HOMEWORK #5: Inverse Laplace Transforms, Poles/Zeros, and {I,F}VT

1. "Simple" Inverse Laplace Transforms

Compute the inverse Laplace Transform of each of the following rational functions of a complex frequency. Completing the square may be required, but partial fraction expansion is unnecessary.

(a) $\boldsymbol{F(s)} = \frac{3}{(2\boldsymbol{s}-5)^5}$

(b) $\boldsymbol{F(s)} = \frac{3\boldsymbol{s}+1}{\boldsymbol{s}+4}$

(c) $\boldsymbol{F(s)} = \frac{\boldsymbol{s}-5}{\boldsymbol{s}^2+4\boldsymbol{s}+5}$

(d) $\boldsymbol{F(s)} = \frac{2\boldsymbol{s}^4+3\boldsymbol{s}^3-\boldsymbol{s}^2+8\boldsymbol{s}+4}{\boldsymbol{s}^3}$

(e) $\boldsymbol{F(s)} = \frac{\boldsymbol{s}(1+e^{-\pi \boldsymbol{s}})}{\boldsymbol{s}^2+4\boldsymbol{s}+5}$

2. Inverse Laplace Transforms via Partial Fraction Expansion

Compute the right sided time functions corresponding to each of the following rational functions of a complex frequency. **Verify all partial fraction expansion results with MATLAB**.

(a) Strictly Proper, Distinct Real Poles

i. $\boldsymbol{F(s)} = \frac{2\boldsymbol{s}^3+33\boldsymbol{s}^2+93\boldsymbol{s}+54}{\boldsymbol{s}(\boldsymbol{s}+1)(\boldsymbol{s}^2+5\boldsymbol{s}+6)}$

(b) Strictly Proper, Repeated Real Poles

i. $\boldsymbol{F(s)} = \frac{2\boldsymbol{s}^2+4\boldsymbol{s}+1}{(\boldsymbol{s}+1)(\boldsymbol{s}+2)^3}$

(c) Strictly Proper, Distinct Complex Poles (**Complex Number Method**)

i. $\boldsymbol{F(s)} = \frac{-\boldsymbol{s}^2+52\boldsymbol{s}+445}{\boldsymbol{s}(\boldsymbol{s}^2+10\boldsymbol{s}+89)}$

ii. $\boldsymbol{F(s)} = \frac{14\boldsymbol{s}^2+56\boldsymbol{s}+152}{(\boldsymbol{s}+6)(\boldsymbol{s}^2+4\boldsymbol{s}+20)}$

(d) Strictly Proper, Distinct Complex Poles (**Real Number Method**)

i. $\boldsymbol{F(s)} = \frac{20\boldsymbol{s}+40}{\boldsymbol{s}(\boldsymbol{s}^2+6\boldsymbol{s}+25)}$

ii. $\boldsymbol{F(s)} = \frac{\boldsymbol{s}+1}{(\boldsymbol{s}+2)(\boldsymbol{s}^2+2\boldsymbol{s}+5)}$

(e) Proper/Improper

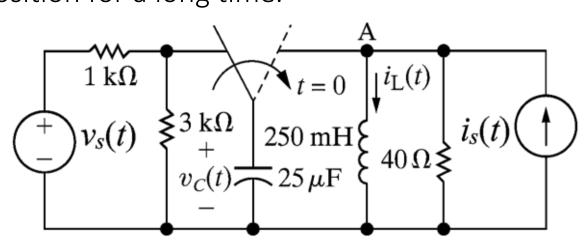
i. $\boldsymbol{F(s)} = \frac{5\boldsymbol{s}^3+20\boldsymbol{s}^2-49\boldsymbol{s}-108}{\boldsymbol{s}^2+7\boldsymbol{s}+10}$

3. Inverse Laplace Transforms, Integro-Differential Equations, and Network Analysis

(a) Consider the second order network shown with $v_s(t) = 100\text{V}$ and $i_s(t) = 100\text{ mA}$. The switch moves to the "right" position after being in the "left" position for a long time.

