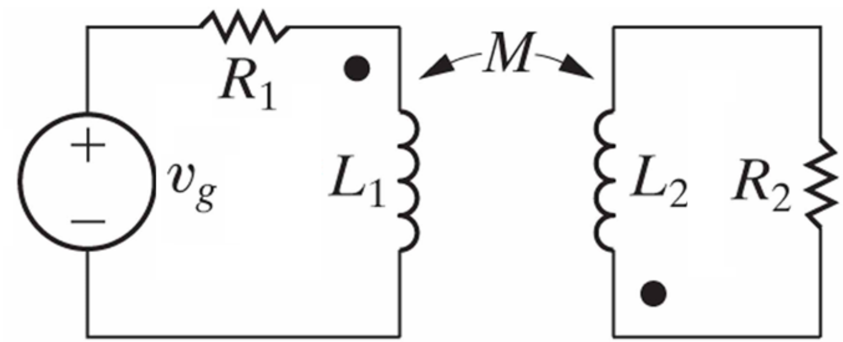
Lecture #2(b): Magnetically Coupled Networks

Examples

ECE 20200: Linear Circuit Analysis II
Steve Naumov (Instructor)

Example #1

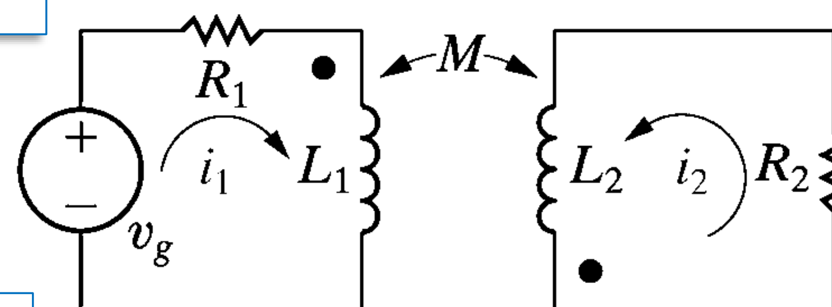
- Write time-domain mesh equations for the network shown
- Solution
  - Assign a set of mesh currents
  - Mesh analysis @ Mesh 1



$$v_g = R_1 i_1 + L_1 \frac{di_1}{dt} \boxed{?}\ M \frac{d}{dt}(?)$$

What sign and current to use for each the mutually induced voltage term?

  - Mesh Analysis @ Mesh 2

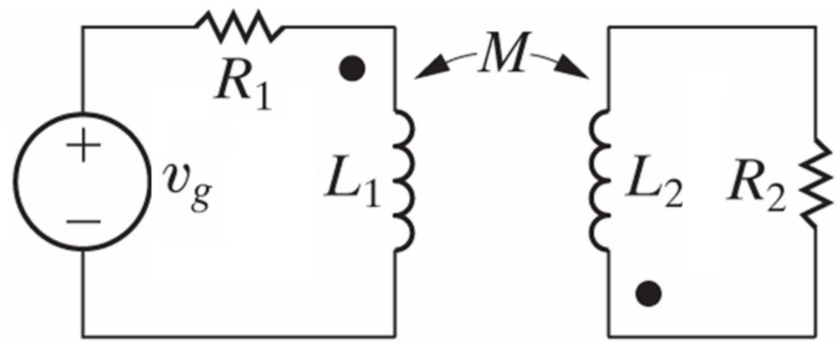


$$0\text{V} = R_2 i_2 + L_2 \frac{di_2}{dt} \boxed{?}\ M \frac{d}{dt}(?)$$

What sign and current to use for each mutually induced voltage term?

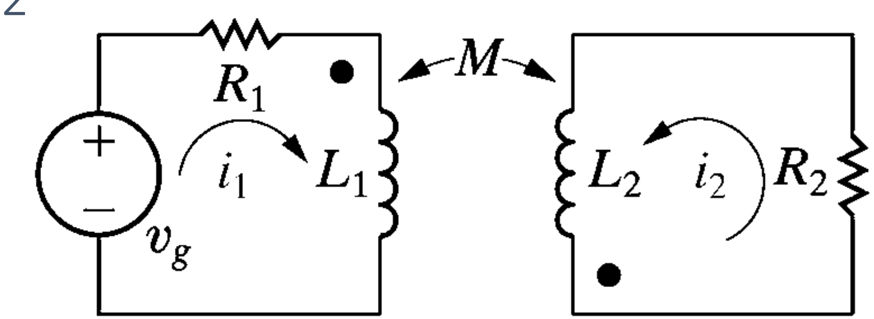
Example #1 (cont'd)

- Write time-domain mesh equations for the network shown
- Solution (cont'd)
  - Assign a set of mesh currents
  - Mesh analysis @ Mesh 1



$$\boxed{v_g = R_1 i_1 + L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}}$$

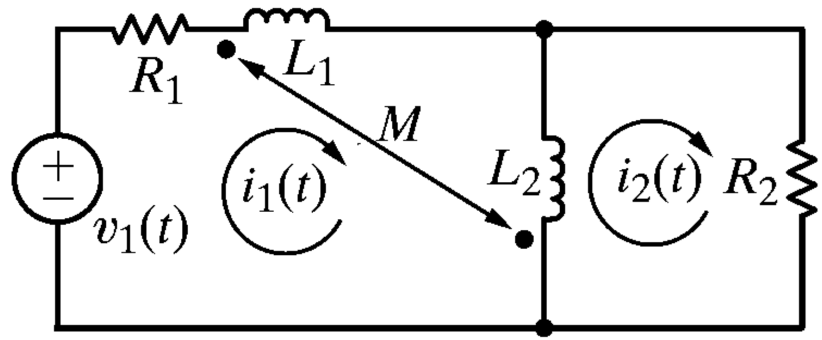
  - Perform Mesh Analysis @ Mesh 2



$$\boxed{0\text{V} = R_2 i_2 + L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}}$$

Example #2

- Write time-domain mesh equations for the network below



- Solution
  - Mesh Analysis @ Mesh #1

$$v_1 = R_1 i_1 + L_1 \frac{di_1}{dt} \boxed{?}\, M \frac{d}{dt}(?) + L_2 \frac{d}{dt}[i_1 - i_2] \boxed{?}\, M \frac{d}{dt}(?)$$

What sign and current to use for each the mutually induced voltage term?

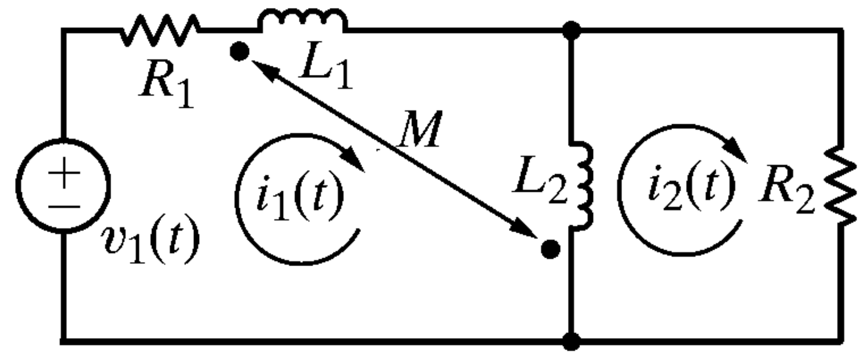
  - Mesh Analysis @ Mesh #2

$$R_2 i_2 + L_2 \frac{d}{dt}[i_2 - i_1] \boxed{?}\, M \frac{d}{dt}(?) = 0\text{V}$$

What sign and current to use for each mutually induced voltage term?

