HOMEWORK #5: Inverse Laplace Transforms, Poles/Zeros, and {I,F}VT (SOLUTIONS)

Due Date: Sunday/Monday, June 12/13, 2016 (midnight)

1. "Simple" Inverse Laplace Transforms

Compute the inverse Laplace Transform of each of the following rational functions of a complex frequency. Completing the square may be required, but partial fraction expansion is unnecessary.

(a) $\boldsymbol{F(s)} = \frac{3}{(2s-5)^5} = \frac{3}{(2s-5)^5} = \frac{3}{(2(s-2.5))^5} = \frac{3}{2^5}\frac{1}{(s-2.5)^5} = \frac{3}{2^5 4!}\frac{4!}{(s-2.5)^5}$

$$\mathcal{L}^{-1}\{\boldsymbol{F(s)}\} = f(t) = \frac{3}{2^5 4!}\mathcal{L}^{-1}\left\{\frac{4!}{(s-2.5)^5}\right\}u(t) = \frac{3}{2^5 4!}e^{2.5t}\mathcal{L}^{-1}\left\{\frac{4!}{s^5}\right\}u(t) = \frac{3}{2^5 4!}e^{2.5t}t^4u(t)$$

$$\boxed{\mathcal{L}^{-1}\{\boldsymbol{F(s)}\} = f(t) = \frac{1}{256}e^{2.5t}t^4u(t)}$$

(b) $\boldsymbol{F(s)} = \frac{3s+1}{s+4} = \frac{3s+1}{s+4} = 3 - \frac{11}{s+4}\ (long\ division)$

$$\mathcal{L}^{-1}\{\boldsymbol{F(s)}\} = f(t) = 3\mathcal{L}^{-1}\{1\}u(t) - 11\mathcal{L}^{-1}\left\{\frac{1}{s+4}\right\}u(t)$$

$$\boxed{\mathcal{L}^{-1}\{\boldsymbol{F(s)}\} = f(t) = 3\delta(t) - 11e^{-4t}u(t)}$$

(c) $\boldsymbol{F(s)} = \frac{s-5}{s^2+4s+5} = \frac{s-5}{s^2+4s+5} = \frac{s-5}{s^2+4s+4-4+5} = \frac{s-5}{(s+2)^2+1} = \frac{(s+2-2)-5}{(s+2)^2+1} = \frac{(s+2)-7}{(s+2)^2+1}$

$$\mathcal{L}^{-1}\{\boldsymbol{F(s)}\} = \mathcal{L}^{-1}\left\{\frac{(s+2)}{(s+2)^2+(1)^2}\right\}u(t) - 7\mathcal{L}^{-1}\left\{\frac{1}{(s+2)^2+(1)^2}\right\}u(t)$$

$$\mathcal{L}^{-1}\{\boldsymbol{F(s)}\} = e^{-2t}\mathcal{L}^{-1}\left\{\frac{s}{s^2+(1)^2}\right\}u(t) - 7e^{-2t}\mathcal{L}^{-1}\left\{\frac{1}{s^2+(1)^2}\right\}u(t)$$

$$\boxed{\mathcal{L}^{-1}\{\boldsymbol{F(s)}\} = f(t) = e^{-2t}[\cos(t) - 7\sin(t)]u(t)}$$

$$\mathcal{L}^{-1}\{\boldsymbol{F(s)}\} = f(t) = e^{-2t}\left[\mathcal{Re}\{e^{jt}\} - \mathcal{Re}\{7e^{j(t-90^\circ)}\}\right]u(t) = e^{-2t}\left[\mathcal{Re}\{e^{jt} - 3e^{j(t-90^\circ)}\}\right]u(t)$$

$$\mathcal{L}^{-1}\{\boldsymbol{F(s)}\} = f(t) = e^{-2t}\left[\mathcal{Re}\{e^{jt} - 7e^{jt}e^{-j90^\circ}\}\right]u(t) = e^{-2t}\left[\mathcal{Re}\{e^{jt}(1-7e^{-j90^\circ})\}\right]u(t)$$

$$\mathcal{L}^{-1}\{\boldsymbol{F(s)}\} = f(t) = e^{-2t}\left[\mathcal{Re}\{e^{jt}(1+j7)\}\right]u(t) = e^{-2t}\left[\mathcal{Re}\{5\sqrt{2}e^{jtan^{-1}(7)}e^{jt}\}\right]u(t)$$

$$\mathcal{L}^{-1}\{\boldsymbol{F(s)}\} = f(t) = e^{-2t}\left[\mathcal{Re}\{5\sqrt{2}e^{j(t+tan^{-1}(7))}\}\right]u(t) \approx e^{-2t}\left[\mathcal{Re}\{7.07e^{j(t+81.87^\circ)}\}\right]u(t)$$

$$\boxed{\mathcal{L}^{-1}\{\boldsymbol{F(s)}\} = f(t) = 5\sqrt{2}e^{-2t}cos(t+\tan^{-1}(7))u(t) \approx 7.07e^{-2t}\cos(t+81.87^\circ)u(t)}$$

(d) $\boldsymbol{F(s)} = \frac{2s^4+3s^3-s^2+8s+4}{s^3} = 2s + 3 - \frac{1}{s} + \frac{8}{s^2} + \frac{4}{s^3}$

$$\mathcal{L}^{-1}\{\boldsymbol{F(s)}\} = f(t) = \mathcal{L}^{-1}\left\{2s + 3 - \frac{1}{s} + \frac{8}{s^2} + \frac{4}{s^3}\right\}$$

$$\mathcal{L}^{-1}\{\boldsymbol{F(s)}\} = f(t) = \left[2\mathcal{L}^{-1}\{s\} + 3\mathcal{L}^{-1}\{1\} - \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + \frac{8}{1!}\mathcal{L}^{-1}\left\{\frac{1!}{s^2}\right\} + \frac{4}{2!}\mathcal{L}^{-1}\left\{\frac{2!}{s^3}\right\}\right]u(t)$$

$$\mathcal{L}^{-1}\{\boldsymbol{F(s)}\} = f(t) = \left[2\frac{d}{dt}\mathcal{L}^{-1}\{1\} + 3\mathcal{L}^{-1}\{1\} - \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + \frac{8}{1!}\mathcal{L}^{-1}\left\{\frac{1!}{s^2}\right\} + \frac{4}{2!}\mathcal{L}^{-1}\left\{\frac{2!}{s^3}\right\}\right]u(t)$$

$$\mathcal{L}^{-1}\{\boldsymbol{F(s)}\} = f(t) = \left[2\frac{d}{dt}\delta(t) + 3\delta(t) - 1 + 8t + 2t^2\right]u(t)$$

$$\boxed{\mathcal{L}^{-1}\{\boldsymbol{F(s)}\} = f(t) = 2\frac{d}{dt}\delta(t) + 3\delta(t) + [2t^2 + 8t - 1]u(t)}$$

