Lecture #6(b): Frequency Response I:
Frequency Response, SSS, and Bode Diagrams: Examples

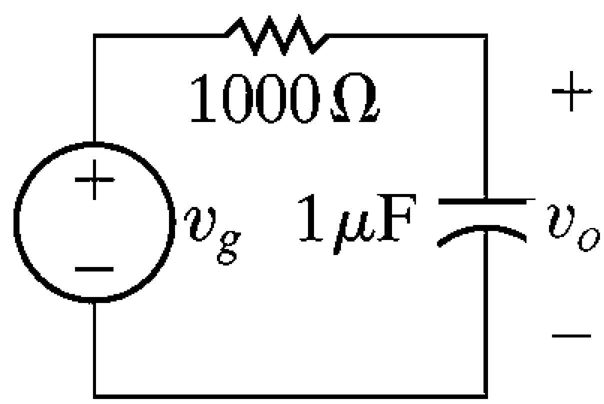
ECE 20200: Linear Circuit Analysis II
Steve Naumov (Instructor)

Lecture #6(b): Frequency Response I:
Frequency Response, SSS, and Bode Diagrams: Examples

Frequency Response and Sinusoidal Steady State

Example #1: 1st Order System Functions, SSS Response

- Consider the relaxed linear first order network having a sinusoidal excitation voltage of $v_g(t)$ for $t \geq 0$. Compute the following:
  - Voltage gain transfer function $\boldsymbol{H_V(s) = V_o(s)/V_g(s)}$
  - Voltage gain frequency response function $\boldsymbol{H_V}(j\omega) = \boldsymbol{V_o}(j\omega)/\boldsymbol{V_g}(j\omega)$
  - Voltage gain frequency response magnitude function $|\boldsymbol{H_V}(j\omega)|$
  - Voltage gain frequency response phase function $\angle \boldsymbol{H_V}(j\omega)$
  - Sinusoidal steady state voltage response $v_{o,sss}(t)$ when
    - $v_g(t) = 2\cos(100t - 30°)\, u(t)$ V
    - $v_g(t) = 4\sqrt{2}\cos(1\text{k}t + 45°)\, u(t)$V
    - $v_g(t) = 6\cos(10\text{k}t + 10°)\, u(t)$V
    - $v_g(t) = 10u(t)$V



Example #1: 1st Order System Functions, SSS Response (Solution)

- Compute the voltage gain transfer function $\boldsymbol{H_V(s) = V_o(s)/V_g(s)}$

$$\boldsymbol{H_V(s)} = \frac{\boldsymbol{V_o(s)}}{\boldsymbol{V_g(s)}} = \frac{\frac{1\text{M}}{\boldsymbol{s}}}{\frac{1\text{M}}{\boldsymbol{s}} + 1\text{k}} = \frac{1\text{M}}{1\text{M} + (1\text{k})\boldsymbol{s}} \rightarrow \boxed{\boldsymbol{H_V(s)} = \frac{1\text{k}}{\boldsymbol{s} + 1\text{k}}}$$

- Compute the voltage gain frequency response $\boldsymbol{H_V}(j\omega) = \boldsymbol{V_o}(j\omega)/\boldsymbol{V_g}(j\omega)$
  - **Method #1**: Compute $\boldsymbol{H_V}(j\omega)$ by $\boldsymbol{H_V(s)}|_{\boldsymbol{s}=j\omega}$

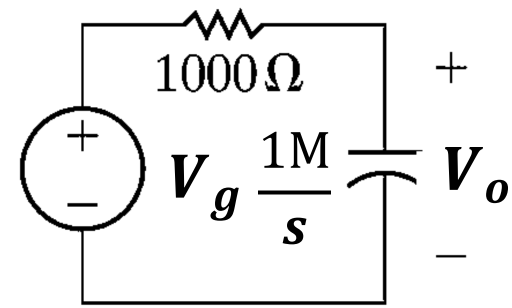
$$\boldsymbol{H_V(s)}\Big|_{\boldsymbol{s}=j\omega} = \left(\frac{\boldsymbol{V_o(s)}}{\boldsymbol{V_g(s)}}\right)\Big|_{\boldsymbol{s}=j\omega} = \left(\frac{1\text{k}}{\boldsymbol{s} + 1\text{k}}\right)\Big|_{\boldsymbol{s}=j\omega}$$

$$\boxed{\boldsymbol{H_V}(j\omega) = \frac{\boldsymbol{V_o}(j\omega)}{\boldsymbol{V_g}(j\omega)} = \frac{1\text{k}}{j\omega + 1\text{k}}}$$

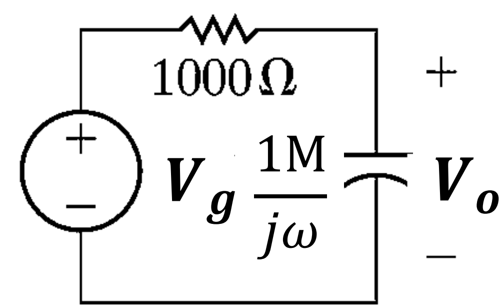
  - **Method #2**: Compute $\boldsymbol{H_V}(j\omega)$ directly in $j\omega$ domain

$$\boxed{\boldsymbol{H_V}(j\omega) = \frac{\frac{1\text{M}}{j\omega}}{\frac{1\text{M}}{j\omega} + 1\text{k}} = \frac{1\text{k}}{j\omega + 1\text{k}}}$$

$\boldsymbol{s}$-domain



$j\omega$-domain



Example #1: 1st Order System Functions, SSS Response (Solution cont'd)

- Compute the magnitude $|\boldsymbol{H_V}(j\omega)|$ of the voltage gain transfer function $\boldsymbol{H_V}(j\omega)$

$$|\boldsymbol{H_V}(j\omega)| = \left|\frac{\boldsymbol{V_o}(j\omega)}{\boldsymbol{V_g}(j\omega)}\right| = \frac{|\boldsymbol{V_o}(j\omega)|}{|\boldsymbol{V_g}(j\omega)|}$$

$$\boxed{|\boldsymbol{H_V}(j\omega)| = \frac{|1\mathrm{k}|}{|j\omega + 1\mathrm{k}|} = \frac{1\mathrm{k}}{\sqrt{\omega^2 + 1\mathrm{M}}}}$$

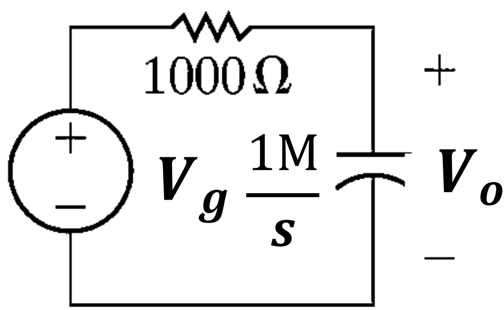
- Compute $\angle\boldsymbol{H_V}(j\omega)$, the phase angle of the voltage gain transfer function $\boldsymbol{H_V}(j\omega)$

$$\angle\boldsymbol{H_V}(j\omega) = \angle\frac{\boldsymbol{V_o}(j\omega)}{\boldsymbol{V_g}(j\omega)}$$

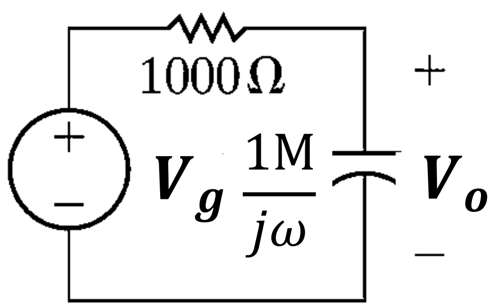
$$\angle\boldsymbol{H_V}(j\omega) = \angle\boldsymbol{V_o}(j\omega) - \angle\boldsymbol{V_g}(j\omega) = 0 - tan^{-1}\left(\frac{\omega}{1\mathrm{k}}\right)$$

$$\boxed{\angle\boldsymbol{H_V}(j\omega) = -\,tan^{-1}\left(\frac{\omega}{1\mathrm{k}}\right)}$$

***s*-domain**



***jω*-domain**



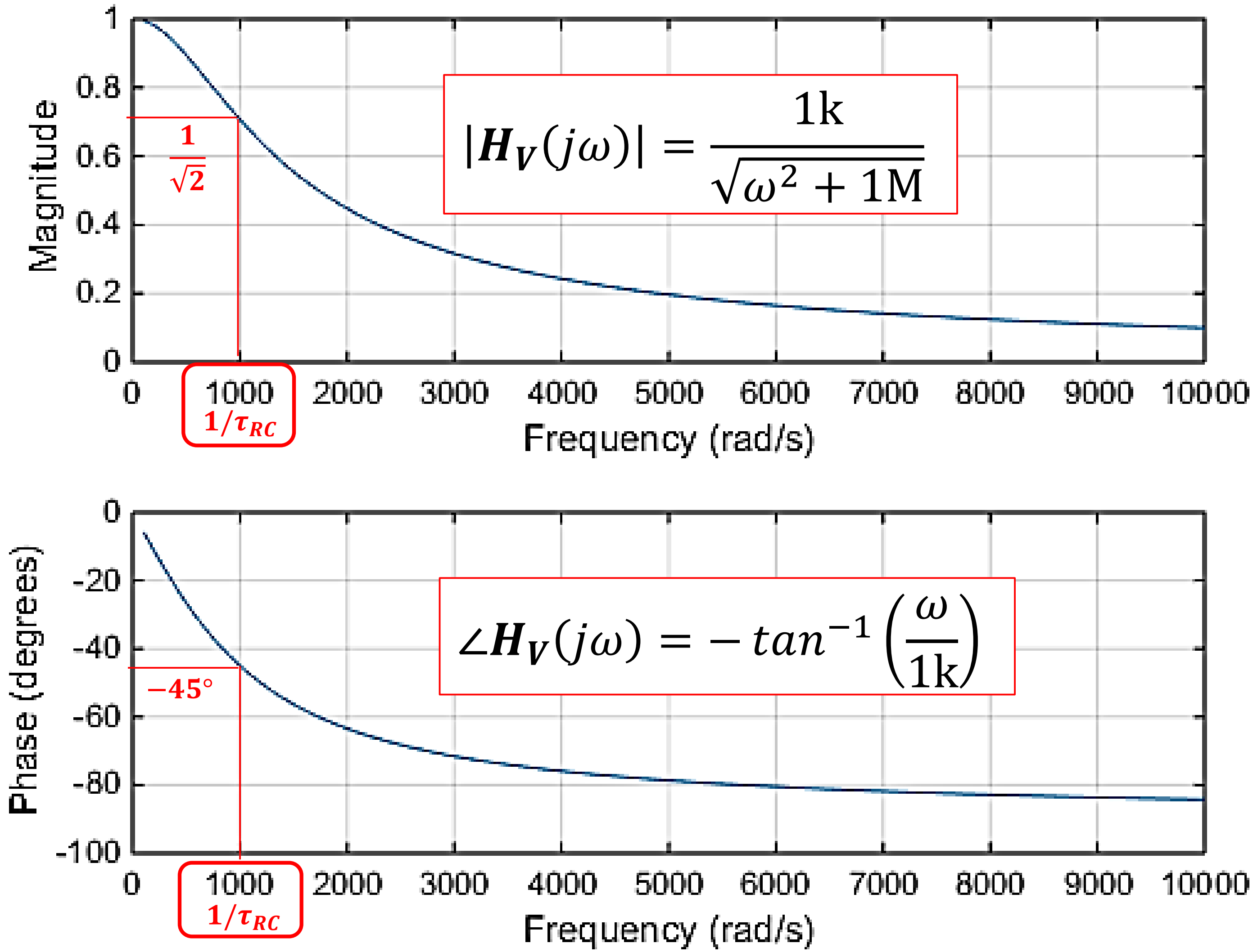
Example #1: 1st Order System Functions, SSS Response (Solution cont'd)

- In MATLAB

```
>> num = 1000;
>> den = [1,1000];
>> [Hjw, omega] = freqs(num, den);
>> mag = abs(Hjw);
>> phase = angle(Hjw)*(180/pi);
>> subplot(2, 1, 1);
>> plot(omega, mag);
>> xlabel('Frequency (rad/s)');
>> ylabel('Magnitude');
>> grid on;
>> subplot(2, 1, 2);
>> plot(omega, phase);
>> xlabel('Frequency (rad/s)');
>> ylabel('Phase (degrees)');
>> grid on;
```

$$H_V(s) = \frac{1\text{k}}{s + 1\text{k}}$$

Example #1: 1st Order System Functions, SSS Response (Solution cont'd)



Example #1: 1st Order System Functions, SSS Response (Solution cont'd)

- Compute $v_{o,sss}(t)$ when $v_g(t) = 2\cos(100t - 30°)\, u(t)\mathrm{V}$

$$v_{o,sss}(t) = 2|\boldsymbol{H_V}(j\omega)| \cos\big(100t - 30° + \angle \boldsymbol{H_V}(j\omega)\big)\, u(t)$$

$$v_{o,sss}(t) = 2|\boldsymbol{H_V}(j100)| \cos\big(100t - 30° + \angle \boldsymbol{H_V}(j100)\big) u(t)$$

$$\boxed{v_{o,sss}(t) = 2\frac{10}{\sqrt{101}}\cos(100t - 30° - tan^{-1}(1/10))u(t)}$$

$$\boxed{v_{o,sss}(t) \approx 1.99\cos(100t - 35.71°)u(t)}$$

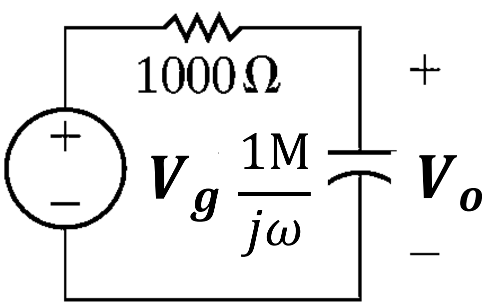
- Compute $v_{o,sss}(t)$ when $v_g(t) = 4\sqrt{2}\cos(1\mathrm{k}t + 45°)\, u(t)\mathrm{V}$

$$v_{o,sss}(t) = 4\sqrt{2}|\boldsymbol{H_V}(j\omega)| \cos\big(1\mathrm{k}t + 45° + \angle \boldsymbol{H_V}(j\omega)\big)\, u(t)$$

$$v_{o,sss}(t) = 4\sqrt{2}|\boldsymbol{H_V}(j1\mathrm{k})| \cos\big(1\mathrm{k}t + 45° + \angle \boldsymbol{H_V}(j1\mathrm{k})\big) u(t)$$

$$v_{o,sss}(t) = 4\sqrt{2}\frac{1}{\sqrt{2}}\cos(1\mathrm{k}t + 45° - tan^{-1}(1))u(t)$$

$$\boxed{v_{o,sss}(t) = 4\cos(1\mathrm{k}t)u(t)}$$



Example #1: 1st Order System Functions, SSS Response (Solution cont'd)

- Compute $v_{o,sss}(t)$ when $v_g(t) = 6\cos(10\text{k}t + 10°)\,u(t)\text{V}$

$$v_{o,sss}(t) = 6|\boldsymbol{H_V}(j\omega)|\cos\big(10\text{k}t + 10° + \angle\boldsymbol{H_V}(j\omega)\big)\,u(t)$$

$$v_{o,sss}(t) = 6|\boldsymbol{H_V}(j10\text{k})|\cos\big(10\text{k}t + 10° + \angle\boldsymbol{H_V}(j10\text{k})\big)u(t)$$

$$\boxed{v_{o,sss}(t) = 6\frac{1}{\sqrt{101}}\cos(10\text{k}t + 10° - tan^{-1}(10))u(t)}$$

$$\boxed{v_{o,sss}(t) \approx 0.597\cos(10\text{k} - 74.29°)u(t)}$$

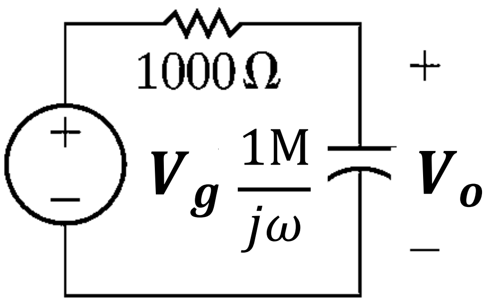
- Compute $v_{o,sss}(t)$ when $v_g(t) = 10u(t)\text{V}$

$$v_{o,sss}(t) = 10|\boldsymbol{H_V}(j\omega)|\cos\big(0t + 0° + \angle\boldsymbol{H_V}(j\omega)\big)\,u(t)$$

$$v_{o,sss}(t) = 10|\boldsymbol{H_V}(j0)|\cos\big(0t + 0° + \angle\boldsymbol{H_V}(j0)\big)u(t)$$

$$v_{o,sss}(t) = 10\frac{1000}{1000}\cos(0t + 0° - tan^{-1}(0))u(t)$$

$$\boxed{v_{o,sss}(t) = 10u(t)\text{V}}$$



Example #2: 2nd Order System Functions, SSS Response

- A current gain transfer function for a second order network is known to have the form

$$\boldsymbol{H_I(s)} = \frac{\boldsymbol{I_o(s)}}{\boldsymbol{I_{in}(s)}} = \frac{\boldsymbol{s}^2}{\boldsymbol{s}^2 + \boldsymbol{s} + 100}$$

Compute the following:

- Current gain frequency response function $\boldsymbol{H_I}(j\omega)$
- $|\boldsymbol{H_I}(j\omega)|$ and $\angle\boldsymbol{H_I}(j\omega)$
- Sinusoidal steady state current response $i_{o,sss}(t)$ when $i_{in}(t) = [1 + 2\cos(10t + 45°) - 10\sin(15t - 30°)]u(t)\text{A}$

Example #2: 2nd Order System Functions, SSS Response (Solution)

- Compute the current gain frequency response function $\boldsymbol{H_I}(j\omega)$

$$\boldsymbol{H_I}(j\omega) = \frac{\boldsymbol{I_o}(j\omega)}{\boldsymbol{I_{in}}(j\omega)} = \frac{(j\omega)^2}{(j\omega)^2 + (j\omega) + 100} = \frac{-\omega^2}{-\omega^2 + j\omega + 100}$$

$$\boxed{\boldsymbol{H_I}(j\omega) = \frac{-\omega^2}{[100 - \omega^2] + j\omega}}$$

- Compute $|\boldsymbol{H_I}(j\omega)|$ and $\angle \boldsymbol{H_I}(j\omega)$

$$|\boldsymbol{H_I}(j\omega)| = \frac{|\boldsymbol{I_o}(j\omega)|}{|\boldsymbol{I_{in}}(j\omega)|} = \frac{|-\omega^2|}{|[100 - \omega^2] + j\omega|} = \frac{\omega^2}{\sqrt{[100 - \omega^2]^2 + \omega^2}}$$

$$\boxed{|\boldsymbol{H_I}(j\omega)| = \frac{\omega^2}{\sqrt{[100 - \omega^2]^2 + \omega^2}}}$$

$$\angle \boldsymbol{H_I}(j\omega) = \angle \boldsymbol{I_o}(j\omega) - \angle \boldsymbol{I_{in}}(j\omega)$$

$$\boxed{\angle \boldsymbol{H_I}(j\omega) = \pm 180° - \tan^{-1}\left(\frac{\omega}{100 - \omega^2}\right)}$$