Example #2 (cont'd)

- Write time-domain mesh equations for the network below



- **Solution (cont'd)**
  - Mesh Analysis @ Mesh #1

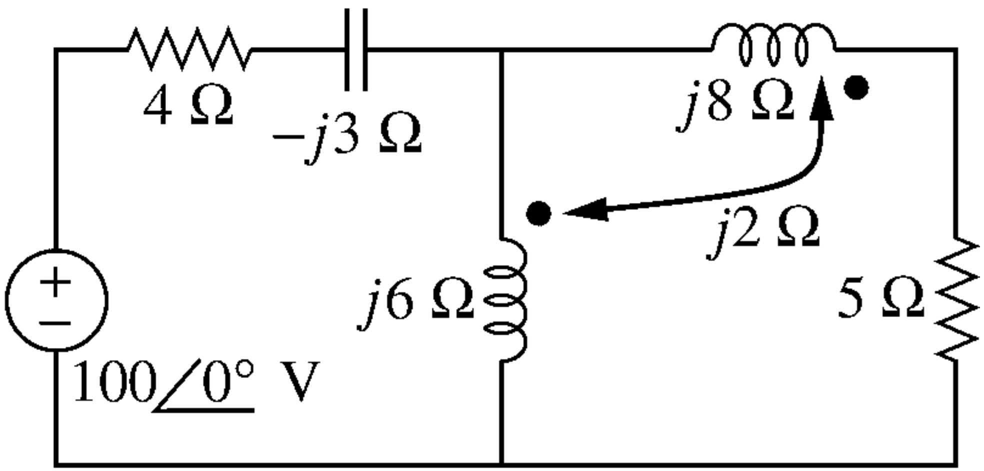
$$\boxed{v_1 = R_1 i_1 + L_1 \frac{di_1}{dt} + M\frac{d}{dt}[i_2 - i_1] + L_2 \frac{d}{dt}[i_1 - i_2] - M\frac{di_1}{dt}}$$

  - Mesh Analysis @ Mesh #2

$$\boxed{R_2 i_2 + L_2 \frac{d}{dt}[i_2 - i_1] + M\frac{di_1}{dt} = 0\text{V}}$$

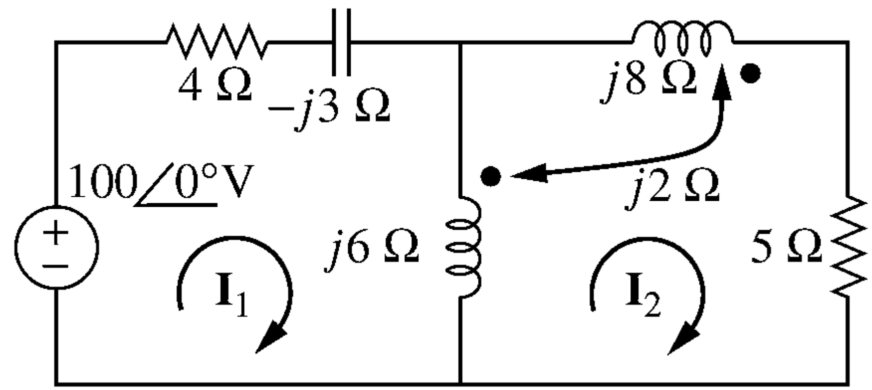
Example #3

- Assume the mutually-inductive network shown below is operating in the SSS at a radian frequency ω.
- Write phasor-domain mesh equations for the network



Example #3 (Solution)

- Write phasor-domain mesh equations for the network



- MCM @ Mesh #1

$$100e^{j0^\circ}\text{V} = (4\Omega - j3\Omega)\boldsymbol{I_1} + j6\Omega(\boldsymbol{I_1} - \boldsymbol{I_2}) \boxed{?}\ j2\Omega(?)$$

What sign and current to use for each the mutually induced voltage term?

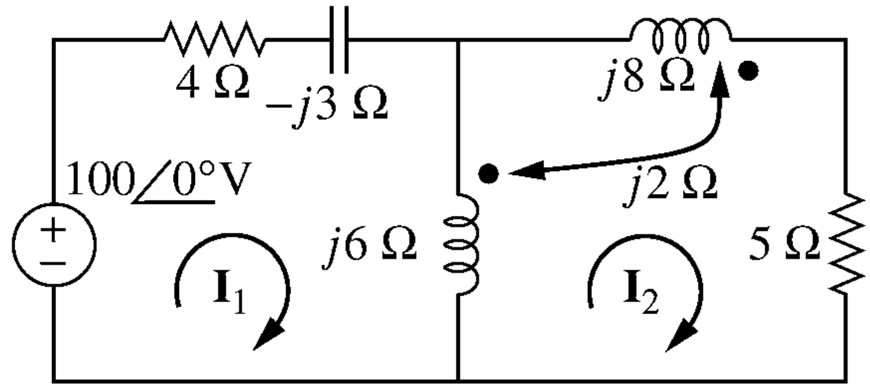
- MCM @ Mesh #2

$$j8\Omega\boldsymbol{I_2} \boxed{?}\ j2(?) + 5\Omega\boldsymbol{I_2} + j6\Omega(\boldsymbol{I_2} - \boldsymbol{I_1}) \boxed{?}\ j2\Omega(?) = 0\text{V}$$

What sign and current to use for each the mutually induced voltage term?

Example #3 (Solution cont'd)

- Write phasor-domain mesh equations for the network



- MCM @ Mesh #1

$$100e^{j0^\circ}\text{V} = (4\Omega - j3\Omega)\boldsymbol{I_1} + j6\Omega(\boldsymbol{I_1} - \boldsymbol{I_2}) - j2\Omega\boldsymbol{I_2}$$

$$\boxed{100e^{j0^\circ}\text{V} = (4\Omega + j3\Omega)\boldsymbol{I_1} - j8\Omega\boldsymbol{I_2}}$$

- MCM @ Mesh #2

$$j8\Omega\boldsymbol{I_2} - j2\Omega(\boldsymbol{I_1} - \boldsymbol{I_2}) + 5\Omega\boldsymbol{I_2} + j6\Omega(\boldsymbol{I_2} - \boldsymbol{I_1}) + j2\Omega\boldsymbol{I_2} = 0\text{V}$$

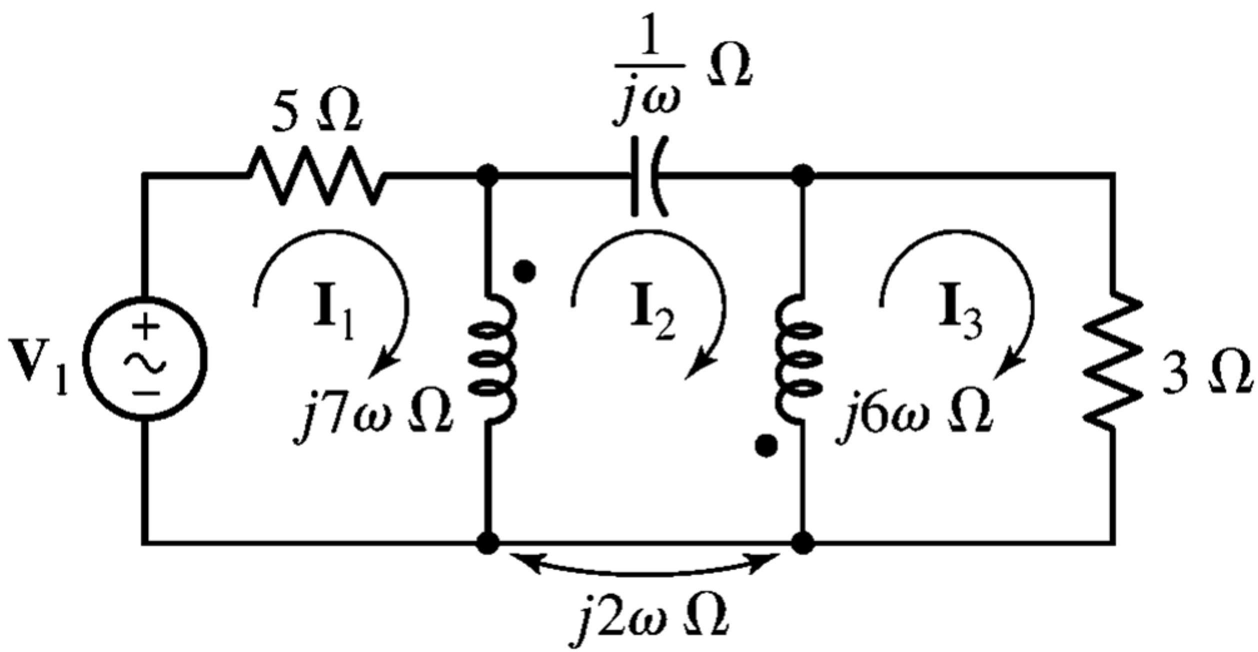
$$\boxed{-j8\Omega\boldsymbol{I_1} + (5\Omega + j18\Omega)\boldsymbol{I_2} = 0\text{V}}$$

$$\boxed{\boldsymbol{I_1} \approx 20.3e^{j3.5^\circ}\ \text{A}}$$

$$\boxed{\boldsymbol{I_2} \approx 8.69e^{j19^\circ}\ \text{A}}$$

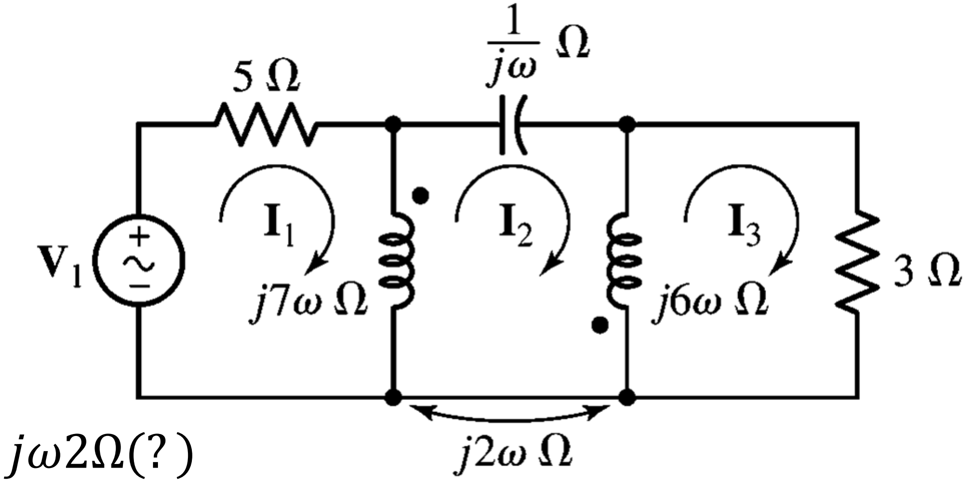
Example #4

- Assume the mutually-inductive network shown below is operating in the SSS at a radian frequency ω.
- Write phasor-domain mesh equations for the network



Example #4 (Solution)

- Write phasor-domain mesh equations for the network



- MCM @ Mesh #1

$$\boldsymbol{V_1} = 5\Omega \boldsymbol{I_1} + j\omega 7\Omega(\boldsymbol{I_1} - \boldsymbol{I_2}) \boxed{?}\ j\omega 2\Omega(?)$$

What sign and current to use for each the mutually induced voltage term?

