]/\boldsymbol{Z_2} = 0\text{A}$
  - Simplify above equation
    $-\boldsymbol{I_{s1}} + (\boldsymbol{V_1} - \boldsymbol{V_{S1}})/\boldsymbol{Z_1} - (N_2/N_1)\boldsymbol{I_{s2}} + [\boldsymbol{V_1} - (N_1/N_2)\boldsymbol{V_{S2}}]/[(N_1/N_2)^2\boldsymbol{Z_2}] = 0\text{A}$
    $\boxed{-\boldsymbol{I_{s1}} + (\boldsymbol{V_1} - \boldsymbol{V_{S1}})/\boldsymbol{Z_1} - \boldsymbol{I_{r_{2\to1}}} + (\boldsymbol{V_1} - \boldsymbol{V_{r_{2\to1}}})/\boldsymbol{Z_{r_{2\to1}}} = 0\text{A}}$




Equivalent Primary Network

- **Reflected Elements:** Elements "appearing" on opposite side of transformer due to ideal transformer operation
  - Impedance $\boldsymbol{Z_{r_{2\to1}}} = (N_1/N_2)^2\boldsymbol{Z_2}$: Secondary impedance seen by primary
  - Current Source $\boldsymbol{I_{r_{2\to1}}} = (N_2/N_1)\boldsymbol{I_{s2}}$: Secondary current source seen by primary
  - Voltage Source $\boldsymbol{V_{r_{2\to1}}} = (N_1/N_2)\boldsymbol{V_{s2}}$: Secondary voltage source seen by primary

# Ideal Transformer – Equivalent Secondary Network

- Equivalent network on secondary side can be established for analysis purposes
  - Sub. for $\boldsymbol{I_1}$ and $\boldsymbol{V_1}$ into primary EQ.

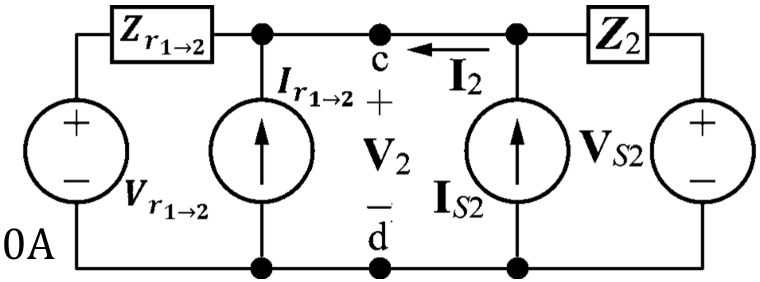
    $-\boldsymbol{I_{s1}} + [(N_1/N_2)\boldsymbol{V_2} - \boldsymbol{V_{s1}}]/\boldsymbol{Z_1} - (N_2/N_1)\boldsymbol{I_2} = 0\text{A}$
  - Solve above for $\boldsymbol{I_2}$

    $-(N_1/N_2)\boldsymbol{I_{s1}} + (N_1/N_2)[(N_1/N_2)\boldsymbol{V_2} - \boldsymbol{V_{s1}}]/\boldsymbol{Z_1} = \boldsymbol{I_2}$
  - Substitute $\boldsymbol{I_2}$ expression into secondary equation

    $-\boldsymbol{I_{s2}} + (\boldsymbol{V_2} - \boldsymbol{V_{s2}})/\boldsymbol{Z_2} - (N_1/N_2)\boldsymbol{I_{s1}} + (N_1/N_2)[(N_1/N_2)\boldsymbol{V_2} - \boldsymbol{V_{s1}}]/\boldsymbol{Z_1}$
  - Simplify above equation

    $-\boldsymbol{I_{s2}} + (\boldsymbol{V_2} - \boldsymbol{V_{s2}})/\boldsymbol{Z_2} - (N_1/N_2)\boldsymbol{I_{s1}} + [\boldsymbol{V_2} - (N_2/N_1)\boldsymbol{V_{s1}}]/[(N_2/N_1)^2\boldsymbol{Z_1}] = 0\text{A}$

    $\boxed{-\boldsymbol{I_{s2}} + (\boldsymbol{V_2} - \boldsymbol{V_{s2}})/\boldsymbol{Z_1} - \boldsymbol{I_{r_{1\to2}}} + (\boldsymbol{V_2} - \boldsymbol{V_{r_{1\to2}}})/\boldsymbol{Z_{r_{1\to2}}} = 0\text{A}}$