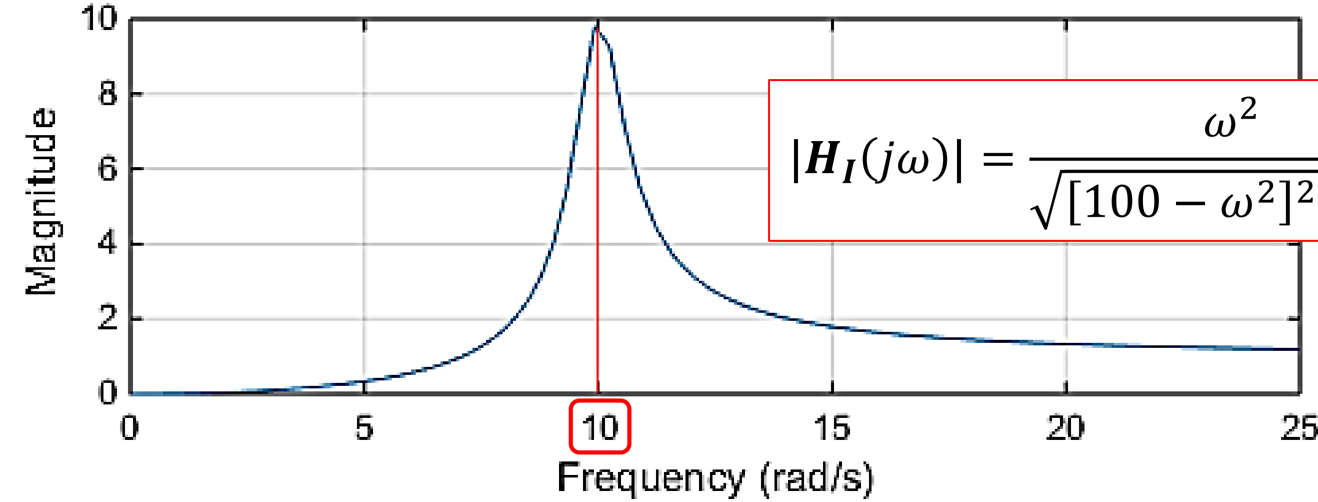
Example #2: 2nd Order System Functions, SSS Response (Solution cont'd)

- In MATLAB

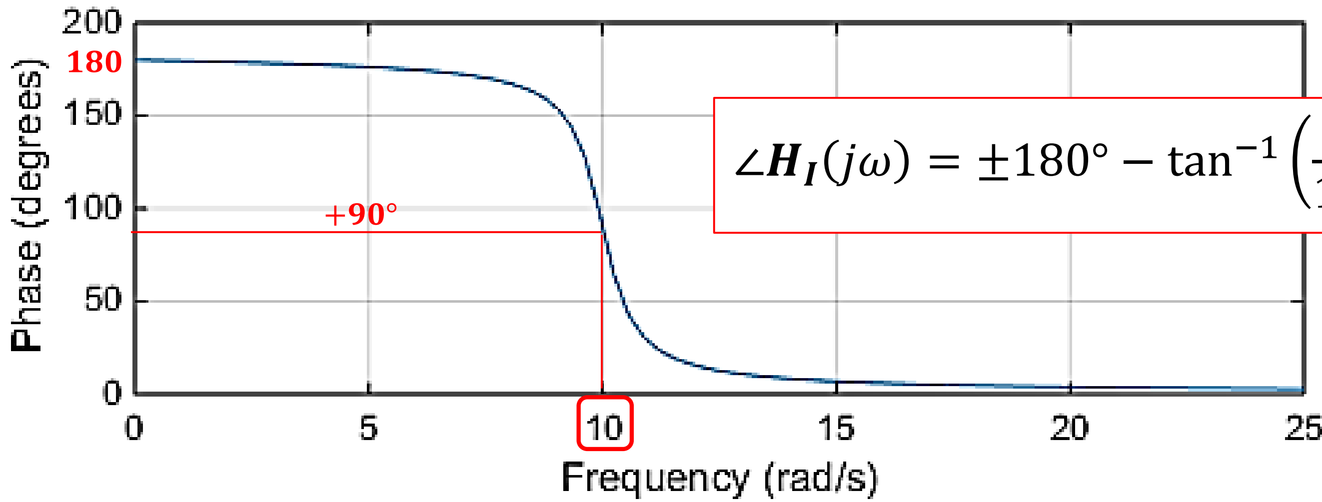
```
>> num = [1,0,0];
>> den = [1,1,100];
>> [Hjw, omega] = freqs(num, den);
>> mag = abs(Hjw);
>> phase = angle(Hjw)*(180/pi);
>> subplot(2,1,1);
>> plot(omega, mag);
>> xlabel('Frequency (rad/s)');
>> ylabel('Magnitude');
>> grid on;
>> xlim([0,25]);
>> subplot(2,1,2);
>> plot(omega, phase);
>> xlabel('Frequency (rad/s)');
>> ylabel('Phase (degrees)');
>> grid on;
>> xlim([0,25]);
```

$$\boldsymbol{H_I(s)} = \frac{\boldsymbol{s}^2}{\boldsymbol{s}^2 + \boldsymbol{s} + 100}$$

Example #2: 2nd Order System Functions, SSS Response (Solution cont'd)



$$|\boldsymbol{H_I}(j\omega)| = \frac{\omega^2}{\sqrt{[100-\omega^2]^2+\omega^2}}$$



$$\angle\boldsymbol{H_I}(j\omega) = \pm 180^\circ - \tan^{-1}\left(\frac{\omega}{100-\omega^2}\right)$$

Example #2: 2nd Order System Functions, SSS Response (Solution cont'd)

- Compute the sinusoidal steady state current response $i_{o,sss}(t)$ when $i_{in}(t) = [1 + 2\cos(10t + 45°) - 10\sin(15t - 30°)]u(t)\text{A}$
  - Since the system is assumed to be linear, we can employ superposition

$$i_{o,sss}(t) = i_{o,sss}^{(1)}(t) + i_{o,sss}^{(2)}(t) - i_{o,sss}^{(3)}(t)$$

$$i_{o,sss}^{(1)}(t) = 1|\boldsymbol{H_I}(j0)|\cos\left(0t + 0° + \angle\boldsymbol{H_I}(j0)\right)u(t)\ \text{A}$$

$$i_{o,sss}^{(1)}(t) = 1(0)\cos(0t + 0° + 180°)\,u(t) \Rightarrow \boxed{i_{o,sss}^{(1)}(t) = 0\text{A}}$$

$$i_{o,sss}^{(2)}(t) = 2|\boldsymbol{H_I}(j10)|\cos\left(10t + 45° + \angle\boldsymbol{H_I}(j10)\right)u(t)\ \text{A}$$

$$i_{o,sss}^{(2)}(t) = 2(10)\cos(10t + 45° + 90°)\,u(t)$$

$$\boxed{i_{o,sss}^{(2)}(t) = 20\cos(10t + 135°)\,u(t)\ \text{A}}$$

Example #2: 2nd Order System Functions, SSS Response (Solution cont'd)

- Compute the sinusoidal steady state current response $i_{o,sss}(t)$ when $i_{in}(t) = [1 + 2\cos(10t + 45°) - 10\sin(15t - 30°)]u(t)\text{A}$
  - Since the system is assumed to be linear, we can employ superposition

$$i_{o,sss}(t) = i_{o,sss}^{(1)}(t) + i_{o,sss}^{(2)}(t) - i_{o,sss}^{(3)}(t)$$

$$i_{o,sss}^{(3)}(t) = 10|\boldsymbol{H_I}(j15)|| \sin\big(15t - 30° + \angle\boldsymbol{H_I}(j15)\big)\, u(t)\ \text{A}$$

$$i_{o,sss}^{(3)}(t) = 10\frac{45}{\sqrt{634}} \sin(15t - 30° + \tan^{-}(3/25))\, u(t)$$

$$\boxed{i_{o,sss}^{(3)}(t) = \frac{225\sqrt{634}}{317} \sin(15t - 30° + \tan^{-}(3/25))\, u(t)}$$

$$\boxed{i_{o,sss}^{(3)}(t) \approx 17.9 \sin(15t - 23.16°)\, u(t)}$$

Example #2: 2nd Order System Functions, SSS Response (Solution cont'd)

- Compute the sinusoidal steady state current response $i_{o,sss}(t)$ when $i_{in}(t) = [1 + 2\cos(10t + 45°) - 10\sin(15t - 30°)]u(t)\text{A}$
  - The complete SSS response $i_{o,sss}(t)$ is therefore

$$\boxed{\begin{aligned} i_{o,sss}(t) &= i_{o,sss}^{(1)}(t) + i_{o,sss}^{(2)}(t) - i_{o,sss}^{(3)}(t) \\ i_{o,sss}(t) &= 0u(t)\text{A} + 20\cos(10t + 135°)\,u(t)\text{ A} \\ &\qquad\qquad -17.9\sin(15t - 23.16°)\,u(t) \end{aligned}}$$

Lecture #6(b): Frequency Response I:
Frequency Response, SSS, and Bode Diagrams: Examples

The Decibel Scale and Interpreting Bode Diagrams

Example #1: Converting dB's to Magnitude

- Calculate the current gain frequency response magnitude $|\boldsymbol{H_I}(j\omega_0)|$ for the following current gain frequency response decibel magnitudes $|\boldsymbol{H_I}(j\omega_0)|_{dB}$
  - $|\boldsymbol{H_I}(j10)|_{dB} = 0.2 \text{ dB}$
  - $|\boldsymbol{H_I}(j100)|_{dB} = 26 \text{ dB}$
  - $|\boldsymbol{H_I}(j1)|_{dB} = -46 \text{ dB}$
- Solution
  - $|\boldsymbol{H_I}(j10)|_{dB} = 0.2 \text{ dB}$

$$20\log_{10}(|\boldsymbol{H_I}(j10)|) = |\boldsymbol{H_I}(j10)|_{dB} = 0.2 \text{ dB}$$
$$\log_{10}(|\boldsymbol{H_I}(j10)|) = 0.01 = 10^{-2}$$
$$|\boldsymbol{H_I}(j10)| = 10^{0.01}$$
$$\boxed{|\boldsymbol{H_I}(j10)| = 1.023 \text{ A/A}}$$

Example #1: Converting dB's to Magnitude

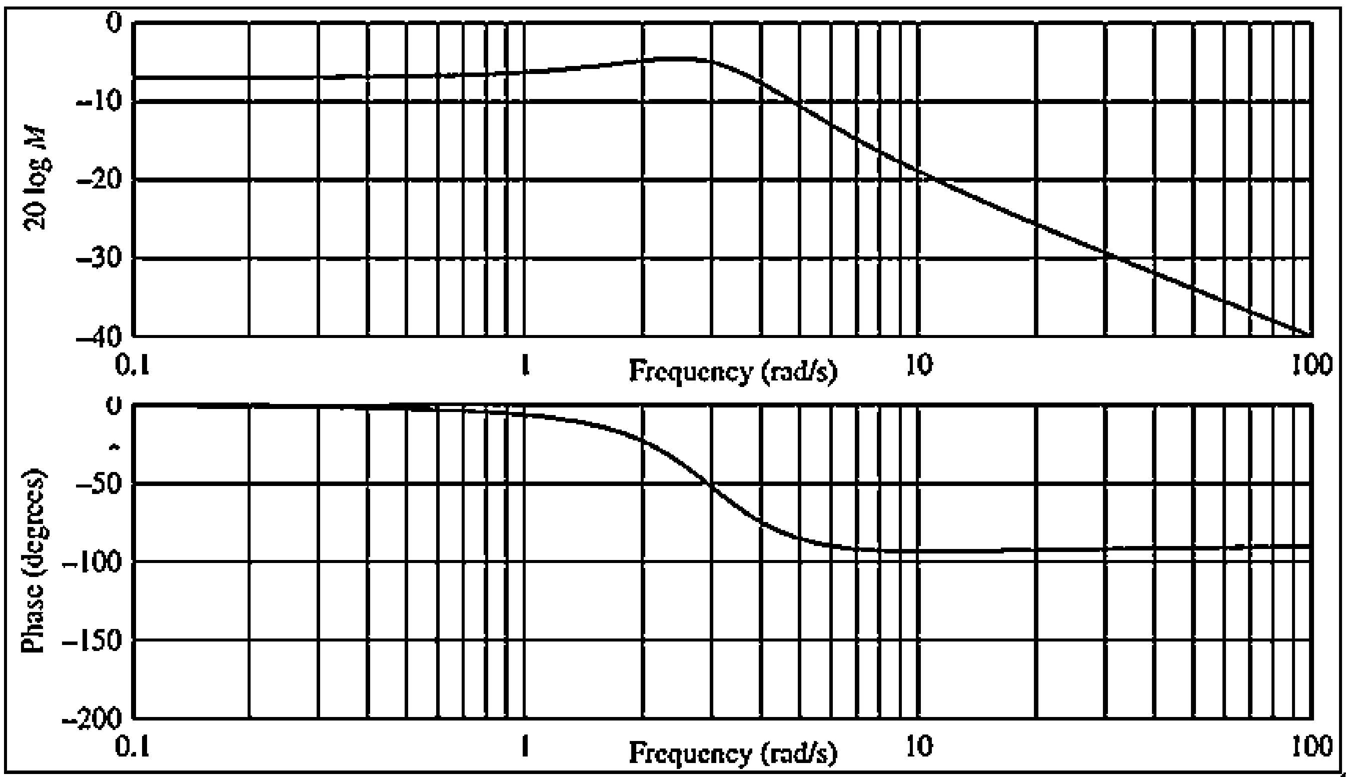
- Calculate the current gain frequency response magnitude $|\boldsymbol{H_I}(j\omega_0)|$ for the following current gain frequency response decibel magnitudes $|\boldsymbol{H_I}(j\omega_0)|_{dB}$
- Solution (cont'd)
  - $|\boldsymbol{H_I}(j100)|_{dB} = 26\text{dB}$

    $$20\log_{10}(|\boldsymbol{H_I}(j100)|) = |\boldsymbol{H_I}(j100)|_{dB} = 26\text{ dB}$$
    $$\log_{10}(|\boldsymbol{H_I}(j100)|) = 13/10 = 1.3$$
    $$|\boldsymbol{H_I}(j100)| = 10^{13/10} = 10^{1.3} \rightarrow \boxed{|\boldsymbol{H_I}(j100)| = 19.95\text{ A/A}}$$
  - $|\boldsymbol{H_I}(j1)|_{dB} = -46\text{ dB}$

    $$20\log_{10}(|\boldsymbol{H_I}(j1)|) = |\boldsymbol{H_I}(j1)|_{dB} = -46\text{ dB}$$
    $$\log_{10}(|\boldsymbol{H_I}(j1)|) = -23/10 = -2.3$$
    $$|\boldsymbol{H_I}(j1)| = 10^{-23/10} = 10^{-2.3} \rightarrow \boxed{|\boldsymbol{H_I}(j1)| = +5\text{ mA/A}}$$

Example #2: Interpreting Exact Bode Diagrams

- Consider the Bode diagram depicting a current gain frequency response function $\boldsymbol{M}(j\omega) = \boldsymbol{I_o}(j\omega)/\boldsymbol{I_i}(j\omega)$. Use the diagram to approximately compute the sinusoidal steady state (SSS) current response $i_{o,sss}(t)$ given $i_i(t) = 20\,cos(2t + 30°)u(t)$ A.

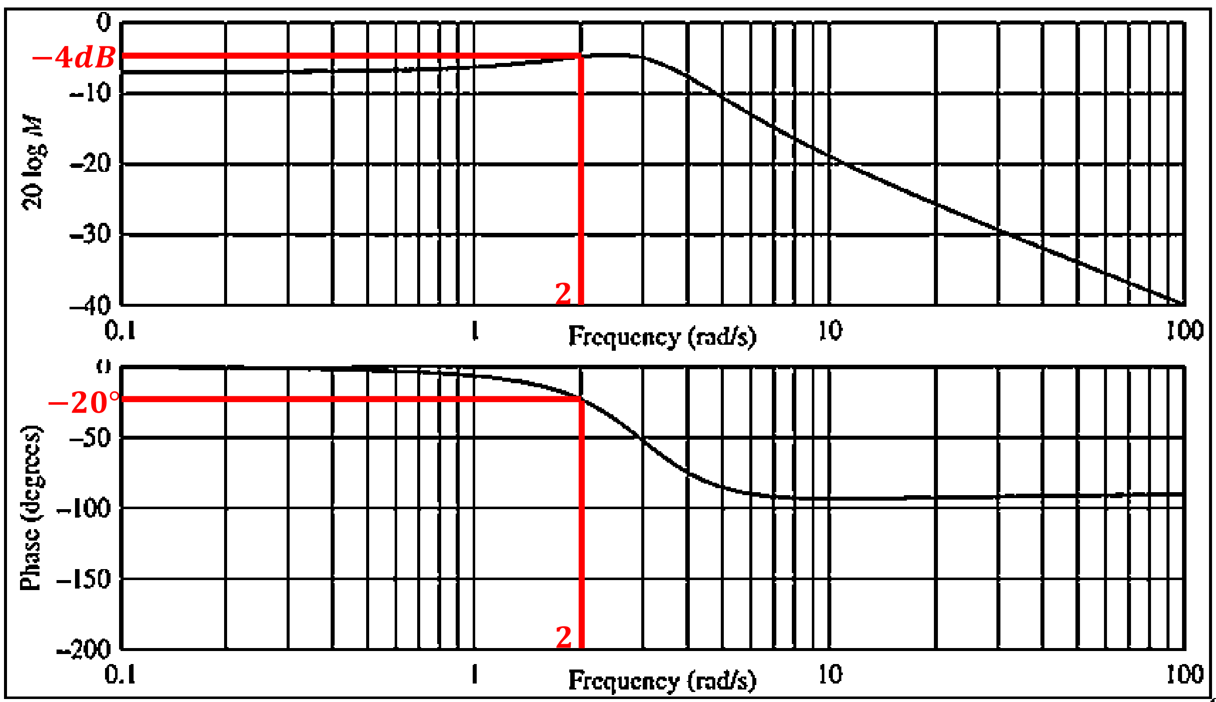


Example #2: Interpreting Exact Bode Diagrams (Solution)

- Compute the SSS current $i_{o,sss}(t)$ given $i_i(t) = 20cos(2t + 30°)u(t)$ A.

$$i_{o,sss}(t) = 20|\boldsymbol{M}(j2)|cos(2t + 30° + \angle\boldsymbol{M}(j2))u(t)$$
$$i_{o,sss}(t) = 20(0.631)cos(2t + 30° - 20°)u(t)$$
$$\boxed{i_{o,sss}(t) = 12.62cos(2t + 10°)u(t)}$$

$$20log_{10}(|\boldsymbol{M}(j2)|) = |\boldsymbol{M}(j2)|_{\mathrm{dB}}$$
$$20log_{10}(|\boldsymbol{M}(j2)|) = -4\mathrm{dB}$$
$$log_{10}(|\boldsymbol{M}(j2)|) = -0.2$$
$$|\boldsymbol{M}(j2)| = 10^{-0.2}$$
$$\boxed{|\boldsymbol{M}(j2)| = 0.631}$$



Lecture #6(b): Frequency Response I:
Frequency Response, SSS, and Bode Diagrams: Examples