(e) $\boldsymbol{F(s)} = \frac{\boldsymbol{s}(1+e^{-\pi \boldsymbol{s}})}{\boldsymbol{s}^2+4\boldsymbol{s}+5}$

$$\boldsymbol{F(s)} = \frac{\boldsymbol{s} + \boldsymbol{s}e^{-\pi\boldsymbol{s}}}{\boldsymbol{s}^2 + 4\boldsymbol{s} + 5} = \frac{\boldsymbol{s} + \boldsymbol{s}e^{-\pi\boldsymbol{s}}}{(\boldsymbol{s}+2)^2 + 1} = \frac{\boldsymbol{s}}{(\boldsymbol{s}+2)^2 + 1} + \frac{\boldsymbol{s}}{(\boldsymbol{s}+2)^2 + 1}e^{-\pi\boldsymbol{s}}$$

$$\boldsymbol{F(s)} = \frac{(\boldsymbol{s}+2) - 2}{(\boldsymbol{s}+2)^2 + 1} + \frac{(\boldsymbol{s}+2-2)}{(\boldsymbol{s}+2)^2 + 1}e^{-\pi\boldsymbol{s}} = \frac{(\boldsymbol{s}+2) - 2}{(\boldsymbol{s}+2)^2 + 1} + e^{-\pi\boldsymbol{s}}\frac{(\boldsymbol{s}+2) - 2}{(\boldsymbol{s}+2)^2 + 1}$$

$$\boldsymbol{F(s)} = \boldsymbol{F_a(s)} + \boldsymbol{F_a(s)}e^{-\pi\boldsymbol{s}}$$

$$\mathcal{L}^{-1}\{\boldsymbol{F(s)}\} = f(t) = \mathcal{L}^{-1}\{\boldsymbol{F_a(s)}\}u(t) + \mathcal{L}^{-1}\{\boldsymbol{F_a(s)}\}u(t)|_{t=t-\pi}$$

$$\mathcal{L}^{-1}\{\boldsymbol{F_a}(s)\} = f_a(t) = \mathcal{L}^{-1}\left\{\frac{(\boldsymbol{s}+2) - 2}{(\boldsymbol{s}+2)^2 + 1}\right\}u(t) = e^{-2t}\mathcal{L}^{-1}\left\{\frac{\boldsymbol{s} - 2}{\boldsymbol{s}^2 + 1}\right\}u(t)$$

$$\mathcal{L}^{-1}\{\boldsymbol{F_a}(s)\} = f_a(t) = e^{-2t}[\cos(t) - 2\sin(t)]u(t)$$

$$\mathcal{L}^{-1}\{\boldsymbol{F_a(s)}\}|_{t=t-\pi} = f_a(t)|_{t=t-\pi} = [e^{-2t}[\cos(t) - 2\sin(t)]u(t)]|_{t=t-\pi}$$

$$\mathcal{L}^{-1}\{\boldsymbol{F_a(s)}\}|_{t=t-\pi} = f_a(t)|_{t=t-\pi} = e^{-2(t-\pi)}[\cos(t-\pi) - 2\sin(t-\pi)]u(t-\pi)$$

$$\boxed{f(t) = e^{-2t}[\cos(t) - 2\sin(t)]u(t) + e^{-2(t-\pi)}[\cos(t-\pi) - 2\sin(t-\pi)]u(t-\pi)}$$

$$\mathcal{L}^{-1}\{\boldsymbol{F_a(s)}\}u(t)|_{t=t-\pi} = f_a(t)|_{t=t-\pi} = e^{2\pi}e^{-2t}[-\cos(t) + 2\sin(t)]u(t-\pi)$$

$$\mathcal{L}^{-1}\{\boldsymbol{F_a(s)}\}u(t)|_{t=t-\pi} = f_a(t)|_{t=t-\pi} = -e^{2\pi}e^{-2t}[\cos(t) - 2\sin(t)]u(t-\pi)$$

$$\mathcal{L}^{-1}\{\boldsymbol{F_a(s)}\}|_{t=t-\pi} = -e^{2\pi}e^{-2t}f_a(t)u(t-\pi)$$

$$\mathcal{L}^{-1}\{\boldsymbol{F(s)}\} = f(t) = e^{-2t}f_a(t)u(t) - e^{2\pi}e^{-2t}f_a(t)u(t-\pi)$$

$$\mathcal{L}^{-1}\{\boldsymbol{F(s)}\} = f(t) = e^{-2t}f_a(t)[u(t) - e^{2\pi}u(t-\pi)]$$

$$\boxed{f(t) = e^{-2t}[\cos(t) - 2\sin(t)][u(t) - e^{2\pi}u(t-\pi)]}$$

2. Inverse Laplace Transforms via Partial Fraction Expansion

Compute the right sided time functions corresponding to each of the following rational functions of a complex frequency. Verify all partial fraction expansion results with MATLAB.

(a) Strictly Proper, Distinct Real Poles

i. $\boldsymbol{F(s)} = \frac{2s^3+33s^2+93s+54}{s(s+1)(s^2+5s+6)}$

$$F(s) = \frac{K_1}{s} + \frac{K_2}{s+1} + \frac{K_3}{s+2} + \frac{K_4}{s+3}$$

$$K_1 = \frac{2s^3+33s^2+93s+54}{(s+1)(s+2)(s+3)}\Big|_{s=0} = 9$$

$$K_2 = \frac{2s^3+33s^2+93s+54}{s(s+2)(s+3)}\Big|_{s=-1} = 4$$

$$K_3 = \frac{2s^3+33s^2+93s+54}{s(s+1)(s+3)}\Big|_{s=-2} = -8$$

$$K_4 = \frac{2s^3+33s^2+93s+54}{s(s+1)(s+2)}\Big|_{s=-3} = -3$$

$$f(t) = [9 + 4e^{-t} - 8e^{-2t} - 3e^{-3t}]u(t)$$

```
>> num = [2,33,93,54];
>> d1 = [1,0];
>> d2 = [1,1];
>> d3 = [1,5,6];
>> den = conv(conv(d1,d2),d3);
>> [r,p,k] = residue(num, den)
r = -3.0000
    -8.0000
     4.0000
     9.0000
p = -3.0000
    -2.0000
    -1.0000
     0
k = []
```

