

Theoretical Foundations of Pre-trained Models

Qi Lei

Princeton University

A.I. is Everywhere



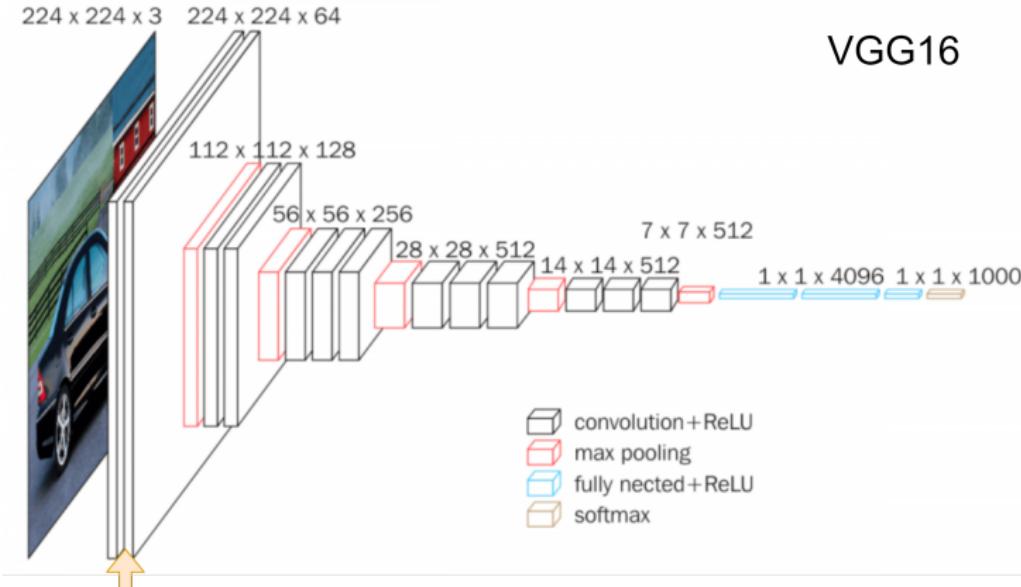
A.I. is Everywhere



A.I. is Everywhere

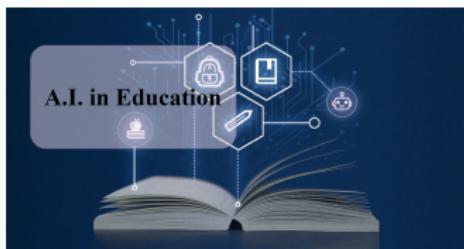


Deep Learning Requires Big Labeled Data



- Deep learning succeeds with abundant labeled data.

Emerging Application Domains Lack Data



Emerging Application Domains Lack Data

Labeled data is lacking: significant costs in money and time.



AI is undergoing a paradigm shift with pre-trained models.

Examples:

- language models: BERT, GPT-3

 SD Times

GPT-3 can now be customized to individual applications

Developers can now fine-tune GPT-3 on their own data, creating a custom version tailored to their application, which allows for faster and...

3 weeks ago



Figure source: Google News.

5/41

AI is undergoing a paradigm shift with pre-trained models.

Examples:

- language models: BERT, GPT-3
- code generation: Codex, AlphaCode

 The New Stack

[When DeepMind's 'AlphaCode' Competed Against Human ...](#)

This month, DeepMind announced that it has also developed a system named AlphaCode to compete in programming competitions, evaluating its...

4 days ago

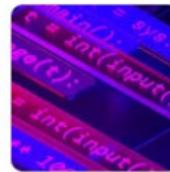


Figure source: Google News.

AI is undergoing a paradigm shift with pre-trained models.

Examples:

- language models: BERT, GPT-3
- code generation: Codex, AlphaCode
- multi-modal pre-trained models: DALL-E, CLIP

 VentureBeat

OpenAI's text-to-image engine, DALL-E, is a powerful visual idea generator

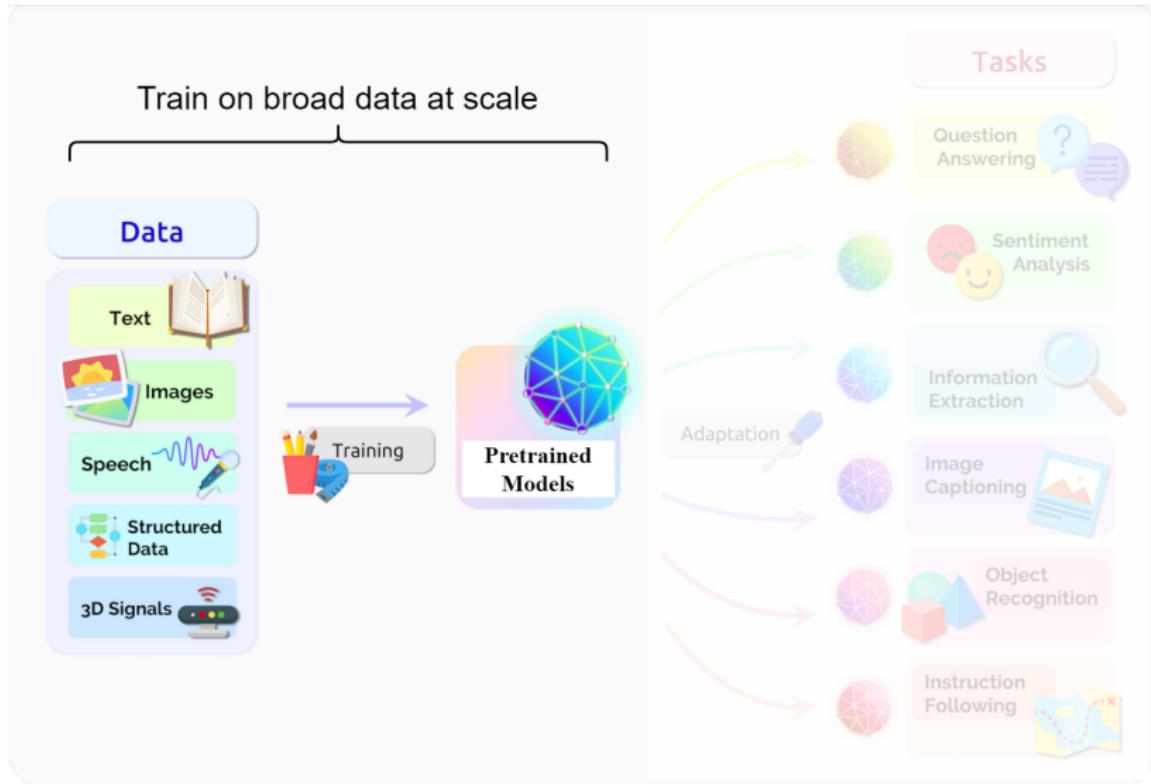
DALL-E is a 12-billion parameter version of the 175 billion parameter GPT-3 natural language processing neural network. GPT-3 "learns" based on...

Jan 16, 2021

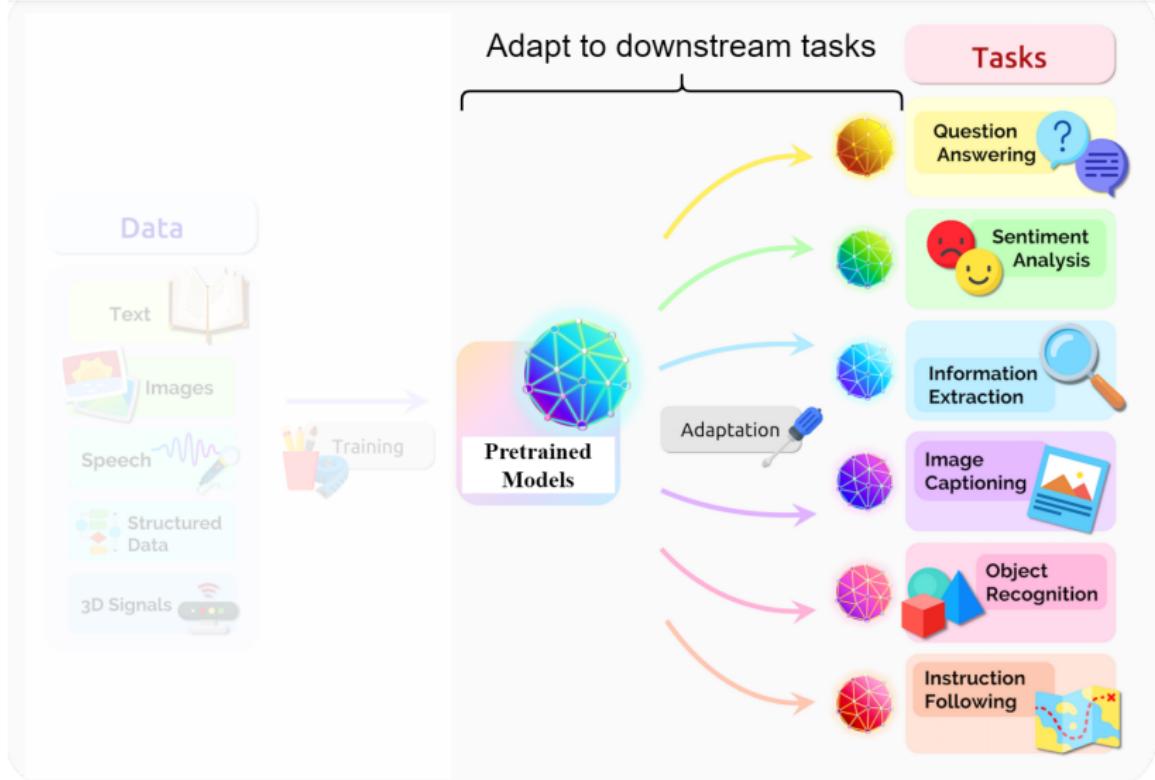


Figure source: Google News.