Equivalent Secondary Network

- **Reflected Elements:** Elements "appearing" on opposite side of transformer due to ideal transformer operation
  - Impedance $\boldsymbol{Z_{r_{1\to2}}} = (N_2/N_1)^2\boldsymbol{Z_1}$: Primary impedance seen by secondary
  - Current Source $\boldsymbol{I_{r_{1\to2}}} = (N_1/N_2)\boldsymbol{I_{s1}}$: Primary current source seen by secondary
  - Voltage Source $\boldsymbol{V_{r_{1\to2}}} = (N_2/N_1)\boldsymbol{V_{s1}}$: Primary voltage source seen by secondary

# Ideal Transformer – SSS Analysis

- KCL analysis of "Typical Ideal Transformer Network" in the SSS
  - KCL @ Node (a) (Primary)

    $$-\boldsymbol{I}_{s1} + (\boldsymbol{V}_1 - \boldsymbol{V}_{S1})/\boldsymbol{Z}_1 + \boldsymbol{I}_1 = 0\text{A}$$

    KCL @ Node (c) (Secondary)

    $$-\boldsymbol{I}_{s2} + (\boldsymbol{V}_2 - \boldsymbol{V}_{S2})/\boldsymbol{Z}_2 + \boldsymbol{I}_2 = 0\text{A}$$

  - Ideal Transformer Voltage Constraint

    $$0\boldsymbol{I}_1 + 0\boldsymbol{I}_2 + (N_2/N_1)\boldsymbol{V}_1 + \boldsymbol{V}_2 = 0\text{V}$$

    Ideal Transformer Current Constraint

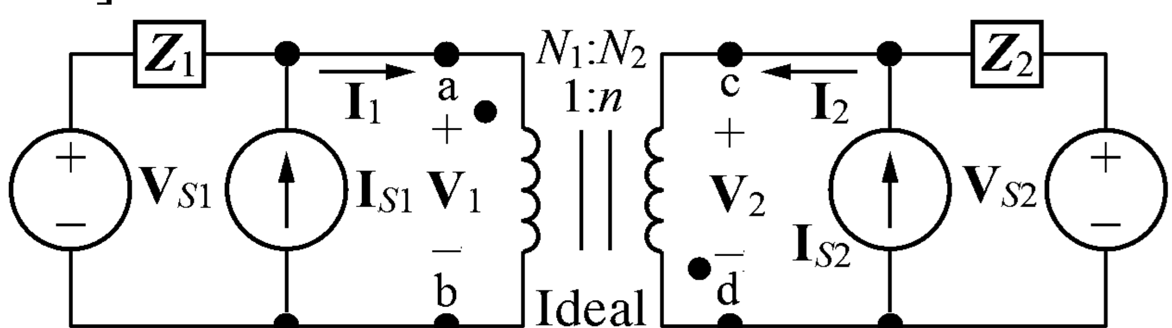
    $$\boldsymbol{I}_1 - (N_2/N_1)\boldsymbol{I}_2 + 0\boldsymbol{V}_1 + 0\boldsymbol{V}_2 = 0\text{A}$$

- Matrix equation governing behavior of typical ideal transformer network

$$\begin{bmatrix} 1 & 0 & \boldsymbol{Z}_1^{-1} & 0 \\ 0 & 1 & 0 & \boldsymbol{Z}_2^{-1} \\ 0 & 0 & N_2/N_1 & 1 \\ 1 & -N_2/N_1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{I}_1 \\ \boldsymbol{I}_2 \\ \boldsymbol{V}_1 \\ \boldsymbol{V}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{I}_{s1} + \boldsymbol{V}_{s1}/\boldsymbol{Z}_1 \\ \boldsymbol{I}_{s2} + \boldsymbol{V}_{s2}/\boldsymbol{Z}_2 \\ 0 \\ 0 \end{bmatrix}$$

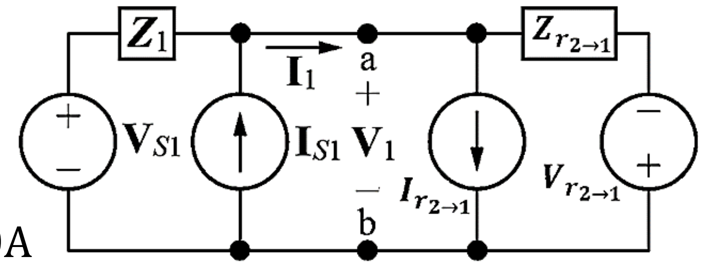
$$\boldsymbol{V}_2/\boldsymbol{V}_1 = -N_2/N_1$$

