HOMEWORK #4: Unilateral Forward Laplace Transforms

1. Unilateral Forward Laplace Transforms

(a) Compute by hand the unilateral Laplace Transform of each of the following time functions. Verify each of your answers using MATLAB and the Symbolic Toolbox.

i. $f(t) = [5 + 2t^5 + 6e^{-3t}]u(t)$

ii. $f(t) = [4\cos(t) + 7\sin(\sqrt{5}t)]u(t)$

iii. $f(t) = \sqrt{t-1}r(t-3)u(t-2)\delta(t-5)$

iv. $f(t) = 3t^4 e^{-10t}u(t)$

v. $f(t) = e^{-2t}\,cos(3t)\,u(t)$

vi. $f(t) = 2e^{-2(t-3)}u(t)$

vii. $f(t) = 2tu(t-4)$

viii. $f(t) = 7(t-4)u(t-2)$

ix. $f(t) = 2e^{-4t}u(t-1)$

x. $f(t) = e^{-2(t+1)}(t+2)u(t-3)$

(b) Consider the plot of function $f(t)$ shown below.

i. Express $f(t)$ as a linear combination of singularity functions. Ensure each term of is expressed in such a way that the Laplace Transform time-shift property may be properly applied.

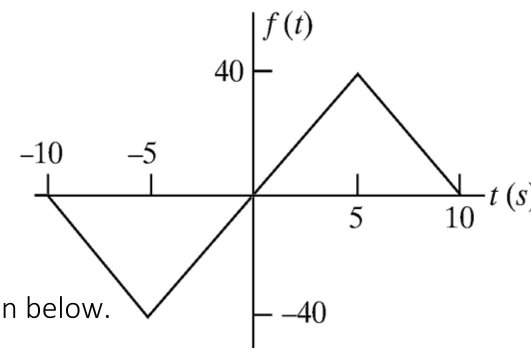
ii. Compute the unilateral Laplace transform of $f(t)$.

iii. Compute and sketch a plot of $f'(t)$.

iv. Compute the unilateral Laplace transform of $f'(t)$.

v. Compute and sketch a plot of $f''(t)$.

vi. Compute the unilateral Laplace transform of $f''(t)$.

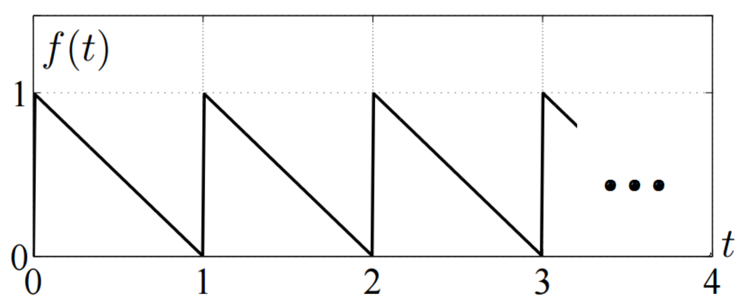


(c) Consider the periodic right-sided time function $f(t)$ shown below.

Express $f(t)$ as an infinite sum of singularity functions.

i. Compute the unilateral Laplace transform of $f(t)$. You may find the following formula vert useful.

$$\sum_{k=0}^{\infty} r^k = \frac{q}{1-r}, -1 < r < 1$$

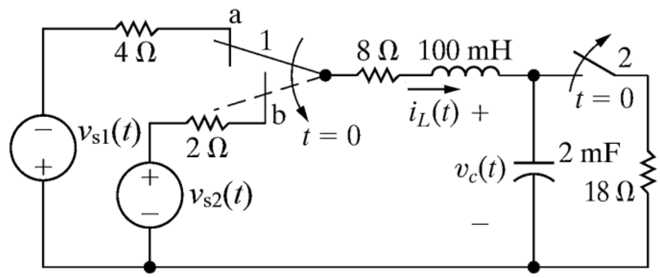


2. Forward Laplace Transform and Integro-Differential Equations

(a) Compute $\boldsymbol{V_o(s)} = \mathcal{L}\{v_o(t)u(t)\}$ for the following integro-differential equation. Identify the characteristic polynomial, the zero state transform component $\boldsymbol{V_{o,ZS}(s)}$ of $\boldsymbol{V_o(s)}$, and the zero input transform component $\boldsymbol{V_{o,ZI}(s)}$ of $\boldsymbol{V_o(s)}$.

$$v_o''(t) + 5v_o'(t) + 6v_o(t) + 2\int_{0^-}^{t} v_o(\lambda)d\lambda = 4\delta(t), \qquad v_o(0^-) = 2\text{V}, v_o'(0^-) = 4\text{ V/s}$$

(b) The second order network shown below has $v_{s1}(t) = 150\text{V}$ and $v_{s2}(t) = 60\text{V}$. At $t = 0$, switch 1 moves to position (b) after being at position (a) for a long time. Simultaneously, switch 2 opens after being closed for a long time.



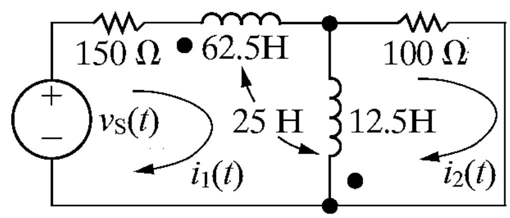
i. Analyze the network at time $t = 0^-$ to compute the state variable values $v_C(0^-)$ and $i_L(0^-)$, respectively.

ii. Analyze the network for $t > 0^-$ using <u>mesh analysis</u> to obtain a single linear second order constant coefficient integro-differential equation that describes the inductor current $i_L(t)u(t)$ for $t > 0^-$.

iii. Take the Laplace Transform of the equation found in (ii) and compute the complete response transform $\boldsymbol{I_L(s)} = \mathcal{L}\{i_L(t)u(t)\}$. identify the characteristic polynomial of the network, the zero state response transform component $\boldsymbol{I_{L,ZS}(s)}$ of $\boldsymbol{I_L(s)}$, and the zero input response transform component $\boldsymbol{I_{L,ZI}(s)}$ of $\boldsymbol{I_L(s)}$.

iv. Write a time-domain expression that relates the capacitor voltage $v_C(t)u(t)$ to the inductor current $i_L(t)u(t)$. Then use the expression compute $\boldsymbol{V_C(s)} = \mathcal{L}\{v_C(t)u(t)\}$.

(c) Consider the magnetically coupled network shown below with $v_s(t) = 625u(t)\text{V}$. Assume the network is in the zero state (i.e. the state variable for each energy storage element is initially zero).



i. Apply <u>mesh analysis</u> to derive two ordinary linear constant-coefficient first order differential equations that govern the behavior of the network.

ii. Take the Laplace Transform of the both equations developed in (i).

iii. Solve for the Laplace Transform of the two mesh currents, i.e., solve for $\boldsymbol{I}_1(\boldsymbol{s}) = \mathcal{L}\{i_1(t)u(t)\}$ and $\boldsymbol{I}_2(\boldsymbol{s}) = \mathcal{L}\{i_2(t)u(t)\}$.