Sketching Bode Diagrams: Real Poles/Zeros

Example #1

- Sketch an approximate Bode diagram of the system function below.

$$\boldsymbol{G}(\boldsymbol{s}) = \frac{200\boldsymbol{s}}{\boldsymbol{s}^2 + 12\boldsymbol{s} + 20} = \frac{200\boldsymbol{s}}{(\boldsymbol{s}+2)(\boldsymbol{s}+10)}$$

- Solution
  - **Step #1**: Identify the poles and zeros of $\boldsymbol{G}(\boldsymbol{s})$

    Zeros: $z_1 = 0 + j0 \rightarrow \boxed{|z_1| = 0}$

    Poles: $p_1 = -2 \rightarrow \boxed{|p_1| = 2}$

    $p_2 = -10 \rightarrow \boxed{|p_2| = 10}$
  - **Step #2**: Represent $\boldsymbol{G}(\boldsymbol{s})$ in standard Bode form

$$\boldsymbol{G}(\boldsymbol{s}) = \left(\frac{200}{(2)(10)}\right)\frac{\boldsymbol{s}}{(\boldsymbol{s}/2+1)(\boldsymbol{s}/10+1)} \rightarrow \boxed{\boldsymbol{G}(\boldsymbol{s}) = 10\frac{\boldsymbol{s}}{(\boldsymbol{s}/2+1)(\boldsymbol{s}/10+1)}}$$

  - **Step #3**: Find frequency response function $\boldsymbol{G}(j\omega)$ in standard Bode form

$$\boxed{\boldsymbol{G}(j\omega) = 10\frac{j\omega}{(j\omega/2+1)(j\omega/10+1)}}$$

Example #1

$$\boxed{\boldsymbol{G}(j\omega) = 10\frac{j\omega}{(j\omega/2+1)(j\omega/10+1)}}$$

- **Solution cont'd**
  - **Step #4(a)**: Compute expression for Decibel magnitude of $\boldsymbol{G}(j\omega)$

$$|\boldsymbol{G}(j\omega)|_{dB} = 20\log_{10}\left(|10\frac{j\omega}{(j\omega/2+1)(j\omega/10+1)}|\right)$$

$$|\boldsymbol{G}(j\omega)|_{dB} = 20\log_{10}(|10j\omega|) - 20\log_{10}(|(j\omega/2+1)(j\omega/10+1)|)$$

$$|\boldsymbol{G}(j\omega)|_{dB} = 20\log_{10}(|10|) + 20\log_{10}(|j\omega|) \\ -20\log_{10}(|j\omega/2+1|) - 20\log_{10}(|j\omega/10+1|)$$

$$\boxed{|\boldsymbol{G}(j\omega)|_{dB} = 20dB + 20\log_{10}(\omega) - 20\log_{10}(|j\omega/2+1|) -20\log_{10}(|j\omega/10+1|)}$$

  - **Step #4(b)**: Find expression for phase angle of $\boldsymbol{G}(j\omega)$

$$\angle\boldsymbol{G}(j\omega) = \angle\left(10\frac{j\omega}{(j\omega/2+1)(j\omega/10+1)}\right)$$

$$\angle\boldsymbol{G}(j\omega) = \angle(10j\omega) - \angle[(j\omega/2+1)\ (j\omega/10+1)]$$

$$\angle\boldsymbol{G}(j\omega) = \angle(10) + \angle(j\omega) - \angle(j\omega/2+1) - \angle(j\omega/10+1)$$

$$\boxed{\angle\boldsymbol{G}(j\omega) = 90^\circ - \angle(j\omega/2+1) - \angle(j\omega/10+1)}$$

Example #1

$$G(j\omega) = 10\frac{j\omega}{(j\omega/2+1)(j\omega/10+1)}$$

- **Solution cont'd**
  - **Step #5(a)**: Note the contributions of each term of $|G(j\omega)|_{dB}$

Term: $\|10\|_{dB}$	**Effects**: $\forall\omega$	**Slope**: 0dB/dec	**Value**: $+20\text{dB}$
Term: $\|j\omega\|_{dB}$	**Effects**: $\forall\omega$	**Slope**: 20dB/dec	**Value** @ $\omega = 1$: 0dB
Term: $-\|j\omega/2+1\|_{dB}$	**Effects**: $\omega > 2$	**Slope**: -20dB/dec	**Value**: N/A
Term: $-\|j\omega/10+1\|_{dB}$	**Effects**: $\omega > 10$	**Slope**: -20dB/dec	**Value**: N/A

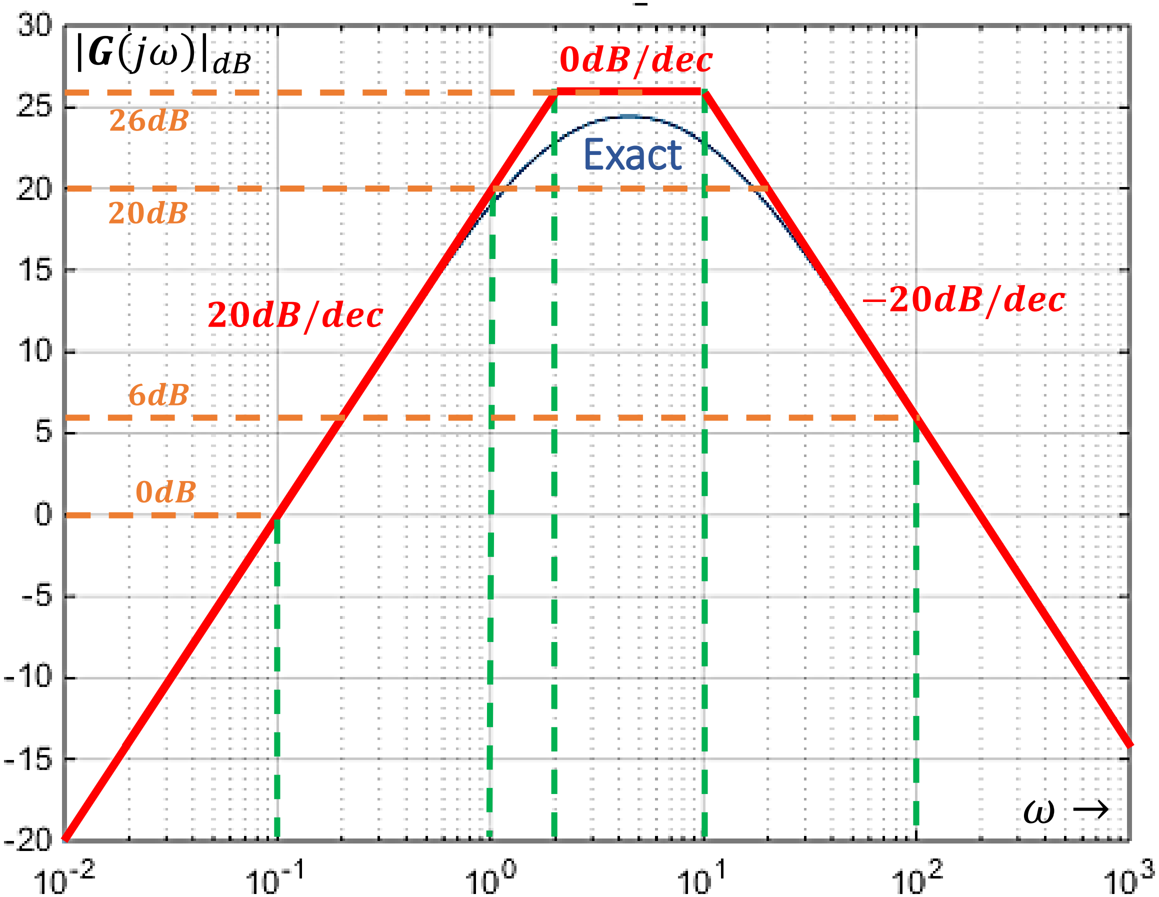
  - **Step #5(b)**: Create table to help sketch $|G(j\omega)|_{dB}$

Magnitude Terms	Radian Frequency $\boldsymbol{\omega}$ (rad/s)				
	Value at $\boldsymbol{\omega = 0.1}$	Slope at $\boldsymbol{\omega > 0.1}$	Slope at $\boldsymbol{\omega > 2}$	Slope at $\boldsymbol{\omega > 10}$	Slope at $\boldsymbol{\omega > 100}$
$\|10\|_{dB}$	$+20\text{dB}$	0dB/dec	0dB/dec	0dB/dec	0dB/dec
$\|j\omega\|_{dB}$	-20dB	20dB/dec	20dB/dec	20dB/dec	20dB/dec
$-\|j\omega/2+1\|_{dB}$	0dB	0dB/dec	-20dB/dec	-20dB/dec	-20dB/dec
$-\|j\omega/10+1\|_{dB}$	0dB	0dB/dec	0dB/dec	-20dB/dec	-20dB/dec
Total	$\mathbf{0dB}$	$\mathbf{20dB/dec}$	$\mathbf{0dB/dec}$	$\mathbf{-20dB/dec}$	$\mathbf{-20dB/dec}$

Example #1

$$G(j\omega) = 10\frac{j\omega}{(j\omega/2+1)(j\omega/10+1)}$$

- **Solution cont'd**
  - **Step #5(c)**: Sketch approximate Decibel magnitude $|G(j\omega)|_{dB}$



Example #1

$$\boldsymbol{G}(j\omega) = 10\frac{j\omega}{(j\omega/2+1)(j\omega/10+1)}$$

- Solution cont'd
  - **Step #6(a)**: Note contribution of each term of $\angle\boldsymbol{G}(j\omega)$

    Term: $\angle 10$ Effects: $\forall\omega$ Slope: $0°/\text{dec}$ Value: $0°$

    Term: $\angle j\omega$ Effects: $\forall\omega$ Slope: $0°/\text{dec}$ Value: $90°$

    Term: $-\angle(j\omega/2+1)$ Effects: $\omega\in[0.2, 20]$ Slope: $-45°/\text{dec}$ Final Value: $-90°$

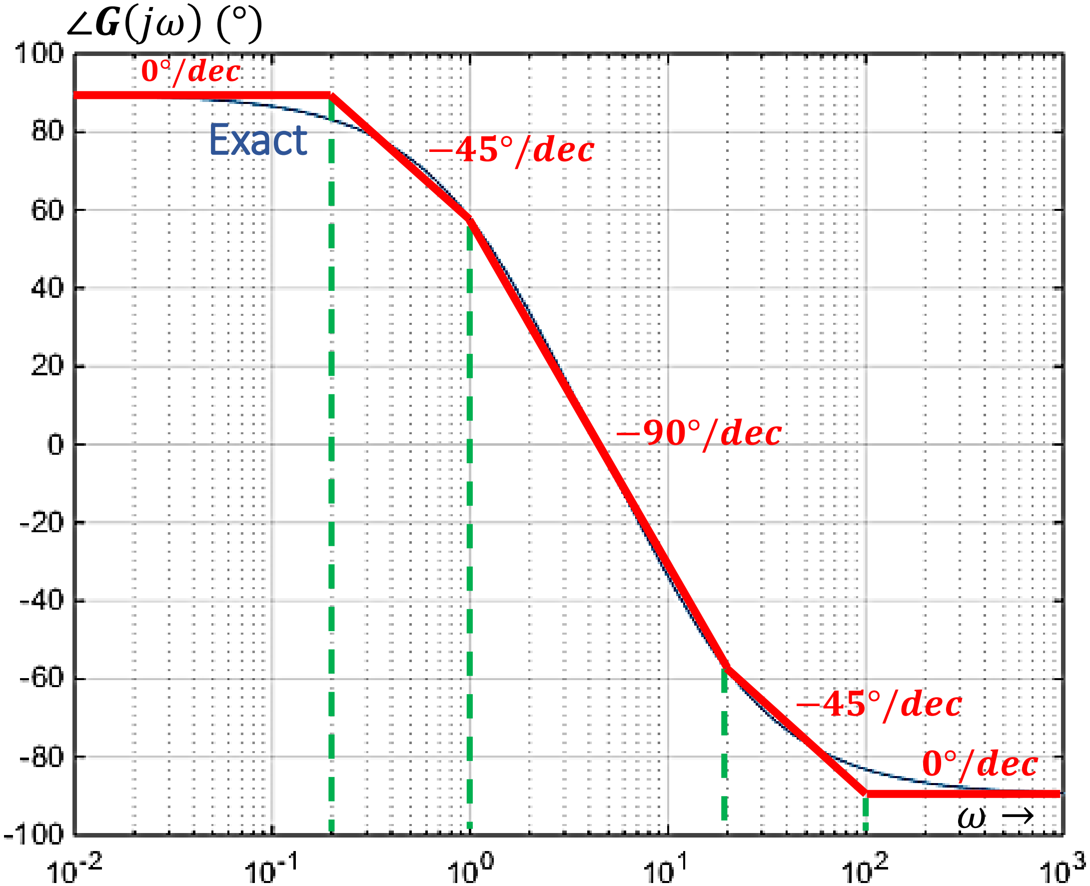
    Term: $-\angle(j\omega/10+1)$ Effects: $\omega\in[1, 100]$ Slope: $-45°/\text{dec}$ Final Value: $-90°$
  - **Step #6(b)**: Create table to help sketch $\angle\boldsymbol{G}(j\omega)$

Phase Terms	Radian Frequency $\boldsymbol{\omega}$ (rad/s)					
	Value at $\boldsymbol{\omega=0.02}$	Slope at $\boldsymbol{\omega>0.2}$	Slope at $\boldsymbol{\omega>1}$	Slope at $\boldsymbol{\omega>20}$	Slope at $\boldsymbol{\omega>10^2}$	Value at $\boldsymbol{\omega=10^3}$
$\angle 10$	$0°$	$0°/\text{dec}$	$0°/\text{dec}$	$0°/\text{dec}$	$0°/\text{dec}$	$0°$
$\angle j\omega$	$90°$	$0°/\text{dec}$	$0°/\text{dec}$	$0°/\text{dec}$	$0°/\text{dec}$	$90°$
$-\angle(j\omega/2+1)$	$0°$	$-45°/\text{dec}$	$-45°/\text{dec}$	$0°/\text{dec}$	$0°/\text{dec}$	$-90°$
$-\angle(j\omega/10+1)$	$0°$	$0°/\text{dec}$	$-45°/\text{dec}$	$-45°/\text{dec}$	$0°/\text{dec}$	$-90°$
Total	$\mathbf{90°}$	$\mathbf{-45°/dec}$	$\mathbf{-90°/dec}$	$\mathbf{-45°/dec}$	$\mathbf{0°/dec}$	$\mathbf{-90°}$

Example #1

$$G(j\omega) = 10\frac{j\omega}{(j\omega/2+1)(j\omega/10+1)}$$

- **Solution cont'd**
  - **Step #6(c)**: Sketch approximate phase angle $\angle G(j\omega)$



Example #1

$$G(s) = \frac{200s}{s^2 + 12s + 20}$$

- Solution cont'd
  - **Step #7**: Plot exact Bode diagram in MATLAB with code below

```
%-----------------------------------------------------
% num - numerator polynomial: 200s + 0
% den - denominator polynomial: s^2 + 12s + 20
%-----------------------------------------------------
num = [200 0];
den = [1 12 20];
%-----------------------------------------------------
% sys_fun - system object specified according to
%           its system (transfer) function tf()
%-----------------------------------------------------
sys_fun = tf(num, den);
%-----------------------------------------------------
% bode() - generates Bode plot (mag, phase) of the freq.
%          response function obtained according to the
%          system function represented by system object
%          object sys_fun
%-----------------------------------------------------
bode(sys_fun);
```

Example #2

- Sketch an approximate Bode diagram of the system function below.

$$\boldsymbol{G}(\boldsymbol{s}) = \frac{5(\boldsymbol{s}+2)}{\boldsymbol{s}^2+10\boldsymbol{s}} = \frac{5(\boldsymbol{s}+2)}{\boldsymbol{s}(\boldsymbol{s}+10)}$$

- Solution
  - **Step #1**: Identify the poles and zeros of $\boldsymbol{G}(\boldsymbol{s})$

Zeros: $|z_1| = -2 \rightarrow \boxed{|z_1| = 2}$

Poles: $p_1 = 0 \rightarrow \boxed{|p_1| = 0}$

$p_2 = -10 \rightarrow \boxed{|p_2| = 10}$

  - **Step #2**: Represent $\boldsymbol{G}(\boldsymbol{s})$ in standard Bode form

$$\boldsymbol{G}(\boldsymbol{s}) = \left(\frac{5(2)}{10}\right)\frac{\boldsymbol{s}/2+1}{\boldsymbol{s}(\boldsymbol{s}/10+1)} \rightarrow \boxed{\boldsymbol{G}(\boldsymbol{s}) = \frac{\boldsymbol{s}/2+1}{\boldsymbol{s}(\boldsymbol{s}/10+1)}}$$

  - **Step #3**: Find frequency response function $\boldsymbol{G}(j\omega)$ in standard Bode form

$$\boxed{\boldsymbol{G}(j\omega) = 1\,\frac{j\omega/2+1}{(j\omega)(j\omega/10+1)}}$$

Example #2

$$\boldsymbol{G}(j\omega) = \frac{j\omega/2 + 1}{(j\omega)(j\omega/10 + 1)}$$

- **Solution cont'd**
  - **Step #4(a)**: Compute expression for Decibel magnitude of $\boldsymbol{G}(j\omega)$

$$|\boldsymbol{G}(j\omega)|_{dB} = 20\log_{10}\left(\left|\frac{j\omega/2+1}{(j\omega)(j\omega/10+1)}\right|\right)$$

$$|\boldsymbol{G}(j\omega)|_{dB} = 20\log_{10}(|j\omega/2+1|) - 20\log_{10}(|(j\omega)(j\omega/10+1)|)$$

$$\boxed{\begin{gathered}|\boldsymbol{G}(j\omega)|_{dB} = 20\log_{10}(|j\omega/2+1|) - 20\log_{10}(|j\omega|) \\ -20\log_{10}(|j\omega/10+1|)\end{gathered}}$$

  - **Step #4(b)**: Find expression for phase angle of $\boldsymbol{G}(j\omega)$

$$\angle\boldsymbol{G}(j\omega) = \angle\left(\frac{j\omega/2+1}{(j\omega)(j\omega/10+1)}\right)$$

$$\angle\boldsymbol{G}(j\omega) = \angle(j\omega/2+1) - \angle[(j\omega)(j\omega/10+1)]$$

$$\angle\boldsymbol{G}(j\omega) = \angle(j\omega/2+1) - \angle(j\omega) - \angle(j\omega/10+1)$$

$$\boxed{\angle\boldsymbol{G}(j\omega) = -90° + \angle(j\omega/2+1) - \angle(j\omega/10+1)}$$

Example #2

$$G(j\omega) = \frac{j\omega/2 + 1}{(j\omega)(j\omega/10 + 1)}$$

- **Solution cont'd**
  - **Step #5(a)**: Note contribution of each term of $|G(j\omega)|_{dB}$

Term: $|1|_{dB}$ Effects: $\forall\omega$ Slope: 0dB/dec Value: 0dB

Term: $-|j\omega|_{dB}$ Effects: $\forall\omega$ Slope: -20dB/dec Value @ $\omega = 1$: $0dB$