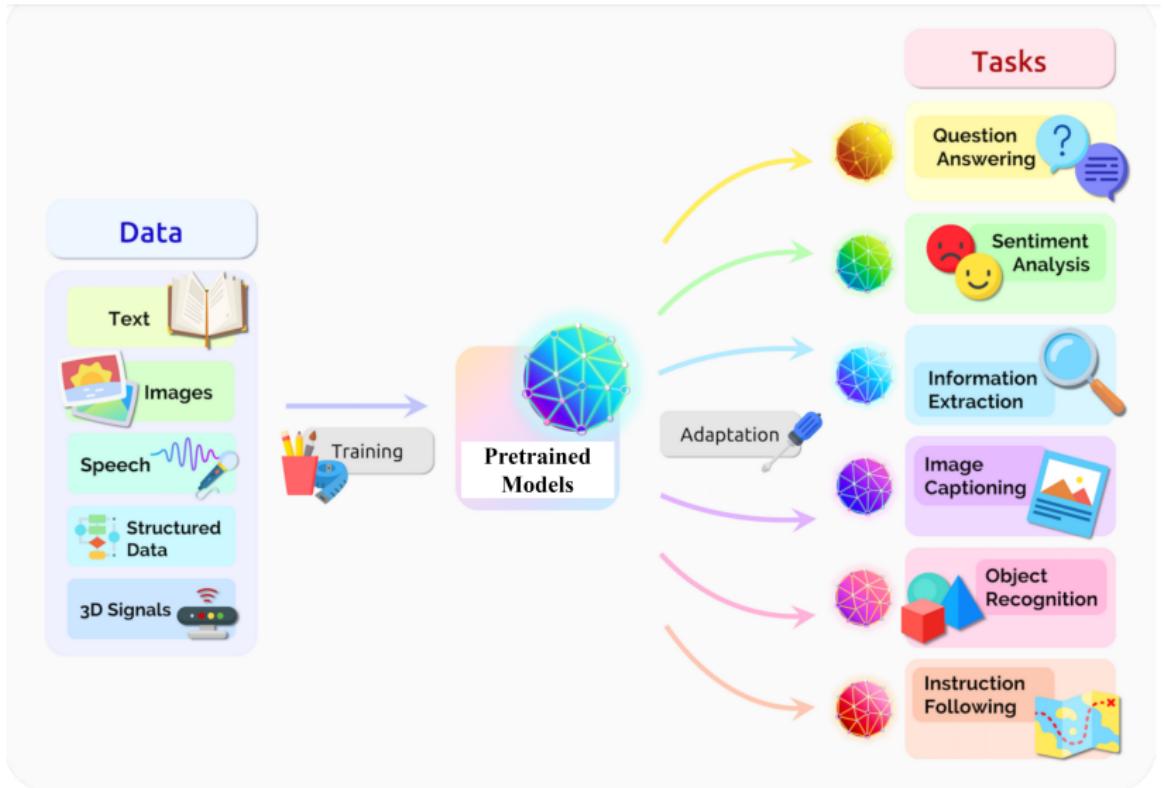
Pre-trained model: any models **trained on broad data at scale** and can be **adapted to a wide range of downstream tasks**.



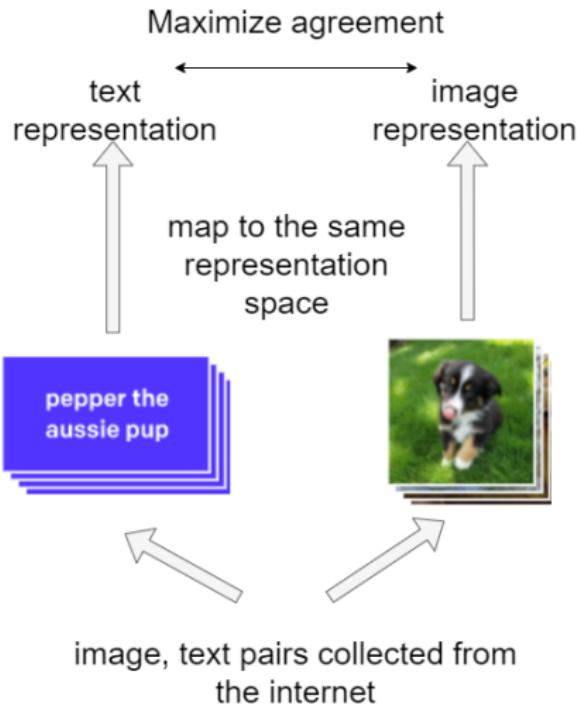
Pre-trained model: any models **trained on broad data at scale** and can be **adapted to a wide range of downstream tasks**.



Pre-trained model: any models **trained on broad data at scale** and can be **adapted to a wide range of downstream tasks**.



Example: CLIP



Example: CLIP

Zero shot image classifier

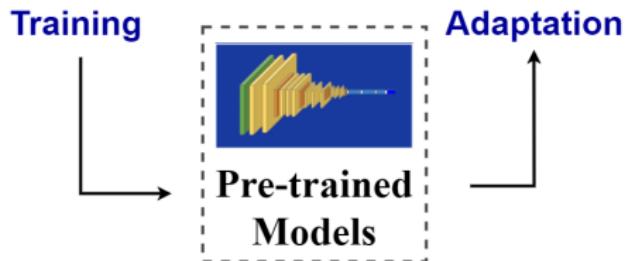
FOOD101

guacamole (90.1%) Ranked 1 out of 101 labels



- ✓ a photo of **guacamole**, a type of food.
- ✗ a photo of **ceviche**, a type of food.
- ✗ a photo of **edamame**, a type of food.
- ✗ a photo of **tuna tartare**, a type of food.
- ✗ a photo of **hummus**, a type of food.

Figure source: OpenAI, <https://clip.backprop.co/>



Theory's Role on
Pre-trained models?



- What training tasks are useful for downstream tasks?
- What algorithm/architecture can identify the useful features?
- How many samples are required?

- guide technical decisions
- reduce trial and error
- forecast outcomes and risks
- inspire new methods

Current learning theory is more mature in **supervised learning**.

- **Big labeled** dataset is necessary to fit deep networks.
- Training and testing data follow the **same distribution**.

Challenge

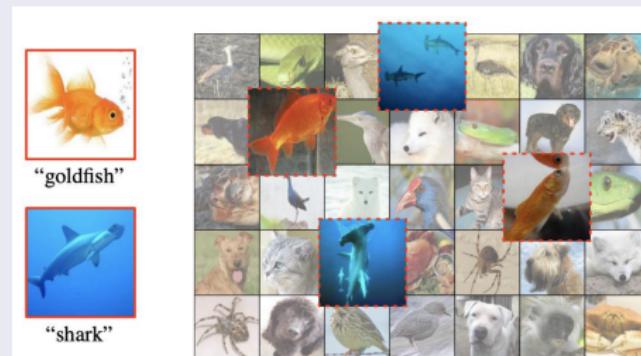
Current learning theory is more mature in **supervised learning**.

- **Big labeled** dataset is necessary to fit deep networks.
- Training and testing data follow the **same distribution**.

Our target

We want to understand how pre-trained model can

- adapt to new tasks **quickly**,



Current learning theory is more mature in **supervised learning**.

- **Big labeled** dataset is necessary to fit deep networks.
- Training and testing data follow the **same distribution**.

Our target

We want to understand how pre-trained model can

- adapt to new tasks **quickly**,
- be learned from **unlabeled samples**,



Current learning theory is more mature in **supervised learning**.

- **Big labeled** dataset is necessary to fit deep networks.
- Training and testing data follow the **same distribution**.

Our target

We want to understand how pre-trained model can

- adapt to new tasks **quickly**,
- be learned from **unlabeled samples**,
- handle **distributional shift** from training to adaptation



