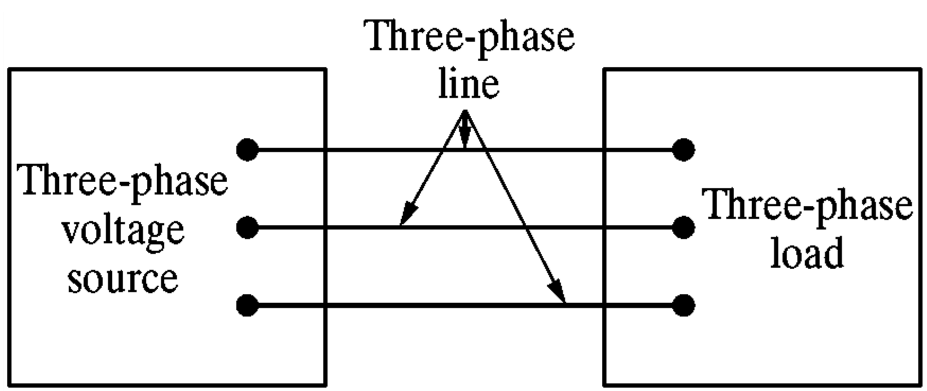
Lecture #1(a): Poly-Phase Networks

Theory

ECE 20200: Linear Circuit Analysis II
Steve Naumov (Instructor)

Overview

- **Balanced poly-phase networks**: A network comprising a **balanced poly-phase source** and a **balanced poly-phase load**
- **Balanced Poly-phase source**: A voltage source that generates k sinusoidal voltage waveforms with these characteristics:
  - Each waveform's **operating frequency is the same**
  - Each waveform's **magnitude is the same**
  - **Consecutive waveform pairs** are **(360/k) degrees apart**.
- **Balanced Poly-phase load**: A load that draws the same power from each voltage waveform
- Most common poly-phase network is a **three-phase network**



Overview (cont'd)

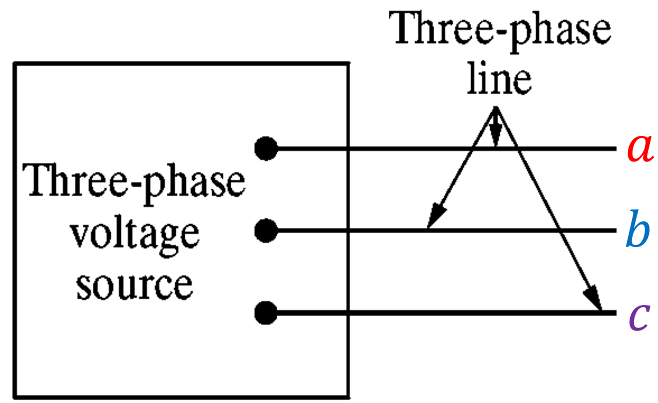
- Why study balanced poly-phase networks?
  - Nearly all electric power is generated and distributed using balanced poly-phase networks.
  - Instantaneous power is constant (not pulsating) and results in uniform power transmission and less vibration in the poly-phase generators.
  - For the same amount of power, a poly-phase network is more economical than a single-phase counterpart.

Lecture #1(a) Poly-Phase Networks
Theory

Balanced 3-Phase Voltage Sources

Balanced 3-Phase Voltages

- A **balanced 3-phase source** generates 3 sinusoidal voltages:
  - Each voltage's magnitude and **frequency is V_ϕ volts and f_ϕ Hz**
  - Consecutive pairs of voltages are $360°/3 = 120°$ apart.

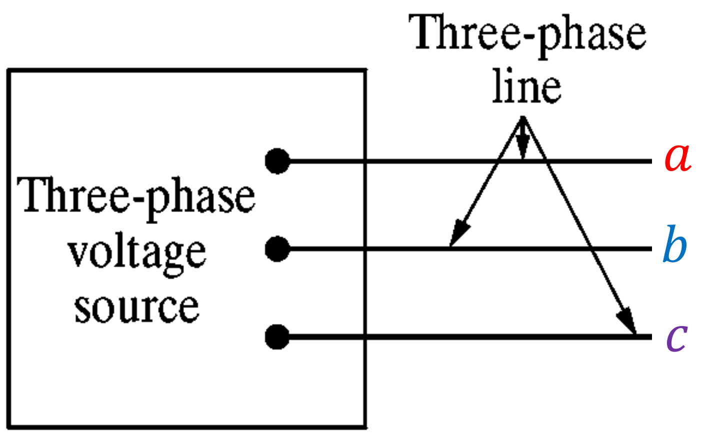


- Each voltage is referred to as $\mathbf{V_a}$ (a-phase voltage), $\mathbf{V_b}$ (b-phase voltage), and $\mathbf{V}_c$ (c-phase voltage)
- $\mathbf{V_b}$ and $\mathbf{V}_c$ relate to $\mathbf{V_a}$ with respect to phase in one of two ways:
  - **Positive Phase Sequence (a-b-c)**
    - $\mathbf{V}_a = V_\phi e^{j\theta_{\phi V}}$ $\quad \mathbf{V}_b = \left(e^{-j120°}\right)\mathbf{V}_a$ $\quad \mathbf{V}_c = \left(e^{+j120°}\right)\mathbf{V}_a$
  - **Negative Phase Sequence (a-c-b)**
    - $\mathbf{V}_a = V_\phi e^{j\theta_{\phi V}}$ $\quad \mathbf{V}_b = \left(e^{+j120°}\right)\mathbf{V}_a$ $\quad \mathbf{V}_c = \left(e^{-j120°}\right)\mathbf{V}_a$

Balanced 3-Phase Voltages(cont'd)

- Independent of the phase sequence, an important characteristic of balanced 3-phase voltages is that:

$$\boxed{\mathbf{V}_a + \mathbf{V}_b + \mathbf{V}_c = 0\text{V}}$$



- Proof (a-b-c sequence)

$$\text{V}_\phi e^{j\theta\phi_V} + \text{V}_\phi e^{j(\theta\phi_V-120°)} + \text{V}_\phi e^{j(\theta\phi_V+120°)} = 0\text{V}$$
$$(1 + e^{-j120} + e^{+j120°})\mathbf{V}_a = 0\text{V}$$
$$(0)\mathbf{V}_a = 0\text{V}$$

- Proof (a-c-b sequence)