$$\boldsymbol{I}_1/\boldsymbol{I}_2 = N_2/N_1$$



# Ideal Transformer – Equivalent Primary Network

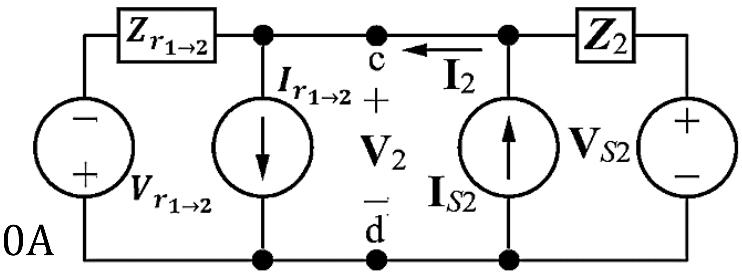
- Equivalent network on primary side can be established for analysis purposes
  - Sub. for $\boldsymbol{I_2}$ and $\boldsymbol{V_2}$ into secondary EQ.
    $$-\boldsymbol{I_{s2}} + [-(N_2/N_1)\boldsymbol{V_1} - \boldsymbol{V_{s2}}]/\boldsymbol{Z_2} + (N_1/N_2)\boldsymbol{I_1} = 0\text{A}$$
  - Solve above for $\boldsymbol{I_1}$
    $$(N_2/N_1)\boldsymbol{I_{s2}} + (N_2/N_1)[(N_2/N_1)\boldsymbol{V_1} + \boldsymbol{V_{s2}}]/\boldsymbol{Z_2} = \boldsymbol{I_1}$$
  - Substitute $\boldsymbol{I_1}$ expression into primary equation
    $$-\boldsymbol{I_{s1}} + (\boldsymbol{V_1} - \boldsymbol{V_{s1}})/\boldsymbol{Z_1} + (N_2/N_1)\boldsymbol{I_{s2}} + (N_2/N_1)[(N_2/N_1)\boldsymbol{V_1} + \boldsymbol{V_{s2}}]/\boldsymbol{Z_2} = 0\text{A}$$
  - Simplify above equation
    $$-\boldsymbol{I_{s1}} + (\boldsymbol{V_1} - \boldsymbol{V_{s1}})/\boldsymbol{Z_1} + (N_2/N_1)\boldsymbol{I_{s2}} + [\boldsymbol{V_1} + (N_1/N_2)\boldsymbol{V_{s2}}]/[(N_1/N_2)^2\boldsymbol{Z_2}] = 0\text{A}$$
    $$\boxed{-\boldsymbol{I_{s1}} + (\boldsymbol{V_1} - \boldsymbol{V_{s1}})/\boldsymbol{Z_1} \boxed{+} \boldsymbol{I_{r_{2\to1}}} + (\boldsymbol{V_1} \boxed{+} \boldsymbol{V_{r_{2\to1}}})/\boldsymbol{Z_{r_{2\to1}}} = 0\text{A}}$$




Equivalent Primary Network

- **Reflected Elements:** Elements "appearing" on opposite side of transformer due to ideal transformer operation
  - Impedance $\boldsymbol{Z_{r_{2\to1}}} = (N_1/N_2)^2\boldsymbol{Z_2}$: Secondary impedance seen on primary
  - Current Source $\boldsymbol{I_{r_{2\to1}}} = -(N_2/N_1)\boldsymbol{I_{s2}}$: Secondary current source seen on primary
  - Voltage Source $\boldsymbol{V_{r_{2\to1}}} = -(N_1/N_2)\boldsymbol{V_{s2}}$: Secondary voltage source seen on primary