Autonomous driving
Trained on sunny weather



Tested on rainy weather

1 Meta-learning

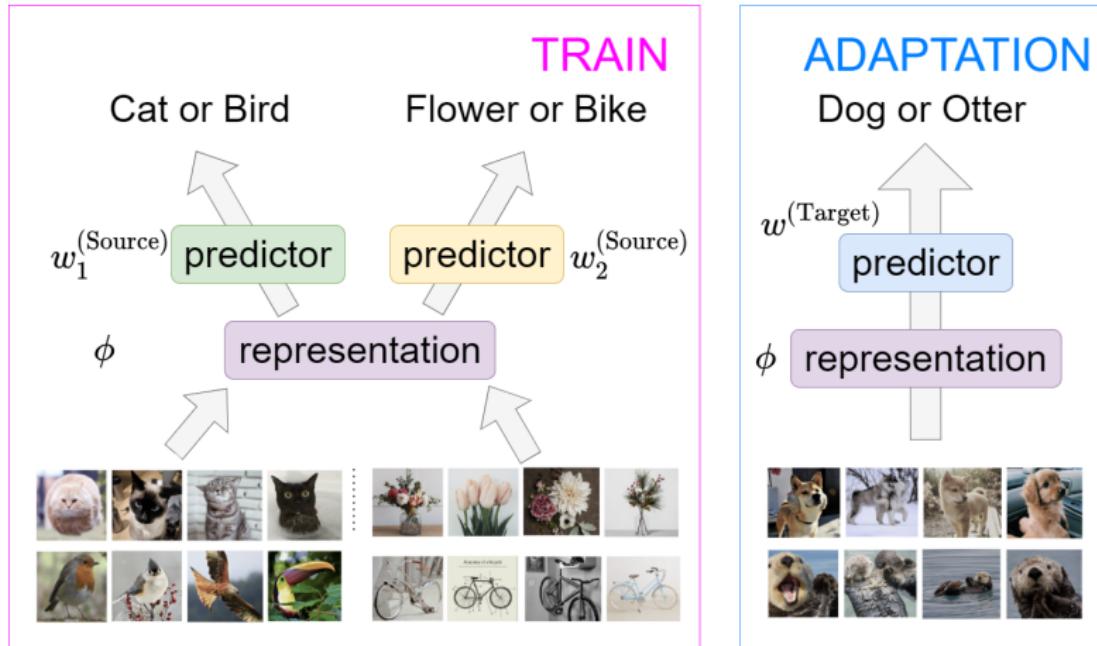
- Meta-Learning with Frozen Representation
- Meta-learning with Fine-tuned Representation

2 Self-Supervised Learning

3 Ongoing and Future Work

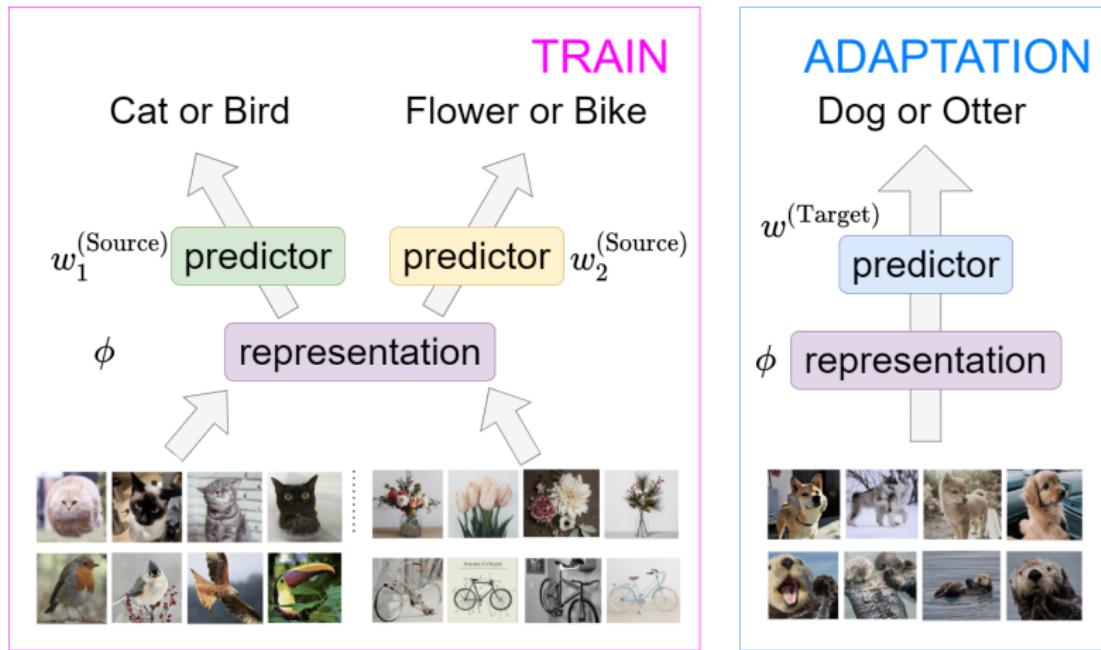
- Domain Adaptation
- Lifelong Learning
- Meta Reinforcement Learning

Learning the Meta-Representation



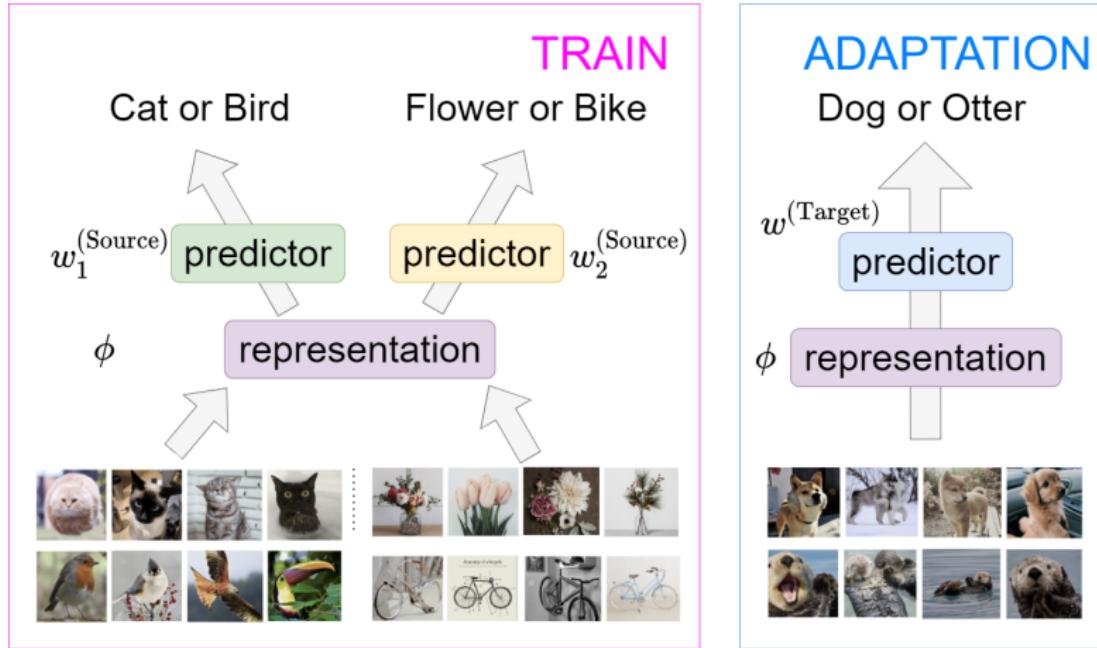
Prototypical network: (Snell et al. 2017), Meta-learning representation: (Javed and White, 2019)

Learning the Meta-Representation



On source tasks: $\hat{\phi} \leftarrow \arg \min_{\phi \in \Phi} \sum_{\text{source tasks } t} \left\{ \min_{w_t} \text{loss}(w_t \circ \phi) \right\}.$

Learning the Meta-Representation



On target task: $\hat{w}^{(\text{Target})} \leftarrow \arg \min_w \text{loss}(w \circ \hat{\phi}).$

How to Quantify Task Similarities?

Why does the learned representation transfer to target task?

How to Quantify Task Similarities?

Why does the learned representation transfer to target task?

- ① Shared good representation across tasks:

There exist predictors w_t , representation function $\phi \in \Phi$,
 $y_t = (w_t)^\top \phi(x) + \text{noise}$ for both source and target tasks.

- ② Is this enough?

How to Quantify Task Similarities?

Why does the learned representation transfer to target task?

- ① Shared good representation across tasks:

There exist predictors w_t , representation function $\phi \in \Phi$,
 $y_t = (w_t)^\top \phi(x) + \text{noise}$ for both source and target tasks.

- ② Is this enough?

Behind the scenes

- ① Shared representation encodes what transfers across the tasks.
- ② Source tasks $\{w_t^{(\text{Source})}\}$ diverse enough to “cover” $w^{(\text{Target})}$.

Importance of Task Diversity

- ① Shared representation encodes what transfers across the tasks.
- ② Diversity of source tasks $\{w_t^{(\text{Source})}\}$
(at least needs to “cover” the target task.)

Task diversity

Source tasks:
Classify types of dogs

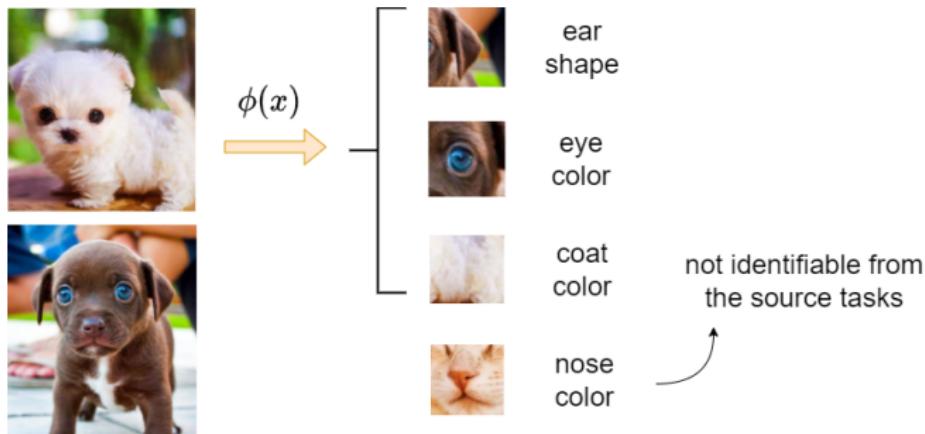


Target task:
Cat or dog?