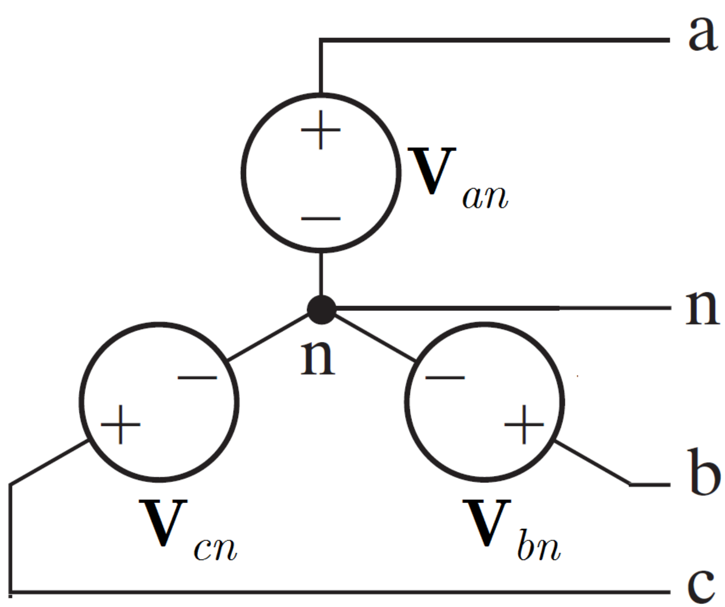
$$\text{V}_\phi e^{j\theta\phi_V} + \text{V}_\phi e^{j(\theta\phi_V+120°)} + \text{V}_\phi e^{j(\theta\phi_V-120°)} = 0\text{V}$$
$$(1 + e^{+j120} + e^{-j120°})\mathbf{V}_a = 0\text{V}$$
$$(0)\mathbf{V}_a = 0\text{V}$$

Balanced 3-phase Voltages

Source Topologies

- Two ideal 3-phase voltage source topologies exist

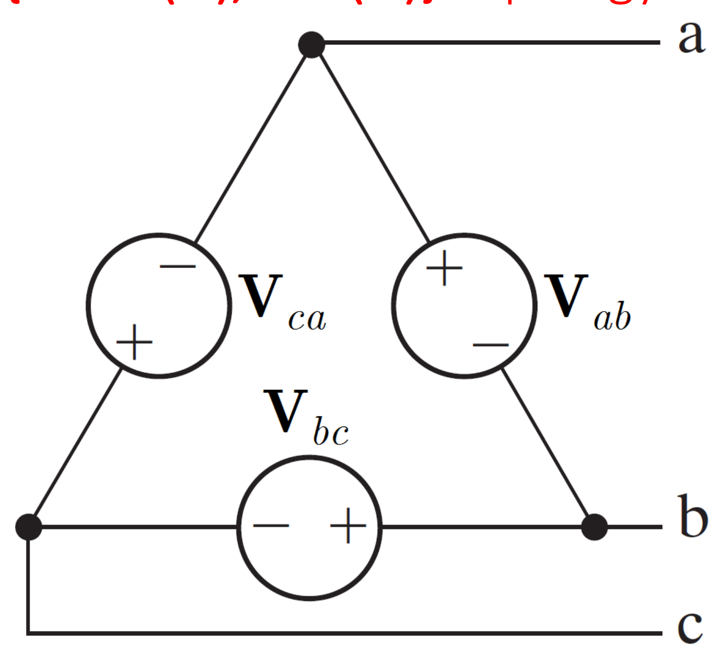
{Wye-(Y), Tee-(T), Star-(∗)} Topology



$\{\mathbf{V}_{an}, \mathbf{V}_{bn}, \mathbf{V}_{cn}\}$ called {phase, line-to-neutral} voltages

$$\boxed{\mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn} = 0\text{V}}$$

{Delta-(Δ), Pie-(π)} Topology



$\{\mathbf{V}_{ab}, \mathbf{V}_{bc}, \mathbf{V}_{ca}\}$ called {line, line-to-line} voltages

$$\boxed{\mathbf{V}_{ab} + \mathbf{V}_{bc} + \mathbf{V}_{ca} = 0\text{V}}$$

Transforming 3-Phase Voltages

Wye-(Y) → Delta-(Δ) Transformations

- Given a a-b-c phase sequence, balanced 3-phase <u>phase voltages</u> $\mathbf{V}_{an}$, $\mathbf{V}_{bn}$, and $\mathbf{V}_{cn}$, the <u>line-to-line voltage</u> $\mathbf{V}_{ab}$ can be computed as follows:

<u>Applying KVL:</u>

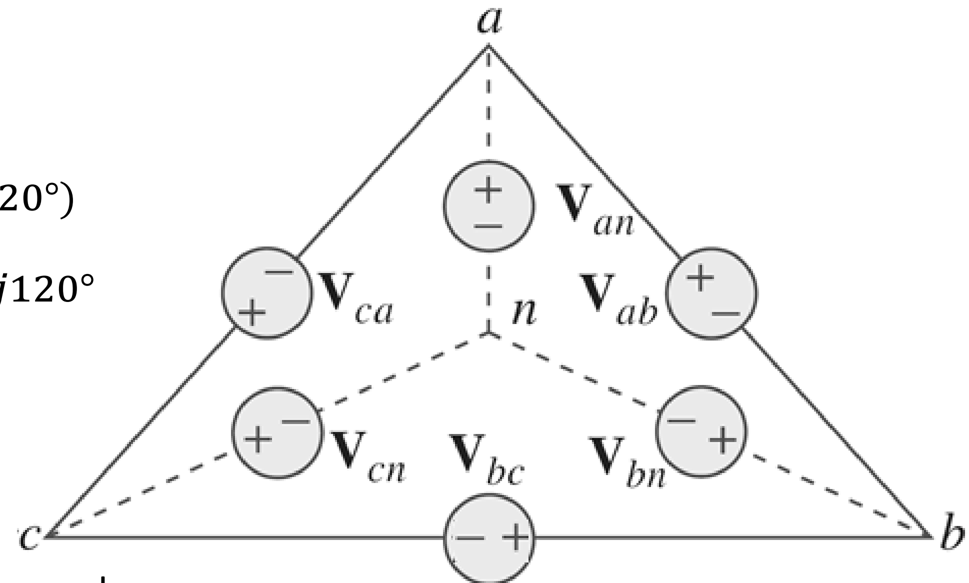
$$\mathbf{V}_{ab} = \mathbf{V}_{an} - \mathbf{V}_{bn}$$

$$\mathbf{V}_{ab} = V_{\phi} e^{j\theta_{\phi V}} - V_{\phi} e^{j(\theta_{\phi V} - 120°)}$$

$$\mathbf{V}_{ab} = V_{\phi} e^{j\theta_{\phi V}} - V_{\phi} e^{j\theta_{\phi V}} e^{-j120°}$$

$$\mathbf{V}_{ab} = \left(1 - e^{-j120°}\right) V_{\phi} e^{j\theta_{\phi V}}$$

$$\boxed{\mathbf{V}_{ab} = \left(\sqrt{3} e^{+j30°}\right) \mathbf{V}_{an}}$$



- Since line-to-line voltages $\mathbf{V}_{bc}$ and $\mathbf{V}_{ca}$ are balanced, we can compute them in terms of $\mathbf{V}_{ab}$:

$$\boxed{\mathbf{V}_{bc} = \left(e^{-j120°}\right) \mathbf{V}_{ab}}$$

$$\boxed{\mathbf{V}_{ca} = \left(e^{+j120°}\right) \mathbf{V}_{ab}}$$

How do equations on this slide change for a negative phase sequence Y source?