(b) Strictly Proper, Repeated Real Poles

i. $\boldsymbol{F(s)} = \frac{2s^2+4s+1}{(s+1)(s+2)^3}$

$$F(s) = \frac{2s^2+4s+1}{(s+1)(s+2)^3} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{(s+2)^2} + \frac{D}{(s+2)^3}$$

$$A = F(s)(s+1)\big|_{s=-1} = -1$$

$$D = F(s)(s+2)^3\big|_{s=-2} = -1$$

$$2s^2+4s+1 = A(s+2)(s^2+4s+4) + B(s+1)(s^2+4s+4) + C(s+1)(s+2) + D(s+1)$$

Equating coefficients :

s^3: $\quad 0 = A + B \longrightarrow B = -A = 1$

s^2: $\quad 2 = 6A + 5B + C = A + C \longrightarrow C = 2 - A = 3$

s^1: $\quad 4 = 12A + 8B + 3C + D = 4A + 3C + D$

$\quad 4 = 6 + A + D \longrightarrow D = -2 - A = -1$

s^0: $\quad 1 = 8A + 4B + 2C + D = 4A + 2C + D = -4 + 6 - 1 = 1$

$$F(s) = \frac{-1}{s+1} + \frac{1}{s+2} + \frac{3}{(s+2)^2} - \frac{1}{(s+2)^3}$$

```
>> num = [2,4,1];
>> d1 = [1,1];
>> d2 = [1,2];
>> den = conv(conv(conv(d2,d2),d2),d1);
>> [r,p,k] = residue(num, den)
r =  1.0000
     3.0000
    -1.0000
    -1.0000
p = -2.0000
    -2.0000
    -2.0000
    -1.0000
k = []
```

$$\boxed{\boldsymbol{f(t) = [-e^{-t} + e^{-2t}(1 + 3t - 0.5t^2)]u(t)}}$$

(c) Strictly Proper, Distinct Complex Poles (**Complex Number Method**)

i. $\boldsymbol{F(s) = \frac{-s^2+52s+445}{s(s^2+10s+89)}}$

$$F(s) = \frac{K_1}{s} + \frac{K_2}{s+5-j8} + \frac{K_2^*}{s+5+j8}$$

$$K_1 = \left.\frac{-s^2+52s+445}{s^2+10s+89}\right|_{s=0} = 5$$

$$K_2 = \left.\frac{-s^2+52s+445}{s(s+5+j8)}\right|_{s=-5+j8} = -3-j2 = 3.6\underline{/-146.31^\circ}$$

$$f(t) = [5 + 7.2e^{-5t}\cos(8t - 146.31^\circ)]u(t)$$

```
>> num = [-1,52, 445];
>> d1 = [1,0];
>> d2 = [1,10,89];
>> den = conv(d1,d2);
>> [r,p,k] = residue(num, den)
r = -3.0000 - 2.0000i
    -3.0000 + 2.0000i
     5.0000 + 0.0000i
p = -5.0000 + 8.0000i
    -5.0000 - 8.0000i
     0.0000 + 0.0000i
k = []
>> r_mag = abs(r)
   r_mag = 3.6056
           3.6056
           5.0000
>> r_theta = angle(r)*(180/pi)
   R_theta = -146.3099
              146.3099
              0
```

ii. $\boldsymbol{F(s) = \frac{14s^2+56s+152}{(s+6)(s^2+4s+20)}}$

$$F(s) = \frac{K_1}{s+6} + \frac{K_2}{s+2-j4} + \frac{K_2^*}{s+2+j4}$$

$$K_1 = \left.\frac{14s^2+56s+152}{s^2+4s+20}\right|_{s=-6} = 10$$

$$K_2 = \left.\frac{14s^2+56s+152}{(s+6)(s+2+j4)}\right|_{s=-2+j4} = 2+j2 = 2.83\underline{/45^\circ}$$

$$f(t) = [10e^{-6t} + 5.66e^{-2t}\cos(4t + 45^\circ)]u(t)$$

```
>> num = [14,56,152];
>> d1 = [1,6];
>> d2 = [1,4,20];
>> den = conv(d1,d2);
>> [r,p,k] = residue(num, den)
r = 10.0000 + 0.0000i
     2.0000 + 2.0000i
     2.0000 - 2.0000i
p = -6.0000 + 0.0000i
    -2.0000 + 4.0000i
    -2.0000 - 4.0000i
k = []
>> r_mag = abs(r)
   r_mag =  10.0000
             2.8284
             2.8284
>> r_theta = angle(r)*(180/pi)
   r_theta = 0
             45.0000
            -45.0000
```

(d) Strictly Proper, Distinct Complex Poles (**Real Number Method**)

i. $\boldsymbol{F(s) = \frac{20s+40}{s(s^2+6s+25)}}$

$$F(s) = \frac{20(s+2)}{s(s^2+6s+25)} = \frac{A}{s} + \frac{Bs+C}{s^2+6s+25}$$

$$20(s+2) = A(s^2+6s+25) + Bs^2 + Cs$$

Equating components,

s^2: $0 = A + B$ or $B = -A$

s: $20 = 6A + C$

constant: $40 = 25A$ or $A = 8/5$, $B = -8/5$, $C = 20 - 6A = 52/5$

$$F(s) = \frac{8}{5s} + \frac{-\frac{8}{5}s + \frac{52}{5}}{(s+3)^2+4^2} = \frac{8}{5s} + \frac{-\frac{8}{5}(s+3) + \frac{24}{5} + \frac{52}{5}}{(s+3)^2+4^2}$$

$$f(t) = \frac{8}{5}u(t) - \frac{8}{5}e^{-3t}\cos 4t + \frac{19}{5}e^{-3t}\sin 4t$$

ii. $\boldsymbol{F(s)} = \frac{s+1}{(s+2)(s^2+2s+5)}$

$$F(s) = \frac{s+1}{(s+2)(s^2+2s+5)} = \frac{A}{s+2} + \frac{Bs+C}{s^2+2s+5}$$

$$A = F(s)(s+2)\Big|_{s=-2} = \frac{-1}{5}$$

$$s+1 = A(s^2+2s+5) + B(s^2+2s) + C(s+2)$$

Equating coefficients :

$$s^2: \quad 0 = A+B \longrightarrow B = -A = \frac{1}{5}$$

$$s^1: \quad 1 = 2A+2B+C = 0+C \longrightarrow C = 1$$

$$s^0: \quad 1 = 5A+2C = -1+2 = 1$$

$$F(s) = \frac{-1/5}{s+2} + \frac{1/5 \cdot s+1}{(s+1)^2+2^2} = \frac{-1/5}{s+2} + \frac{1/5(s+1)}{(s+1)^2+2^2} + \frac{4/5}{(s+1)^2+2^2}$$

$$\underline{f(t) = (\mathbf{-0.2e^{-2t} + 0.2e^{-t}\cos(2t) + 0.4e^{-t}\sin(2t))u(t)}}$$

(e) Proper/Improper

i. $\boldsymbol{F(s)} = \frac{5s^3+20s^2-49s-108}{s^2+7s+10}$

$$F(s) = \frac{5s^3+20s^2-49s-108}{s^2+7s+10}$$

$$\begin{array}{r}
5s - 15 \\
s^2+7s+10 \enclose{longdiv}{5s^3+20s^2-49s-108} \\
5s^3+35s^2+50s \\
\hline
-15s^2-99s-108 \\
-15s^2-105s-150 \\
\hline
6s+42
\end{array}$$

$$F(s) = 5s - 15 + \frac{K_1}{s+2} + \frac{K_2}{s+5}$$

$$K_1 = \frac{6s+42}{s+5}\Big|_{s=-2} = 10$$

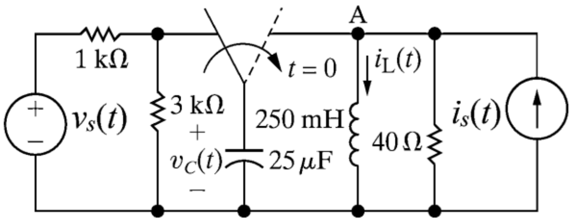
$$K_2 = \frac{6s+42}{s+2}\Big|_{s=-5} = -4$$

$$f(t) = 5\delta'(t) - 15\delta(t) + [10e^{-2t} - 4e^{-5t}]u(t)$$

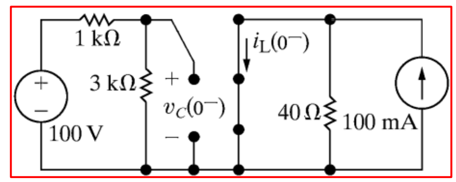
```
>> num = [5,20,-49,-108];
>> den = [1,7,10];
>> [r,p,k] = residue(num, den)
r = -4
     10
p = -5
    -2
k =  5   -15
```

3. Inverse Laplace Transforms, Integro-Differential Equations, and Network Analysis

(a) Consider the second order network shown with $v_s(t) = 100\text{V}$ and $i_s(t) = 100\text{ mA}$. The switch moves to the "right" position after being in the "left" position for a long time.