Term: $|j\omega/2 + 1|_{dB}$ Effects: $\omega > 2$ Slope: 20dB/dec Value: N/A

Term: $-|j\omega/10 + 1|_{dB}$ Effects: $\omega > 10$ Slope: -20dB/dec Value: N/A

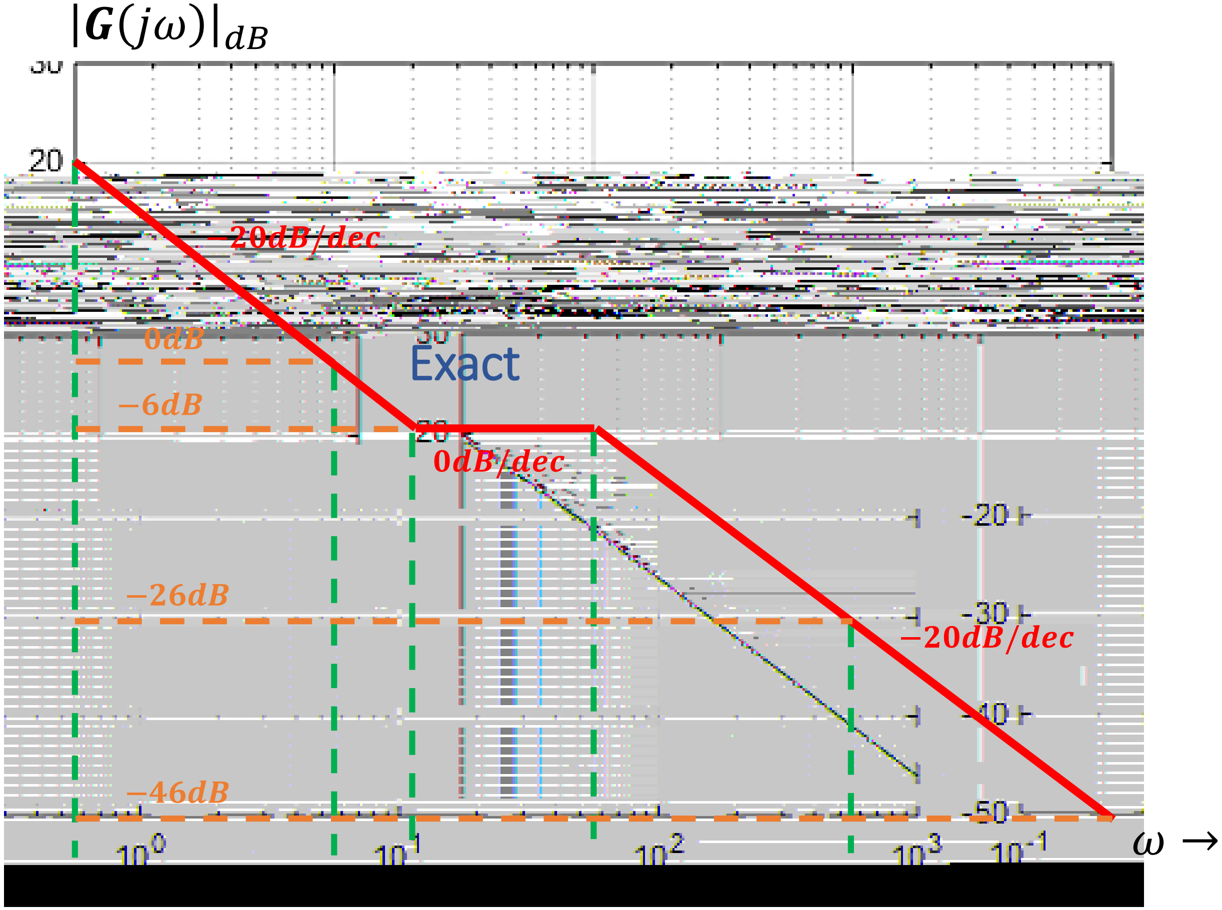
  - **Step #5(b)**: Create table to help sketch $|G(j\omega)|_{dB}$

Magnitude Terms	Radian Frequency ω (rad/s)					
	Value at $\boldsymbol{\omega = 0.1}$	Slope at $\boldsymbol{\omega > 0.1}$	Slope at $\boldsymbol{\omega > 1}$	Slope at $\boldsymbol{\omega > 2}$	Slope at $\boldsymbol{\omega > 10}$	Slope at $\boldsymbol{\omega > 100}$
$\lvert 1\rvert_{dB}$	0dB	0dB/dec	0dB/dec	0dB/dec	0dB/dec	0dB/dec
$-\lvert j\omega\rvert_{dB}$	20dB	-20dB/dec	-20dB/dec	-20dB/dec	-20dB/dec	-20dB/dec
$\lvert j\omega/2+1\rvert_{dB}$	0dB	0dB/dec	0dB/dec	20dB/dec	20dB/dec	20dB/dec
$-\lvert j\omega/10+1\rvert_{dB}$	0dB	0dB/dec	0dB/dec	0dB/dec	-20dB/dec	-20dB/dec
Total	**$\mathbf{20}$dB**	**$\mathbf{-20}$dB/dec**	**$\mathbf{-20}$dB/dec**	**$\mathbf{0}$dB/dec**	**$\mathbf{-20}$dB/dec**	**$\mathbf{-20}$dB/dec**

Example #2

$$G(j\omega) = \frac{j\omega/2 + 1}{(j\omega)(j\omega/10 + 1)}$$

- **Solution cont'd**
  - **Step #5(c)**: Sketch approximate Decibel magnitude $|G(j\omega)|_{dB}$



Example #2

$$G(j\omega) = \frac{j\omega/2 + 1}{(j\omega)(j\omega/10 + 1)}$$

- **Solution cont'd**
  - **Step #6(a)**: Note contribution of each term of $\angle G(j\omega)$

    Term: $\angle 1$ Effects: $\forall\omega$ Slope: $0°/\text{dec}$ Value: $0°$

    Term: $-\angle j\omega$ Effects: $\forall\omega$ Slope: $0°/\text{dec}$ Value: $-90°$

    Term: $\angle(j\omega/2 + 1)$ Effects: $\omega \in [0.2, 20]$ Slope: $45°/\text{dec}$ Final Value: $90°$

    Term: $-\angle(j\omega/10 + 1)$ Effects: $\omega \in [1, 100]$ Slope: $-45°/\text{dec}$ Final Value: $-90°$

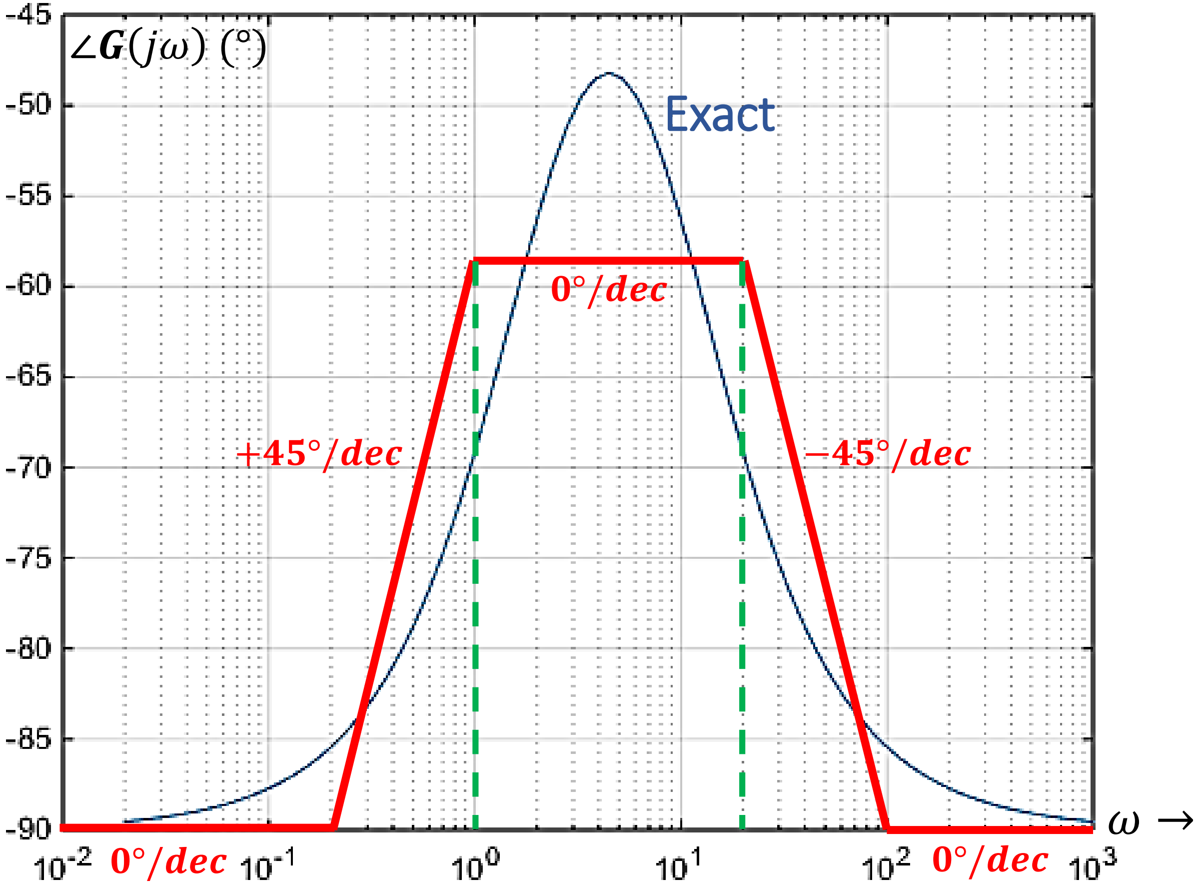
  - **Step #6(b)**: Create table to help sketch $\angle G(j\omega)$

Phase Terms	Radian Frequency $\boldsymbol{\omega}$ (rad/s)					
	Value at $\boldsymbol{\omega = 0.02}$	Slope at $\boldsymbol{\omega > 0.2}$	Slope at $\boldsymbol{\omega > 1}$	Slope at $\boldsymbol{\omega > 20}$	Slope at $\boldsymbol{\omega > 10^2}$	Value at $\boldsymbol{\omega > 10^3}$
$\angle 1$	$0°$	$0°/\text{dec}$	$0°/\text{dec}$	$0°/\text{dec}$	$0°/\text{dec}$	$0°$
$-\angle j\omega$	$-90°$	$0°/\text{dec}$	$0°/\text{dec}$	$0°/\text{dec}$	$0°/\text{dec}$	$-90°$
$\angle(j\omega/2 + 1)$	$0°$	$45°/\text{dec}$	$45°/\text{dec}$	$0°/\text{dec}$	$0°/\text{dec}$	$90°$
$-\angle(j\omega/10 + 1)$	$0°$	$0°/\text{dec}$	$-45°/\text{dec}$	$-45°/\text{dec}$	$0°/\text{dec}$	$-90°$
Total	$\boldsymbol{-90°}$	$\boldsymbol{45°/\text{dec}}$	$\boldsymbol{0°/\text{dec}}$	$\boldsymbol{-45°/\text{dec}}$	$\boldsymbol{0°/\text{dec}}$	$\boldsymbol{-90°}$

Example #2

$$\boldsymbol{G}(j\omega) = \frac{j\omega/2 + 1}{(j\omega)(j\omega/10 + 1)}$$

- **Solution cont'd**
  - **Step #6(c)**: Sketch approximate phase angle $\angle\boldsymbol{G}(j\omega)$



Example #2

$$G(s) = \frac{5(s+2)}{s^2 + 10s}$$

- **Solution cont'd**
  - **Step #7**: Plot exact Bode diagram in MATLAB with code below

```
%-----------------------------------------------------
% num - numerator polynomial: 5s + 10
% den - denominator polynomial: s^2 + 10s
%-----------------------------------------------------
num = [5 10];
den = [1 10 0];
%-----------------------------------------------------
% sys_fun - system object specified according to
%           its system (transfer) function tf()
%-----------------------------------------------------
sys_fun = tf(num, den);
%-----------------------------------------------------
% bode() - generates Bode plot (mag, phase) of the freq.
%          response function obtained according to the
%          system function represented by system object
%          object sys_fun
%-----------------------------------------------------
bode(sys_fun);
```

Example #3

- Sketch an approximate Bode diagram of the system function below.

$$\boldsymbol{G}(\boldsymbol{s}) = \frac{\boldsymbol{s}+10}{\boldsymbol{s}(s^2+10s+25)} = \frac{\boldsymbol{s}+10}{\boldsymbol{s}(\boldsymbol{s}+5)^2}$$

- Solution
  - **Step #1**: Identify the poles and zeros of $\boldsymbol{G}(\boldsymbol{s})$ in standard Bode form

Zeros: $|z_1| = -10 \rightarrow \boxed{|z_1| = 10}$

Poles: $p_1 = 0 \rightarrow \boxed{|p_1| = 0}$

$p_{2,3} = -5 \rightarrow \boxed{|p_{2,3}| = 5}$

  - **Step #2**: Represent $\boldsymbol{G}(\boldsymbol{s})$ in standard Bode form

$$G(s) = \left(\frac{10}{(1)(5)(5)}\right)\frac{\boldsymbol{s}/10+1}{\boldsymbol{s}(\boldsymbol{s}/5+1)^2} \rightarrow \boxed{G(s) = 0.4\frac{\boldsymbol{s}/10+1}{\boldsymbol{s}(\boldsymbol{s}/5+1)^2}}$$

  - **Step #3**: Find frequency response function $\boldsymbol{G}(j\omega)$ in standard Bode form

$$\boxed{\boldsymbol{G}(j\omega) = 0.4\frac{j\omega/10+1}{(j\omega)(j\omega/5+1)^2}}$$

Example #3

$$\boldsymbol{G}(j\omega) = 0.4\frac{j\omega/10+1}{(j\omega)(j\omega/5+1)^2}$$

- Solution cont'd
  - **Step #4(a)**: Compute expression for Decibel magnitude of $\boldsymbol{G}(j\omega)$

$$|\boldsymbol{G}(j\omega)|_{dB} = 20\log_{10}\left(|0.4\frac{j\omega/10+1}{(j\omega)(j\omega/5+1)^2}|\right)$$

$$|\boldsymbol{G}(j\omega)|_{dB} = 20\log_{10}(|0.4(j\omega/10+1)|) - 20\log_{10}\left(\left|(j\omega)(j\omega/5+1)^2\right|\right)$$

$$|\boldsymbol{G}(j\omega)|_{dB} = 20\log_{10}(|0.4|) + 20\log_{10}(|j\omega/10+1|) - 20\log_{10}(|j\omega|) - 40\log_{10}(|j\omega/5+1|)$$

$$\boxed{|\boldsymbol{G}(j\omega)|_{dB} = -8\text{dB} + 20\log_{10}(|j\omega/10+1|) - 20\log_{10}(|j\omega|) - 40\log_{10}(|j\omega/5+1|)}$$

  - **Step #4(b)**: Find expression for phase angle of $\boldsymbol{G}(j\omega)$

$$\angle\boldsymbol{G}(j\omega) = \angle\left(0.4[j\omega/10+1]/[(j\omega)(j\omega/5+1)^2]\right)$$

$$\angle\boldsymbol{G}(j\omega) = \angle[(0.4)(j\omega/10+1)] - \angle[(j\omega)(j\omega/5+1)^2]$$

$$\angle\boldsymbol{G}(j\omega) = \angle(0.4) + \angle(j\omega/10+1) - \angle(j\omega) - 2\angle(j\omega/5+1)$$

$$\boxed{\angle\boldsymbol{G}(j\omega) = -90^\circ + \angle(j\omega/10+1) - 2\angle(j\omega/5+1)}$$

Example #3

$$\boldsymbol{G}(j\omega) = 0.4\frac{j\omega/10 + 1}{(j\omega)(j\omega/5 + 1)^2}$$

- **Solution cont'd**
  - **Step #5(a)**: Note contribution of each term of $|\boldsymbol{G}(j\omega)|_{dB}$

Term: $|0.4|_{dB}$ Effects: $\forall\omega$ Slope: 0dB/dec Value: -8dB

Term: $-|j\omega|_{dB}$ Effects: $\forall\omega$ Slope: -20dB/dec Value @ $\omega = 1$: 0dB

Term: $-2|j\omega/5 + 1|_{dB}$ Effects: $\omega > 5$ Slope: -40dB/dec Value: N/A

Term: $|j\omega/10 + 1|_{dB}$ Effects: $\omega > 10$ Slope: 20dB/dec Value: N/A

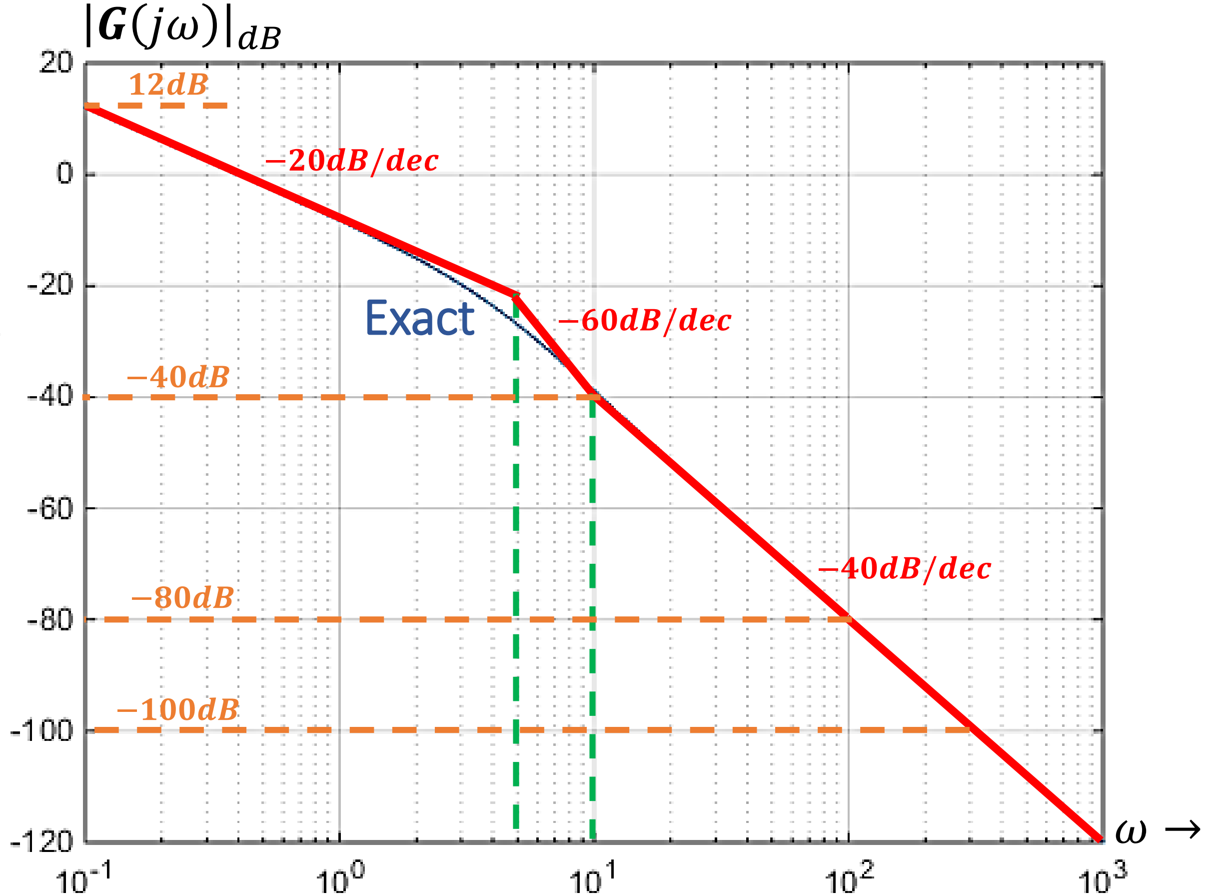
  - **Step #5(b)**: Create table to help sketch $|\boldsymbol{G}(j\omega)|_{dB}$

