Density layering in rotating stratified turbulence

Cécile Le Dizes, Jim McElwaine, Claudia Cenedese, Pascale Garaud

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Motivation

Mixing in the ocean / atmosphere / stars

- Small-scale mixing events necessary to close the global buoyancy budget.

- But expensive to model : large range of scales

- Some characteristics :
  - Anisotropy and intermittency
  - Layering

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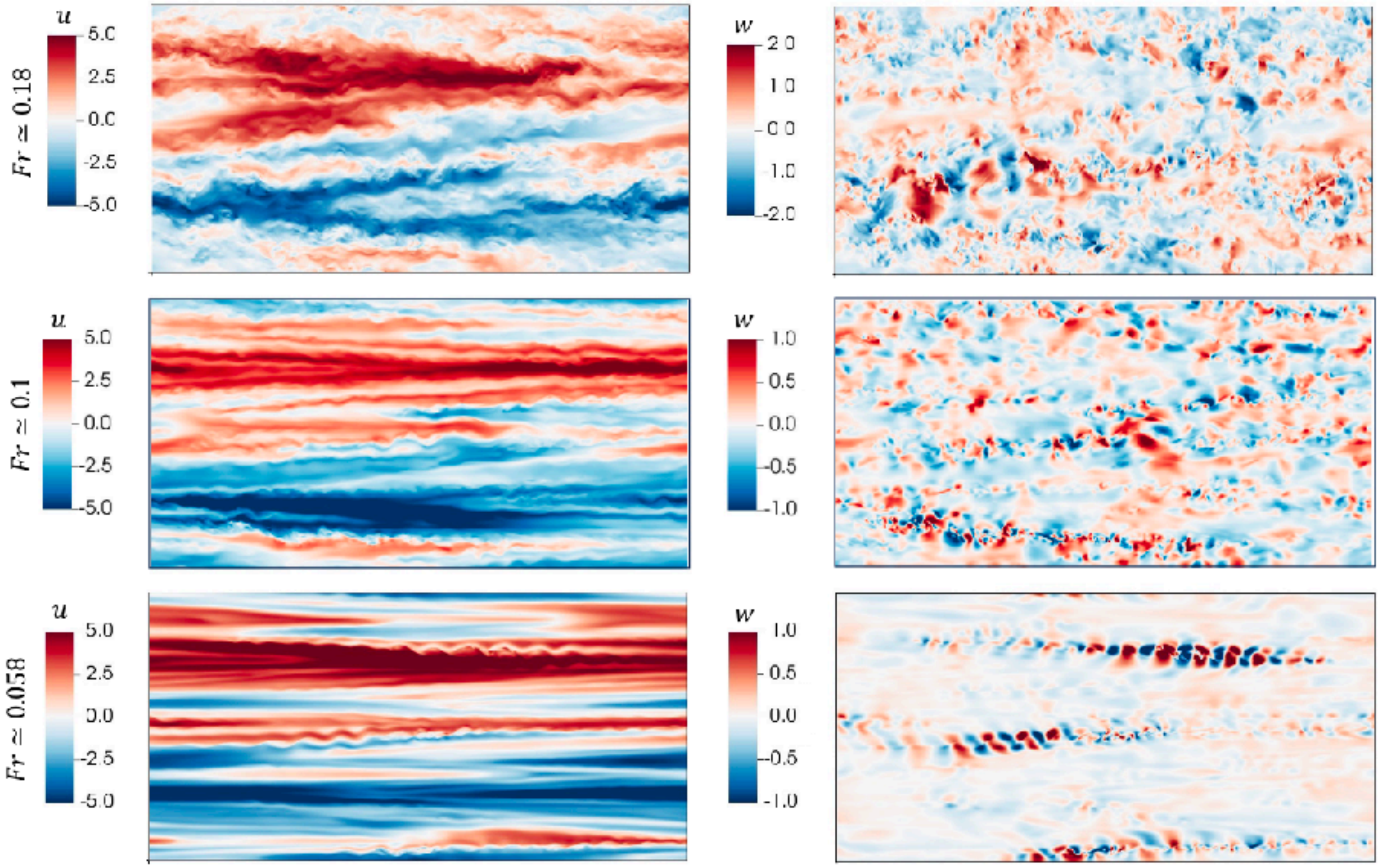
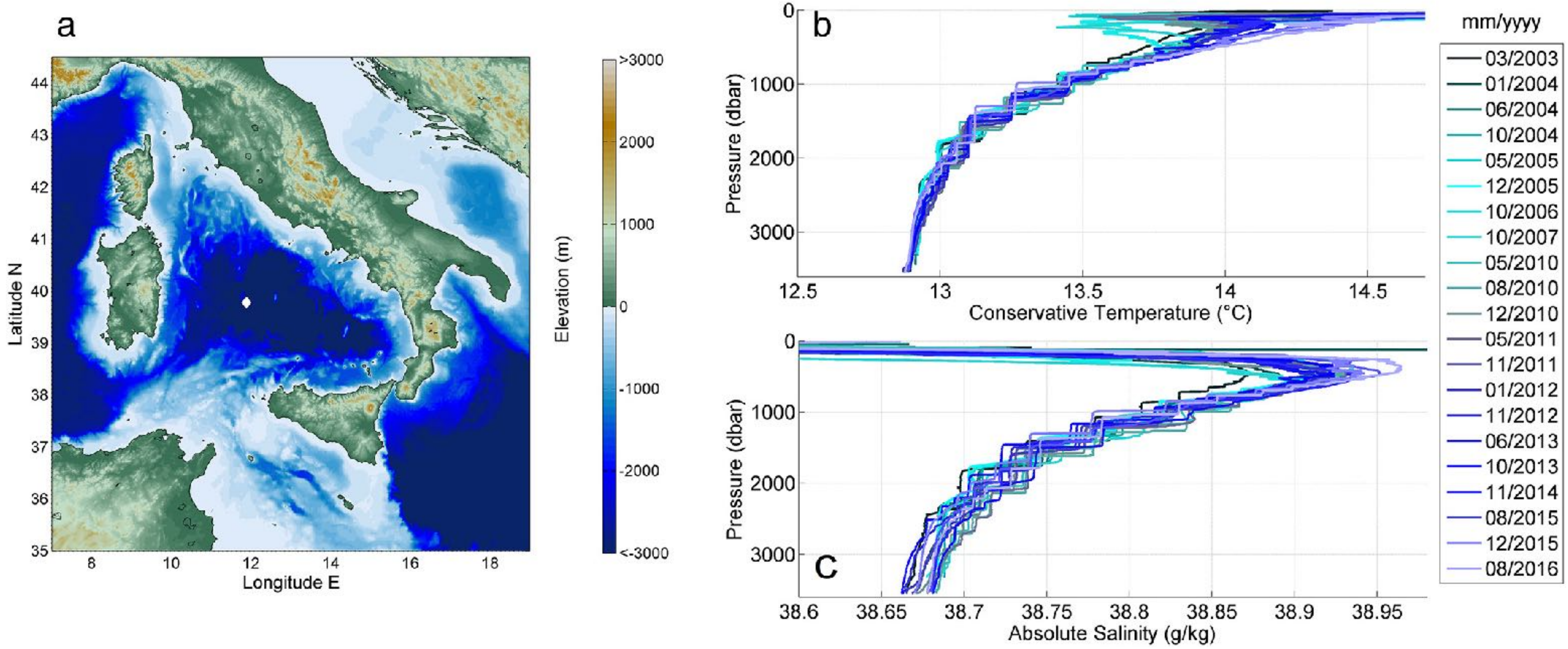


Horizontal slice of u and w for Re = 1000, Pe = 100, with increasing stratification increasing, from Garaud et al (2024)

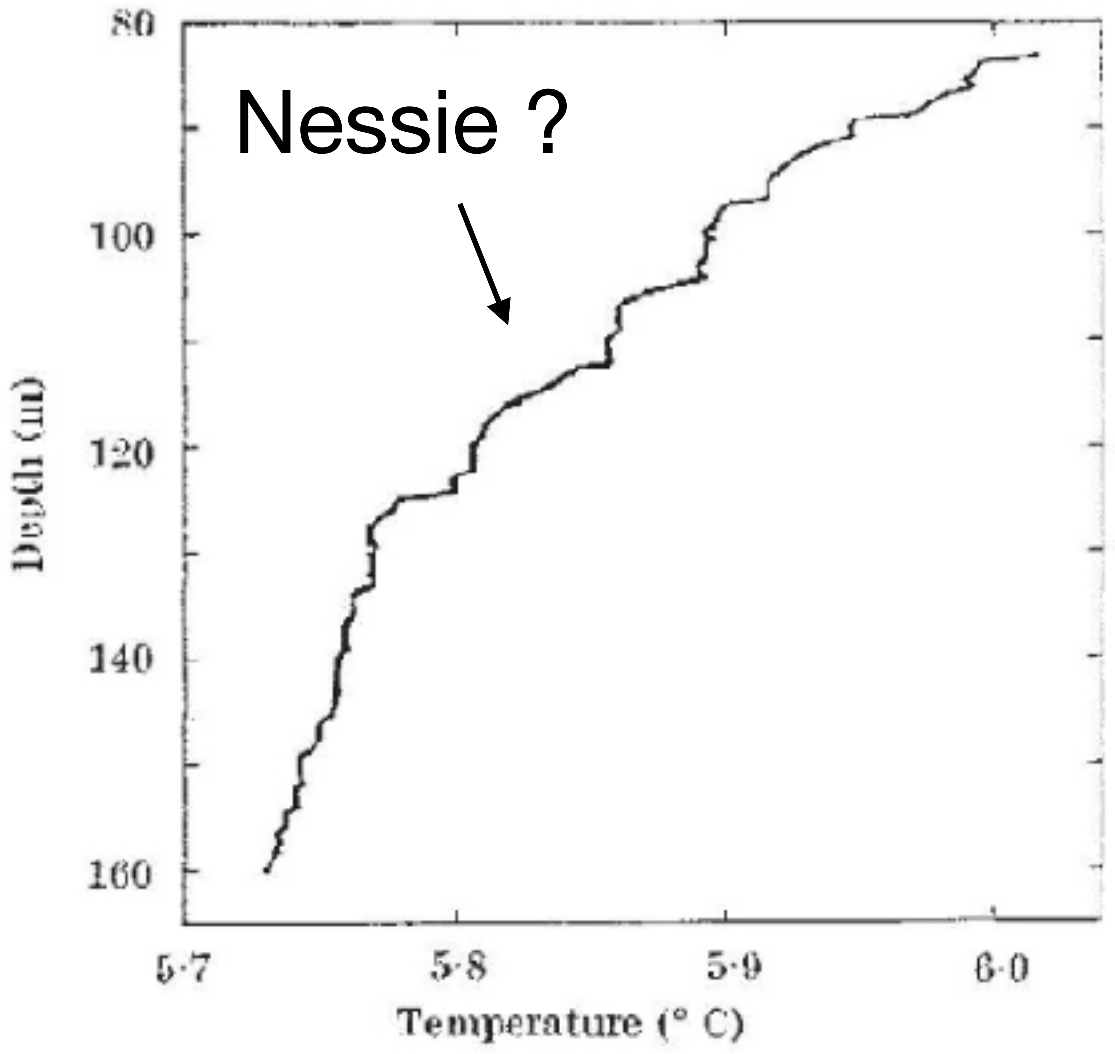
Staircases in the ocean

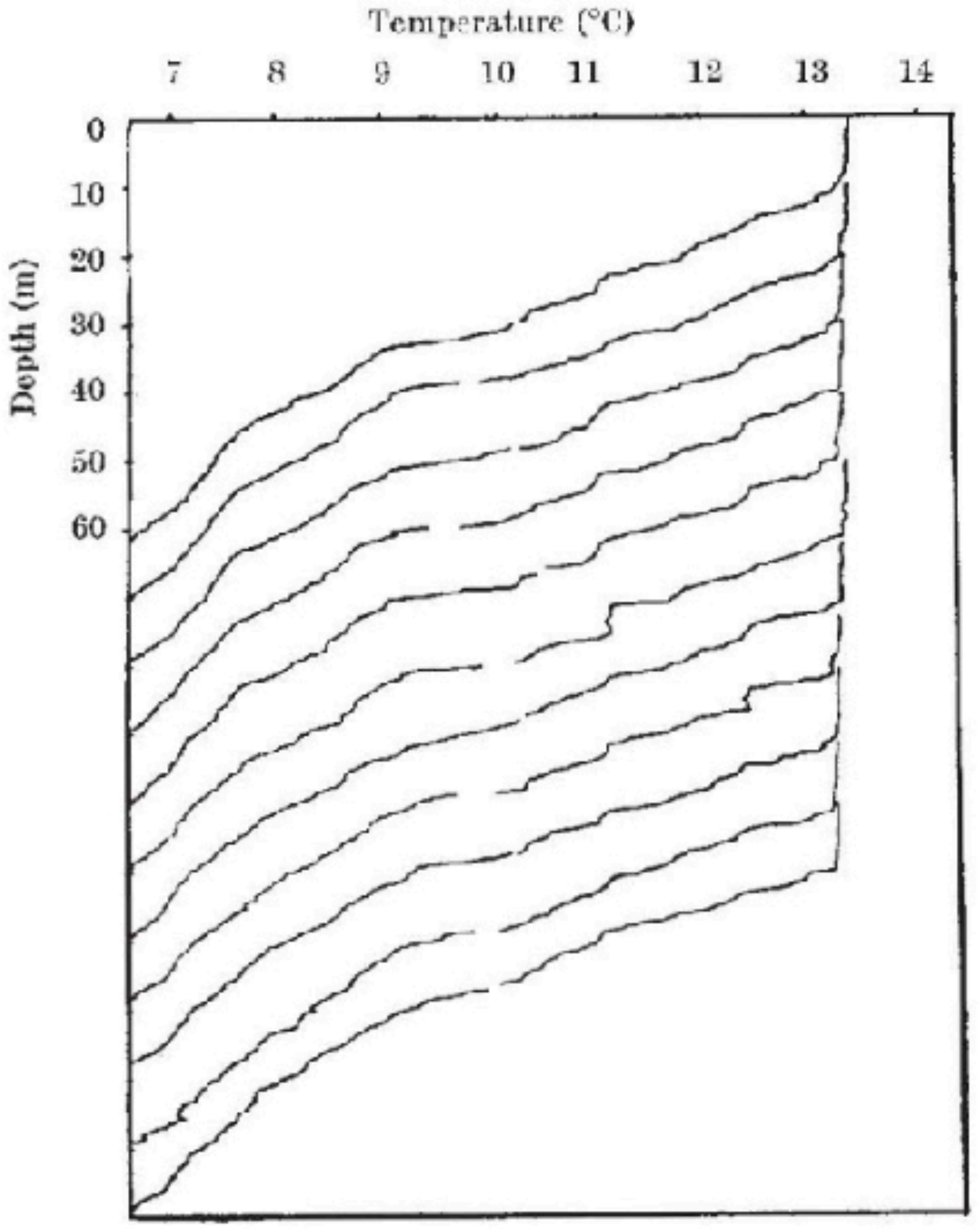
Double-diffusivity cases



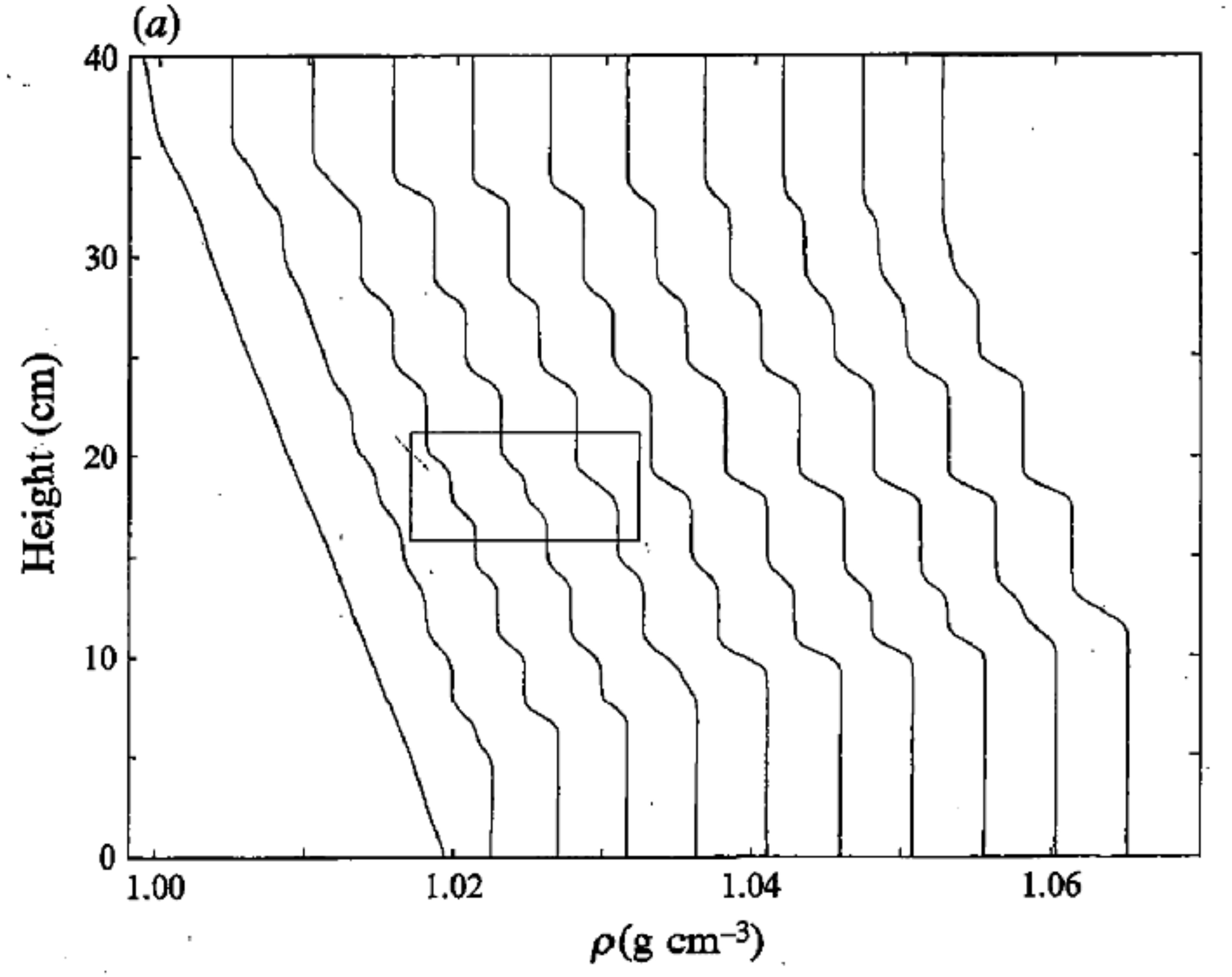
Profiles of conservative temperature (°C) and absolute sanity (g/kg), in the central Tyrrhenian station (from Durante et al (2019))

Staircases
Single-diffusivity cases





Temperature profiles measured in the Loch Ness, from Simpson & Woods (1970)



Density profiles of stably stratified fluid stirred with a vertical rod, from Park et al (1994)

Staircases

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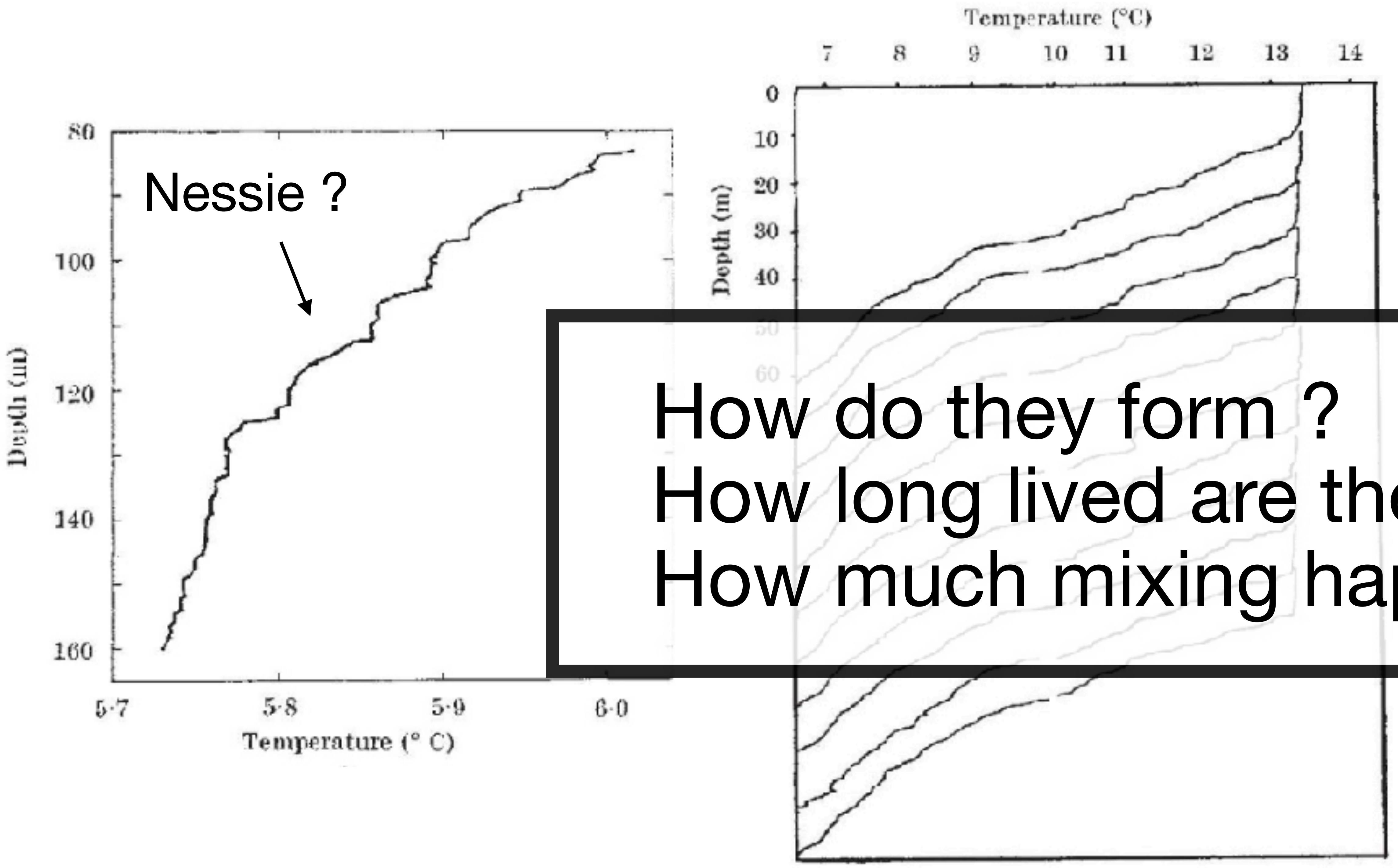
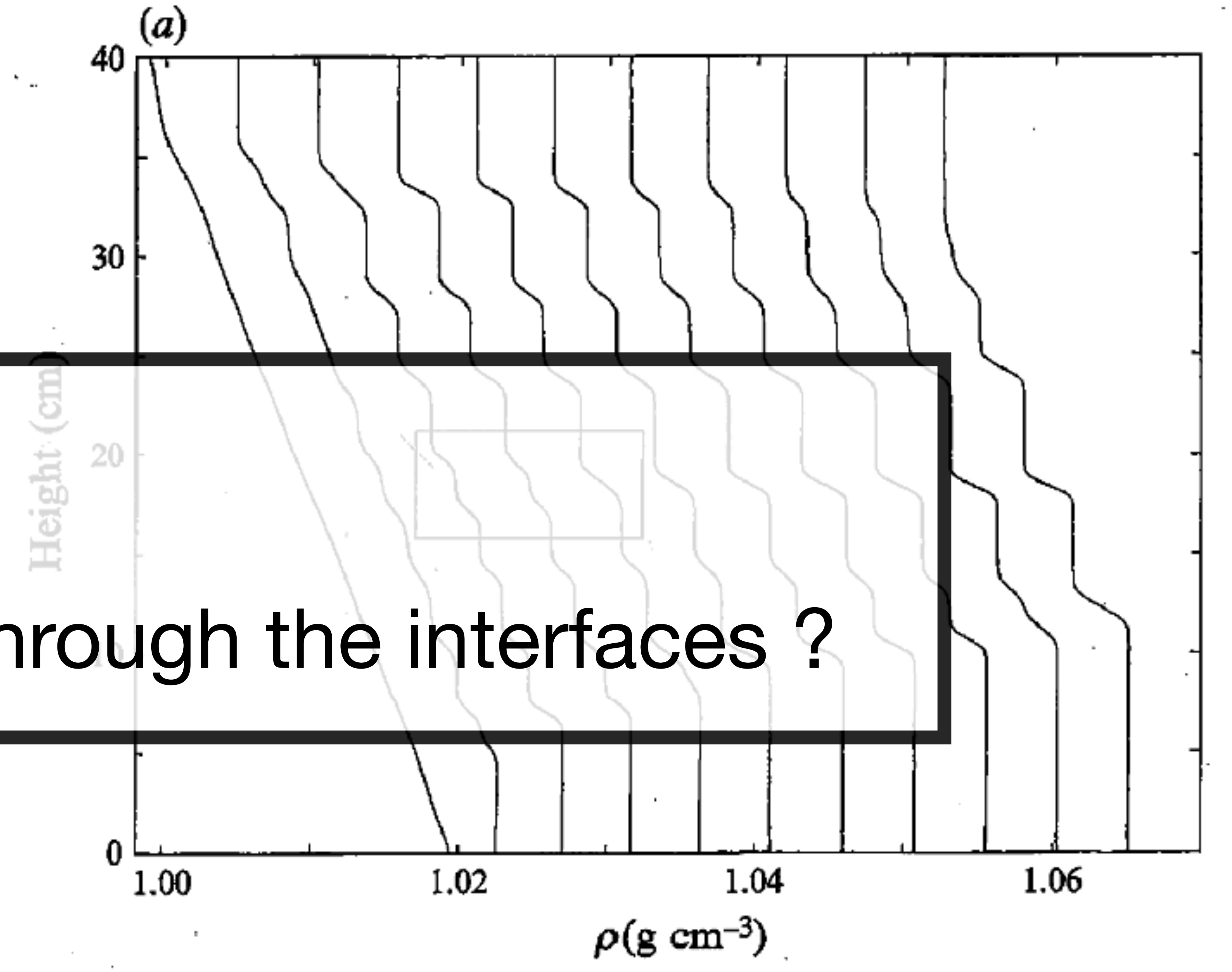




How do they form ?
How long lived are they ?
How much mixing happens through the interfaces ?

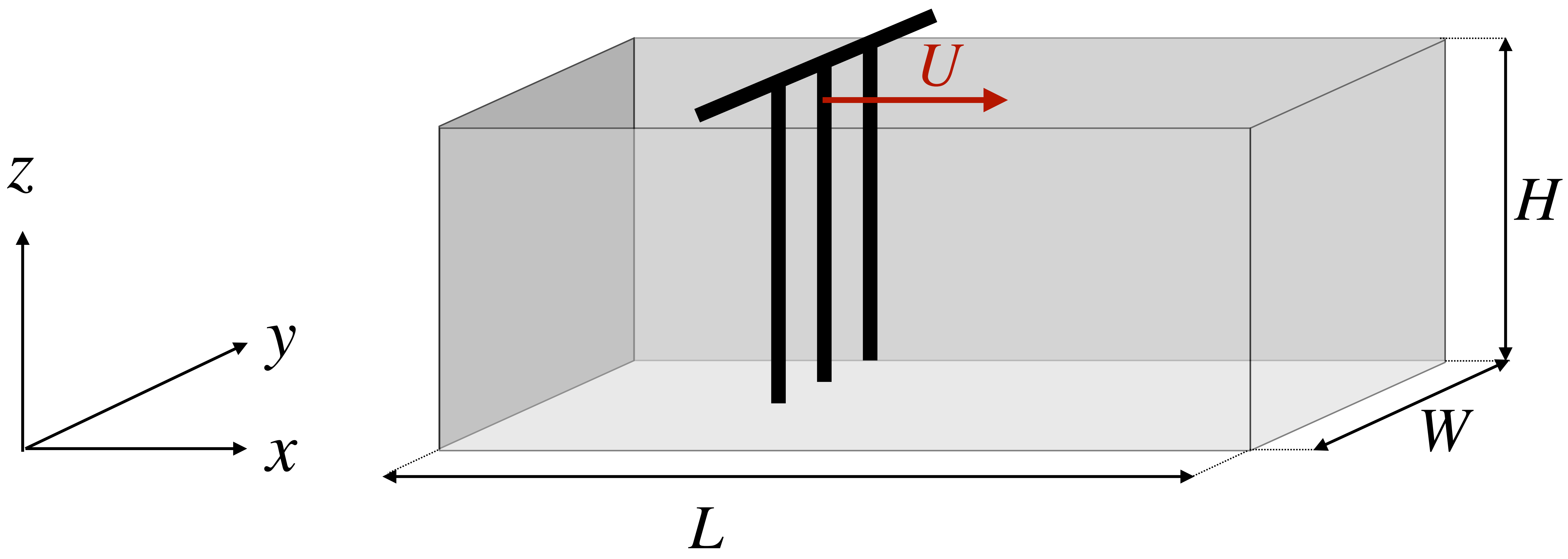
Temperature profiles measured in the Loch Ness, from Simpson & Woods (1970)

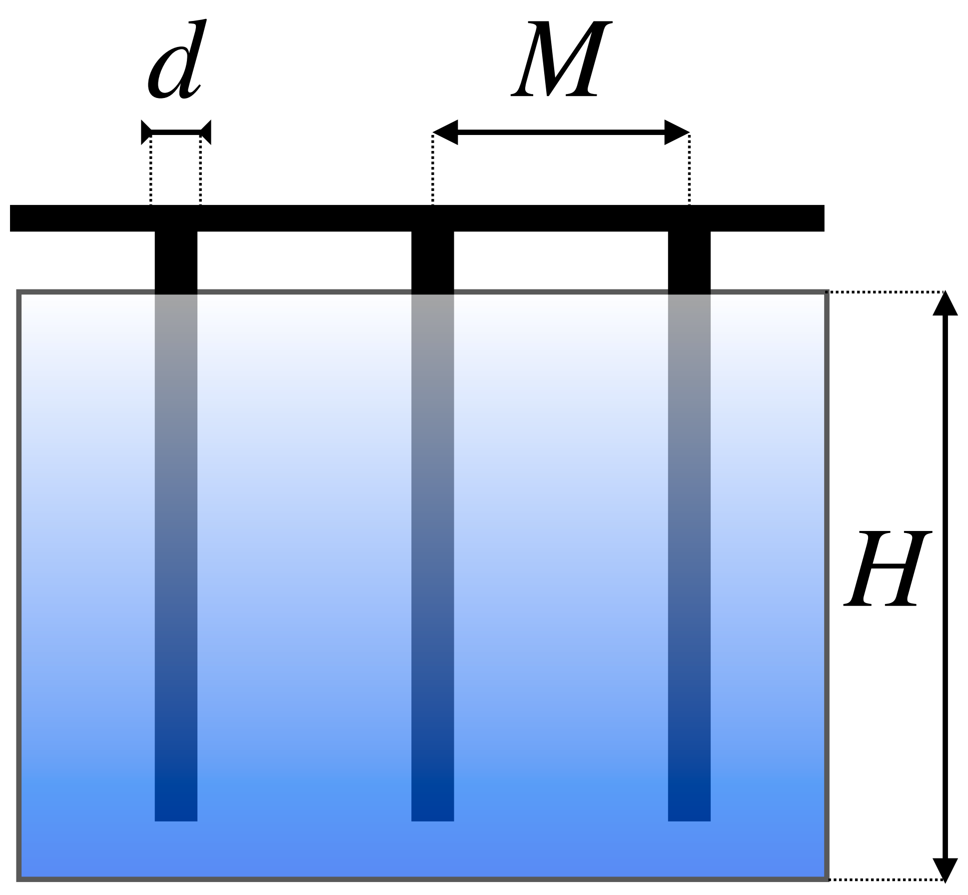
Density profiles of stably stratified fluid stirred with a vertical rod, from Park et al (1994)

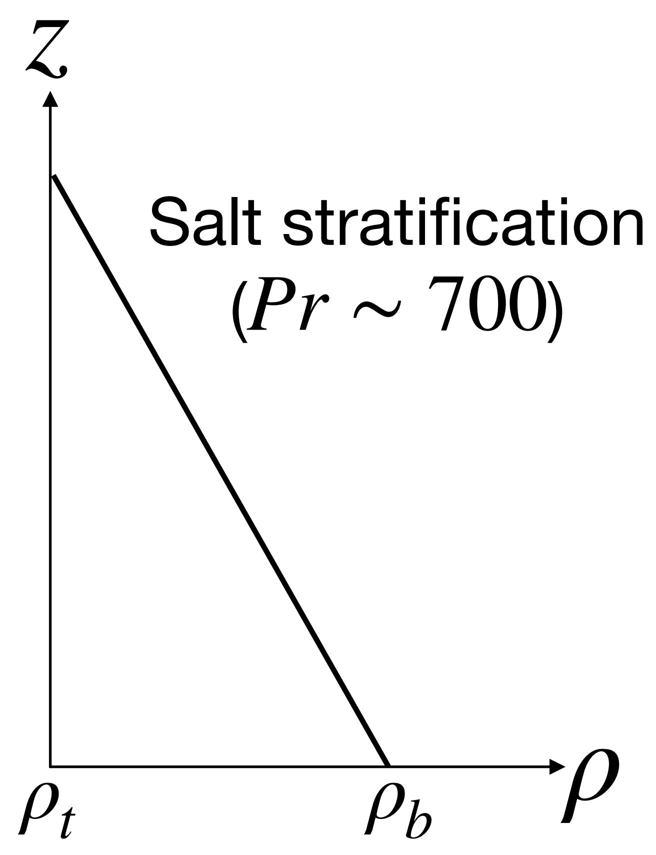
Original experiments

Set-up

Ruddick et al (1989)
Park et al (1994)
Holford & Linden (1999)







Length scales
$d,\ M,\ L,\ H,\ W$

Velocity
U

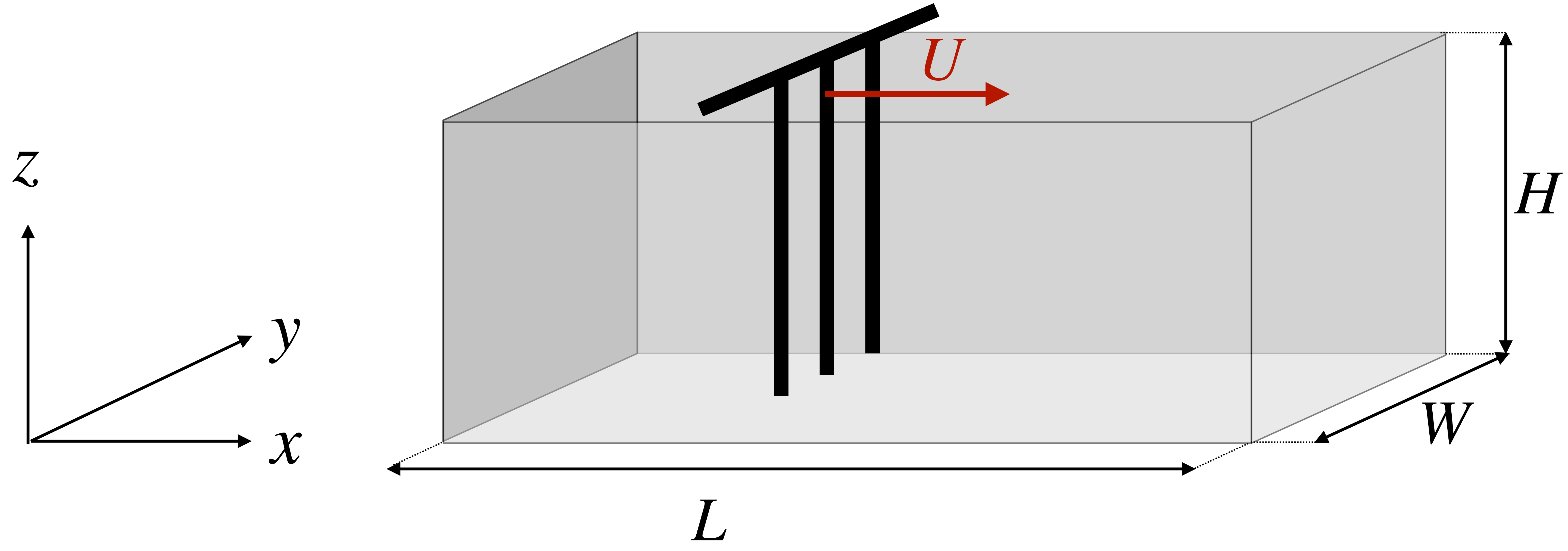
Buoyancy gradient
$$N_0^2 \equiv -\frac{g}{\rho_0}\frac{\rho_t - \rho_b}{H}$$

