Lecture #1(b) Poly-Phase Networks

Examples

ECE 20200: Linear Circuit Analysis II
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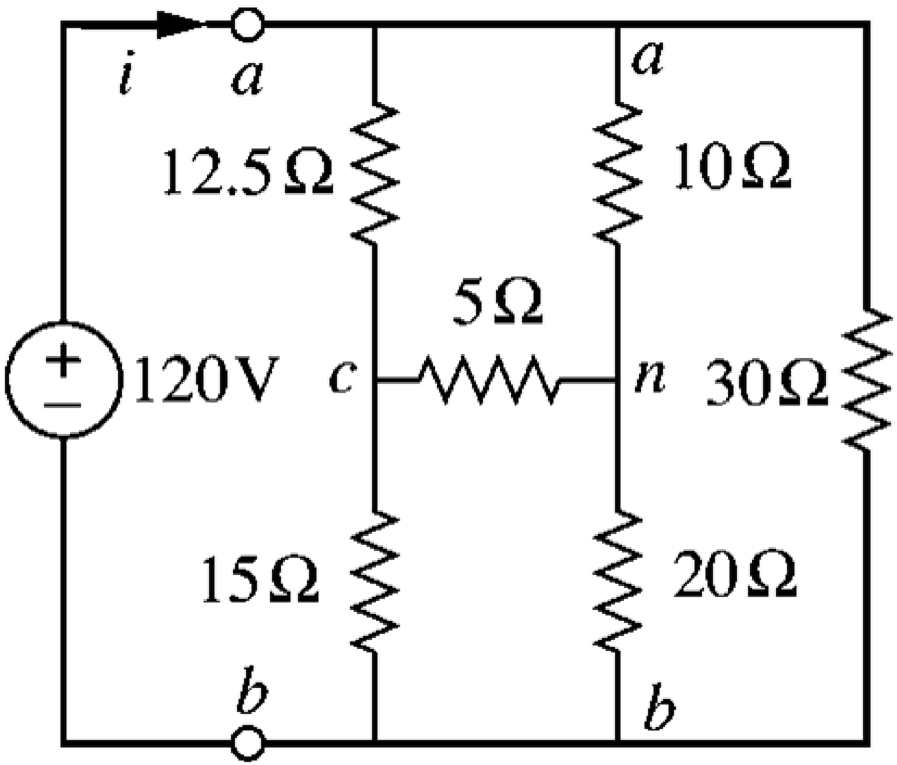
Lecture #1(b) Poly-Phase Networks
Examples

Υ *and* Δ *Impedance Networks*

Example #1

- Assume the resistive network operates in the DC steady state.
  - Determine the equivalent resistance R_{ab} seen by the source
  - Compute current i using the equivalent resistance R_{ab}



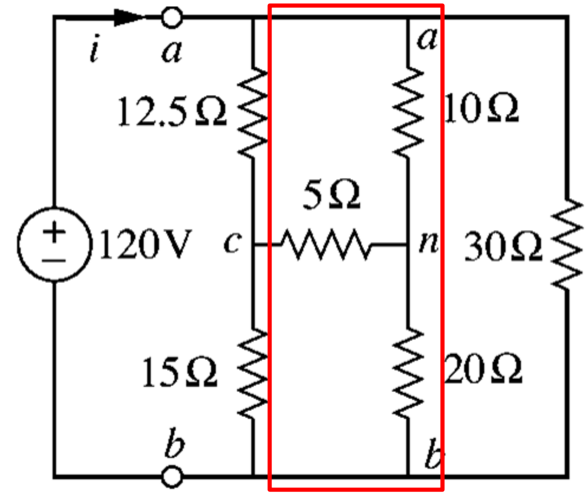
Example #1 (SOLUTION)

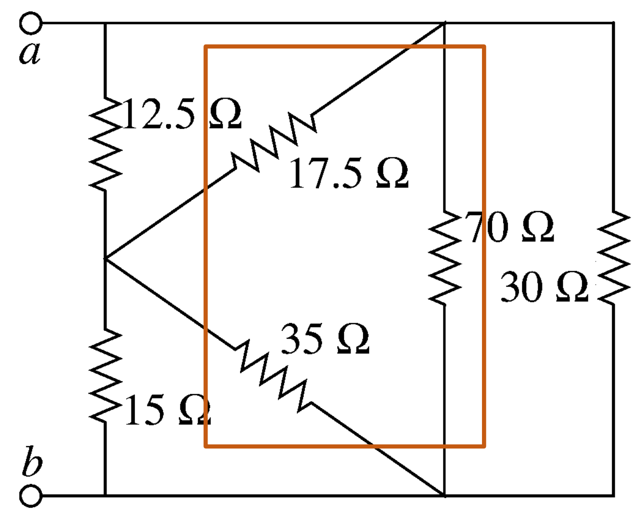
- Determine R_{ab} seen by the source
  - The 3 red-enclosed resistors are in a Y config.

    $R_{an} = 10\Omega$, $R_{bn} = 20\Omega$, and $R_{cn} = 5\Omega$
  - Transform the three resistors into a delta config

    $$R_\Delta = R_{an}R_{bn} + R_{bn}R_{cn} + R_{cn}R_{an}$$
    $$R_\Delta = (10\Omega)(20\Omega) + (20\Omega)(5\Omega) + (5\Omega)(10\Omega)$$
    $$\boxed{R_\Delta = 350\Omega^2}$$

    $$R_{ab} = R_\Delta / R_{cn} = 350/5 \rightarrow \boxed{R_{ab} = 70\Omega}$$
    $$R_{bc} = R_\Delta / R_{an} = 350/10 \rightarrow \boxed{R_{bc} = 35\Omega}$$
    $$R_{ca} = R_\Delta / R_{bn} = 350/20 \rightarrow \boxed{R_{ca} = 17.5\Omega}$$
  - The new equivalent resistance network is shown here

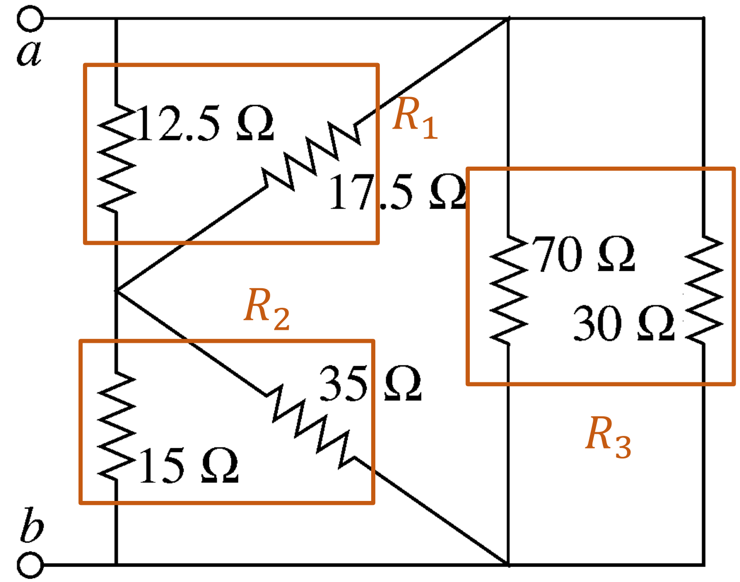




Example #1 (SOLUTION cont'd)

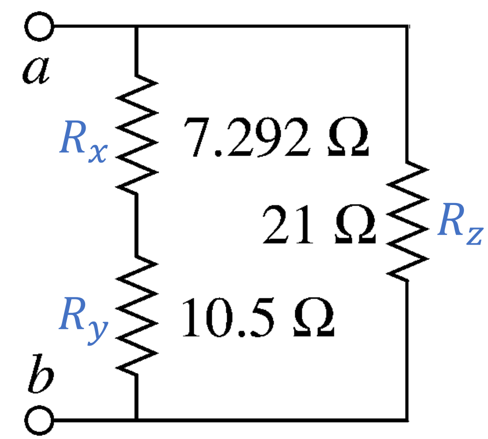
- Determine R_{ab} seen by the source
  - Now, there are three pairs of resistors in parallel with each other

$$R_1 = (12.5\Omega)||(17.5\Omega) \rightarrow \boxed{R_1 = 7.292\Omega}$$
$$R_2 = (15\Omega)||(35\Omega) \rightarrow \boxed{R_2 = 10.5\Omega}$$
$$R_3 = (70\Omega)||(30\Omega) \rightarrow \boxed{R_3 = 21\Omega}$$



  - The reduced equivalent network is shown. Finally, R_{ab} can be computed as

$$R_{ab} = \left(R_x + R_y\right)\left|\right|R_z = (7.292\Omega + 10.5\Omega)\left|\right|(21\Omega)$$
$$R_{ab} = (17.792\Omega)||(21\Omega) \rightarrow \boxed{R_{ab} = 9.632\Omega}$$

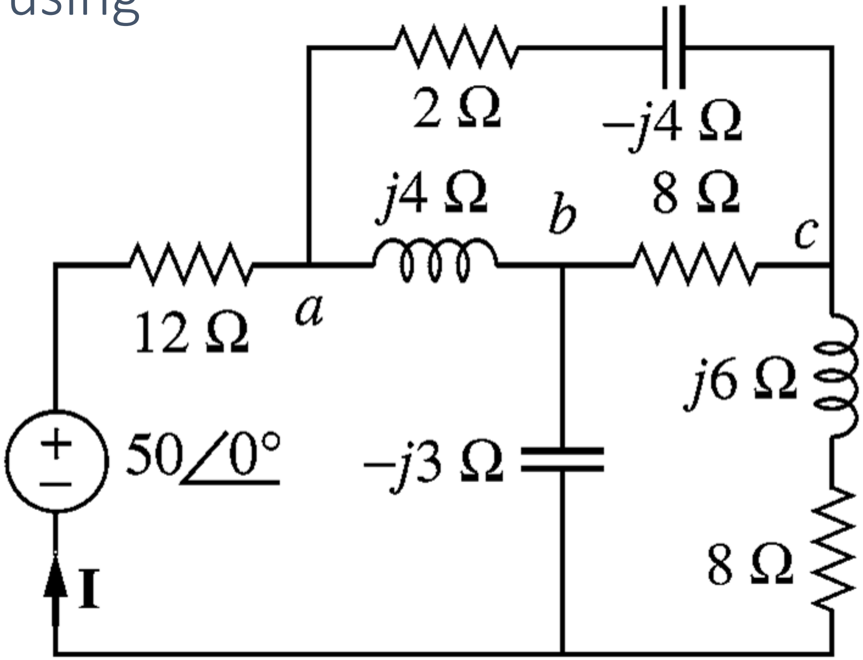


- Compute current i using the equivalent resistance R_{ab}

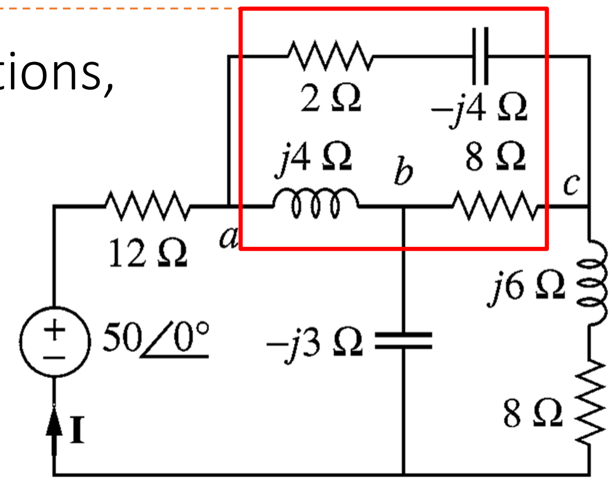
$$i = \frac{V_{src}}{R_{ab}} = \frac{120\text{V}}{9.632\Omega} \rightarrow \boxed{i = 12.458\text{A}}$$

Example #2

- Assume the network is operating in the sinusoidal steady state (SSS). Compute the following
  - Use Δ-to-Y transformations, Y-to-Δ transformations, or both to compute the equivalent impedance seen by the load
  - Compute the phasor current $\mathbf{I}$ using the equivalent impedance



Example #2 (SOLUTION)



- Use Δ-to-Y transformations, Y-to-Δ transformations, or both to compute the equivalent impedance seen by the load
  - Red-enclosed impedances form a Δ config.