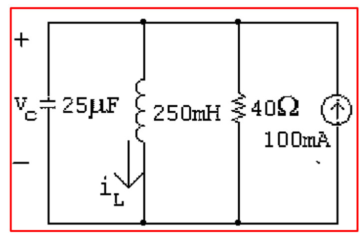
i. Analyze the network at time $t = 0^-$ to compute the state variable values $v_C(0^-)$ and $i_L(0^-)$.



$$v_C(0^-) = \frac{3\text{k}\Omega}{3\text{k}\Omega + 1\text{k}\Omega}(100\text{V}) = (3/4)(100\text{V}) \rightarrow \boxed{v_C(0^-) = 75\text{V}}$$

$$i_L(0^-) = \frac{40\Omega}{40\Omega + 0\Omega}(100\text{mA}) = (1)(100\text{mA}) \rightarrow \boxed{i_L(0^-) = 100\text{mA}}$$

ii. Analyze the network for $t > 0^-$ using <u>nodal analysis</u> at node A to obtain an integro-differential equation that describes the voltage $v_C(t)u(t)$ for $t > 0^-$.



Nodal Analysis @ Node A:

$$-i_S(t) + i_R(t) + i_C(t) + i_L(t) = 0\text{A}$$

$$\frac{v_C(t) - 0\text{V}}{R} + C\frac{d}{dt}[v_C(t) - 0\text{V}] + i_L(0^-) + \frac{1}{L}\int_{0^-}^{t}[v_C(x) - 0\text{V}]dx = i_S(t)$$

$$\boxed{\frac{dv_C(t)}{dt} + \frac{v_C(t)}{RC} + \frac{i_L(0^-)}{C} + \frac{1}{LC}\int_{0^-}^{t} v_C(x)dx = \frac{i_S(t)}{C}}$$

$$\frac{1}{RC} = \frac{1}{(40\Omega)(25\mu F)} = 1\text{k} \qquad \frac{1}{C} = \frac{1}{(25\mu F)} = 40\text{k} \qquad \frac{1}{LC} = \frac{1}{(25\mu F)(250mH)} = 160\text{k}$$

$$\boxed{\frac{dv_C(t)}{dt} + (1\text{k})v_C(t) + (40\text{k})i_L(0^-) + (160\text{k})\int_{0^-}^{t} v_C(x)dx = (40\text{k})i_S(t)}$$

iii. Take the Laplace Transform of the equation found in (ii) and compute the complete capacitor voltage response transform $\boldsymbol{V_C(s)} = \mathcal{L}\{v_C(t)u(t)\}$. As part of your computation, identify the characteristic polynomial of $\boldsymbol{V_C(s)}$, the zero state component $\boldsymbol{V_{C,ZS}(s)}$ of $\boldsymbol{V_C(s)}$, and the zero input component $\boldsymbol{V_{C,ZI}(s)}$ of $\boldsymbol{V_C(s)}$.

$$\mathcal{L}\left[1\text{k}v_C(t) + \frac{dv_C(t)}{dt} + (40\text{k})i_L(0^-) + 160\text{k}\int_{0^-}^{t} v_C(x)dx = (40\text{k})i_S(t)\right]$$

$$1\text{k}\mathcal{L}[v_C(t)] + \mathcal{L}\left[\frac{dv_C(t)}{dt}\right] + 40\text{k}i_L(0^-)\mathcal{L}[1] + 160\text{k}\mathcal{L}\left[\int_{0^-}^{t} v_C(x)dx\right] = (40\text{k})\mathcal{L}[i_S(t)]$$

$$1\text{k}\boldsymbol{V_C(s)} + [\boldsymbol{sV_C(s)} - v_C(0^-)] + \frac{40\text{k}i_L(0^-)}{\boldsymbol{s}} + 160\text{k}\left[\frac{\mathbf{V_C(s)}}{\boldsymbol{s}}\right] = (40\text{k})\boldsymbol{I_S(s)}$$

$$\boldsymbol{V_C(s)}\left(1\text{k} + \boldsymbol{s} + \frac{160\text{k}}{\boldsymbol{s}}\right) = (40\text{k})\boldsymbol{I_S(s)} + v_C(0^-) - \frac{40\text{k}i_L(0^-)}{\boldsymbol{s}}$$

$$\boldsymbol{V_C(s)}(\boldsymbol{s}^2 + 1\text{k}\boldsymbol{s} + 160\text{k}) = (40\text{k})\boldsymbol{sI_S(s)} + \boldsymbol{s}v_C(0^-) - 40\text{k}i_L(0^-)$$

$$\boldsymbol{V_C(s)} = \boldsymbol{V_{C,ZS}(s)} + \boldsymbol{V_{C,ZI}(s)} = \frac{(40\text{k})\boldsymbol{s}}{\boldsymbol{s}^2 + 1\text{k}\boldsymbol{s} + 160\text{k}}\boldsymbol{I_S(s)} + \frac{\boldsymbol{s}v_C(0^-) - 40\text{k}i_L(0^-)}{\boldsymbol{s}^2 + 1\text{k}\boldsymbol{s} + 160\text{k}}$$

$$\boldsymbol{V_C(s)} = \boldsymbol{V_{C,ZS}(s)} + \boldsymbol{V_{C,ZI}(s)} = \frac{(40\text{k})\boldsymbol{s}}{\boldsymbol{s}^2 + 1\text{k}\boldsymbol{s} + 160\text{k}}\left(\frac{100\text{m}}{\boldsymbol{s}}\right) + \frac{\boldsymbol{s}(75) - 40\text{k}(100\text{m})}{\boldsymbol{s}^2 + 1\text{k}\boldsymbol{s} + 160\text{k}}$$

$$\boldsymbol{V_C(s)} = \boldsymbol{V_{C,ZS}(s)} + \boldsymbol{V_{C,ZI}(s)} = \frac{4\text{k}}{\boldsymbol{s}^2 + 1\text{k}\boldsymbol{s} + 160\text{k}} + \frac{75\boldsymbol{s} - 4\text{k}}{\boldsymbol{s}^2 + 1\text{k}\boldsymbol{s} + 160\text{k}}$$

$$\boxed{\boldsymbol{V_{C,ZS}(s)} = \frac{4\text{k}}{\boldsymbol{s}^2 + 1\text{k}\boldsymbol{s} + 160\text{k}}} \qquad \boxed{\boldsymbol{V_{C,ZI}(s)} = \frac{75\boldsymbol{s} - 4\text{k}}{\boldsymbol{s}^2 + 1\text{k}\boldsymbol{s} + 160\text{k}} = 25\frac{3\boldsymbol{s} - 160}{\boldsymbol{s}^2 + 1\text{k}\boldsymbol{s} + 160\text{k}}}$$

$$\boldsymbol{V_C(s)} = \frac{4\text{k} + 75\boldsymbol{s} - 4\text{k}}{\boldsymbol{s}^2 + 1\text{k}\boldsymbol{s} + 160\text{k}} \to \boxed{\boldsymbol{V_C(s)} = \frac{75\boldsymbol{s}}{\boldsymbol{s}^2 + 1\text{k}\boldsymbol{s} + 160\text{k}}}$$