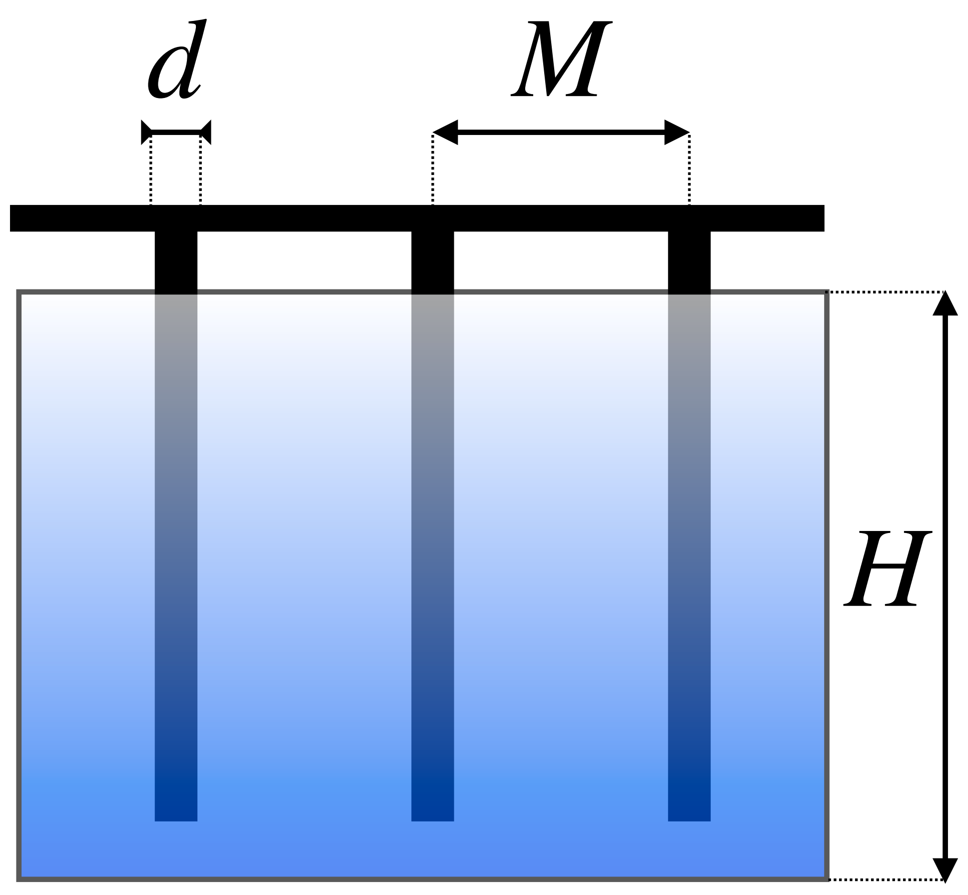
Others
ν, κ

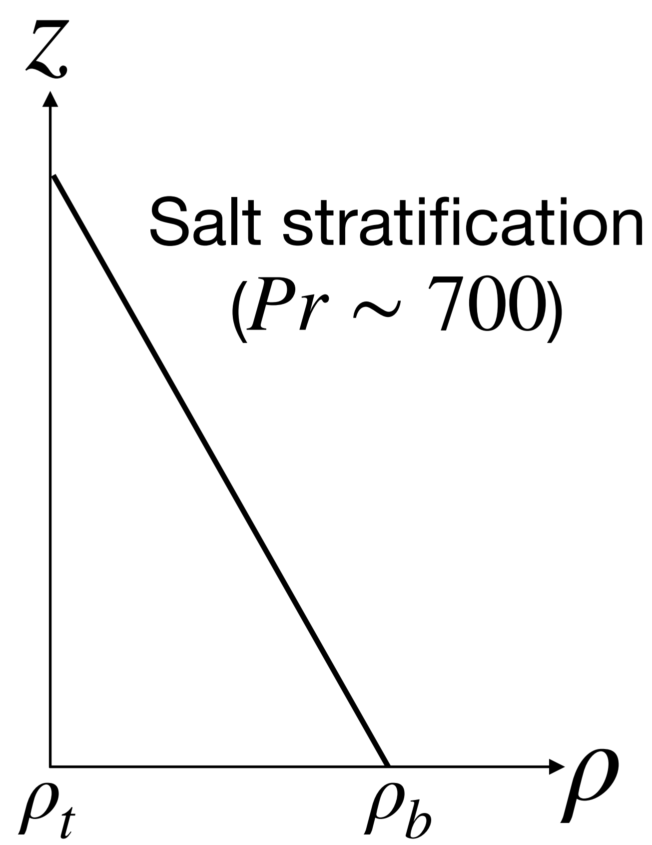
Original experiments

Dimensionless parameters

$$U_0 \equiv U(1 - d/M);\; L_0 \equiv \sqrt{dM};\; N_0^2 \equiv -\frac{g}{\rho_0}\frac{\rho_t - \rho_b}{H}$$







Grid solidity $S = d/M$	Reynold number $Re_0 = U_0 L_0/\nu$	Richardson number $Ri_0 = N_0^2 L_0^2/U_0^2$	Prandtl number $Pr \equiv \frac{\nu}{\kappa} = 700$

Original experiments

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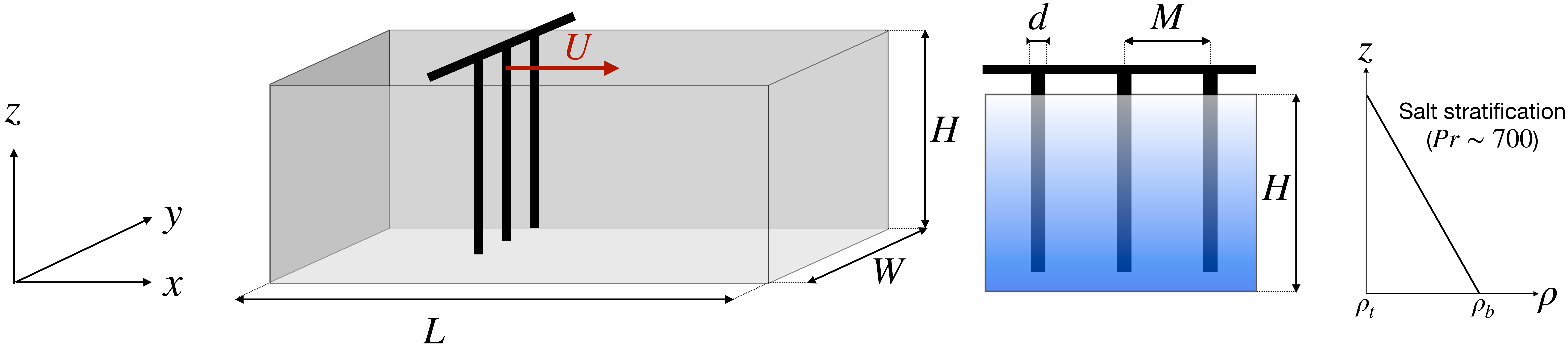

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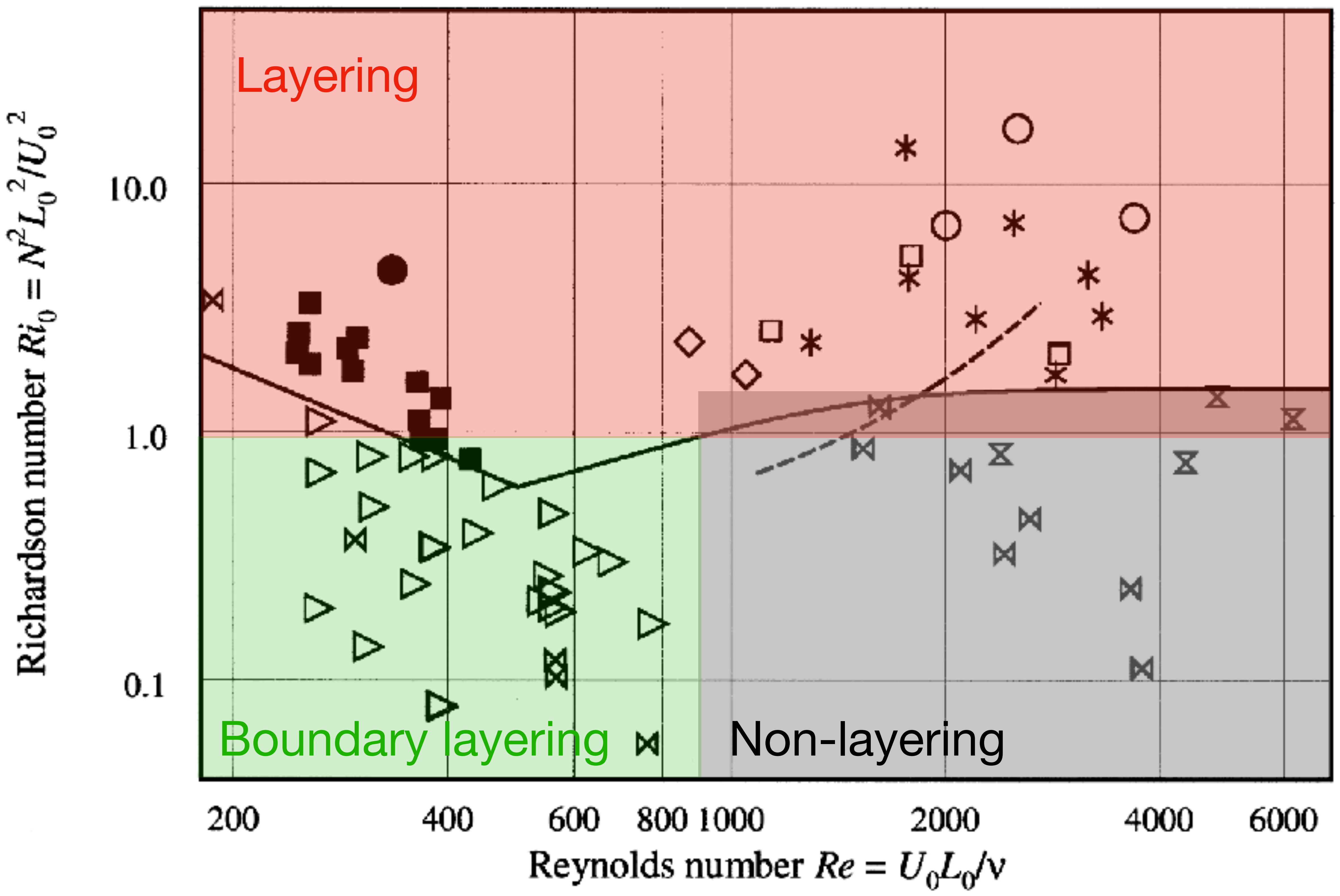
Original experiments

Layering



Regime diagram, from Holford & Linden (1999)

Symbol	d (cm)	M (cm)	S	n_{bars}	L
■	0.65	3.25	0.20	8	5–20 M
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⋈ ⧗ Non-layering experiments for experiments * and for the others

▷ Boundary layering

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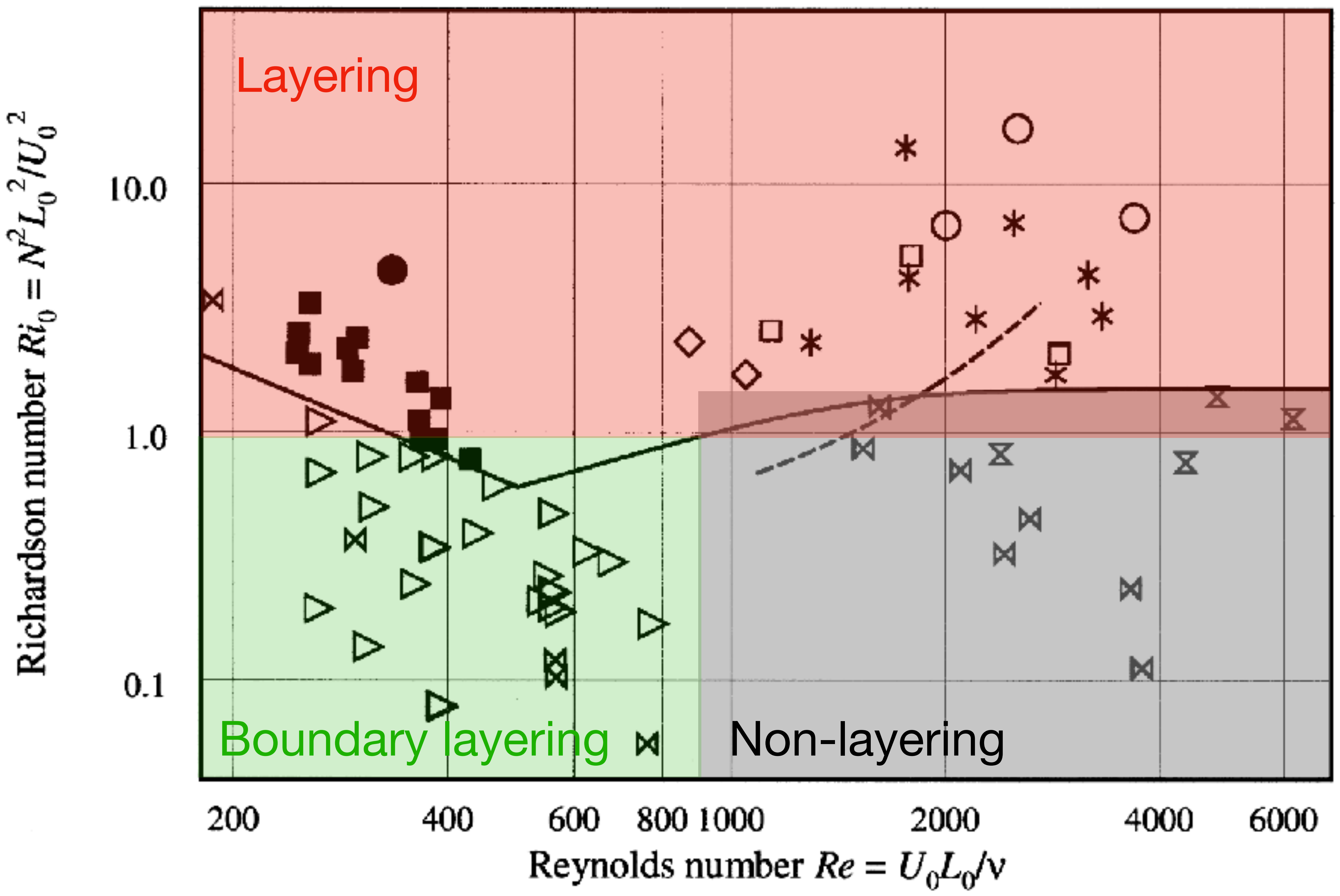

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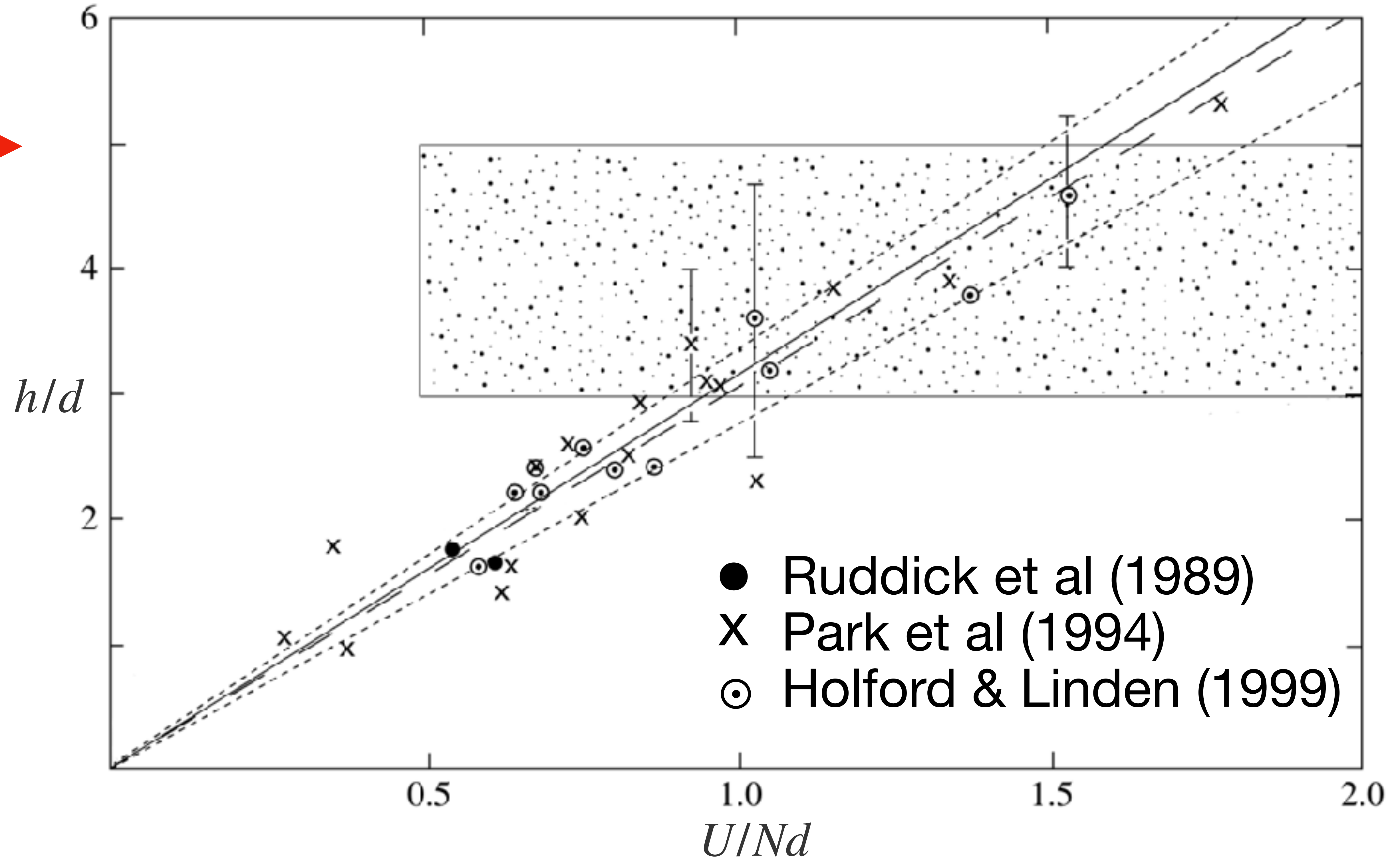


Variation of the layer thickness h/d with $U/Nd = Fr_h$, adapted from Thorpe (2016)

Possible mechanism for layer formation

Phillips / Posmentier 'instability'

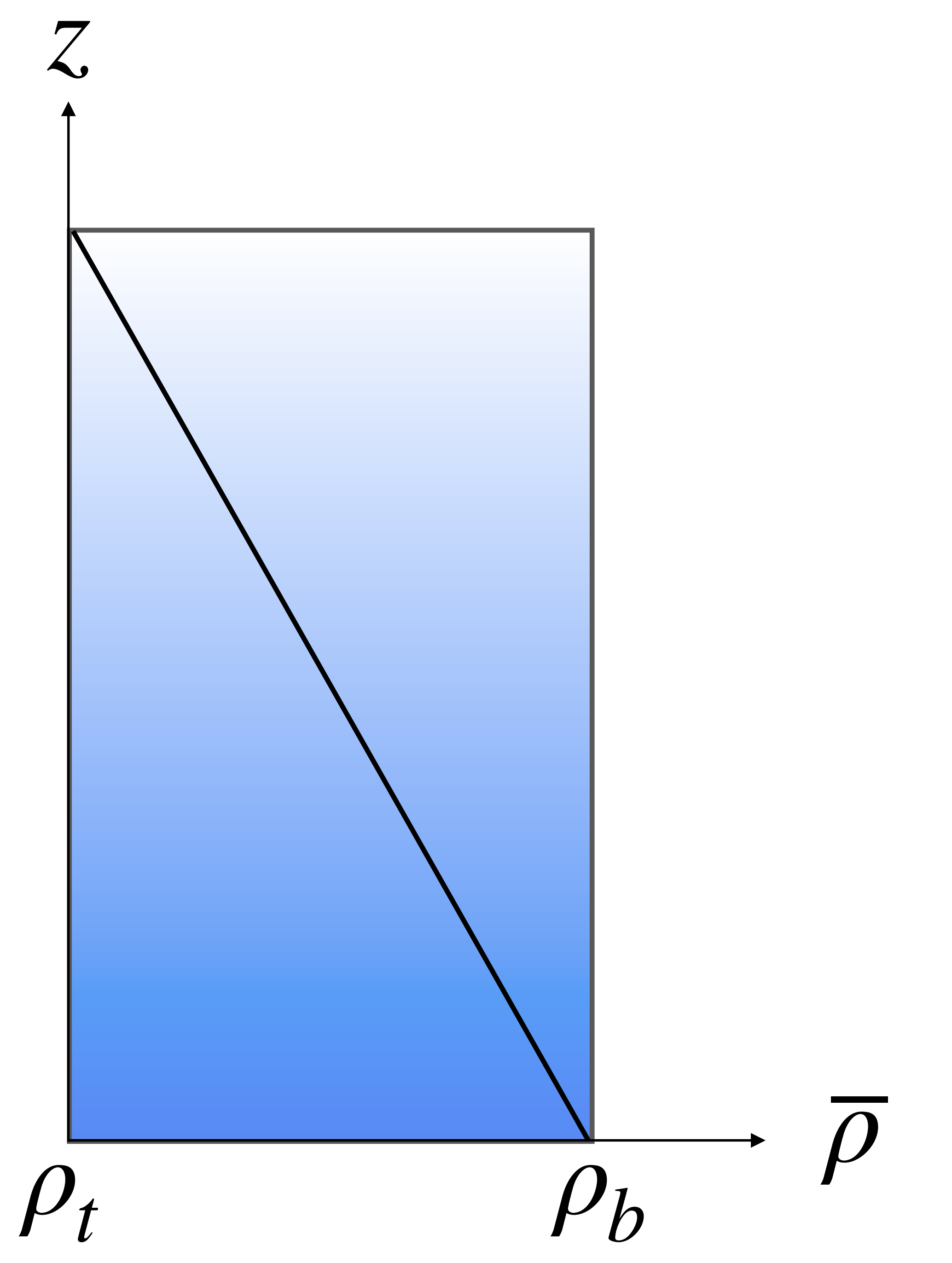
Consider a linear mean density profile $N^2 = -\dfrac{g}{\rho_0}\dfrac{d\bar{\rho}}{dz}$.

The buoyancy equation for $b = -g(\rho - \rho_0)/\rho_0$ gives :

$$\frac{\partial b}{\partial t} + \mathbf{u}\,.\,\nabla b = \kappa \nabla^2 b$$

Averaging horizontally : $\dfrac{\partial \bar{b}}{\partial t} + \dfrac{\partial}{\partial z}\overline{wb} = \kappa \dfrac{\partial^2 \bar{b}}{\partial z^2}$

$$\Rightarrow \quad \frac{\partial \bar{b}}{\partial t} = \frac{\partial \mathcal{F}_b}{\partial z} \quad \text{with} \quad \mathcal{F}_b = \kappa \frac{\partial \bar{b}}{\partial z} - \overline{wb}$$



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Phillips / Posmentier 'instability'

If we assume $\mathscr{F}_b = \mathscr{F}_b\left(\frac{\partial \bar{b}}{\partial z}\right)$, then

$$\frac{\partial \bar{b}}{\partial t} = \mathscr{F}'_b\left(\frac{\partial \bar{b}}{\partial z}\right)\frac{\partial^2 \bar{b}}{\partial z^2}$$

If $\mathscr{F}'_b\left(\frac{\partial \bar{b}}{\partial z}\right) < 0$, the system is anti-diffusive and we have density layering.

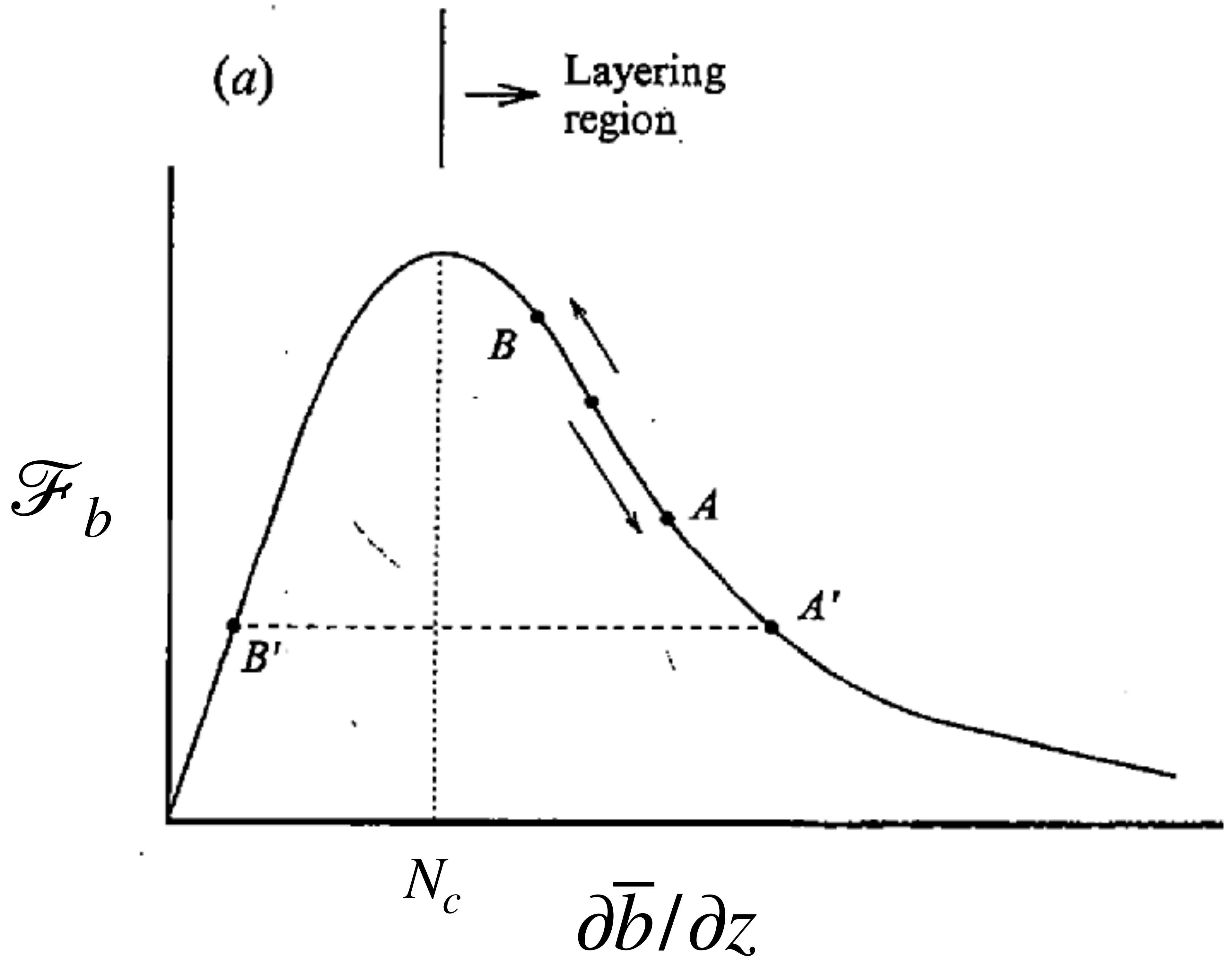

Figure adapted from Park et al (1994)

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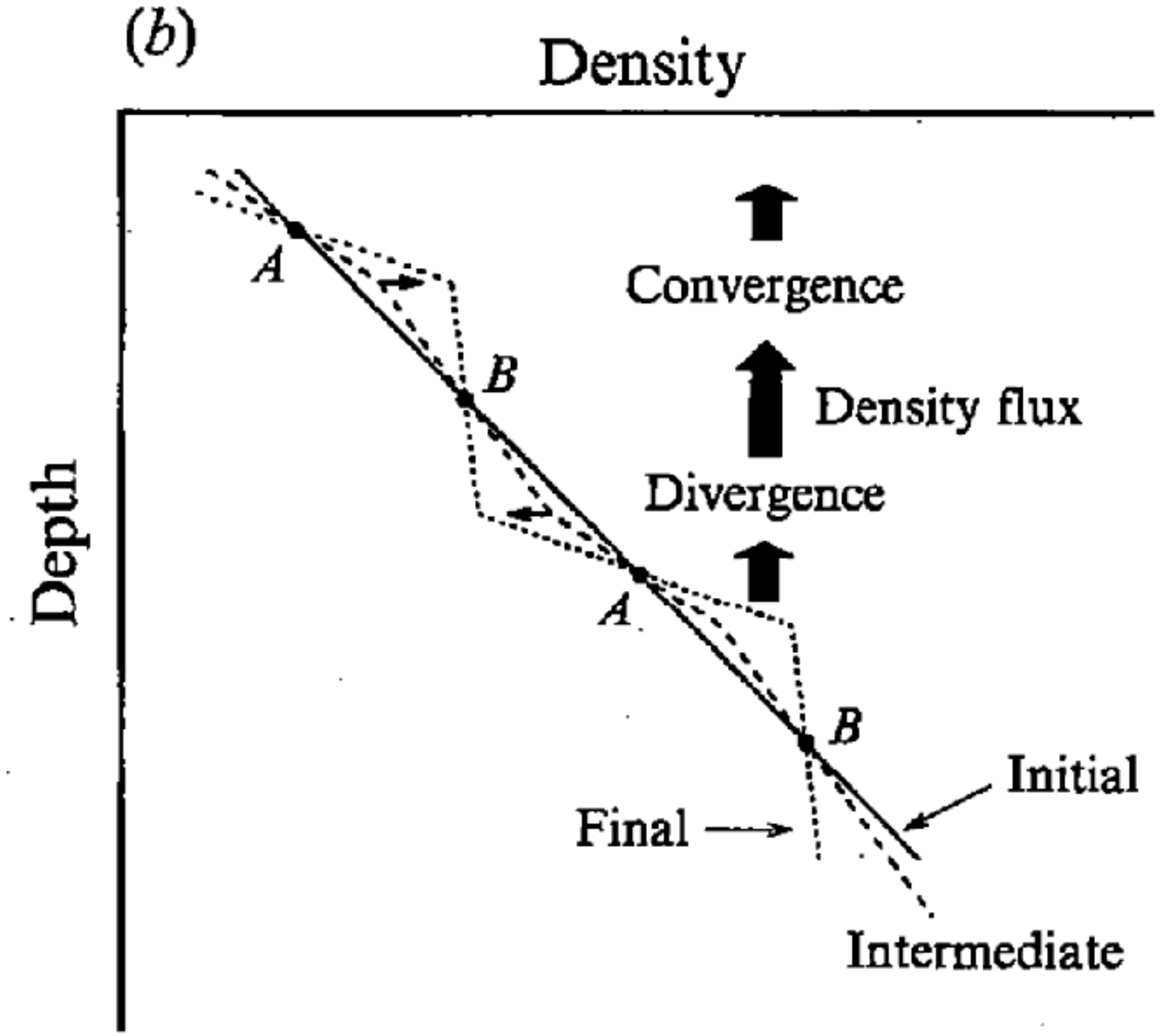

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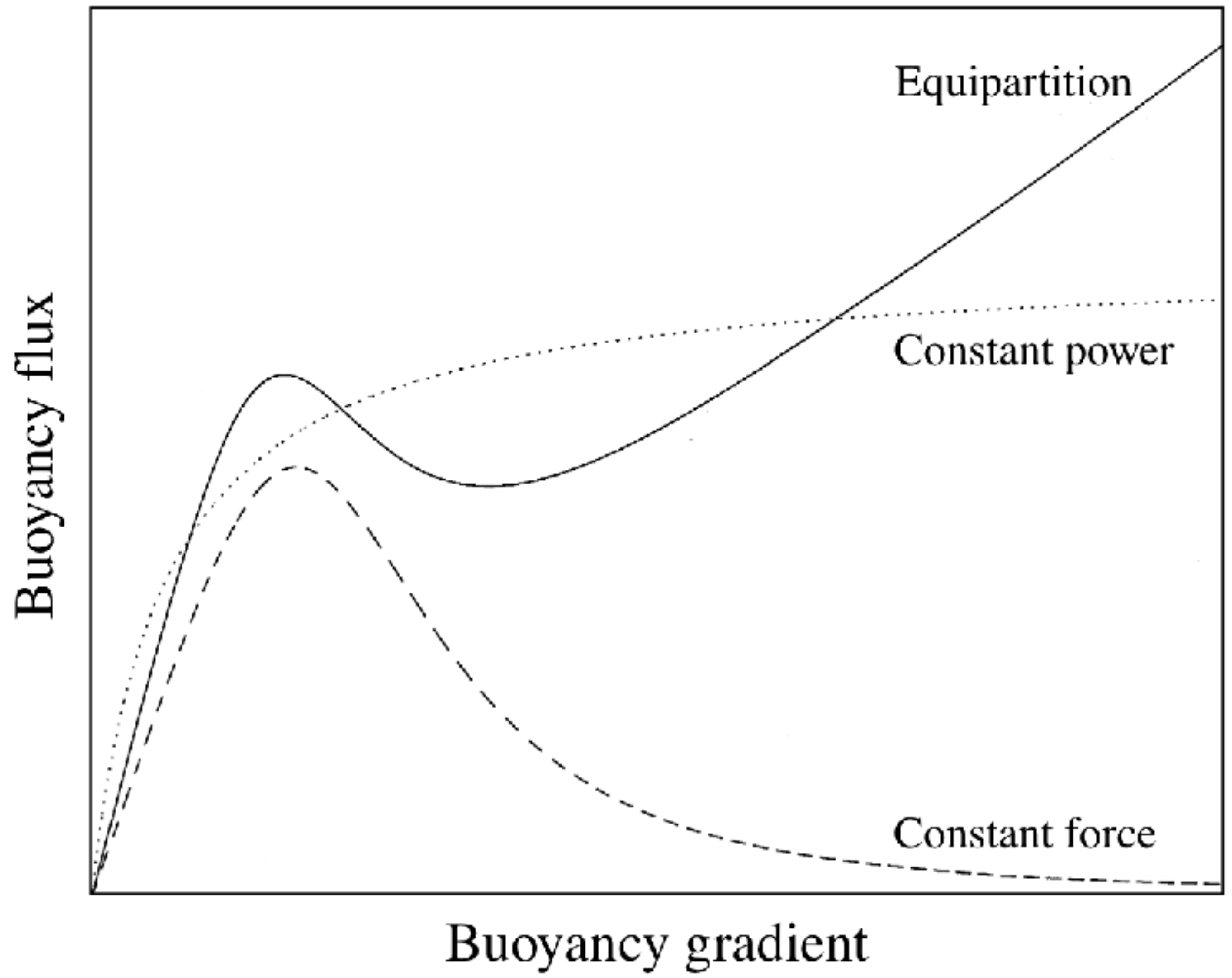
Possible mechanism for layer formation

Balmforth, Llewellyn-Smith and Young



N-shaped flux gradient law, from Balmforth et al (1997)

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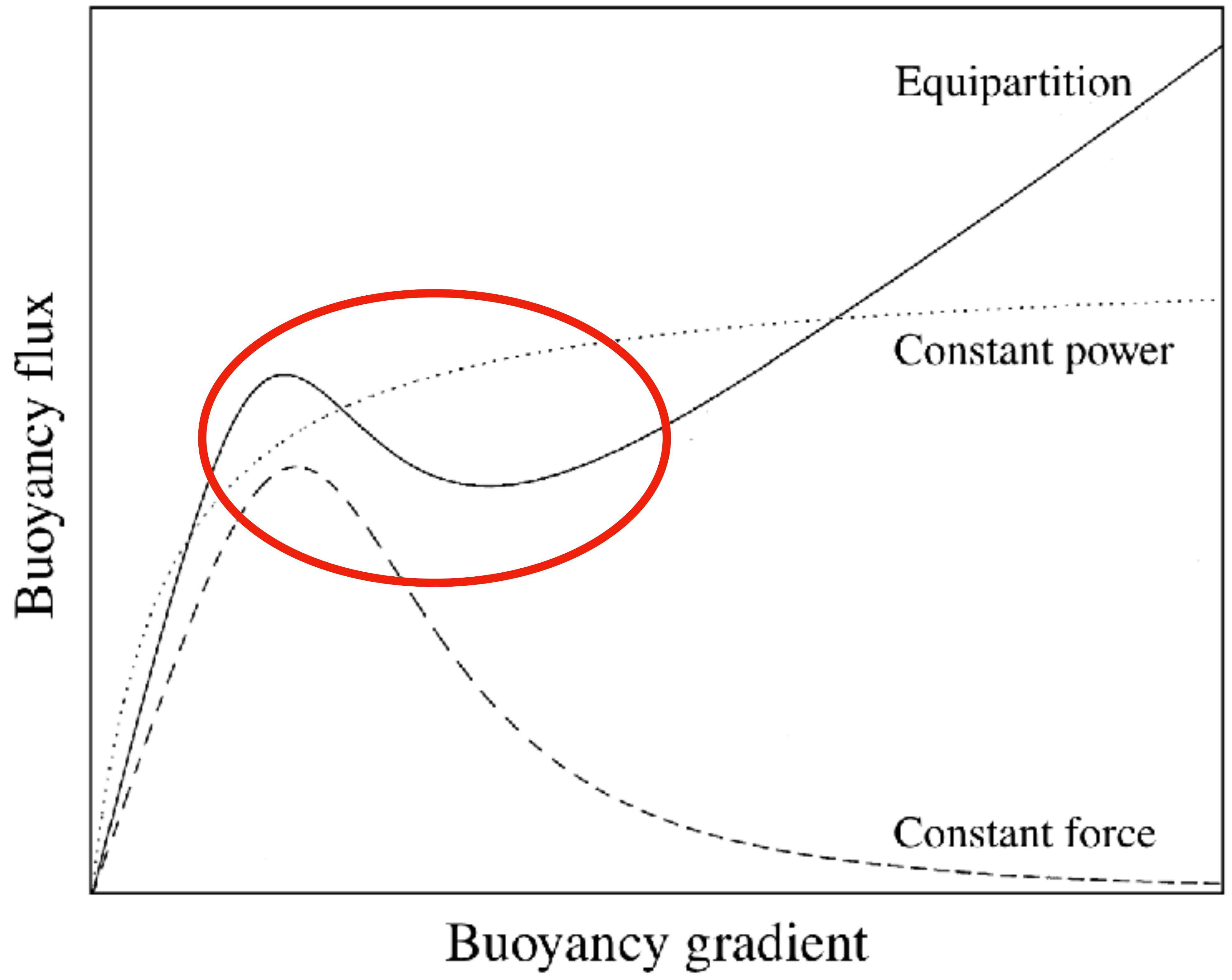