Transforming 3-Phase Voltages
Delta-(Δ) → Wye-(Y) Transformations

- Given an a-b-c sequence, balanced 3-phase line-to-line voltages $\mathbf{V}_{ab}$, $\mathbf{V}_{bc}$, and $\mathbf{V}_{ca}$, the a-phase voltage $\mathbf{V}_{an}$ can be computed as follows:

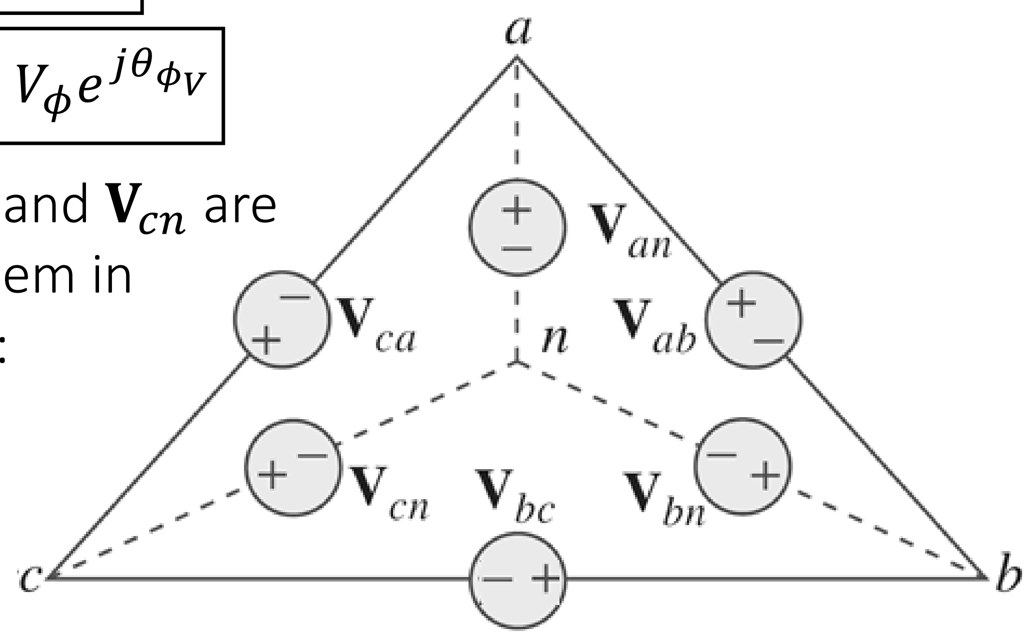
$$\boxed{\mathbf{V}_{an} = \mathbf{V}_{ab} / \left(\sqrt{3}e^{+j30^\circ}\right) = V_\phi e^{j\theta_{\phi V}}}$$

$$\boxed{\mathbf{V}_{an} = \mathbf{V}_{ab}\left[\left(\sqrt{3}/3\right)e^{-j30^\circ}\right] = V_\phi e^{j\theta_{\phi V}}}$$

- Since the phase voltages $\mathbf{V}_{bn}$ and $\mathbf{V}_{cn}$ are balanced, we can compute them in terms of a-phase voltage $\mathbf{V}_{ab}$:

$$\boxed{\mathbf{V}_{bc} = \left(e^{-j120^\circ}\right)\mathbf{V}_{ab}}$$

$$\boxed{\mathbf{V}_{ca} = \left(e^{+j120^\circ}\right)\mathbf{V}_{ab}}$$



How do equations on this slide change for a negative phase sequence Y source?

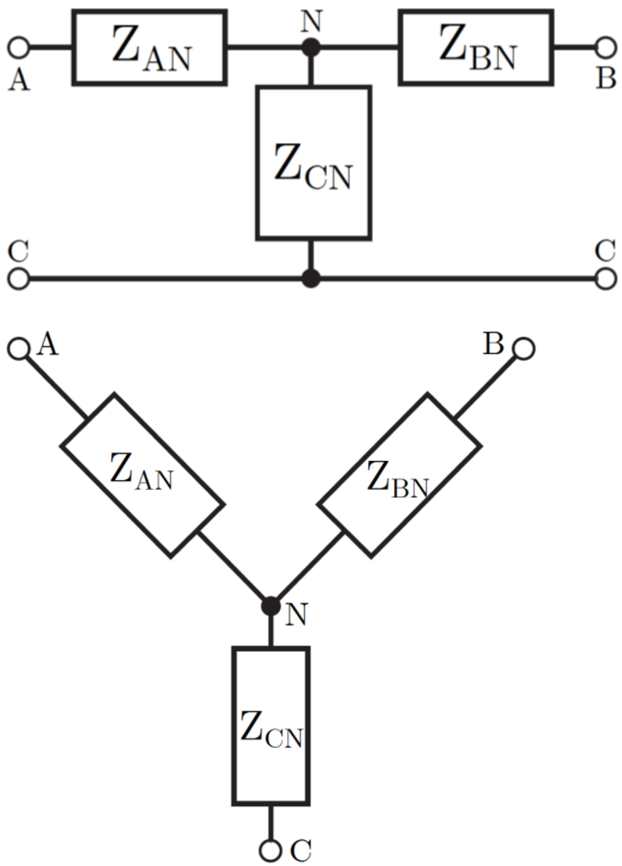
Lecture #1(a) Poly-Phase Networks
Theory