    $Z_{ab} = j4\Omega$, $Z_{bc} = 8\Omega$, and $Z_{ca} = 2 - j4\Omega$
  - Transform the 3 impedances to a Y-config

    $Z_y = Z_{ab} + Z_{bc} + Z_{ca}$ (denominator)

    $Z_y = j4\Omega + 8\Omega + 2\Omega - j4\Omega \rightarrow \boxed{Z_y = 10\Omega}$

    $Z_{an} = Z_{ab}Z_{ca}/Z_y = [(j4)(2 - j4)]/10 \rightarrow \boxed{Z_{an} = 1.6 + j0.8\Omega}$

    $Z_{bn} = (Z_{ab}Z_{bc})/Z_y = [(j4)(8)]/10 \rightarrow \boxed{Z_{bn} = j3.2\Omega}$

    $Z_{cn} = (Z_{bc}Z_{ca})/Z_y = [(8)(2 - j4)]/10 \rightarrow \boxed{Z_{cn} = 1.6 - j3.2\Omega}$

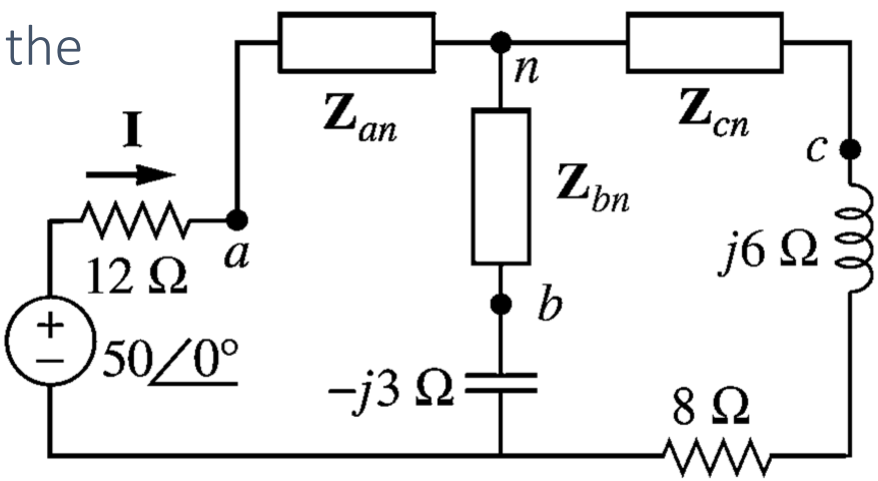
Example #2 (SOLUTION cont'd)

- Use Δ-to-Y transformations, Y-to-Δ transformations, or both to compute the equivalent impedance seen by the load
  - The equivalent schematic after the transformation is shown below
  - Now, Z_{eq} seen by the source can now be computed

$$Z_{eq} = (Z_{cn} + 8\Omega + j6\Omega)||(Z_{bn} - j3\Omega) + (Z_{an} + 12\Omega)$$
$$Z_{eq} = (1.6\Omega - j3.2\Omega + 8\Omega + j6\Omega)||(j3.2\Omega - j3\Omega) + (1.6\Omega + j0.8\Omega + 12\Omega)$$
$$\boxed{Z_{eq} = 13.6\Omega + j1\Omega = 13.64e^{j4.2^\circ}\Omega}$$

- Compute the phasor current $\mathbf{I}$ using the equivalent impedance

$$\mathbf{I} = \frac{\mathbf{V}_{src}}{Z_{eq}} = \frac{50e^{j0^\circ}\mathrm{V}}{13.64e^{j4.2^\circ}\Omega}$$
$$\boxed{\mathbf{I} = 3.66 - j0.269\mathrm{A} = 3.67e^{-j4.2^\circ}\mathrm{A}}$$



Lecture #1(b) Poly-Phase Networks
Examples

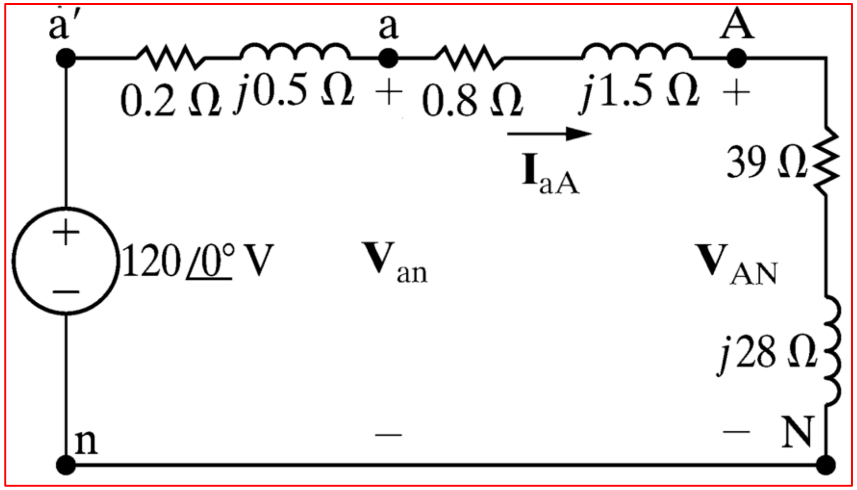
Balanced 3-Phase Networks

Example #1

- A balanced 3-phase Y source with a-b-c sequence has an impedance of $Z_s = 0.2 + j0.5\ \Omega/\phi$ (Ohm's per phase) and an voltage of $\mathbf{V}_{a'n} = 120e^{j0^\circ}\ \text{V}$ (rms). The source drives a balanced 3-phase Y load with an impedance of $Z_\text{Y} = 39 + j28\ \Omega/\phi$. The line impedance connecting source to the load is $Z_\ell = 0.8 + j1.5\ \Omega/\phi$. Use a-phase internal source voltage as the reference.
  - Draw the a-phase equivalent network of the Y-Y system
  - Calculate the transmission line currents for each phase
  - Calculate the phase voltages at the load
  - Calculate the line-to-line voltages at the load's terminals
  - Calculate the phase voltages at the source's terminals
  - Calculate the line-to-line voltages at the source's terminals
  - Calculate the total complex power, average power, and reactive power absorbed by the load
  - Calculate the total complex power, average power, and reactive power lost in the three transmission lines
  - Calculate the total complex power, average power, and reactive power lost in the 3-phase source

Example #1 (SOLUTION)

- Draw the a-phase equivalent network of the Y-Y system
- Calculate transmission line currents currents for each phase
  - First compute the a-phase equivalent impedance Z_ϕ



$$Z_\phi = Z_s + Z_\ell + Z_Y$$

$$Z_\phi = (0.2 + 0.8 + 39) + j(0.5 + 1.5 + 28) \rightarrow \boxed{Z_\phi = 40 + j30\ \Omega}$$

  - Next compute the a-phase transmission line current

$$\mathbf{I}_{aA} = \mathbf{V}_{a'n}/Z_\phi = [120e^{j0^\circ}V(rms)]/(40 + j30\Omega) \rightarrow \boxed{\mathbf{I}_{aA} \approx 2.4e^{-j36.89^\circ}\mathrm{A}(rms)}$$

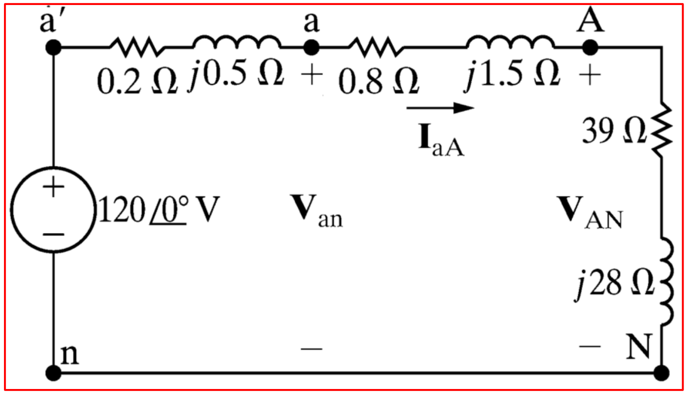
  - Finally, compute the b- and c-phase line currents (a-b-c) sequence

$$\mathbf{I}_{bB} = \left(e^{-j120^\circ}\right)\mathbf{I}_{aA} \rightarrow \boxed{\mathbf{I}_{bB} \approx 2.4e^{-j156.89^\circ}\ \mathrm{A}(rms)}$$

$$\mathbf{I}_{cC} = \left(e^{+j120^\circ}\right)\mathbf{I}_{aA} \rightarrow \boxed{\mathbf{I}_{cC} \approx 2.4e^{+j83.11^\circ}\ \mathrm{A}(rms)}$$

Example #1 (SOLUTION cont'd)

- Calculate the phase voltages at the load
  - First, compute the A-phase load voltage



$$\mathbf{V}_{AN} = I_{aA}Z_Y \approx \left[2.4e^{-j36.89^\circ}\mathrm{A}_{(rms)}\right](39 + j28\Omega)$$

$$\boxed{\mathbf{V}_{AN} \approx 115.23e^{-j1.19^\circ}\mathrm{V(rms)}}$$

  - Now, compute B- and C-phase load voltages

$$\mathbf{V}_{BN} = \left(e^{-j120^\circ}\right)\mathbf{V}_{AN} \rightarrow \boxed{\mathbf{V}_{BN} \approx 115.23e^{-j121.19^\circ}\ \mathrm{V}(rms)}$$

$$\mathbf{V}_{CN} = \left(e^{+j120^\circ}\right)\mathbf{V}_{AN} \rightarrow \boxed{\mathbf{V}_{CN} \approx 115.23e^{+j118.81^\circ}\ \mathrm{V}(rms)}$$