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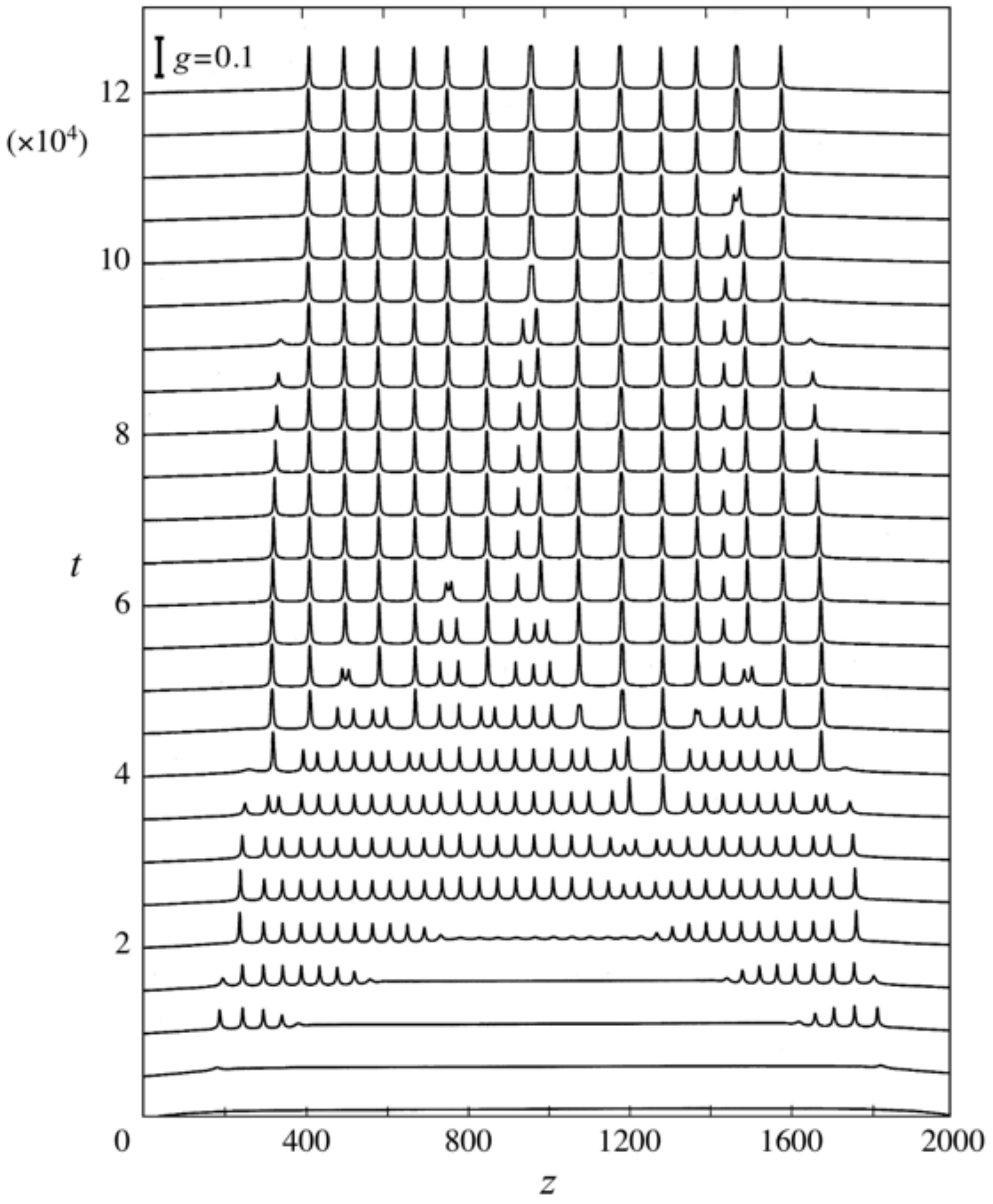
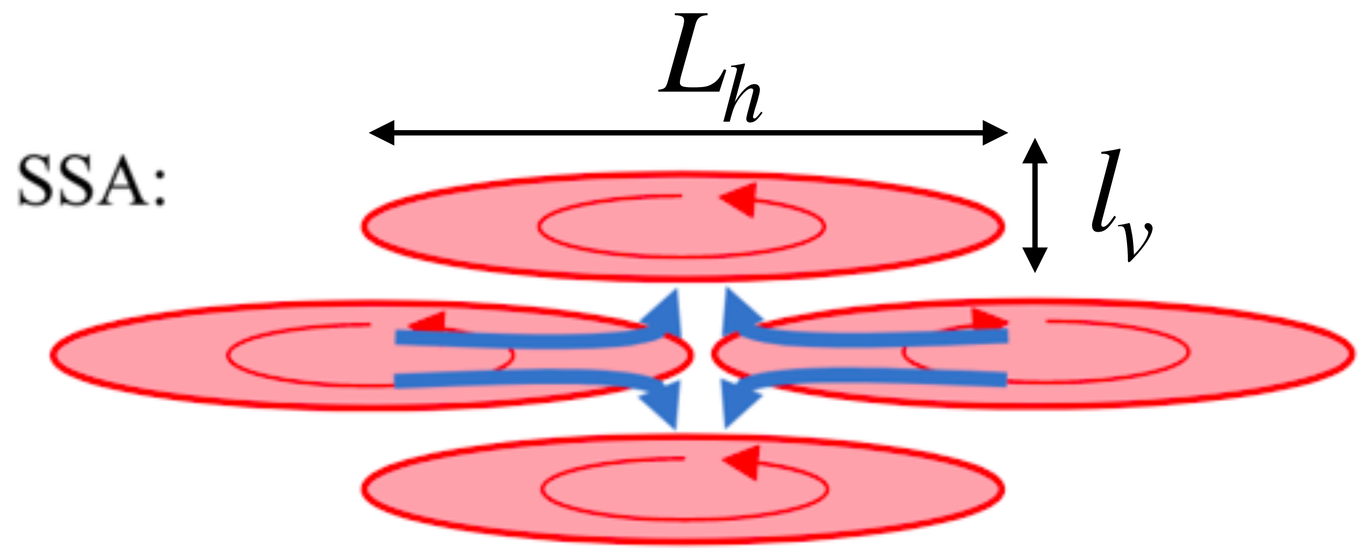


Scaling of the layer thickness

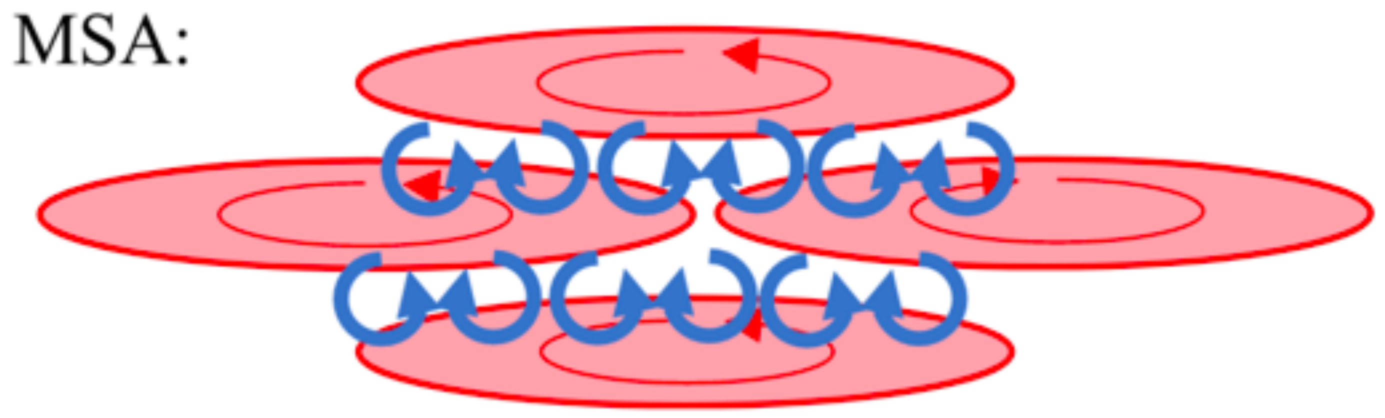
Asymptotic analysis for $Fr_h = U/NL_h \ll 1;\ Re = UL_h/\nu \gg 1$

- Single scale analysis : Billant & Chomaz (2001)



$$l_v \sim U/N$$
$$w \sim U^2/NL_h$$
$$b \sim UN$$

- Multi-scale analysis : Chini et al (2022)

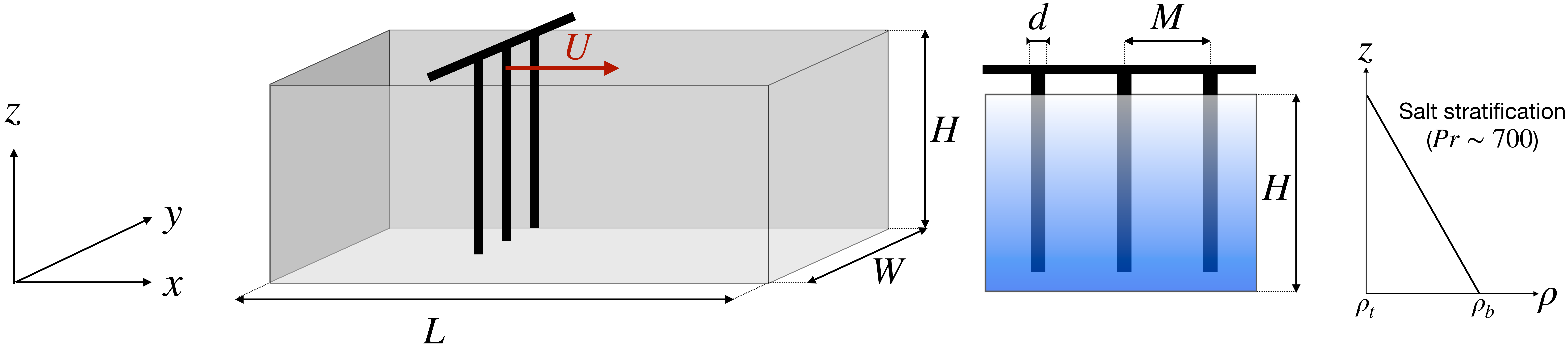


$$l_v \sim U/N$$
$$w' \sim U^{3/2}(NL_h)^{-1/2} \qquad \Rightarrow \quad \overline{b'w'} \sim U^3/L_h$$
$$b' \sim U^{3/2}N^{1/2}L_h^{-1/2}$$

The non-rotating lab experiment

The experiments

Dimensionless parameters



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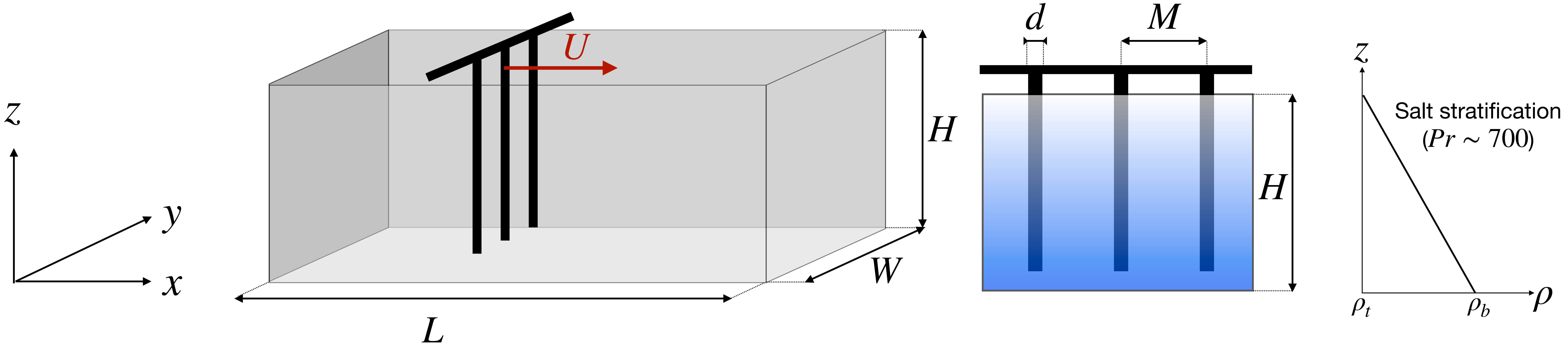

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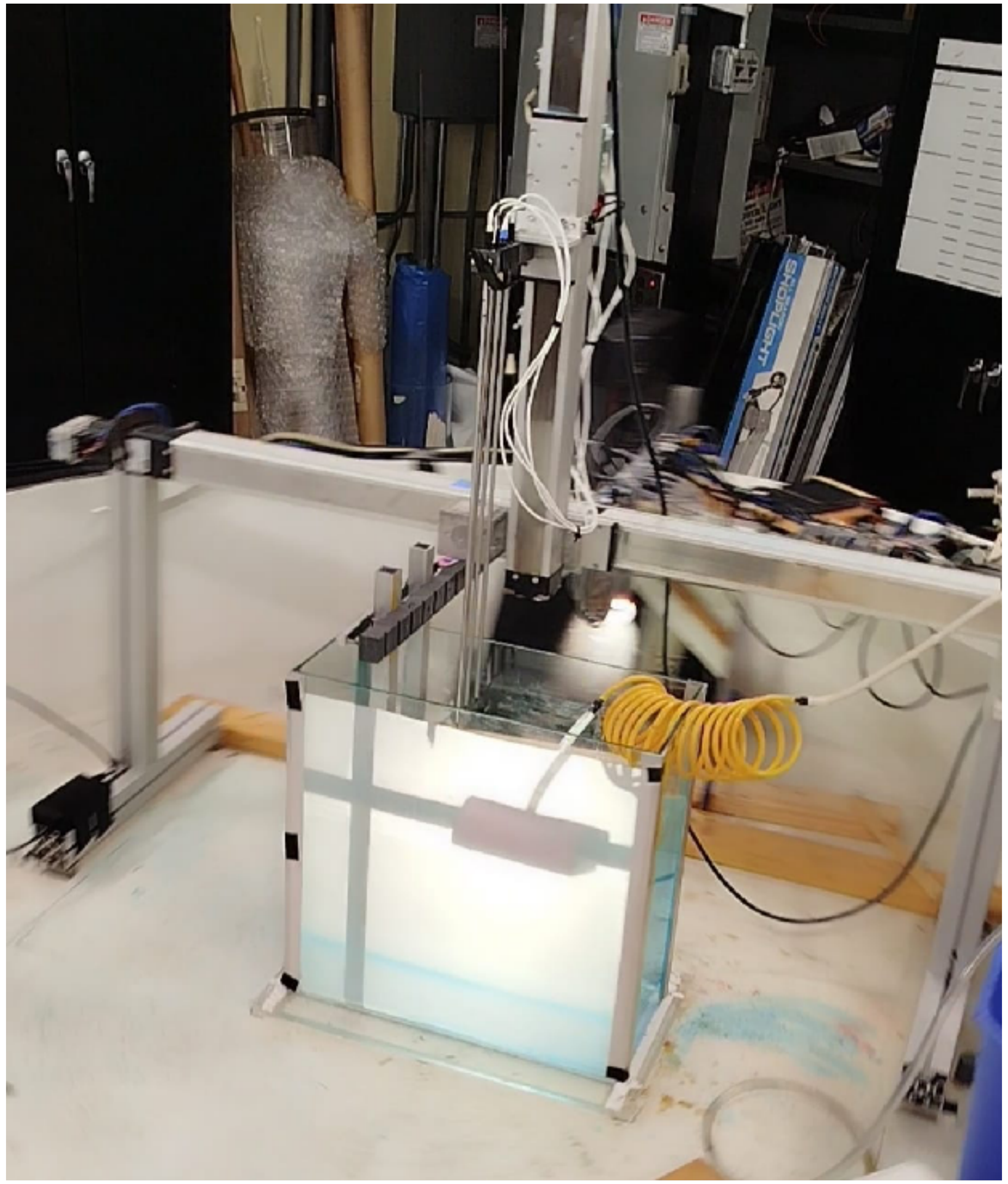

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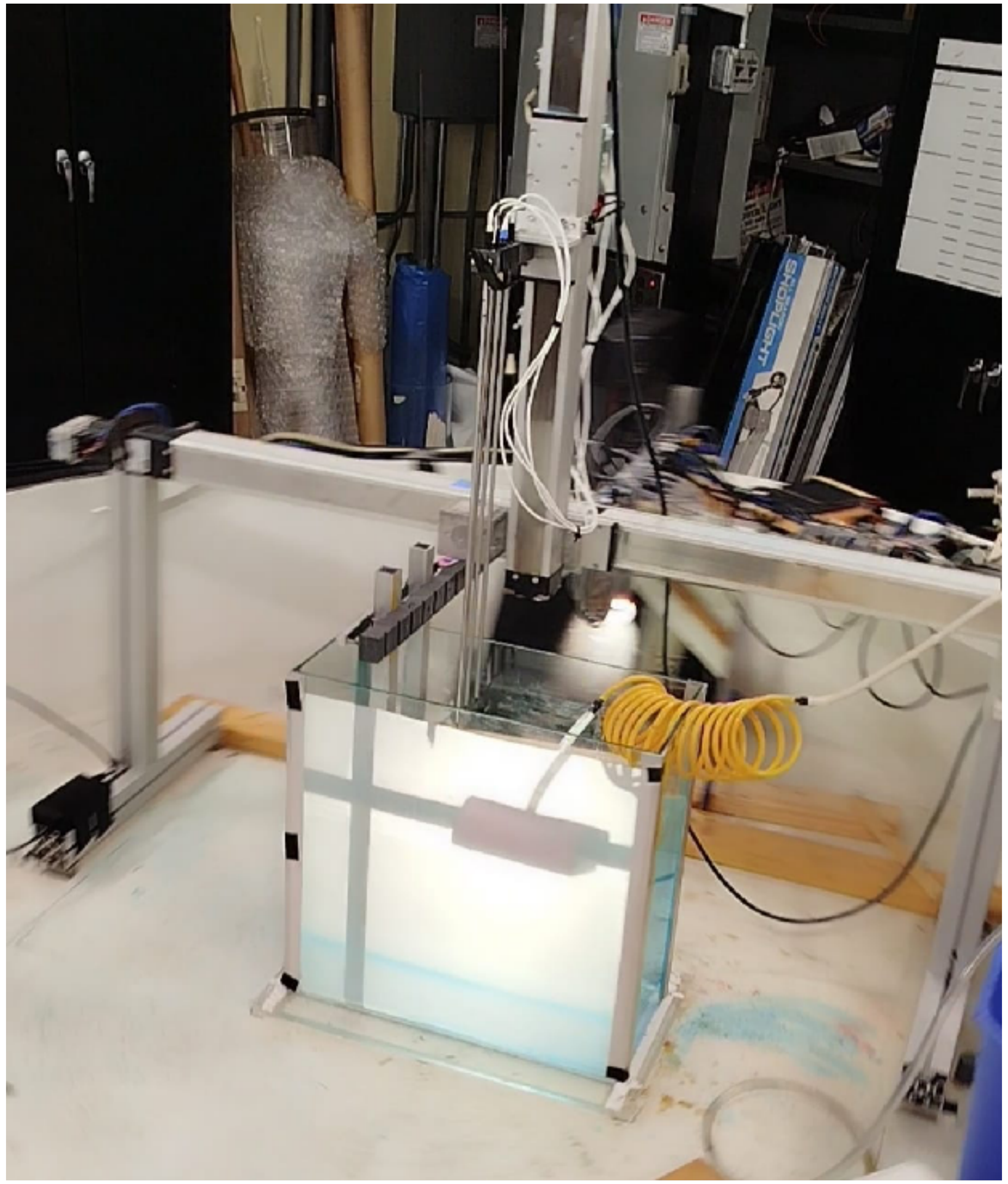
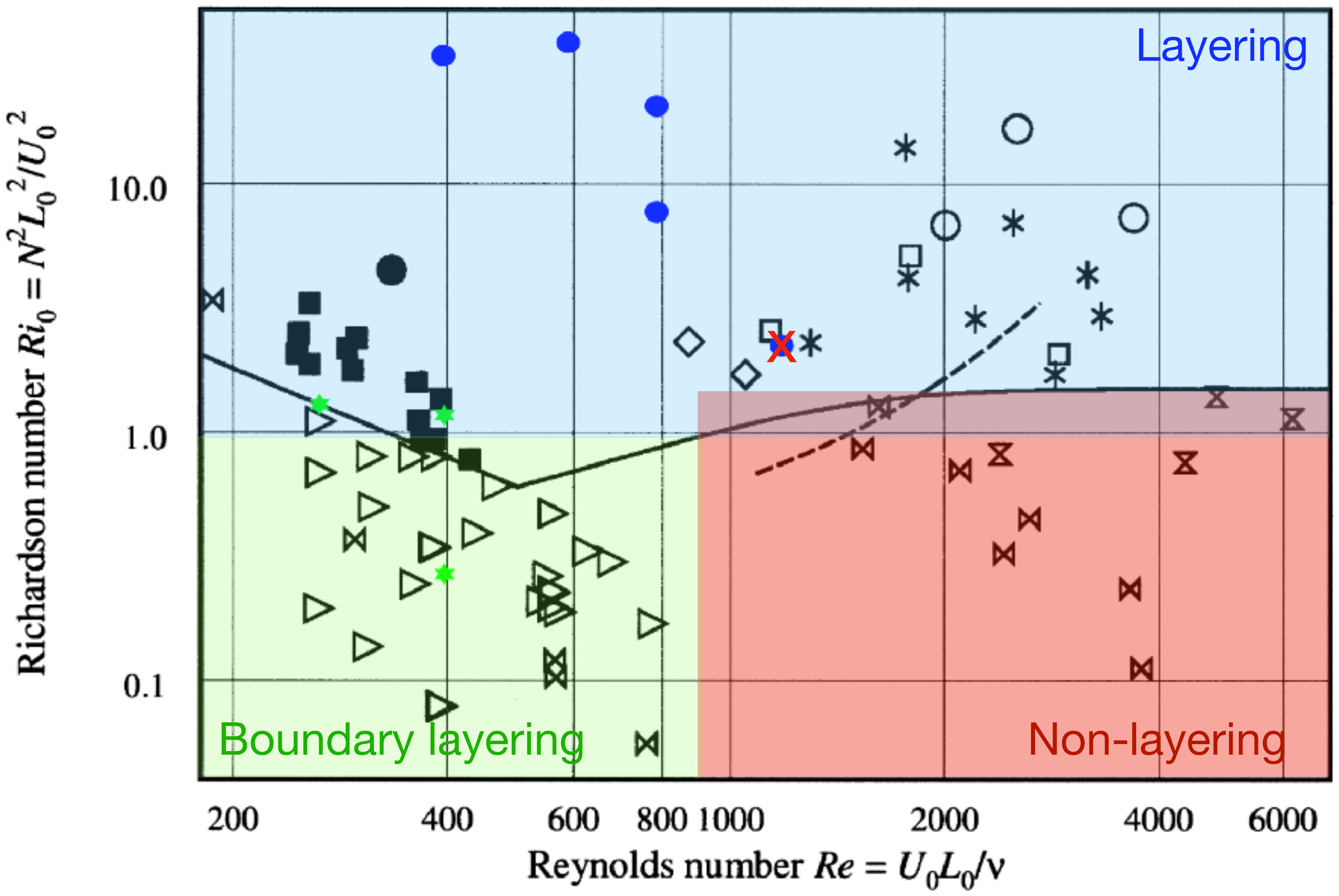


Original experiments

Layering



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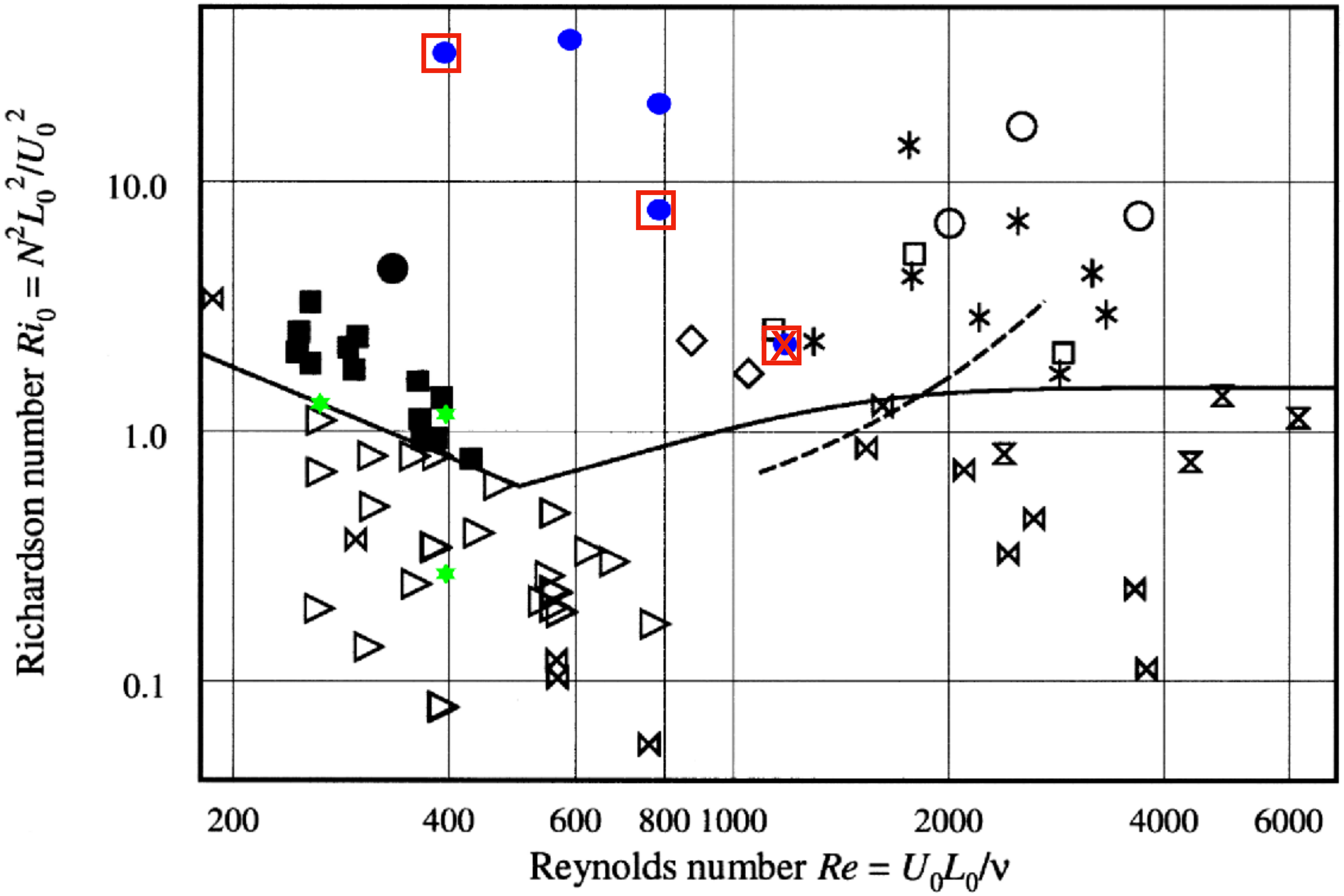
WHOI experiments

- ● $d = 2.5$ cm; $M = 11$ cm
- ★ $d = 0.6$ cm; $M = 3$ cm
- X Non-layering

The experiments

Parameter space



- $d = 2.5$ cm; $M = 11$ cm
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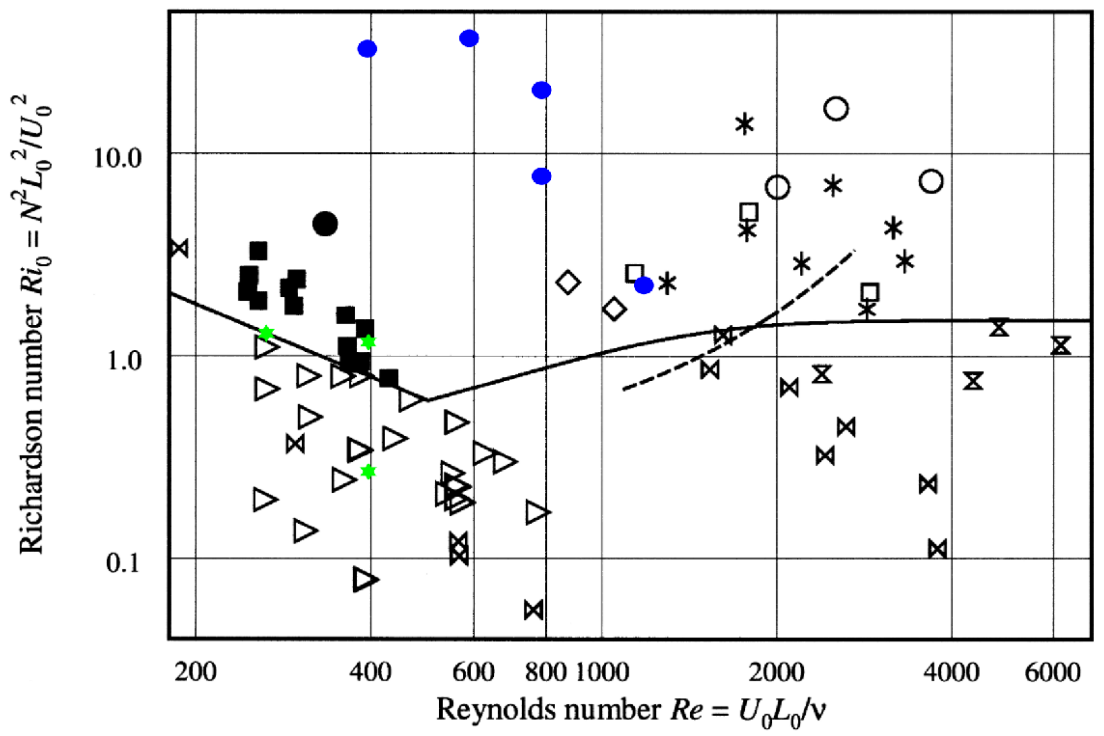
Layer formation

Low Froude, moderate to high Reynolds



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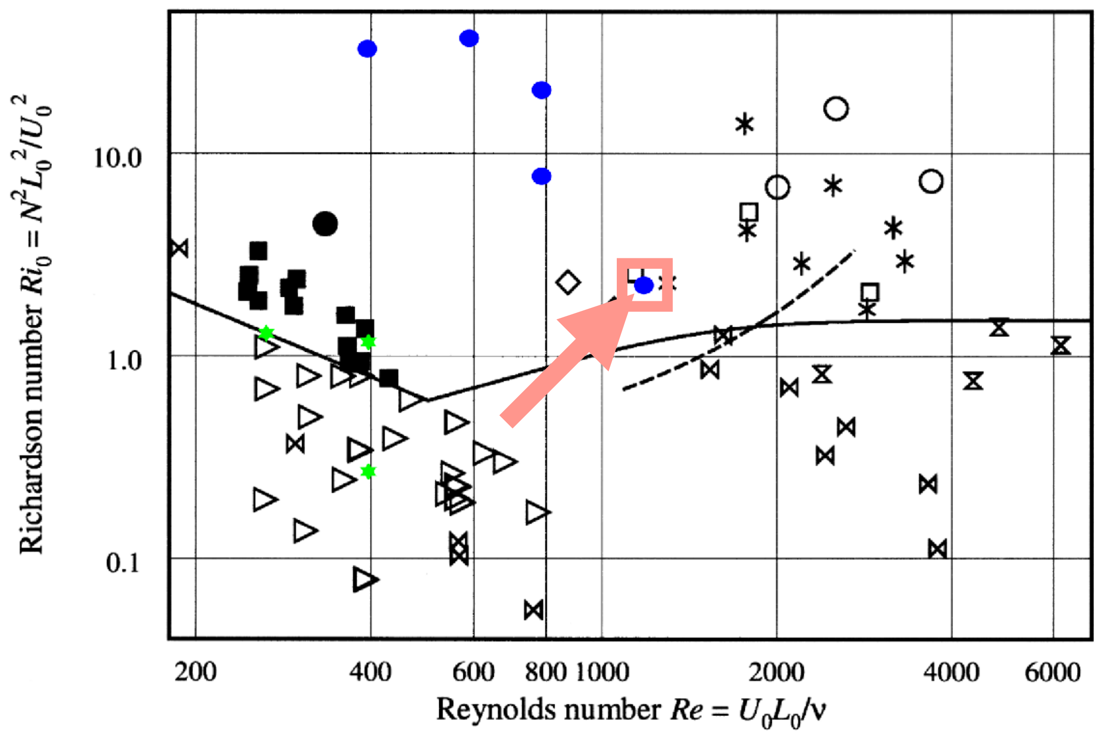

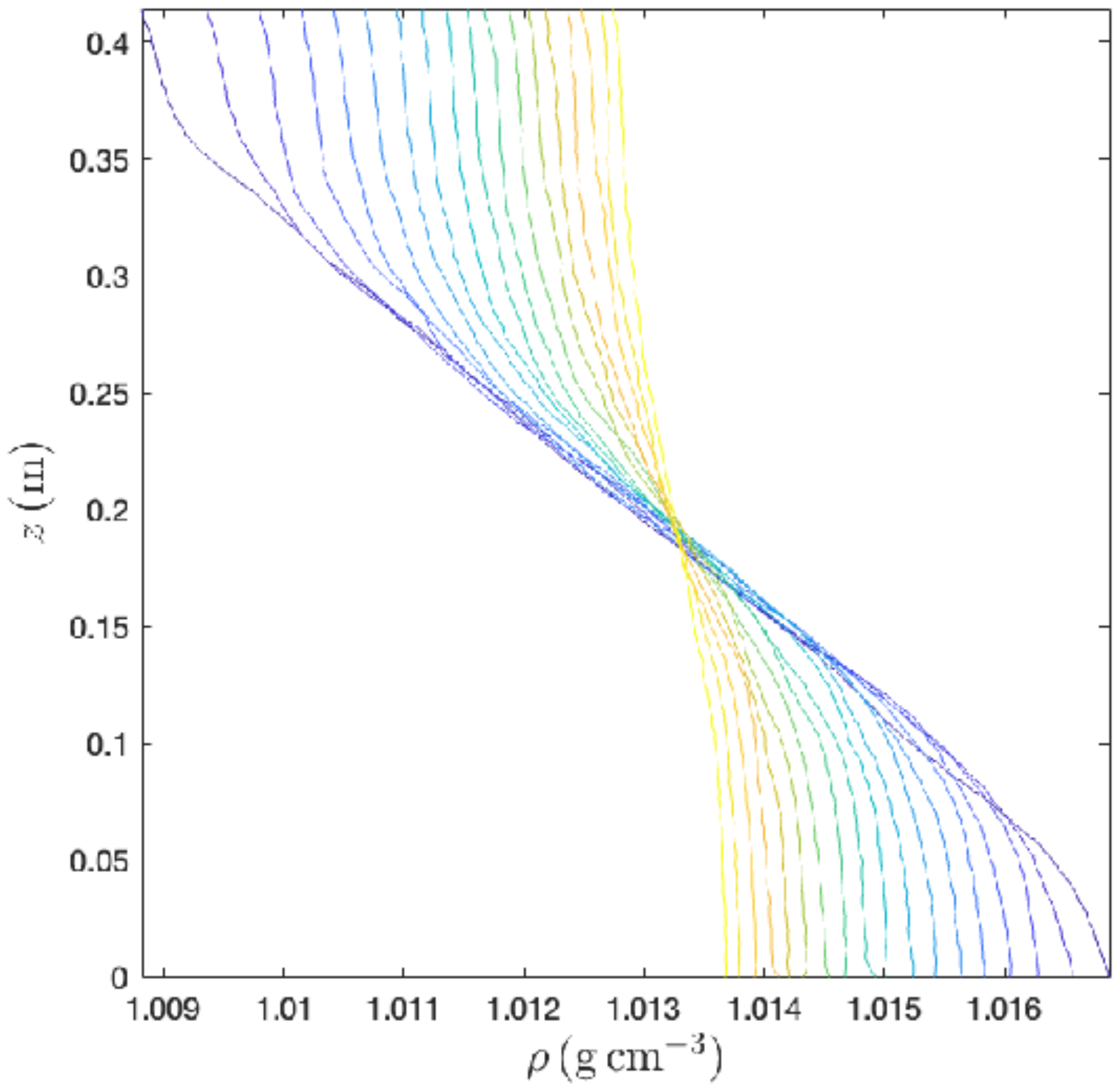

$Re_M = 3200;\ Ri_M = 5.5$

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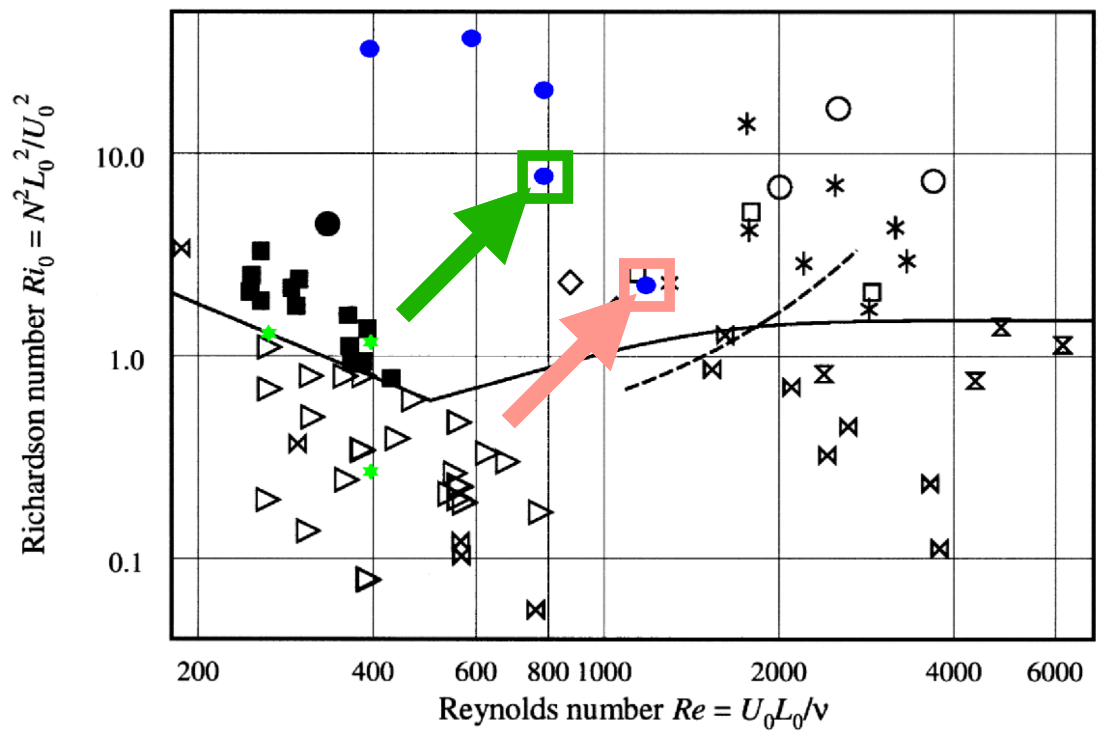