- MCM @ Mesh #2

$$j\omega 7\Omega(\boldsymbol{I_2} - \boldsymbol{I_1}) \boxed{?}\ j\omega 2\Omega(?) - j\omega^{-1}\Omega \boldsymbol{I_2} + j\omega 6\Omega(\boldsymbol{I_2} - \boldsymbol{I_3}) \boxed{?}\ j\omega 2\Omega(?) = 0\text{V}$$

What sign and current to use for each the mutually induced voltage term?

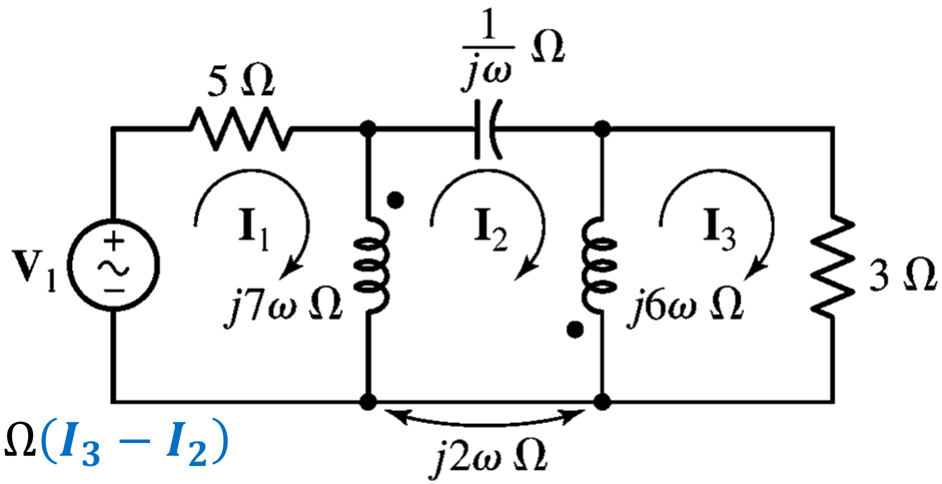
- MCM @ Mesh #3

$$3\Omega \boldsymbol{I_3} \boxed{?}\ j\omega 6\Omega(?) + j\omega 2\Omega(\boldsymbol{I_1} - \boldsymbol{I_2}) = 0\text{V}$$

What sign and current to use for each the mutually induced voltage term?

Example #4 (Solution cont'd)

- Write phasor-domain mesh equations for the network



- MCM @ Mesh #1

$$\boldsymbol{V_1} = 5\Omega \boldsymbol{I_1} + j\omega 7\Omega(\boldsymbol{I_1} - \boldsymbol{I_2}) + j\omega 2\Omega(\boldsymbol{I_3} - \boldsymbol{I_2})$$

$$\boxed{\boldsymbol{V_1} = \boldsymbol{I_1}(5\Omega + j\omega 7\Omega) - \boldsymbol{I_2}(j\omega 9\Omega) + \boldsymbol{I_3}(j\omega 2\Omega)}$$

- MCM @ Mesh #2

$$j\omega 7\Omega(\boldsymbol{I_2} - \boldsymbol{I_1}) - j\omega 2\Omega(\boldsymbol{I_3} - \boldsymbol{I_2}) - j\omega^{-1}\Omega \boldsymbol{I_2} + j\omega 6\Omega(\boldsymbol{I_2} - \boldsymbol{I_3}) - j\omega 2\Omega(\boldsymbol{I_1} - \boldsymbol{I_2}) = 0\text{V}$$

$$\boxed{-\boldsymbol{I_1}(j\omega 9\Omega) + \boldsymbol{I_2} j(\omega 17\Omega - \omega^{-1}\Omega) - \boldsymbol{I_3}(j\omega 8\Omega) \ = 0\text{V}}$$

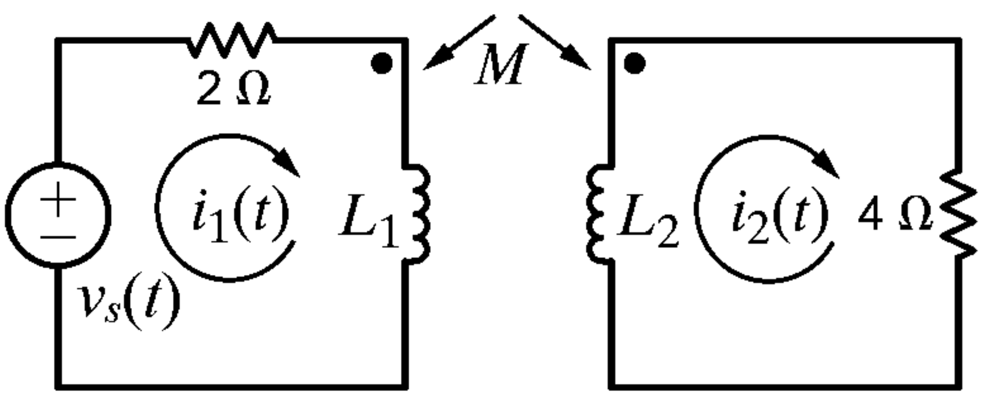
- MCM @ Mesh #3

$$3\Omega \boldsymbol{I_3} + j\omega 6\Omega(\boldsymbol{I_3} - \boldsymbol{I_2}) + j\omega 2\Omega(\boldsymbol{I_1} - \boldsymbol{I_2}) = 0\text{V}$$

$$\boxed{\boldsymbol{I_1}(j\omega 2\Omega) - \boldsymbol{I_2}(j\omega 8\Omega) + \boldsymbol{I_3}(3\Omega + j\omega 6\Omega) = 0\text{V}}$$

Example #5

- Assume the network shown below is operating in the SSS with $v_S(t) = 24\cos(120\pi t)V$. It is known that $L_1 = 2.653\text{mH}$, $L_2 = 10.61\text{mH}$, and that the two coils are perfectly coupled.
  - Determine the mutual inductance between the two coils
  - Write phasor-domain mesh equations for the network
  - Compute the phasor mesh currents
  - Compute the energy dissipated by the coils at $t = 5ms$ *after* the network enters SSS



Example #5 (**Solution**)

- Determine the mutual inductance between the two coils

$$M = k\sqrt{L_1 L_2} = (1)\sqrt{(2.653\text{mH})(10.61\text{mH})} \rightarrow \boxed{M = 5.31\text{mH}}$$

- The phasor domain equivalent network is shown below.
- Write phasor-domain mesh equations for the network
  - MCM @ Mesh #1

$$24e^{j0^\circ}\text{V} = 2\Omega \boldsymbol{I_1} + j1\Omega \boldsymbol{I_1} - j2\Omega \boldsymbol{I_2}$$

$$\boxed{24e^{j0^\circ}\text{V} = (2\Omega + j1\Omega)\boldsymbol{I_1} - j2\Omega \boldsymbol{I_2}}$$

  - MCM @ Mesh #2

$$0\text{V} = 4\Omega \boldsymbol{I_2} + j4\Omega \boldsymbol{I_2} - j2\Omega \boldsymbol{I_1}$$

$$\boxed{0\text{V} = -j2\Omega \boldsymbol{I_1} + (4\Omega + j4\Omega)\boldsymbol{I_2}}$$

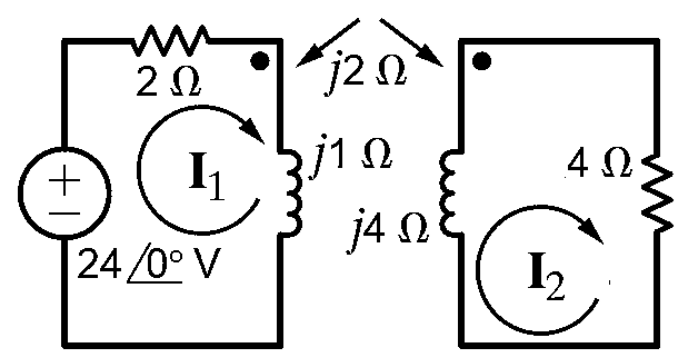
- Solve the mesh equations and compute the phasor mesh currents

$$\boxed{\boldsymbol{I_1} = 9.41e^{-j11.31^\circ}\text{A}} \qquad \boxed{\boldsymbol{I_2} = 3.33e^{j33.69^\circ}\text{A}}$$

- The time-domain SSS mesh currents are

$$\boxed{i_{1,sss}(t) = 9.41\cos(120\pi t - 11.31^\circ)\text{ A}}$$

$$\boxed{i_{2,sss}(t) = 3.33\cos(120\pi t + 33.69^\circ)\text{ A}}$$



Example #5 (Solution cont'd)

- Compute the energy dissipated by the coils at $t = 5ms$ *after* the network enters SSS

Use **minus sign** since the coupling is subtractive!