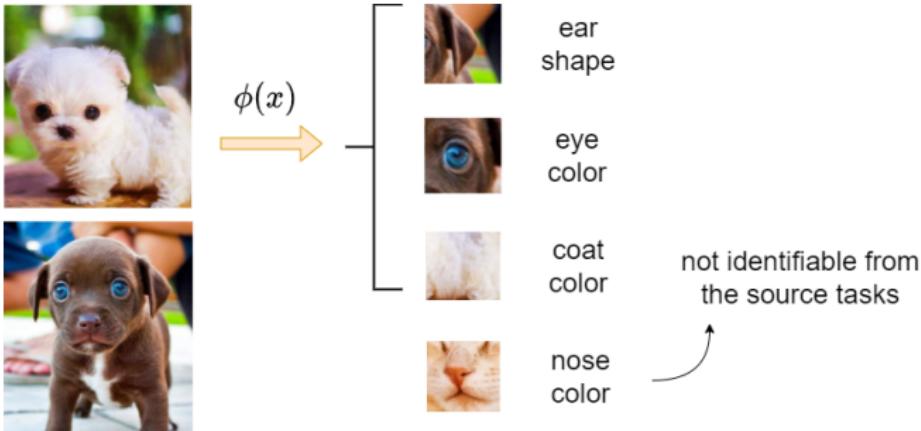
Importance of Task Diversity

- ① Shared representation encodes what transfers across the tasks.
- ② Diversity of source tasks $\{w_t^{(\text{Source})}\}$
(at least needs to “cover” the target task.)



Importance of Task Diversity

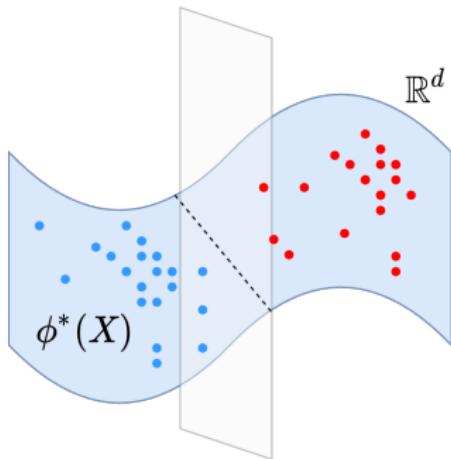
- ① Shared representation encodes what transfers across the tasks.
- ② Diversity of source tasks $\{w_t^{(\text{Source})}\}$
(at least needs to “cover” the target task.)



Mathematically speaking, $w^{(\text{Target})} \in \text{span}\{w_1^{(\text{Source})}, \dots, w_{ne}^{(\text{Source})}\}$.

Setup:

- Shared representation:
 $y_t = w_t^\top \phi(x_t) + \text{noise}$
- Representation layer is of dimension k (We assume k is small)



Theorem 1 (Informal)

We only need $O(k)$ labeled samples from target domain to get small **test error**.

In contrast, supervised learning requires samples up to the **complexity of the function class**. (500 vs. 10^8 .)

Theorem 1

With shared representation and task diversity,

$$\begin{aligned} & \text{Test Error}(\hat{w}^{(\text{Target})} \circ \hat{\phi}) \\ & \leq \text{Representation Error} + \text{Adaptation Error} \end{aligned}$$

- Representation error: how well you learn representation layer ϕ
- Adaptation error: how well you learn target predictor $w^{(\text{Target})}$

Main Result on Meta Representation

Theorem 1

With shared representation and task diversity,

$$\begin{aligned} & \text{Test Error}(\hat{w}^{(\text{Target})} \circ \hat{\phi}) \\ & \leq \text{Representation Error} + \text{Adaptation Error} \\ & \lesssim \underbrace{\frac{\mathcal{C}(\Phi)}{n_S n_e}}_{\text{representation error}} + \underbrace{\frac{k}{n_T}}_{\text{adaptation error}}. \end{aligned}$$

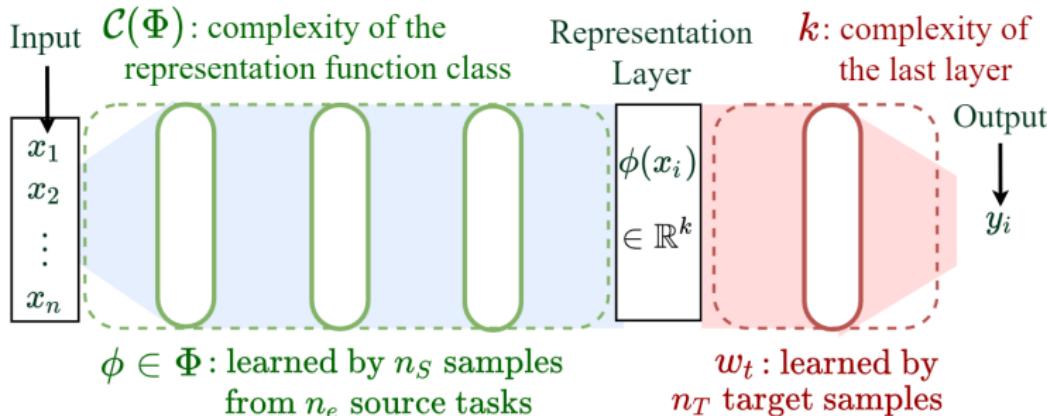
- Representation error: how well you learn representation layer ϕ
- Adaptation error: how well you learn target predictor $w^{(\text{Target})}$

Main Result on Meta Representation

Theorem 1

With shared representation and task diversity,

$$\text{Test Error}(\hat{w}^{(\text{Target})} \circ \hat{\phi}) \lesssim \underbrace{\frac{\mathcal{C}(\Phi)}{n_S n_e}}_{\text{representation error}} + \underbrace{\frac{k}{n_T}}_{\text{adaptation error}}.$$



Main Result on Meta Representation

Theorem 1

With shared representation and task diversity,

$$\text{Test Error}(\hat{w}^{(\text{Target})} \circ \hat{\phi}) \lesssim \underbrace{\frac{\mathcal{C}(\Phi)}{n_S n_e}}_{\text{representation error}} + \underbrace{\frac{k}{n_T}}_{\text{adaptation error}}.$$

Baselines:

- Supervised learning:

$$\text{Test error} \leq \frac{\mathcal{C}(w \circ \Phi)}{n_T}.$$

- Maurer et al. 2016:

$$\text{Test error} \leq \frac{\mathcal{C}(\Phi)}{\sqrt{n_e}} + \frac{k}{n_T}.$$

Theorem 1

With shared representation and task diversity,

$$\text{Test Error}(\hat{w}^{(\text{Target})} \circ \hat{\phi}) \lesssim \underbrace{\frac{\mathcal{C}(\Phi)}{n_S n_e}}_{\text{representation error}} + \underbrace{\frac{k}{n_T}}_{\text{adaptation error}}.$$

Meta-learning handles distributional shift:

- Covariate shift is allowed.
Source and target data can come from different marginal distribution.

Theorem 1

With shared representation and task diversity,

$$\text{Test Error}(\hat{w}^{(\text{Target})} \circ \hat{\phi}) \lesssim \underbrace{\frac{\mathcal{C}(\Phi)}{n_S n_e}}_{\text{representation error}} + \underbrace{\frac{k}{n_T}}_{\text{adaptation error}}.$$

Meta-learning handles distributional shift:

- Covariate shift is allowed.
Source and target data can come from different marginal distribution.

1 Meta-learning

- Meta-Learning with Frozen Representation
- Meta-learning with Fine-tuned Representation

2 Self-Supervised Learning

3 Ongoing and Future Work

- Domain Adaptation
- Lifelong Learning
- Meta Reinforcement Learning

What if there is misspecification in the representation?
(Namely, the representation is only approximately shared.)

What if there is misspecification in the representation?
(Namely, the representation is only approximately shared.)

- Previously: $y_t = w_t^\top \phi(x) + \text{noise}$.

What if there is misspecification in the representation?
(Namely, the representation is only approximately shared.)

- Previously: $y_t = w_t^\top \phi(x) + \text{noise}$.
- Now: $y_t = w_t^\top \phi_t(x) + \text{noise}$, ϕ_t is γ -close to ϕ .

What if there is misspecification in the representation?
(Namely, the representation is only approximately shared.)

- Previously: $y_t = w_t^\top \phi(x) + \text{noise}$.
- Now: $y_t = w_t^\top \phi_t(x) + \text{noise}$, ϕ_t is γ -close to ϕ .

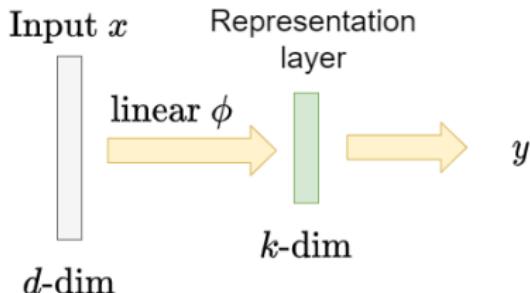
Question

Does METAREP (previous algorithm) still work?

If not, how should we modify the algorithm?

Failure with Previous Algorithm: METAREP

Instantiation in linear setting:



When $\gamma = 0$ (no misspecification), METAREP requires at most $O(k)$ samples on the target task.

Instantiation in linear setting:

When $\gamma = 0$ (no misspecification), METAREP requires at most $O(k)$ samples on the target task.

Theorem: However, when $\gamma > 0$, METAREP requires at least $\Omega(d)$ samples on the target task.

Instantiation in linear setting:

When $\gamma = 0$ (no misspecification), METAREP requires at most $O(k)$ samples on the target task.