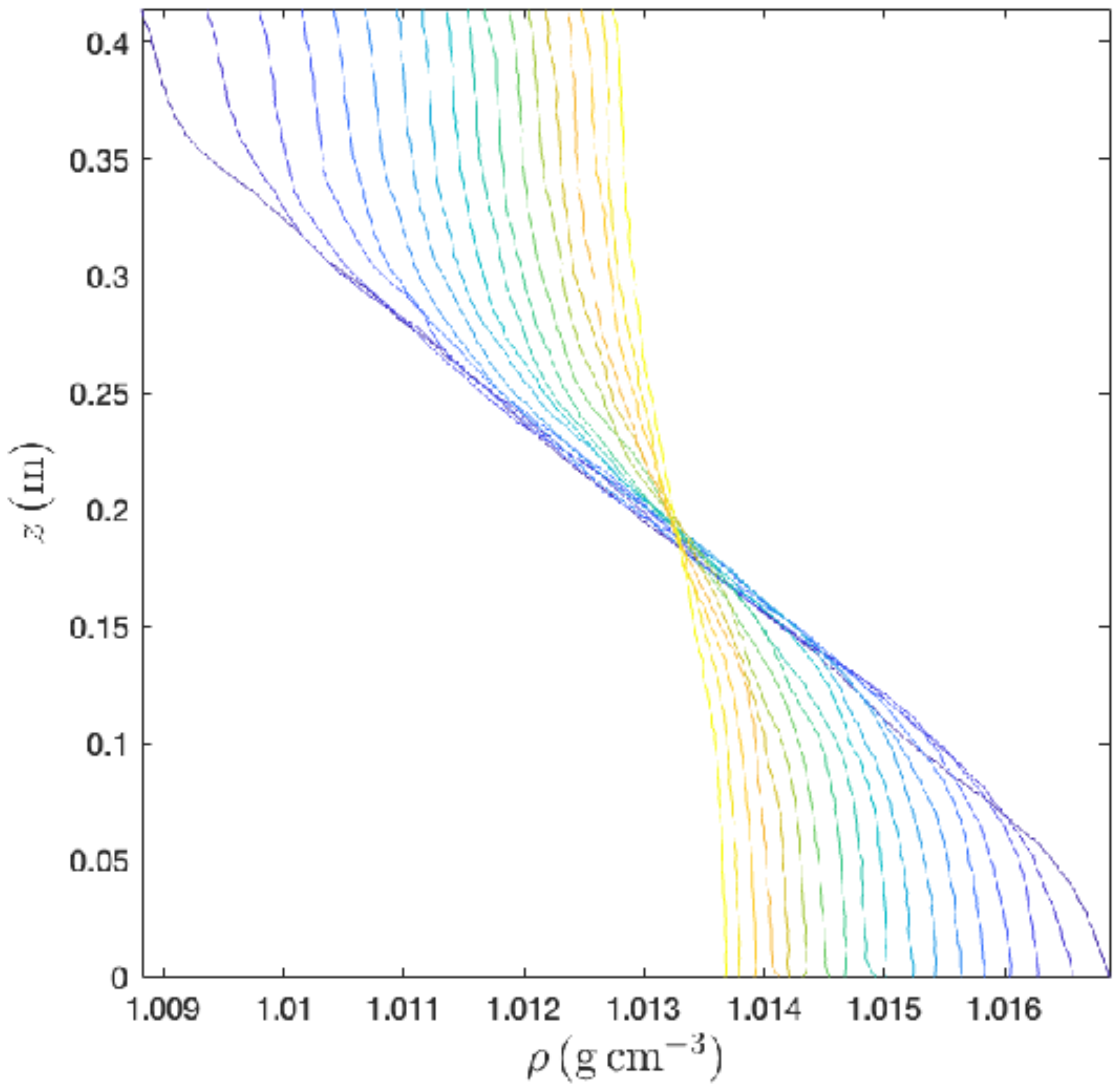

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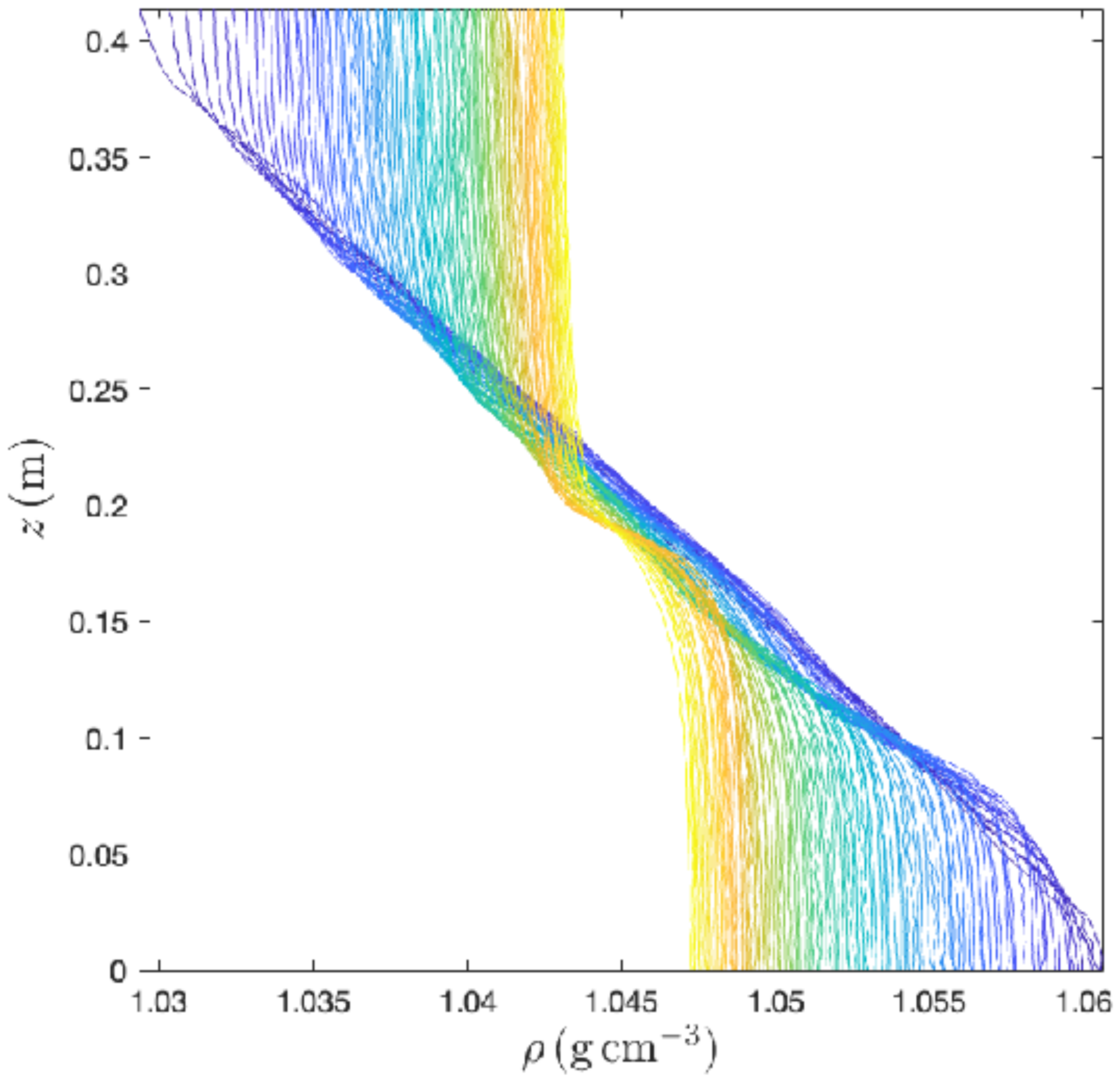

$Re_M = 2120;\;\; Ri_M = 20$

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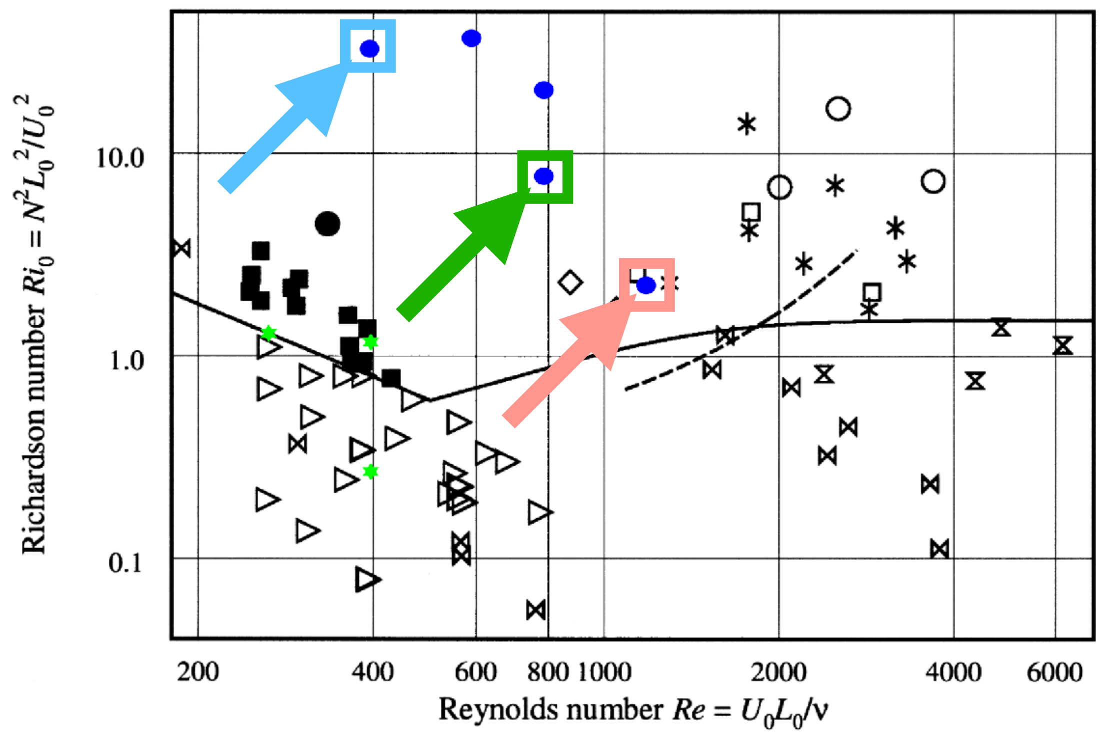

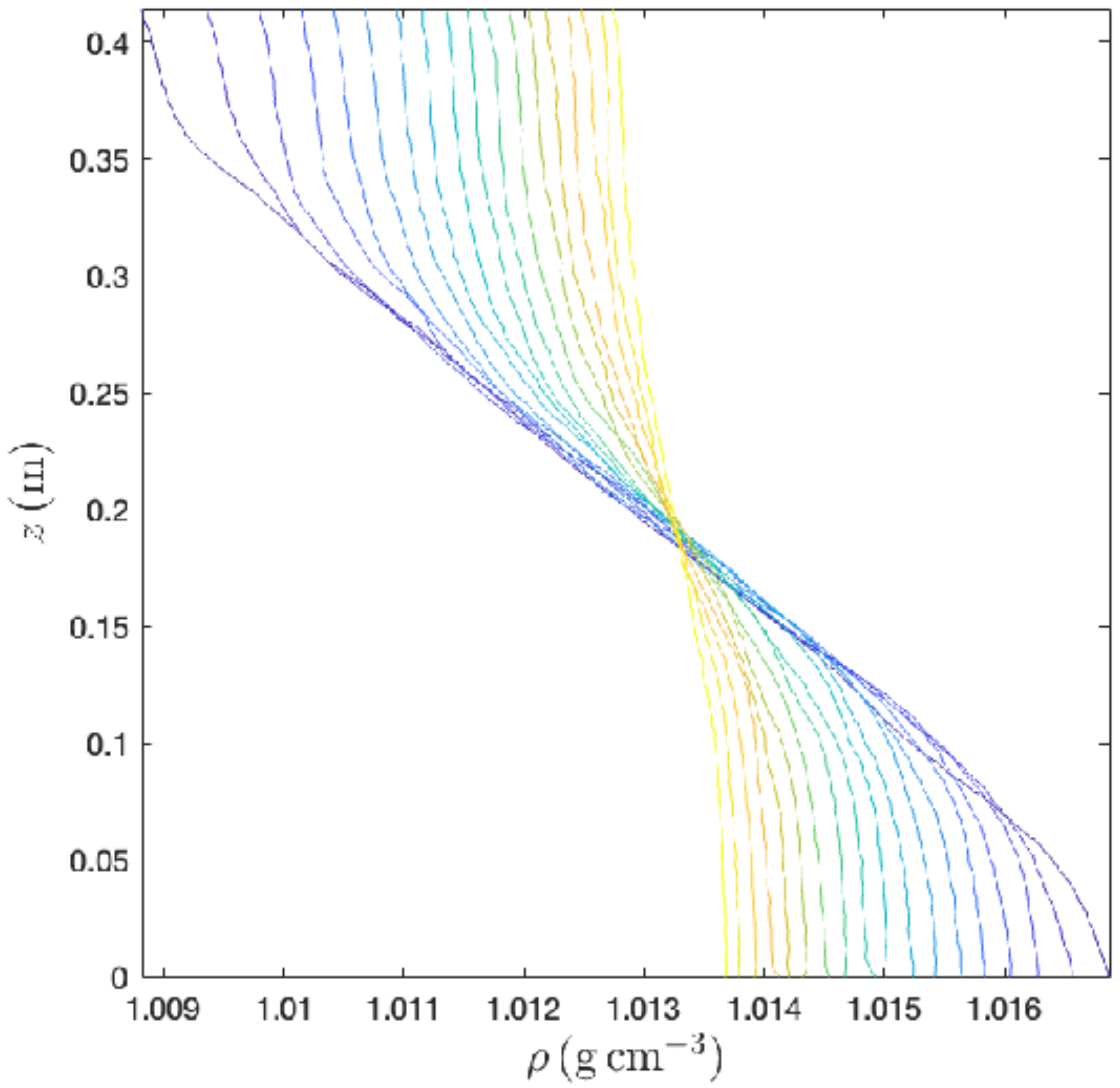

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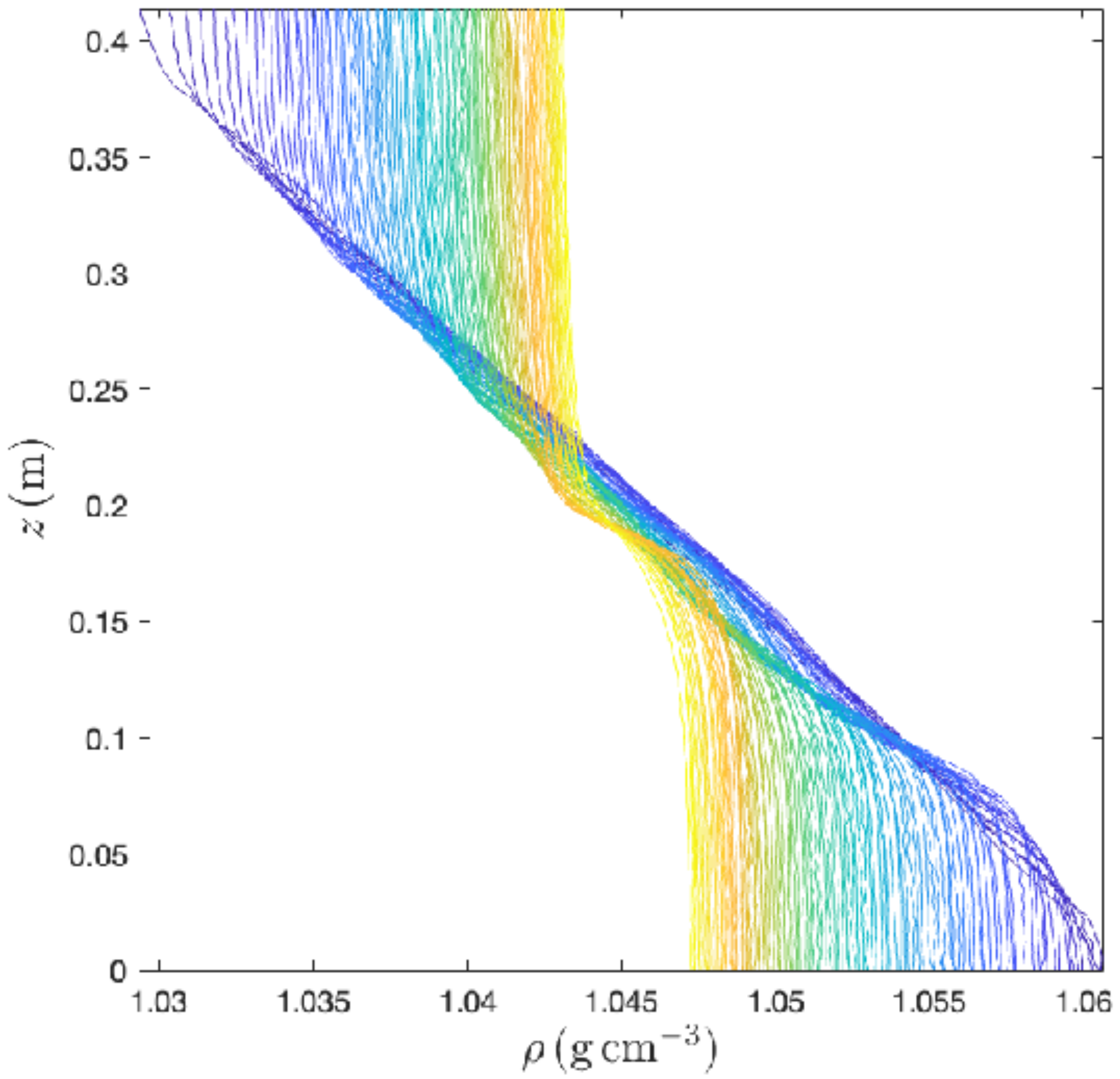

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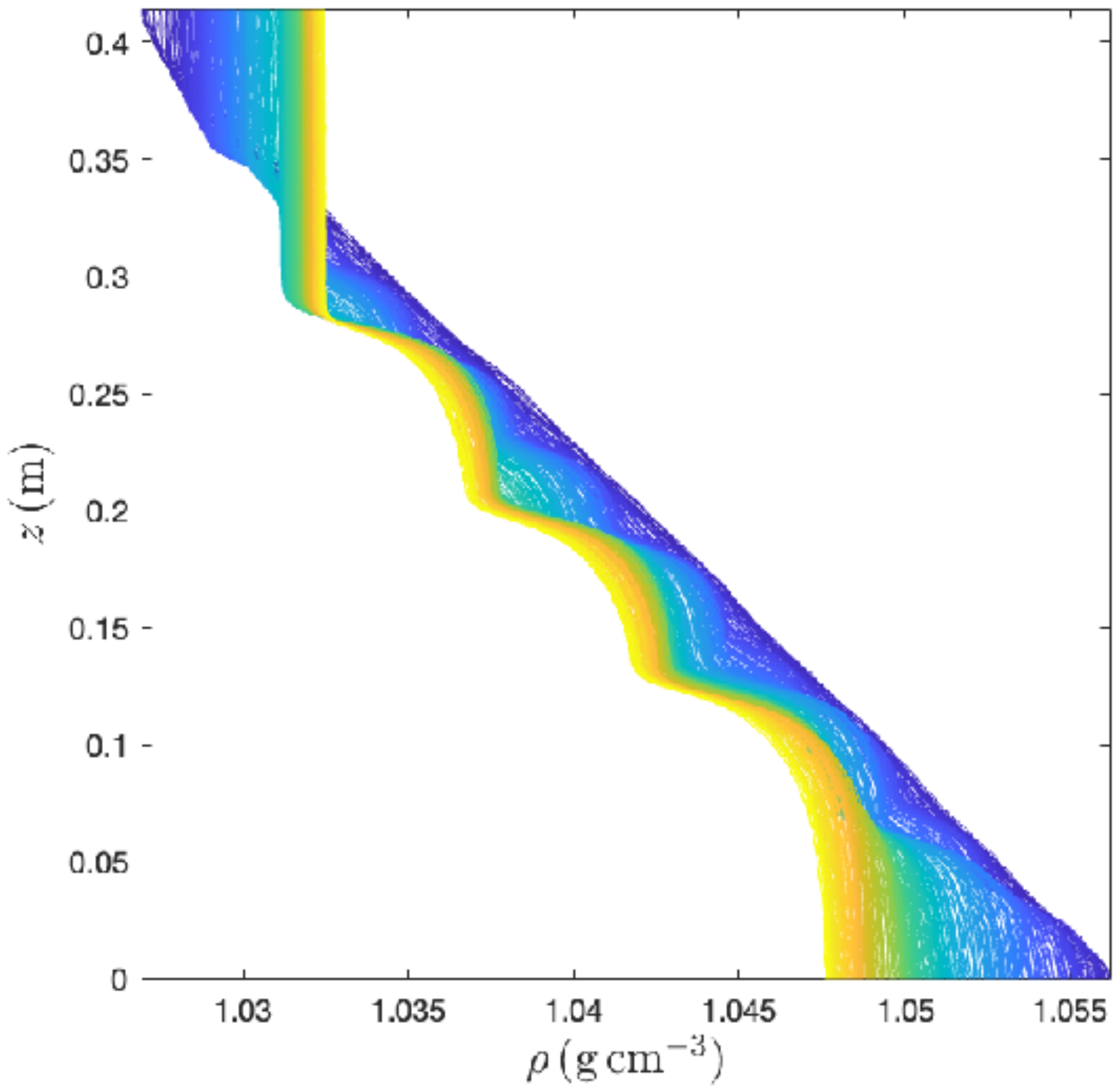
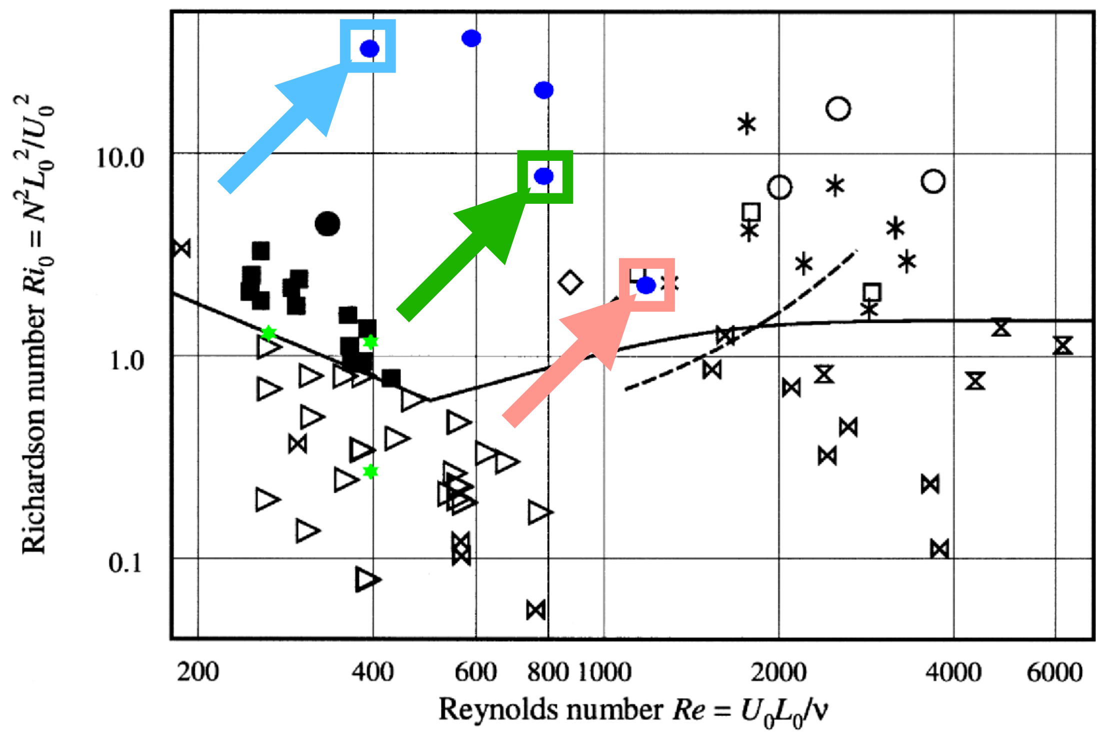
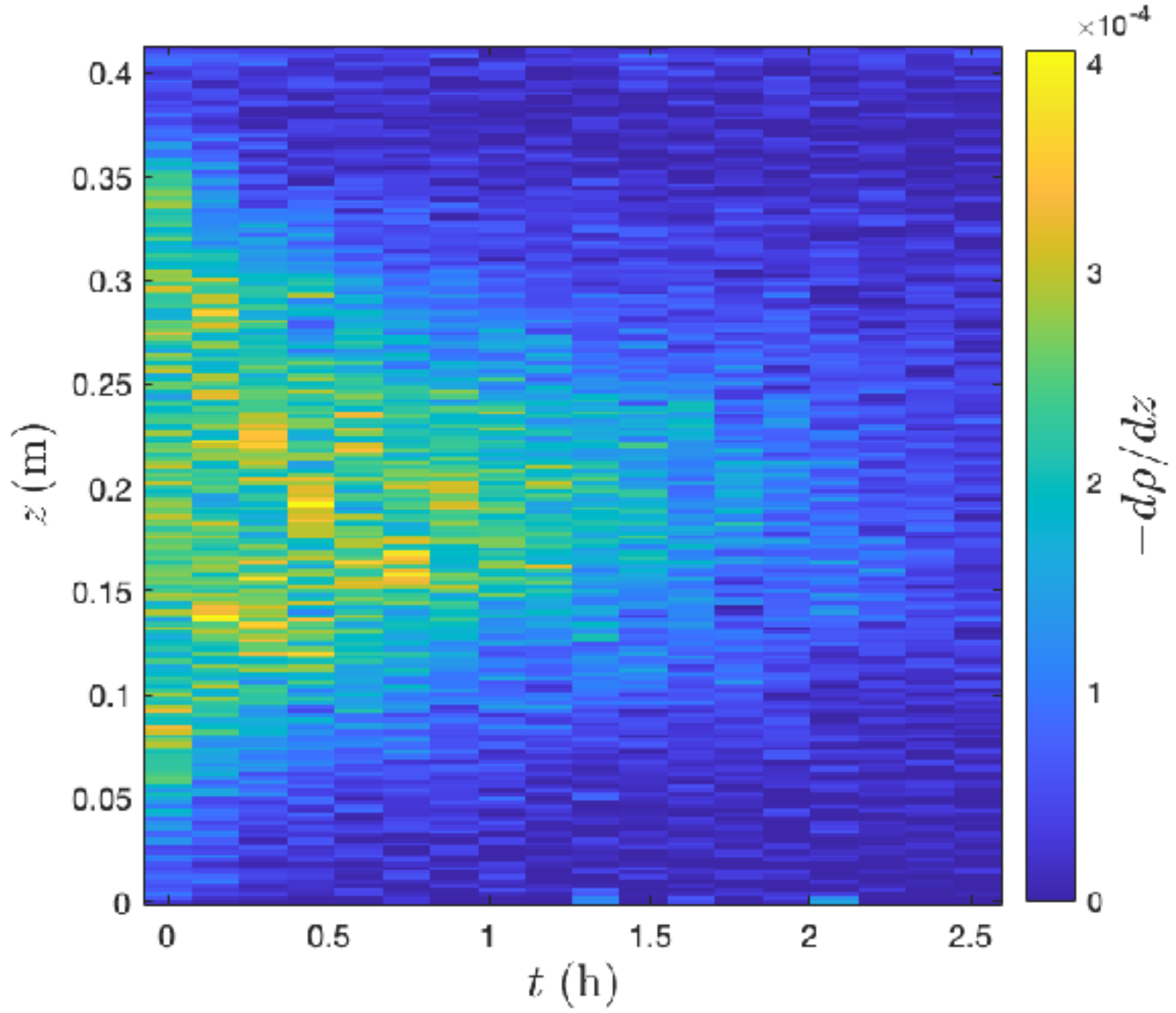


$Re_M = 1069;\ Ri_M = 80$

Layer formation

Low Froude, moderate to high Reynolds





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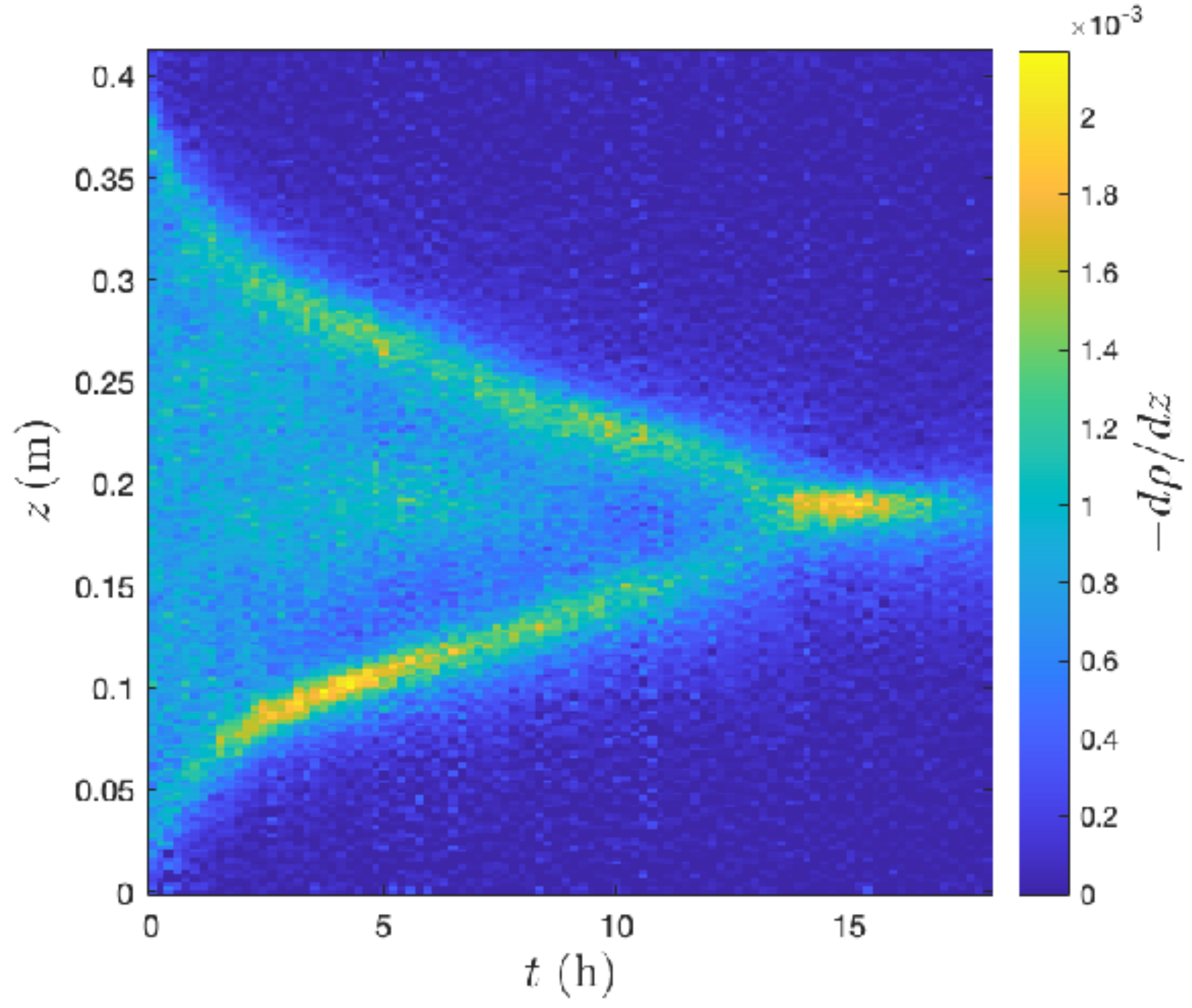

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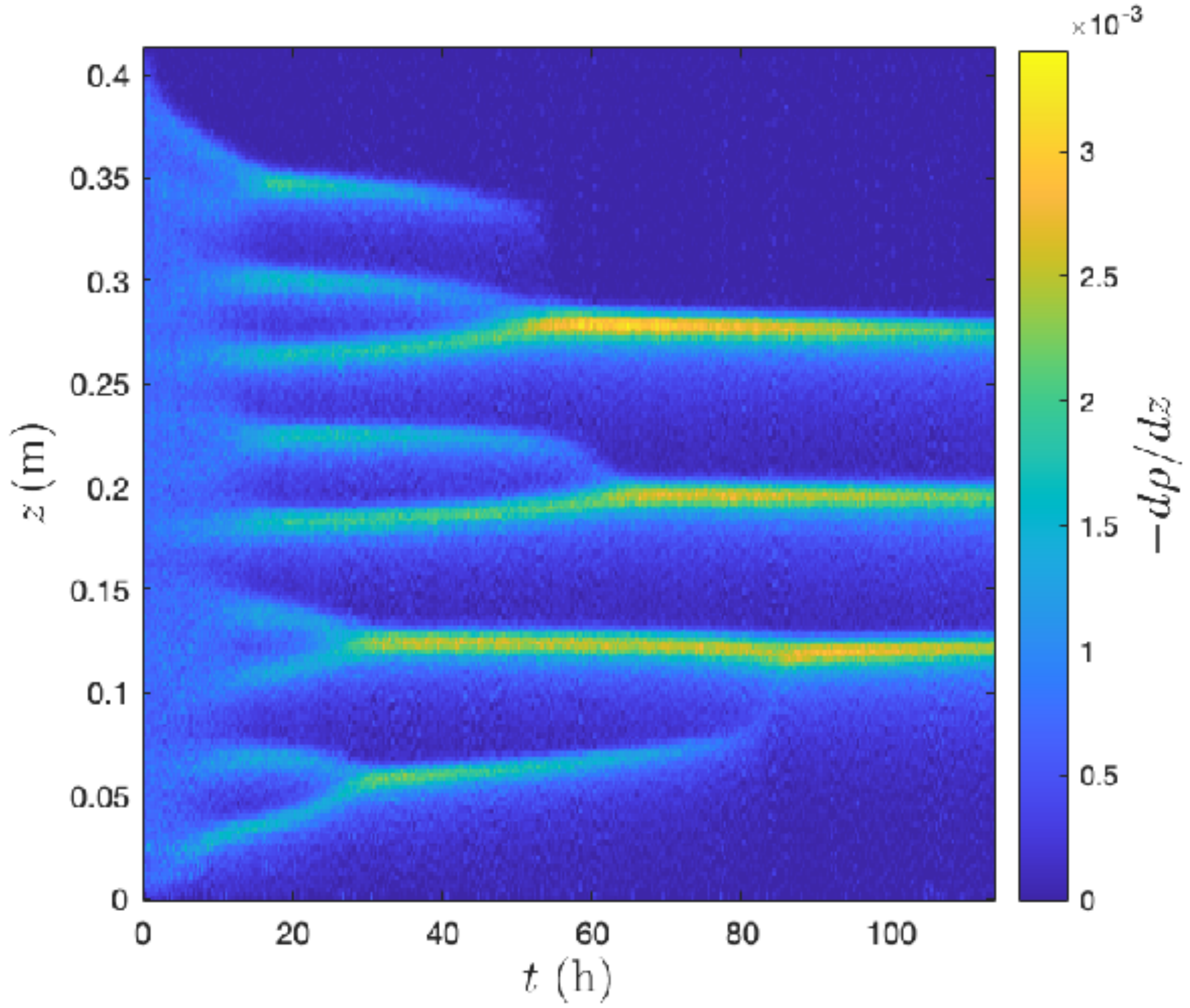

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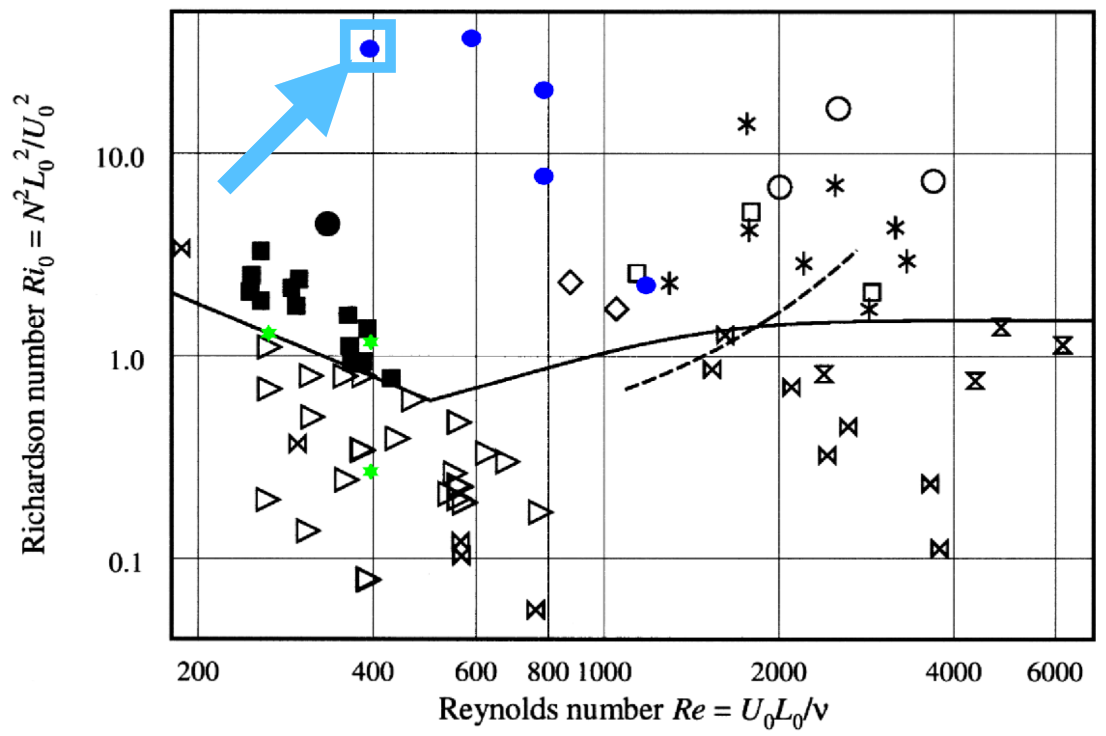
Layer formation

High Ri, moderate to high Re



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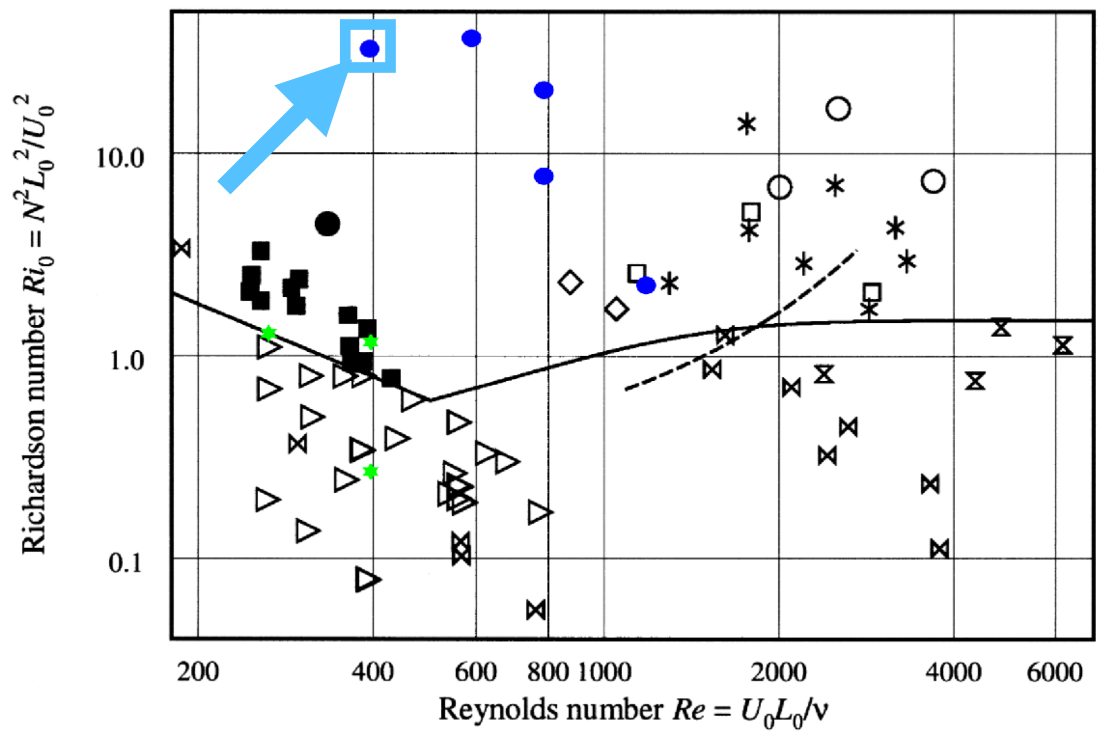