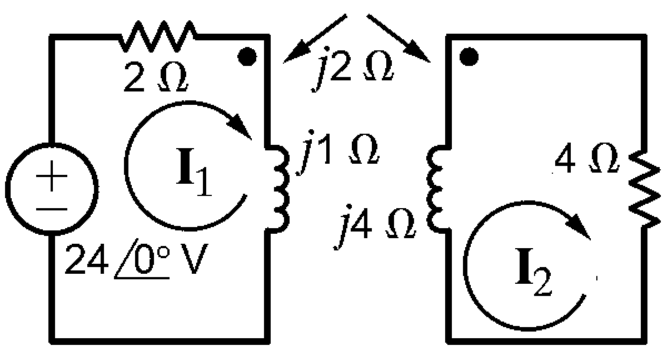
$$w(t) = 0.5L_1 i_1^2(t) + 0.5L_2 i_2^2(t) - M i_1(t) i_2(t)$$
$$w(5ms) = 0.5L_1 i_1^2(5ms) + 0.5L_1 i_2^2(5ms) - M i_1(5ms) i_2(5ms)$$

$$i_1(5ms) = 9.41\cos[120\pi(5ms) - 11.31°]\,\text{A} \Rightarrow \boxed{i_1(5ms) = -1.10\text{A}}$$

$$i_2(5ms) = 3.33\cos[120\pi(5ms) + 33.69°]\,\text{A} \Rightarrow \boxed{i_2(5ms) = -2.61\text{A}}$$

$$w(5ms) = 0.5(2.653\text{mH})(-1.10\text{A})^2 + 0.5(10.61\text{mH})(-2.61A)^2 - (5.31\text{mH})(-1.10\text{A})(-2.61\text{A})$$

$$\boxed{w(5ms) = 22.5\ \text{m}J}$$

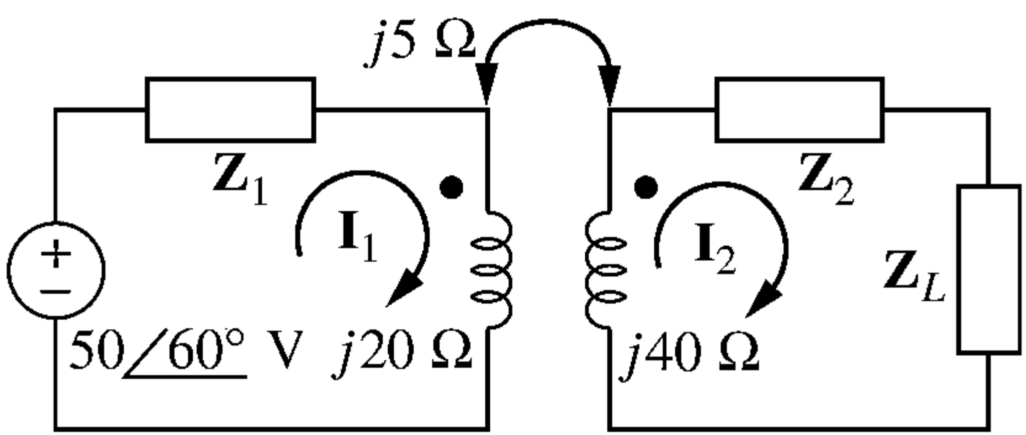


Example #6 (Linear Transformers, Impedance)

- Consider the network below operating in the SSS and comprising a linear transformer. The impedances are given as:

  $\boldsymbol{Z_1} = 60\Omega - j100\Omega \qquad \boldsymbol{Z_2} = 30\Omega + j40\Omega \qquad \boldsymbol{Z_L} = 80\Omega + j60\Omega$

  - Find impedance $\boldsymbol{Z_{r,2\to1}}$ reflected from the secondary to primary coil
  - Find impedance $\boldsymbol{Z_{IN}}$ seen by the source.
  - Find impedance $\boldsymbol{Z_{r,1\to2}}$ reflected from the primary to the secondary coil
  - Find impedance $\boldsymbol{Z_{TH}}$ seen by the load impedance $\boldsymbol{Z_L}$



Example #6 (Linear Transformers, Impedance) (Solution)

- Find impedance $\boldsymbol{Z}_{\boldsymbol{r},\mathbf{2}\to\mathbf{1}}$ reflected from the secondary to the primary coil

$$\boldsymbol{Z}_{r,2\to1} = \frac{|(j\omega M\Omega)|^2}{\boldsymbol{Z}_{22}} = \left|\frac{(j\omega M)}{\boldsymbol{Z}_{22}}\right|^2 \boldsymbol{Z}_{22}^{*} = \left|\frac{(j\omega M)}{\boldsymbol{Z}_2 + \boldsymbol{Z}_L + j40\Omega}\right|^2 (\boldsymbol{Z}_2 + \boldsymbol{Z}_L + j40\Omega)^{*}$$

$$\boldsymbol{Z}_{r,2\to1} = \frac{|(j5\Omega)|^2}{|110\Omega + j140\Omega|^2}(110\Omega - j140\Omega) \Rightarrow \boxed{\boldsymbol{Z}_{r,2\to1} = (1268)^{-1}(110 - j140)\Omega}$$

- Find the impedance $\boldsymbol{Z}_{\boldsymbol{in}}$ seen by the source.

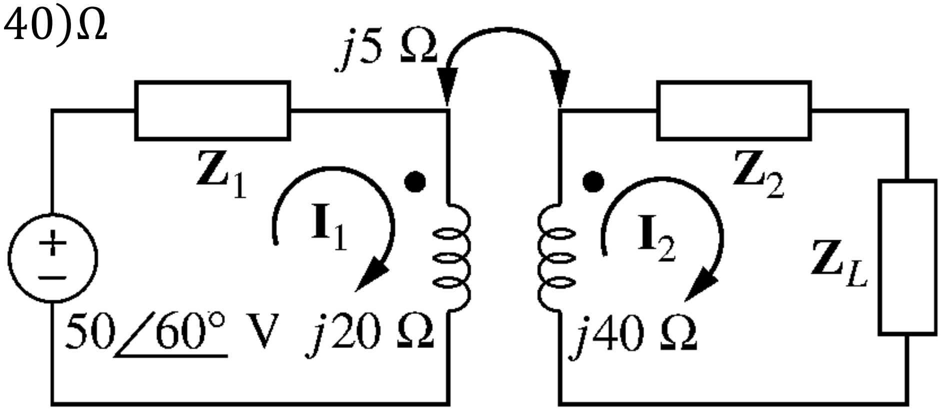
$$\boldsymbol{Z}_{IN} = \boldsymbol{Z}_{11} + \boldsymbol{Z}_{r,2\to1} = (\boldsymbol{Z}_1 + j20\Omega) + \boldsymbol{Z}_{r,2\to1}$$

$$\boldsymbol{Z}_{IN} = (60\Omega - j100\Omega + j20\Omega) + (1268)^{-1}(110 - j140)\Omega$$

$$\boldsymbol{Z}_{IN} = 60 - j80 + (1268)^{-1}(110 - j140)\Omega$$

$$\boxed{\boldsymbol{Z}_{IN} = \frac{1}{317}\left[\frac{38{,}095}{2}\Omega - j25{,}395\Omega\right]}$$

$$\boxed{\boldsymbol{Z}_{IN} \approx 60.09\Omega - j80.11\Omega}$$



Example #6 (Linear Transformers, Impedance) (Solution cont'd)