# Ideal Transformer – Equivalent Secondary Network

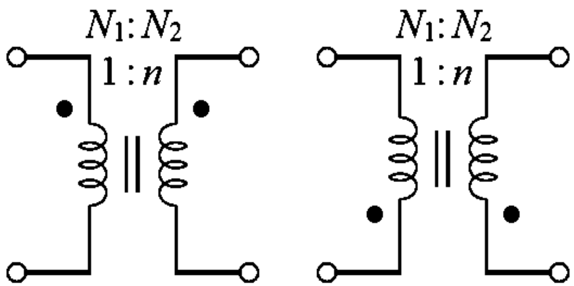



Equivalent Secondary Network

- Equivalent network on secondary side can be established for analysis purposes
  - Sub. for $\boldsymbol{I_1}$ and $\boldsymbol{V_1}$ into primary EQ.
    $$-\boldsymbol{I_{s1}} + [-(N_1/N_2)\boldsymbol{V_2} - \boldsymbol{V_{s1}}]/\boldsymbol{Z_1} + (N_2/N_1)\boldsymbol{I_2} = 0\text{A}$$
  - Solve above for $\boldsymbol{I_2}$
    $$(N_1/N_2)\boldsymbol{I_{s1}} - (N_1/N_2)[-(N_1/N_2)\boldsymbol{V_2} - \boldsymbol{V_{s1}}]/\boldsymbol{Z_1} = \boldsymbol{I_2}$$
  - Substitute $\boldsymbol{I_2}$ expression into secondary equation
    $$-\boldsymbol{I_{s2}} + (\boldsymbol{V_2} - \boldsymbol{V_{s2}})/\boldsymbol{Z_2} + (N_1/N_2)\boldsymbol{I_{s1}} + (N_1/N_2)[(N_1/N_2)\boldsymbol{V_2} + \boldsymbol{V_{s1}}]/\boldsymbol{Z_1}$$
  - Simplify above equation
    $$-\boldsymbol{I_{s2}} + (\boldsymbol{V_2} - \boldsymbol{V_{s2}})/\boldsymbol{Z_2} + (N_1/N_2)\boldsymbol{I_{s1}} + [\boldsymbol{V_2} + (N_2/N_1)\boldsymbol{V_{s1}}]/[(N_2/N_1)^2\boldsymbol{Z_1}] = 0\text{A}$$
    $$\boxed{-\boldsymbol{I_{s2}} + (\boldsymbol{V_2} - \boldsymbol{V_{s2}})/\boldsymbol{Z_1} \boxed{+} \boldsymbol{I_{r_{1\to2}}} + (\boldsymbol{V_2} \boxed{+} \boldsymbol{V_{r_{1\to2}}})/\boldsymbol{Z_{r_{1\to2}}} = 0\text{A}}$$
- **Reflected Elements:** Elements "appearing" on opposite side of transformer due to ideal transformer operation
  - Impedance $\boldsymbol{Z_{r_{1\to2}}} = (N_2/N_1)^2\boldsymbol{Z_1}$: Primary impedance seen by secondary
  - Current Source $\boldsymbol{I_{r_{1\to2}}} = -(N_1/N_2)\boldsymbol{I_{s1}}$: Primary current source by secondary
  - Voltage Source $\boldsymbol{V_{r_{1\to2}}} = -(N_2/N_1)\boldsymbol{V_{s1}}$: Primary voltage source seen by secondary

# Reflecting Elements – Summary (1/2)

- Rules for Reflecting Elements <u>to Primary Side</u>




Rule for Impedances

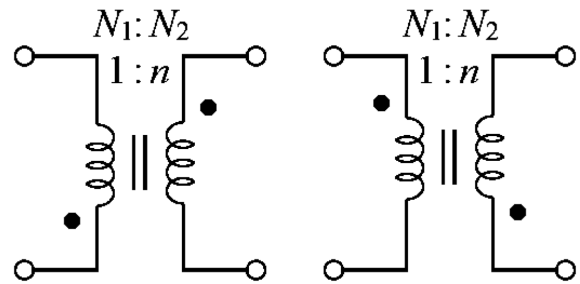
$$\boxed{\boldsymbol{Z}_{\boldsymbol{r}_{2\to 1}} = \boldsymbol{Z}_2(N_1/N_2)^2 = \boldsymbol{Z}_2/n^2}$$

Rule for Voltage Sources

$$\boxed{\boldsymbol{V}_{\boldsymbol{r}_{2\to 1}} = \boldsymbol{V}_{s2}(N_1/N_2) = \boldsymbol{V}_{s2}/n}$$

Rule for Current Sources

$$\boxed{\boldsymbol{I}_{\boldsymbol{r}_{2\to 1}} = \boldsymbol{I}_{s2}(N_2/N_1) = n\boldsymbol{I}_{s2}}$$




Rule for Impedances

$$\boxed{\boldsymbol{Z}_{\boldsymbol{r}_{2\to 1}} = \boldsymbol{Z}_2(N_1/N_2)^2 = \boldsymbol{Z}_2/n^2}$$

Rule for Voltage Sources

$$\boxed{\boldsymbol{V}_{\boldsymbol{r}_{2\to 1}} = -\boldsymbol{V}_{s2}(N_1/N_2) = -\boldsymbol{V}_{s2}/n}$$