- Calculate the line-to-line voltages at the load's terminals (A-B-C sequence)
  - Use factor $\left(\sqrt{3}e^{j30^\circ}\right)$ to compute the A-to-B line-to-line load voltage

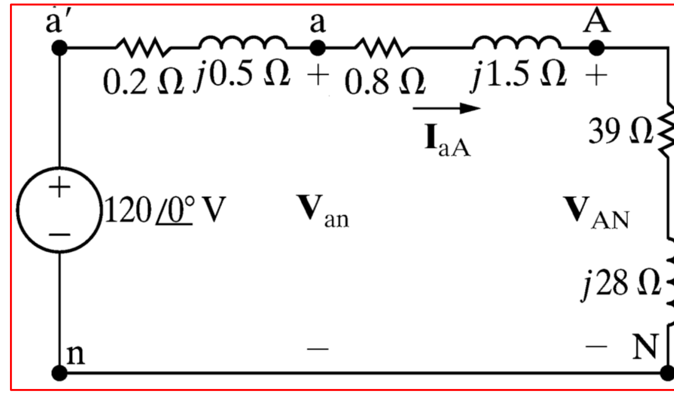
$$\mathbf{V}_{AB} = \left(\sqrt{3}e^{j30^\circ}\right)\mathbf{V}_{AN} \rightarrow \boxed{\mathbf{V}_{AB} \approx 199.58e^{j28.81^\circ}\ \mathrm{V}(rms)}$$

  - Next, compute the B-C and C-A load voltages (A-B-C sequence)

$$\mathbf{V}_{BC} = \left(e^{-j120^\circ}\right)\mathbf{V}_{AB} \rightarrow \boxed{\mathbf{V}_{BC} \approx 199.58e^{-j91.19^\circ}\ \mathrm{V}(rms)}$$

$$\mathbf{V}_{CA} = \left(e^{+j120^\circ}\right)\mathbf{V}_{AB} \rightarrow \boxed{\mathbf{V}_{CA} \approx 199.58e^{+j148.81^\circ}\ \mathrm{V}(rms)}$$

Example #1 (SOLUTION cont'd)



- Calculate phase voltages at source's terminals
  - First, compute the a-phase voltage at the source terminal

$$\mathbf{V}_{an} = \mathbf{V}_{a'n} - \mathbf{I}_{aA} Z_S$$

$$\mathbf{V}_{an} \approx 120e^{j0^\circ}\mathrm{V} - \left[2.4e^{-j36.89^\circ}\mathrm{A}\right](0.2 + j0.5\Omega)$$

$$\boxed{\mathbf{V}_{an} \approx 118.90e^{-j0.32^\circ}\mathrm{V}(rms)}$$

  - Now, compute b- and c-phase voltages at source's terminals (a-b-c seq.)

$$\mathbf{V}_{bn} = \left(e^{-j120^\circ}\right)\mathbf{V}_{an} \rightarrow \boxed{\mathbf{V}_{bn} \approx 118.90e^{-j120.32^\circ}\ \mathrm{V}(rms)}$$

$$\mathbf{V}_{cn} = \left(e^{+j120^\circ}\right)\mathbf{V}_{an} \rightarrow \boxed{\mathbf{V}_{cn} \approx 118.90e^{j119.68^\circ}\ \mathrm{V}(rms)}$$

- Calculate the line-to-line voltages and at the source's terminals
  - First compute a-b line source terminal voltage (a-b-c sequence)

$$\mathbf{V}_{ab} = \left(\sqrt{3}e^{j30^\circ}\right)\mathbf{V}_{an} \rightarrow \boxed{\mathbf{V}_{ab} \approx 205.94e^{j29.68^\circ}\ \mathrm{V}(rms)}$$

  - Next, compute the b-c and c-a load voltages (a-b-c sequence)

$$\mathbf{V}_{bc} = \left(e^{-j120^\circ}\right)\mathbf{V}_{ab} \rightarrow \boxed{\mathbf{V}_{bc} \approx 205.94e^{-j90.32^\circ}\ \mathrm{V}(rms)}$$

$$\mathbf{V}_{ca} = \left(e^{+j120^\circ}\right)\mathbf{V}_{ab} \rightarrow \boxed{\mathbf{V}_{ca} \approx 205.94e^{+j149.68^\circ}\ \mathrm{V}(rms)}$$

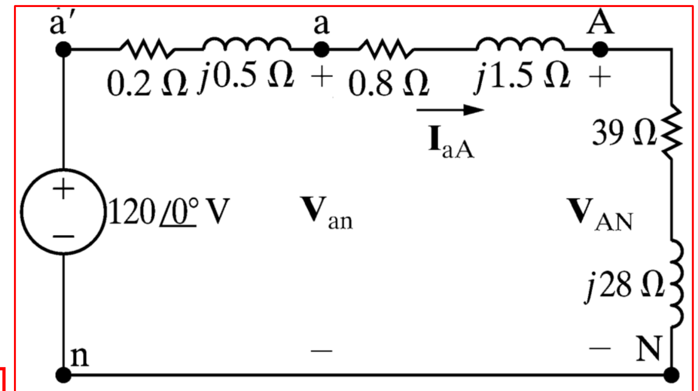
Example #1 (SOLUTION cont'd)

- Calculate the total complex power, average power, and reactive power absorbed by the load
  - First, calculate load's a-phase complex power

$$\mathbf{S}_{AN} = \mathbf{V}_{AN}\mathbf{I}_{\mathrm{aA}}^{*} = |\mathbf{I}_{\mathrm{aA}}^{*}|^2 Z_Y = |\mathbf{V}_{AN}|^2/Z_Y^{*}$$

$$\mathbf{S}_{AN} = \left(115.23e^{-j1.19^\circ}\mathrm{V_{rms}}\right)\left(2.4e^{-j36.89^\circ}\mathrm{A_{rms}}\right)^{*}$$

$$\boxed{\mathbf{S}_{AN} = 224.58 + j161.38\ \mathrm{VA} = 276.55e^{j35.7^\circ}\mathrm{VA}}$$



  - Next, compute the total complex power absorbed by the load

$$\mathbf{S}_{tot,load} = 3\boldsymbol{S}_{AN} = 3(224.58 + j161.38\ \mathrm{VA}) = 3\left(276.55e^{j35.7^\circ}\ \mathrm{VA}\right)$$

$$\boxed{\mathbf{S}_{tot,load} = 673.75 + j484.14\ \mathrm{VA} = 829.66e^{j35.7^\circ}\ \mathrm{VA}}$$

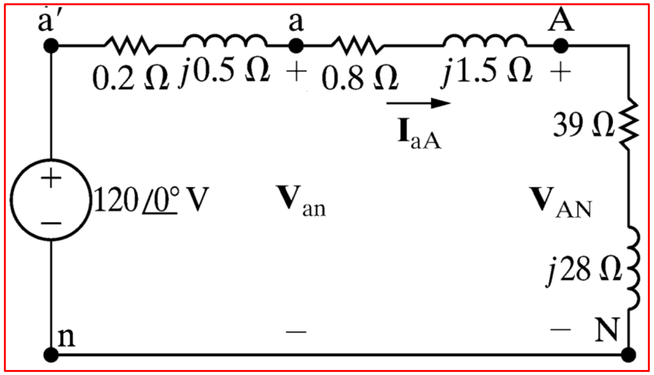
  - Finally, compute the total average and reactive power of the load

$$P_{tot,load} = \boldsymbol{Re}\left(\mathbf{S}_{tot,load}\right) \rightarrow \boxed{P_{tot,load} = 673.75\mathrm{W}}$$

$$Q_{tot,load} = \boldsymbol{Im}\left(\mathbf{S}_{tot,load}\right) \rightarrow \boxed{Q_{tot,load} = 484.14\ \mathrm{VARs}}$$

Example #1 (SOLUTION cont'd)

- Calculate the total complex power, average power, and reactive power lost in the three transmission lines
  - First, calculate a-phase line's complex power



$$\mathbf{S}_{aA} = (\mathbf{V}_{an} - \mathbf{V}_{AN})\mathbf{I}_{\mathrm{aA}}^{*} = |\mathbf{I}_{\mathrm{aA}}^{*}|^2 Z_\ell = \frac{|\mathbf{V}_{an} - \mathbf{V}_{AN}|^2}{Z_\ell^*}$$

$$\mathbf{S}_{aA} = \left|2.4e^{-j36.89°}\mathrm{A_{rms}}\right|^2 (0.8 + j1.5\ \Omega)$$

$$\boxed{\mathbf{S}_{aA} = 4.61 + j8.64\mathrm{VA} = 9.79e^{j61.93°}\mathrm{VA}}$$

  - Next, compute the total complex power lost in the three lines

$$\mathbf{S}_{tot,line} = 3\boldsymbol{S}_{aA} = 3(4.61 + j8.64\mathrm{VA}) = 3\left(9.79e^{j61.93°}\mathrm{VA}\right)$$

$$\boxed{\mathbf{S}_{tot,line} = 13.82 + j25.92\ \mathrm{VA} = 29.38e^{j61.93°}\ \mathrm{VA}}$$

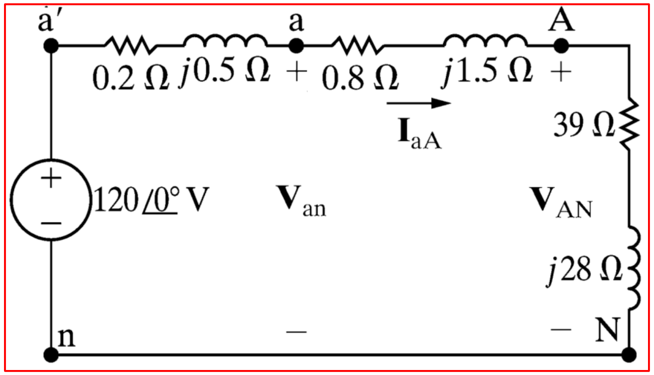
  - Finally, compute the total average and reactive power of the lines

$$P_{tot,line} = \boldsymbol{Re}\left(\mathbf{S}_{tot,line}\right) \rightarrow \boxed{P_{tot,line} = 13.82\ \mathrm{W}}$$

$$Q_{tot,line} = \boldsymbol{Im}\left(\mathbf{S}_{tot,line}\right) \rightarrow \boxed{Q_{tot,line} = 25.92\ \mathrm{VARs}}$$