- Find impedance $\boldsymbol{Z_{r,1\to 2}}$ reflected from the primary to the secondary coil

$$\boldsymbol{Z_{r,1\to 2}} = \frac{|(j\omega M\Omega)|^2}{\boldsymbol{Z_{11}}} = \left|\frac{(j\omega M)}{\boldsymbol{Z_{11}}}\right|^2 \boldsymbol{Z_{11}^*} = \left|\frac{(j\omega M)}{\boldsymbol{Z_1} + j20\Omega}\right|^2 (\boldsymbol{Z_1} + j20\Omega)^*$$

$$\boldsymbol{Z_{r,1\to 2}} = \frac{|(j5\Omega)|^2}{|60\Omega - j80\Omega|^2}(60\Omega + j80\Omega) \Rightarrow \boxed{\boldsymbol{Z_{r,1\to 2}} = 0.15\Omega + j0.2\Omega}$$

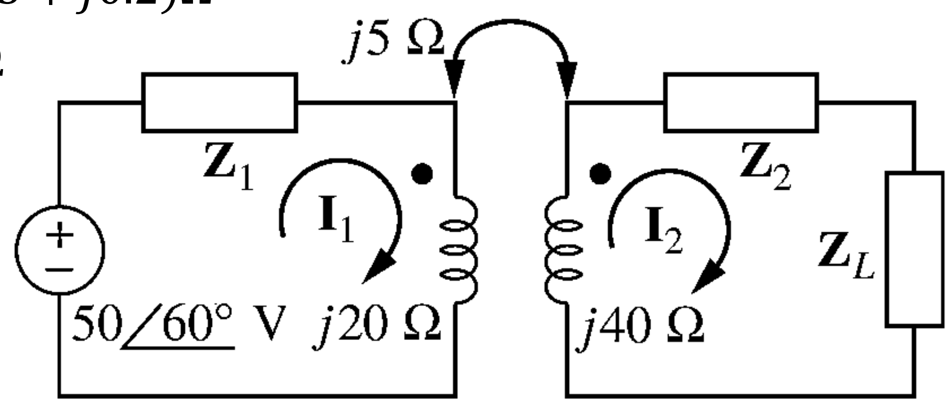
- Find the impedance $\boldsymbol{Z_{TH}}$ seen by the load.

$$\boldsymbol{Z_{TH}} = (\boldsymbol{Z_{22}} - \boldsymbol{Z_L}) + \boldsymbol{Z_{r,1\to 2}} = (\boldsymbol{Z_2} + j40\Omega) + \boldsymbol{Z_{r,1\to 2}}$$

$$\boldsymbol{Z_{TH}} = (30\Omega + j40\Omega + j40\Omega) + (0.15 + j0.2)\Omega$$

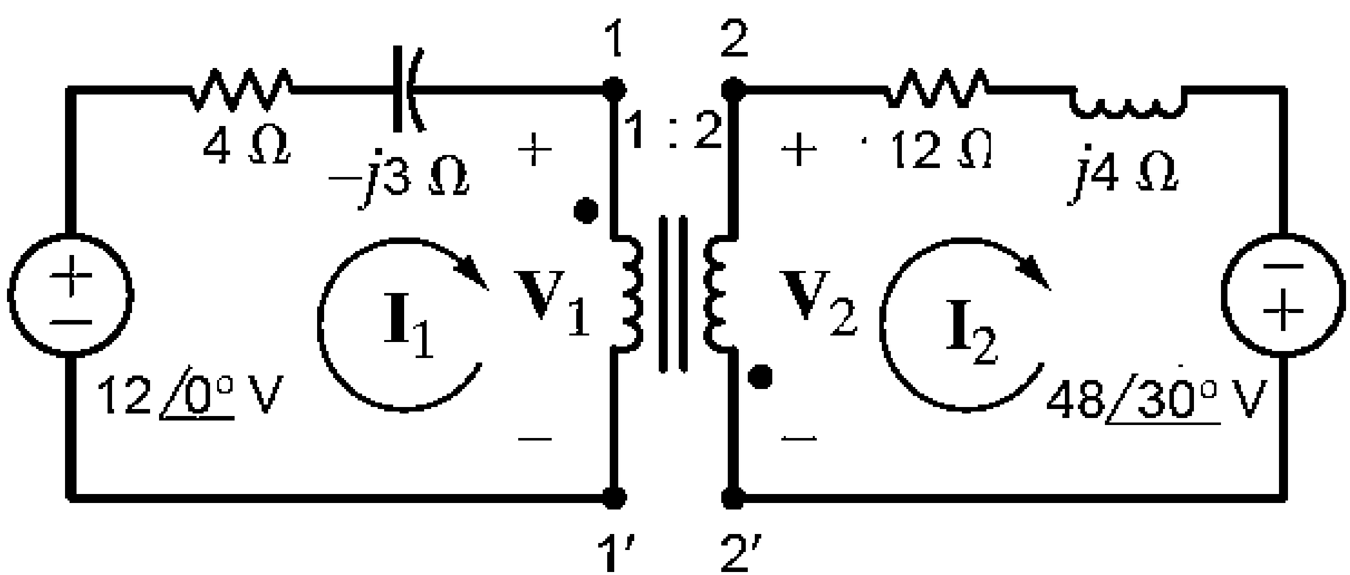
$$\boldsymbol{Z_{TH}} = 30\Omega + j80\Omega + 0.15\Omega + j0.2\Omega$$

$$\boxed{\boldsymbol{Z_{TH}} = 30.15\Omega + j80.2\Omega}$$



Example #7 (Ideal Transformer, Reflecting Elements)

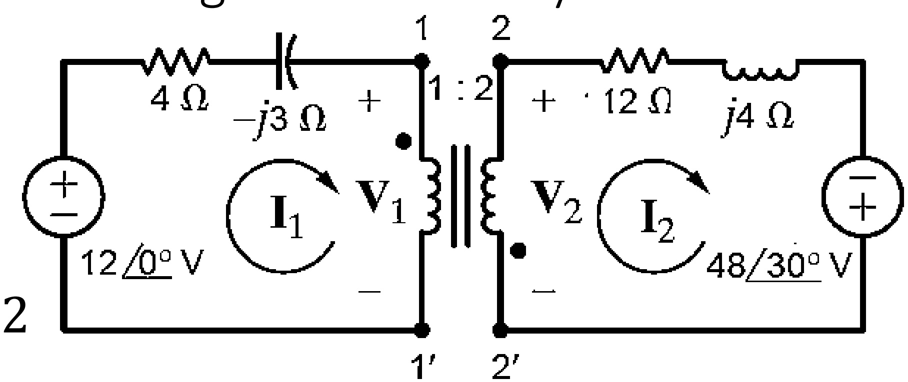
- The ideal transformer network is operating in the SSS.
  - Draw the equivalent network that results from reflecting the entire secondary side network to the primary side
  - Draw the equivalent network that results from reflecting the entire primary side network to the secondary side



Example #7 (Ideal Transformer, Reflecting Elements) (Solution)

- Draw the network that results from reflecting the secondary network to the primary side
  - Transformer *i-v* constraint dictates how the scaling factors

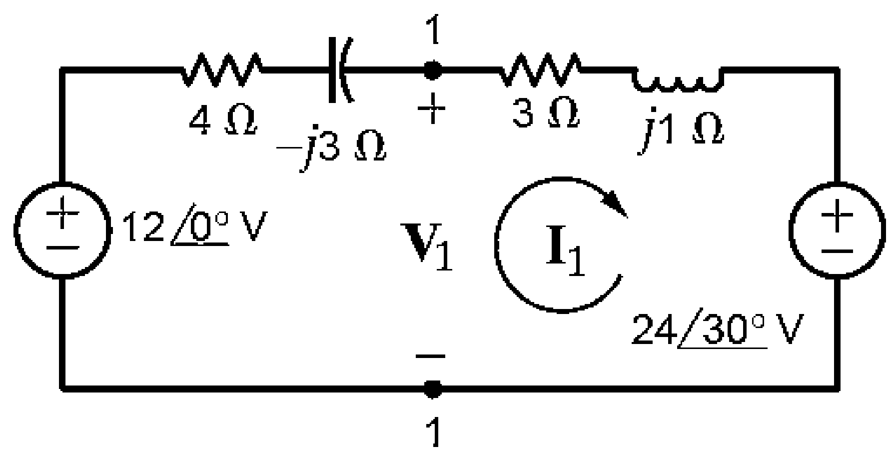
$$\frac{\boldsymbol{V_2}}{\boldsymbol{V_1}} = -\frac{N_2}{N_1} = -2 \qquad \frac{\boldsymbol{I_1}}{\boldsymbol{I_2}} = -\frac{N_2}{N_1} = -2$$