Magnitude Terms	Radian Frequency $\boldsymbol{\omega}$ (rad/s)				
	Value at $\boldsymbol{\omega = 0.1}$	Slope at $\boldsymbol{\omega > 0.1}$	Slope at $\boldsymbol{\omega > 5}$	Slope at $\boldsymbol{\omega > 10}$	Slope at $\boldsymbol{\omega > 100}$
$\|0.4\|_{dB}$	-8dB	0dB/dec	0dB/dec	0dB/dec	0dB/dec
$-\|j\omega\|_{dB}$	20dB	-20dB/dec	-20dB/dec	-20dB/dec	-20dB/dec
$-2\|j\omega/5 + 1\|_{dB}$	0dB	0dB/dec	-40dB/dec	-40dB/dec	-40dB/dec
$\|j\omega/10 + 1\|_{dB}$	0dB	0dB/dec	0dB/dec	20dB/dec	20dB/dec
Total	**12dB**	**-20dB/dec**	**-60dB/dec**	**-40dB/dec**	**-40dB/dec**

Example #3

$$G(j\omega) = 0.4\frac{j\omega/10+1}{(j\omega)(j\omega/5+1)^2}$$

- **Solution cont'd**
  - **Step #5(c)**: Sketch approximate Decibel magnitude $|G(j\omega)|_{dB}$



Example #3

$$G(j\omega) = 0.4\frac{j\omega/10+1}{(j\omega)(j\omega/5+1)^2}$$

- Solution cont'd
  - **Step #6(a)**: Note contribution of each term of $\angle G(j\omega)$

Term: $\angle 0.4$ Effects: $\forall\omega$ Slope: $0°/\text{dec}$ Value: $0°$

Term: $-\angle j\omega$ Effects: $\forall\omega$ Slope: $0°/\text{dec}$ Value: $-90°$

Term: $-2\angle(j\omega/5+1)$ Effects: $\omega \in [0.5, 50]$ Slope: $-90°/\text{dec}$ Final Value: $-180°$

Term: $\angle(j\omega/10+1)$ Effects: $\omega \in [1, 100]$ Slope: $45°/\text{dec}$ Final Value: $90°$

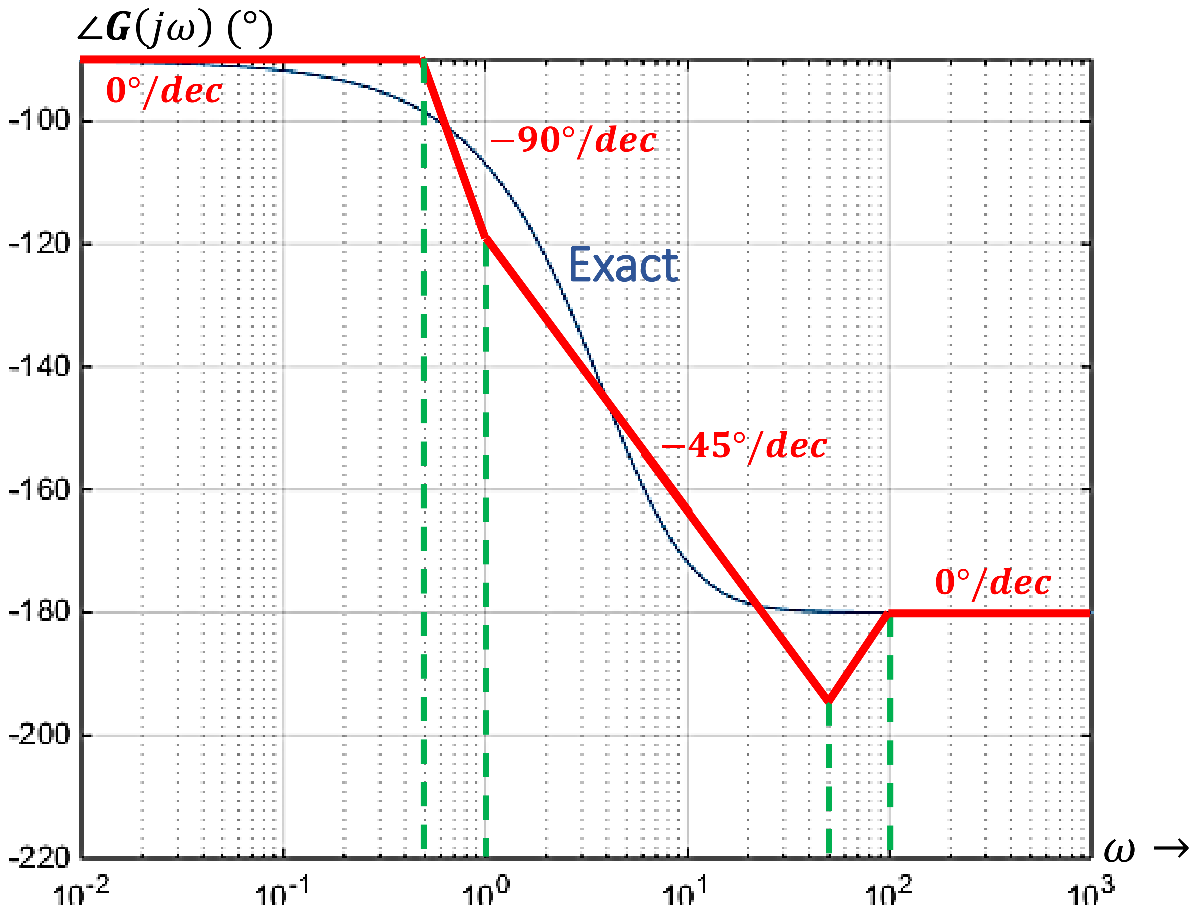
  - **Step #6(b)**: Create table to help sketch $\angle G(j\omega)$

Phase Terms	Radian Frequency $\boldsymbol{\omega}$ (rad/s)					
	Value at $\boldsymbol{\omega = 0.05}$	Slope at $\boldsymbol{\omega > 0.5}$	Slope at $\boldsymbol{\omega > 1}$	Slope at $\boldsymbol{\omega > 50}$	Slope at $\boldsymbol{\omega > 10^2}$	Value at $\boldsymbol{\omega = 10^3}$
$\angle 0.4$	$0°$	$0°/\text{dec}$	$0°/\text{dec}$	$0°/\text{dec}$	$0°/\text{dec}$	$0°$
$-\angle j\omega$	$-90°$	$0°/\text{dec}$	$0°/\text{dec}$	$0°/\text{dec}$	$0°/\text{dec}$	$-90°$
$-2\angle(j\omega/5+1)$	$0°$	$-90°/\text{dec}$	$-90°/\text{dec}$	$0°/\text{dec}$	$0°/\text{dec}$	$-180°$
$\angle(j\omega/10+1)$	$0°$	$0°/\text{dec}$	$45°/\text{dec}$	$45°/\text{dec}$	$0°/\text{dec}$	$90°$
Total	$\boldsymbol{-90°}$	$\boldsymbol{-90°/\text{dec}}$	$\boldsymbol{-45°/\text{dec}}$	$\boldsymbol{45°/\text{dec}}$	$\boldsymbol{0°/\text{dec}}$	$\boldsymbol{-180°}$

Example #3

$$\boldsymbol{G}(j\omega) = 0.4\frac{j\omega/10+1}{(j\omega)(j\omega/5+1)^2}$$

- **Solution cont'd**
  - **Step #6(c)**: Sketch approximate phase angle $\angle\boldsymbol{G}(j\omega)$



Example #3

$$G(s) = \frac{s + 10}{s(s^2 + 10s + 25)}$$

- Solution cont'd
  - **Step #7**: Plot exact Bode diagram in MATLAB with code below

```
%-----------------------------------------------------
% num - numerator polynomial: s + 10
% den - denominator polynomial: s^3 + 10s^2 + 25s + 0
%-----------------------------------------------------
num = [1 10];
den = [1 10 25 0];
%-----------------------------------------------------
% sys_fun - system object specified according to
%           its system (transfer) function tf()
%-----------------------------------------------------
sys_fun = tf(num, den);
%-----------------------------------------------------
% bode() - generates Bode plot (mag, phase) of the freq.
%          response function obtained according to the
%          system function represented by system object
%          object sys_fun
%-----------------------------------------------------
bode(sys_fun);
```

Example #4

- Sketch an approximate Bode diagram of the system function below.

$$\boldsymbol{G}(\boldsymbol{s}) = \frac{(\boldsymbol{s}+10)(\boldsymbol{s}+100)^2}{10\boldsymbol{s}^2(\boldsymbol{s}+10^3)}$$

- Solution
  - **Step #1**: Identify the poles and zeros of $\boldsymbol{G}(\boldsymbol{s})$

Zeros: $|z_1| = -10 \rightarrow \boxed{|z_1| = 10}$

$|z_{2,3}| = -100 \rightarrow \boxed{|z_{2,3}| = 10^2}$

Poles: $p_{1,2} = 0 \rightarrow \boxed{|p_{1,2}| = 0}$

$p_3 = -1k \rightarrow \boxed{|p_3| = 10^3}$

  - **Step #2**: Represent $\boldsymbol{G}(\boldsymbol{s})$ in standard Bode form

$$\boldsymbol{G}(\boldsymbol{s}) = \left(\frac{(10^1)(10^2)(10^2)}{(10)(10^3)}\right)\frac{\left(\frac{\boldsymbol{s}}{10}+1\right)\left(\frac{\boldsymbol{s}}{10^2}+1\right)^2}{\boldsymbol{s}^2\left(\frac{\boldsymbol{s}}{10^3}+1\right)} \rightarrow \boxed{\boldsymbol{G}(\boldsymbol{s}) = 10\frac{\left(\frac{\boldsymbol{s}}{10}+1\right)\left(\frac{\boldsymbol{s}}{10^2}+1\right)^2}{\boldsymbol{s}^2\left(\frac{\boldsymbol{s}}{10^3}+1\right)}}$$

  - **Step #3**: Find frequency response function $\boldsymbol{G}(j\omega)$ in standard Bode form

$$\boxed{\boldsymbol{G}(j\omega) = 10\frac{(j\omega/10+1)(j\omega/10^2+1)^2}{(j\omega)^2(j\omega/10^3+1)}}$$

Example #4

$$\boldsymbol{G}(j\omega) = 10\,\frac{(j\omega/10+1)\left(j\omega/10^2+1\right)^2}{(j\omega)^2(j\omega/10^3+1)}$$

- Solution cont'd
  - **Step #4(a)**: Compute expression for Decibel magnitude of $\boldsymbol{G}(j\omega)$

$$|\boldsymbol{G}(j\omega)|_{dB} = 20\log_{10}\left(\left|10\,\frac{(j\omega/10+1)\left(j\omega/10^2+1\right)^2}{(j\omega)^2(j\omega/10^3+1)}\right|\right)$$

$$|\boldsymbol{G}(j\omega)|_{dB} = 20\log_{10}(|10|) + 20\log_{10}(|j\omega/10+1|) + 40\log_{10}\left(\left|j\omega/10^2+1\right|\right) \\ -40\log_{10}(|j\omega|) - 20\log_{10}\left(|j\omega/10^3+1|\right)$$

$$\boxed{|\boldsymbol{G}(j\omega)|_{dB} = 20dB + 20\log_{10}(|j\omega/10+1|) + 40\log_{10}\left(\left|j\omega/10^2+1\right|\right) -40\log_{10}(|j\omega|) - 20\log_{10}\left(|j\omega/10^3+1|\right)}$$

  - **Step #4(b)**: Find expression for phase angle of $\boldsymbol{G}(j\omega)$

$$\angle\boldsymbol{G}(j\omega) = \angle\left(10\,\frac{(j\omega/10+1)\left(j\omega/10^2+1\right)^2}{(j\omega)^2(j\omega/10^3+1)}\right)$$

$$\angle\boldsymbol{G}(j\omega) = \angle(10) + \angle(j\omega/10+1) + 2\angle\left(j\omega/10^2+1\right) - 2\angle(j\omega) - \angle\left(j\omega/10^3+1\right)$$

$$\boxed{\angle\boldsymbol{G}(j\omega) = -180^\circ + \angle(j\omega/10+1) + 2\angle\left(j\omega/10^2+1\right) - \angle\left(j\omega/10^3+1\right)}$$

Example #4

$$\boldsymbol{G}(j\omega) = 10\,\frac{(j\omega/10+1)\left(j\omega/10^2+1\right)^2}{(j\omega)^2(j\omega/10^3+1)}$$

- **Solution cont'd**
  - **Step #5(a)**: Note contribution of each term of $|\boldsymbol{G}(j\omega)|_{dB}$

Term: $|10|_{dB}$	Effects: $\forall\omega$	Slope: 0dB/dec	Value: 20dB
Term: $-2|j\omega|_{dB}$	Effects: $\forall\omega$	Slope: -40dB/dec	Value @ $\omega = 1$: 0dB
Term: $|j\omega/10+1|_{dB}$	Effects: $\omega > 10$	Slope: 20dB/dec	Value: N/A
Term: $2\left|j\omega/10^2+1\right|_{dB}$	Effects: $\omega > 10^2$	Slope: 40dB/dec	Value: N/A
Term: $-\left|j\omega/10^3+1\right|_{dB}$	Effects: $\omega > 10^3$	Slope: -20dB/dec	Value: N/A

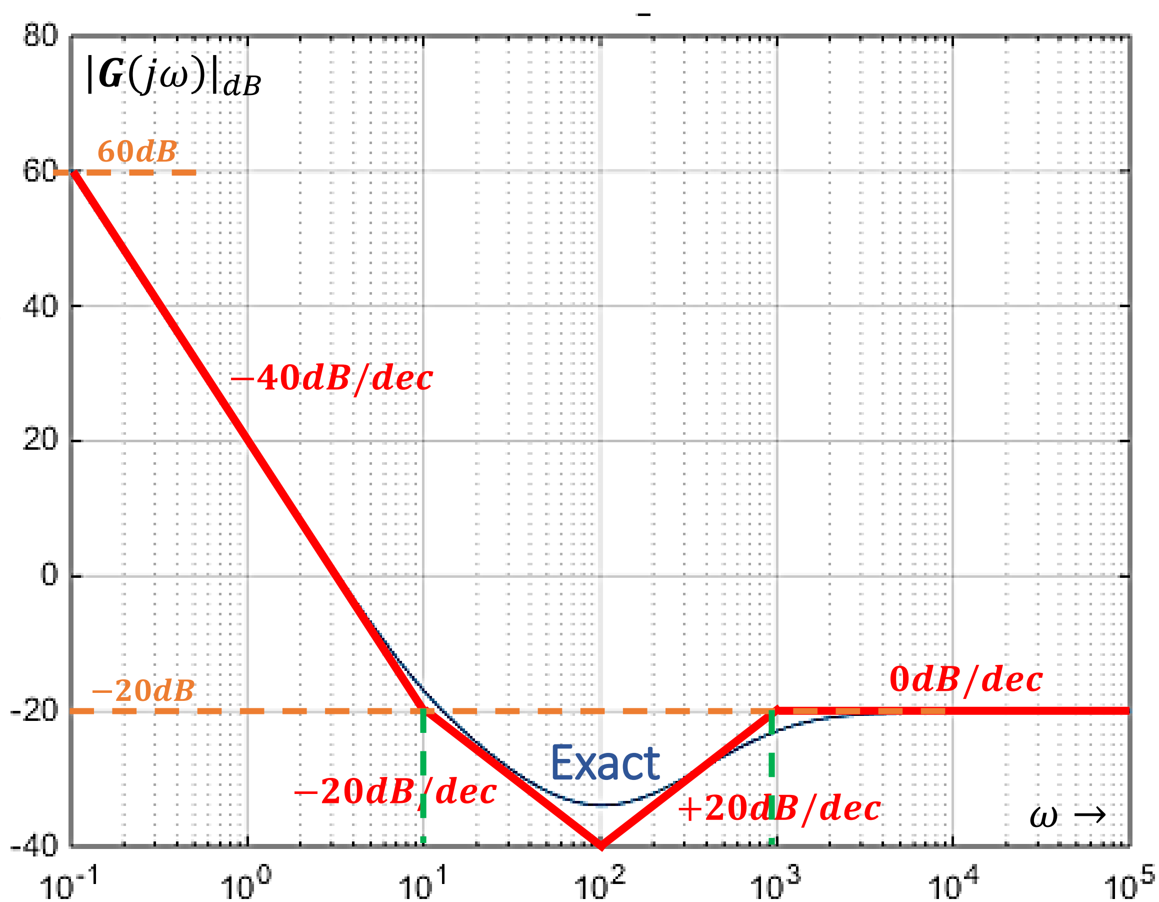
  - **Step #5(b)**: Create table to help sketch $|\boldsymbol{G}(j\omega)|_{dB}$

Magnitude Terms	Radian Frequency $\boldsymbol{\omega}$ (rad/s)					
	Value at $\boldsymbol{\omega = 0.1}$	Slope at $\boldsymbol{\omega > 0.1}$	Slope at $\boldsymbol{\omega > 10}$	Slope at $\boldsymbol{\omega > 10^2}$	Slope at $\boldsymbol{\omega > 10^3}$	Slope at $\boldsymbol{\omega > 10^4}$
$|10|_{dB}$	20dB	0dB/dec	0dB/dec	0dB/dec	0dB/dec	0dB/dec
$-2|j\omega|_{dB}$	40dB	-40dB/dec	-40dB/dec	-40dB/dec	-40dB/dec	-40dB/dec
$|j\omega/10+1|_{dB}$	0dB	0dB/dec	20dB/dec	20dB/dec	20dB/dec	20dB/dec
$2\left|j\omega/10^2+1\right|_{dB}$	0dB	0dB/dec	0dB/dec	40dB/dec	40dB/dec	40dB/dec
$-\left|j\omega/10^3+1\right|_{dB}$	0dB	0dB/dec	0dB/dec	0dB/dec	-20dB/dec	-20dB/dec
Total	$\mathbf{60dB}$	$\mathbf{-40dB/dec}$	$\mathbf{-20dB/dec}$	$\mathbf{20dB/dec}$	$\mathbf{0dB/dec}$	$\mathbf{0dB/dec}$

Example #4

$$G(j\omega) = 10\frac{(j\omega/10+1)(j\omega/100+1)^2}{(j\omega)^2(j\omega/1k+1)}$$

- **Solution cont'd**
  - **Step #5(c)**: Sketch approximate Decibel magnitude $|G(j\omega)|_{dB}$



Example #4

$$\boldsymbol{G}(j\omega) = 10\frac{(j\omega/10+1)(j\omega/100+1)^2}{(j\omega)^2(j\omega/10^3+1)}$$

- Solution cont'd
  - **Step #6(a)**: Note contribution of each term of $\angle \boldsymbol{G}(j\omega)$