Example #1 (SOLUTION cont'd)

- Calculate the total complex power, average power, and reactive power lost in source
  - First, calculate a-phase line's complex power



$$\mathbf{S}_{a'a} = (\mathbf{V}_{a'n} - \mathbf{V}_{an})\mathbf{I}_{aA}^{*} = |\mathbf{I}_{aA}^{*}|^2 Z_s = \frac{|\mathbf{V}_{a'n} - \mathbf{V}_{an}|^2}{Z_s^{*}}$$

$$\mathbf{S}_{a'a} = \left|2.4e^{-j36.89^\circ}\mathrm{A_{rms}}\right|^2 (0.2 + j0.5\ \Omega)$$

$$\boxed{\mathbf{S}_{a'a} = 1.15 + j2.88\mathrm{VA} = 3.10e^{j68.20^\circ}\mathrm{VA}}$$

  - Next, compute the total complex power lost in the source

$$\mathbf{S}_{tot,source} = 3\mathbf{S}_{a'a} = 3(1.15 + j2.88\mathrm{VA}) = 3\left(3.10e^{j68.20^\circ}\mathrm{VA}\right)$$

$$\boxed{\mathbf{S}_{tot,source} = 3.46 + j8.64\ \mathrm{VA} = 9.31e^{j68.20^\circ}\ \mathrm{VA}}$$

  - Finally, compute the total average and reactive power lost in the source

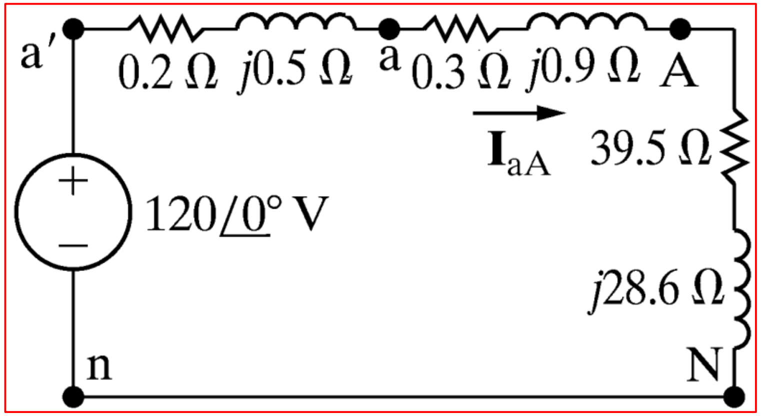
$$P_{tot,source} = \boldsymbol{Re}\left(\mathbf{S}_{tot,source}\right) \to \boxed{P_{tot,source} = 3.46\ \mathrm{W}}$$

$$Q_{tot,source} = \boldsymbol{Im}\left(\mathbf{S}_{tot,source}\right) \to \boxed{Q_{tot,source} = 8.64\ \mathrm{VARs}}$$

Example #2

- A balanced 3-phase Y source with a-b-c sequence has an impedance of $Z_s = 0.2 + j0.5\ \Omega/\phi$ and a voltage of $\mathbf{V}_{a'n} = 120e^{j0^\circ}$ V (rms). The source drives a balanced 3-phase Δ load with an impedance of $Z_\Delta = 118.5 + j85.8\ \Omega/\phi$. The line impedance connecting source to the load is $Z_\ell = 0.3 + j0.9\ \Omega/\phi$. Use a-phase internal source voltage as the reference.
  - Draw the a-phase Y-Y equivalent network of the Y-Δ system
  - Calculate the line currents for each phase
  - Calculate the phase voltages at the load terminals
  - Calculate the phase currents of the load
  - Calculate the line-to-line voltages at the source's terminals
  - Calculate the <u>total</u> complex power, average power, and reactive power <u>absorbed</u> by the source
  - Calculate the <u>total</u> apparent power of the load
  - Calculate the power factor of the load

Example #2 (SOLUTION)



- Draw the a-phase Y-Y equivalent network of the Y-Δ system
  - Transform Z_Δ load to equivalent Z_Y load

    $Z_Y = Z_\Delta/3 = (118.5 + j85.8\Omega)/3$

    $\boxed{Z_Y = 39.5 + j28.6\ \Omega}$

- Calculate line currents for each phase
  - First compute the a-phase equivalent impedance Z_ϕ

    $Z_\phi = Z_s + Z_\ell + Z_Y$

    $Z_\phi = (0.2 + 0.3 + 39.5) + j(0.5 + 0.9 + 28.6) \rightarrow \boxed{Z_\phi = 40 + j30\ \Omega}$
  - Next compute the a-phase transmission line current

    $\mathbf{I}_{aA} = \mathbf{V}_{a'n}/Z_\phi = [120e^{j0^\circ}V(rms)]/(40 + j30\Omega) \rightarrow \boxed{\mathbf{I}_{aA} \approx 2.4e^{-j36.89^\circ}\mathrm{A}(rms)}$
  - Finally, compute the b- and c-phase line currents (a-b-c) sequence

    $\mathbf{I}_{bB} = \left(e^{-j120^\circ}\right)\mathbf{I}_{aA} \rightarrow \boxed{\mathbf{I}_{bB} \approx 2.4e^{-j156.89^\circ}\ \mathrm{A}(rms)}$

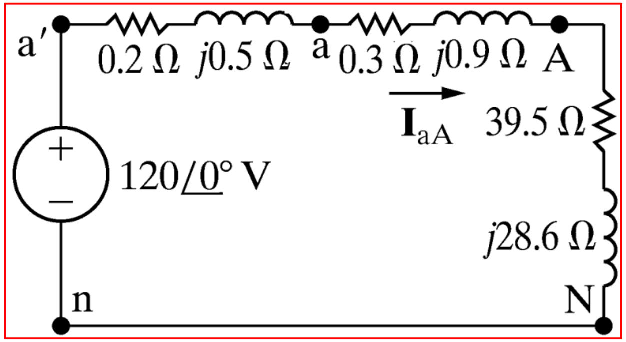
    $\mathbf{I}_{cC} = \left(e^{+j120^\circ}\right)\mathbf{I}_{aA} \rightarrow \boxed{\mathbf{I}_{cC} \approx 2.4e^{+j83.11^\circ}\ \mathrm{A}(rms)}$

Example #2 (SOLUTION cont'd)

- Calculate the phase voltages at the load
  - First, compute the A-phase load voltage

$$\mathbf{V}_{AN} = I_{aA}Z_Y \approx \left[2.4e^{-j36.89^\circ}\mathrm{A}_{(rms)}\right](39.5 + j28.6\Omega)$$

$$\boxed{\mathbf{V}_{AN} \approx 117.04e^{-j0.96^\circ}\mathrm{V}(rms)}$$



  - Now, compute B- and C-phase load voltages

$$\mathbf{V}_{BN} = \left(e^{-j120^\circ}\right)\mathbf{V}_{AN} \rightarrow \boxed{\mathbf{V}_{BN} \approx 117.04e^{-j120.96^\circ}\ \mathrm{V}(rms)}$$

$$\mathbf{V}_{CN} = \left(e^{+j120^\circ}\right)\mathbf{V}_{AN} \rightarrow \boxed{\mathbf{V}_{CN} \approx 117.04e^{+j119.04^\circ}\ \mathrm{V}(rms)}$$

- Calculate the line-to-line voltages at the load's terminals
  - Use factor $\left(\sqrt{3}e^{j30^\circ}\right)$ to compute the A-B load voltage (A-B-C sequence)

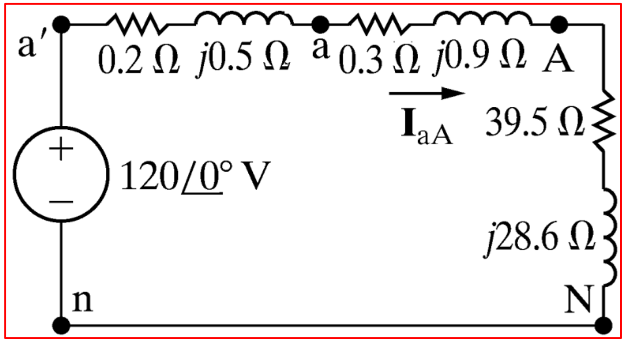
$$\mathbf{V}_{AB} = \left(\sqrt{3}e^{j30^\circ}\right)\mathbf{V}_{AN} \rightarrow \boxed{\mathbf{V}_{AB} \approx 202.72e^{j29.04^\circ}\ \mathrm{V}(rms)}$$

  - Next, compute the B-C and C-A load voltages (A-B-C sequence)

$$\mathbf{V}_{BC} = \left(e^{-j120^\circ}\right)\mathbf{V}_{AB} \rightarrow \boxed{\mathbf{V}_{BC} \approx 202.72e^{-j90.96^\circ}\ \mathrm{V}(rms)}$$

$$\mathbf{V}_{CA} = \left(e^{+j120^\circ}\right)\mathbf{V}_{AB} \rightarrow \boxed{\mathbf{V}_{CA} \approx 202.72e^{+j149.04^\circ}\ \mathrm{V}(rms)}$$

Example #2 (SOLUTION cont'd)



- Calculate phase currents of the load
  - Compute the a-phase current at load terminals

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{Z_{\Delta}} = (202.72e^{j29.04^{\circ}}\text{V})/(118.5 + j85.8\Omega)$$

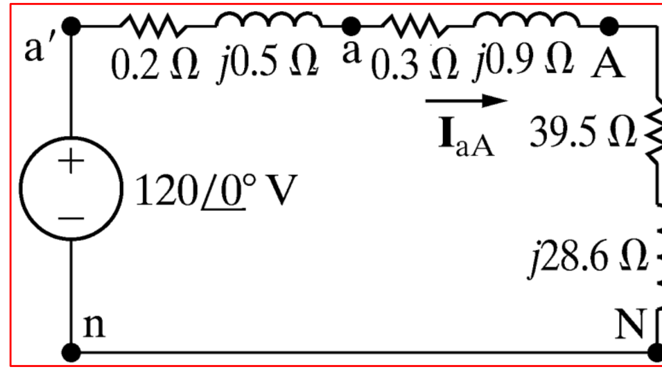
$$\boxed{\mathbf{I}_{AB} \approx 1.39e^{-j6.87^{\circ}}\text{A}(rms)}$$

  - Now, compute B- and C-phase load currents (a-b-c seq.)

$$\mathbf{I}_{BC} = \left(e^{-j120^{\circ}}\right)\mathbf{I}_{AB} \rightarrow \boxed{\mathbf{I}_{BC} \approx 1.39e^{-j126.87^{\circ}}\text{A}(rms)}$$

$$\mathbf{I}_{CA} = \left(e^{-j120^{\circ}}\right)\mathbf{I}_{CA} \rightarrow \boxed{\mathbf{I}_{CA} \approx 1.39e^{j113.13^{\circ}}\text{A}(rms)}$$