Delta (Pie) and Wye (Tee) Impedance Networks

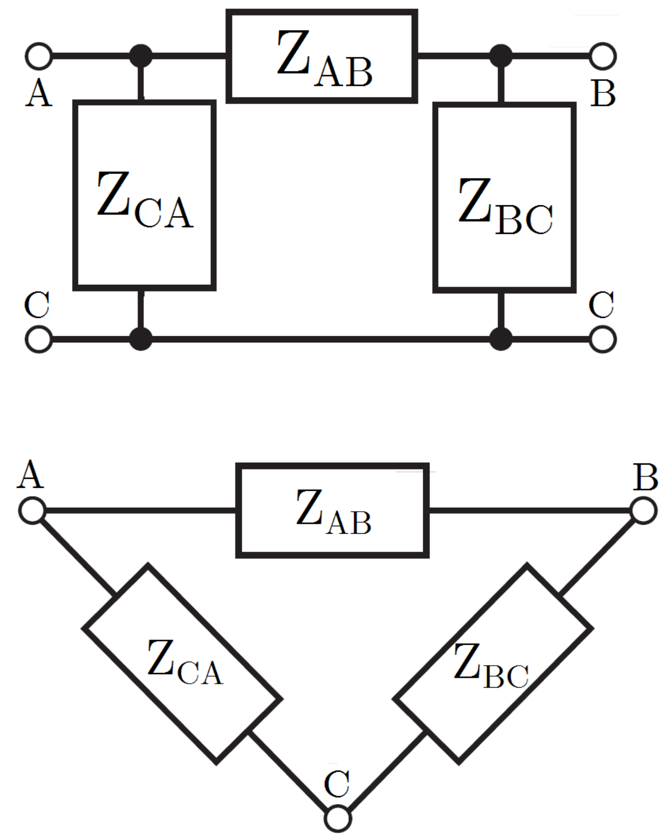
Wye-(Y) and Delta-(Δ) Impedance Networks

- There are two important load types for poly-phase networks

{Wye-(Y), Tee-(T), Star-(∗)} Topology



Delta-(Δ) or Pie-(π) Topology



Wye-(Y) and Delta-(Δ) Impedance Networks

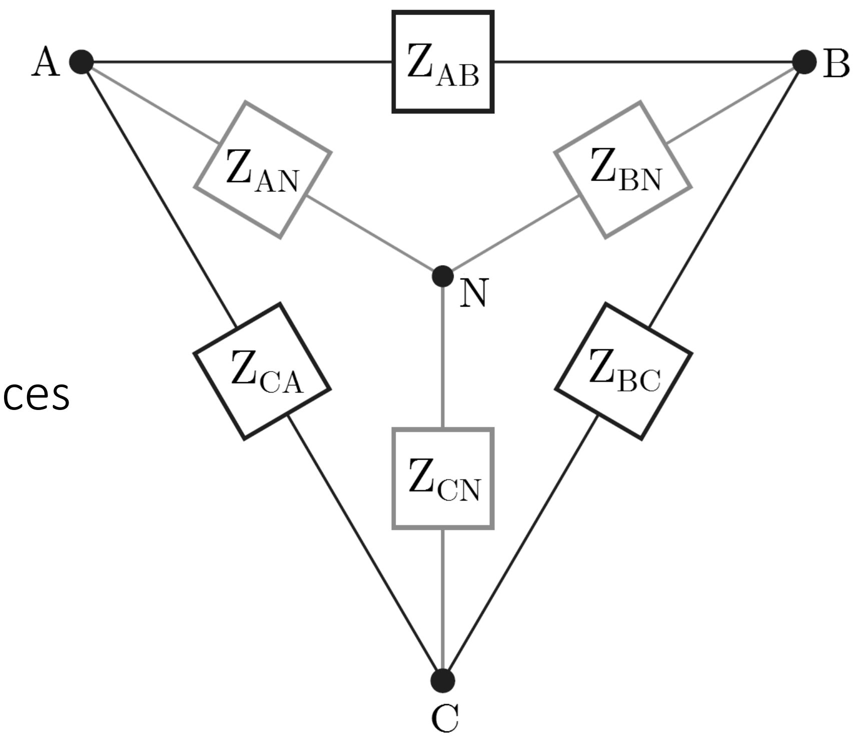
Transforming Impedances of Unbalanced Loads

- Transforming Wye-Y → Delta-Δ Impedances

$$\boxed{\begin{aligned} Z_{AB} &= \frac{Z_{AN}Z_{BN} + Z_{BN}Z_{CN} + Z_{CN}Z_{AN}}{Z_{CN}} \\ Z_{BC} &= \frac{Z_{AN}Z_{BN} + Z_{BN}Z_{CN} + Z_{CN}Z_{AN}}{Z_{AN}} \\ Z_{CA} &= \frac{Z_{AN}Z_{BN} + Z_{BN}Z_{CN} + Z_{CN}Z_{AN}}{Z_{BN}} \end{aligned}}$$

- Transforming Delta-Δ → Wye-Y Impedances

$$\boxed{\begin{aligned} Z_{AN} &= \frac{Z_{AB}Z_{CA}}{Z_{AB} + Z_{BC} + Z_{CA}} \\ Z_{BN} &= \frac{Z_{BC}Z_{AB}}{Z_{AB} + Z_{BC} + Z_{CA}} \\ Z_{CN} &= \frac{Z_{CA}Z_{BC}}{Z_{AB} + Z_{BC} + Z_{CA}} \end{aligned}}$$



Balanced 3-Phase Loads

Voltage and Current Transformations

- **Balanced Poly-phase load**: Load draws same power from each phase
  - To meet requirement, the each load impedance must be equal!
  - A Wye-Y load is balanced when $\mathrm{Z_{AN}} = \mathrm{Z_{BN}} = \mathrm{Z_{CN}} = \mathrm{Z_Y}$
  - A Delta-Δ load is balanced when $\mathrm{Z_{AB}} = \mathrm{Z_{BC}} = \mathrm{Z_{CA}} = \mathrm{Z_\Delta}$
- Transforming Balanced Loads
  - Wye-Y → Delta-Δ