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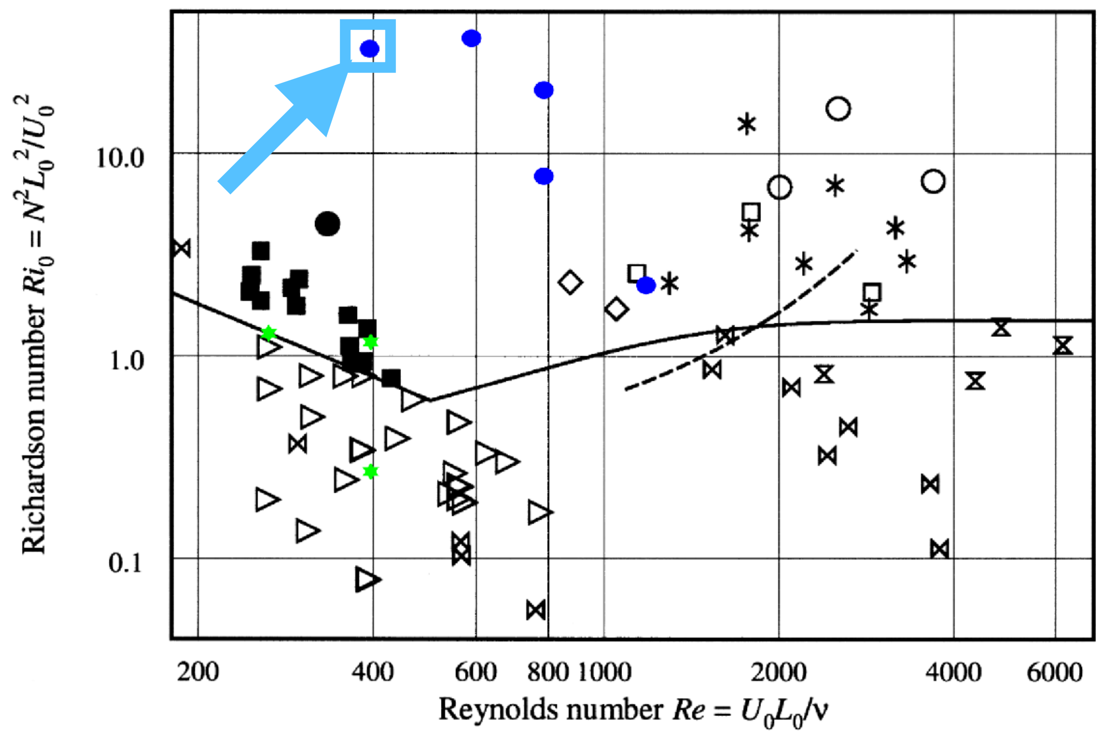

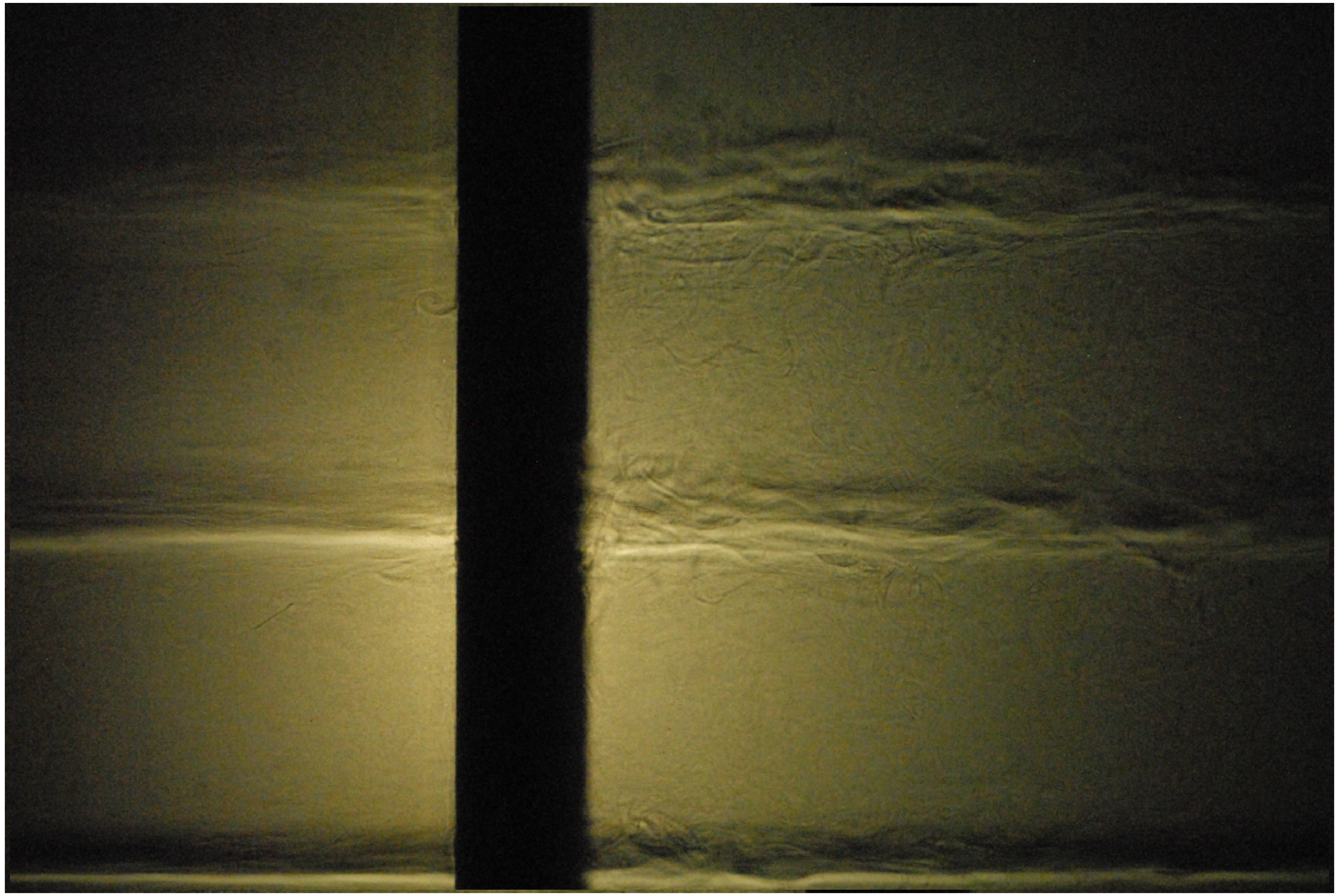

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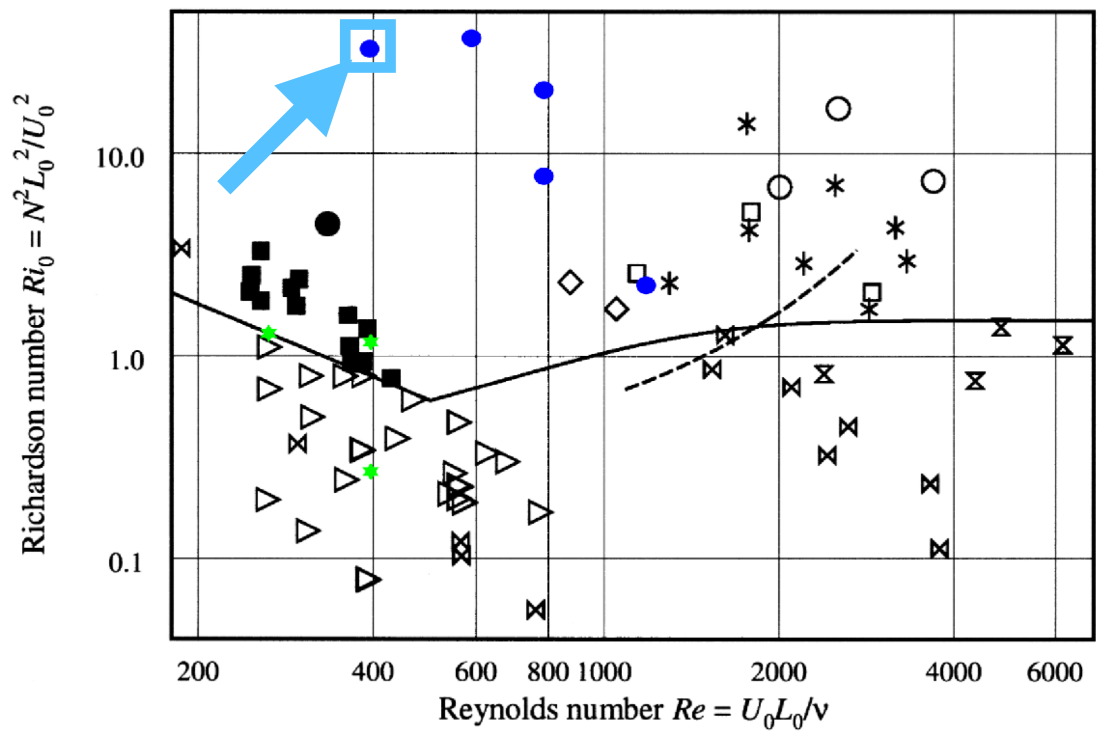

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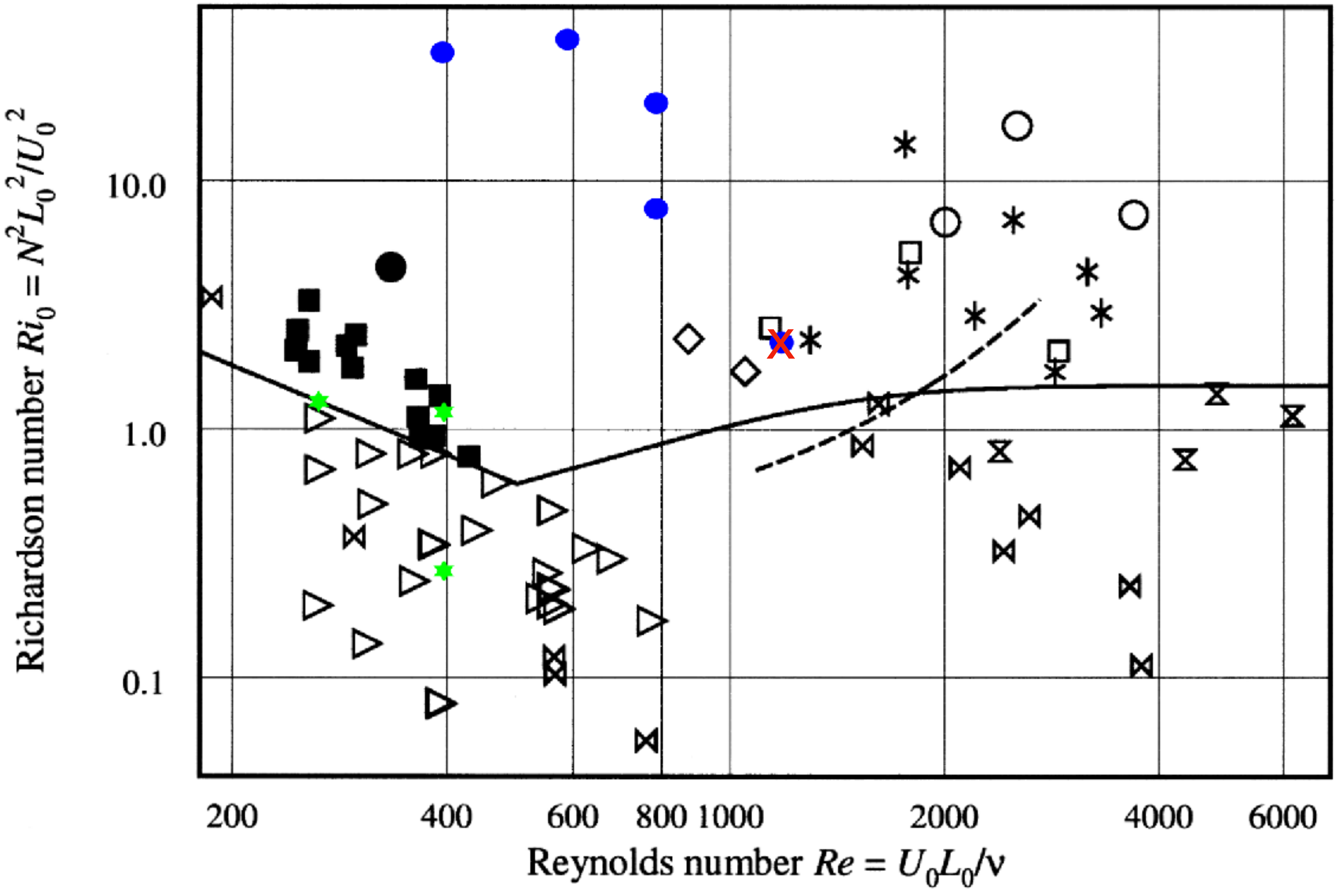
The experiments

Boundary layering at lower Richardson ?



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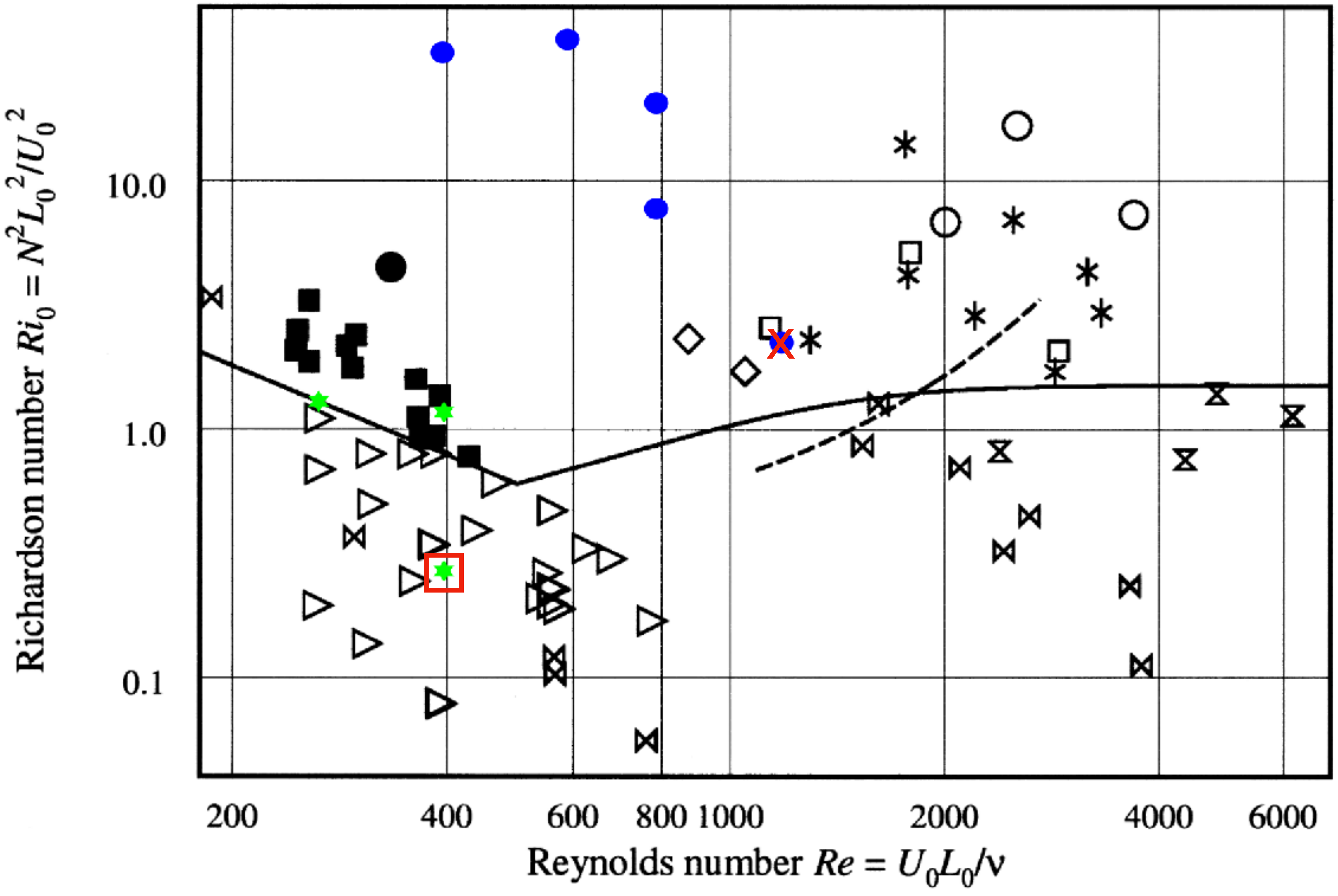
The experiments

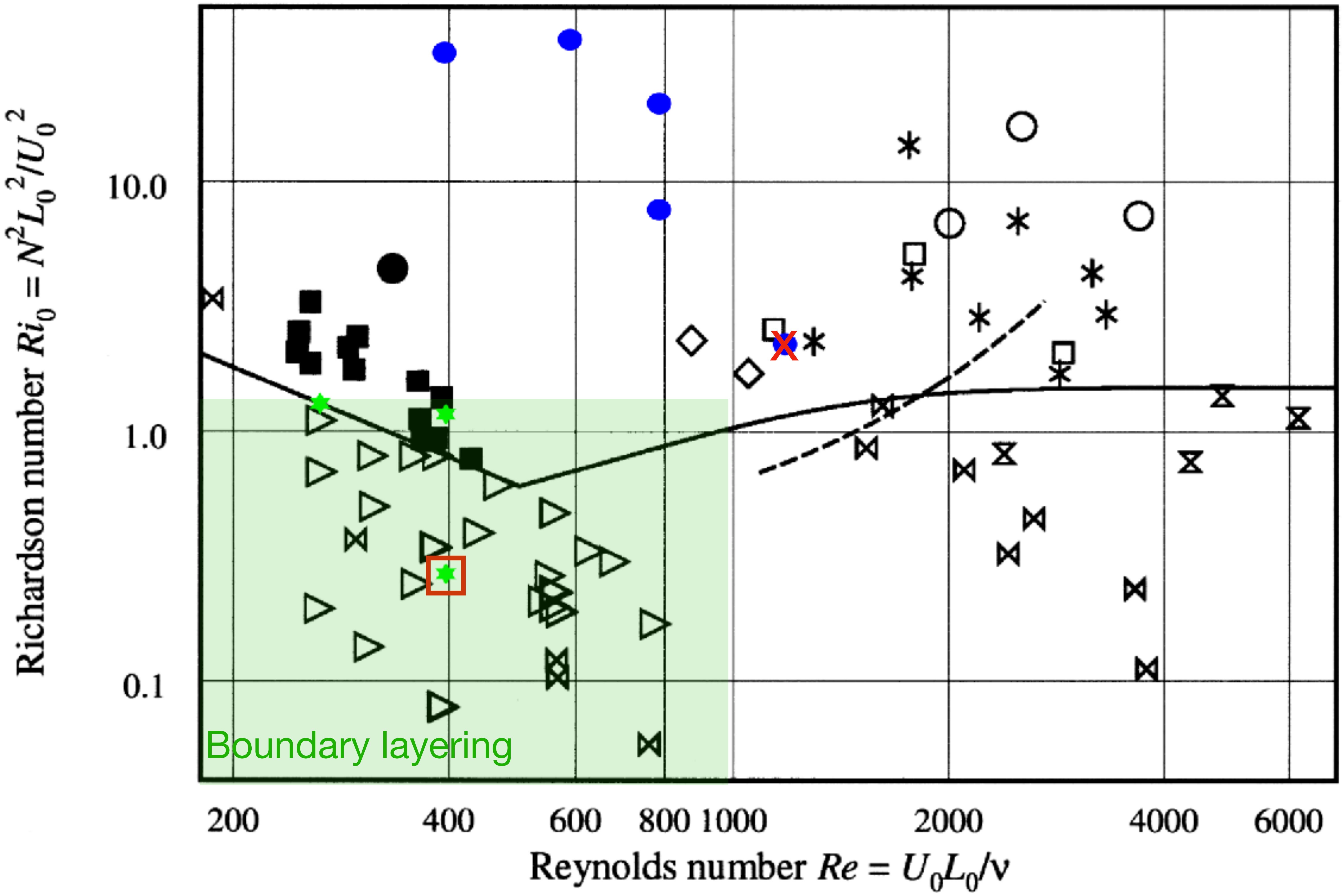
Boundary layering at lower Richardson ?



- ● $d = 2.5$ cm; $M = 11$ cm
- ★ $d = 0.6$ cm; $M = 3$ cm
- x Non-layering

The experiments

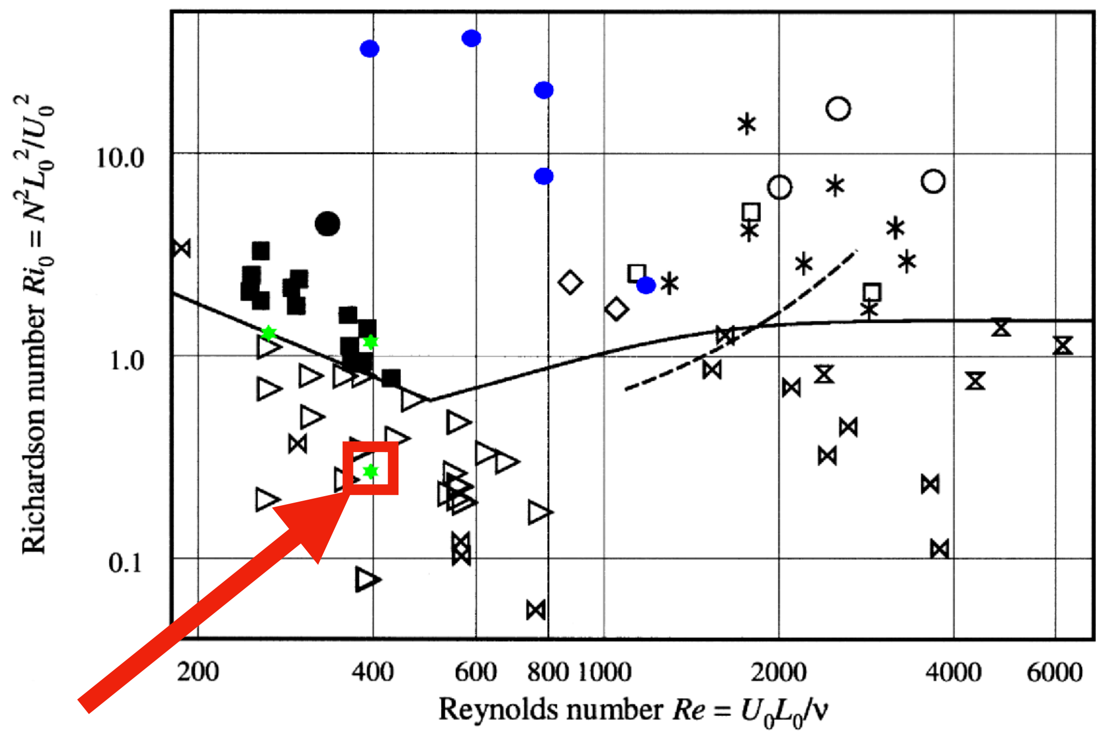
Boundary layering at lower Richardson ?



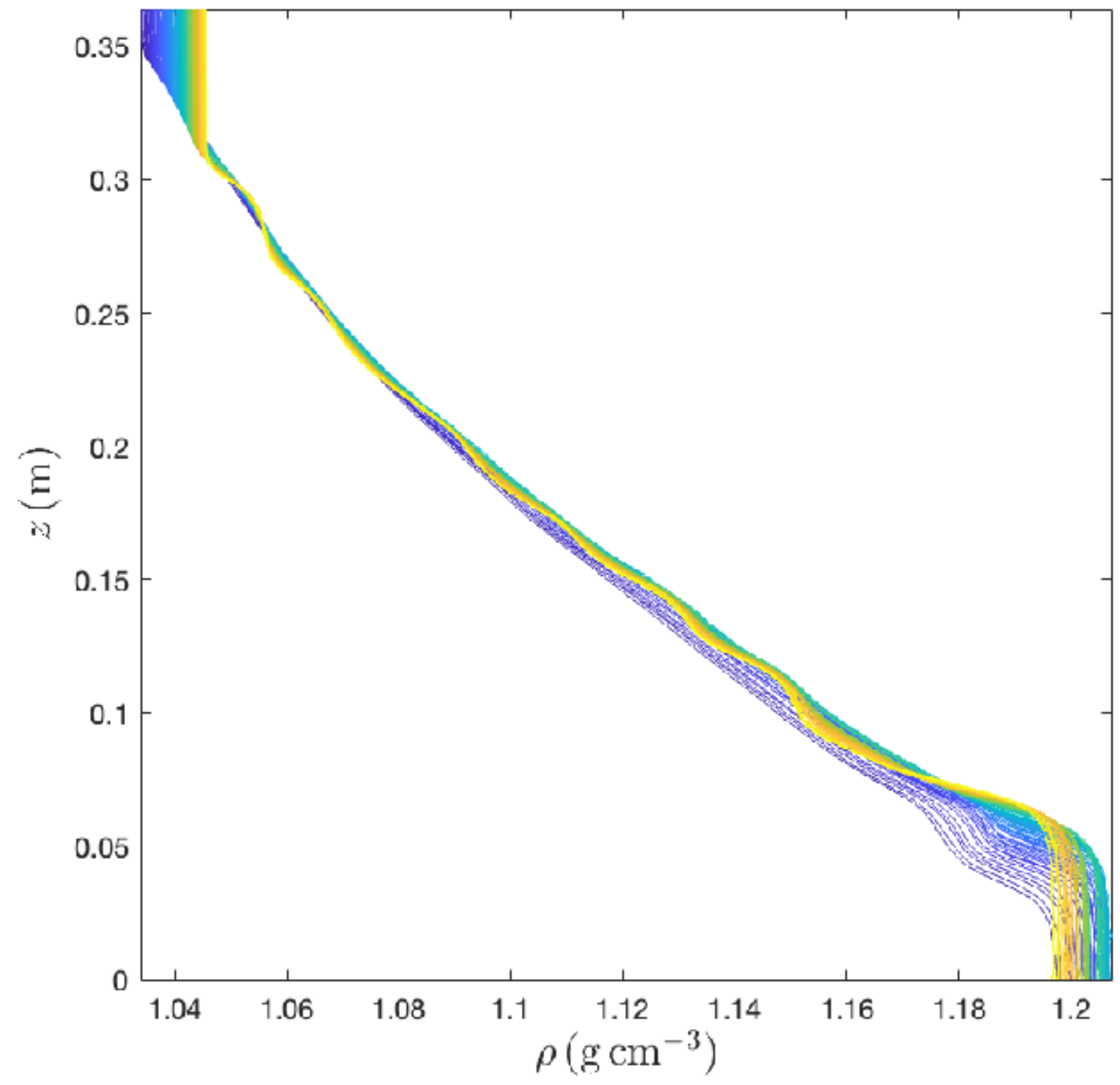
- ● $d = 2.5$ cm; $M = 11$ cm
- ★ $d = 0.6$ cm; $M = 3$ cm
- x Non-layering

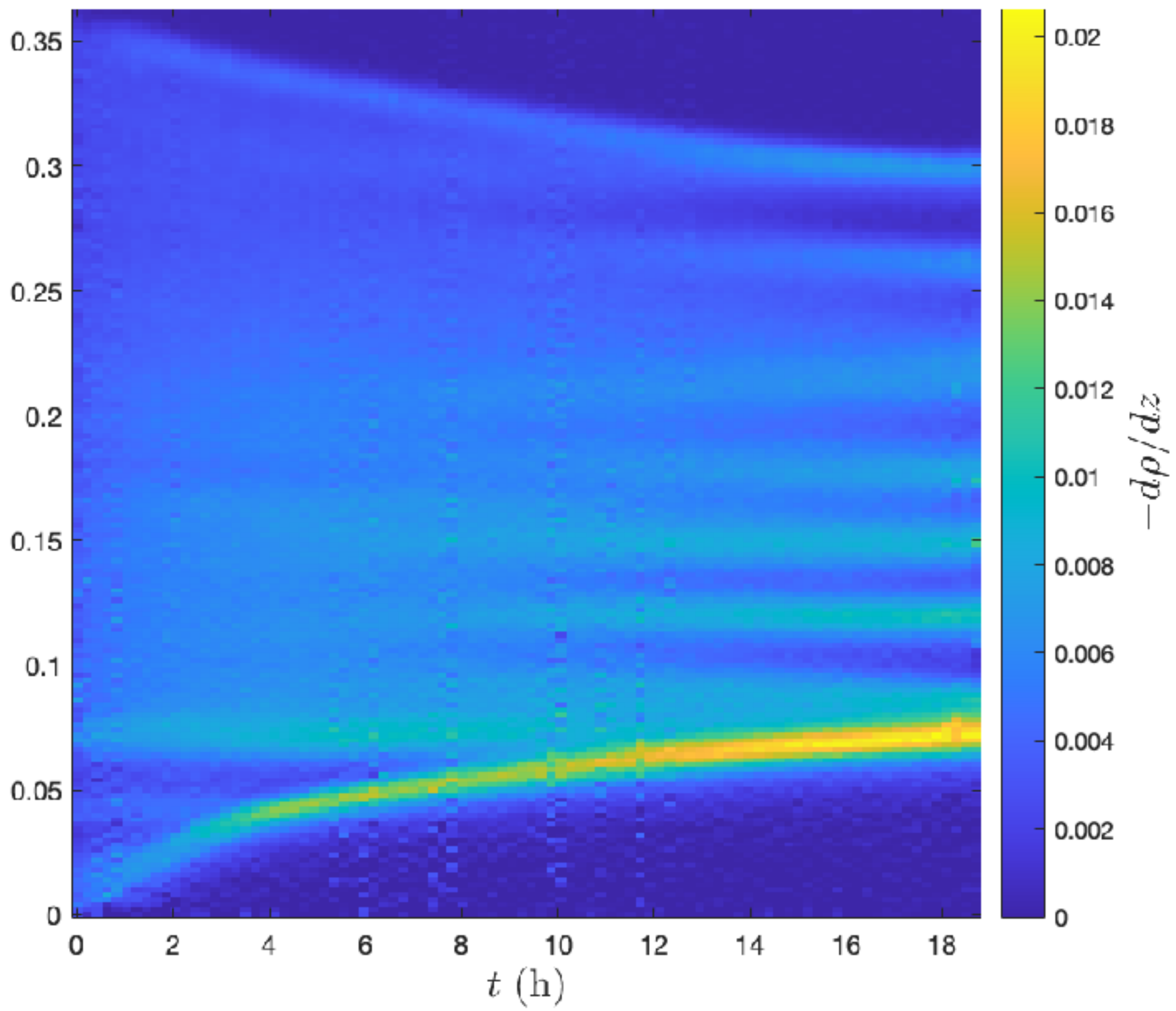
Layer formation

Boundary layering at lower Richardson ?



$Re_M = 1200;\ Ri_M \sim 3.4$





Smaller layer size
$h \sim 1 - 2\ U/N$

The non-rotating DNS

DNS

Dimensional equations

- Horizontally forced flow with uniform stratification $N^2 = g\alpha \; d\overline{T}_0/dz$

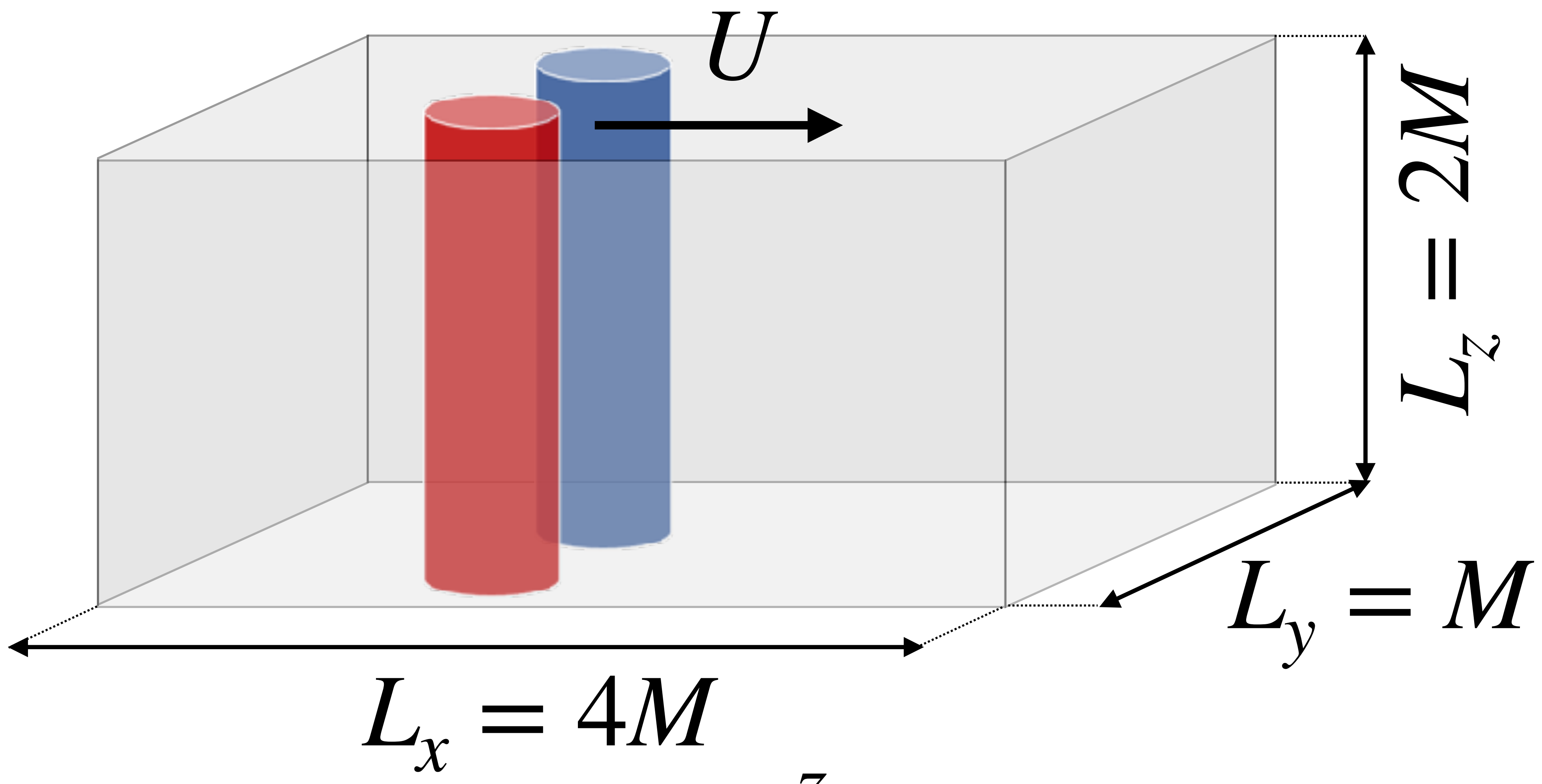
$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \,.\, \nabla \mathbf{u} = -\frac{1}{\rho_0} \nabla p + b\mathbf{e_z} + \nu \nabla^2 \mathbf{u} + \mathbf{F}$$

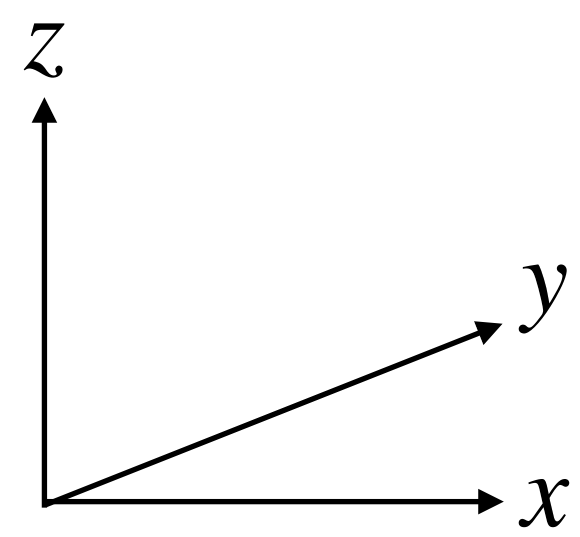
$$\nabla \,.\, \mathbf{u} = 0$$

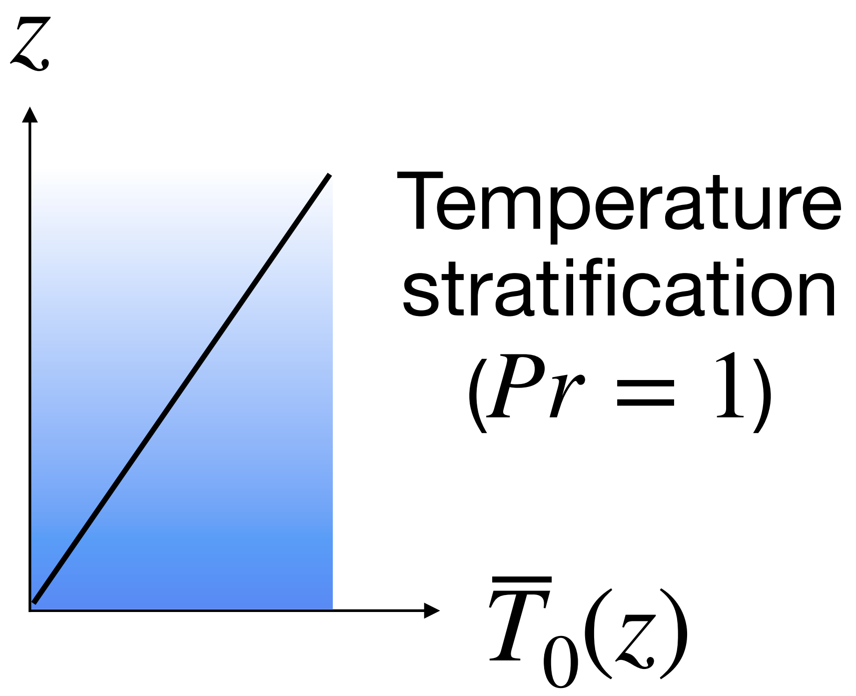
$$\frac{\partial b}{\partial t} + \mathbf{u} \,.\, \nabla b + N^2 w = \kappa \nabla^2 b$$

with $b = g\alpha(T - \overline{T}_0)$.

- Triply periodic boundary conditions on $\mathbf{u}, b, p$ and $Pr \equiv \nu/\kappa = 1$.