Example #2 (SOLUTION cont'd)



- Calculate the line-to-line voltages at the source's terminals
  - First, compute the a-phase voltage at the source's terminals

$$\mathbf{V}_{an} = \mathbf{V}_{a'n} - \mathbf{I}_{aA}Z_S$$
$$\mathbf{V}_{an} \approx 120e^{j0^\circ}\mathrm{V} - \left[2.4e^{-j36.89^\circ}\mathrm{A}\right](0.2 + j0.5\Omega)$$
$$\boxed{\mathbf{V}_{an} \approx 118.90e^{-j0.32^\circ}\mathrm{V}(rms)}$$

  - Next, compute a-b line source terminal voltage

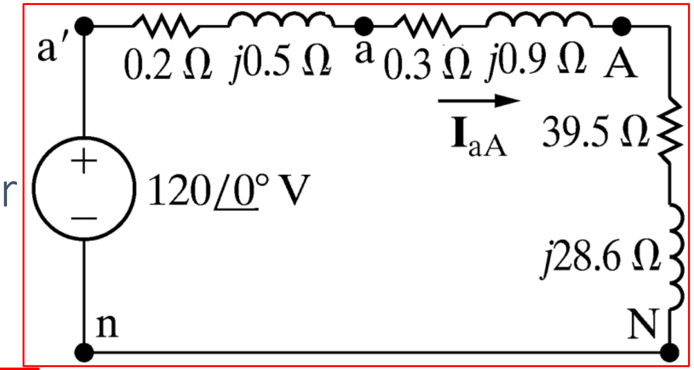
$$\mathbf{V}_{ab} = \left(\sqrt{3}e^{j30^\circ}\right)\mathbf{V}_{AN} \approx \left(\sqrt{3}e^{j30^\circ}\right)\left(118.90e^{-j0.32^\circ}\mathrm{V}\right)$$
$$\boxed{\mathbf{V}_{ab} \approx 205.94e^{j29.68^\circ}\ \mathrm{V}(rms)}$$

  - Finally, compute the b-c and c-a load voltages (a-b-c sequence)

$$\mathbf{V}_{bc} = \left(e^{-j120^\circ}\right)\mathbf{V}_{ab} \rightarrow \boxed{\mathbf{V}_{bc} \approx 205.94e^{-j90.32^\circ}\ \mathrm{V}(rms)}$$
$$\mathbf{V}_{ca} = \left(e^{+j120^\circ}\right)\mathbf{V}_{ab} \rightarrow \boxed{\mathbf{V}_{ca} \approx 205.94e^{+j149.68^\circ}\ \mathrm{V}(rms)}$$

Example #2 (SOLUTION cont'd)

- Calculate total complex power, average power, and reactive power absorbed by the source
  - First, calculate source's a-phase complex power



$$\mathbf{S}_{a'n} = -\mathbf{V}_{a'n}\mathbf{I}_{aA}^* = -|\mathbf{I}_{aA}^*|^2 Z_\phi = -\left|\mathbf{V}_{a'n}\right|^2 / Z_\phi^*$$

$$\mathbf{S}_{a'n} = -\left(120e^{j0°}\mathrm{V_{rms}}\right)\left(2.4e^{-j36.89°}\mathrm{A_{rms}}\right)^*$$

$$\boxed{\mathbf{S}_{a'n} = -230.34 - j172.81\text{ VA} = 288e^{-j143.11°}\text{VA}}$$

  - Next, compute the total complex power absorbed by the source

$$\mathbf{S}_{tot,source} = 3\mathbf{S}_{a'n} = 3(-230.34 - j172.81\text{ VA}) = 3\left(288e^{-j143.11°}\text{VA}\right)$$

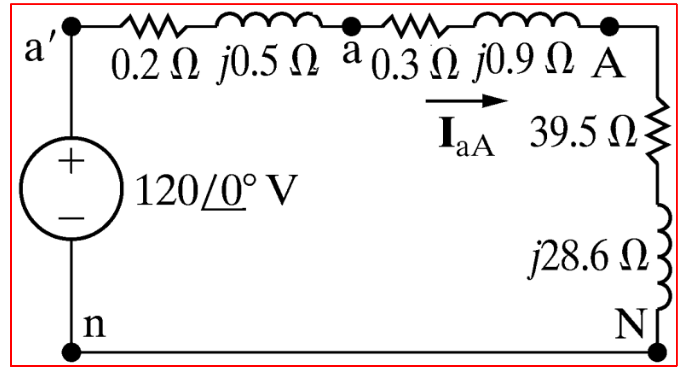
$$\boxed{\mathbf{S}_{tot,source} = -691.02 - j518.64\text{ VA} = 864e^{-j143.11°}\text{ VA}}$$

  - Finally, compute the total average and reactive power absorbed by the source

$$P_{tot,source} = \boldsymbol{Re}\left(\mathbf{S}_{tot,source}\right) \rightarrow \boxed{P_{tot,source} = -691.02\text{ W absorb}}$$

$$Q_{tot,source} = \boldsymbol{Im}\left(\mathbf{S}_{tot,source}\right) \rightarrow \boxed{Q_{tot,source} = -518.64\text{ VARs absorb}}$$

Example #2 (SOLUTION cont'd)



- Calculate the total apparent power of the load
  - First, calculate A-phase complex power of load

$$\mathbf{S}_{AB} = \mathbf{V}_{AB}\mathbf{I}_{AB}^{*} = |\mathbf{V}_{AB}|^2/Z_{\Delta}^{*}$$

$$\mathbf{S}_{AB} = \left|202.72e^{j29.04^\circ}\ \mathrm{V}_{rms}\right|^2/(118.5 + j85.8\ \Omega)^{*}$$

$$\boxed{\mathbf{S}_{AB} = 227.52 + j164.74\ \mathrm{VA} = 280.90e^{j35.91^\circ}\mathrm{VA}}$$

  - Next, compute the total complex power absorbed by the load

$$\mathbf{S}_{tot,load} = 3\mathbf{S}_{AB} = 3(227.52 + j164.74\ \mathrm{VA}) = 3\left(280.90e^{j35.91^\circ}\mathrm{VA}\right)$$

$$\boxed{\mathbf{S}_{tot,load} = 682.56 + j494.21\ \mathrm{VA} = 842.69e^{j35.91^\circ}\ \mathrm{VA}}$$

  - Finally, compute the total apparent power of the load

$$S_{tot,load} = |\boldsymbol{S}_{tot,load}| \rightarrow \boxed{S_{tot,load} = 842.69\ \mathrm{VA}}$$

- Calculate the power factor of the load

$$pf_{load} = \cos(\theta_s) = \cos(\theta_{Z_\Delta}) = \frac{P_{tot,load}}{|\boldsymbol{S}_{tot,load}|}$$

$$pf_{load} = \cos(35.91^\circ) = \frac{682.56\ \mathrm{W}}{842.69\ \mathrm{VA}} \rightarrow \boxed{pf_{load} = 0.81\ lagging}$$

Example #3

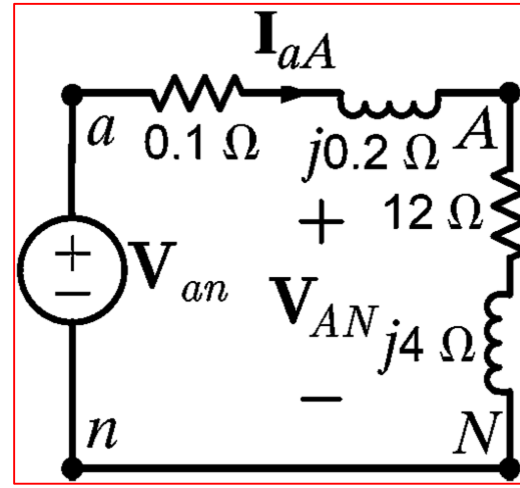
- A balanced 3-phase Δ source with a-b-c sequence has a negligible internal impedance and an a-phase terminal voltage of $\mathbf{V}_{ab} = 208e^{j0^\circ}$ V (rms). The source drives a balanced 3-phase Y load with an impedance of $Z_{\mathrm{Y}} = 12 + j4\ \Omega/\phi$. The line impedance connecting source and load is $Z_\ell = 0.1 + j0.2\ \Omega/\phi$.
  - Draw the single-phase equivalent network of the 3-phase network
  - Calculate the transmission line currents for each phase
  - Calculate the phase voltages at the load terminals
  - Calculate the phase currents of the load
  - Calculate the line-to-line currents for the source
  - Calculate the <u>total</u> complex power, average power, and reactive power <u>delivered</u> by the source
  - Calculate the <u>total</u> apparent power <u>absorbed</u> of the load

Example #3 (SOLUTION)

- Draw single-phase equivalent network of the system
  - Transform the Δ source of $\mathbf{V}_{ab} = 208e^{j0°}$ V to equivalent Y source

$$\mathbf{V}_{ab} = \left(\sqrt{3}e^{j30°}\right)\mathbf{V}_{an} \rightarrow \mathbf{V}_{an} = \mathbf{V}_{ab}/\left(\sqrt{3}e^{j30°}\right)$$

$$\mathbf{V}_{an} = \frac{208e^{j0°}\text{ V}}{\left(\sqrt{3}e^{j30°}\right)} \rightarrow \boxed{\mathbf{V}_{\text{an}} = 120.09e^{-j30°}\text{V (rms)}}$$



- Calculate the transmission line currents

$$\mathbf{I}_{aA} = \mathbf{V}_{an}/Z_{\phi} = (120.09e^{-j30°}\text{V (rms)})/(12.1\Omega + j4.2\Omega)$$

$$\boxed{I_{aA} = 9.38e^{-j49.14°}\text{A (rms)}}$$

$$\mathbf{I}_{bB} = \left(e^{-j120°}\right)\boldsymbol{I}_{aA} = \left(e^{-j120°}\right)\left(9.38e^{-j49.14°}\text{A (rms)}\right)$$

$$\boxed{I_{bB} = 9.38e^{-j169.14°}\text{A (rms)}}$$

$$\mathbf{I}_{cC} = \left(e^{+j120°}\right)\boldsymbol{I}_{aA} = \left(e^{+j120°}\right)\left(9.38e^{-j49.14°}\text{A (rms)}\right)$$

$$\boxed{I_{cC} = 9.38e^{j70.86°}\text{A (rms)}}$$

Example #3 (SOLUTION cont'd)

- Calculate the phase voltages at the load terminals

$$\mathbf{V}_{AN} = \mathbf{I}_{aA} Z_Y = \left(9.38 e^{-j49.14°}\,\mathrm{A}\right)(12\Omega + j4\Omega)$$