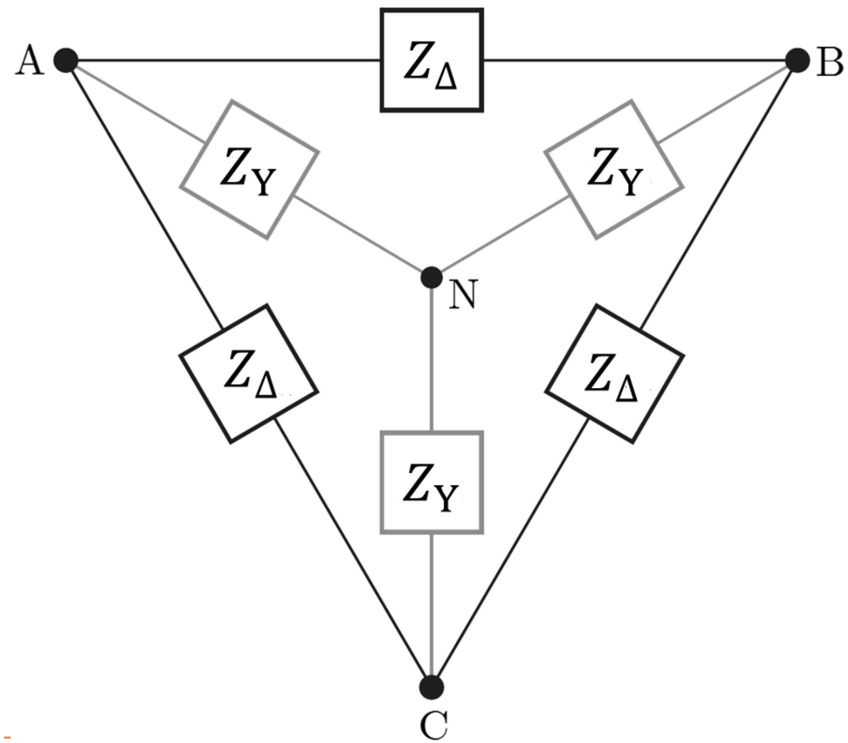
$$Z_\Delta = \frac{Z_Y Z_Y + Z_Y Z_Y + Z_Y Z_Y}{Z_Y}$$

$$\boxed{Z_\Delta = 3Z_Y}$$

  - Delta-Δ → Wye-Y Loads

$$Z_Y = \frac{Z_\Delta Z_\Delta}{Z_\Delta + Z_\Delta + Z_\Delta}$$

$$\boxed{Z_Y = Z_\Delta/3}$$



Overview of 3-Phase Networks

- Since 3-phase sources and loads can each have either a Wye-Y or Delta-D topology, four combinations of sources and loads exist
- Configurations of 3-phase networks

Common/ practical

Source	Load
Y	Y
Y	Δ
Δ	Y
Δ	Δ

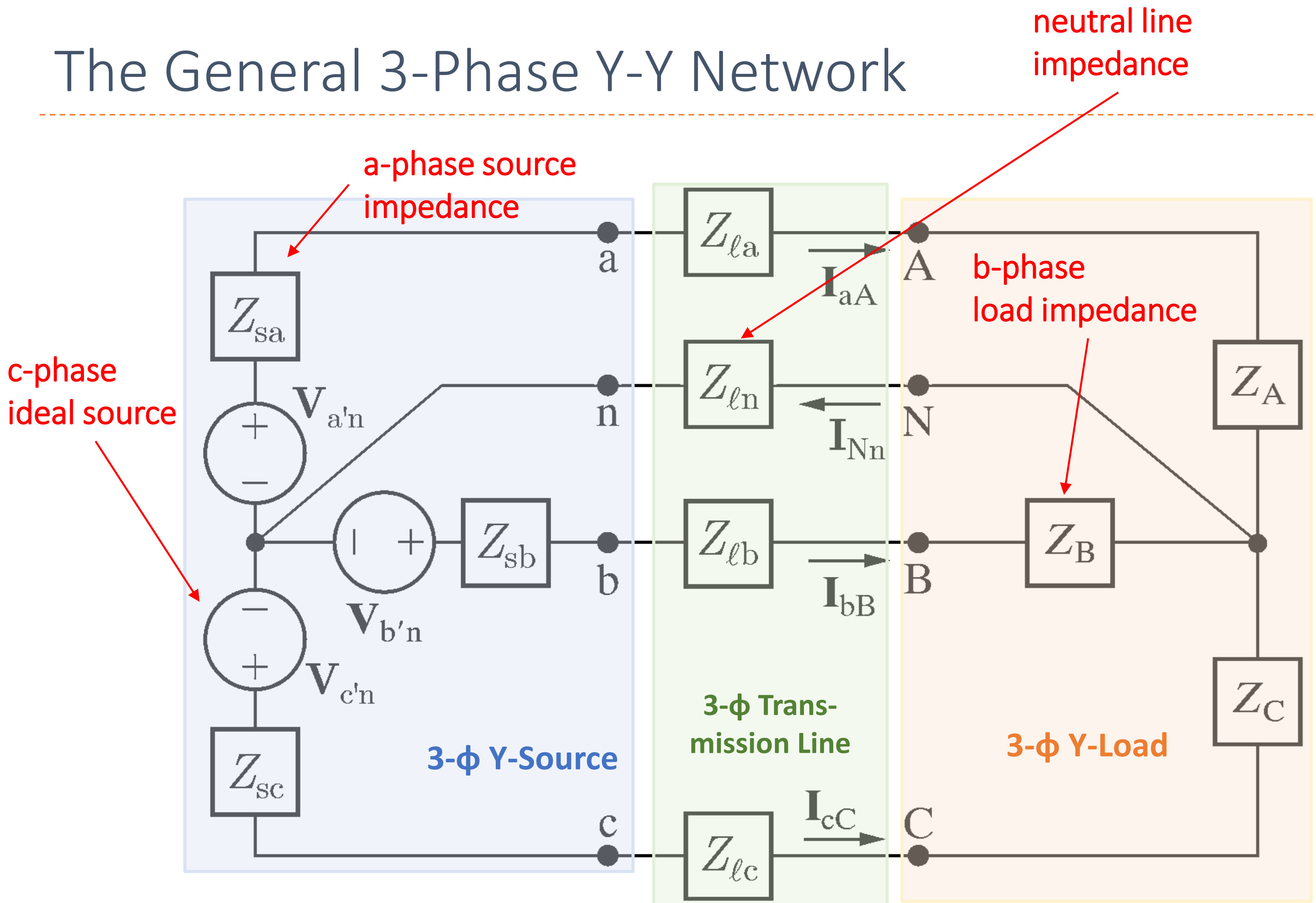
Common/ practical

Lecture #1(a) Poly-Phase Networks

Theory

The Balanced Y-Y *Network*

The General 3-Phase Y-Y Network



A Balanced Y-Y 3-phase Network: Analysis

- Perform nodal analysis @ at node N:

$$\frac{\mathbf{V}_N - \mathbf{V}_{a'n}}{Z_S + Z_\ell + Z_Y} + \frac{\mathbf{V}_N - \mathbf{V}_{b'n}}{Z_S + Z_\ell + Z_Y} + \frac{\mathbf{V}_N - \mathbf{V}_{c'n}}{Z_S + Z_\ell + Z_Y} + \frac{\mathbf{V}_N - 0\text{V}}{Z_{\ell n}} = 0\text{A}$$

$$\mathbf{V}_N\left[\frac{3}{Z_\phi} + \frac{1}{Z_{\ell n}}\right] = \frac{1}{Z_\phi}\left[\mathbf{V}_{a'n} + \mathbf{V}_{b'n} + \mathbf{V}_{c'n}\right] \rightarrow \mathbf{V}_N\left[\frac{3}{Z_\phi} + \frac{1}{Z_{\ell n}}\right] = \frac{1}{Z_\phi}[0\text{V}]$$

