$$V = \{ H^1_R \mid w(0) = \mu \} \times \mathbb{R}$$

$$\langle -A(t) y, y \rangle_V + \lambda \|y\|_Y$$

$$= b \underbrace{\left(\frac{w(0)}{2}\right)^2}_{\mu} + \varepsilon \|\partial_x w\|^2_{L^2}$$

$\Leftrightarrow \| \ \|_{H^1}$

$$+ \frac{1}{2} \frac{\dot{b}}{b} \|y\|_Y + \lambda \|y\|$$

$\lambda > \frac{\dot{b}}{b}$, $b > \alpha$ $\dot{b}$ borné de signe quelconque

on a $-A$ $V-Y$ coercive (1.31) p 178 Da Prato Bensoussan.

$$w = \frac{u}{\sqrt{b}} \qquad \mu = \frac{m}{\sqrt{b}}$$

$$\partial_t w = b^{-1/2}\,\partial_t u - \frac{1}{2}\dot{b}\,b^{-3/2}\,u$$

$$= b^{-1/2}\left(b\,\partial_x u + \varepsilon\,\partial_{xx} u\right) - \frac{1}{2}\dot{b}\,b^{-3/2}\,u$$

$$= b\,\partial_x w + \varepsilon\,\partial_{xx} w - \frac{1}{2}\frac{\dot{b}}{b} w$$

$$\dot{\mu} = b^{-1/2}\dot{m} - \frac{1}{2}\dot{b}\,b^{-3/2}\,m$$

$$= b^{-1/2}(\partial_x u) - \frac{1}{2}\frac{\dot{b}}{b}\mu = \partial_x w - \frac{1}{2}\frac{\dot{b}}{b}\mu.$$

$$D(A(t)) = \{ H^1_R \quad w(0) = \mu \}$$

$$(A(t)y_1, y_2)_y = \left(b\,\partial_x w_1 + \varepsilon\,\partial^2_{xx} w_1 - \frac{1}{2}\frac{\dot{b}}{b}w_1, w_2\right)$$

$$+ \varepsilon\left(\partial_x w_1(0) - \frac{1}{2}\frac{\dot{b}}{b}\mu_1, \mu_2\right)$$

$$= b(\partial_x w_1, w_2) - \varepsilon(\partial_x w_1, \partial_x w_2)$$

$$- \frac{1}{2}\frac{\dot{b}}{b}(w_1, w_2) - \frac{1}{2}\frac{\dot{b}}{b}(\mu_1, \mu_2)$$

$$- \varepsilon\,\cancel{\partial_x w_1(0)\,w_2(0)} + \varepsilon\,\cancel{\partial_x w_1(0)\,\mu_2}$$