Characteristic polynomial is the denominator of $\boldsymbol{V_{C,ZI}(s)}$. Therefore, the characteristic polynomial is $\boxed{\boldsymbol{s}^2 + 1\text{k}\boldsymbol{s} + 160\text{k}}$. This means the integro-differential equation models the behavior of a second order system, which is what the network is because it contains one independent capacitor and one independent inductor.

iv. Compute the complete capacitor voltage response $v_C(t)u(t)$ by taking the inverse Laplace Transform of the complete capacitor voltage response transform $\boldsymbol{V_C(s)}$.

$$\boldsymbol{V_C(s)} = \frac{75\boldsymbol{s}}{\boldsymbol{s}^2 + 1\text{k}\boldsymbol{s} + 160\text{k}} = \frac{75\boldsymbol{s}}{(\boldsymbol{s} + 800)(\boldsymbol{s} + 200)} = \frac{r_1}{\boldsymbol{s} + 800} + \frac{r_2}{\boldsymbol{s} + 200}$$

$$75\boldsymbol{s} = r_1(\boldsymbol{s} + 200) + r_2(\boldsymbol{s} + 800)$$

Compute r_1: Substitute for s the pole $p_1 = -800$ into above expression

$$75(-800) = r_1(-800 + 200) + r_2(-800 + 800) \to \boxed{r_1 = 75(-800)/(-600) = 100}$$

Compute r_2: Substitute for s the pole $p_2 = -200$ into above expression

$$75(-200) = r_1(-200 + 200) + r_2(-200 + 800) \to \boxed{r_2 = 75(-200)/600 = -25}$$

$$\boldsymbol{V_C(s)} = \frac{100}{\boldsymbol{s} + 800} + \frac{-25}{\boldsymbol{s} + 200} \to v_C(t) = \mathcal{L}^{-1}\{\boldsymbol{V_C(s)}\} = \mathcal{L}^{-1}\left\{\frac{100}{\boldsymbol{s} + 800} + \frac{-25}{\boldsymbol{s} + 200}\right\}u(t)$$

$$v_C(t) = \mathcal{L}^{-1}\{\boldsymbol{V_C(s)}\} = 100\mathcal{L}^{-1}\left\{\frac{1}{\boldsymbol{s} + 800}\right\}u(t) - 25\mathcal{L}^{-1}\left\{\frac{1}{\boldsymbol{s} + 200}\right\}u(t)$$

$$\boxed{v_C(t) = \mathcal{L}^{-1}\{\boldsymbol{V_C(s)}\} = [100e^{-800t} - 25e^{-200t}]u(t)}$$

v. Write an expression that relates the complete inductor current response $i_L(t)u(t)$ to the complete capacitor voltage response $v_C(t)u(t)$. Then use the expression for $i_L(t)u(t)$ to compute complete inductor current response transform $\boldsymbol{I_L(s)} = \mathcal{L}\{i_L(t)u(t)\}$.

$$i_L(t) = i_L(0^-) + \frac{1}{L}\int_{0^-}^{t} v_C(x)dx = 100\text{mA} + \frac{1}{250\text{mH}}\int_{0^-}^{t} v_C(x)dx$$

$$i_L(t) = 0.1 + 4\int_{0^-}^{t} v_C(x)dx \rightarrow \mathcal{L}\left[i_L(t) = 0.1 + 4\int_{0^-}^{t} v_C(x)dx\right]$$

$$\mathcal{L}[i_L(t)] = 0.1\mathcal{L}[1] + 4\mathcal{L}\left[\int_{0^-}^{t} v_C(x)dx\right] \rightarrow \boldsymbol{I_L(s)} = \frac{0.1}{\boldsymbol{s}} + 4\left[\frac{\mathbf{V_C}(\boldsymbol{s})}{\boldsymbol{s}}\right] = \frac{1}{\boldsymbol{s}}[0.1 + 4\mathbf{V_C}(\boldsymbol{s})]$$

$$\boldsymbol{I_L(s)} = \frac{1}{\boldsymbol{s}}\left[\frac{1}{10} + 4\frac{75\boldsymbol{s}}{\boldsymbol{s}^2 + 1\text{k}\boldsymbol{s} + 160\text{k}}\right] = \frac{1}{\boldsymbol{s}}\left(\frac{1}{10} + \frac{300\boldsymbol{s}}{\boldsymbol{s}^2 + 1\text{k}\boldsymbol{s} + 160\text{k}}\right)$$

$$\boldsymbol{I_L(s)} = \frac{1}{\boldsymbol{s}}\left(\frac{\boldsymbol{s}^2 + 1\text{k}\boldsymbol{s} + 160\text{k} + 300(10)\boldsymbol{s}}{10(\boldsymbol{s}^2 + 1\text{k}\boldsymbol{s} + 160\text{k})}\right) \rightarrow \boxed{\boldsymbol{I_L(s)} = \frac{1}{10}\left[\frac{\boldsymbol{s}^2 + 4\text{k}\boldsymbol{s} + 160\text{k}}{\boldsymbol{s}(\boldsymbol{s}^2 + 1\text{k}\boldsymbol{s} + 160\text{k})}\right]}$$

vi. Compute the complete inductor current response $i_L(t)u(t)$ by taking the inverse Laplace Transform of the complete inductor current response transform $\boldsymbol{I_L(s)}$.

$$\boldsymbol{I_L(s)} = \frac{1}{10}\frac{\boldsymbol{s}^2 + 4\text{k}\boldsymbol{s} + 160\text{k}}{\boldsymbol{s}(\boldsymbol{s}^2 + 1\text{k}\boldsymbol{s} + 160\text{k})} = \frac{1}{10}\frac{\boldsymbol{s}^2 + 4\text{k}\boldsymbol{s} + 160\text{k}}{\boldsymbol{s}(\boldsymbol{s} + 800)(\boldsymbol{s} + 200)} = \frac{r_0}{\boldsymbol{s}} + \frac{r_1}{\boldsymbol{s} + 800} + \frac{r_2}{\boldsymbol{s} + 200}$$

$$\frac{1}{10}(\boldsymbol{s}^2 + 4\text{k}\boldsymbol{s} + 160\text{k}) = r_0(\boldsymbol{s} + 800)(\boldsymbol{s} + 200) + r_1\boldsymbol{s}(\boldsymbol{s} + 200) + r_2\boldsymbol{s}(\boldsymbol{s} + 800)$$