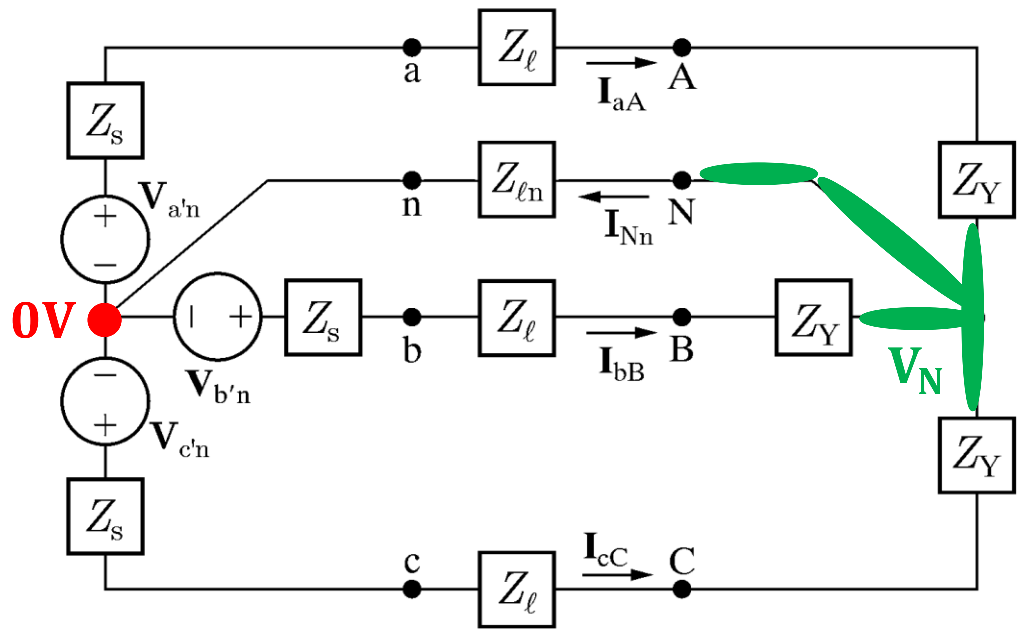
$$\boxed{\mathbf{V}_N = 0\text{V}}$$

- Major conclusions:

$$\boxed{\mathbf{V}_{Nn} = 0\text{V}}$$

$$\boxed{\mathbf{I}_{Nn} = 0\text{A} = \mathbf{I}_{aA} + \mathbf{I}_{bB} + \mathbf{I}_{cC}}$$

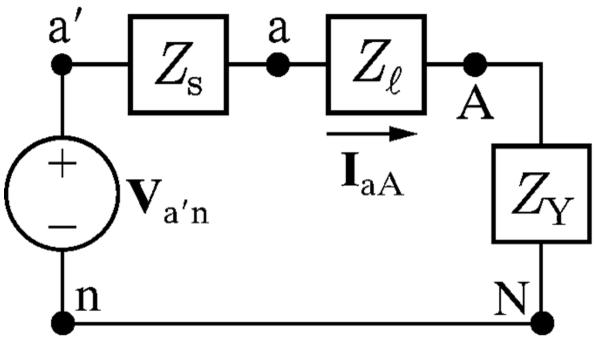
- Consequence:
  - Can remove neutral line from <u>this</u> configuration or can replace neutral line with a short between nodes {n,N}



A Balanced Y-Y 3-phase Network

Preferred Analysis Method: Single-phase equivalent network

- **Single-phase equivalent network:**



  - An equivalent network for a single phase (e.g. a-phase) that gives same V-I values on previous slide.
    - The a-phase line current $\mathbf{I}_{aA}$: $\mathbf{I}_{aA} = \mathbf{V}_{a'n}/Z_\phi$
    - The a-phase load voltage $\mathbf{V}_{AN}$: $\mathbf{V}_{AN} = \mathbf{I}_{aA} Z_Y$
    - The a-phase terminal source voltage $\mathbf{V}_{an}$:

$$\mathbf{V}_{an} = V_{a'n} - \mathbf{I}_{aA} Z_S = \mathbf{I}_{aA}(Z_\ell + Z_Y) = (\mathbf{V}_{a'n}/Z_\phi)(Z_\ell + Z_Y)$$

  - Line-to-line load voltages ($\mathbf{V}_{AB}, \mathbf{V}_{BC}, \mathbf{V}_{CA}$) and the line-to-line terminal source voltages ($\mathbf{V}_{ab}, \mathbf{V}_{bc}, \mathbf{V}_{ca}$) can be derived using conversion factor $\left(\sqrt{3}e^{\pm j30°}\right)$ and the source's phase sequence
  - For example (assuming a-b-c sequence):

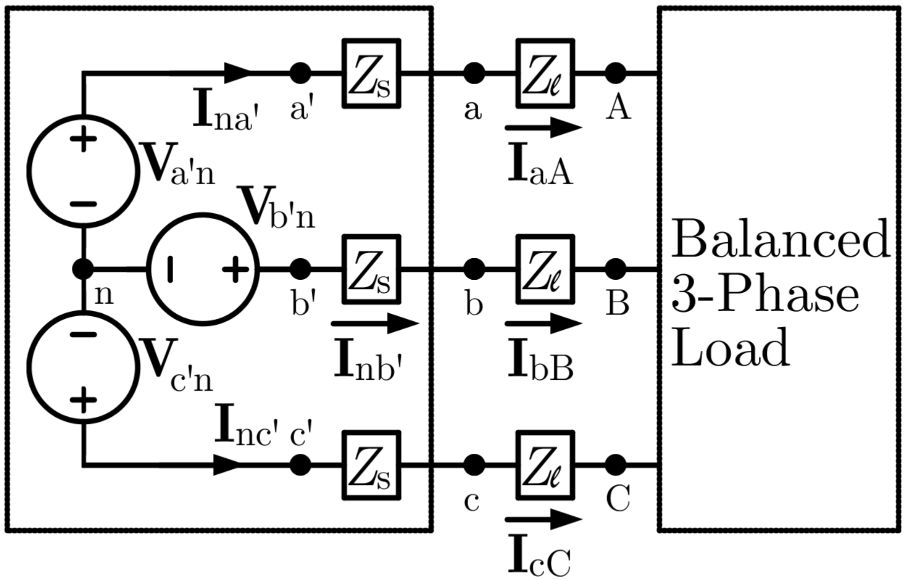
$$\mathbf{V}_{AB} = \left(\sqrt{3}e^{j30°}\right)\mathbf{V}_{AN} \qquad \mathbf{V}_{ab} = \left(\sqrt{3}e^{j30°}\right)\mathbf{V}_{an}$$