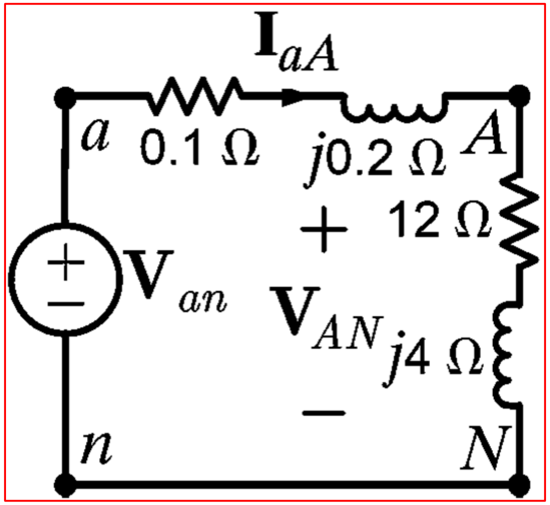
$$\boxed{\mathbf{V}_{AN} = 118.65 e^{-j30.71°}\,\mathrm{V\ (rms)}}$$

$$\mathbf{V}_{BN} = \left(e^{-j120°}\right)\mathbf{V}_{AN} = \left(e^{-j120°}\right)\left(118.65 e^{-j30.71°}\,\mathrm{V}\right)$$

$$\boxed{\mathbf{V}_{BN} = 118.65 e^{-j150.71°}\,\mathrm{V\ (rms)}}$$

$$\mathbf{V}_{CN} = \left(e^{-j120°}\right)\mathbf{V}_{AN} = \left(e^{j120°}\right)\left(118.65 e^{-j30.71°}\,\mathrm{V}\right)$$

$$\boxed{\mathbf{V}_{CN} = 118.65 e^{j89.29°}\,\mathrm{V\ (rms)}}$$



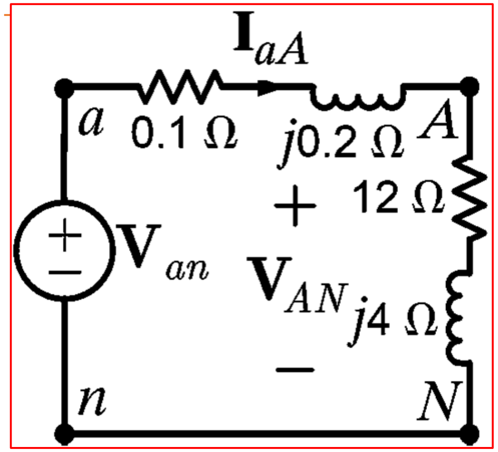
- Calculate the phase currents of the load

$$\mathbf{I}_{AN} = \mathbf{I}_{aA} \rightarrow \boxed{I_{AN} = 9.38 e^{-j49.14°}\,\mathrm{A\ (rms)}}$$

$$\mathbf{I}_{BN} = \mathbf{I}_{bB} \rightarrow \boxed{I_{BN} = 9.38 e^{-j169.14°}\,\mathrm{A\ (rms)}}$$

$$\mathbf{I}_{CN} = \mathbf{I}_{cC} \rightarrow \boxed{I_{CN} = 9.38 e^{j70.86°}\,\mathrm{A\ (rms)}}$$

Example #3 (SOLUTION cont'd)



- Calculate the line-to-line voltage at the load

$$\mathbf{V}_{AB} = \left(\sqrt{3}e^{j30^\circ}\right)\mathbf{V}_{AN} = \left(\sqrt{3}e^{j30^\circ}\right)\left(118.65e^{-j30.71^\circ}\mathrm{V}\right)$$

$$\boxed{\mathbf{V}_{AB} = 205.51e^{-j0.71^\circ}\ \text{(rms)}}$$

$$\mathbf{V}_{BC} = \left(e^{-j120^\circ}\right)\mathbf{V}_{AN} \rightarrow \boxed{\mathbf{V}_{BC} = 205.51e^{-j120.71^\circ}\ \mathrm{A}\ \text{(rms)}}$$

$$\mathbf{V}_{CA} = \left(e^{j120^\circ}\right)\mathbf{V}_{AN} \rightarrow \boxed{\mathbf{V}_{BC} = 205.51e^{j119.25^\circ}\ \mathrm{A}\ \text{(rms)}}$$

- Calculate the line-to-line (phase) currents for the source

$$\mathbf{I}_{aA} = \mathbf{I}_{ba} - \mathbf{I}_{ac} = \left(1 - e^{j120^\circ}\right)\mathbf{I}_{ba} \rightarrow \mathbf{I}_{aA} = \left(\sqrt{3}e^{-j30^\circ}\right)\mathbf{I}_{ba}$$

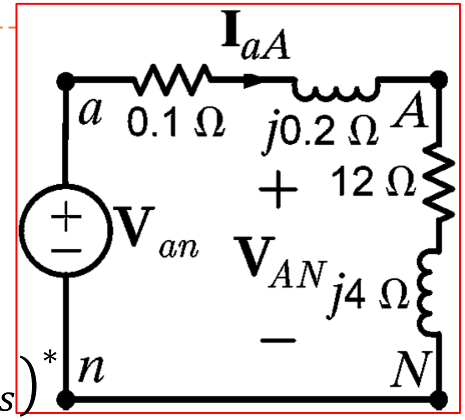
$$\mathbf{I}_{ba} = \mathbf{I}_{aA}/\left(\sqrt{3}e^{-j30^\circ}\right) = \left(9.38e^{-j49.14^\circ}\mathrm{A}\right)/\left(\sqrt{3}e^{-j30^\circ}\right)$$

$$\boxed{\mathbf{I}_{ba} = 5.42e^{-j19.14^\circ}\mathrm{A}\ \text{(rms)}}$$

$$\mathbf{I}_{cb} = \left(e^{-j120^\circ}\right)\mathbf{I}_{ba} =\rightarrow \boxed{\mathbf{I}_{cb} = 5.42e^{-j139.14^\circ}\ \mathrm{A}\ \text{(rms)}}$$

$$\mathbf{I}_{ac} = \left(e^{-j120^\circ}\right)\mathbf{I}_{ba} \rightarrow \boxed{\mathbf{I}_{ac} = 5.42e^{j100.86^\circ}\ \mathrm{A}\ \text{(rms)}}$$

Example #3 (SOLUTION cont'd)



- Calculate the total complex power, average power, and reactive power delivered by the source
  - First, compute total complex power supplied by source

$$\boldsymbol{S}_{\boldsymbol{tot,src}} = 3\boldsymbol{S}_{\boldsymbol{ab}} = (+)3\mathbf{V}_{ab}\mathbf{I}_{ba}^{*}$$

$$\boldsymbol{S}_{\boldsymbol{tot,src}} = 3\mathbf{V}_{ab}\mathrm{I}_{ba}^{*} = (3)\left(208e^{j0^\circ}\mathrm{V}_{rms}\right)\left(5.42e^{-j19.14^\circ}A_{rms}\right)^{*}$$

$$\boxed{\boldsymbol{S}_{\boldsymbol{tot,src}} = 3382.08e^{j19.14^\circ}\text{VA supplied} = 3195.12 + j1108.91\text{VA supplied}}$$

  - Next, compute total average and reactive power delivered by source

$$P_{tot,src} = \mathbf{Re}\left(\boldsymbol{S}_{\boldsymbol{tot,src}}\right) \rightarrow \boxed{P_{tot,src} = 3195.12\ \mathrm{W}}$$

$$Q_{tot,src} = \mathbf{Im}\left(\boldsymbol{S}_{\boldsymbol{tot,src}}\right) \rightarrow \boxed{Q_{tot,src} = 1108.91\ \mathrm{VARs}}$$

- Calculate the total apparent power absorbed of the load

$$S_{tot,AN} = 3|\boldsymbol{S}_{\boldsymbol{AN}}| = (+)3|\mathbf{V}_{AN}\mathbf{I}_{AN}^{*}|$$

$$S_{tot,AN} = (3)\left|\left(118.65e^{-j30.71^\circ}\mathrm{V}_{\mathrm{rms}}\right)\left(9.38e^{-j49.14^\circ}\mathrm{A}_{\mathrm{rms}}\right)^{*}\right|$$

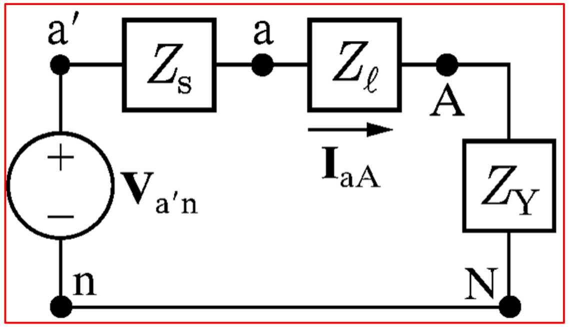
$$\boxed{S_{tot,AN} = 3338.82\ \mathrm{VA}}$$

Example #4

- A balanced 3-phase Δ-source with a-b-c sequence has a source impedance of $Z_S = 0.1 + j0.2\ \Omega/\phi$ and an ideal voltage of $\mathbf{V}_{a'b} = 240e^{j10^\circ}$ V (rms). The source drives a balanced Δ-load with impedance $Z_\Delta = 30 - j20\ \Omega/\phi$. The line impedance connecting the source and load is $Z_\ell = 0.6 + j0.5\ \Omega/\phi$.
  - Draw the single-phase equivalent network of the system
  - Calculate the transmission line currents for each phase
  - Calculate the phase voltages at the load terminals
  - Calculate the phase currents of the load
  - Calculate the line-to-line currents through the source
  - Calculate the line-to-line voltages at the source terminals

Example #4 (SOLUTION)

- Draw single-phase equivalent network of the system



  - Transform the Δ source of $\mathbf{V}_{a'b} = 240e^{j10°}$ V (rms) to equivalent Y internal source voltage $\mathbf{V}_{a'n}$

$$\mathbf{V}_{a'b} = \left(\sqrt{3}e^{j30°}\right)\mathbf{V}_{an} \rightarrow \mathbf{V}_{a'n} = \mathbf{V}_{a'b}/\left(\sqrt{3}e^{j30°}\right)$$

$$\mathbf{V}_{a'n} = \frac{240e^{j10°}\text{ V}}{\left(\sqrt{3}e^{j30°}\right)} \rightarrow \boxed{\mathbf{V}_{a'n} = 80\sqrt{3}e^{-j20°}\text{V (rms)}}$$

  - Transform the Δ source impedance of $Z_{s\Delta} = 0.1\Omega + j0.2\Omega$ to an equivalent Y source impedance $Z_{s\text{Y}}$

$$Z_{s\text{Y}} = Z_{s\Delta}/3 = (0.1\Omega + j0.2\Omega)/3 \rightarrow \boxed{Z_{s\text{Y}} = 33.3\text{ m}\Omega + j66.7\text{ m}\Omega}$$