Theorem: However, when $\gamma > 0$, METAREP requires at least $\Omega(d)$ samples on the target task.

- No improvement over supervised learning that requires $O(d)$ samples.

Instantiation in linear setting:

When $\gamma = 0$ (no misspecification), METAREP requires at most $O(k)$ samples on the target task.

Theorem: However, when $\gamma > 0$, METAREP requires at least $\Omega(d)$ samples on the target task.

- No improvement over supervised learning that requires $O(d)$ samples.
- Previous algorithm is extremely fragile!

- ① Use source tasks to find ϕ as an initialization.
- ② Fine-tune each representation ϕ_t starting from ϕ that tolerates mis-specification.

- ① Use source tasks to find ϕ as an initialization.
- ② Fine-tune each representation ϕ_t starting from ϕ that tolerates mis-specification.

$$\min_{\phi} \min_{\|\phi_t - \phi\| \leq \gamma, w_t} \sum_{\text{Task } t} \text{loss}(w_t, \phi_t),$$

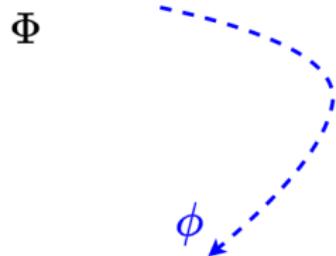
- ① Use source tasks to find ϕ as an initialization.
- ② Fine-tune each representation ϕ_t starting from ϕ that tolerates mis-specification.

$$\Phi$$

$$\min_{\phi} \min_{\|\phi_t - \phi\| \leq \gamma, w_t} \sum_{\text{Task } t} \text{loss}(w_t, \phi_t),$$

- ① Use source tasks to find ϕ as an initialization.
- ② Fine-tune each representation ϕ_t starting from ϕ that tolerates mis-specification.

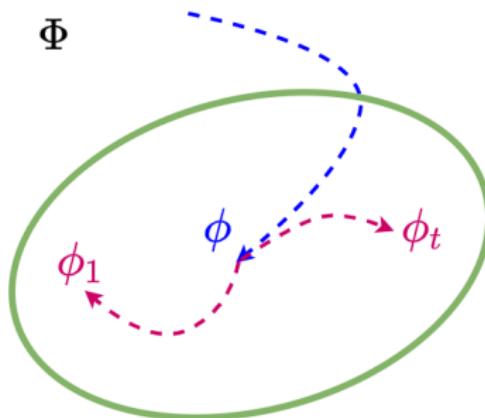
$$\min_{\phi} \min_{\|\phi_t - \phi\| \leq \gamma, w_t} \sum_{\text{Task } t} \text{loss}(w_t, \phi_t),$$



Modified Algorithm: FINE-TUNEDREP

- ① Use source tasks to find ϕ as an initialization.
- ② Fine-tune each representation ϕ_t starting from ϕ that tolerates mis-specification.

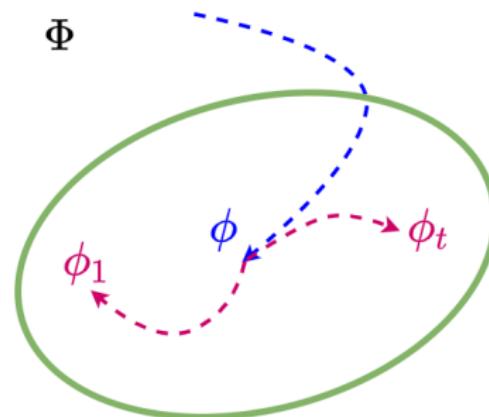
$$\min_{\phi} \min_{\|\phi_t - \phi\| \leq \gamma, w_t} \sum_{\text{Task } t} \text{loss}(w_t, \phi_t),$$



Modified Algorithm: FINE-TUNEDREP

- 1 Use source tasks to find ϕ as an initialization.
- 2 Fine-tune each representation ϕ_t starting from ϕ that tolerates mis-specification.

$$\min_{\phi} \min_{\|\phi_t - \phi\| \leq \gamma, w_t} \sum_{\text{Task } t} \text{loss}(w_t, \phi_t),$$



Theorem 2 (Informal)

When adapting ϕ to target task, it requires $O(k) + O(\gamma^2)$ training samples from target domain.

Theorem 2

For **general function classes**, under similar settings,

$$\text{Test Error} \lesssim \text{METAREP Error (when } \gamma = 0) + O\left(\frac{\gamma}{\sqrt{n_T}}\right).$$

- γ measures mis-specification in representation ϕ
- n_T : number of samples from target training set

Theorem 2

For **general function classes**, under similar settings,

$$\text{Test Error} \lesssim \text{METAREP Error (when } \gamma = 0) + O\left(\frac{\gamma}{\sqrt{nT}}\right).$$

- We need $O(k) + O(\gamma^2)$ samples from target domain.

Baselines:

- Supervised learning and METAREP need $\mathcal{C}(w \circ \Phi)$ samples from target domain.
- $\mathcal{C}(w \circ \Phi)$: Complexity of the function class for the whole network.

1 Meta-learning

- Meta-Learning with Frozen Representation
- Meta-learning with Fine-tuned Representation

2 Self-Supervised Learning

3 Ongoing and Future Work

- Domain Adaptation
- Lifelong Learning
- Meta Reinforcement Learning

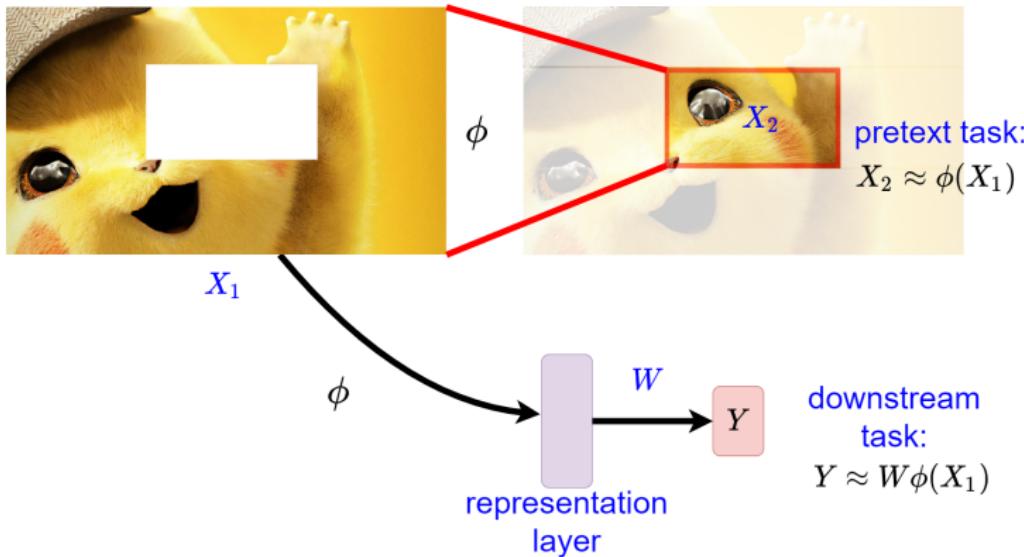
Create your own labels

Supervised representation learning needs labels from related tasks.
What if this isn't available?

Create pseudo-labels from the input data.

Type I: reconstruction-based SSL

Reconstructing part of the input from the other part



Context encoder: (Pathak et al. 2016)

Other examples: Masked Autoencoder: (He et al., 2021), Colorization: (Zhang et al., 2016)

Type I: reconstruction-based SSL

Reconstructing part of the input from the other part

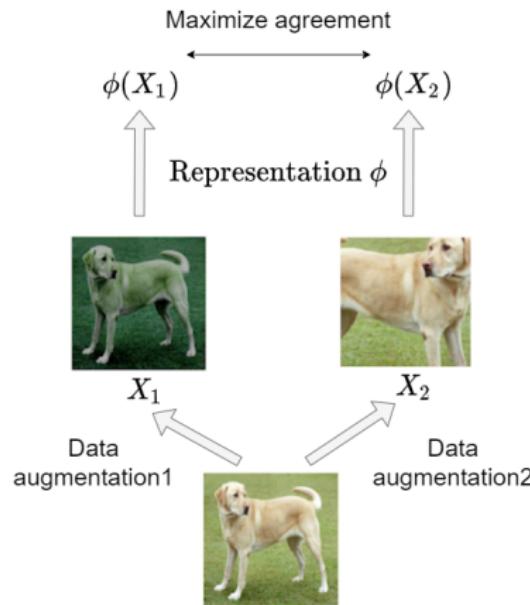


BERT: (Devlin et al., 2018)

Other examples: Masked Autoencoder: (He et al., 2021), Colorization: (Zhang et al., 2016)