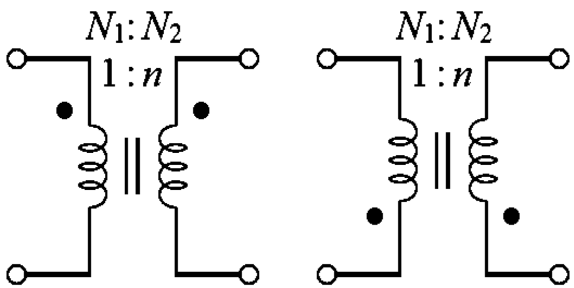
Rule for Current Sources

$$\boxed{\boldsymbol{I}_{\boldsymbol{r}_{2\to 1}} = -\boldsymbol{I}_{s2}(N_2/N_1) = -n\boldsymbol{I}_{s2}}$$

- Notes About Reflection
  - Order of elements must be preserved when reflecting
  - Reflection rules also apply to dependent sources
  - <u>Cannot be applied</u> if primary and secondary is <u>electrically coupled</u>!

# Reflecting Elements – Summary (2/2)

- Rules for Reflecting Elements <u>to Secondary Side</u>




Rule for Impedances

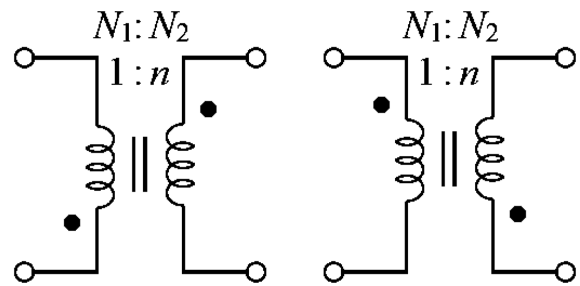
$$\boxed{\boldsymbol{Z}_{\boldsymbol{r}_{\mathbf{1}\to\mathbf{2}}} = \boldsymbol{Z}_{\mathbf{1}}(N_2/N_1)^2 = n^2\boldsymbol{Z}_{\mathbf{1}}}$$

Rule for Voltage Sources

$$\boxed{\boldsymbol{V}_{\boldsymbol{r}_{\mathbf{1}\to\mathbf{2}}} = \boldsymbol{V}_{\boldsymbol{s}\mathbf{1}}(N_2/N_1) = n\boldsymbol{V}_{\boldsymbol{s}\mathbf{1}}}$$

Rule for Current Sources

$$\boxed{\boldsymbol{I}_{\boldsymbol{r}_{\mathbf{1}\to\mathbf{2}}} = \boldsymbol{I}_{\boldsymbol{s}\mathbf{1}}(N_1/N_2) = \boldsymbol{I}_{\boldsymbol{s}\mathbf{1}}/n}$$




Rule for Impedances

$$\boxed{\boldsymbol{Z}_{\boldsymbol{r}_{\mathbf{1}\to\mathbf{2}}} = \boldsymbol{Z}_{\mathbf{1}}(N_2/N_1)^2 = n^2\boldsymbol{Z}_{\mathbf{1}}}$$

Rule for Voltage Sources

$$\boxed{\boldsymbol{V}_{\boldsymbol{r}_{\mathbf{1}\to\mathbf{2}}} = -\boldsymbol{V}_{\boldsymbol{s}\mathbf{1}}(N_2/N_1) = -n\boldsymbol{V}_{\boldsymbol{s}\mathbf{1}}}$$

Rule for Current Sources

$$\boxed{\boldsymbol{I}_{\boldsymbol{r}_{\mathbf{1}\to\mathbf{2}}} = -\boldsymbol{I}_{\boldsymbol{s}\mathbf{1}}(N_1/N_2) = -\boldsymbol{I}_{\boldsymbol{s}\mathbf{1}}/n}$$

- Notes About Reflection
  - Order of elements must be preserved when reflecting
  - Reflection rules also apply to dependent sources
  - <u>Cannot be applied</u> if primary and secondary is <u>electrically coupled</u>!

# Lecture Summary

- This set of slides presented the following
  - Mutual Inductance
    - Defining Mutual Conductance
    - Dotted Sign Convention
    - Energy Considerations
    - Analysis of Magnetically Coupled Networks
  - The Linear Transformer
    - SSS Analysis
    - Reflected Impedances
  - The Ideal Transformer
    - Element Constraints
    - Dot Convention
    - SSS Analysis
    - Reflecting Elements