  - Solve the i-v constraint for primary values

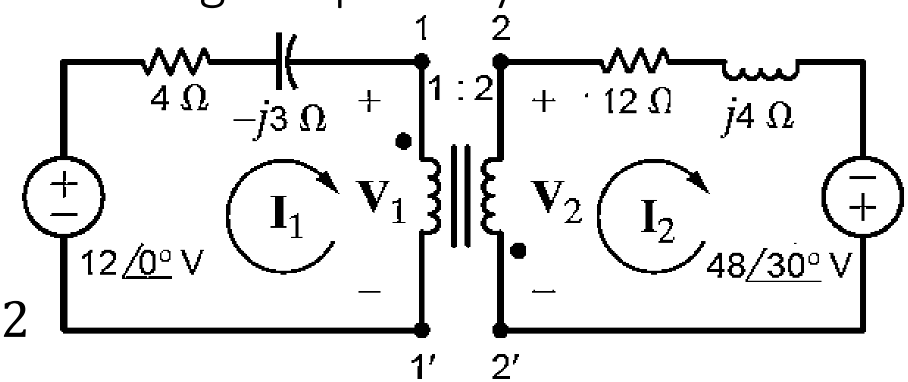
$$\boldsymbol{V_1} = -0.5\boldsymbol{V_2} \quad \boldsymbol{I_1} = -2\boldsymbol{I_2} \qquad \boldsymbol{Z_1} = \boldsymbol{V_1}/\boldsymbol{I_1} = 0.25\boldsymbol{Z_2}$$

  - Since the dot markings are on opposite terminals, **sources reflect with a minus sign**
  - The corresponding primary equivalent network is shown here



Example #7 (Ideal Transformer, Reflecting Elements) (Solution cont'd)

- Draw the network that results from reflecting the primary network to the secondary side
  - Transformer *i-v* constraint dictates how the scaling factors

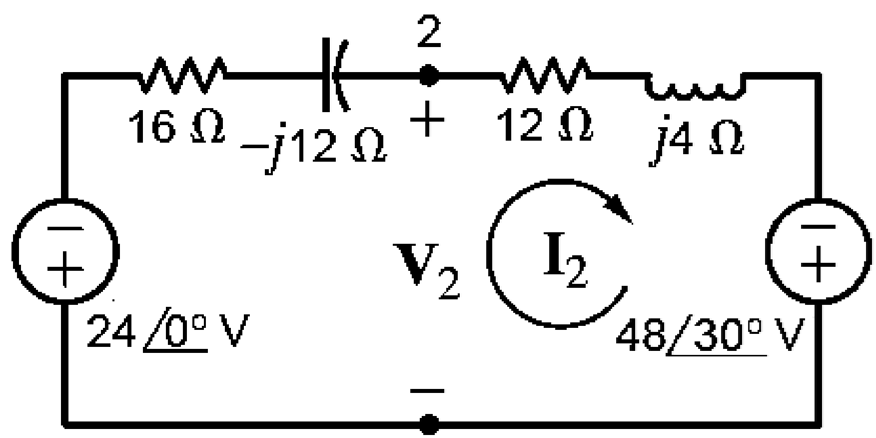


$$\frac{\boldsymbol{V_2}}{\boldsymbol{V_1}} = -\frac{N_2}{N_1} = -2 \qquad \frac{\boldsymbol{I_1}}{\boldsymbol{I_2}} = -\frac{N_2}{N_1} = -2$$

  - To reflect to the secondary, solve the *i-v* constraint for secondary values

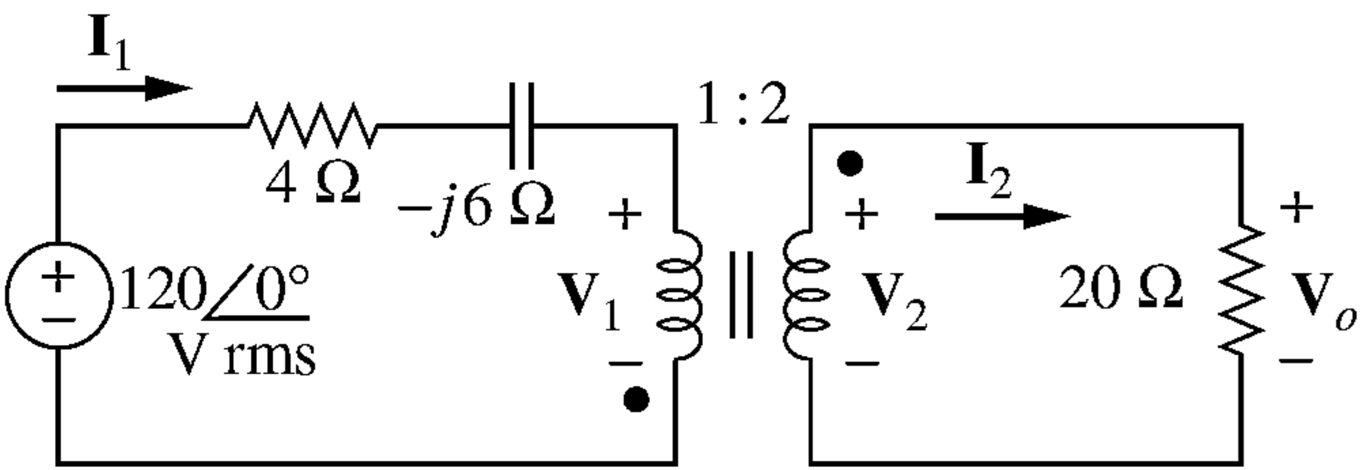
$$\boldsymbol{V_2} = -2\boldsymbol{V_1} \qquad \boldsymbol{I_2} = -0.5\boldsymbol{I_1} \qquad \boldsymbol{Z_2} = \boldsymbol{V_2}/\boldsymbol{I_2} = 4\boldsymbol{Z_1}$$

  - Since the dot markings are on opposite terminals, <u>**sources reflect with a minus sign**</u>
  - The corresponding secondary equivalent network is shown here



Example #8 (Ideal Transformer)

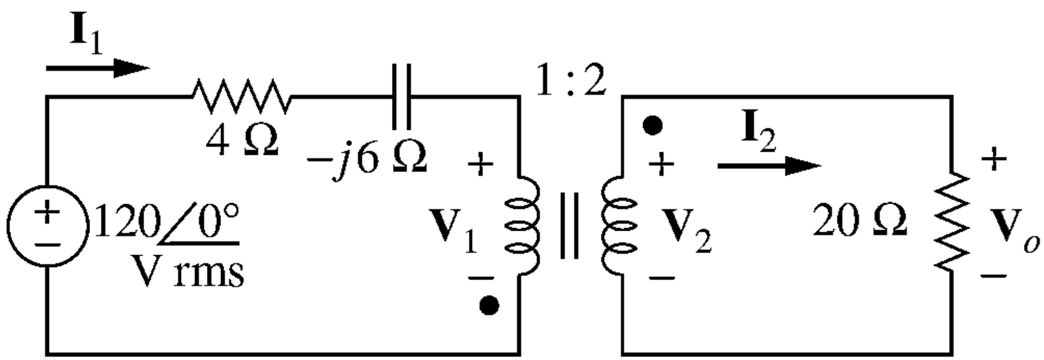
- The ideal transformer network is operating in the SSS. Find the following
  - The source current $\mathbf{I_1}$
  - The output voltage $\mathbf{V_o}$
  - The complex power $\mathbf{S_s}$ <u>supplied</u> by the source



Example #8 (Ideal Transformer) (Solution)

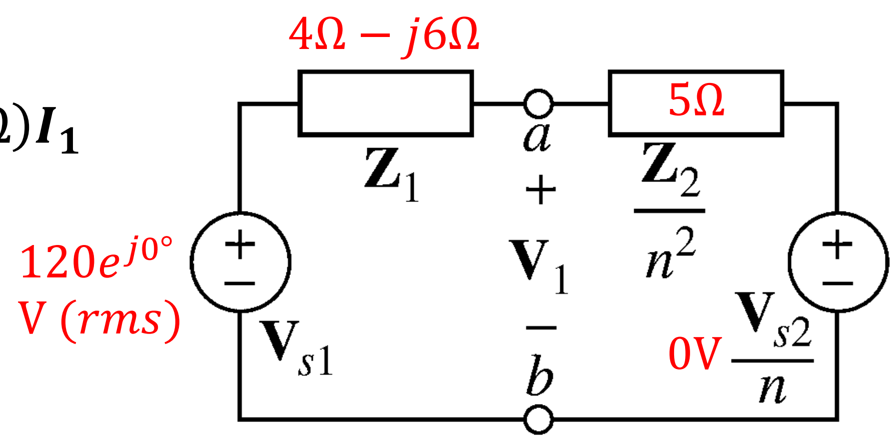
- Find the current $\boldsymbol{I_1}$
  - Reflect 20Ω resistor to primary

$$\boldsymbol{Z_{r,2\to 1}} = 20\Omega(N_1/N_2)^2$$
$$\boldsymbol{Z_{r,2\to 1}} = (20)(0.25) \ = 5\Omega$$



  - Draw primary equivalent a shown on the right
  - Use mesh analysis on equivalent network above to solve for $\boldsymbol{I_1}$

$$120e^{j0}\text{ V}(rms) = (4\Omega - j6\Omega + 5\Omega)\boldsymbol{I_1}$$
$$\boldsymbol{I_1} = \frac{120e^{j0^\circ}\text{V } rms}{9\Omega - j6\Omega}$$
$$\boldsymbol{I_1} = (1/13)(120 + j80)\text{ A } rms$$
$$\boldsymbol{I_1} \approx 11.09e^{j33.69^\circ}\text{A } rms$$