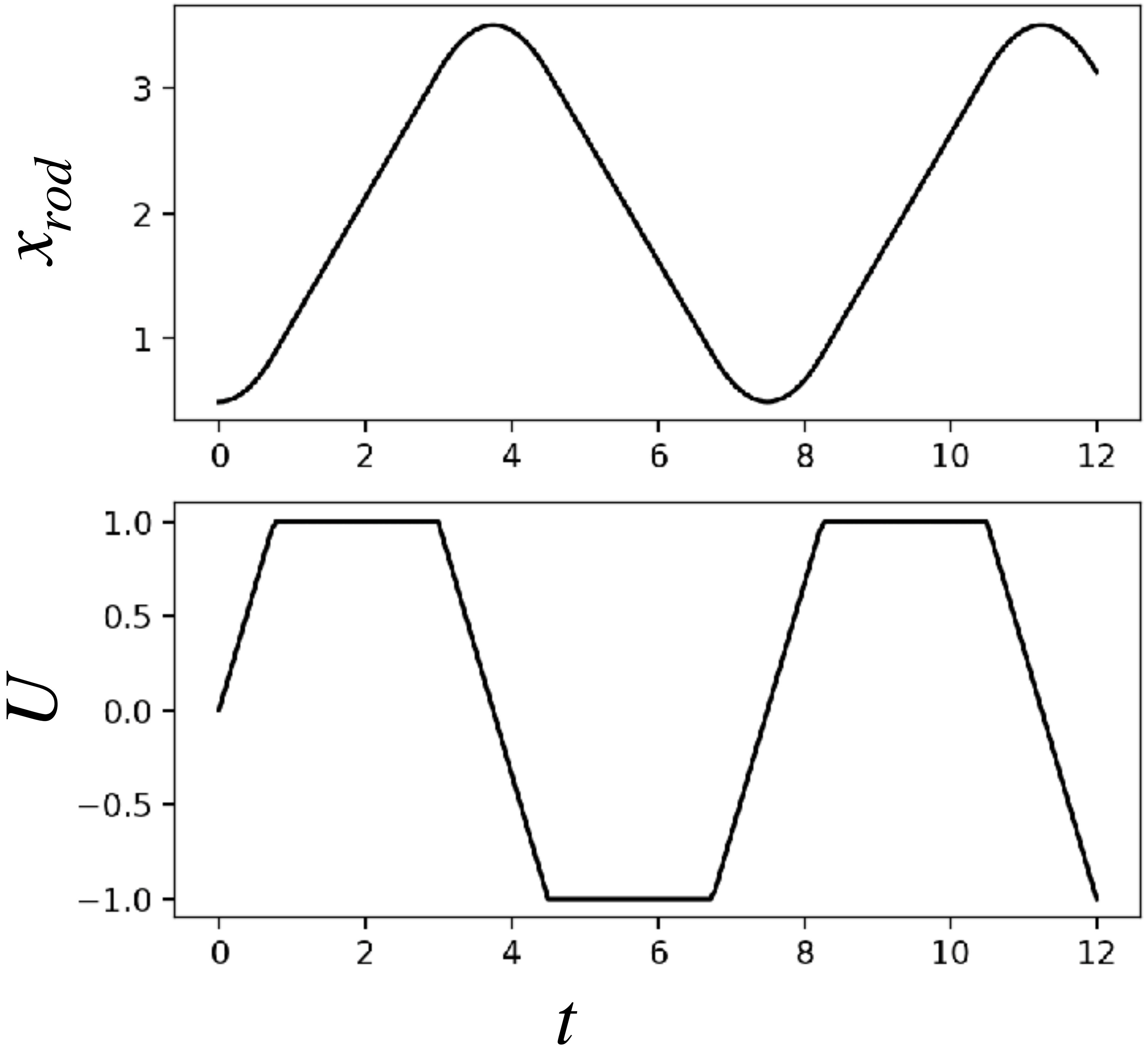


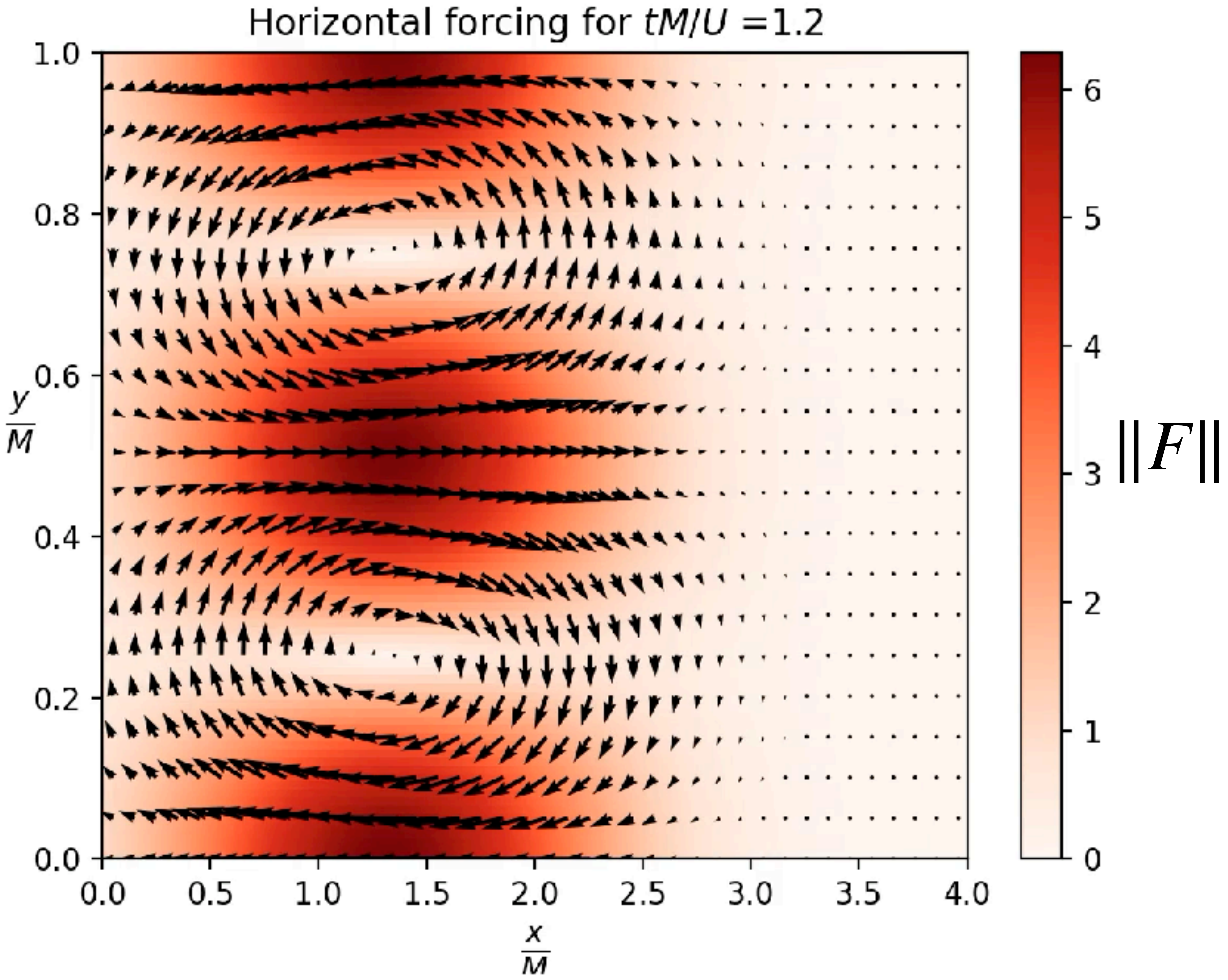




DNS

Horizontal forcing





DNS

Dimensionless equations

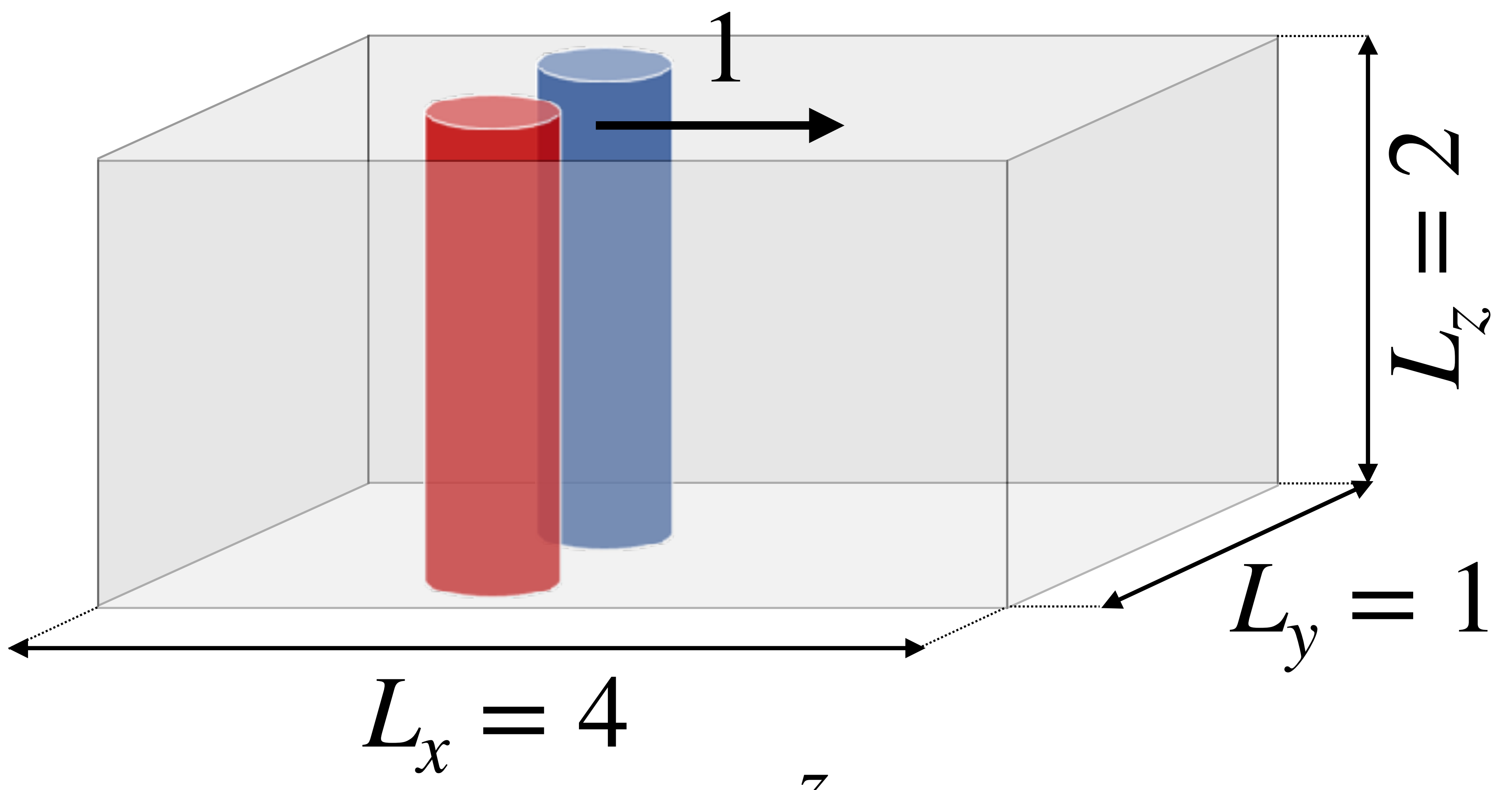
- Non-dimensionalize using $L_h = M$, $[\mathbf{u}] = U$, $[t] = L_h/U$, $[b] = L_h N^2$, $[p] = \rho_0 U^2$

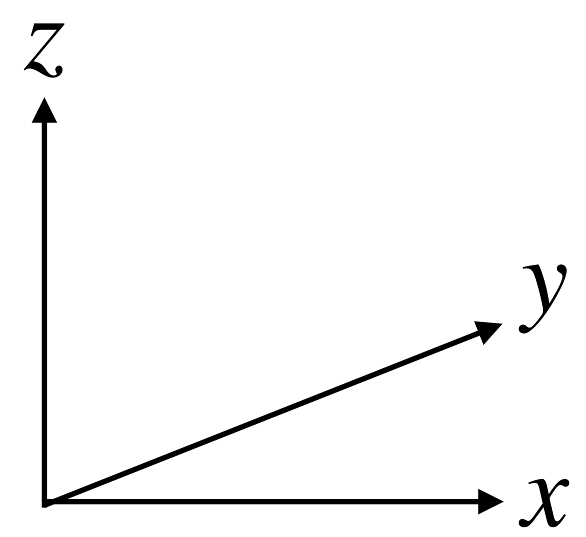
$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \,.\, \nabla \mathbf{u} = -\nabla p + \ Ri\ b\mathbf{e_z} + \frac{1}{Re}\nabla^2 \mathbf{u} + \mathbf{F}$$

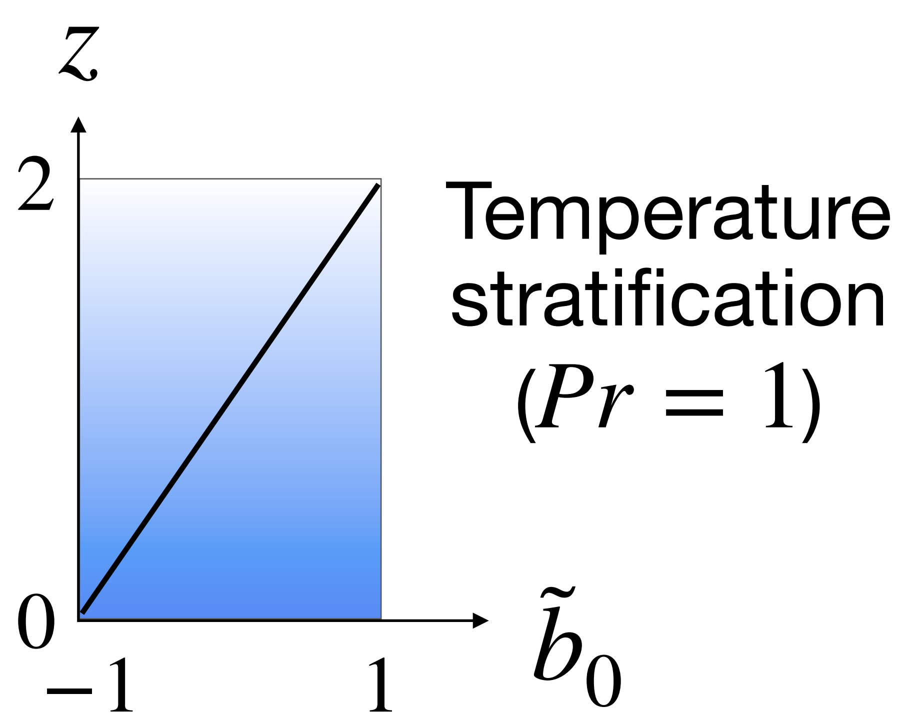
$$\nabla \,.\, \mathbf{u} = 0$$

$$\frac{\partial b}{\partial t} + \mathbf{u} \,.\, \nabla b + w = \frac{1}{Pe}\nabla^2 b$$

with $Re = \dfrac{UM}{\nu}$, $Pe = \dfrac{UM}{\kappa} = Re$, $Ri = \dfrac{N^2 M^2}{U^2}$.





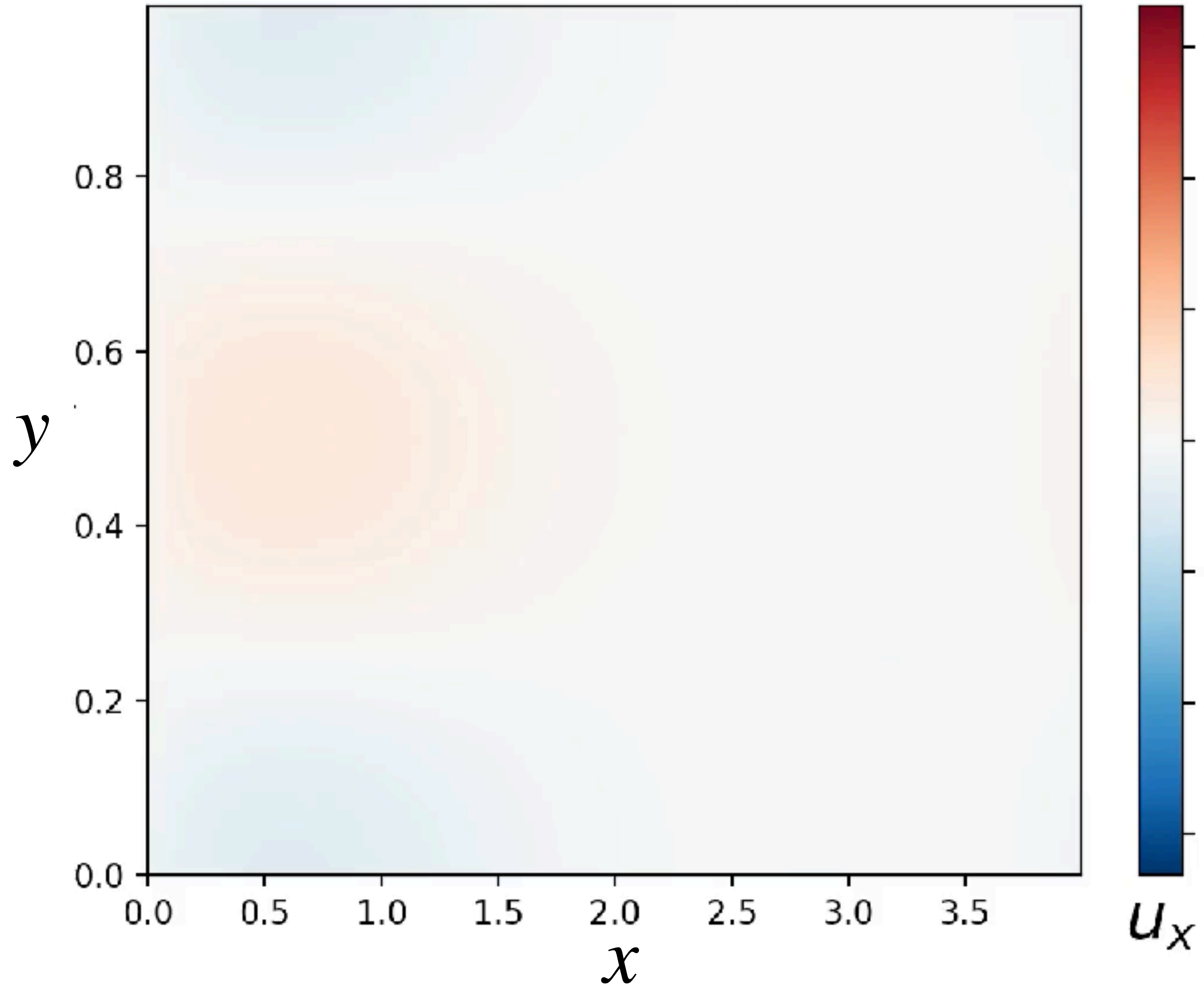


DNS

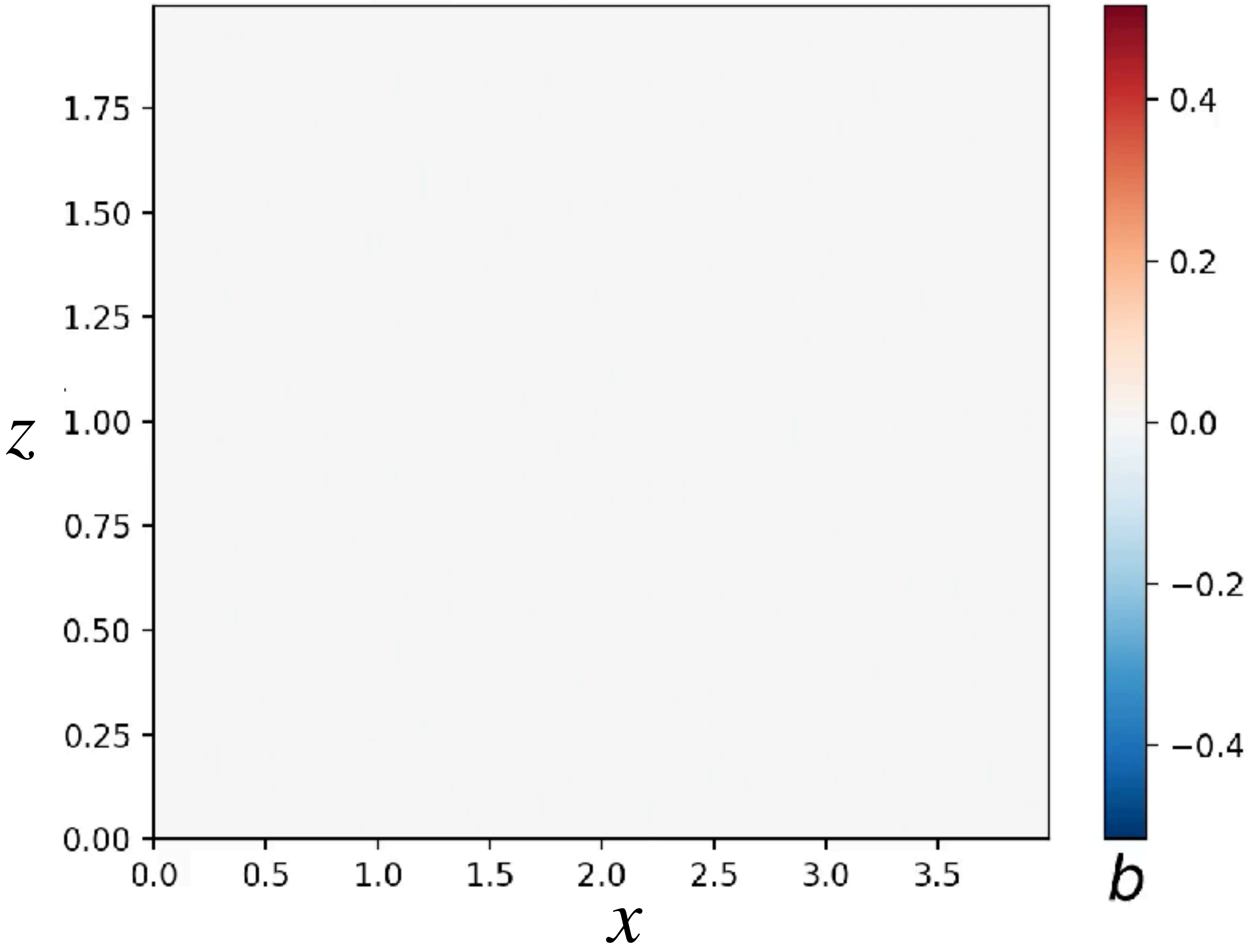
Results

$$Re_M = 800; \; Ri_M = 60 \;\; \Rightarrow Re_b \sim 10$$

Top view



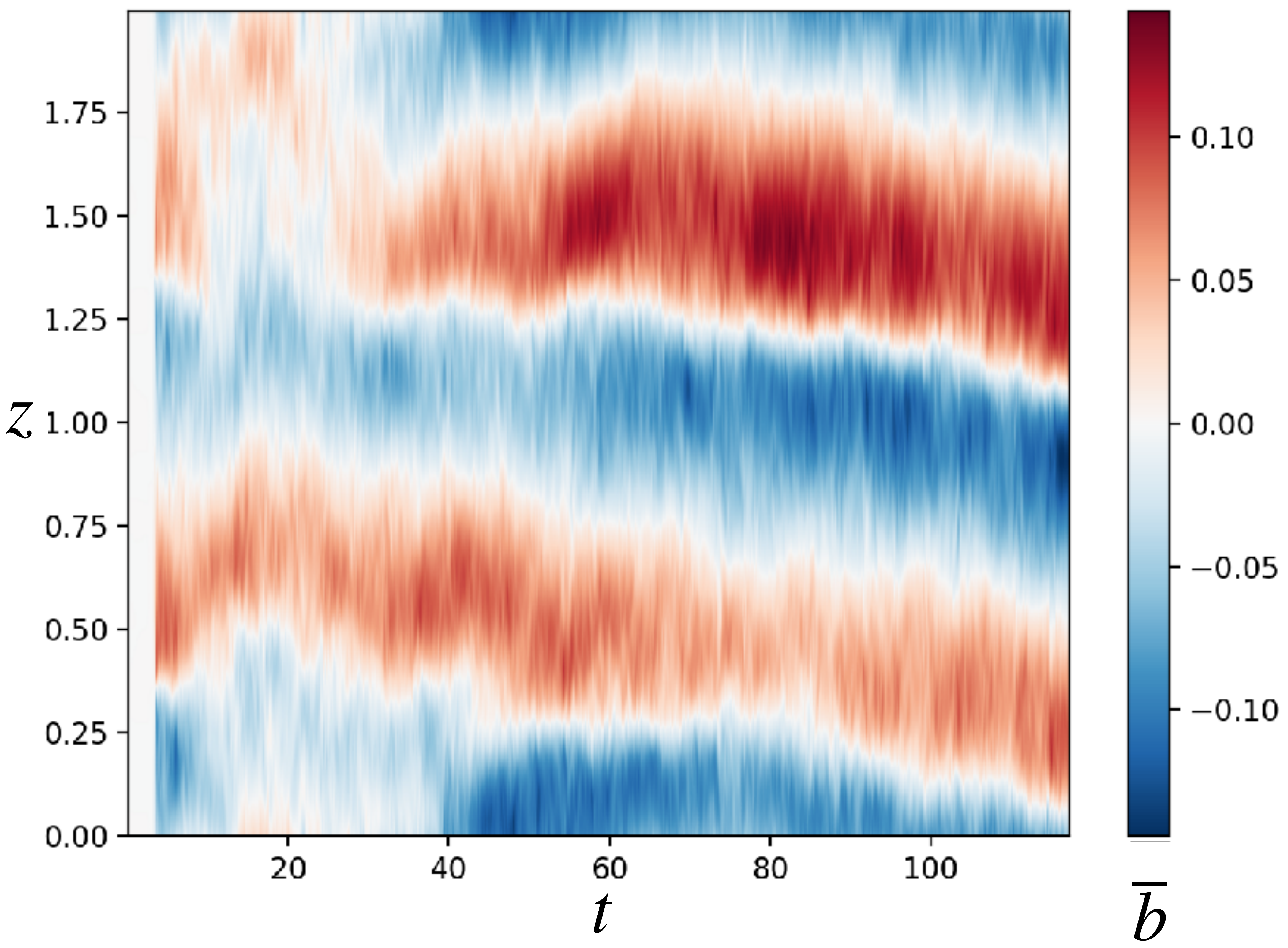
Side view

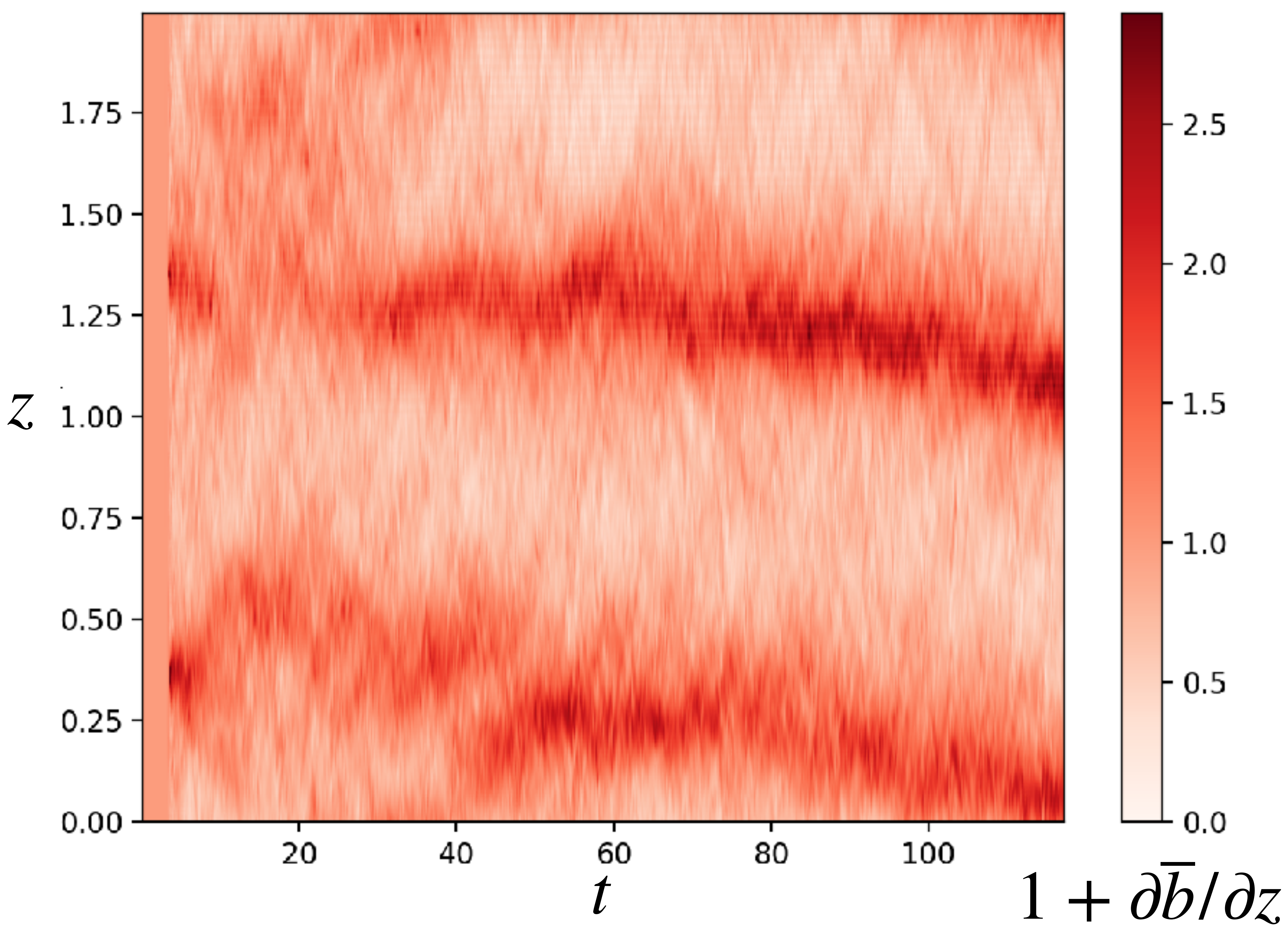


DNS

Horizontally-averaged profiles

$$Re_M = 800; \;\; Ri_M = 60 \;\; \Rightarrow Re_b \sim 10$$





DNS

Horizontally-averaged profiles

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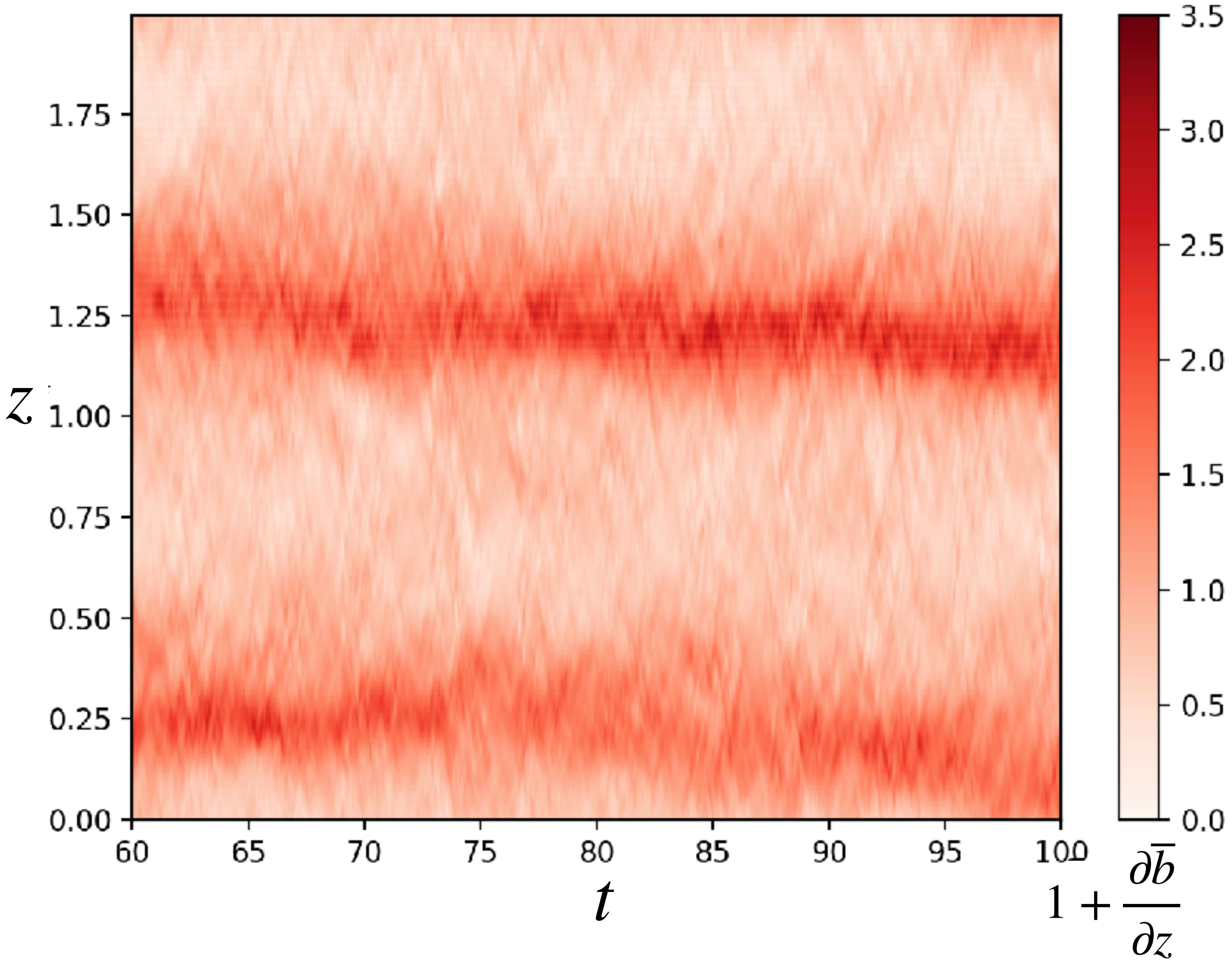