    Term: $\angle 10$ Effects: $\forall\omega$ Slope: $0°/\text{dec}$ Value: $0°$

    Term: $-2\angle j\omega$ Effects: $\forall\omega$ Slope: $0°/\text{dec}$ Value: $-180°$

    Term: $\angle(j\omega/10+1)$ Effects: $\omega \in [1, 10^2]$ Slope: $45°/\text{dec}$ Final Value: $90°$

    Term: $2\angle(j\omega/10^2+1)$ Effects: $\omega \in [10, 10^3]$ Slope: $90°/\text{dec}$ Final Value: $180°$

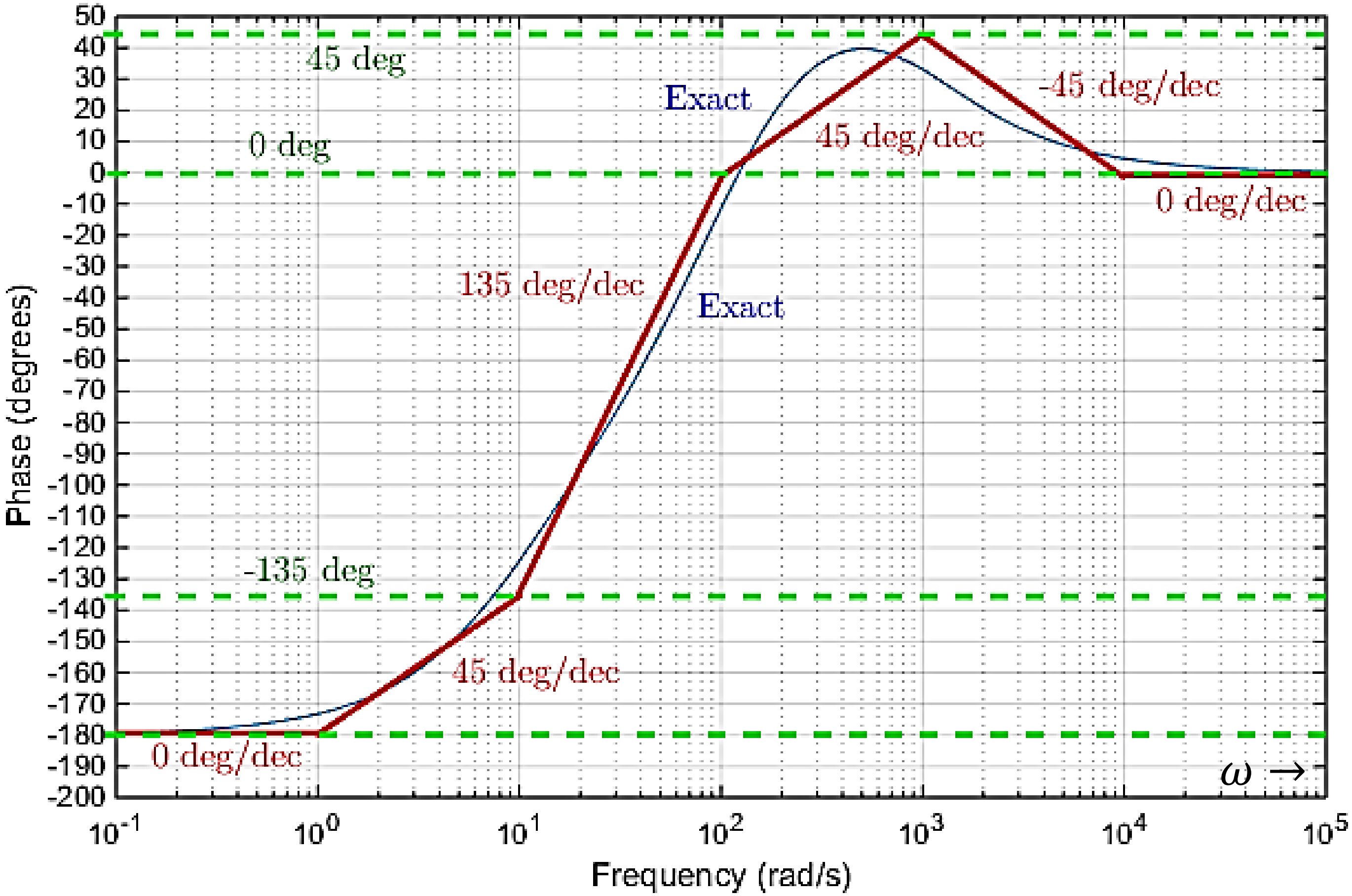
    Term: $-\angle(j\omega/10^3+1)$ Effects: $\omega \in [10^2, 10^4]$ Slope: $-45°/\text{dec}$ Final Value: $-90°$
  - **Step #6(b)**: Create table to help sketch $\angle \boldsymbol{G}(j\omega)$

Phase Terms	Radian Frequency $\boldsymbol{\omega}$ (rad/s)						
	Value at $\boldsymbol{\omega = 0.1}$	Slope at $\boldsymbol{\omega > 1}$	Slope at $\boldsymbol{\omega > 10}$	Slope at $\boldsymbol{\omega > 10^2}$	Slope at $\boldsymbol{\omega > 10^3}$	Slope at $\boldsymbol{\omega > 10^4}$	Value at $\boldsymbol{\omega = 10^5}$
$\angle 10$	$0°$	$0°/\text{dec}$	$0°/\text{dec}$	$0°/\text{dec}$	$0°/\text{dec}$	$0°/\text{dec}$	$0°$
$-2\angle j\omega$	$-180°$	$0°/\text{dec}$	$0°/\text{dec}$	$0°/\text{dec}$	$0°/\text{dec}$	$0°/\text{dec}$	$-180°$
$\angle(j\omega/10+1)$	$0°$	$45°/\text{dec}$	$45°/\text{dec}$	$0°/\text{dec}$	$0°/\text{dec}$	$0°/\text{dec}$	$90°$
$2\angle(j\omega/10^2+1)$	$0°$	$0°/\text{dec}$	$90°/\text{dec}$	$90°/\text{dec}$	$0°/\text{dec}$	$0°/\text{dec}$	$180°$
$-\angle(j\omega/10^3+1)$	$0°$	$0°/\text{dec}$	$0°/\text{dec}$	$-45°/\text{dec}$	$-45°/\text{dec}$	$0°/\text{dec}$	$-90°$
Total	$\boldsymbol{-180°}$	$\boldsymbol{45°}$**/dec**	$\boldsymbol{135°}$**/dec**	$\boldsymbol{45°}$**/dec**	$\boldsymbol{-45°}$**/dec**	$\boldsymbol{0°}$**/dec**	$\boldsymbol{0°}$

Example #4

$$G(j\omega) = 10\,\frac{(j\omega/10+1)(j\omega/100+1)^2}{(j\omega)^2(j\omega/1k+1)}$$

- **Solution cont'd**
  - **Step #6(c)**: Sketch of $\angle G(j\omega)$ (°)



Example #4

$$G(s) = \frac{(s+10)(s+100)^2}{10s^2(s+10^3)}$$

- Solution cont'd
  - **Step #7**: Plot exact Bode diagram in MATLAB with code below

```
%------------------------------------------------------
% construct numerator and denominator polynomials
%------------------------------------------------------
n1 = [1,10];  n2 = [1,100];
num = conv(conv(n2,n2),n1);
d1 = 10*[1,0,0];  d2 = [1, 1e3];
den = conv(d1,d2);
%------------------------------------------------------
% sys_fun - system object specified according to
%           its system (transfer) function tf()
%------------------------------------------------------
sys_fun = tf(num, den);
%------------------------------------------------------
% bode() - generates Bode plot (mag, phase) of the freq.
%          response function obtained according to the
%          system function represented by system object
%          object sys_fun
%------------------------------------------------------
bode(sys_fun);
```

Lecture #6(b): Frequency Response I:
Frequency Response, SSS, and Bode Diagrams: Examples

Sketching Bode Diagrams: Complex Poles/Zeros

Example #1

- Sketch an approximate Bode diagram of the system function below.

$$G(s) = \frac{10^7 s^2}{(s+10)^2(s^2+20s+10^4)}$$

- Solution cont'd
  - **Step #1(a)**: Identify real poles and zeros of $G(s)$

$$\boxed{|z_{1,2}| = 0} \qquad \boxed{|p_{1,2}| = 10}$$

  - **Step #1(b)**: Identify the undamped natural frequency ω_n and damping ratio ζ of complex poles and zeros of $G(s)$

$$\boxed{\omega_n = \sqrt{10^4} = 100} \qquad 2\zeta\omega_n = 20 \rightarrow \boxed{\zeta = 20/(2\omega_n) = 0.1}$$

Peaking occurs since $\zeta < 1/\sqrt{2}$

  - **Step #2**: Represent $G(s)$ in standard Bode form

$$G(s) = \left(\frac{10^7}{(10)^2(10^4)}\right)\frac{s^2}{(s/10+1)^2[(s/100)^2+(20/10^4)s+1]}$$

$$\boxed{G(s) = 10\frac{s^2}{(s/10+1)^2[(s/100)^2+(20/10^4)s+1]}}$$

Example #1

- Sketch an approximate Bode diagram of the system function below.

$$\boldsymbol{G}(\boldsymbol{s}) = \frac{10^7 \boldsymbol{s}^2}{(\boldsymbol{s}+10)^2(\boldsymbol{s}^2+20\boldsymbol{s}+10^4)}$$

- Solution cont'd
  - **Step #3**: Find the frequency response function $\boldsymbol{G}(j\omega)$ of $\boldsymbol{G}(\boldsymbol{s})$

$$\boxed{\boldsymbol{G}(j\omega) = 10\frac{(j\omega)^2}{(j\omega/10+1)^2[(j\omega/100)^2+j(20/10^4)\omega+1]}}$$

  - **Step #4(a)**: Compute expression for Decibel magnitude of $\boldsymbol{G}(j\omega)$

$$\begin{aligned}|\boldsymbol{G}(j\omega)|_{dB} = {} & 20\log_{10}(|10|) + 40\log_{10}(|j\omega|) - 40\log_{10}(|j\omega/10+1|) \\ & -20\log_{10}\left(|(j\omega/100)^2+j(20/10^4)\omega+1|\right)\end{aligned}$$

  - **Step #4(b)**: Compute expression for phase angle of $\boldsymbol{G}(j\omega)$

$$\boxed{\angle\boldsymbol{G}(j\omega) = \angle(10) + 2\angle(j\omega) - 2\angle(j\omega/10+1) - \angle[(j\omega/100)^2+j(20/10^4)\omega+1]}$$

$$G(j\omega) = 10\frac{(j\omega)^2}{\left(\frac{j\omega}{10}+1\right)^2\left[\left(\frac{j\omega}{100}\right)^2 + j\left(\frac{20}{10^4}\right)\omega + 1\right]}$$

Example #1

- Solution cont'd
  - **Step #5(a)**: Note contribution of each term of $|\boldsymbol{G}(j\omega)|_{dB}$

Term: $|10|_{dB}$	**Effects**: $\forall\omega$	**Slope**: 0dB/dec	**Value**: 20dB
Term: $2|j\omega|_{dB}$	**Effects**: $\forall\omega$	**Slope**: 40dB/dec	**Value** @ $\omega = 1$: 0dB
Term: $-2|j\omega/10+1|_{dB}$	**Effects**: $\omega > 10$	**Slope**: -40dB/dec	**Value**: N/A
Term: $-\left|\left(\frac{j\omega}{100}\right)^2 + j\left(\frac{20}{10^4}\right)\omega + 1\right|_{dB}$	**Effects**: $\omega > 10^2$	**Slope**: -40dB/dec	**Value at** @ $\omega = 10^2$: 14dB

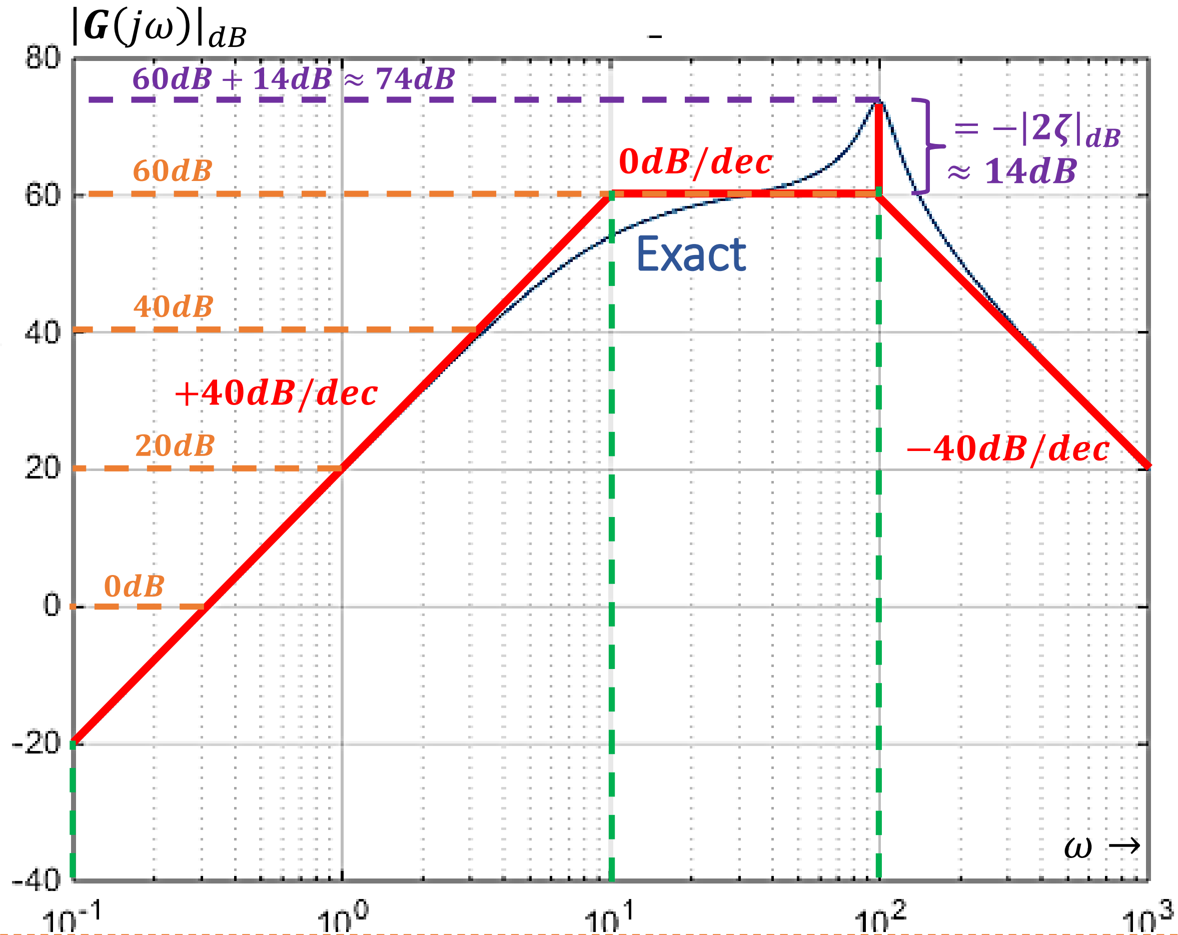
  - **Step #5(b)**: Create table to help sketch $|\boldsymbol{G}(j\omega)|_{dB}$

Magnitude Terms	Radian Frequency $\boldsymbol{\omega}$ (rad/s)				
	Value at $\boldsymbol{\omega = 0.1}$	Slope at $\boldsymbol{\omega > 0.1}$	Slope at $\boldsymbol{\omega > 10}$	Slope at $\boldsymbol{\omega > 10^2}$	Slope at $\boldsymbol{\omega > 10^3}$
$|10|_{dB}$	20dB	0dB/dec	0dB/dec	0dB/dec	0dB/dec
$2|j\omega|_{dB}$	-40dB	40dB/dec	40dB/dec	40dB/dec	40dB/dec
$-2|j\omega/10+1|_{dB}$	0dB	0dB/dec	-40dB/dec	-40dB/dec	-40dB/dec
$-\left|(j\omega/100)^2 + j(20/10^4)\omega + 1\right|_{dB}$	0dB	0dB/dec	0dB/dec	-40dB/dec	-40dB/dec
Total	$\boldsymbol{-20}$**dB**	$\boldsymbol{40}$**dB/dec**	$\boldsymbol{0}$**dB/dec**	$\boldsymbol{-40}$**dB/dec**	$\boldsymbol{-40}$**dB/dec**

Example #1

$$\boldsymbol{G}(j\omega) = 10\frac{(j\omega)^2}{\left(\frac{j\omega}{10}+1\right)^2\left[\left(\frac{j\omega}{100}\right)^2 + j\left(\frac{20}{10^4}\right)\omega + 1\right]}$$

- **Solution cont'd**
  - **Step #5(c)**: Sketch approximate Decibel magnitude $|\boldsymbol{G}(j\omega)|_{dB}$



Example #1

$$G(j\omega) = 10\frac{(j\omega)^2}{\left(\frac{j\omega}{10}+1\right)^2\left[\left(\frac{j\omega}{100}\right)^2 + j\left(\frac{20}{10^4}\right)\omega + 1\right]}$$

- Solution cont'd
  - **Step #6(a)**: Note contribution of each term of $\angle \boldsymbol{G}(j\omega)$

Term: $\angle 10$ Effects: $\forall\omega$ Slope: $0°/\text{dec}$ Value: $0°$

Term: $2\angle j\omega$ Effects: $\forall\omega$ Slope: $0°/\text{dec}$ Value: $180°$

Term: $-2\angle(j\omega/10+1)$ Effects: $\omega \in [1, 10^2]$ Slope: $-90°/\text{dec}$ Final Value: $-180°$

Term: $-\angle\left[\left(j\omega/10^2\right)^2 + j(20/10^4)\omega + 1\right]$

Effects: $\omega \in \left[10^{-\zeta}10^2, 10^{\zeta}10^2\right] \approx [80, 126]$ Slope: $(-90/\zeta)°/\text{dec} = -900°/\text{dec}$
Final Value: $-180°$

