# HOMEWORK #3: Basic Signal Waveforms

1. Expressing Functions In Terms of Singularity Functions

(a) Express the following functions of time using a linear combination of singularity functions.

i. $v_1(t) = \begin{cases} 3 & t < 1 \\ -2 & 1 < t < 2 \\ 0 & elsewhere \end{cases}$

ii. $x(t) = \begin{cases} t-1 & 1 < t < 2 \\ 1 & 2 < t < 3 \\ -t+4 & 3 < t < 4 \\ 0 & elsewhere \end{cases}$

(b) Consider the plot of each of the following functions of time shown on the right. Express each as a linear combination of singularity functions. Simplify each expression as much as possible.
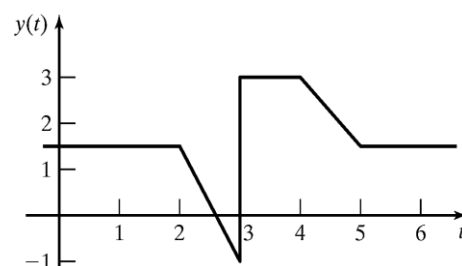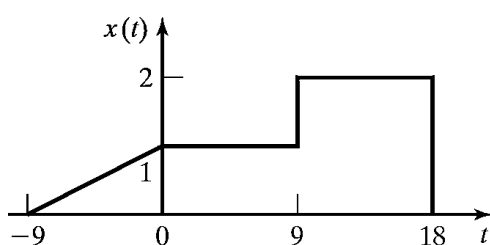
2. Sketching Waveforms Involving Singularity Functions

(a) Sketch each of the following functions by hand. Clearly label each sketches. Use MATLAB to plot $f_1(t)$ through $f_4(t)$ and use MATLAB's output to verify your hand sketches.

i. $\frac{d}{dt}x(t)$ (from question Q1(b))

   $\frac{d}{dt}y(t)$ (from question Q1(b))

ii. $f_1(t) = u(2t - 6)$

iii. $f_2(t) = u(t - 4) - u(t - 1)$

iv. $f_3(t) = 2u(t - 2)u(3 - t)$

v. $f_4(t) = r(t) - r(t - 1) - u(t - 2) - r(t - 2) + r(t - 3) + u(t - 4)$

vi. $f_5(t) = \frac{sin\left(t + \frac{\pi}{3}\right)}{t^2 + 1}\delta(t)$

vii. $f_6(t) = [e^{2t^2} - e^{-2t^2}][\delta(t + 1) - \delta(t - 1)]$

(b) Sketch each of the following "similar" time functions. Clearly label each sketch. Note the similarities and differences among the four waveforms.

i. $f_1(t) = e^{-2t}u(t)$

ii. $f_2(t) = e^{-2t}u(t - 1)$

iii. $f_3(t) = e^{-2(t-1)}u(t)$

iv. $f_4(t) = e^{-2(t-1)}u(t - 1)$

3. Evaluating Derivative/Integral Expressions Involving Singularity Functions

(a) Apply the sifting property of the Dirac Delta/Impulse to evaluate each of the following integrals.

    i. $\int_{-3}^{2} \cos(t)\, \delta(t)dt$

    iii. $\int_{-3}^{1} \ln(t)\, \delta(t-2)dt$

    v. $\int_{-3+}^{4^-} e^{-5t}\, \delta(t-4)dt$

    ii. $\int_{-3}^{2} t^2\, \delta(t-1)dt$

    iv. $\int_{-3}^{5^+} \sin(t)\, \delta(t-5)dt$

    vi. $\int_{-3-}^{3^+} e^{t^2}\, \delta(t+3)dt$

(b) Evaluate the following expressions involving time derivatives and singularity functions.

    i. $f_1(t) = [u(t+1)u(t-1)]'$

    iii. $f_3(t) = \left[\sin(4t)\, u\left(t - \frac{\pi}{8}\right)\right]$

    ii. $f_2(t) = [r(t-6)u(t-2)]'$