HOMEWORK #3: Basic Signal Waveforms (Selected Answers)

1. Expressing Functions In Terms of Singularity Functions

(a) Express the following functions of time using a linear combination of singularity functions.

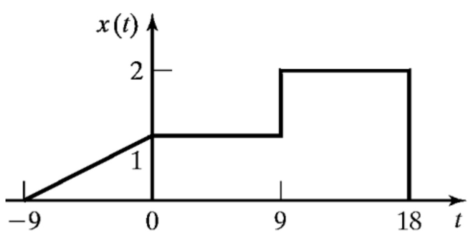
i. $v_1(t) = \begin{cases} 3 & t < 1 \\ -2 & 1 < t < 2 \\ 0 & elsewhere \end{cases}$

$\boxed{v_1(t) = 3 - 5u(t-1) + 2u(t-2)}$ or $\boxed{v_1(t) = 5u(1-t) - 2u(2-t)}$

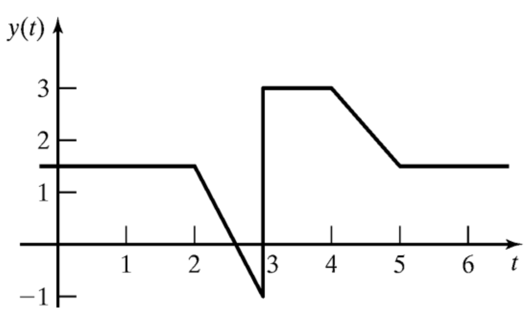
ii. $x(t) = \begin{cases} t-1 & 1 < t < 2 \\ 1 & 2 < t < 3 \\ -t+4 & 3 < t < 4 \\ 0 & elsewhere \end{cases}$

$\boxed{x(t) = r(t-1) - r(t-2) - r(t-3) + r(t-4)}$

(b) Consider the plot of each of the following functions of time shown on the right. Express each as a linear combination of singularity functions. Simplify each expression as much as possible.



$$\boxed{x(t) = \frac{1}{9}r(t+9) - \frac{1}{9}r(t) + u(t-9) - 2u(t-18)}$$



$$\boxed{y(t) = 1.5 - 2.5r(t-2) + 2.5r(t-3) + 4u(t-3) - 1.5r(t-4) + 1.5r(t-5)}$$

2. Sketching Waveforms Involving Singularity Functions

(a) Sketch each of the following functions by hand. Clearly label each sketches. Use MATLAB to plot $f_1(t)$ through $f_4(t)$ and use MATLAB's output to verify your hand sketches.

i. $\frac{d}{dt}x(t)$ (from question Q1(b)).

$$\boxed{x'(t) = (1/9)u(t+9) - (1/9)u(t) + \delta(t-9) - 2\delta(t-18)}$$

$\frac{d}{dt}y(t)$ (from question Q1(b))

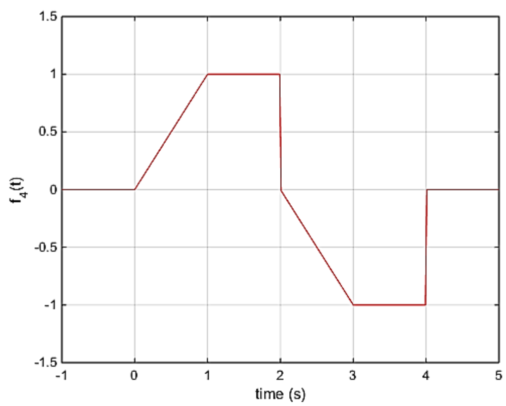
$$\boxed{y'(t) = -2.5u(t-2) + 2.5u(t-3) + 4\delta(t-3) - 1.5u(t-4) + 1.5u(t-5)}$$

ii. $f_1(t) = u(2t-6)$ (No answer)

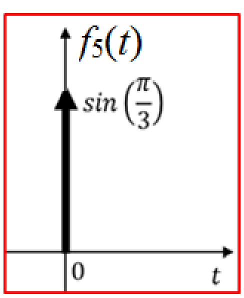
iii. $f_2(t) = u(t-4) - u(t-1)$ (No answer)

iv. $f_3(t) = 2u(t-2)u(3-t)$ (No answer)

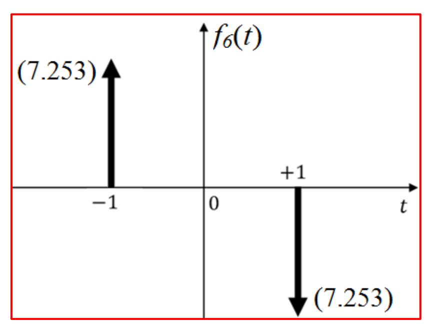
v. $f_4(t) = r(t) - r(t-1) - u(t-2) - r(t-2) + r(t-3) + u(t-4)$



vi. $f_5(t) = \frac{\sin\left(t+\frac{\pi}{3}\right)}{t^2+1}\delta(t)$



vii. $f_6(t) = [e^{2t^2} - e^{-2t^2}][\delta(t+1) - \delta(t-1)]$



(b) Sketch each of the following "similar" time functions. Clearly label each sketch. Note the similarities and differences among the four waveforms.