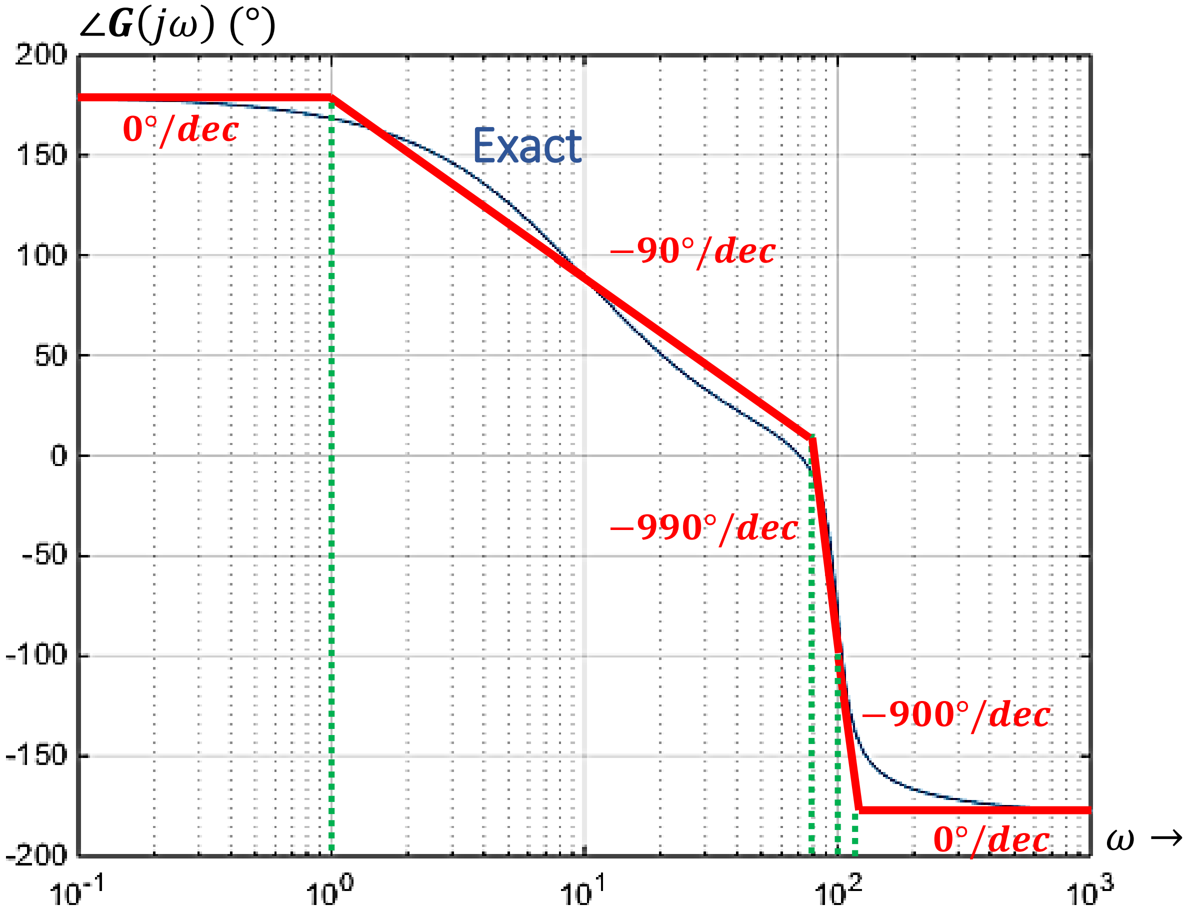
  - **Step #6(b)**: Create table to help sketch $\angle \boldsymbol{G}(j\omega)$

Phase Terms	Radian Frequency $\boldsymbol{\omega}$ (rad/s)					
	Value at $\boldsymbol{\omega = 0.1}$	Slope at $\boldsymbol{\omega > 1}$	Slope at $\boldsymbol{\omega > 80}$	Slope at $\boldsymbol{\omega > 10^2}$	Slope at $\boldsymbol{\omega > 126}$	Value at $\boldsymbol{\omega = 10^3}$
$\angle 10$	$0°$	$0°/\text{dec}$	$0°/\text{dec}$	$0°/\text{dec}$	$0°/\text{dec}$	$0°$
$2\angle j\omega$	$180°$	$0°/\text{dec}$	$0°/\text{dec}$	$0°/\text{dec}$	$0°/\text{dec}$	$180°$
$-2\angle(j\omega/10+1)$	$0°$	$-90°/\text{dec}$	$-90°/\text{dec}$	$0°/\text{dec}$	$0°/\text{dec}$	$-180°$
$-\angle[(j\omega/100)^2 + j(20/10^4)\omega + 1]$	$0°$	$0°/\text{dec}$	$-900°/\text{dec}$	$-900°/\text{dec}$	$0°/\text{dec}$	$-180°$
Total	$\mathbf{180°}$	$\mathbf{-90°/dec}$	$\mathbf{-990°/dec}$	$\mathbf{-900°/dec}$	$\mathbf{-0°/dec}$	$\mathbf{-180°}$

Example #1

$$G(j\omega) = 10\frac{(j\omega)^2}{\left(\frac{j\omega}{10}+1\right)^2\left[\left(\frac{j\omega}{100}\right)^2 + j\left(\frac{20}{10^4}\right)\omega + 1\right]}$$

- **Solution cont'd**
  - **Step #6(c)**: Sketch of $\angle G(j\omega)$ (°)



Example #1

$$G(s) = \frac{10^7 s^2}{(s+10)^2(s^2+20s+10^4)}$$

- Solution cont'd
  - **Step #7**: Plot exact Bode diagram in MATLAB with code below

```
%----------------------------------------------------------
% construct numerator and denominator polynomials
%----------------------------------------------------------
num = 1e7*[1 0 0];                   %numerator polynomial
den1 = [1 10];                       %1st denominator factor
den2 = [1 20 1e4];                   %2nd denominator factor
den = conv( conv(den1,den1), den2); %den is product of factors
%----------------------------------------------------------
% sys_fun - system object specified according to
%           its system (transfer) function tf()
%----------------------------------------------------------
sys_fun = tf(num, den);
%----------------------------------------------------------
% bode() - generates Bode plot (mag, phase) of the freq.
%          response function obtained according to the
%          system function represented by system object
%          object sys_fun
%----------------------------------------------------------
bode(sys_fun);
```

Example #2

- Sketch an approximate Bode diagram of the system function below.

$$G(s) = \frac{-10^6(s^2 + 4s + 100)}{s^2(s + 10^3)^2}$$

- Solution cont'd
  - **Step #1(a)**: Identify real poles and zeros of $G(s)$

$$\boxed{|p_{1,2}| = 0} \qquad \boxed{|p_{3,4}| = 10^3}$$

  - **Step #1(b)**: Identify the undamped natural frequency ω_n and damping ratio ζ of complex poles and zeros of $G(s)$

$$\boxed{\omega_n = \sqrt{10^2} = 10} \qquad 2\zeta\omega_n = 4 \ \rightarrow \boxed{\zeta = 4/(2\omega_n) = 0.2}$$

Peaking occurs since $\zeta < 1/\sqrt{2}$

  - **Step #2**: Represent $G(s)$ in standard Bode form

$$G(s) = \left(\frac{(-10^6)(10^2)}{(10^3)(10^3)}\right)\frac{(s/10)^2 + (4/10^2)s + 1}{s^2(s/10^3 + 1)^2}$$

$$\boxed{G(s) = -10^2\frac{(s/10)^2 + (4/10^2)s + 1}{s^2(s/10^3 + 1)^2}}$$

Example #2

- Sketch an approximate Bode diagram of the system function below.

$$\boldsymbol{G}(\boldsymbol{s}) = \frac{-10^6(\boldsymbol{s}^2 + 4\boldsymbol{s} + 100)}{\boldsymbol{s}^2(\boldsymbol{s} + 10^3)^2}$$

- Solution cont'd
  - **Step #3**: Find the frequency response function $\boldsymbol{G}(j\omega)$ of $\boldsymbol{G}(\boldsymbol{s})$

$$\boxed{\boldsymbol{G}(j\omega) = -10^2 \frac{(j\omega/10)^2 + j\left(4/10^2\right)\omega + 1}{(j\omega)^2(j\omega/10^3 + 1)^2}}$$

  - **Step #4(a)**: Compute expression for Decibel magnitude of $\boldsymbol{G}(j\omega)$

$$\begin{aligned}|\boldsymbol{G}(j\omega)|_{dB} = {} & 20\log_{10}\left(|-10^2|\right) + 20\log_{10}\left(\left|(j\omega/10)^2 + j\left(20/10^2\right)\omega + 1\right|\right) \\ & -40\log_{10}(|j\omega|) - 40\log_{10}\left(|j\omega/10^3 + 1|\right)\end{aligned}$$

  - **Step #4(b)**: Compute expression for phase angle of $\boldsymbol{G}(j\omega)$

$$\begin{aligned}\angle\boldsymbol{G}(j\omega) = {} & \angle\left(-10^2\right) + \angle\left[(j\omega/10)^2 + j\left(4/10^2\right)\omega + 1\right] \\ & -2\angle(j\omega) - 2\angle\left(j\omega/10^3 + 1\right)\end{aligned}$$

Example #2

$$G(j\omega) = -10^2 \frac{(j\omega/10)^2 + j(4/10^2)\omega + 1}{(j\omega)^2(j\omega/10^3 + 1)^2}$$

- **Solution cont'd**
  - **Step #5(a)**: Note contribution of each term of $|G(j\omega)|_{dB}$

    Term: $|-10^2|_{dB}$ Effects: $\forall\omega$ Slope: 0dB/dec Value: 40dB

    Term: $-2|j\omega|_{dB}$ Effects: $\forall\omega$ Slope: -40dB/dec Value @ $\omega = 1$: 0dB

    Term: $\left|\left(\frac{j\omega}{10}\right)^2 + j\left(\frac{4}{10^2}\right)\omega + 1\right|_{dB}$ Effects: $\omega > 10$ Slope: 40dB/dec Value at @ $\omega = 10$: -8dB

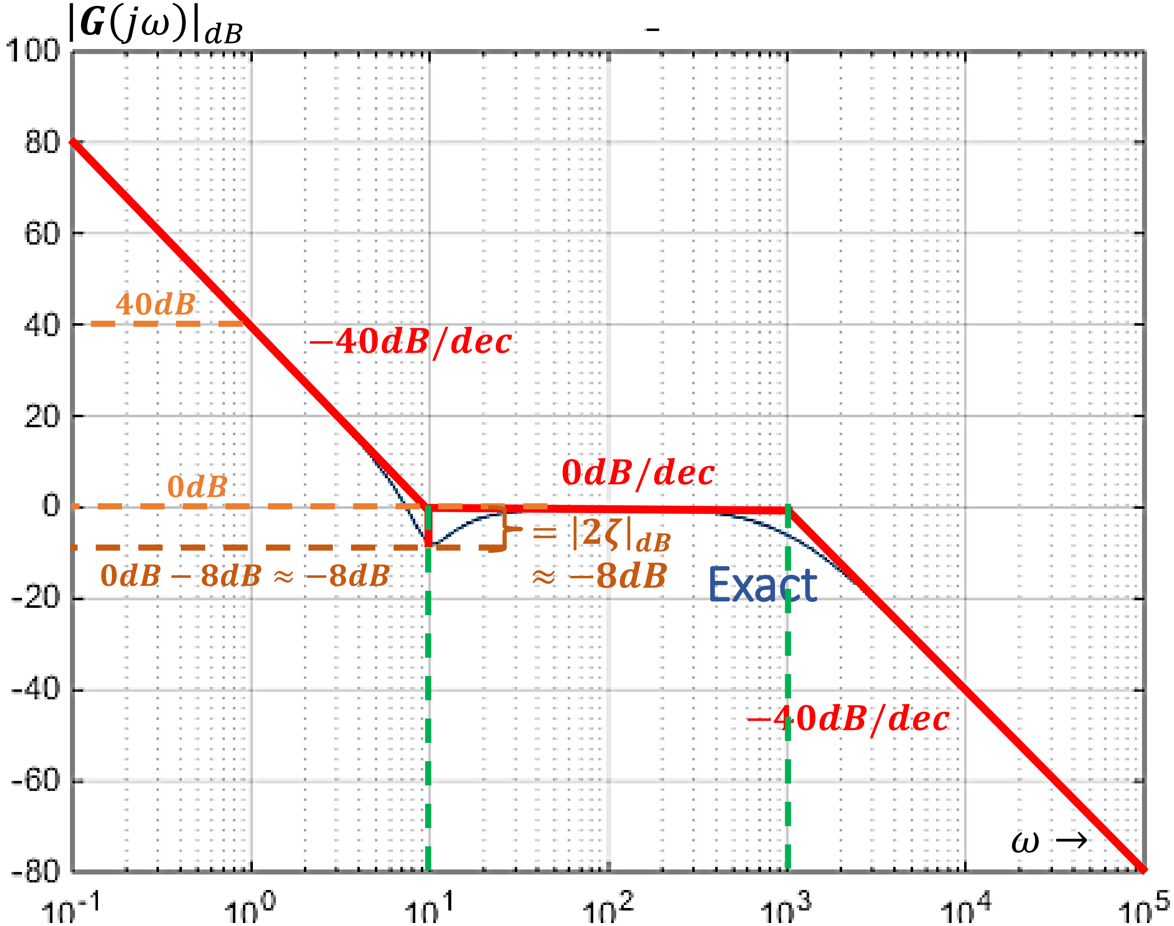
    Term: $-2|j\omega/10^3 + 1|_{dB}$ Effects: $\omega > 10^3$ Slope: -40dB/dec Value: N/A
  - **Step #5(b)**: Create table to help sketch $|G(j\omega)|_{dB}$

Magnitude Terms	Radian Frequency ω (rad/s)				
	Value at $\omega = 0.1$	Slope at $\omega > 0.1$	Slope at $\omega > 10$	Slope at $\omega > 10^3$	Slope at $\omega > 10^4$
$|-10^2|_{dB}$	40dB	0dB/dec	0dB/dec	0dB/dec	0dB/dec
$-2|j\omega|_{dB}$	40dB	-40dB/dec	-40dB/dec	-40dB/dec	-40dB/dec
$|(j\omega/10)^2 + j(4/10^2)\omega + 1|_{dB}$	0dB	0dB/dec	40dB/dec	40dB/dec	40dB/dec
$-2|j\omega/10^3 + 1|_{dB}$	0dB	0dB/dec	0dB/dec	-40dB/dec	-40dB/dec
Total	**80dB**	**-40dB/dec**	**0dB/dec**	**-40dB/dec**	**-40dB/dec**

Example #2

$$G(j\omega) = -10^2 \frac{(j\omega/10)^2 + j(4/10^2)\omega + 1}{(j\omega)^2(j\omega/10^3+1)^2}$$

- Solution cont'd
  - **Step #5(c)**: Sketch approximate Decibel magnitude $|G(j\omega)|_{dB}$



Example #2

$$G(j\omega) = -10^2 \frac{(j\omega/10)^2 + j(4/10^2)\omega + 1}{(j\omega)^2(j\omega/10^3 + 1)^2}$$

- Solution cont'd
  - **Step #6(a)**: Note contribution of each term of $\angle G(j\omega)$

Term: $\angle -10^2$ Effects: $\forall\omega$ Slope: $0°/\text{dec}$ Value: $\pm 180°$

Term: $-2\angle j\omega$ Effects: $\forall\omega$ Slope: $0°/\text{dec}$ Value: $-180°$

Term: $\angle[(j\omega/10)^2 + j(4/10^2)\omega + 1]$ Effects: $\omega \in [10^{-\zeta}10, 10^{\zeta}10] \approx [6, 16]$

Slope: $(90/\zeta)°/\text{dec} = 450°/\text{dec}$ Final Value: $180°$

Term: $-2\angle(j\omega/10^3 + 1)$ Effects: $\omega \in [10^2, 10^4]$ Slope: $-90°/\text{dec}$ Final Value: $-180°$

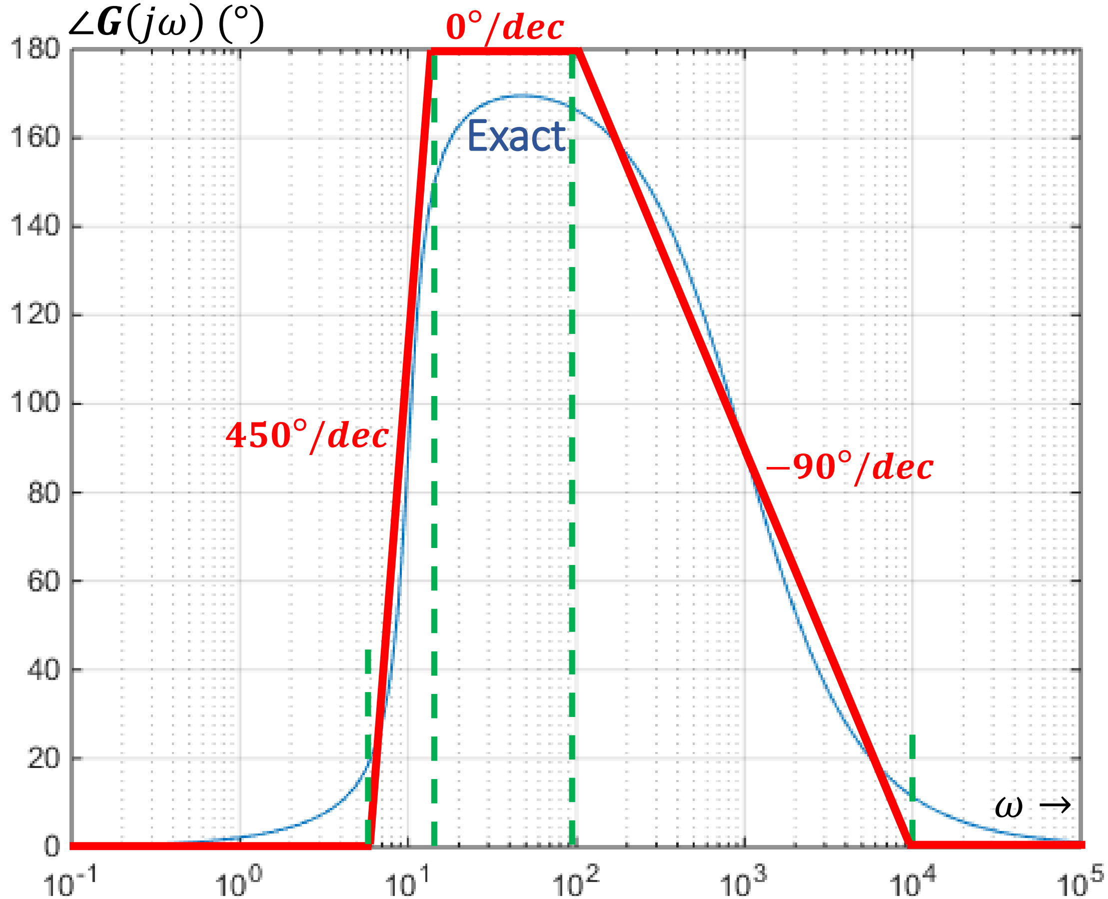
  - **Step #6(b)**: Create table to help sketch $\angle G(j\omega)$

Phase Terms	Radian Frequency $\boldsymbol{\omega}$ (rad/s)					
	Value at $\boldsymbol{\omega = 0.1}$	Slope at $\boldsymbol{\omega > 6}$	Slope at $\boldsymbol{\omega > 16}$	Slope at $\boldsymbol{\omega > 10^2}$	Slope at $\boldsymbol{\omega > 10^4}$	Value at $\boldsymbol{\omega = 10^5}$
$\angle -10^2$	$\pm 180°$	$0°/\text{dec}$	$0°/\text{dec}$	$0°/\text{dec}$	$0°/\text{dec}$	$\pm 180°$
$-2\angle j\omega$	$-180°$	$0°/\text{dec}$	$0°/\text{dec}$	$0°/\text{dec}$	$0°/\text{dec}$	$-180°$
$\angle[(j\omega/100)^2 + j(20/10^4)\omega + 1]$	$0°$	$450°/\text{dec}$	$0°/\text{dec}$	$0°/\text{dec}$	$0°/\text{dec}$	$+180°$
$-2\angle(j\omega/10^3 + 1)$	$0°$	$0°/\text{dec}$	$0°/\text{dec}$	$-90°/\text{dec}$	$0°/\text{dec}$	$-180°$
Total	$\mathbf{0°}$	$\mathbf{450°/dec}$	$\mathbf{0°/dec}$	$\mathbf{-90°/dec}$	$\mathbf{0°/dec}$	$\mathbf{0°}$