Compute r_0: Substitute for s the pole $p_0 = \mathbf{0}$ into above expression

$$\frac{1}{10}(160\text{k}) = r_0(800)(200) + r_1(0) + r_2(0) \rightarrow r_0 = \frac{160\text{k}}{(10)160\text{k}} = \frac{1}{10}$$

Compute r_1: Substitute for s the pole $p_1 = -800$ into above expression

$$\frac{1}{10}((-800)^2 + 4\text{k}(-800) + 160\text{k}) = r_0(0) + r_1(-800)(-800 + 200) + r_2(0)$$

$$r_1 = \frac{((-800)^2 + 4\text{k}(-800) + 160\text{k})}{10(-800)(-600)} = -\frac{1}{2}$$

Compute r_2: Substitute for s the pole $p_2 = -200$ into above expression

$$\frac{1}{10}((-200)^2 + 4\text{k}(-200) + 160\text{k}) = r_0(0) + r_1(0) + r_2(-200)(-200 + 800)$$

$$r_2 = \frac{((-200)^2 + 4\text{k}(-200) + 160\text{k})}{10(-200)(600)} = \frac{1}{2}$$

$$\boldsymbol{I_L(s)} = \frac{\frac{1}{10}}{\boldsymbol{s}} + \frac{-\frac{1}{2}}{\boldsymbol{s} + 800} + \frac{\frac{1}{2}}{\boldsymbol{s} + 200} = \frac{0.1}{\boldsymbol{s}} + \frac{-0.5}{\boldsymbol{s} + 800} + \frac{0.5}{\boldsymbol{s} + 200}$$

$$i_L(t) = \mathcal{L}^{-1}\{\boldsymbol{I_L(s)}\} = \mathcal{L}^{-1}\left\{\frac{0.1}{s} + \frac{-0.5}{s + 800} + \frac{0.5}{s + 200}\right\}u(t)$$

$$i_L(t) = \mathcal{L}^{-1}\{\boldsymbol{I_L(s)}\} = 0.1\mathcal{L}^{-1}\left\{\frac{1}{s}\right\}u(t) - 0.5\mathcal{L}^{-1}\left\{\frac{1}{s + 800}\right\}u(t) + 0.5\mathcal{L}^{-1}\left\{\frac{1}{s + 200}\right\}u(t)$$

$$\boxed{i_L(t) = \mathcal{L}^{-1}\{\boldsymbol{I_L(s)}\} = [0.1 - 0.5e^{-800t} + 0.5e^{-200t}]u(t)}$$

4. Pole-Zero Representation of Rational Functions and Pole-Zero Diagrams

(a) Consider the rational function $\boldsymbol{F(s)} = \boldsymbol{N(s)}/\boldsymbol{D(s)}$ of a complex frequency.

$$\boldsymbol{F(s)} = \frac{(8s+40)(4s^2+8s+36)}{(2s+14)(s+3)(s^2+5s+6)}$$

Upon algebraically manipulating $\boldsymbol{F(s)}$, it may be re-written as

$$\boldsymbol{F(s)} = \frac{(8)(s+5)(4)(s^2+2s+9)}{(2)(s+7)(s+3)(s+2)(s+3)} = \boxed{16\frac{(s+5)(s^2+2s+9)}{(s+7)(s+3)^2(s+2)}}$$

i. Compute the scale factor K. By inspection, the scale (gain) factor is $\boxed{K = 16}$

ii. Compute the poles (finite, infinite) of $\boldsymbol{F(s)}$.

$\boldsymbol{F(s)}$ has four finite poles: $\boxed{p_1 = -7 + j0}$, $\boxed{p_{2,3} = -3 + j0}$, and $\boxed{p_4 = -2 + j0}$.

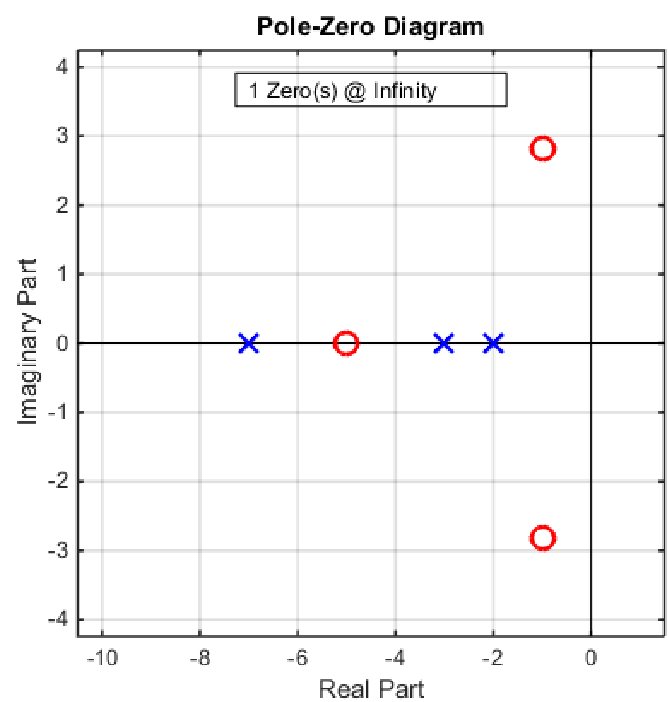
iii. Compute the zeros (finite, infinite) of $\boldsymbol{F(s)}$.

$\boldsymbol{F(s)}$ has three finite zeros: $\boxed{z_1 = -5 + j0}$, and $\boxed{z_{2,3} = -1 \pm j2\sqrt{2}}$.

Also, not that if $s \to \infty$, $\boldsymbol{F(s)} \to 0$. So, there is an infinite zero $\boxed{z_4 = \infty}$. This must be true since the # of poles must equal the # of zeros.

iv. Sketch the pole-zero diagram for $\boldsymbol{F(s)}$. Include any infinite poles and zeros in your sketch. Then, use MATLAB and the **`pzplot2()`** user-defined function file from Blackboard Learn to create a pole-zero diagram of $\boldsymbol{F(s)}$.

```
>> n1 = [8, 40];
>> n2 = [4, 8, 36];
>> num = conv(n1, n2);
>> d1 = [2, 14];
>> d2 = [1, 3];
>> d3 = [1, 5, 6];
>> den = conv(conv(d1, d2), d3);
>> [p, z] = pzplot2(num, den)
p = -7.0000
    -3.0000
    -3.0000
    -2.0000
z = -5.0000 + 0.0000i
    -1.0000 + 2.8284i
    -1.0000 - 2.8284i
```



(b) Consider the pole-zero diagram of $\boldsymbol{F}(\boldsymbol{s}) = \boldsymbol{N}(\boldsymbol{s})/\boldsymbol{D}(\boldsymbol{s})$ shown. Compute the expression for $\boldsymbol{F}(\boldsymbol{s})$ if $\boldsymbol{F}(150) = \frac{400}{41}$.