Example #8 (Ideal Transformer) (Solution cont'd)

- Find voltage $\mathbf{V_o}$
  - First use the ideal transformer i-v characteristic to find $\boldsymbol{I_2}$

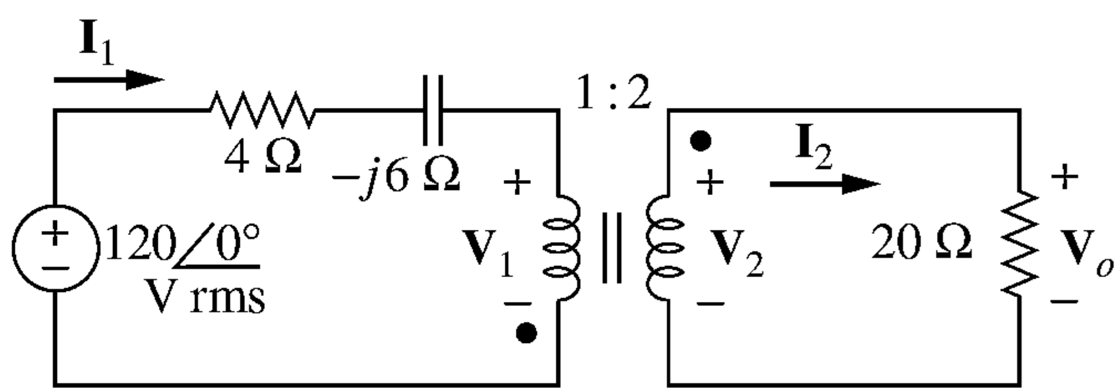
$$\frac{\boldsymbol{I_1}}{\boldsymbol{I_2}} = -\frac{N_2}{N_1} = -n \Rightarrow \boldsymbol{I_2} = -\boldsymbol{I_1}\frac{N_1}{N_2} = -\frac{1}{n}\boldsymbol{I_1}$$

$$\boldsymbol{I_2} = -\left(\frac{1}{2}\right)\left(\frac{1}{13}\right)(120 + j80)\ \mathrm{A}\ rms$$

$$\boldsymbol{I_2} = -(1/13)(60 + j40)\ \mathrm{A}\ rms \approx 5.55e^{-j146.31^\circ}\mathrm{A}\ rms$$

  - Then use the ideal transformer i-v characteristic to find $\boldsymbol{V_o}$

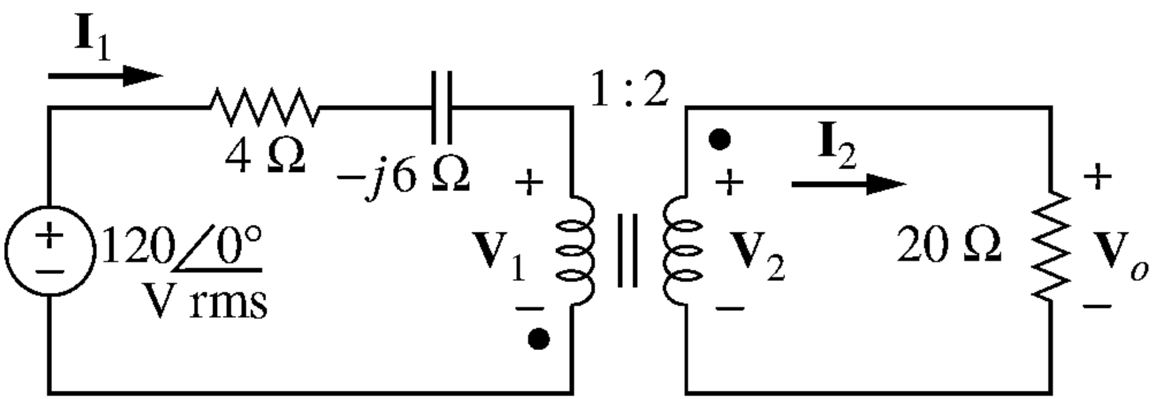
$$\boldsymbol{V_o} = \boldsymbol{I_2}(20\Omega) = -(20/13)(60 + j40)\ \mathrm{V}\ rms \approx 110.94e^{-j146.31^\circ}\mathrm{A}\ rms$$



Example #8 (Ideal Transformer) (Solution cont'd)

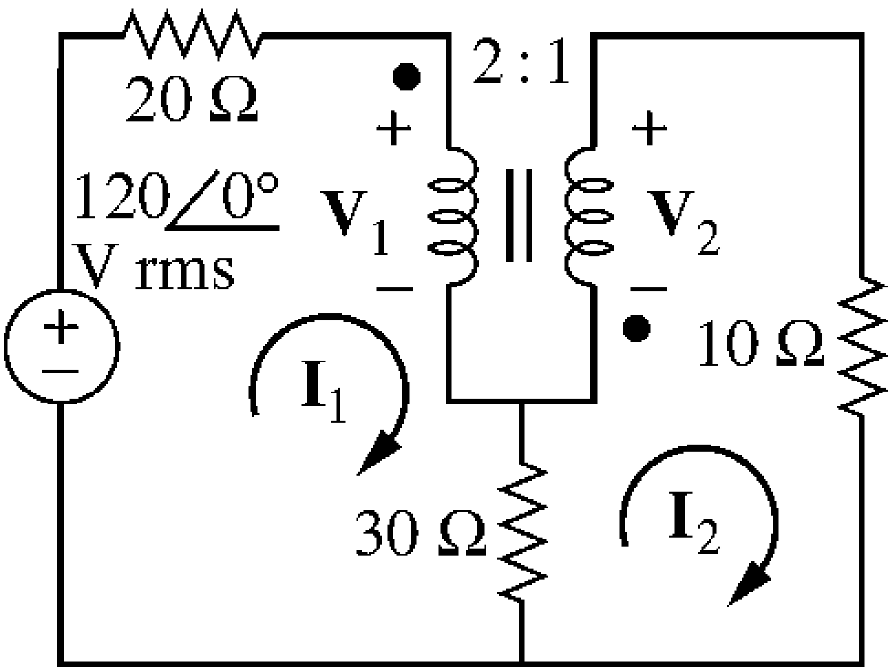
- Find the complex power **supplied** by the source

$$\boldsymbol{S_S} = +\boldsymbol{V_S}\boldsymbol{I_1^*} = \left(120e^{j0^\circ}\ V\ rms\right)\left((1/13)(120 + j80)\ A\ rms\right)^*$$
$$\boldsymbol{S_S} = +(1/13)\left(120e^{j0^\circ}\ V\ rms\right)(120 - j80A\ rms)$$
$$\boldsymbol{S_S} = +(1/13)(14{,}400 - j9600)\ \mathrm{VA} \approx 1331.28e^{-j33.69^\circ}\ \mathrm{VA}$$



Example #9 Electrically Coupled Ideal Transformer

- The ideal transformer network is operating in the SSS. Find the average power absorbed by the 10Ω resistor



Example #9 Electrically Coupled Ideal Transformer (Solution)

- Find the average power absorbed by the $R = 10\Omega$
  - Can't reflect since primary and secondary are electrically coupled
  - Use mesh analysis and i-v ideal transformer constraints for analysis
  - Mesh @ Mesh #1

$$120 = \boldsymbol{I_1}(20\Omega) + \boldsymbol{V_1} + (30\Omega)(\boldsymbol{I_1} - \boldsymbol{I_2})$$

$$\boxed{120 = (50\Omega)\boldsymbol{I_1} - (30\Omega)\boldsymbol{I_2} + \boldsymbol{V_1}}$$

Mesh @ Mesh #2

$$0\text{V} = (10\Omega)\boldsymbol{I_2} + (30\Omega)(\boldsymbol{I_2} - \boldsymbol{I_1}) - \boldsymbol{V_2}$$

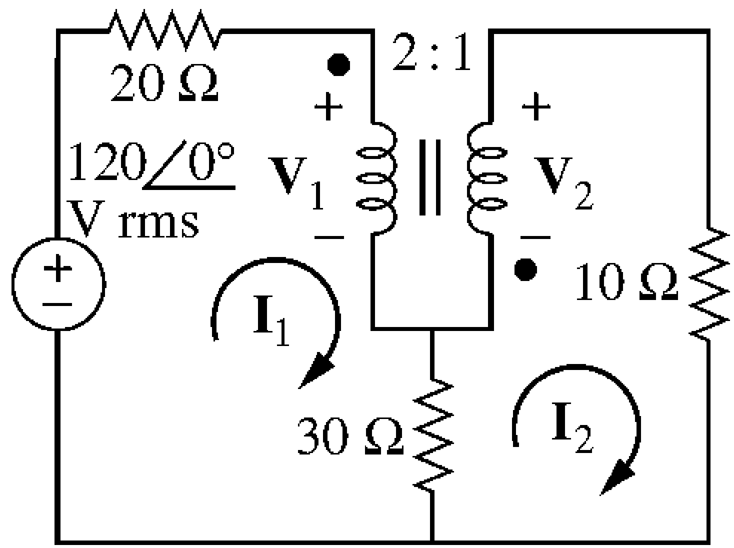
$$\boxed{0\text{V} = -(30\Omega)\boldsymbol{I_1} + (40\Omega)\boldsymbol{I_2} - \boldsymbol{V_2}}$$