  - Transform the Δ load impedance of $Z_\Delta = 30\Omega - j20\Omega$ to an equivalent Y load impedance Z_Y

$$Z_\text{Y} = Z_\Delta/3 = (30\Omega - j20\Omega)/3 \rightarrow \boxed{Z_\text{Y} = 10\Omega - j6.7\Omega}$$

  - Note, the line impedance remains unchanged

Example #4 (SOLUTION cont'd)

- Compute the transmission line currents

$$\mathbf{I}_{\mathrm{aA}} = \mathbf{V}_{a'n}/Z_{\phi} = \mathbf{V}_{a'n}/(Z_{s\mathrm{Y}} + Z_{l} + Z_{\mathrm{Y}})$$

$$\mathbf{I}_{\mathrm{aA}} = \left[80\sqrt{3}e^{-j20^{\circ}}\mathrm{V\ (rms)}\right]/(10.63\Omega - j6.1\Omega)$$

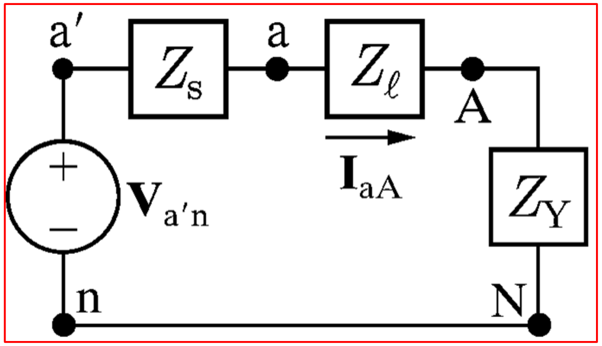
$$\boxed{\mathbf{I}_{\mathrm{aA}} = 11.31e^{j9.85^{\circ}}\ \mathrm{A\ (rms)}}$$

$$\mathbf{I}_{bB} = \left(e^{-j120^{\circ}}\right)\boldsymbol{I}_{aA} = \left(e^{-j120^{\circ}}\right)[11.31e^{j9.85^{\circ}}\ \mathrm{A\ (rms)}]$$

$$\boxed{\mathbf{I}_{bB} = 11.31e^{-j110.15^{\circ}}\mathrm{A\ (rms)}}$$

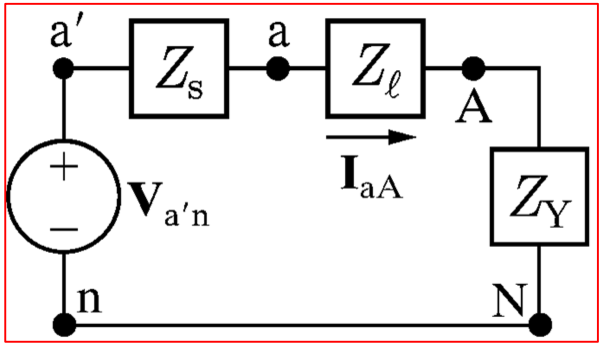
$$\mathbf{I}_{cC} = \left(e^{+j120^{\circ}}\right)\boldsymbol{I}_{aA} = \left(e^{+j120^{\circ}}\right)[11.31e^{j9.85^{\circ}}\ \mathrm{A\ (rms)}]$$

$$\boxed{\mathbf{I}_{cC} = 11.31e^{j29.85^{\circ}}\mathrm{A\ (rms)}}$$



Example #4 (SOLUTION cont'd)

- Calculate the phase voltages at the load terminals
  - First, compute the terminal voltage of the equivalent Y load



$$\mathbf{V}_{AN} = \mathbf{I}_{aA}\mathrm{Z}_{\mathrm{Y}} = \left[11.31e^{j9.85^\circ}\ \mathrm{A}\right]\left[10\Omega - j6.7\Omega\right]$$

$$\boxed{\mathbf{V}_{AN} = 136.14e^{-j23.97^\circ}\ \mathrm{V\ (rms)}}$$

$$\mathbf{V}_{BN} = \left(e^{-j120^\circ}\right)\mathbf{V}_{AN} \rightarrow \boxed{\mathbf{V}_{BN} = 136.14e^{-j143.97^\circ}\ \mathrm{V\ (rms)}}$$

$$\mathbf{V}_{CN} = \left(e^{+j120^\circ}\right)\mathbf{V}_{AN} \rightarrow \boxed{\mathbf{V}_{CN} = 136.14e^{j96.03^\circ}\ \mathrm{V\ (rms)}}$$

  - Now, compute the terminal voltage of the actual delta load

$$\mathbf{V}_{AB} = \left(\sqrt{3}e^{j30^\circ}\right)\mathbf{V}_{AN} = \left(\sqrt{3}e^{j30^\circ}\right)\left[136.14e^{-j23.97^\circ}\ \mathrm{V\ (rms)}\right]$$

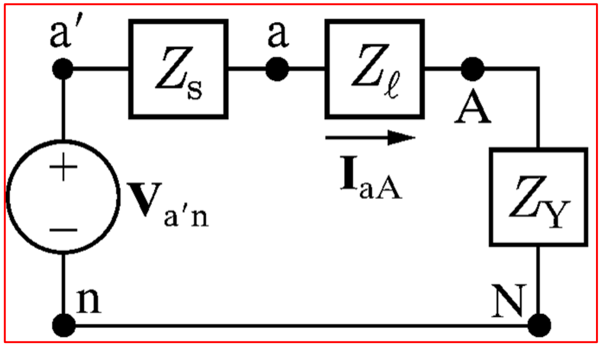
$$\boxed{\mathbf{V}_{AB} = 235.80e^{j6.03^\circ}\ \mathrm{V\ (rms)}}$$

$$\mathbf{V}_{BC} = \left(e^{-j120^\circ}\right)\mathbf{V}_{AB} \rightarrow \boxed{\mathbf{V}_{BC} = 235.80e^{-j113.97^\circ}\ \mathrm{V\ (rms)}}$$

$$\mathbf{V}_{CA} = \left(e^{+j120^\circ}\right)\mathbf{V}_{AB} \rightarrow \boxed{\mathbf{V}_{CA} = 235.80e^{j126.03^\circ}\ \mathrm{V\ (rms)}}$$

Example #4 (SOLUTION cont'd)

- Calculate the phase currents of the load
  - One way to compute the load's phase currents is to use the load's phase (line-to-line) voltages



$$\mathbf{I}_{AB} = \mathbf{V}_{AB}/Z_\Delta = \left[235.80e^{j6.03^\circ}\text{ V}\right]/[30\Omega - j20\Omega]$$

$$\boxed{\mathbf{I}_{AB} = 6.54e^{-j39.72^\circ}\text{ A (rms)}}$$

$$\mathbf{I}_{BC} = \left(e^{-j120^\circ}\right)\mathbf{I}_{AB} \rightarrow \boxed{\mathbf{I}_{BC} = 6.54e^{-j80.28^\circ}\text{ A (rms)}}$$

$$\mathbf{I}_{CA} = \left(e^{+j120^\circ}\right)\mathbf{I}_{AB} \rightarrow \boxed{\mathbf{I}_{CA} = 6.54e^{j159.72^\circ}\text{ A (rms)}}$$

  - One can also compute the load's phase current using $\mathbf{I}_{aA}$

$$\mathbf{I}_{aA} = \mathbf{I}_{AB} - \mathbf{I}_{CA} = \mathbf{I}_{AB} - \mathbf{I}_{AB}e^{j120^\circ} = \left(\sqrt{3}e^{-j30^\circ}\right)\mathbf{I}_{AB}$$

$$\mathbf{I}_{AB} = \frac{\mathbf{I}_{aA}}{\left(\sqrt{3}e^{-j30^\circ}\right)} = \frac{11.31e^{j9.85^\circ}\text{ A rms}}{\sqrt{3}e^{-j30^\circ}} \rightarrow \boxed{\mathbf{I}_{AB} \approx 6.54e^{-j39.72^\circ}\text{ A (rms)}}$$

$$\mathbf{I}_{BC} = \left(e^{-j120^\circ}\right)\mathbf{I}_{AB} \rightarrow \boxed{\mathbf{I}_{BC} = 6.54e^{-j80.28^\circ}\text{ A (rms)}}$$

$$\mathbf{I}_{CA} = \left(e^{+j120^\circ}\right)\mathbf{I}_{AB} \rightarrow \boxed{\mathbf{I}_{CA} = 6.54e^{j159.72^\circ}\text{ A (rms)}}$$

Example #4 (SOLUTION cont'd)

- Calculate the phase currents through the source
  - Compute the source's phase currents using the transmission line current

$$\mathbf{I}_{aA} = \mathbf{I}_{ba} - \mathbf{I}_{ac} = \left(1 - e^{j120^\circ}\right)\mathbf{I}_{ba}$$

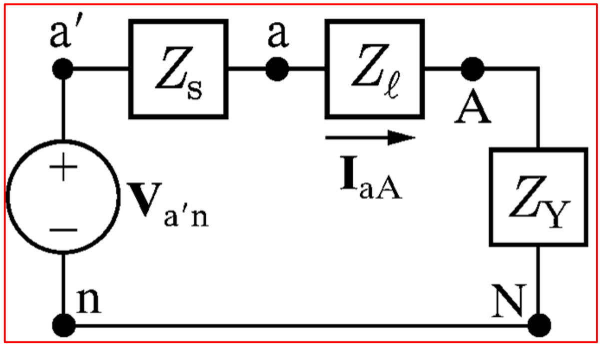
$$\mathbf{I}_{aA} = \left(\sqrt{3}e^{-j30^\circ}\right)\mathbf{I}_{ba} \rightarrow \mathbf{I}_{ba} = \mathbf{I}_{aA}/\left(\sqrt{3}e^{-j30^\circ}\right)$$

$$\mathbf{I}_{ba} = \left(11.31e^{j9.85^\circ}\ \mathrm{A\ (rms)}\right)/\left(\sqrt{3}e^{-j30^\circ}\right)$$

$$\boxed{\mathbf{I}_{ba} = 6.54e^{-j39.72^\circ}\ \mathrm{A\ (rms)}}$$

$$\mathbf{I}_{cb} = \left(e^{-j120^\circ}\right)\mathbf{I}_{ba} \rightarrow \boxed{\mathbf{I}_{cb} = 6.54e^{-j80.28^\circ}\ \mathrm{A\ (rms)}}$$