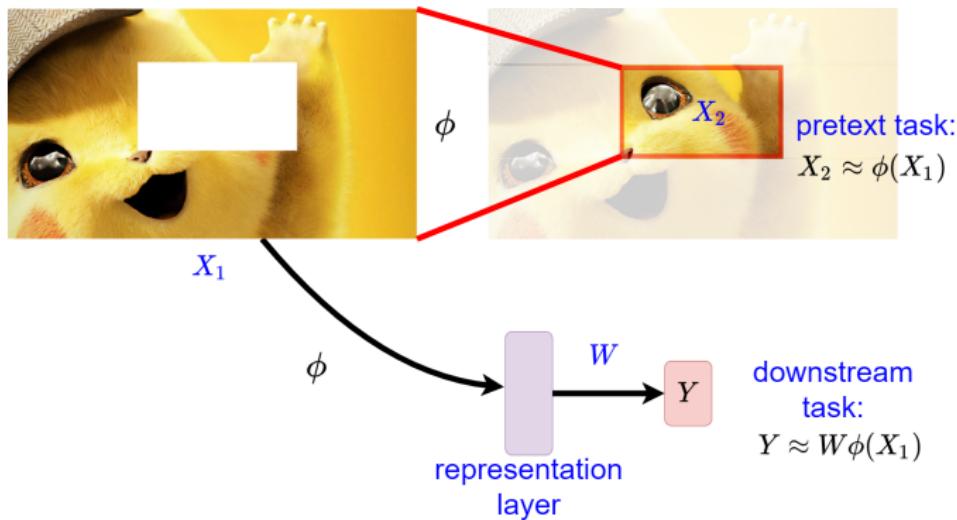
Type II: similarity-based SSL

Enforcing two views of the same data to have similar representation

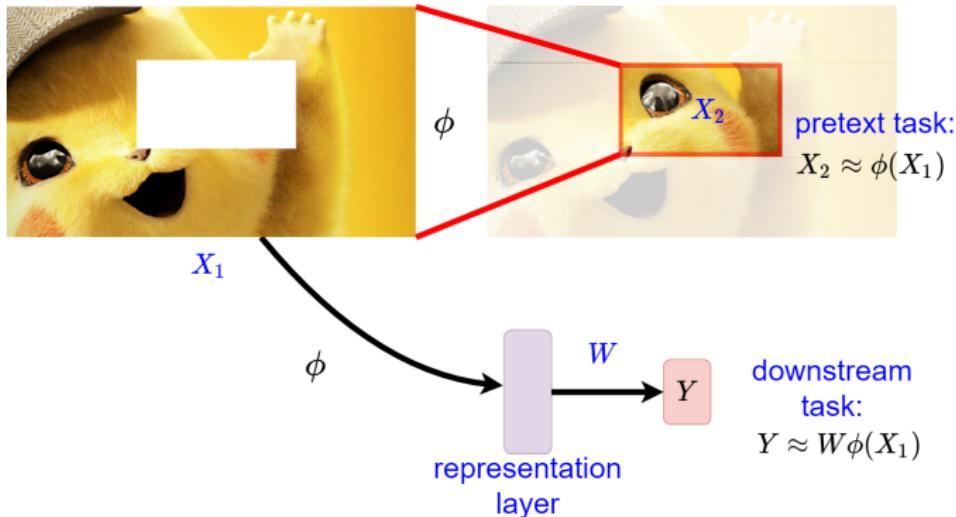


Examples: SimSiam: (Chen et al., 2021), CLIP: (Radford et al., 2021) ,
SimCLR: (Chen et al., 2020)

- ① Label Y with k classes.
- ② Unmasked image X_1 and masked image X_2 .



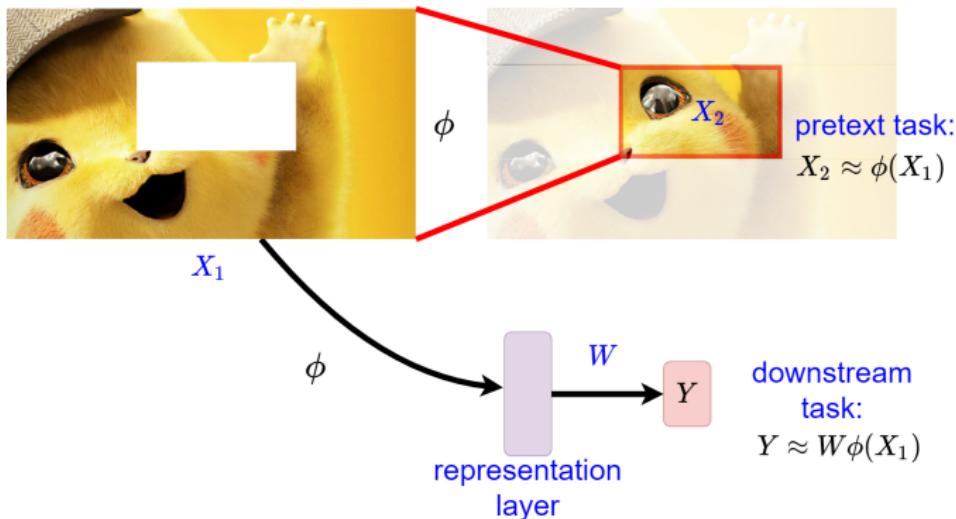
- ① Label Y with k classes.
- ② Unmasked image X_1 and masked image X_2 .



- ③ Key intuition: Pretext tasks should help us reduce irrelevant features/forget information that is not necessary to predict Y

Ideal Scenario

- ① Label Y with k classes.
- ② Unmasked image X_1 and masked image X_2 .



- ③ Ideal scenario: $X_1 \rightarrow Y \rightarrow X_2$
 $\iff X_1 \perp X_2 | Y$

A Thought Experiment

- $X_1 \perp X_2 | Y$
- Image colorization for photos of desert, forest, sea

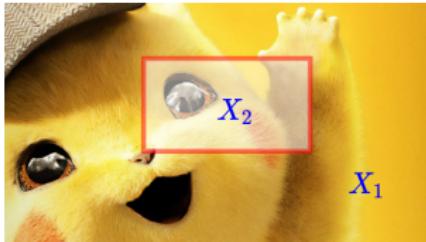


A Thought Experiment

- $X_1 \perp X_2 | Y$
- Image colorization for photos of desert, forest, sea



- Image inpainting:



Setting:

- ① k -class labels Y .
- ② Representation ϕ , last layer W^* .

Compare this procedure to ground truth classifier f^* :

Theorem 3

- ① (No representation error.) If $X_1 \perp X_2 | Y$,

$$f^* = W^* \phi(X_1).$$

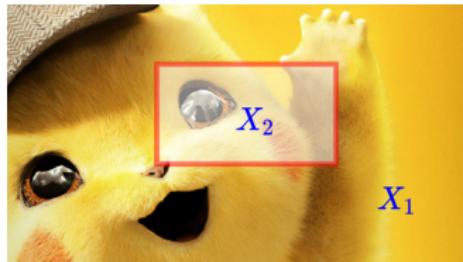
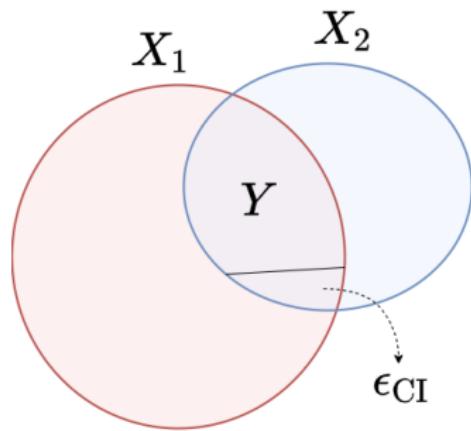
- ② Only need $O(k)$ labeled samples.

Remark: Only need k samples instead of Rademacher complexity of function class.

No representation error: if $X_1 \rightarrow Y \rightarrow X_2$, (i.e., $X_1 \perp X_2 | Y$),
 then $f^* = W^* \phi(X_1)$.

$$\begin{aligned} \phi(\cdot) &:= \mathbb{E}[X_2|X_1] \quad \overbrace{\quad}^{\text{tower property}} \quad \mathbb{E}[\mathbb{E}[X_2|X_1, Y]|X_1] \overbrace{\quad}^{\text{CI}} \quad \mathbb{E}[\mathbb{E}[X_2|Y]|X_1] \\ &= \sum_{y=1}^k \mathbb{E}[X_2|Y=y] P(Y=y|X_1) =: \mathbf{A}^\top f(X_1), \end{aligned}$$

Here $f(x_1)_y := P(Y=y|X_1=x_1), y=1 \cdots, k$
 \mathbf{A} satisfies $\mathbf{A}_{y,:} = \mathbb{E}[X_2|Y=y]$.



Characterizing Approximate Conditional Independence

Define $\epsilon_{CI} = \mathbb{E}_{X_1} \|\mathbb{E}[X_2|X_1] - \mathbb{E}_Y[\mathbb{E}[X_2|Y]|X_1]\|^2$.

- This is 0 if $X_1 \perp X_2 | Y$.
- ϵ_{CI} can be viewed as quantification of extra shared features between X_1 and X_2 are not captured by Y (spurious feature not reducible by SSL)

Characterizing Approximate Conditional Independence

Define $\epsilon_{CI} = \mathbb{E}_{X_1} \|\mathbb{E}[X_2|X_1] - \mathbb{E}_Y[\mathbb{E}[X_2|Y]|X_1]\|^2$.

- This is 0 if $X_1 \perp X_2 | Y$.
- ϵ_{CI} can be viewed as quantification of extra shared features between X_1 and X_2 are not captured by Y (spurious feature not reducible by SSL)

$$\text{Test Error} \lesssim \underbrace{\frac{k}{n_L}}_{\text{adaptation error}} + \underbrace{\epsilon_{CI}}_{\text{representation error}}$$

Characterizing Approximate Conditional Independence

Define $\epsilon_{CI} = \mathbb{E}_{X_1} \|\mathbb{E}[X_2|X_1] - \mathbb{E}_Y[\mathbb{E}[X_2|Y]|X_1]\|^2$.

