3D modelling of internal tides generation

Internal tides are waves generated by the interaction of tidal currents with seafloor topography. Understanding their energetic directional dependence is crucial to identify areas of enhanced mixing. We present here a semi-analytical method using Green's functions for 3D modelling of internal tides generation, **not restricted to weak topographies.**

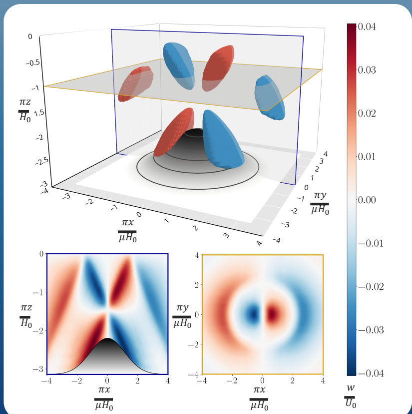


Figure 2 : Internal waves vertical velocity field for the test case, reconstructed with 20 modes: isocontours $w/U_0 = \pm 0.03$ (top); horizontal and vertical slices (bottom)

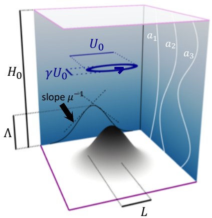
Cécile LE DIZES*, Matthieu MERCIER*, Nicolas GRISOUARD**, Olivier THUAL*

*Institut de Mécanique des fluides de Toulouse, **University of Toronto

The problem to solve

Internal tide generated on idealized topography, under Boussinesq assumption:

$$\mu^2(z)\,\nabla_H^2 w - w_{,zz} = 0 \quad \text{with } \mu^{-1}(z) = \sqrt{\frac{\omega^2-f^2}{N(z)^2-\omega^2}} \text{ the beams' slope.} \qquad (1)$$



Tidal forcing

$$\vec{U}_0 = U_0 e^{-i\omega t}(\vec{e}_x + i\gamma\vec{e}_y)$$

Boundary conditions

- Top-lid : $w_{|z=0} = 0$
- Bottom : $w_{|z=-H_0+h} = (\vec{U}_0 + \vec{u}_H).\vec{\nabla}_H h$

Vertical modes

$$[w, f^2N^{-2}b](\vec{r},z) = \sum [w_n, b_n](\vec{r})\, a_n(z)$$
$$[u,v,p](\vec{r},z) = \sum k_n^{-1}[u_n, v_n, p_n](\vec{r})\, a_n{}'(z)$$

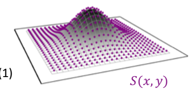
with $\frac{d^2a_n}{dz^2} + \mu^2(z)k_n^2 a_n = 0.$

Boundary integral equation

The topography is associated with an **unknown distribution** $S(x,y)$ of source points, which is used as a forcing term in equation (1).

- Solution of the problem:

$$w = \mathcal{G} * S$$
$$[u,v,p] = [\mathcal{G}_u, \mathcal{G}_v, \mathcal{G}_p] * S$$



where $\mathcal{G}$ is the Green's function associated with (1) and $\mathcal{G}_u, \mathcal{G}_v, \mathcal{G}_p$ given functions.

- Computing the source distribution:

$$\boxed{\vec{U}_0.\vec{\nabla}_H h_{|\vec{r}} = [\mathcal{G} - h_{,x}(\vec{r})\,\mathcal{G}_u - h_{,y}(\vec{r})\,\mathcal{G}_v] * S} \qquad (2)$$

Test case : Internal waves generated by an unidirectional tide on a subcritical Gaussian with height ratio $\delta = \Lambda/H_0 = 0.3$ and criticality $\epsilon = 0.5$.

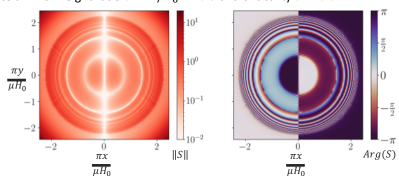


Figure 1 : Source distribution for the test case, solved with 160 modes.

Influence of Coriolis frequency on energy flux direction

- **Depth-integrated energy flux:**

$$\vec{J} = \sum \vec{J}_n = \sum \Re e\left\{\frac{\Gamma_n}{2} p_n^* u_n\right\} \qquad \text{with } \vec{u}_n = -i\begin{pmatrix} p_{n,r} + i\frac{f}{\omega}\frac{p_{n,\theta}}{r} \\ \frac{p_{n,\theta}}{r} - i\frac{f}{\omega}p_{n,r}\end{pmatrix}_{(\vec{e}_r,\vec{e}_\theta)}$$

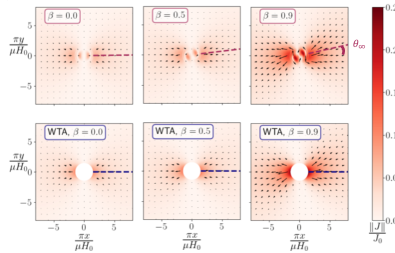


Figure 3 : Energy density flux for $\delta = 0.3$ and $\epsilon = 0.5$, reconstructed with 10 modes.

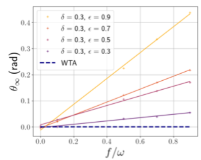
Far from the topography, using asymptotic expansion :

$$\vec{J}_n = \frac{\Gamma_n}{16\,\pi k_n r}\left\{\left(k_n|f_n|^2(\theta) + \frac{f}{\omega}\frac{\Re e\left(f_n^*(\theta)f_n'(\theta)\right)}{r}\right)\vec{e}_r + \frac{f}{\omega}\frac{|f_n|^2(\theta)}{2r}\vec{e}_\theta\right\}$$

with $f_n(\theta) = \mathcal{F}[\sigma_n](k_n\vec{e}_r)$ the Fourier transform of $\sigma_n = i\frac{a_n(h(\vec{r}))}{k_n\Gamma_n}S(\vec{r})$.

For **WTA :** $\sigma_n^{WTA} = i\frac{a'_n(-1)}{k_n\Gamma_n}\vec{U}_0.\vec{\nabla}_H h$ and $f_n^{WTA}(\theta) = C_n\cos\theta.$

- **Direction of maximum radial flux:**



Shifted from the barotropic tide direction, and controlled by :

- f/ω
- the shape of the topography

Conclusion and perspectives

- Efficient model for 3D generation of internal tide
- Significant influence of Coriolis frequency on the direction of propagation
- Consider asymmetrical topographic effects.
- Investigate the effects of elliptic tide and variable stratification