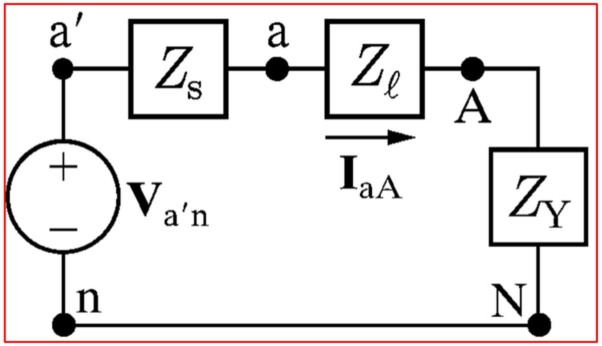
$$\mathbf{I}_{ac} = \left(e^{+j120^\circ}\right)\mathbf{I}_{ba} \rightarrow \boxed{\mathbf{I}_{ac} = 6.54e^{j159.72^\circ}\ \mathrm{A\ (rms)}}$$



Example #4 (SOLUTION cont'd)

- Calculate the line-to-line (phase) voltages at the source terminals
  - First, compute $\mathbf{V}_{an}$ of single-phase equivalent network

$$\mathbf{V}_{an} = (Z_l + Z_Y)\mathbf{I}_{aA} = V_{a'n} - \mathbf{I}_{aA}Z_{sY}$$
$$\mathbf{V}_{an} = (10.6\Omega - j6.2\Omega)\left(11.31e^{j9.85^\circ}\text{ A rms}\right)$$
$$\boxed{\mathbf{V}_{an} = 138.88e^{-j20.47^\circ}\text{V (rms)}}$$



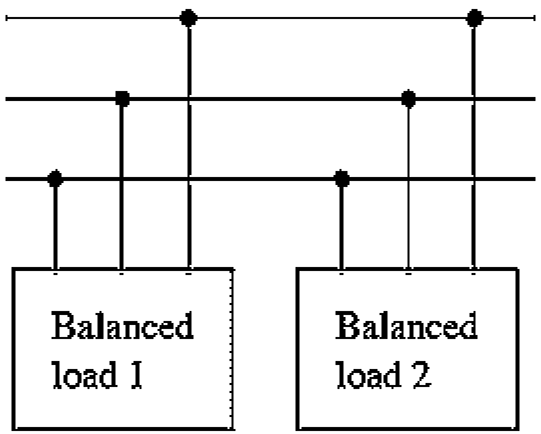
  - Now, use $\mathbf{V}_{an}$ to compute the phase voltages at the source terminals

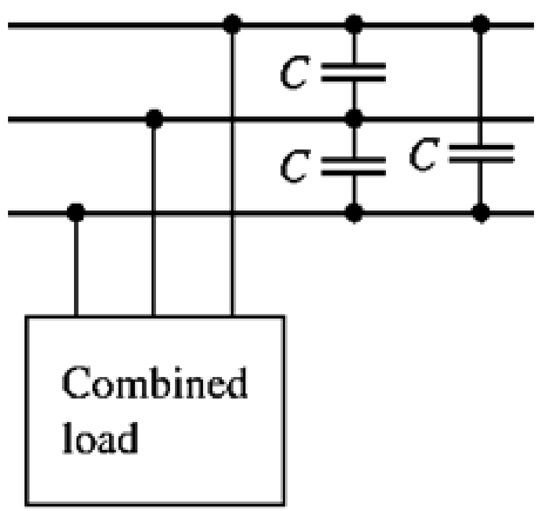
$$\mathbf{V}_{ab} = \left(\sqrt{3}e^{j30^\circ}\right)\mathbf{V}_{an} = \left(\sqrt{3}e^{j30^\circ}\right)\left(138.62e^{-j20.35^\circ}\text{V (rms)}\right)$$
$$\boxed{\mathbf{V}_{ab} = 240.6e^{j9.52^\circ}\text{ V rms}}$$
$$\mathbf{V}_{bc} = \left(e^{-j120^\circ}\right)\mathbf{V}_{ab} \rightarrow \boxed{\mathbf{V}_{bc} = 240.6e^{-j110.47^\circ}\text{ V (rms)}}$$
$$\mathbf{V}_{ca} = \left(e^{+j120^\circ}\right)\mathbf{V}_{ab} \rightarrow \boxed{\mathbf{V}_{ca} = 240.6e^{j129.53^\circ}\text{ V (rms)}}$$

Example #5

- Two balanced loads are connected to $240\ \mathrm{kV}$ (rms) 3-phase lines.
  - Load 1 absorbs $30\ \mathrm{kW}$ at a 0.6 lagging pf
  - Load 2 absorbs $45\ \mathrm{kVAR}$ at a 0.8 lagging pf

  Assume an *a-b-c* phase sequence for the source:
  - Compute the total complex, real, and reactive power absorbed by the combined load
  - Compute the combined load's power factor
  - Compute the kVAR rating for 3 capacitors, that when placed in parallel with the combined load, will raise combined load's pf to 0.9 lagging.
  - Compute the capacitance of each capacitor assuming $f = 60\mathrm{Hz}$ 3-phase lines.





Example #5 (SOLUTION)

- Compute total {complex, real, reactive} power absorbed by combined load
  - First, compute total complex power of Load 1: $30\text{ kW} @ pf = 0.6$ lag

$$P_1 = |\boldsymbol{S_1}|\cos(\theta_1) = |\boldsymbol{S_1}|pf_1 \rightarrow \boxed{P_1 = 30\text{kW}}$$

$$Q_1 = |\boldsymbol{S_1}|\sin(\theta_1) = P_1\frac{\sin(\theta_1)}{\cos(\theta_1)} = P_1\tan(\theta_1) = P_1\tan(\cos^{-1}(pf_1))$$

$$Q_1 = (30\text{kW})\tan(\cos^{-1}(0.6)) \rightarrow \boxed{Q_1 = 40\text{kVARs}}$$

$$\mathbf{S}_1 = P_1 + jQ_1 \rightarrow \boxed{\mathbf{S}_1 = 30 + j40\text{ kVA} = 50e^{j53.13^\circ}\text{kVA}}$$

  - Next, compute total complex power of Load 2: $45\text{ kVAR} @ pf = 0.8$ lag

$$Q_2 = |\boldsymbol{S_2}|\sin(\theta_2) = |\boldsymbol{S_2}|rf_1 \rightarrow \boxed{Q_2 = 45\text{kVAR}}$$

$$P_2 = |\boldsymbol{S_2}|\cos(\theta_2) = Q_2\frac{\cos(\theta_2)}{\sin(\theta_2)} = Q_2\cot(\theta_2) = Q_2\cot(\cos^{-1}(pf_2))$$

$$P_2 = (45\text{kVARs})\cot(\cos^{-1}(0.8)) \rightarrow \boxed{P_2 = 60\text{kW}}$$

$$\mathbf{S}_2 = P_2 + jQ_2 \rightarrow \boxed{\mathbf{S}_2 = 60 + j45\text{ kVA} = 75e^{j36.87^\circ}\text{kVA}}$$

Example #5 (SOLUTION cont'd)

- Compute total {complex, real, reactive} power absorbed by combined load
  - Next, compute total complex power of combined load

$$\mathbf{S}_{comb} = \boldsymbol{S}_1 + \boldsymbol{S}_2 = (30 + j40\text{ kVA}) + (60 + j45\text{ kVA})$$

$$\boxed{\mathbf{S}_{comb} = 90 + j85\text{ kVA} = 123.79e^{j43.36°}\text{kVA}}$$

  - Finally, compute total real and reactive power of combined load

$$P_{comb} = \boldsymbol{Re}(\mathbf{S}_{comb}) \rightarrow \boxed{P_{comb} = 90\text{ kW absorb}}$$

$$Q_{comb} = \boldsymbol{Im}(\mathbf{S}_{comb}) \rightarrow \boxed{Q_{comb} = 85\text{ kVARs absorb}}$$

- Compute the combined load's power factor

$$pf_{comb} = \cos(\theta_{s_{comb}}) = \frac{P_{comb}}{|\boldsymbol{S}_{comb}|} = \cos(43.36°) = \frac{90\text{ kW}}{123.79\text{ kVA}}$$

$$\boxed{pf_{comb} = 0.747\ lagging}$$

Example #5 (SOLUTION cont'd)

- Compute the quadrature (kVAR) power rating for 3 capacitors, that when placed in parallel with the combined load, will raise combined load's power factor to 0.9 lagging.
  - First, compute the new power angle (use a + sign for lagging power factor)

$$\theta_{new} = (+)\cos^{-1}(pf_{new}) = (+)\cos^{-1}(0.9) \rightarrow \boxed{\theta_{new} = 25.84°}$$

  - Next, compute the new quadrature power of the combined load while ensuring the average power of the combined load is unchanged!

$$Q_{new} = |S_{new}|\sin(\theta_{new}) = \frac{P_{new}}{\cos(\theta_{new})}\sin(\theta_{new}) = P_{old}\tan(\theta_{new})$$

$$Q_{new} = P_{old}\cos^{-1}(pf_{new}) = (90\text{kW})\tan(\cos^{-}(0.9)) \rightarrow \boxed{Q_{new} = 43{,}489\text{kVAR}}$$

  - Finally, compute the corrective reactive power for the 3 caps as the difference between the new and old reactive powers of the combined load

$$Q_{corr} = Q_{3caps} = Q_{new} - Q_{old}$$

$$Q_{corr} = (43.489 - 85.0)\text{kVARs} \rightarrow \boxed{Q_{corr} = Q_{3caps} = -41.411\text{KVARs}}$$

Example #5 (SOLUTION cont'd)

- Compute the capacitance of each capacitor assuming $f = 60\text{Hz}$ 3-phase lines.
  - First, note that $Q_{corr} = Q_{3caps} = -41.411\text{KVARs}$ is for all 3 capacitors. So, the corrective reactive power for one capacitor is computed as:

    $$Q_{1cap} = Q_{3caps}/3 = (-41.411\text{kVARs})/3$$

    $$\boxed{Q_{1cap} = -13.804\text{kVARs}}$$

  - Next, compute the corresponding corrective reactance of 1 cap:

    $$Q_{1cap} = \frac{\left|\boldsymbol{V}_{\boldsymbol{line,rms}}\right|^2}{X_{1cap}} \rightarrow X_{1cap} = \frac{\left|\boldsymbol{V}_{\boldsymbol{line,rms}}\right|^2}{Q_{1cap}} = \frac{(240\text{kV})^2}{-13.804\text{kVAR}}$$

    $$\boxed{X_{1cap} = -4.173\text{k}\Omega}$$

  - Next, compute the corresponding corrective reactance of 1 cap as follows:

    $$X_{1cap} = -\frac{1}{\omega C} \rightarrow C = -\frac{1}{\omega X_{1cap}} = -\frac{1}{2\pi f X_{1cap}}$$

    $$C = -\frac{1}{2\pi(60\text{Hz})(-4.173\text{k}\Omega)} \rightarrow \boxed{C = 635.65\text{pF}}$$