- This is 0 if $X_1 \perp X_2 | Y$.
- ϵ_{CI} can be viewed as quantification of extra shared features between X_1 and X_2 are not captured by Y (spurious feature not reducible by SSL)

$$\text{Test Error} \lesssim \underbrace{\frac{k}{n_L}}_{\text{adaptation error}} + \underbrace{\epsilon_{CI}}_{\text{representation error}}$$

- ERM would need $n_L \asymp \text{Complexity of Function Class.}$

Characterizing Approximate Conditional Independence

Define $\epsilon_{CI} = \mathbb{E}_{X_1} \|\mathbb{E}[X_2|X_1] - \mathbb{E}_Y[\mathbb{E}[X_2|Y]|X_1]\|^2$.

- This is 0 if $X_1 \perp X_2 | Y$.
- ϵ_{CI} can be viewed as quantification of extra shared features between X_1 and X_2 are not captured by Y (spurious feature not reducible by SSL)

$$\text{Test Error} \lesssim \underbrace{\frac{k}{n_L}}_{\text{adaptation error}} + \underbrace{\epsilon_{CI}}_{\text{representation error}}$$

- ERM would need $n_L \asymp \text{Complexity of Function Class}$.
- Both terms are tight in $\frac{k}{n_L} + \epsilon_{CI}$ in some scenarios

Characterizing Approximate Conditional Independence

Define $\epsilon_{CI} = \mathbb{E}_{X_1} \|\mathbb{E}[X_2|X_1] - \mathbb{E}_Y[\mathbb{E}[X_2|Y]|X_1]\|^2$.

- This is 0 if $X_1 \perp X_2 | Y$.
- ϵ_{CI} can be viewed as quantification of extra shared features between X_1 and X_2 are not captured by Y (spurious feature not reducible by SSL)

$$\text{Test Error} \lesssim \underbrace{\frac{k}{n_L}}_{\text{adaptation error}} + \underbrace{\epsilon_{CI}}_{\text{representation error}}$$

- ERM would need $n_L \asymp \text{Complexity of Function Class}$.
- Both terms are tight in $\frac{k}{n_L} + \epsilon_{CI}$ in some scenarios
- Also applies to similarity-based SSL

Implications on pretext selection

- Design pretext tasks such that X_1 and X_2 have smaller dependence (given Y)

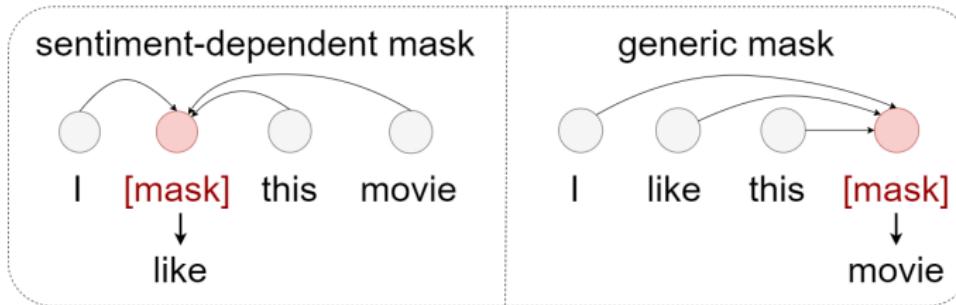
Empirical Implications

Implications on pretext selection

- Design pretext tasks such that X_1 and X_2 have smaller dependence (given Y)

Applications:

- Image: image classification [He et al. 2021]
- Text: sentiment analysis [Zhang and Hashimoto, 2021]

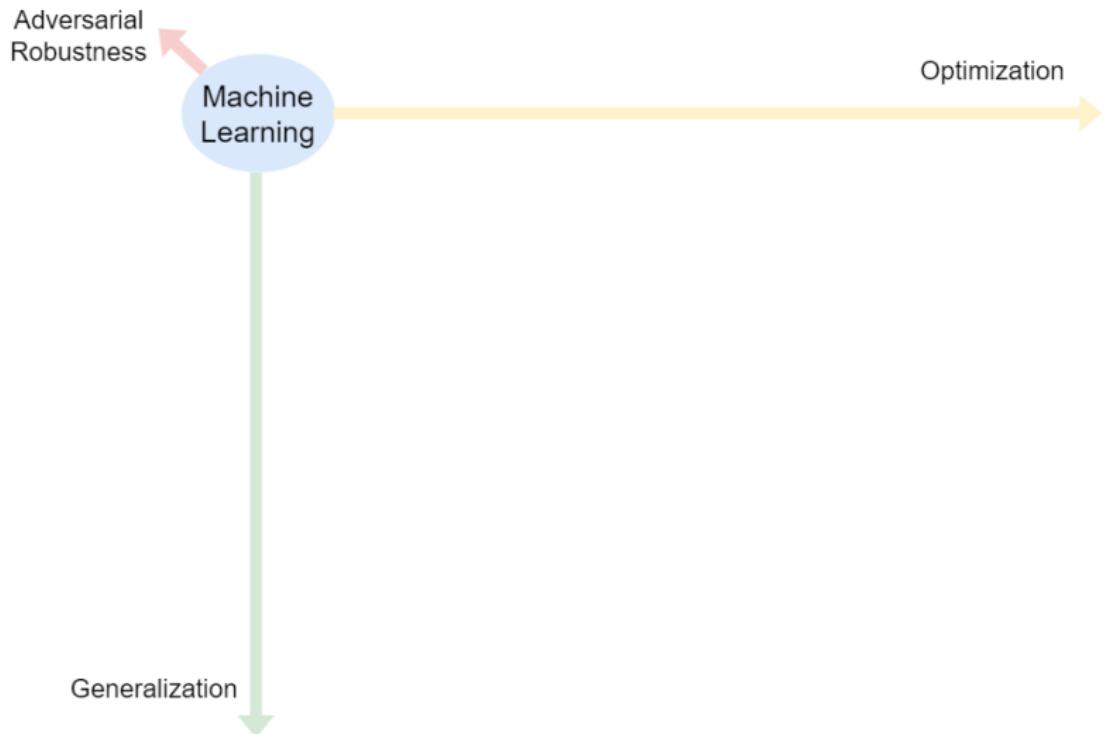


- Audio: speech recognition [Zaiem et al., 2021]

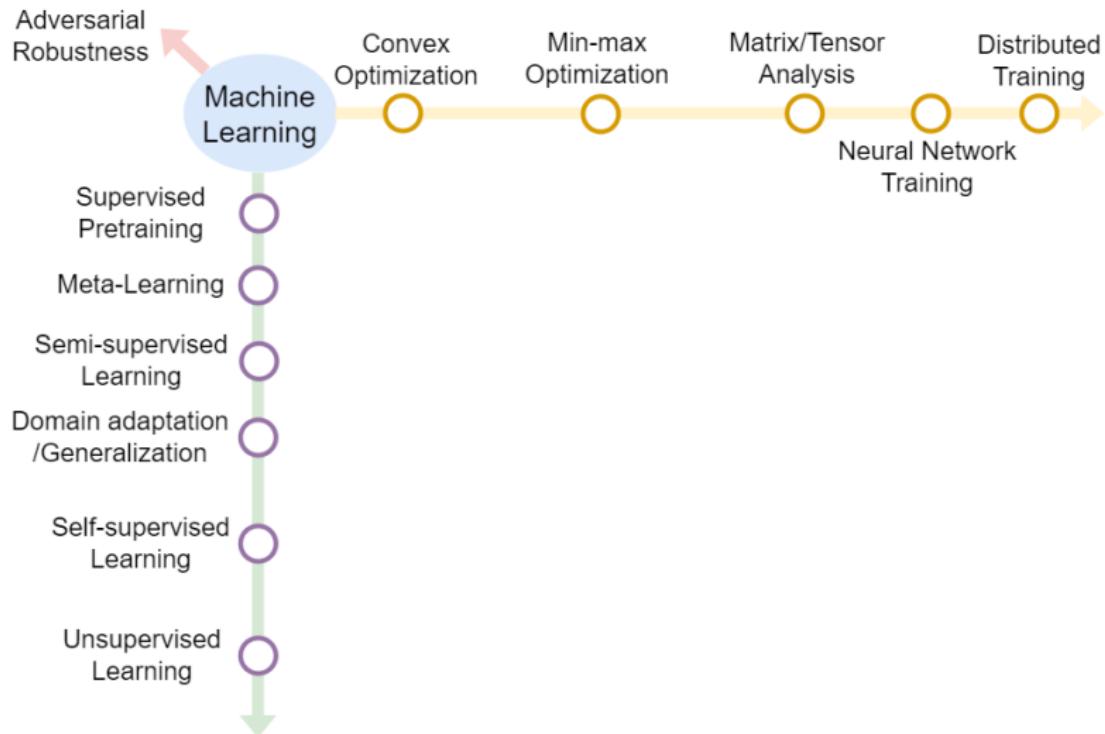
Predicting what you already know helps: Provable self-supervised learning,
NeurIPS 2021

