

## Homework #6 Solution

### Problem 1)

It's easiest to do this problem by first computing the connectivity matrix  $C$ , in which each element  $c_{a,b}$  is equal to the edge weights in the graph  $= 4/(n^2 - \text{mod}(n,2))$ , where  $n$  is the number of connections on each net. This is somewhat confusing for  $c_{1,2}$  and  $c_{1,4}$ , because there are two nets connecting these instances. In these cases, use the sum of the weights for each net.

Vertex	1	2	3	4	5	6	7	8	9	10
1	0	1.25	0.25	0.5	0.25	0	0	0	0.25	0
2	1.25	0	0	0.25	0.25	0.5	0	0	0	0.5
3	0.25	0	0	0.25	0	0	0	0	0.25	0
4	0.5	0.25	0.25	0	0.25	0	0	1	0.25	0
5	0.25	0.25	0	0.25	0	1	0	1	1	0
6	0	0.5	0	0	1	0	0	0	0	0.5
7	0	0	0	0	0	0	0	1	0	0
8	0	0	0	1	1	0	1	0	0	0
9	0.25	0	0.25	0.25	1	0	0	0	0	0
10	0	0.5	0	0	0	0.5	0	0	0	0

(a)

The two partitions are  $A = \{1,2,3,4,5\}$  &  $B = \{6,7,8,9,10\}$

$$\begin{aligned}
 T &= c_{1,9} + c_{2,6} + c_{2,10} + c_{3,9} + c_{4,8} + c_{4,9} + c_{5,6} + c_{5,8} + c_{5,9} \\
 &= 0.25 + 0.5 + 0.5 + 0.25 + 1 + 0.25 + 1 + 1 + 1 \\
 &= 5.75
 \end{aligned}$$

(b)

$$W_a = W_1(\text{XOR}) + W_2(\text{FF}) + W_3(\text{XOR}) + W_4(\text{AND}) + W_5(\text{MUX}) = 47$$

$$W_b = W_6(\text{FF}) + W_7(\text{AND}) + W_8(\text{OR}) + W_9(\text{AND}) + W_{10}(\text{AND}) = 39$$

**$W_b < W_a < 48$ , this partition is admissible**

(c)

$$D_2 = E_2 - I_2 = c_{2,6} + c_{2,10} - c_{1,2} - c_{2,4} - c_{2,5} = 0.5 + 0.5 - 1.25 - 0.25 - 0.25 = -0.75$$

Since node 7 connects to node 8 only, we can effectively ignore the cost of node 7 if we move it with node 8

$$D_{7,8} = D_8 = E_8 - I_8 = c_{8,4} + c_{8,5} = 2$$

$$\text{Gain} = g = D_8 + D_2 - 2 c_{2,8} = 2 - 0.75 = 1.25$$

(d)

$$W_a = W_1(\text{XOR}) + W_3(\text{XOR}) + W_4(\text{AND}) + W_5(\text{MUX}) + W_7(\text{AND}) + W_8(\text{OR}) = 38$$

$$W_b = W_2(\text{FF}) + W_6(\text{FF}) + W_9(\text{AND}) + W_{10}(\text{AND}) = 48$$

**This partition is admissible**

(e)

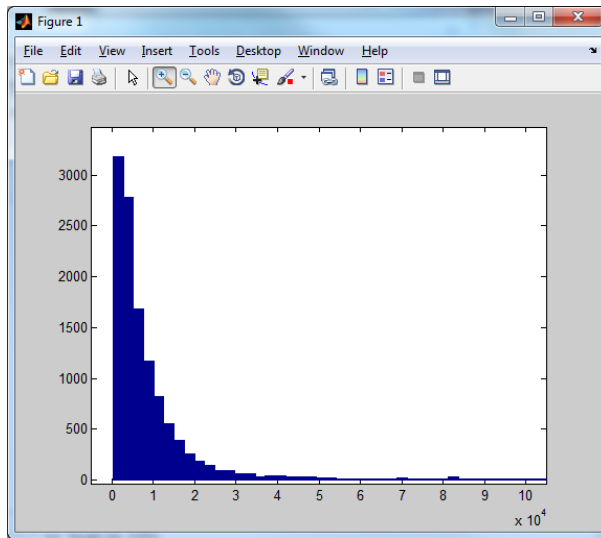
Acceptable partitions must have  $T < 4.5$ . Some acceptable partitions are given below:

A	B	T	$W_a$	$W_b$
$\{1,3,4,5,7,8,9\}$	$\{2,6,10\}$	$c_{1,2} + c_{2,4} + c_{2,5} + c_{5,6} = 2.75$	43	43
$\{1,2,4,7,8\}$	$\{3,5,6,9,10\}$	$c_{1,3} + c_{1,5} + c_{1,9} + c_{2,5} + c_{2,6} + c_{2,10} + c_{3,4} + c_{4,5} + c_{4,9} + c_{5,8} = 3.75$	42	44
$\{1,2,3,4,10\}$	$\{5,6,7,8,9\}$	$c_{1,5} + c_{1,9} + c_{2,6} + c_{3,9} + c_{4,5} + c_{4,8} + c_{4,9} + c_{6,10} = 3.25$	45	41

### Problem 3)

The histogram generated for the wires in the FIR\_cascade\_routed.def file is actually quite boring. Nearly all nets are in the smallest bin. If you run the script in Matlab and execute the command “hist(w,500)” and zoom in, you’ll see the following figure. The average reported is 15474. Dividing this by 2000 (“UNITS DISTANCE MICRONS” in the DEF file header), we get an average wire-length of

$$L_{avg} = 15474/2000 = 7.737 \mu\text{m}$$



Donath’s method calculation:

$$L_{avg} = \begin{cases} d_{avg} \cdot \frac{2}{9} \left( 7 \frac{C^{p-(1/2)} - 1}{4^p - (1/2)} - \frac{1 - C^{p-(3/2)}}{1 - 4^{p-(3/2)}} \right) \cdot \frac{1 - 4^{p-1}}{1 - C^{p-1}} & \text{for } p \neq \frac{1}{2} \\ d_{avg} \cdot \frac{2}{9} \left( 7 \log_4 C - \frac{1 - C^{p-(3/2)}}{1 - 4^{p-(3/2)}} \right) \cdot \frac{1 - 4^{p-1}}{1 - C^{p-1}} & \text{for } p = \frac{1}{2} \end{cases}$$

Core area is  $128 \mu\text{m} \times 128 \mu\text{m}$ . Total number of components in the DEF file is 14516. 2907 of those are filler cells, which gives us  $C = 14516 - 2907 = 11609$ . Therefore, we can approximate  $d_{avg}$  as  $d_{avg} = \sqrt{128 \cdot 128 / 11609} = 1.19 \mu\text{m}$ . Plugging these numbers into Donath’s method, we get the following:

$$L_{avg} = 13.43 \mu\text{m} \text{ for } p=3/4$$

$$L_{avg} = 6.13 \mu\text{m} \text{ for } p=1/2$$

As we might expect, the measured average lies between the Donath’s method predictions for  $p=1/2$  and  $p=3/4$ .