Lecture #1(a) Poly-Phase Networks
Theory

Analysis of Balanced 3-Phase Sources

Balanced 3-Phase Networks

Y-Connected Sources



- Ideal a-Phase Source Voltage: $\mathbf{V}_{a'n}$
- Transmission Line Current: $\mathbf{I}_{aA}$
- Other values are computed as follows:
  - a-Phase Current ($\mathbf{I}_{na'}$):

    $$\boxed{\mathbf{I}_{na'} = \mathbf{I}_{aA}}$$
  - a-Phase Terminal Voltage ($\mathbf{V}_{an}$):

    $$\boxed{\mathbf{V}_{an} = \mathbf{V}_{a'n} - \mathbf{I}_{aA}Z_s}$$
  - a-to-b Line-to-Line Voltage ($\mathbf{V}_{ab}$): Using KVL…

    $$\mathbf{V}_{ab} = \mathbf{V}_{an} - \mathbf{V}_{bn} = \mathbf{V}_{an} - \mathbf{V}_{an}e^{\mp j120°} = \left(1 - e^{\mp j120°}\right)\mathbf{V}_{an}$$

    $$\boxed{\mathbf{V}_{ab} = \left(\sqrt{3}e^{j\pm 30°}\right)\mathbf{V}_{an} = \left(\sqrt{3}e^{\pm j30°}\right)\left(\mathbf{V}_{a'n} - \mathbf{I}_{aA}Z_s\right)}$$
  - a-to-b Line-to-Line Current ($\mathbf{I}_{ab}$): N/A

Balanced 3-Phase Networks

Δ-Connected Sources

- Ideal a-Phase Source Voltage: $\mathbf{V}_{a'b}$
- Transmission Line Current: $\mathbf{I}_{aA}$
- Other values are computed as follows
  - a-to-b Terminal Line-to-Line Voltage ($\mathbf{V}_{ab}$):

$$\boxed{\mathbf{V}_{ab} = \mathbf{V}_{a'b} - \mathbf{I}_{ba} Z_s = \left(\sqrt{3} e^{\pm j30°}\right) \mathbf{V}_{an}}$$

  - a-Phase Voltage ($\mathbf{V}_{ab}$):

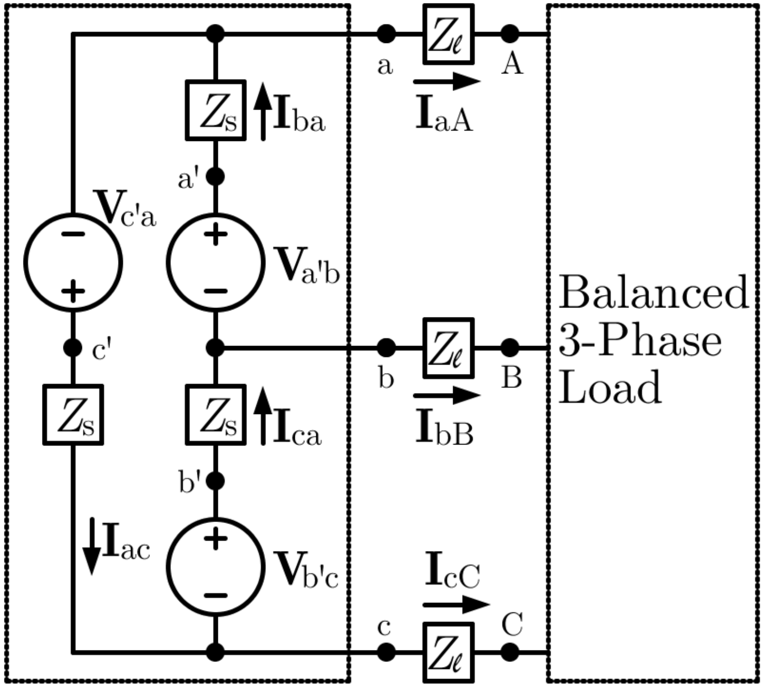
$$\boxed{\mathbf{V}_{ab} = \mathbf{V}_{a'b} - \mathbf{I}_{ba} Z_s = \left(\sqrt{3} e^{\pm j30°}\right) \mathbf{V}_{an}}$$

  - a-Phase Current ($\mathbf{I}_{ba}$): Use KCL @ (a)

$$\mathbf{I}_{aA} = \mathbf{I}_{ba} - \mathbf{I}_{ac} = \mathbf{I}_{ba} - \mathbf{I}_{ba} e^{\pm j120°} = \left(1 - e^{\pm j120°}\right) \mathbf{I}_{ba} = \left(\sqrt{3} e^{\mp j30°}\right) \mathbf{I}_{ba}$$

$$\boxed{\mathbf{I}_{ba} = \mathbf{I}_{aA} / \left(\sqrt{3} e^{\mp j30°}\right)}$$

  - b-to-a Line-to-Line Current ($\mathbf{I}_{ba}$): $\boxed{\mathbf{I}_{ba} = (\mathbf{V}_{a'b} - \mathbf{V}_{ab})/Z_s = \mathbf{I}_{aA}/(\sqrt{3} e^{\mp j30°})}$