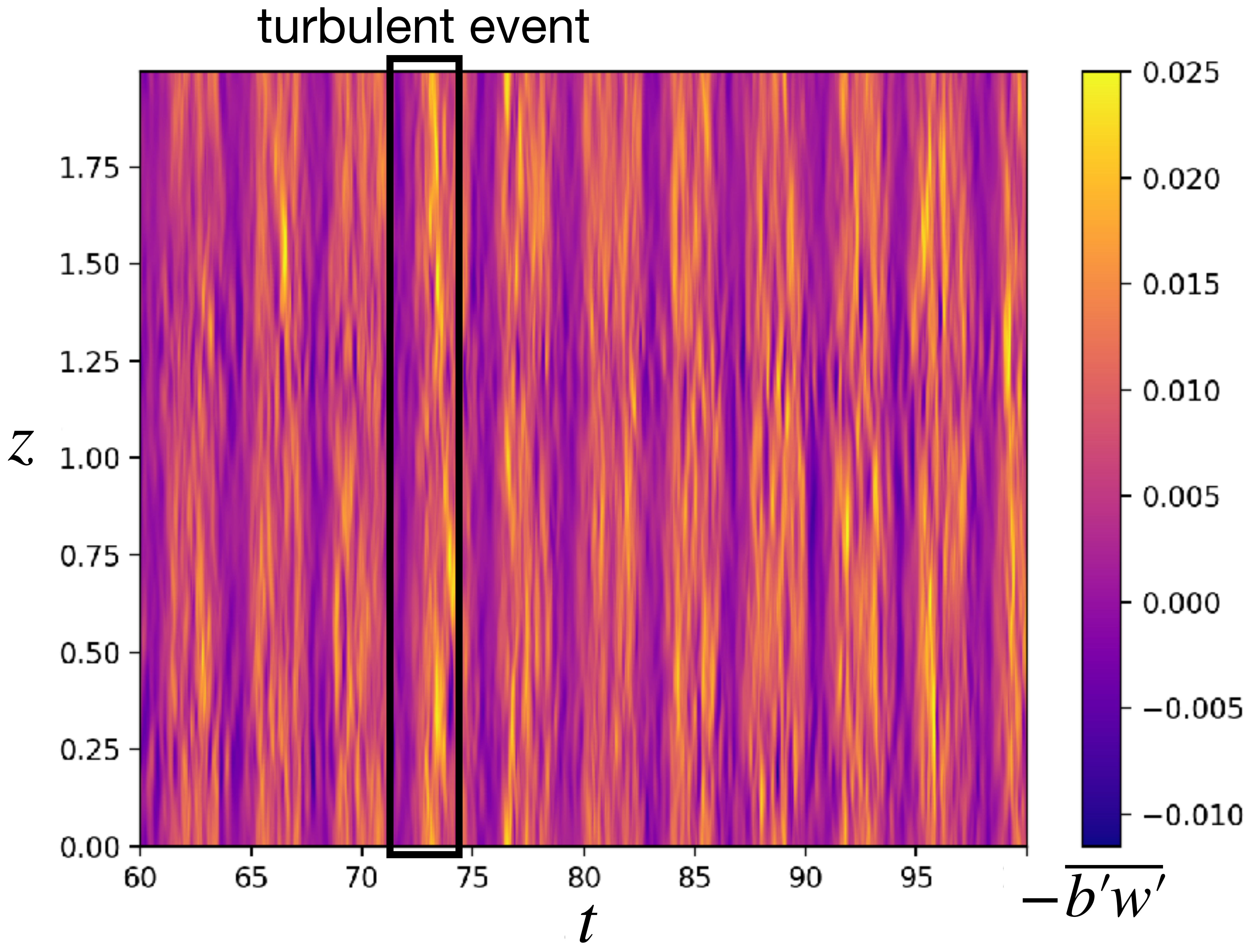

DNS

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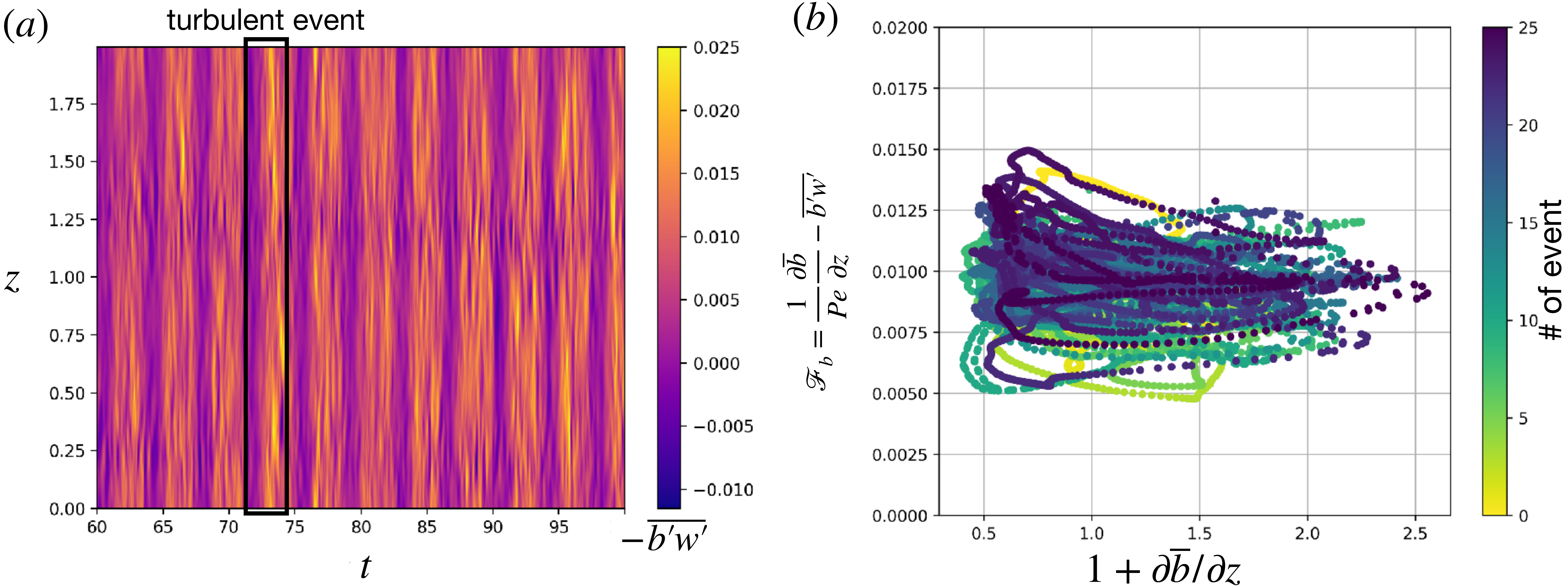
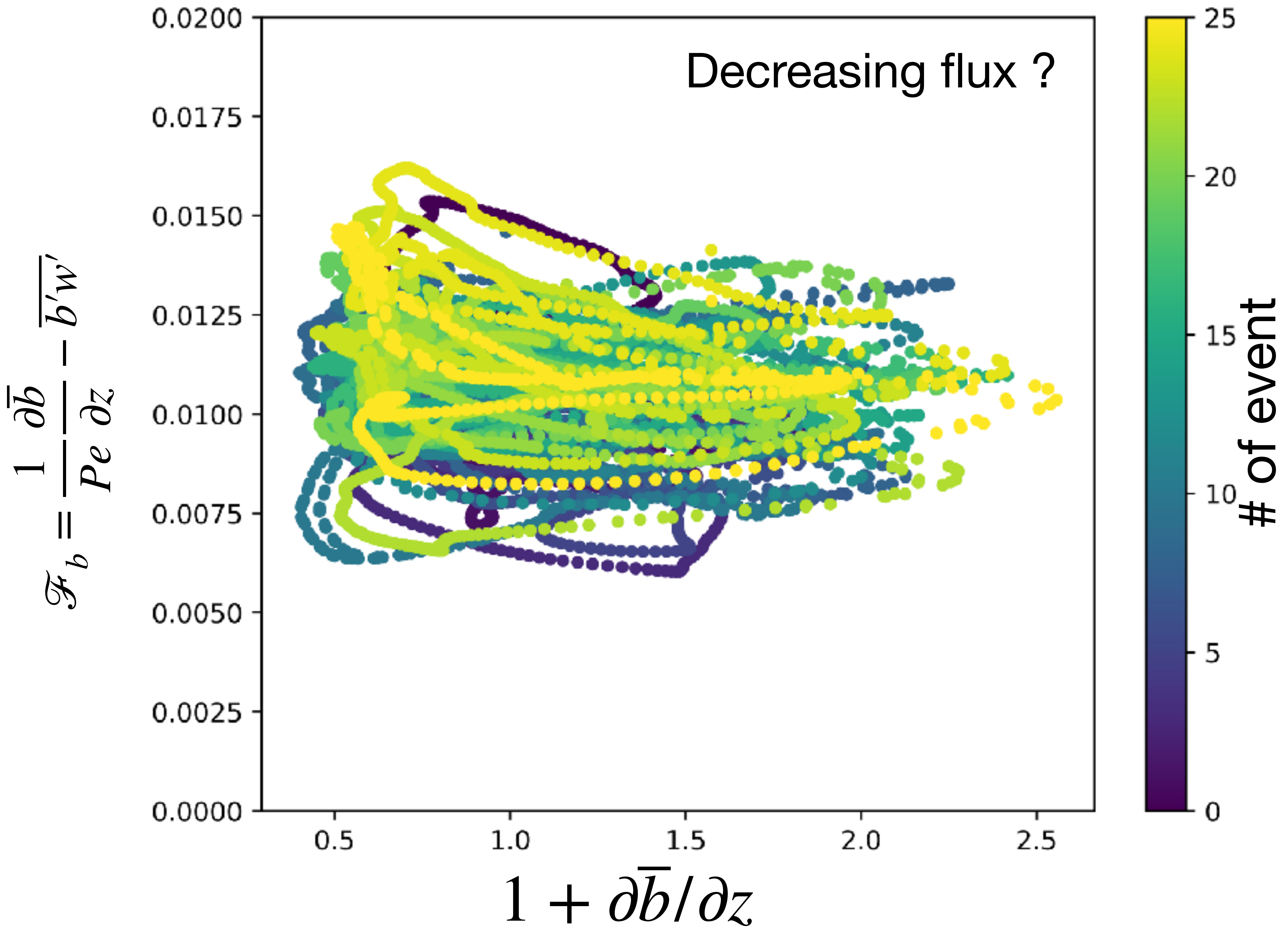
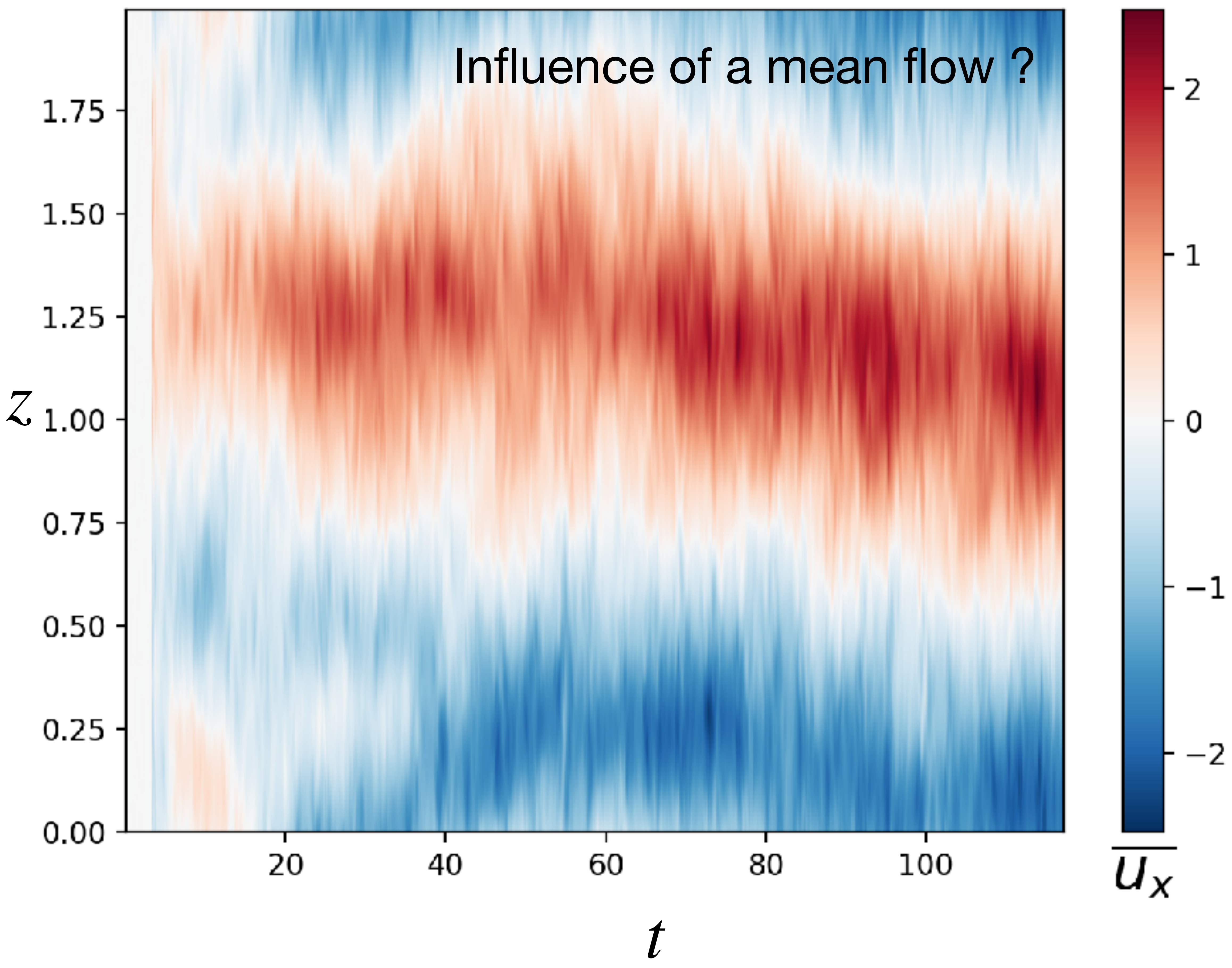


DNS

Results

$$Re_M = 800;\ \ Ri_M = 60\ \ \Rightarrow Re_b \sim 10$$





DNS
Results

- Able to create ‘layers’ in the DNS, but none were completely mixed. They may need more time to develop ?
- Correlation between region of turbulence and lower gradients ; decrease of the buoyancy flux ?
- Artificial mean flow may have an influence on the layers formation.

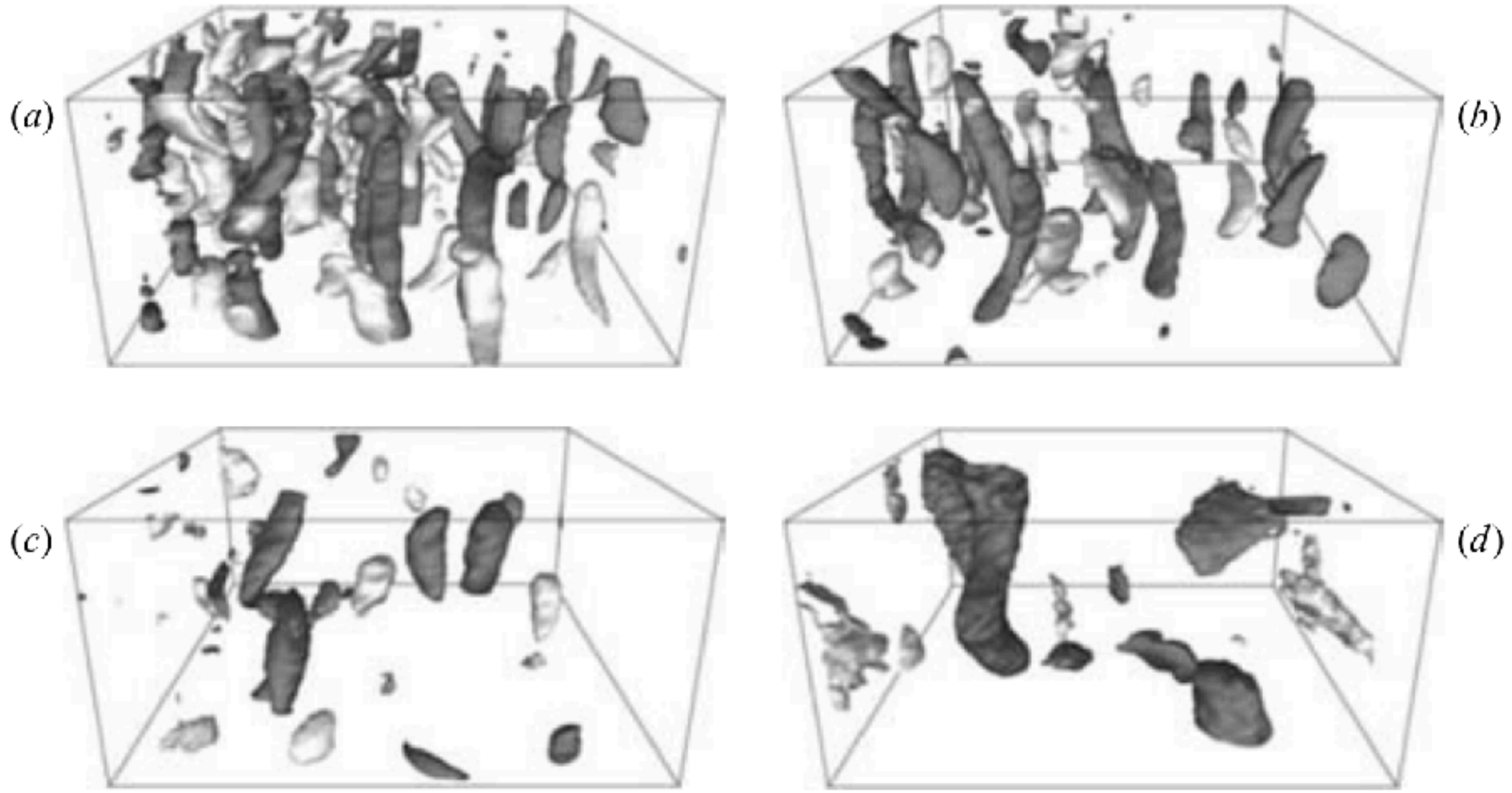
Influence of rotation

Influence of rotation

Motivation

- Mesoscales in the ocean and turbulence in stellar interiors influenced both by stratification and rotation

- Antagonist effects :
    - Stratification tends to layering → 'pancake' eddies
    - Rotation alone causes invariance in the vertical → columnar eddies



$Ro_M = 0.28;\ Fr_M = 0.04;\ f/N = 0.15$

Iso-surfaces of ω_z at different times for $Re_M = 4500$, from Praud et al (2005)

Influence of rotation

Motivation

- Mesoscales in the ocean and turbulence in stellar interiors influenced both by stratification and rotation

- Antagonist effects :
    - Stratification tends to layering → 'pancake' eddies
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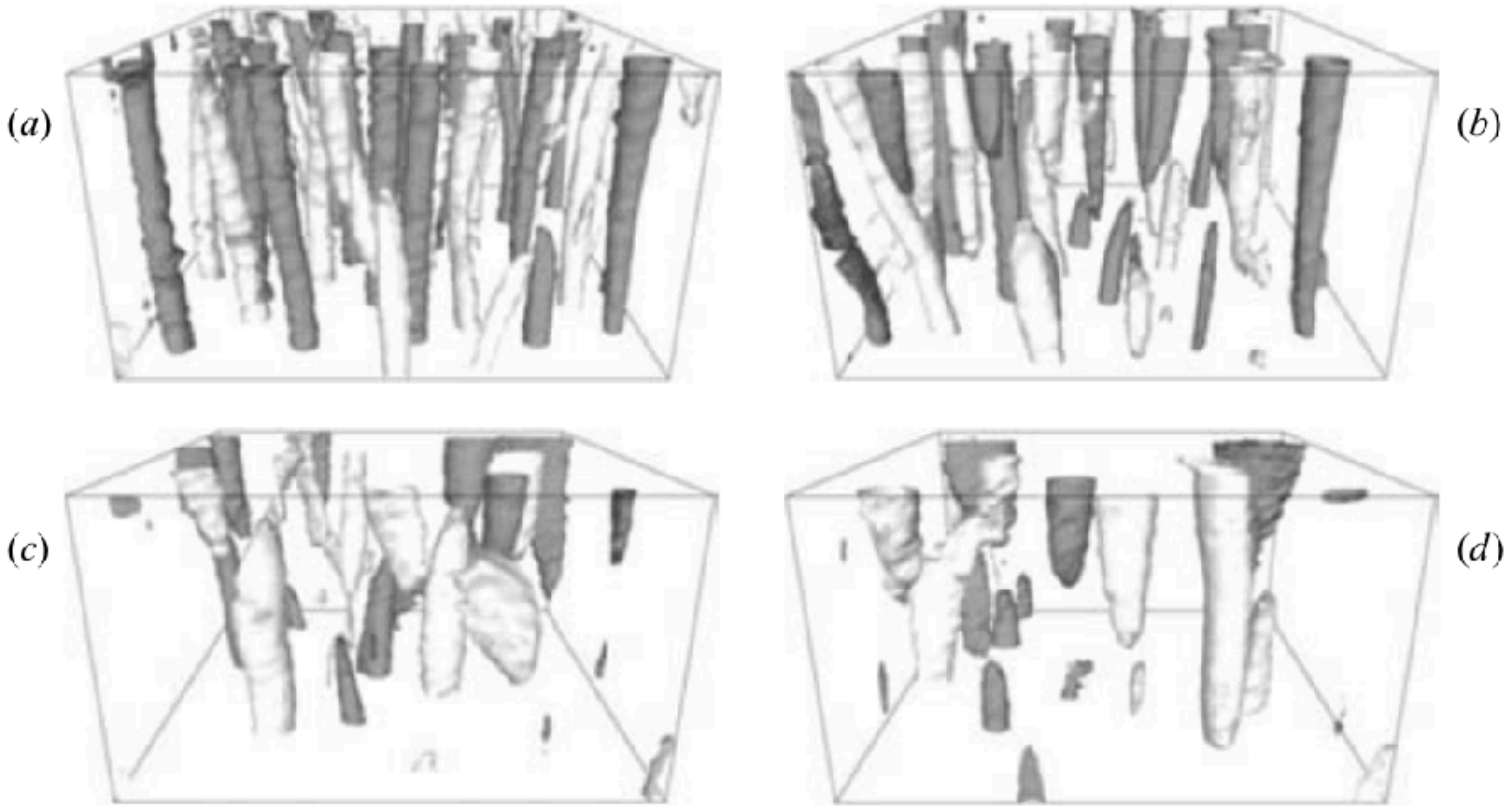


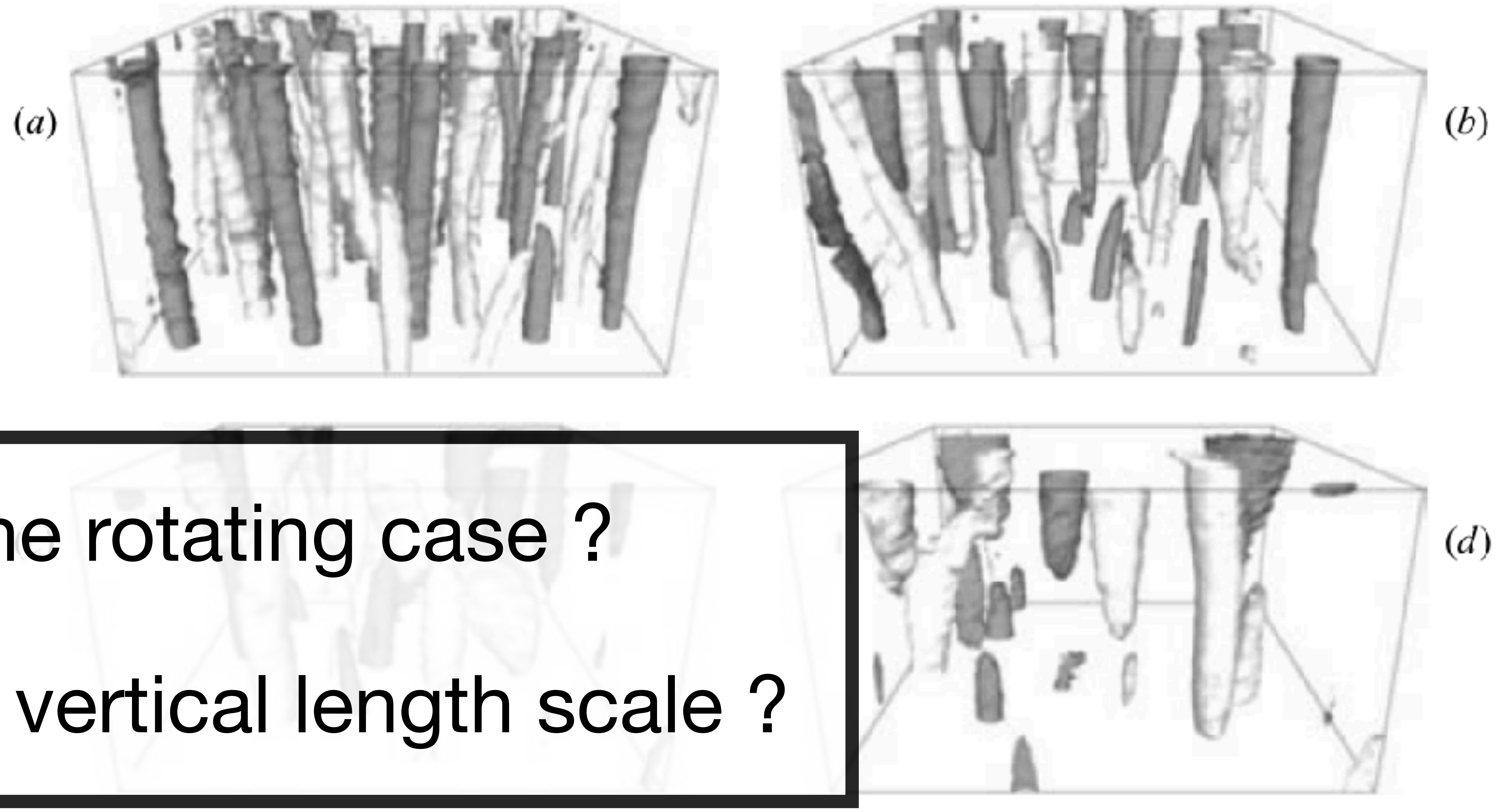
$Ro_M = 0.07;\ Fr_M = 0.09;\ f/N = 1.2$

Iso-surfaces of ω_z at different times for $Re_M = 4500$, from Praud et al (2005)

Influence of rotation

Motivation

- Mesoscales in the ocean and turbulence in stellar interiors influenced both by stratification and rotation



Do layers still form in the rotating case ?

If they do, what is their vertical length scale ?

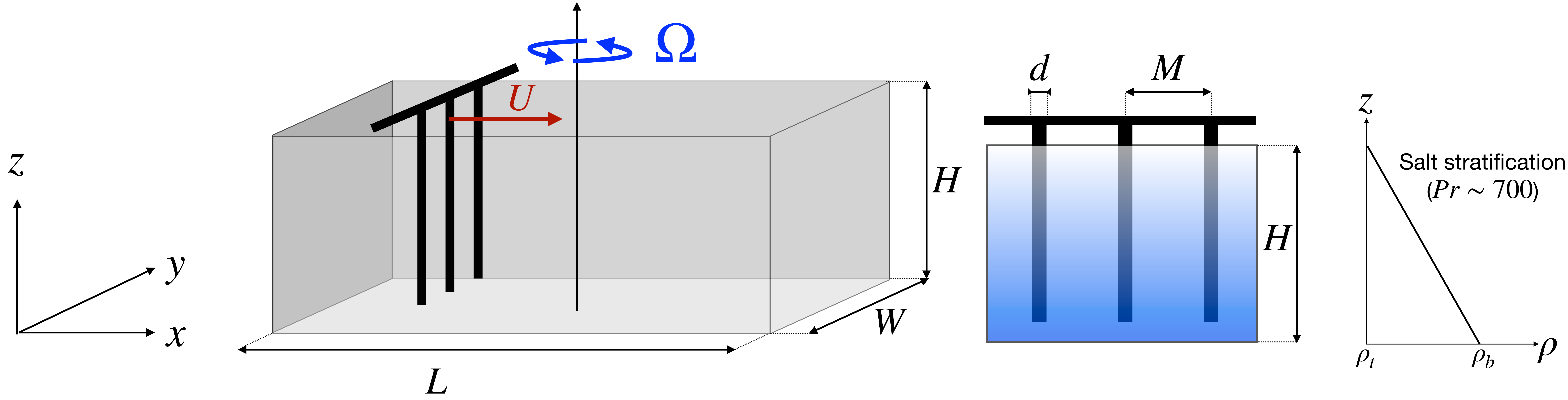
- Antagonist effects :
  - Stratification tends to layering → 'pancake' eddies
  - Rotation alone causes invariance in the vertical → columnar eddies

$Ro_M = 0.07;\ Fr_M = 0.09;\ f/N = 1.2$

Iso-surfaces of ω_z at different times for $Re_M = 4500$, from Praud et al (2005)

Rotating experiments

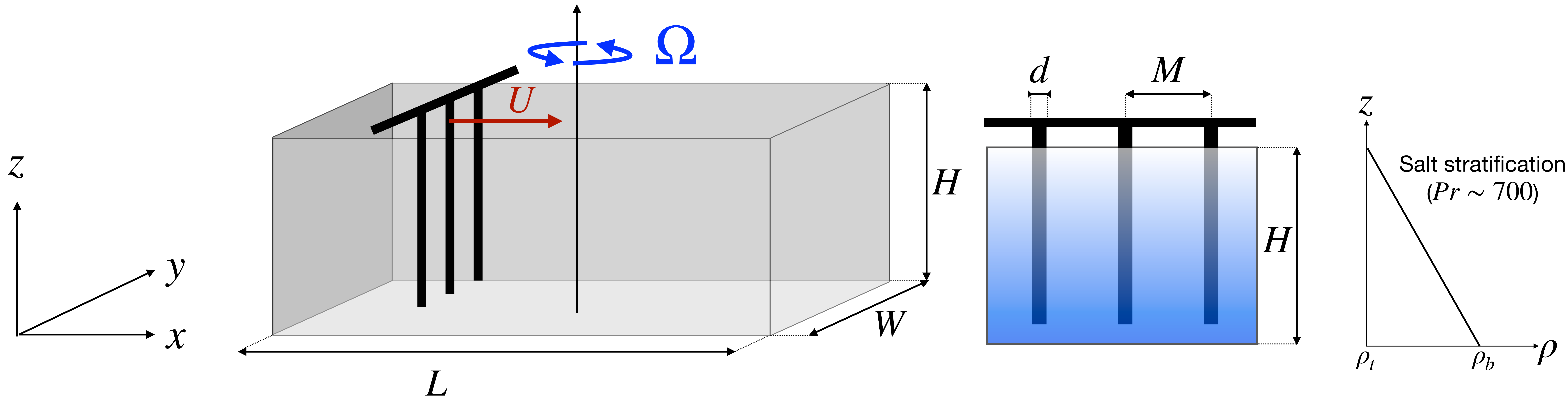
Dimensionless parameters



Reynold number $Re_M = UM/\nu$	Richardson number $Ri_M = N_0^2 M^2/U^2$	Rossby number $Ro_M = U/Mf$	Frequency ratio f/N

Rotating experiments

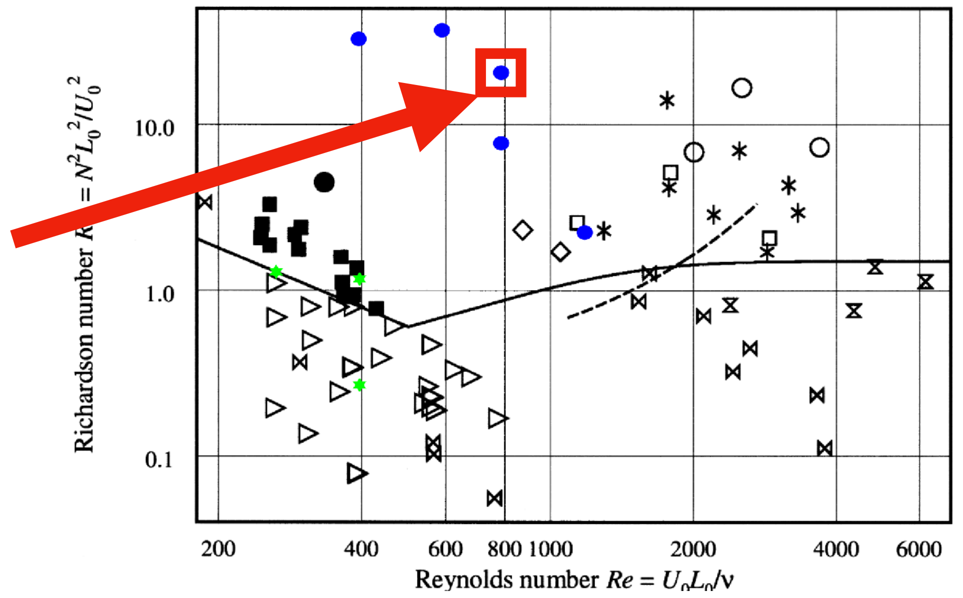
Dimensionless parameters

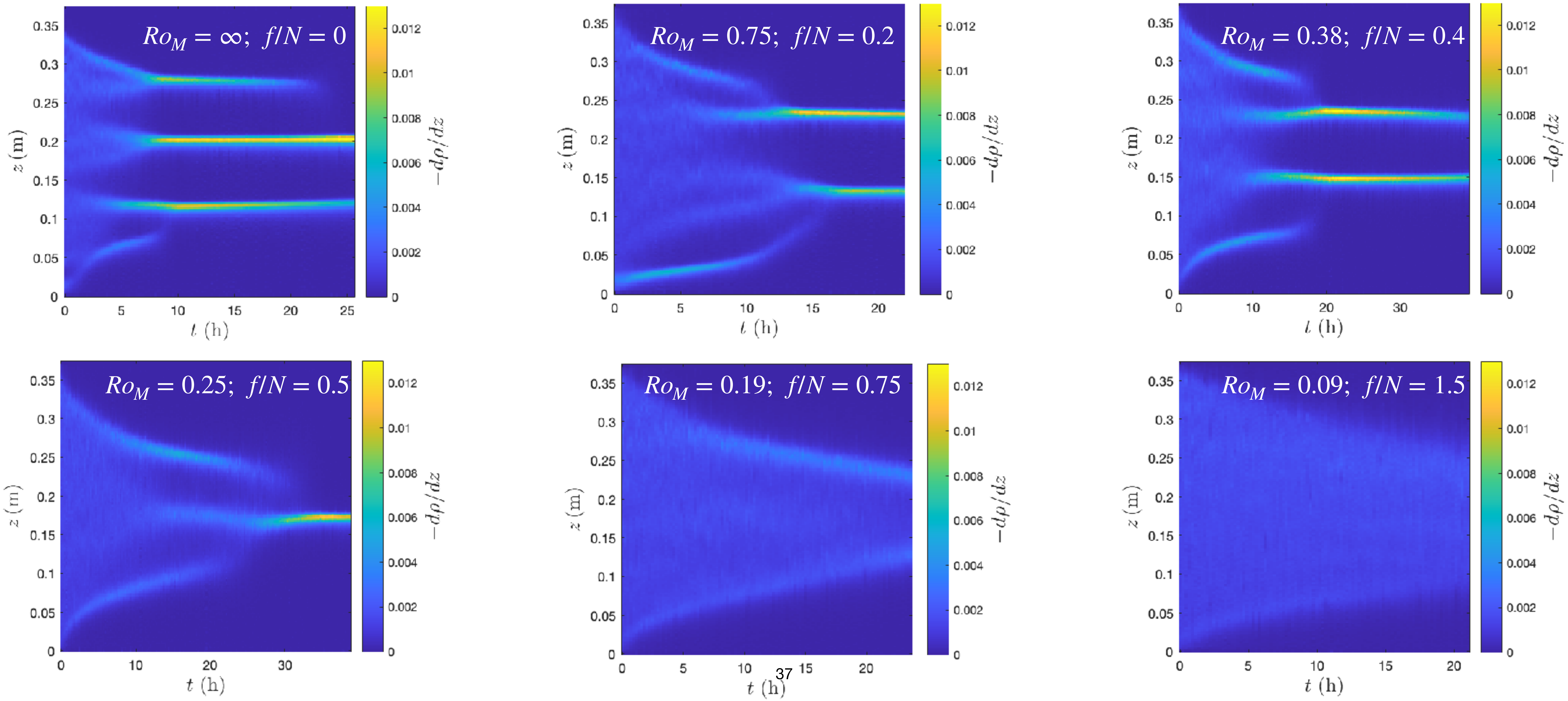


Reynold number $Re_M = UM/\nu$	Richardson number $Ri_M = N_0^2 M^2/U^2$	Rossby number $Ro_M = U/Mf$	Frequency ratio f/N

Layering ?

$Re_M = 2100;\ Ri_M \sim 50$

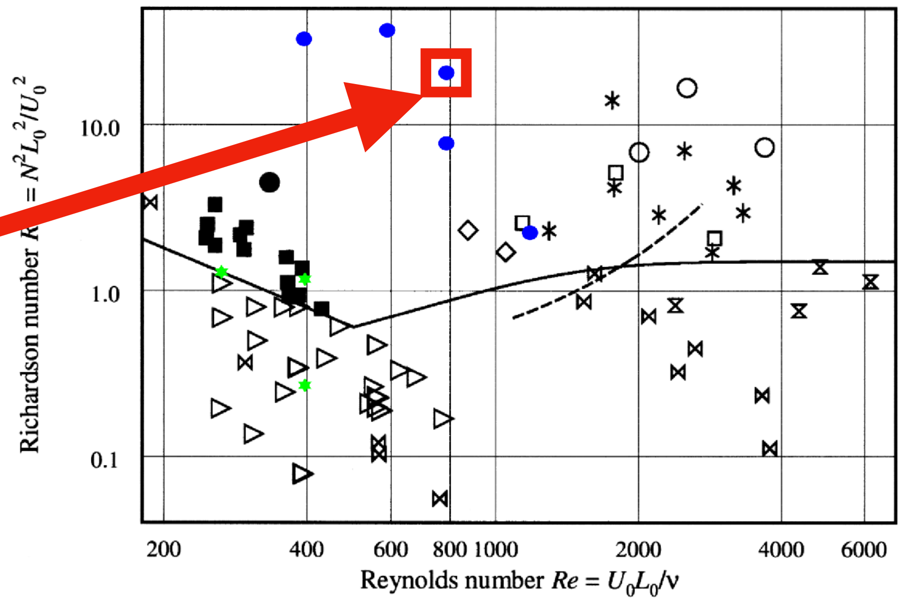




The experiments

Flow visualisation

$Re_M = 2100;\ \ Ri_M \sim 50$



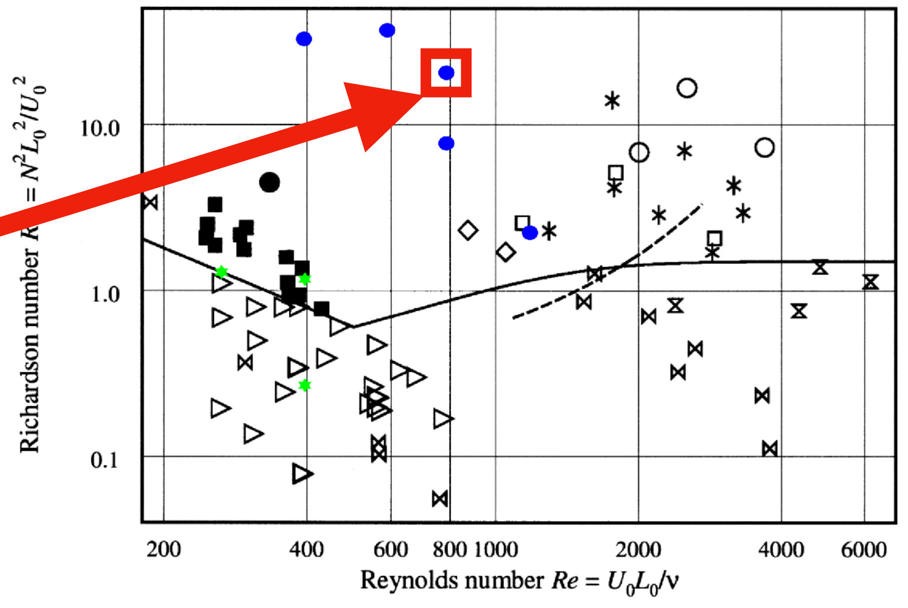
$Ro_M = \infty;\ \ f/N = 0$

$Ro_M = 0.09;\ \ f/N = 1.5$

The experiments

Flow visualisation

$Re_M = 2100;\ Ri_M \sim 50$



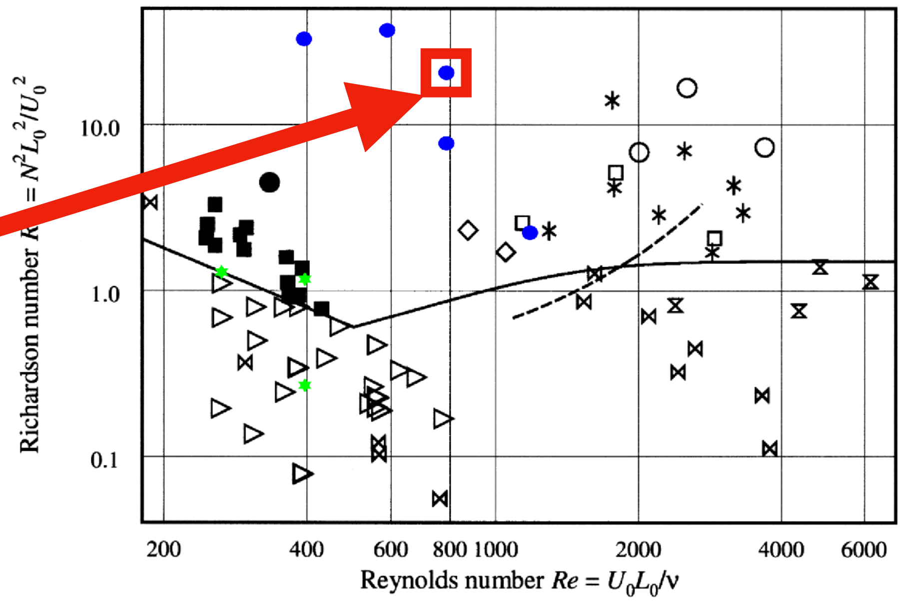
$Ro_M = \infty;\ f/N = 0$

$Ro_M = 0.09;\ f/N = 1.5$

The experiments

Flow visualisation

$Re_M = 2100; \;\; Ri_M \sim 50$



$Ro_M = \infty; \;\; f/N = 0$

$Ro_M = 0.09; \;\; f/N = 1.5$

DNS

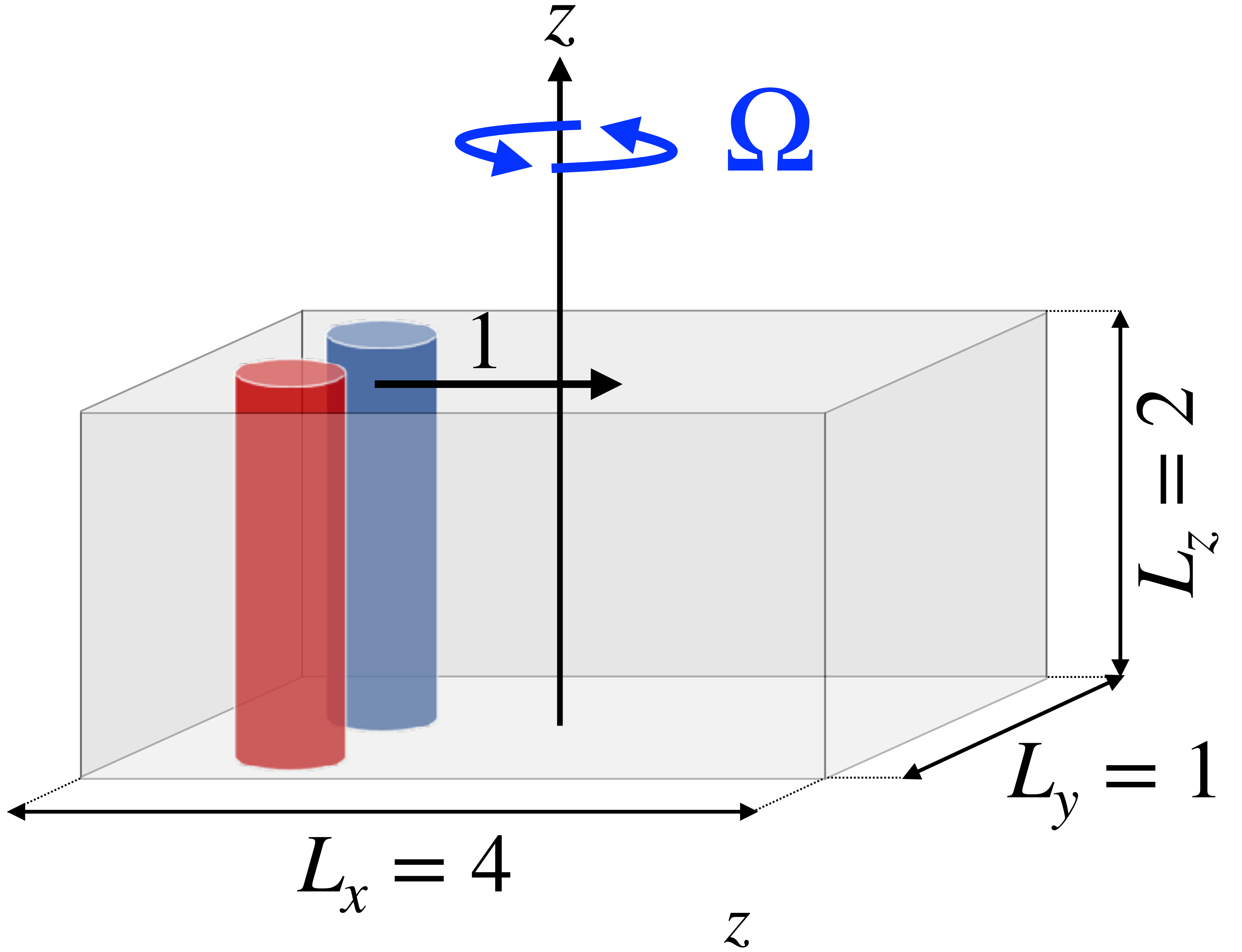
Adding rotation

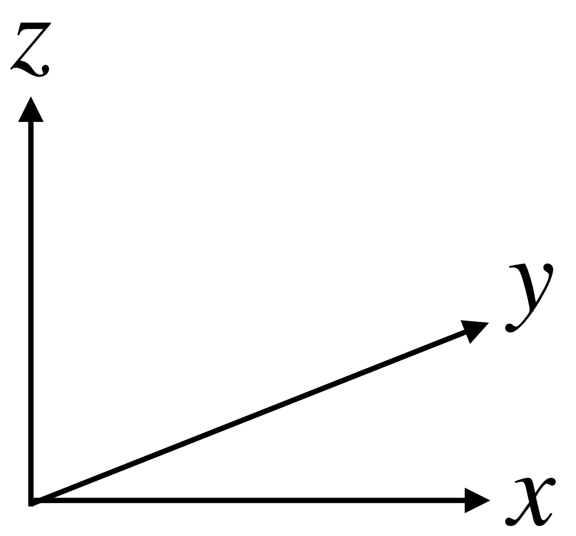
$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \, . \, \nabla \mathbf{u} + \frac{\mathbf{e_z} \times \mathbf{u}}{Ro} = -\nabla p + Ri \; b\mathbf{e_z} + \frac{1}{Re} \nabla^2 \mathbf{u} + \mathbf{F}$$

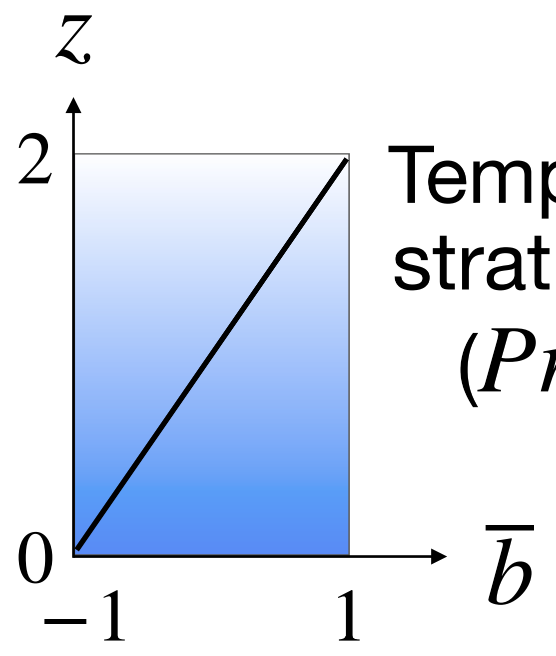
$$\nabla \, . \, \mathbf{u} = 0$$

$$\frac{\partial b}{\partial t} + \mathbf{u} \, . \, \nabla b + w = \frac{1}{Pe} \nabla^2 b$$

with $Re = \dfrac{UM}{\nu}$, $Pe = \dfrac{UM}{\kappa}$, $Ri = \dfrac{N^2 M^2}{M^2}$; $Ro = \dfrac{U}{2\Omega M}$.