i. Analyze the network at time $t = 0^-$ to compute the state variable values $v_C(0^-)$ and $i_L(0^-)$.

ii. Analyze the network for $t > 0^-$ using nodal analysis at node A to obtain an integro-differential equation that describes the voltage $v_C(t)u(t)$ for $t > 0^-$.



iii. Take the Laplace Transform of the equation found in (ii) and compute the complete capacitor voltage response transform $\boldsymbol{V_C(s)} = \mathcal{L}\{v_C(t)u(t)\}$. As part of your computation, identify the characteristic polynomial of $\boldsymbol{V_C(s)}$, the zero state component $\boldsymbol{V_{C,ZS}(s)}$ of $\boldsymbol{V_C(s)}$, and the zero input component $\boldsymbol{V_{C,ZI}(s)}$ of $\boldsymbol{V_C(s)}$.

iv. Compute the complete capacitor voltage response $v_C(t)u(t)$ by taking the inverse Laplace Transform of the complete capacitor voltage response transform $\boldsymbol{V_C(s)}$.

v. Write an expression that relates the complete inductor current response $i_L(t)u(t)$ to the complete capacitor voltage response $v_C(t)u(t)$. Then use the expression for $i_L(t)u(t)$ to compute complete inductor current response transform $\boldsymbol{I_L(s)} = \mathcal{L}\{i_L(t)u(t)\}$.

vi. Compute the complete inductor current response $i_L(t)u(t)$ by taking the inverse Laplace Transform of the complete inductor current response transform $\boldsymbol{I_L(s)}$.

4. Pole-Zero Representation of Rational Functions and Pole-Zero Diagrams

(a) Consider the rational function $\boldsymbol{F(s)} = \boldsymbol{N(s)}/\boldsymbol{D(s)}$ of a complex frequency variable shown below.

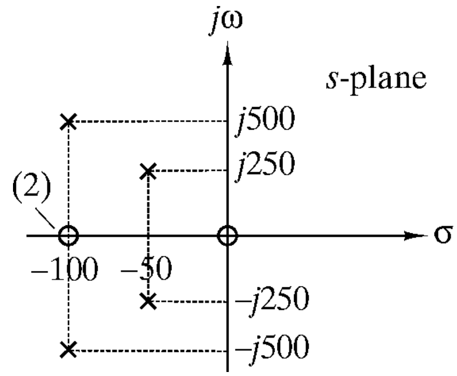
$$\boldsymbol{F}(\boldsymbol{s}) = \frac{(8\boldsymbol{s}+40)(4\boldsymbol{s}^2+8\boldsymbol{s}+36)}{(2\boldsymbol{s}+14)(\boldsymbol{s}+3)(\boldsymbol{s}^2+5\boldsymbol{s}+6)}$$

i. Compute the scale (gain) factor K.

ii. Find the poles (finite, infinite) of $\boldsymbol{F(s)}$.

iii. Find the zeros (finite, infinite) of $\boldsymbol{F(s)}$.

iv. Sketch the pole-zero diagram for $\boldsymbol{F(s)}$. Include any infinite poles and zeros in your sketch. Then, use MATLAB and the **`pzplot2()`** user-defined function file from Blackboard Learn to create a pole-zero diagram of $\boldsymbol{F(s)}$.



(b) Consider the pole-zero diagram of $\boldsymbol{F(s)} = \boldsymbol{N(s)}/\boldsymbol{D(s)}$ shown above (on the right). Compute the expression for $\boldsymbol{F(s)}$ if $\boldsymbol{F}(150) = \frac{400}{41}$.

5. Initial and Final Value Theorems

Compute, if possible, $f_k(0^+)$ and $f_k(\infty)$ of the right-sided time function corresponding to each of the following rational functions of a complex frequency. If it is not possible, briefly explain why.

(a) $\boldsymbol{F_1(s)} = \frac{s+3}{s^2+s}$ (b) $\boldsymbol{F_2(s)} = \frac{5}{(s+1)(s^2+9)}$ (c) $\boldsymbol{F_3(s)} = \frac{3s^3+6s^2+12s+3}{s(s+3)^2}$