$$\boldsymbol{F}(\boldsymbol{s}) = K\frac{(\boldsymbol{s}+0)(\boldsymbol{s}+100)^2}{(\boldsymbol{s}+50-j250)(\boldsymbol{s}+50+j250)(\boldsymbol{s}+100-j500)(\boldsymbol{s}+100+j500)}$$

$$\boldsymbol{F}(\boldsymbol{s}) = K\frac{\boldsymbol{s}(\boldsymbol{s}+100)^2}{(\boldsymbol{s}^2+100\boldsymbol{s}+65000)(\boldsymbol{s}^2+200\boldsymbol{s}+260000)}$$

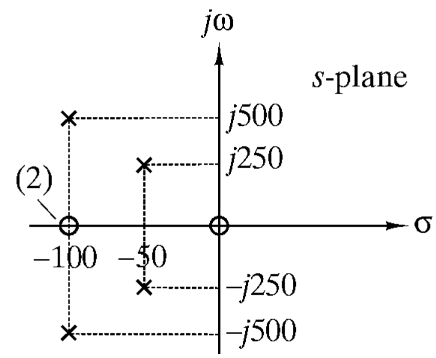
Compute K using the information that $F(150) = 400/41$.

$$\boldsymbol{F}(150) = \frac{400}{41} = K\frac{150(150+100)^2}{((150)^2+100(150)+65000)((150)^2+200(150)+260000)}$$

$$K = \frac{400}{41}\left[\frac{150(150+100)^2}{((150)^2+100(150)+65000)((150)^2+200(150)+260000)}\right]^{-1}$$

$\boxed{K = \frac{1}{3}100\times10^3}$. Putting the pieces together yields.

$$\boxed{\boldsymbol{F}(\boldsymbol{s}) = \frac{1}{3}100\times10^3\frac{\boldsymbol{s}(\boldsymbol{s}+100)^2}{(\boldsymbol{s}^2+100\boldsymbol{s}+65000)(\boldsymbol{s}^2+200\boldsymbol{s}+260000)}}$$



5. Initial and Final Value Theorems

Compute, if possible, $f_k(0^+)$ and $f_k(\infty)$ of the right-sided time function corresponding to each of the following rational functions of a complex frequency. If it is not possible, briefly explain why.

(a) $\boldsymbol{F_1}(\boldsymbol{s}) = \frac{\boldsymbol{s}+3}{\boldsymbol{s}^2+\boldsymbol{s}}$

We can apply the IVT since $F_1(s)$ is strictly proper. Therefore,

$$f_1(0^+) = \lim_{s\to\infty}\boldsymbol{s}\boldsymbol{F_1}(s) = \lim_{s\to\infty}\frac{\boldsymbol{s}(\boldsymbol{s}+3)}{\boldsymbol{s}(\boldsymbol{s}+1)} = \lim_{s\to\infty}\frac{(\boldsymbol{s}+3)}{(\boldsymbol{s}+1)} = \lim_{s\to\infty}\frac{\boldsymbol{s}\left(1+\frac{3}{\boldsymbol{s}}\right)}{\boldsymbol{s}\left(1+\frac{1}{\boldsymbol{s}}\right)} = \lim_{s\to\infty}\frac{\left(1+\frac{3}{\boldsymbol{s}}\right)}{\left(1+\frac{1}{\boldsymbol{s}}\right)} = \boxed{1}$$

$F_1(s)$ has a single finite pole at $\boldsymbol{s} = p_1 = 0+j0$ and a single finite pole at $s = p_2 = -1+j0$, which is located in the left half of the s-plane. Therefore, we are free to apply the FVT since all the poles of $F_1(s)$ are in the open left half except for a single pole at $p = 0$. Applying the FVT yields:

$$f_1(\infty) = \lim_{s\to0}\boldsymbol{s}\boldsymbol{F_1}(\boldsymbol{s}) = \lim_{s\to0}\frac{\boldsymbol{s}(\boldsymbol{s}+3)}{\boldsymbol{s}(\boldsymbol{s}+1)} = \lim_{s\to0}\frac{(\boldsymbol{s}+3)}{(\boldsymbol{s}+1)} = \frac{(0+3)}{(0+1)} = \boxed{3}$$

Does the above answer make sense? The finite pole p_1 of $\boldsymbol{F_1}(\boldsymbol{s})$ tells us that $f_1(t)$ has a term that is a constant right-sided time function (i.e. DC signal) that neither converges or diverges as $t\to\infty$. The finite pole p_2 of $\boldsymbol{F_1}(\boldsymbol{s})$ tells us that $f_1(t)$ also has a term that is a decaying exponential right-sided time function converging to 0 as $t\to\infty$. Therefore, as $t\to\infty$, the final value $f_1(\infty)$ is determined by the term that is a constant time function (i.e. DC signal) corresponding to the pole p_1. The answer makes sense!

(b) $\boldsymbol{F_2(s)} = \frac{5}{(s+1)(s^2+9)}$

We can apply the IVT since $F_2(s)$ is strictly proper. Therefore,

$$f_2(0^+) = \lim_{s\to\infty} \boldsymbol{sF_2(s)} = \lim_{s\to\infty} \frac{5\boldsymbol{s}}{(\boldsymbol{s}+1)(\boldsymbol{s}^2+9)} = \lim_{s\to\infty} \frac{5\boldsymbol{s}}{\boldsymbol{s}\left(1+\frac{1}{\boldsymbol{s}}\right)\boldsymbol{s}^2\left(1+\frac{9}{\boldsymbol{s}^2}\right)}$$

$$f_2(0^+) = \lim_{s\to\infty} \boldsymbol{sF_2(s)} = \lim_{s\to\infty} \frac{5\boldsymbol{s}}{\boldsymbol{s}^3\left(1+\frac{1}{\boldsymbol{s}}\right)\left(1+\frac{9}{\boldsymbol{s}^2}\right)} = \lim_{s\to\infty} \frac{5}{\boldsymbol{s}^2\left(1+\frac{1}{\boldsymbol{s}}\right)\left(1+\frac{9}{\boldsymbol{s}^2}\right)}$$

$$f_2(0^+) = \frac{5}{(\infty)(1+1/\infty)(1+9/\infty^2)} = \frac{5}{(\infty)(1+0)(1+0)} \to \boxed{f_2(0^+) = 0}$$

$\boldsymbol{F_2(s)}$ has a finite pole at $\boldsymbol{s} = p_1 = -1 + j0$ located in the left half of the $\boldsymbol{s}$-plane. $\boldsymbol{F_2(s)}$ also has a finite set of imaginary conjugate poles $p_{2,3} = \pm j3$ located on the imaginary axis. Therefore, we cannot apply the FVT since poles $p_{2,3} = \pm j3$ are not located in the left half of the s-plane.