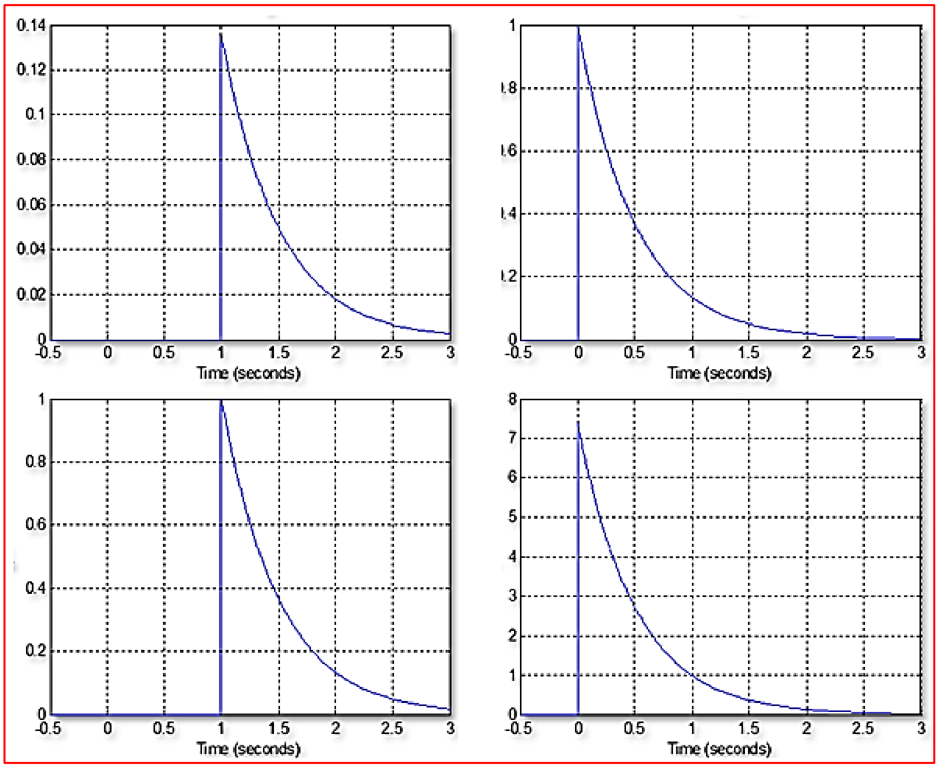
i. $f_1(t) = e^{-2t}u(t)$

ii. $f_2(t) = e^{-2t}u(t-1)$

iii. $f_3(t) = e^{-2(t-1)}u(t)$

iv. $f_4(t) = e^{-2(t-1)}u(t-1)$

In random order, the sketches are shown below



3. Evaluating Derivative and Integral Expressions Involving Singularity Functions

(a) Apply the sifting property of the Dirac Delta/Impulse to evaluate each of the following integrals.

i. $\int_{-3}^{2} \cos(t)\,\delta(t)dt = \boxed{1}$

ii. $\int_{-3}^{2} t^2\,\delta(t-1)dt = \boxed{1}$

iii. $\int_{-3}^{1} \ln(t)\,\delta(t-2)dt = \boxed{0}$

iv. $\int_{-3}^{5^+} sin(t)\,\delta(t-5)dt = \boxed{\sin(5) \approx -0.96}$

v. $\int_{-3^+}^{4^-} e^{-5t}\,\delta(t-4)dt = \boxed{0}$

vi. $\int_{-3^-}^{3^+} e^{t^2}\,\delta(t+3)dt \approx \boxed{8103.08}$

(b) Evaluate the following expressions involving time derivatives and singularity functions.

i. $f_1(t) = [u(t+1)u(t-1)]'$ $\boxed{f_1(t) = \delta(t-1)}$

ii. $f_2(t) = [r(t-6)u(t-2)]'$ $\boxed{f_2(t) = u(t-6)}$

iii. $f_3(t) = \left[\sin(4t)\,u\left(t-\frac{\pi}{8}\right)\right]'$ $\boxed{f_3(t) = 4\cos(4t)\,u\left(t-\frac{\pi}{8}\right) + \delta\left(t-\frac{\pi}{8}\right)}$