Example #2

$$G(j\omega) = -10^2 \frac{(j\omega/10)^2 + j(4/10^2)\omega + 1}{(j\omega)^2(j\omega/10^3 + 1)^2}$$

- **Solution cont'd**
  - **Step #6(c)**: Sketch of $\angle G(j\omega)$ (°)



Example #2

$$G(s) = \frac{-10^6(s^2 + 4s + 100)}{s^2(s + 10^3)^2}$$

- Solution cont'd
  - **Step #7**: Plot exact Bode diagram in MATLAB with code below

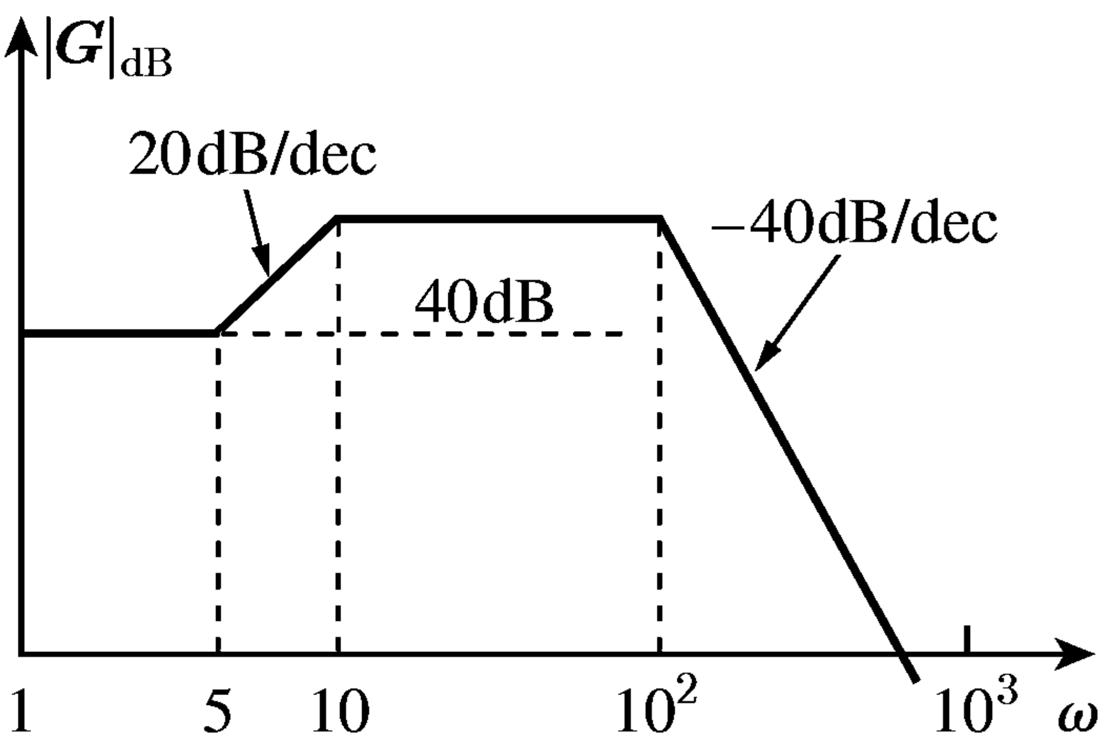
```
%-----------------------------------------------------
% construct numerator and denominator polynomials
%-----------------------------------------------------
num = -1*1e6*[1 4 100];           %numerator polynomial
den1 = [1 0 0];                   %1st denominator factor
den2 = [1 1e3];                   %2nd denominator factor
den = conv(den1, conv(den2,den2)); %den is product of factors
%-----------------------------------------------------
% sys_fun - system object specified according to
%           its system (transfer) function tf()
%-----------------------------------------------------
sys_fun = tf(num, den);
%-----------------------------------------------------
% bode() - generates Bode plot (mag, phase) of the freq.
%          response function obtained according to the
%          system function represented by system object
%          object sys_fun
%-----------------------------------------------------
bode(sys_fun);
```

Lecture #6(b): Frequency Response I:
Frequency Response, SSS, and Bode Diagrams: Examples

Frequency Response Functions from Bode Sketches

Example #1

- What is the frequency response function $\boldsymbol{G}(j\omega)$ associated with the asymptotic approximate Bode diagram below. You may assume the poles and zeros are all real. Express $\boldsymbol{G}(j\omega)$ in standard normalized Bode form.



Example #1 (Solution)

- What is $\boldsymbol{G}(j\omega)$ associated with the asymptotic approximate Bode diagram.
  - Determine poles and zeros

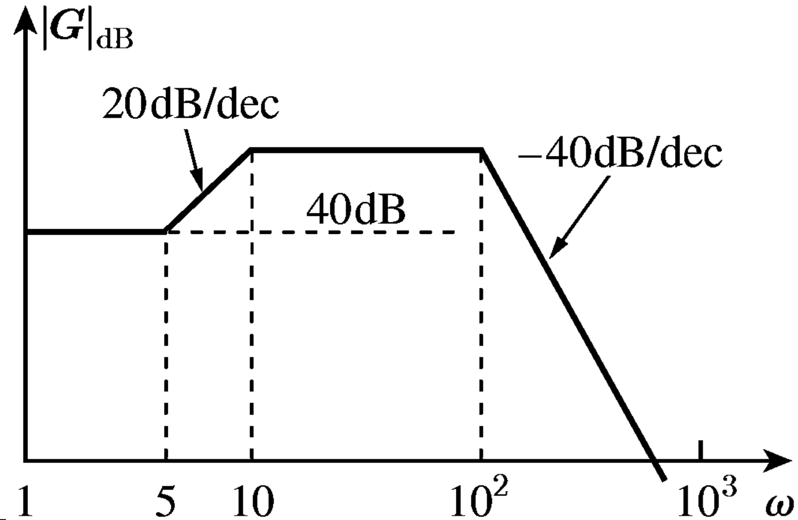
    $|z_1| = 5$

    $|p_1| = 10, \ |p_{2,3}| = 100$
  - Express $\boldsymbol{G}(j\omega)$ in Bode form to within a constant K_0

$$\boxed{\boldsymbol{G}(j\omega) = \frac{K_0(j\omega/5+1)}{(j\omega/10+1)(j\omega/10^2+1)^2}}$$

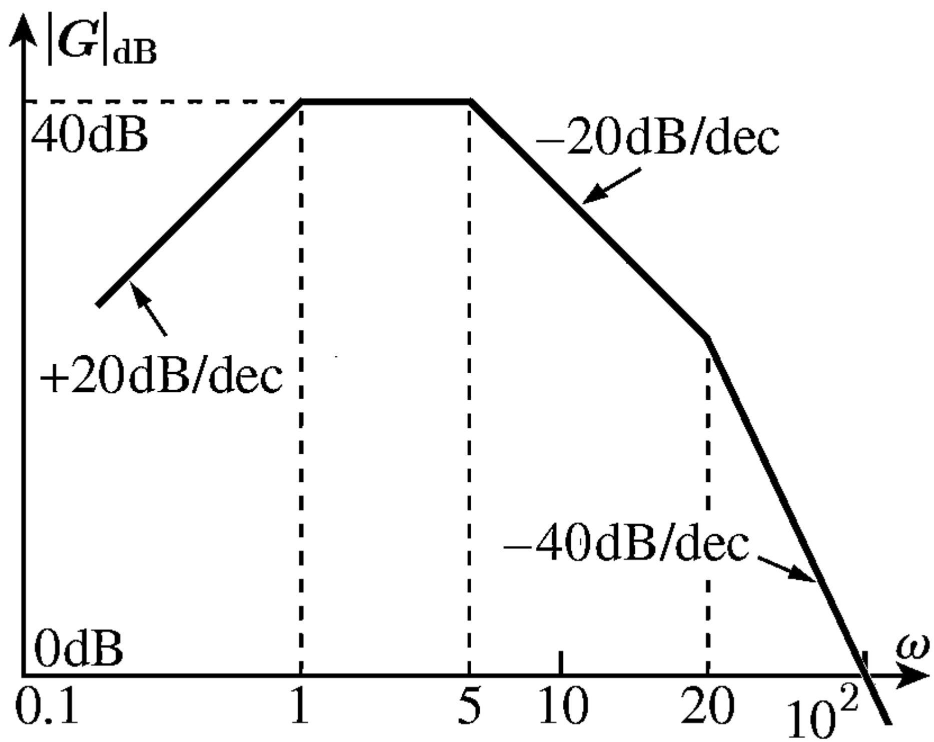
  - Compute constant K_0 knowning $|\boldsymbol{G}(j1)|_{dB} = 40dB$
    - At $\omega = 1$, the only term in $|\boldsymbol{G}(j\omega)|_{dB}$ that is “on” is $|K_0|_{dB}$. Therefore,

$$|\boldsymbol{G}(j1)|_{dB} = 40dB = |K_0|_{dB}$$
$$|K_0|_{dB} = 40dB \rightarrow \log_{10}(|K_0|) = 2$$
$$|K_0| = 10^2 \rightarrow \boxed{K_0 = \pm 100}$$



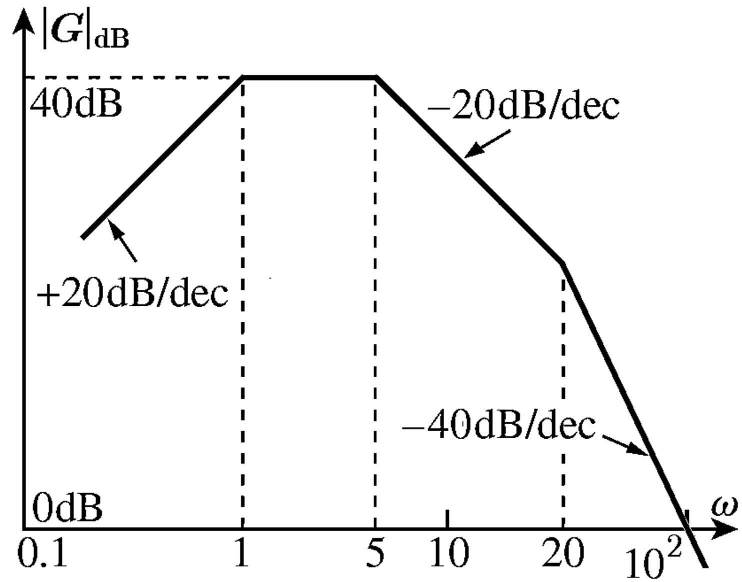
Example #2

- What is the frequency response function $\boldsymbol{G}(j\omega)$ associated with the asymptotic approximate Bode diagram below. You may assume the poles and zeros are all real. Express $\boldsymbol{G}(j\omega)$ in standard normalized Bode form.



Example #2 (Solution)

- What is $\boldsymbol{G}(j\omega)$ associated with the asymptotic approximate Bode diagram.



  - Determine poles and zeros

    $|z_1| = 0$

    $|p_1| = 1, |p_2| = 5, \ |p_3| = 20$

  - Express $\boldsymbol{G}(j\omega)$ in Bode form to within a constant K_0

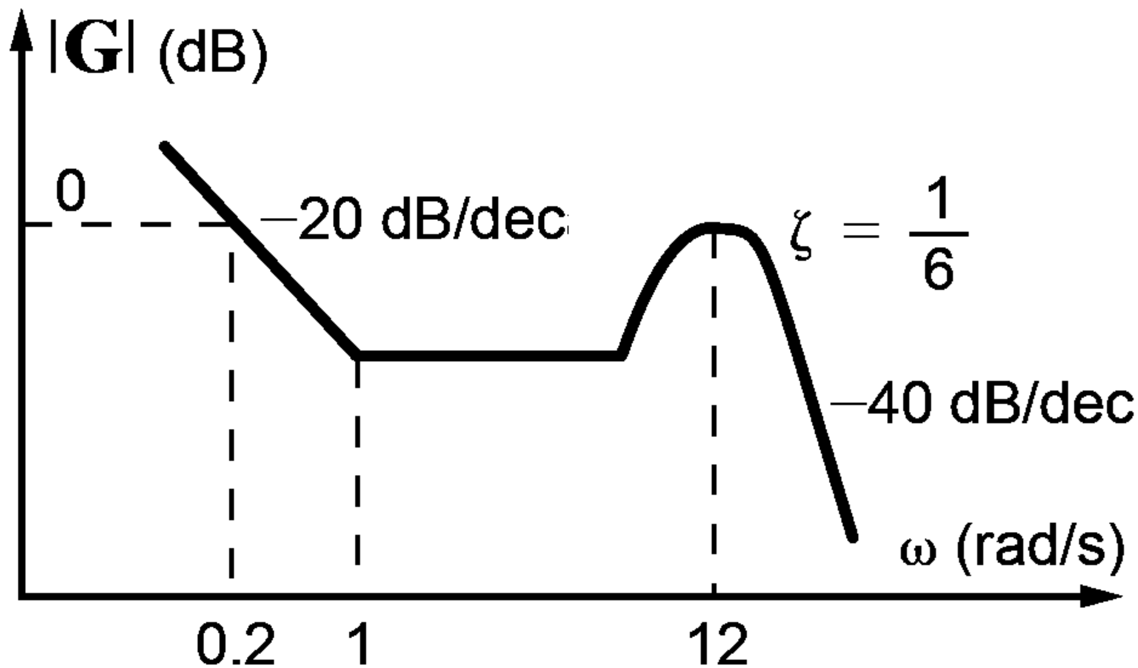
$$\boldsymbol{G}(j\omega) = \frac{K_0 j\omega}{(j\omega + 1)(j\omega/5 + 1)(j\omega/20 + 1)}$$

  - Compute constant K_0 knowning $|\boldsymbol{G}(j0.1)|_{dB} = 20dB$
    - At $\omega = 0.1$, the terms in $|\boldsymbol{G}(j\omega)|_{dB}$ that are "on" are $|K_0|_{dB}$ and $|j\omega|_{dB}$.

$$|\boldsymbol{G}(j0.1)|_{\mathrm{dB}} = 20\mathrm{dB} = |K_0|_{\mathrm{dB}} + |j0.1|_{dB}$$
$$|K_0|_{\mathrm{dB}} = 20\mathrm{dB} - 20\log_{10}(0.1) = 40\mathrm{dB} \rightarrow |K_0|_{\mathrm{dB}} = 40\mathrm{dB}$$
$$\log_{10}(|K_0|) = 2 \rightarrow |K_0| = 10^2 \rightarrow \boxed{K_0 = \pm 100}$$

Example #3

- What is the frequency response function $\boldsymbol{G}(j\omega)$ associated with the asymptotic approximate Bode diagram below. Express $\boldsymbol{G}(j\omega)$ in standard normalized Bode form.

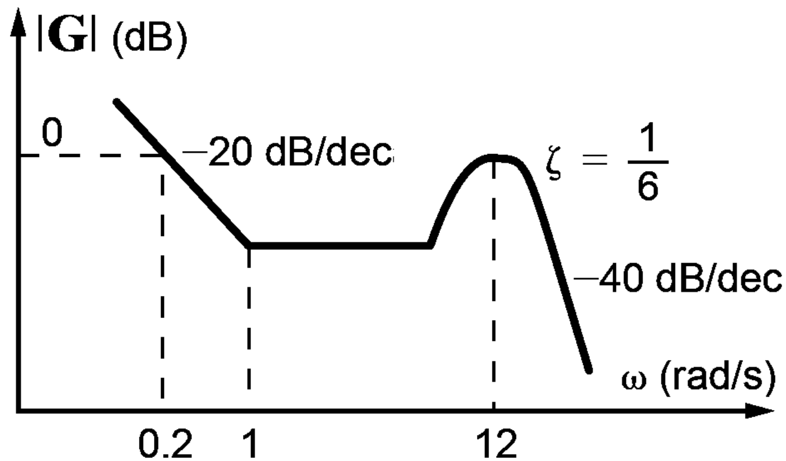


Example #3 (Solution)

- What is $\boldsymbol{G}(j\omega)$ associated with the asymptotic approximate Bode diagram.
  - Determine real poles and zeros

    $|z_1| = 1 \quad |p_1| = 0$
  - Determine ω_n and ζ of any complex conjugate poles and zeros
    - Pair of complex conjugate poles

      $\omega_n = 12 \qquad \zeta = 1/6$
  - Express $\boldsymbol{G}(j\omega)$ in Bode form to within a constant K_0

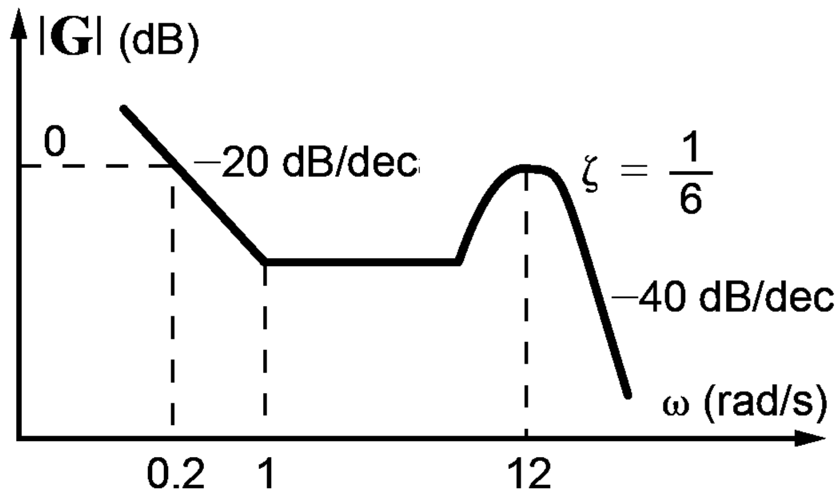


$$\boldsymbol{G}(j\omega) = \frac{K_0(j\omega + 1)}{(j\omega)[(j\omega/\omega_n)^2 + j\omega(2\zeta/\omega_n) + 1]}$$

$$\boldsymbol{G}(j\omega) = \frac{K_0(j\omega + 1)}{(j\omega)[(j\omega/12)^2 + j\omega(2(1/6)/12) + 1]}$$

Example #3 (Solution)

- What is $\boldsymbol{G}(j\omega)$ associated with the approximate Bode diagram.
  - Compute constant K_0 knowning $|\boldsymbol{G}(j0.2)|_{dB} = 0\text{dB}$
    - At $\omega = 0.2$, the terms in $|\boldsymbol{G}(j\omega)|_{dB}$ that are "on" are $|K_0|_{dB}$ and $-|j\omega|_{dB}$.
    - Therefore,



$$|\boldsymbol{G}(j0.2)|_{\text{dB}} = 0\text{dB} = |K_0|_{\text{dB}} - |j0.2|_{\text{dB}}$$
$$|K_0|_{\text{dB}} = 0\text{dB} + 20\log_{10}(0.2)$$
$$|K_0|_{\text{dB}} = 20\log_{10}(0.2)$$
$$\log_{10}(|K_0|) = \log_{10}(0.2)$$
$$|K_0| = 10^{\log_{10}(0.2)} = 0.2$$
$$\boxed{K_0 = \pm 0.2}$$

$$\boldsymbol{G}(j\omega) = \frac{\pm 0.2(j\omega + 1)}{(j\omega)[(j\omega/12)^2 + j\omega(1/36) + 1]}$$