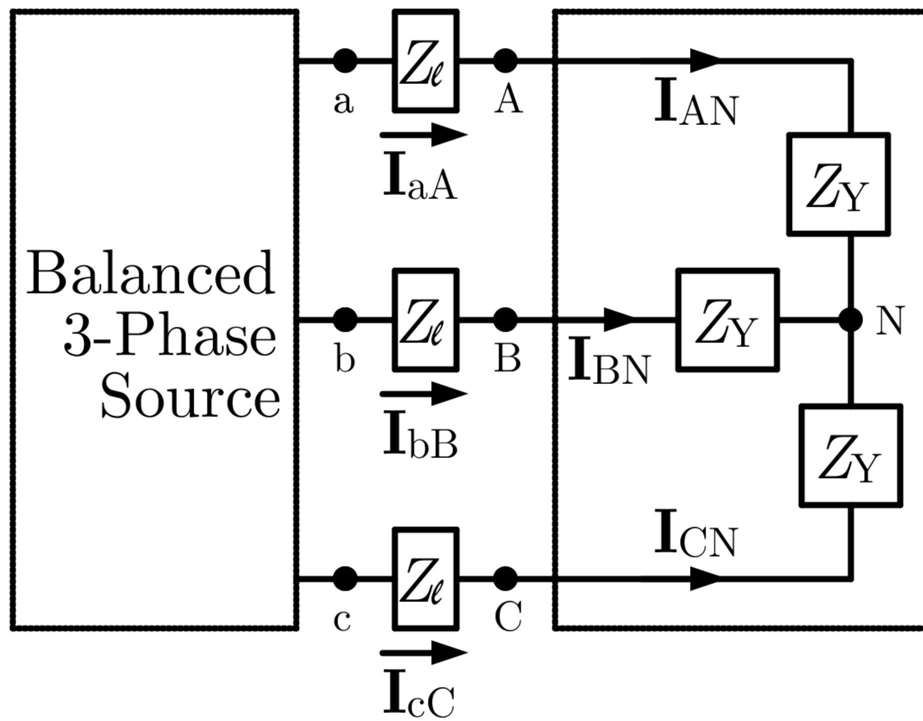
Lecture #1(a) Poly-Phase Networks

Theory

Analysis of Balanced 3-Phase Loads

Balanced 3-Phase Networks
Y-*Connected Loads*

- Transmission Line Current: $\mathbf{I}_{aA}$
- Other related values can be computed as follows
  - A-Phase Current ($\mathbf{I}_{AN}$):
    $\boxed{\mathbf{I}_{AN} = \mathbf{I}_{aA}}$
  - A-Phase Voltage ($\mathbf{V}_{AN}$):
    $\boxed{\mathbf{V}_{AN} = \mathbf{I}_{AN} Z_Y = \mathbf{I}_{aA} Z_Y}$
  - A-to-B Line-to-Line Voltage $\mathbf{V}_{AB}$: Using KVL…



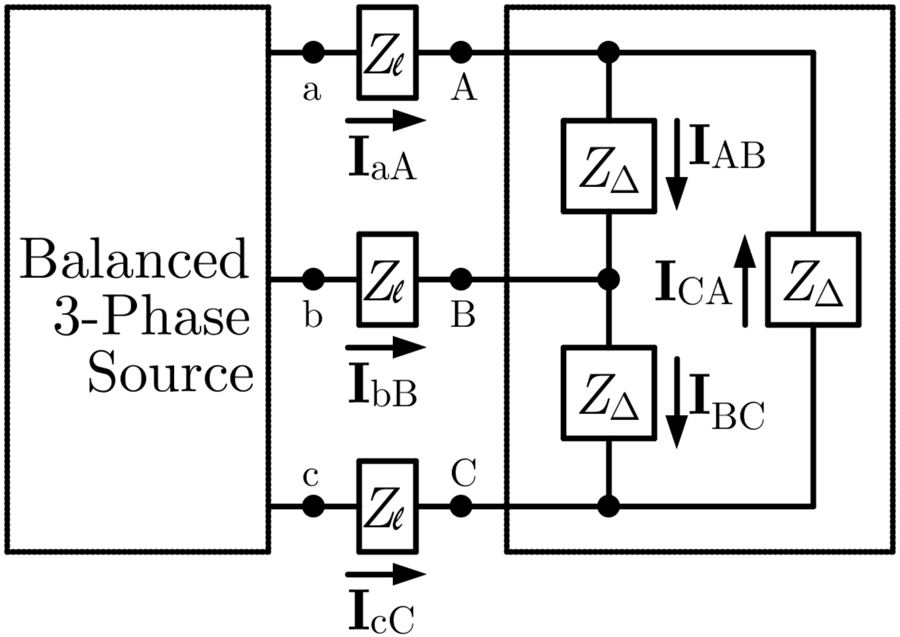
$$\mathbf{V}_{AB} = \mathbf{V}_{AN} - \mathbf{V}_{BN} = \mathbf{V}_{AN} - \mathbf{V}_{AN} e^{\mp j120^\circ} = \left(1 - e^{\mp j120^\circ}\right)\mathbf{V}_{AN}$$

$$\boxed{\mathbf{V}_{AB} = \left(\sqrt{3} e^{\pm j30^\circ}\right)\mathbf{V}_{AN}}$$

  - A-to-B Line-to-Line Current: $\mathbf{I}_{AB} \rightarrow N/A$

Balanced 3-Phase Networks

Δ-Connected Loads



- Transmission line current: $\mathbf{I}_{aA}$
- Other related values can be computed as follows
  - A-Phase Voltage ($\mathbf{V}_{AB}$):

$$\boxed{\mathbf{V}_{AB} = \left(\sqrt{3}e^{\pm j30^\circ}\right)\mathbf{V}_{AN}}$$

  - A-to-B Line-to-Line Voltage ($\mathbf{V}_{AB}$):

$$\boxed{\mathbf{V}_{AB} = \left(\sqrt{3}e^{\pm j30^\circ}\right)\mathbf{I}_{aA}(Z_\Delta/3)}$$

  - A-Phase Current ($\mathbf{I}_{AB}$): Use KCL @ (A)

$$\mathbf{I}_{aA} = \mathbf{I}_{AB} - \mathbf{I}_{CA} = \mathbf{I}_{AB} - \mathbf{I}_{AB}e^{\pm j120^\circ} = \left(1 - e^{\pm j120^\circ}\right)\mathbf{I}_{AB} = \left(\sqrt{3}e^{\mp j30^\circ}\right)\mathbf{I}_{AB}$$

$$\boxed{\mathbf{I}_{AB} = \mathbf{V}_{AB}/Z_\Delta = \mathbf{I}_{aA}/\left(\sqrt{3}e^{\mp j30^\circ}\right)}$$

  - A-to-B Line-to-Line Current ($\mathbf{I}_{AB}$): $\boxed{\mathbf{I}_{AB} = \mathbf{V}_{AB}/Z_\Delta = \mathbf{I}_{aA}/\left(\sqrt{3}e^{\mp j30^\circ}\right)}$