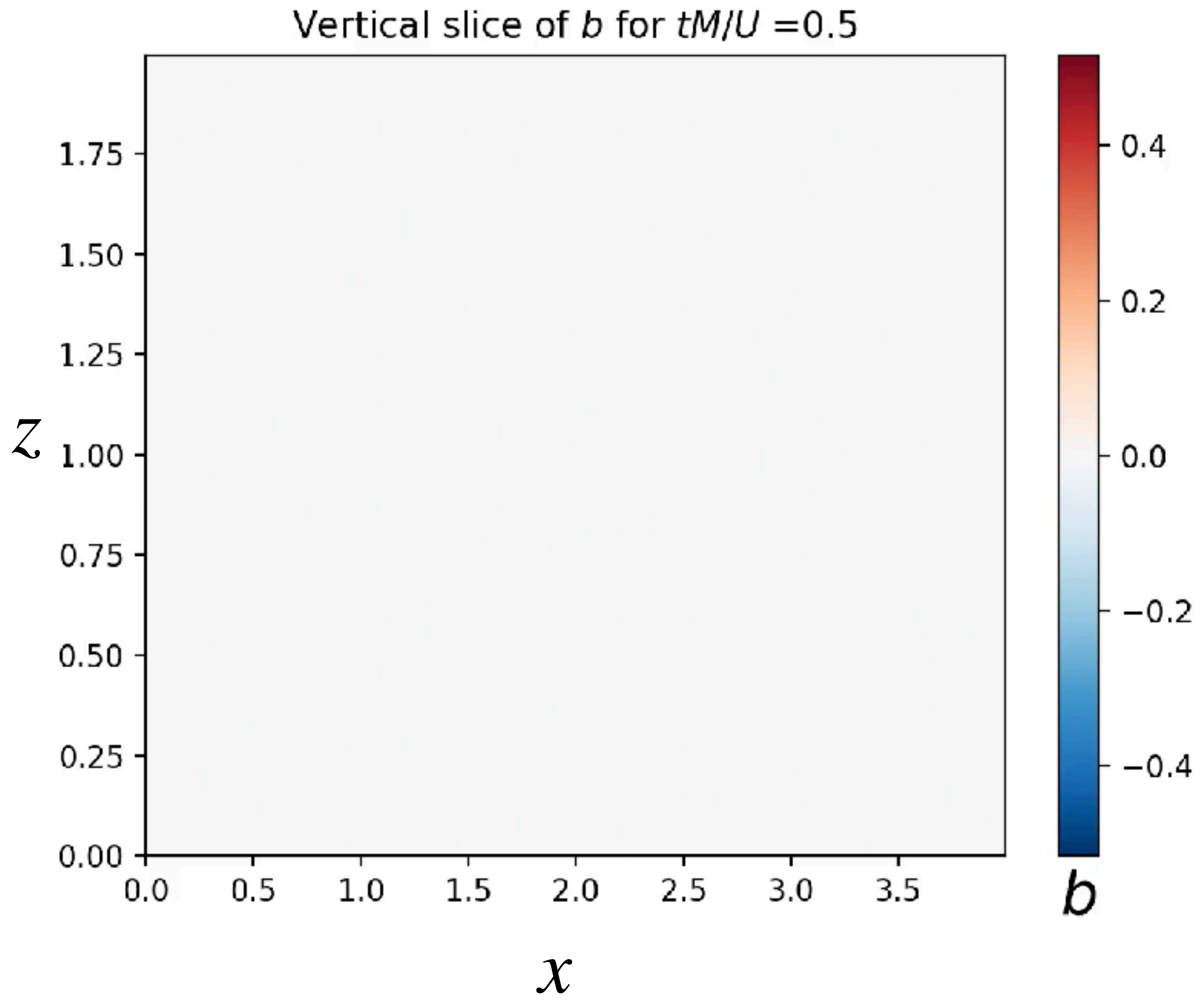




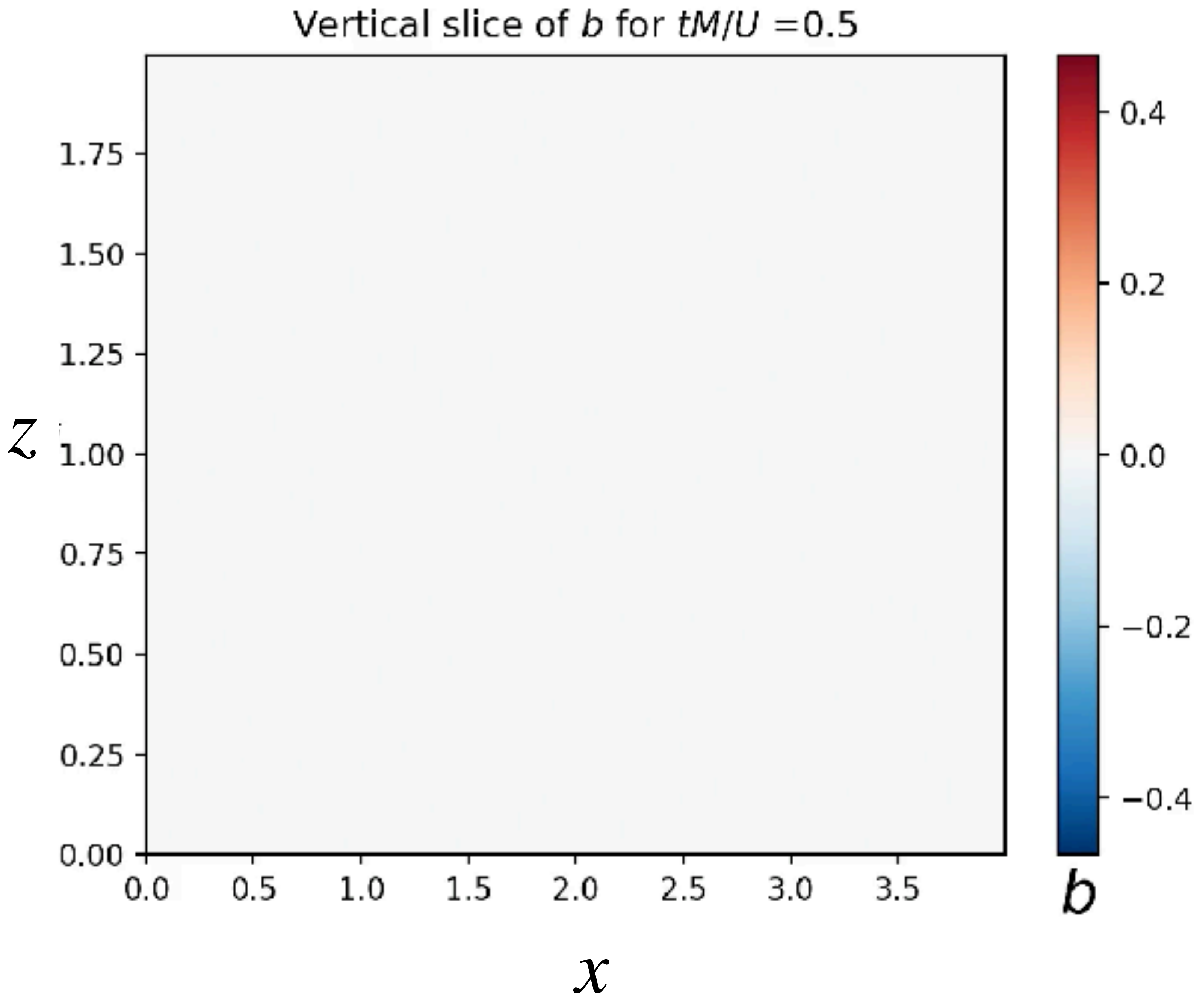
DNS

Side view

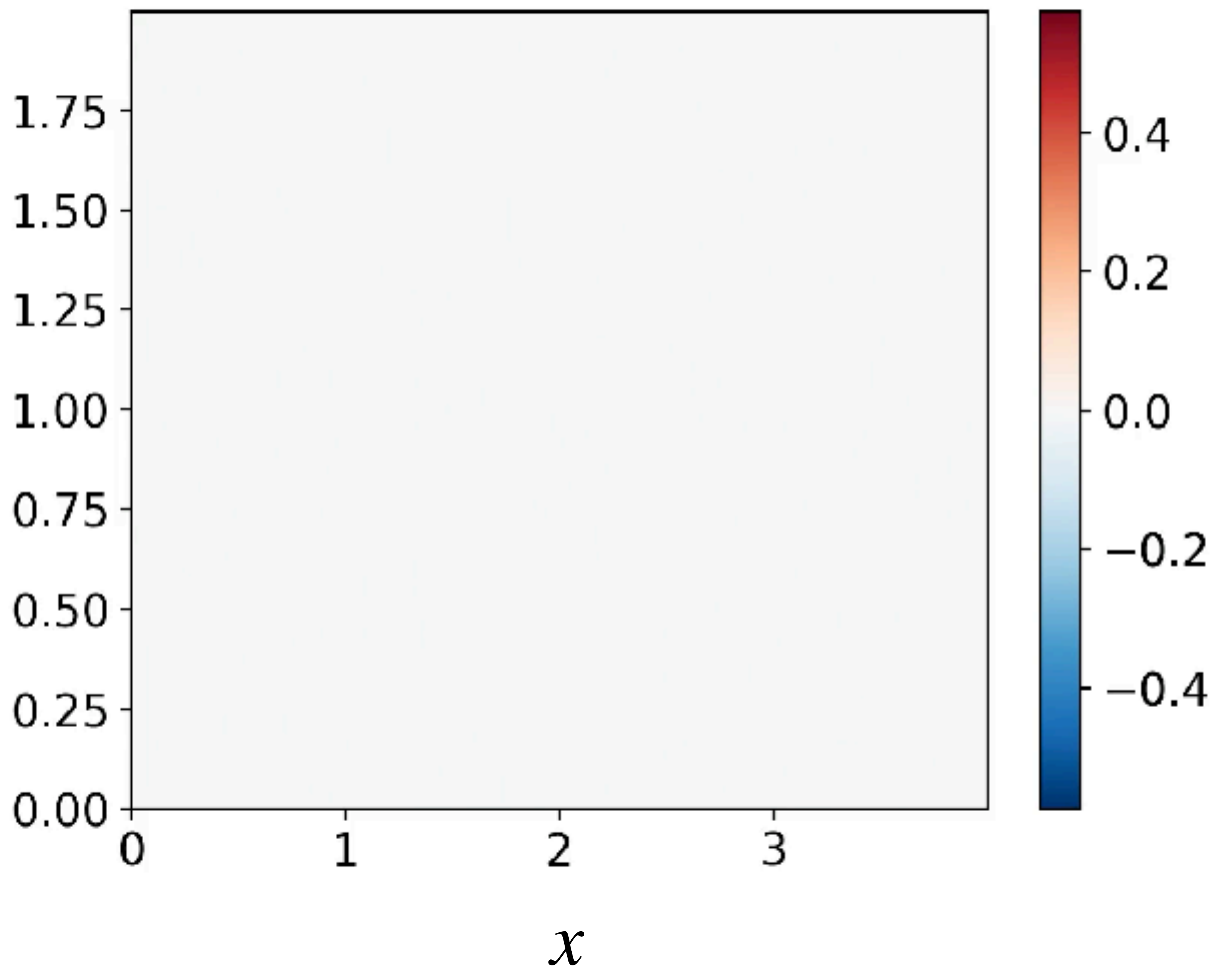
$$Re = 800; \;\; Ri = 60 \;\; \Rightarrow Re_b \sim 10$$



$Ro_M = \infty; \;\; f/N = 0$



$Ro_M = 1; \;\; f/N = 0.13$

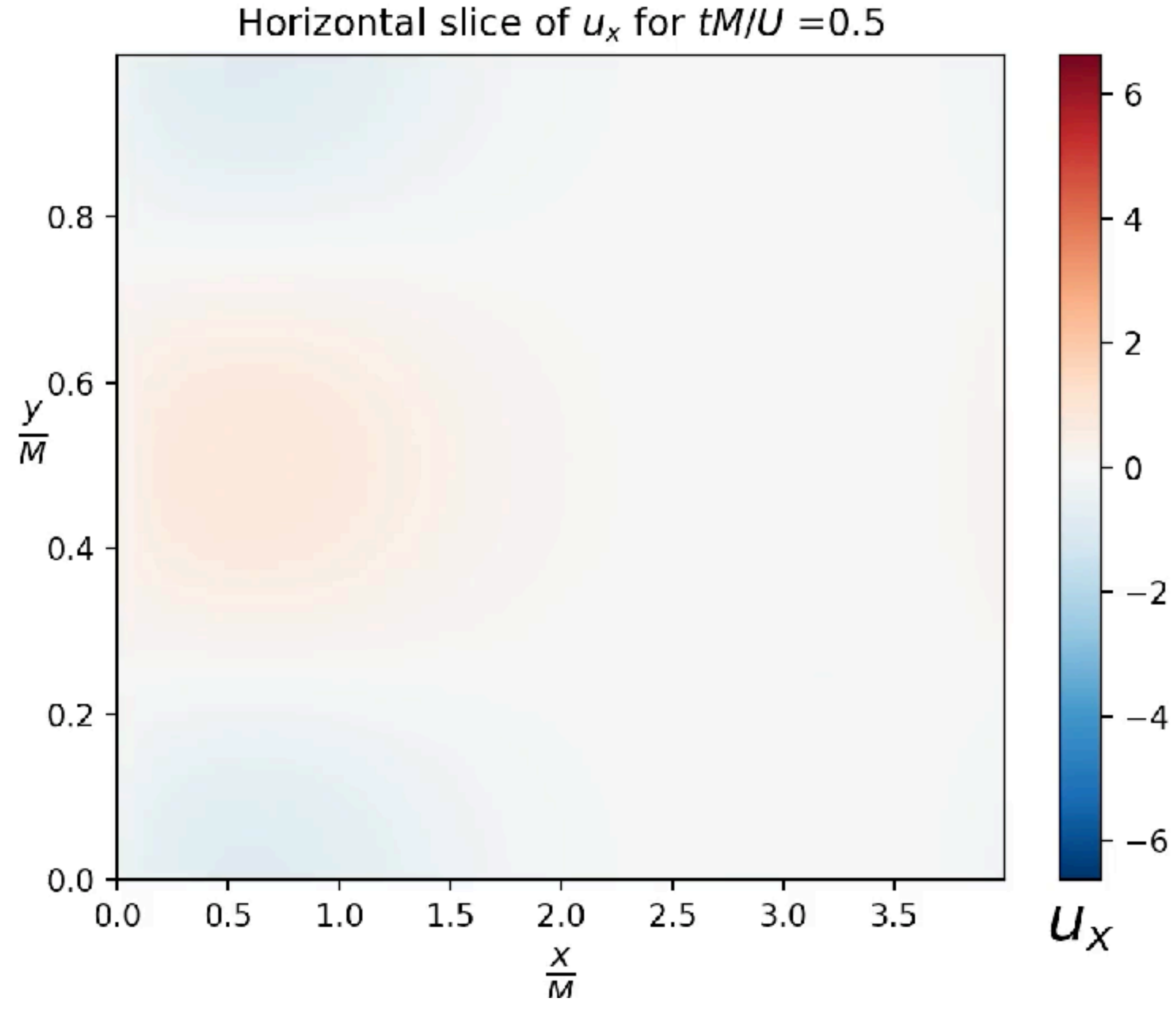


$Ro_M = 0.12; \;\; f/N = 0.8$

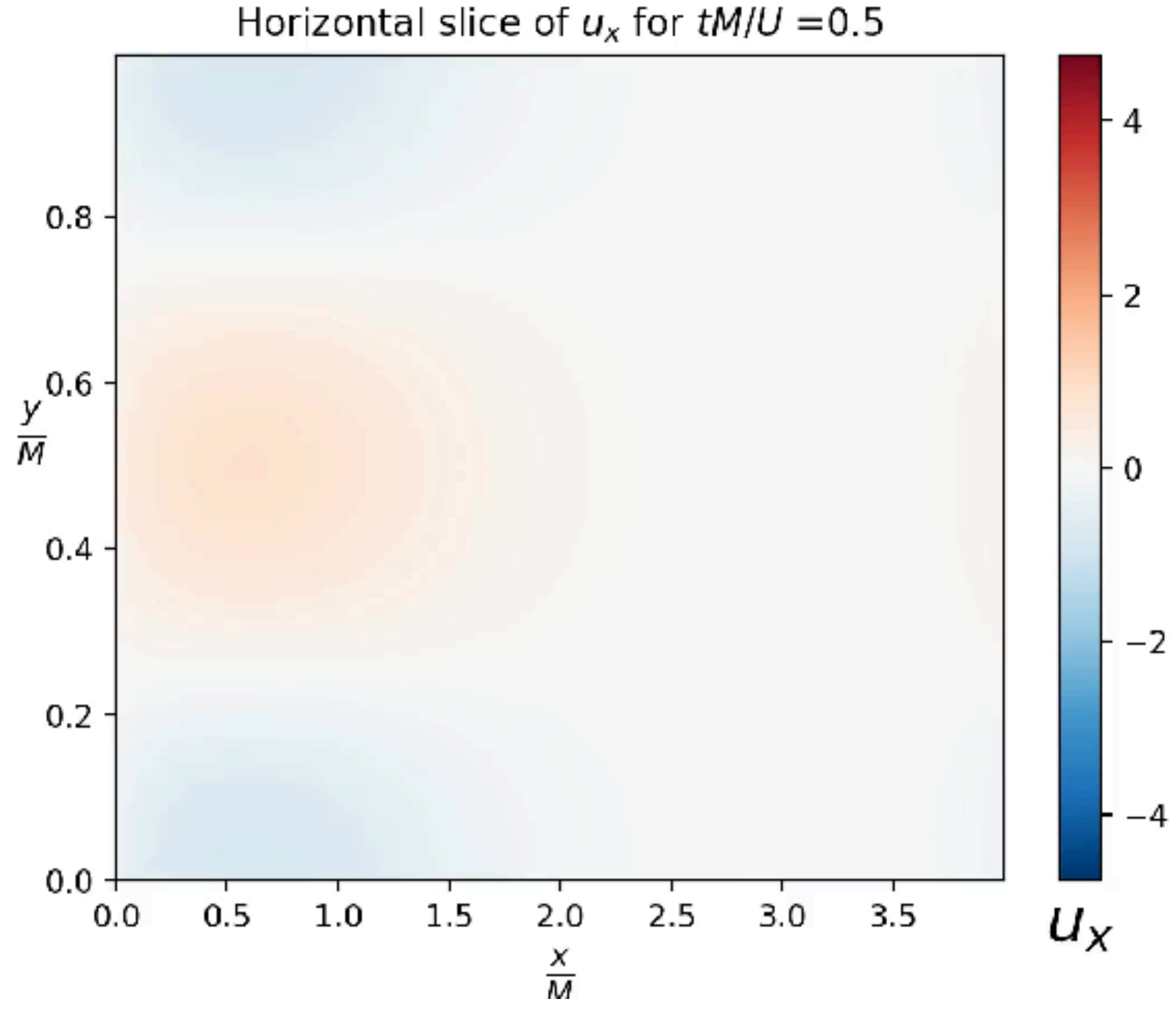
DNS

Top view

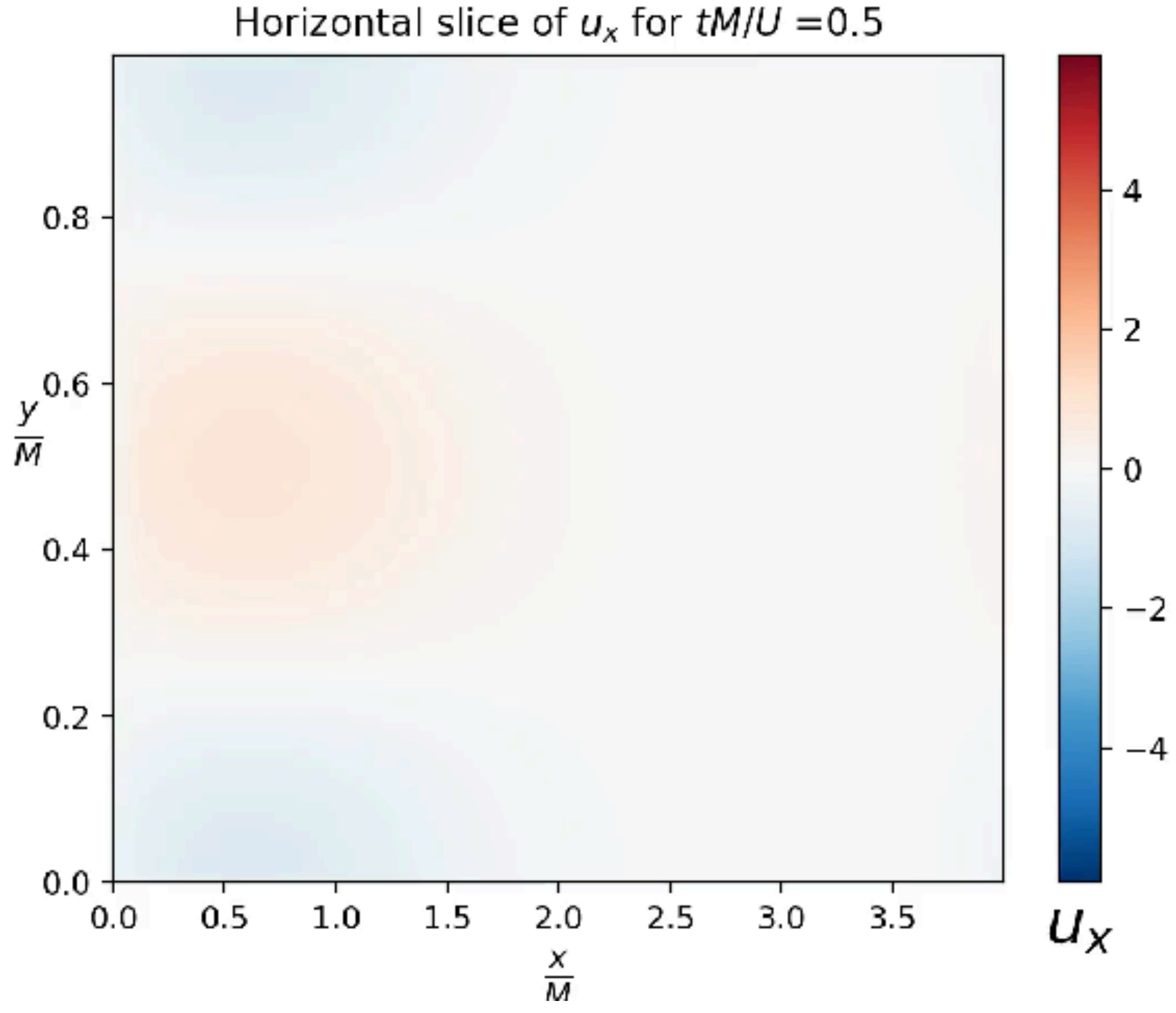
$Re = 800;\ Ri = 60\ \Rightarrow Re_b \sim 10$



$Ro_M = \infty;\ f/N = 0$



$Ro_M = 1;\ f/N = 0.13$



$Ro_M = 0.12;\ f/N = 0.8$

DNS

Horizontally-averaged profiles

$Re = 800;\; Ri = 60 \;\Rightarrow Re_b \sim 10$

$Ro_M = \infty;\; f/N = 0$ $\qquad$ $Ro_M = 1;\; f/N = 0.13$ $\qquad$ $Ro_M = 0.12;\; f/N = 0.8$

DNS

Horizontally-averaged profiles

$Re = 800;\ Ri = 60\ \Rightarrow Re_b \sim 10$

$Ro_M = \infty;\ f/N = 0$ $\quad$ $Ro_M = 1;\ f/N = 0.13$ $\quad$ $Ro_M = 0.12;\ f/N = 0.8$

DNS

Horizontally-averaged profiles

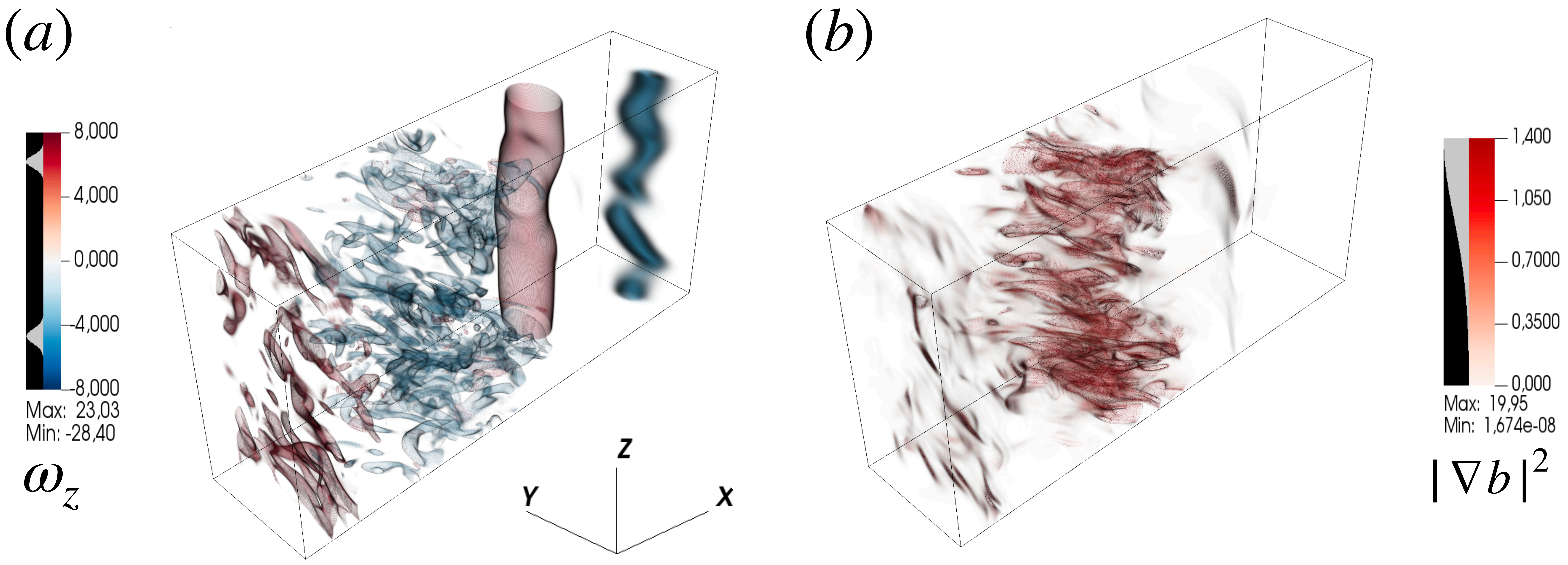
$Re = 800;\ Ri = 60\ \Rightarrow Re_b \sim 10$

$Ro_M = \infty;\ f/N = 0$ $\quad Ro_M = 1;\ f/N = 0.13$ $\quad Ro_M = 0.12;\ f/N = 0.8$

$Ro_M = 0.12;\ f/N = 0.8$

DNS

Vertical vorticity



Conclusion

- Layering inhibited by strong rotation rates
- Very different structures of the flow :
  - when $f/N = O(1)$, flow dominated by vertically invariant cyclonic vortices.
  - Only part of the domain turbulent
- The layers scale does not seem to be very affected by the rotation, but would need more data points to confirm.

Perspectives

Non-rotating case

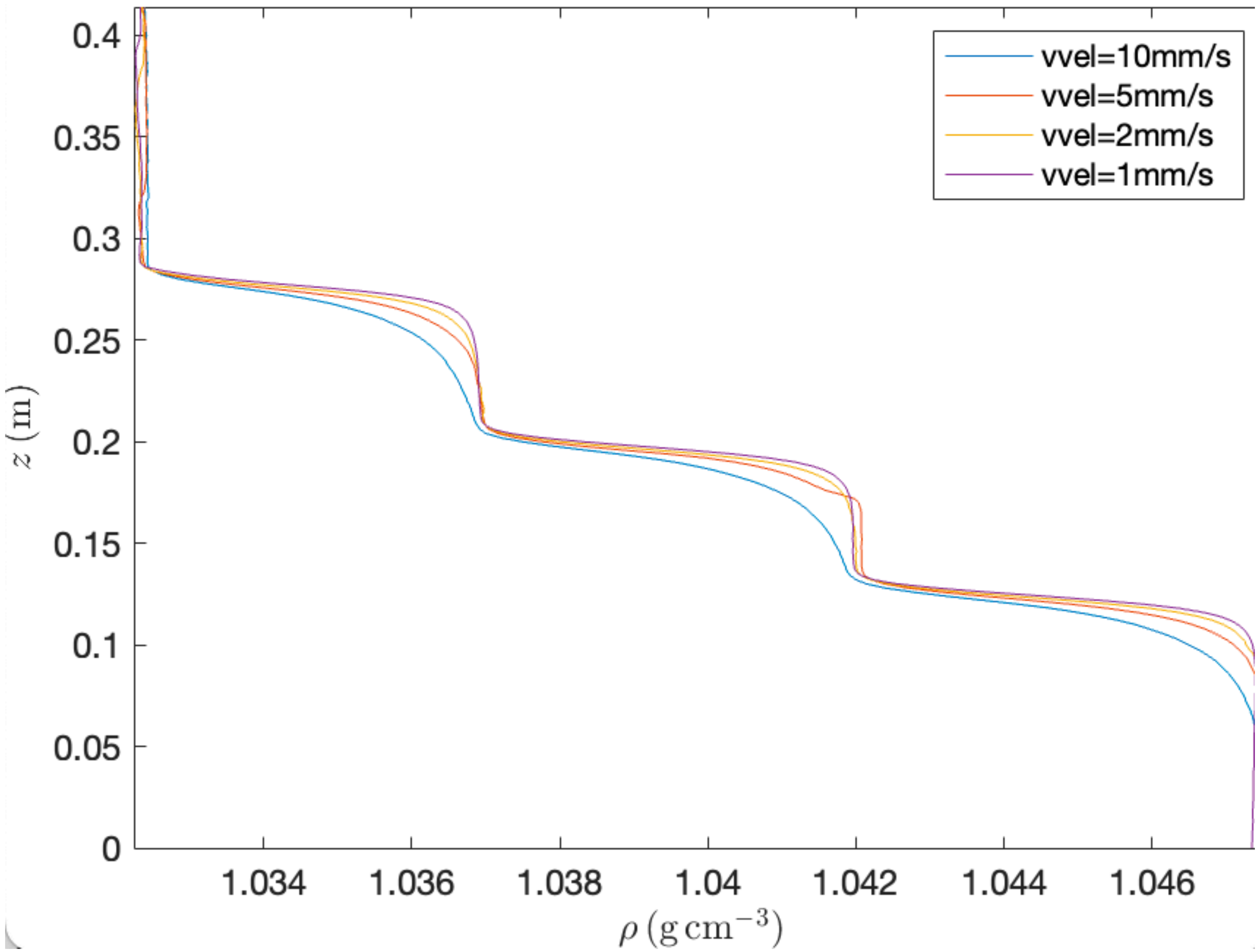
- More data in the boundary layering regime
- Flow visualisation and turbulence measurements : PIV ? Synthetic schlieren ?
- Put walls in the DNS

Rotating case

- Influence of the Rossby number ?
- Flow visualization : PIV tricky in the rotating case
- Put walls in the DNS

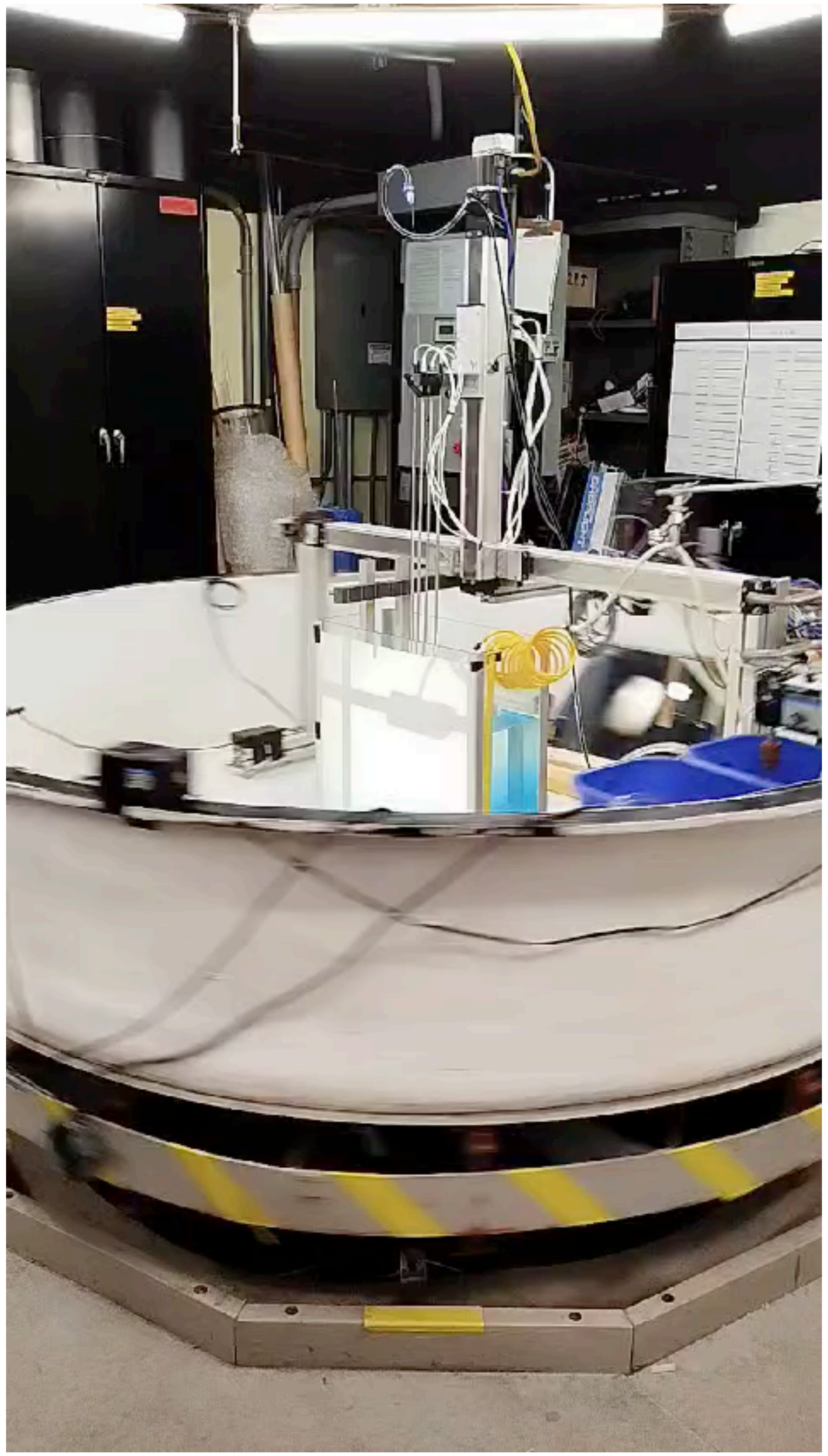
Appendix

Asymmetry of the steps



- Consequence of the probe dragging lighter fluid down

Appendix
Filling up the tank



Appendix
Filling up the tank