  - i-v characteristic for ideal transformer

$$\frac{\boldsymbol{V_2}}{\boldsymbol{V_1}} = -\frac{N_2}{N_1} = -n \qquad \frac{\boldsymbol{I_1}}{\boldsymbol{I_2}} = -\frac{N_2}{N_1} = -n$$

$$\boxed{\frac{\boldsymbol{V_2}}{\boldsymbol{V_1}} = -\frac{1}{2}} \qquad \boxed{\frac{\boldsymbol{I_1}}{\boldsymbol{I_2}} = -\frac{1}{2}}$$

Based on current and voltage references



Example #9 Electrically Coupled Ideal Transformer (Solution cont'd)

- Find the average power absorbed by $R = 10\Omega$
  - Solving the 4 equation, 4 unknown system yields the following values for the primary and secondary voltages and currents

$$\boxed{\boldsymbol{I_1} = (8/22)\ e^{j0^\circ}\ \text{A rms}} \qquad \boxed{\boldsymbol{V_1} = 80e^{j0^\circ}\ \text{V rms}}$$

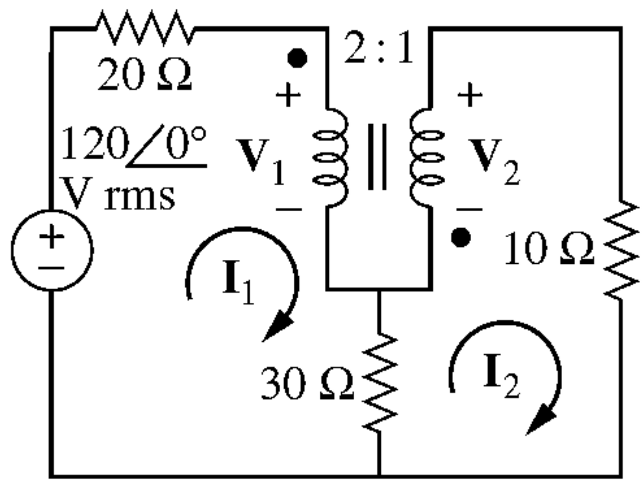
$$\boxed{\boldsymbol{I_2} = (8/11)\ e^{j180^\circ}\ \text{A rms}} \qquad \boxed{\boldsymbol{V_2} = 40e^{j0^\circ}\ \text{A rms}}$$

  - The average power absorbed by the $R = 10\Omega$ resistor is

$$P_{10} = \left|\boldsymbol{I_{2,rms}}\right|^2 R_{10}$$

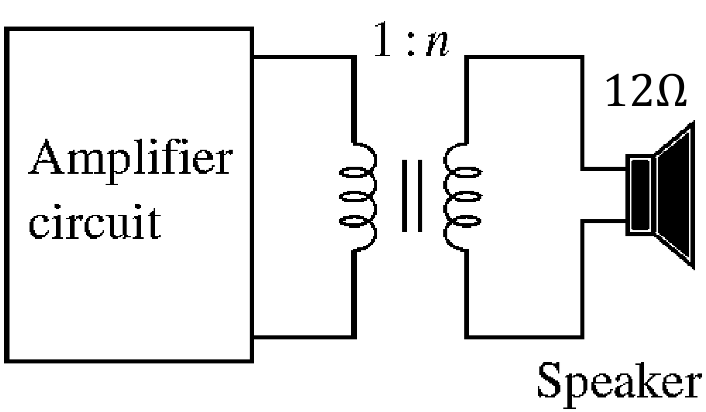
$$P_{10} = (8/11\ \text{A}\ rms)^2(10\Omega)\ \text{W}$$

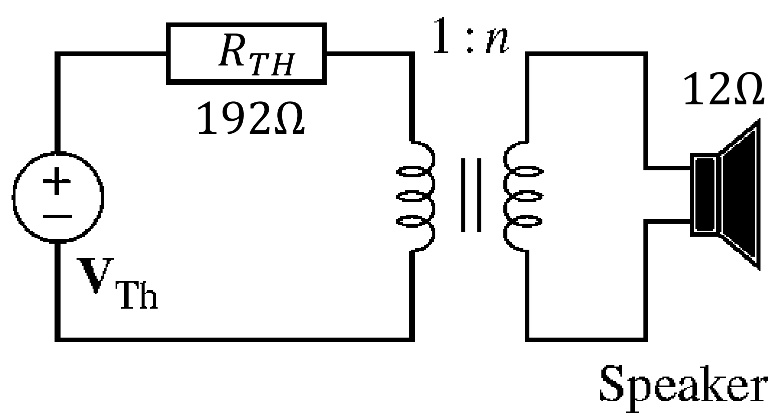
$$\boxed{P_{10} = \frac{640}{121}\text{W} \approx 5.29\text{W}}$$



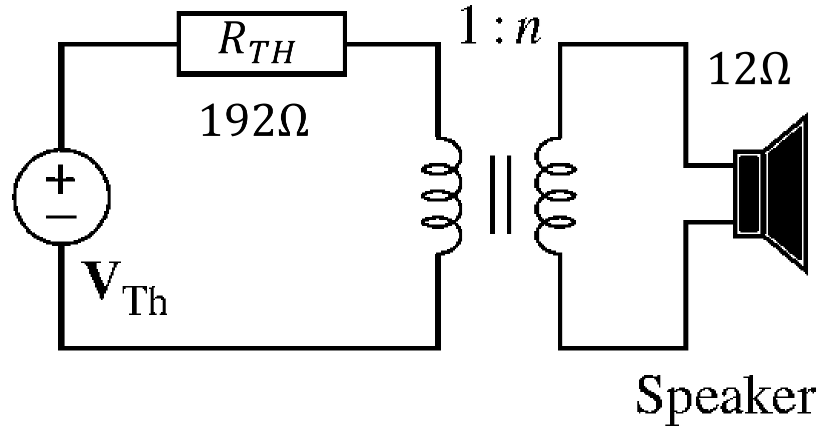
Example #10 (Ideal Transformer, Impedance Matching)

- We want to achieve maximum power transfer from an audio amplifier to a speaker system. The Thevenin resistance of the amplifier is $R_{TH} = 192\Omega$, but the speaker's resistance is only $R_s = 12\Omega$.
- We can use an ideal transformer to "reflect" the resistance of the speaker to the primary side and in the process scale it so that the reflected resistance is equal to the amplifier Thevenin resistance such that maximum power can be transferred by the amplifier to the speaker.
- What should the turns ratio be such that maximum power transfer is achieved to the speaker?





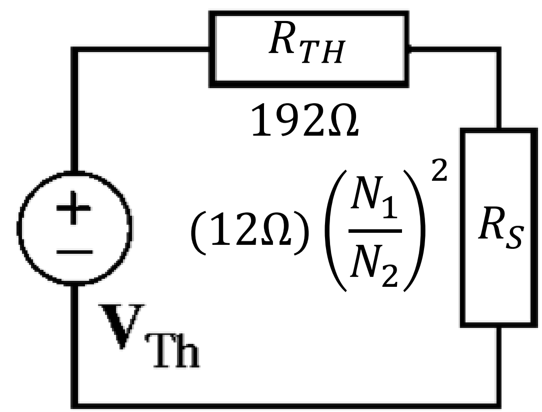
Example #10 (Ideal Transformer, Impedance Matching) (Solution)



- What should the turns ratio be such that maximum power is supplied by the amplifier to the speaker?
  - Reflect the speaker's resistance to the primary side.

$$R_{r,2\to1} = R_s\left(\frac{N_1}{N_2}\right)^2 = (12\Omega)\left(\frac{N_1}{N_2}\right)^2$$

  - Eliminate the ideal transformer and draw the equivalent network



  - Max power transfer for pure resistive loads requires

$$R_{TH} = R_{r,2\to1} = 192\Omega = \left(\frac{N_1}{N_2}\right)^2 (12\Omega)$$

  - For max. power transfer, the turns ratio must be

$$\frac{N_2}{N_1} = n = \sqrt{\frac{12\Omega}{192\Omega}} = \sqrt{\frac{1}{16}} \Rightarrow \boxed{n = \frac{1}{4}} \rightarrow$$

Requires a step-down ideal transformer