Applying the FVT would yield the following erroneous answer:

$$f_2(\infty) = \lim_{s\to 0} \boldsymbol{sF_2(s)} = \lim_{s\to 0} \frac{5\boldsymbol{s}}{(\boldsymbol{s}+1)(\boldsymbol{s}^2+9)} = \frac{5(0)}{((0)+1)((0)^2+9)} = \boxed{0}\ WRONG!!$$

The above does not make sense for the following reason. Finite pole p_1 of $\boldsymbol{F_2(s)}$ tells us that $f_2(t)$ has a term that is a decaying exponential time function converging to 0 as $\boldsymbol{t} \to \infty$. . Finite poles $p_{2,3}$ of $\boldsymbol{F_2(s)}$ tell us that $f_2(t)$ has a term that is purely sinusoidal (an un-damped sinusoid) for all time $\boldsymbol{t} > 0$, which neither diverges nor converges as $\boldsymbol{t} \to \infty$. Therefore, as $\boldsymbol{t} \to \infty$, the final value is determined by the term that is purely sinusoidal, which does not have a single finite final value.

(c) $\boldsymbol{F_3(s)} = \frac{3s^3+6s^2+12s+3}{s(s+3)^2}$

We start this problem by finding $f_3(0^+)$ and $f_3(\infty)$ by first finding $f_3(t)$. After applying partial fraction expansion to $\boldsymbol{F_3(s)}$ and taking the inverse LT, it is determined that $f_3(t)$ is

$$\boxed{f_3(t) = 3\delta(t) + \left[\frac{1}{3} + e^{-3t}\left(20t - \frac{37}{3}\right)\right]u(t)}$$

Finding $f_3(0^+)$ using $f_3(t)$

$$f_3(0^+) = 3\delta(0^+) + \left[\frac{1}{3} + e^{-3(0^+)}\left(20(0^+) - \frac{37}{3}\right)\right]u(0^+) = \boxed{f_3(0^+) = -12}$$

Finding $f_3(\infty)$ using $f_3(t)$

$$f_3(\infty) = 3\delta(\infty) + \left[\frac{1}{3} + e^{-3(\infty)}\left(20(\infty) - \frac{37}{3}\right)\right]u(\infty) = \boxed{f_3(\infty) = \frac{1}{3}}$$

Now let's see how we can "try" to obtain the above results using IVT and FVT.

We cannot apply the IVT directly to $\boldsymbol{F_3(s)}$ because $\boldsymbol{F_3(s)}$ is not strictly proper. Applying the IVT directly to $\boldsymbol{F_3(s)}$ yields the following erroneous results:

$$f_3(0^+) = \lim_{s\to\infty} \boldsymbol{sF_3(s)} = \lim_{s\to\infty} \frac{\boldsymbol{s}(3\boldsymbol{s}^3+6\boldsymbol{s}^2+12\boldsymbol{s}+3)}{\boldsymbol{s}(\boldsymbol{s}+3)^2} = \lim_{s\to\infty} \frac{3\boldsymbol{s}^3+6\boldsymbol{s}^2+12\boldsymbol{s}+3}{\boldsymbol{s}^2+6\boldsymbol{s}+9}$$

$$f_3(0^+) = \lim_{s\to\infty} \frac{s^3\left(1+\frac{6}{s}+\frac{12}{s^2}+\frac{3}{s^3}\right)}{s^2\left(1+\frac{6}{s}+\frac{9}{s^2}\right)} = \lim_{s\to\infty} \frac{s\left(1+\frac{6}{s}+\frac{12}{s^2}+\frac{3}{s^3}\right)}{\left(1+\frac{6}{s}+\frac{9}{s^2}\right)} = \frac{(\infty)(1)}{(1)} = \infty\ WRONG!$$

The above answer is incorrect since we know from the start of this solution that $\boxed{f_3(0^+) = -12}$:

To perform the IVT to $F_3(s)$, we must first perform long division to find $\boldsymbol{P_3(s) = R_3(s)/D_3(s)}$. After performing long division, $\boldsymbol{F_3(s)}$ can be expressed as follows:

$$\boldsymbol{F_3(s)} = \frac{3s^3+6s^2+12s+3}{s(s+3)^2} = \boldsymbol{Q_3(s)} + \boldsymbol{P_3(s)} = \boldsymbol{Q_3(s)} + \frac{\boldsymbol{R_3(s)}}{\boldsymbol{D_3(s)}} = 3 + \frac{-12s^2-15s+3}{s(s+3)^2}$$

Applying the IVT directly to $\boldsymbol{P_3(s) = R_3(s)/D_3(s)}$ yields the following correct result:

$$f_3(0^+) = \lim_{s\to\infty} s\boldsymbol{P_3(s)} = \lim_{s\to\infty} \frac{s(-12s^2-15s+3)}{s(s+3)^2} = \lim_{s\to\infty} \frac{-12s^2-15s+3}{s^2+6s+9}$$

$$f_3(0^+) = \lim_{s\to\infty} \frac{s^2\left(-12-\frac{15}{s}+\frac{3}{s^2}\right)}{s^2\left(1+\frac{6}{s}+\frac{9}{s^2}\right)} = \lim_{s\to\infty} \frac{\left(-12-\frac{15}{s}+\frac{3}{s^2}\right)}{\left(1+\frac{6}{s}+\frac{9}{s^2}\right)} = \frac{-12}{+1} = -12$$

The above answer is the correct result since we know from the start of this solution that $\boxed{f_3(0^+) = -12}$:

$\boldsymbol{F_3(s)}$ has a finite pole at $\boldsymbol{s} = p_1 = 0 + j0$ of multiplicity 1 located at the origin of the s-plane. $\boldsymbol{F_3(s)}$ also has two finite poles $\boldsymbol{s} = p_{2,3} = -3 + j0$ located in the left half of the s-plane. Therefore, we can apply the FVT since poles $p_{2,3}$ are located in the left half of the s-plane and p_1 is a single finite pole (i.e. multiplicity 1) at the origin of the $\boldsymbol{s}$-plane.

Applying the FVT yields:

$$f_3(\infty) = \lim_{s\to 0} s\boldsymbol{F_3}(s) = \lim_{s\to 0} \frac{s(3s^3+6s^2+12s+3)}{s(s+3)^2} = \frac{(3(0)^3+6(0)^2+12(0)+3)}{((0)+3)^2} = \boxed{\frac{1}{3}}$$

Does the above answer make sense? The finite pole p_1 of $F_3(s)$ tells us that $f_1(t)$ has term that is a constant right-sided time function (i.e. DC signal) that neither converges or diverges as $t \to \infty$. The finite poles $p_{2,3}$ of $\boldsymbol{F_3(s)}$ tells us that $f_3(t)$ also has a term that is a decaying exponential right-sided time function multiplied by a first order polynomial in t. Although its multiplied by a first-order polynomial in t, the term converges to 0 as $t \to \infty$ because the decaying exponential dominates the first order polynomial for large values of time. Therefore, as $t \to \infty$, the final value $f_3(\infty)$ is determined by the term that is a constant right-sided time function (i.e. DC signal) corresponding to the pole p_1. The answer makes sense! By the way, the answer also makes sense since we know from the start of this solution that $\boxed{f_3(\infty) = \frac{1}{3}}$.