wrap-up

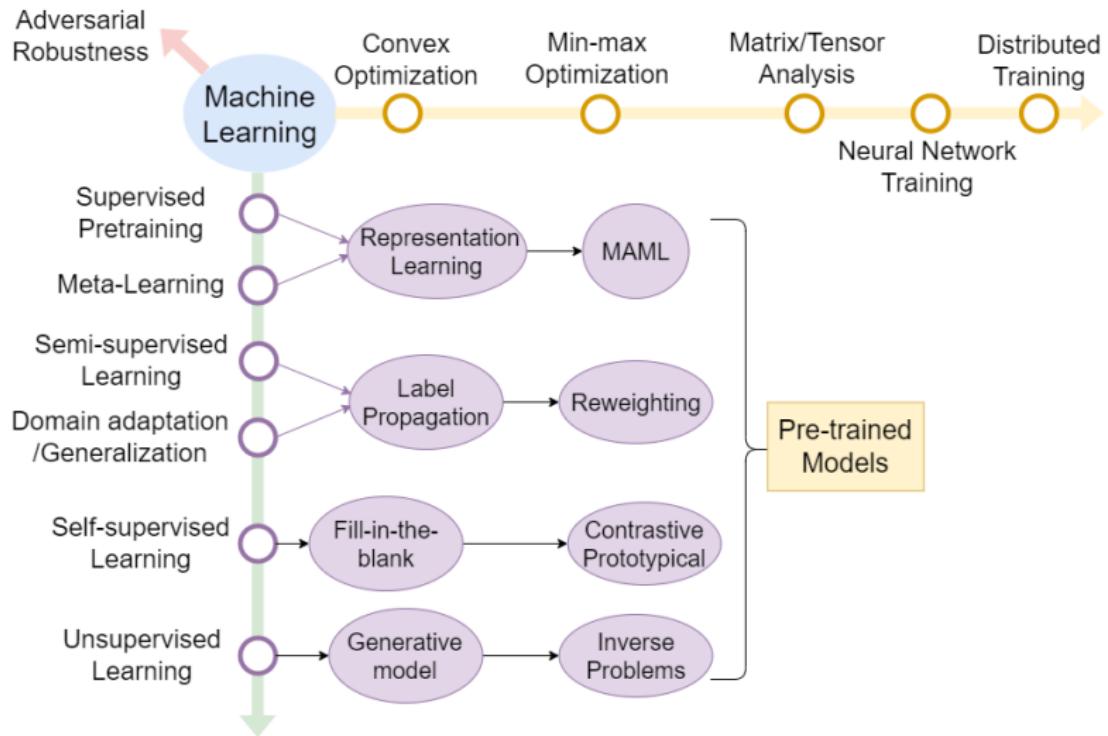
Overview of My Research Contributions



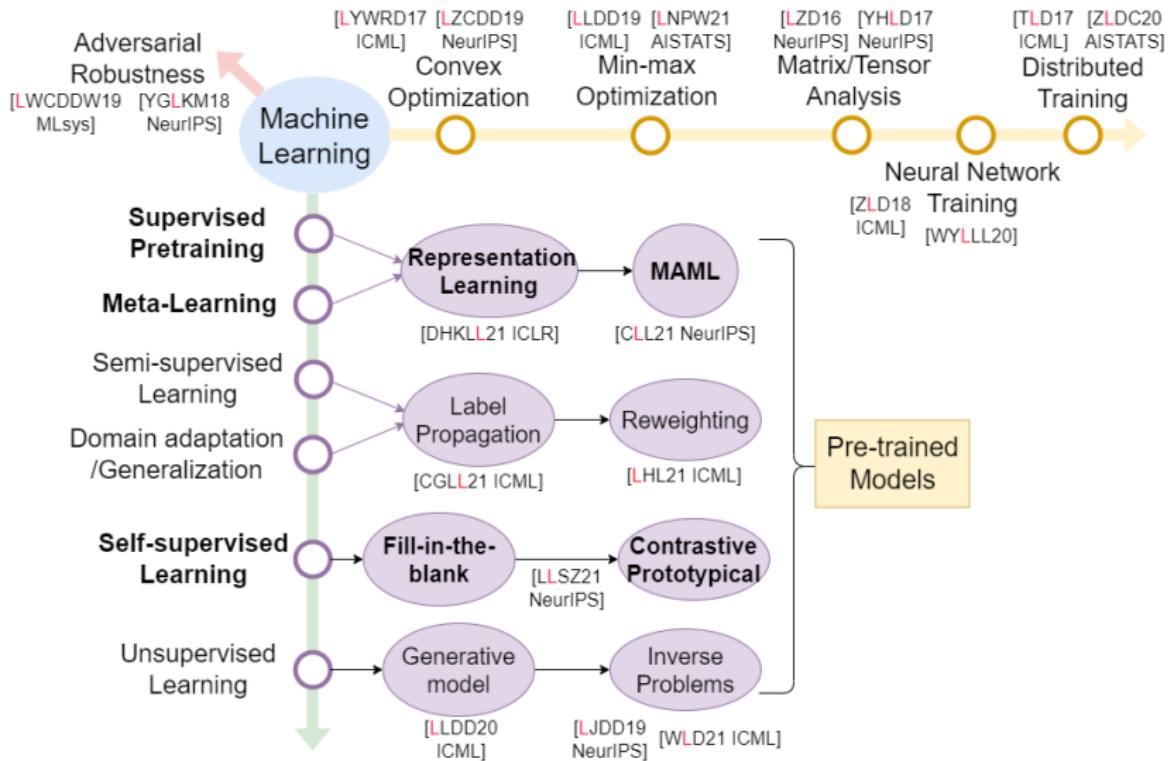
Overview of My Research Contributions



Overview of My Research Contributions



Overview of My Research Contributions



Outlook



A.I. in Healthcare



A.I. and Law



A.I. in Education

Pre-trained model:
Expanding the capability of AI to
broader disciplines.



A.I. in Healthcare



A.I. and Law



A.I. in Education

Pre-trained model:
Expanding the capability of AI to
broader disciplines.

I will work towards accelerating us to
their “ImageNet” moment.

Remaining challenges:

- Optimization
 - Convergence analysis
 - Training and testing time speed up
- Robustness
 - Handle different types of distribution shift
 - Adversarial robustness (Security issues)
- Applications
 - Reinforcement learning
 - Medical image
- Trustworthy AI
 - Fairness
 - Interpretability
- Evaluation
 - How to evaluate the model without target data
- Other technical details

Remaining challenges:

- Optimization
 - Convergence analysis
 - Training and testing time speed up
- Robustness
 - Handle different types of distribution shift
 - Adversarial robustness (Security issues)
- Applications
 - Reinforcement learning
 - Medical image
- Trustworthy AI
 - Fairness
 - Interpretability
- Evaluation
 - How to evaluate the model without target data
- Other technical details

1 Meta-learning

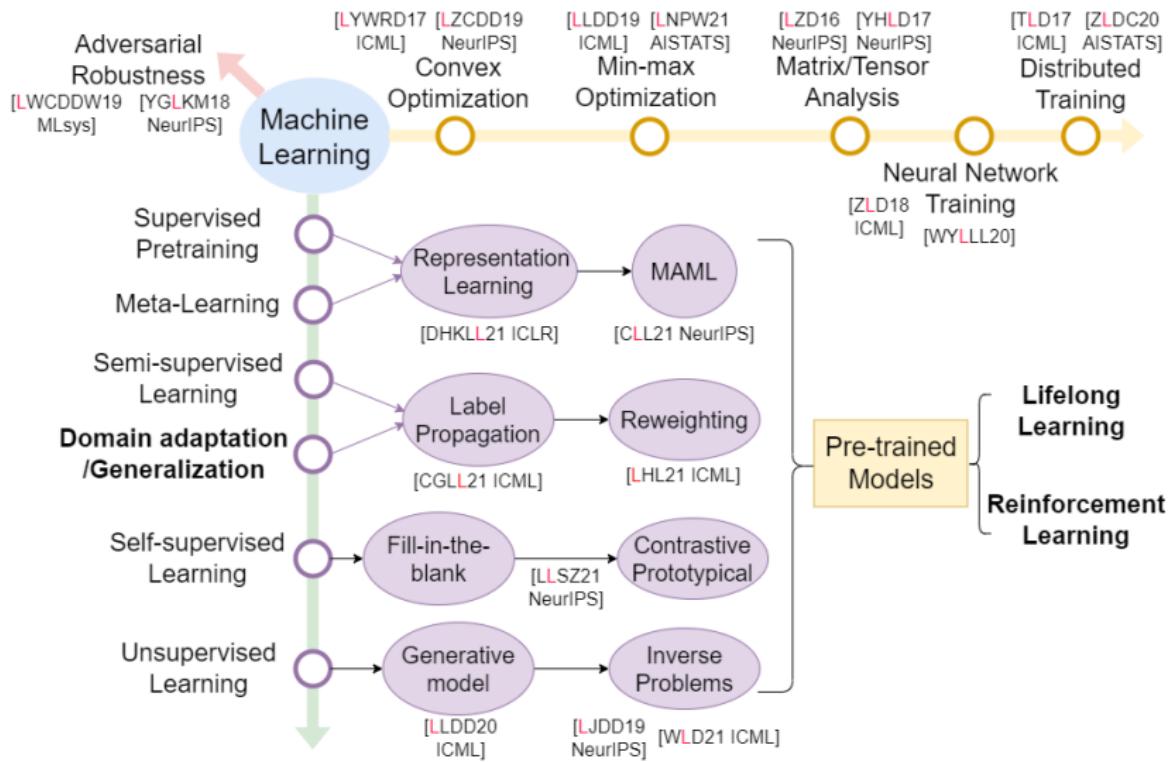
- Meta-Learning with Frozen Representation
- Meta-learning with Fine-tuned Representation

2 Self-Supervised Learning

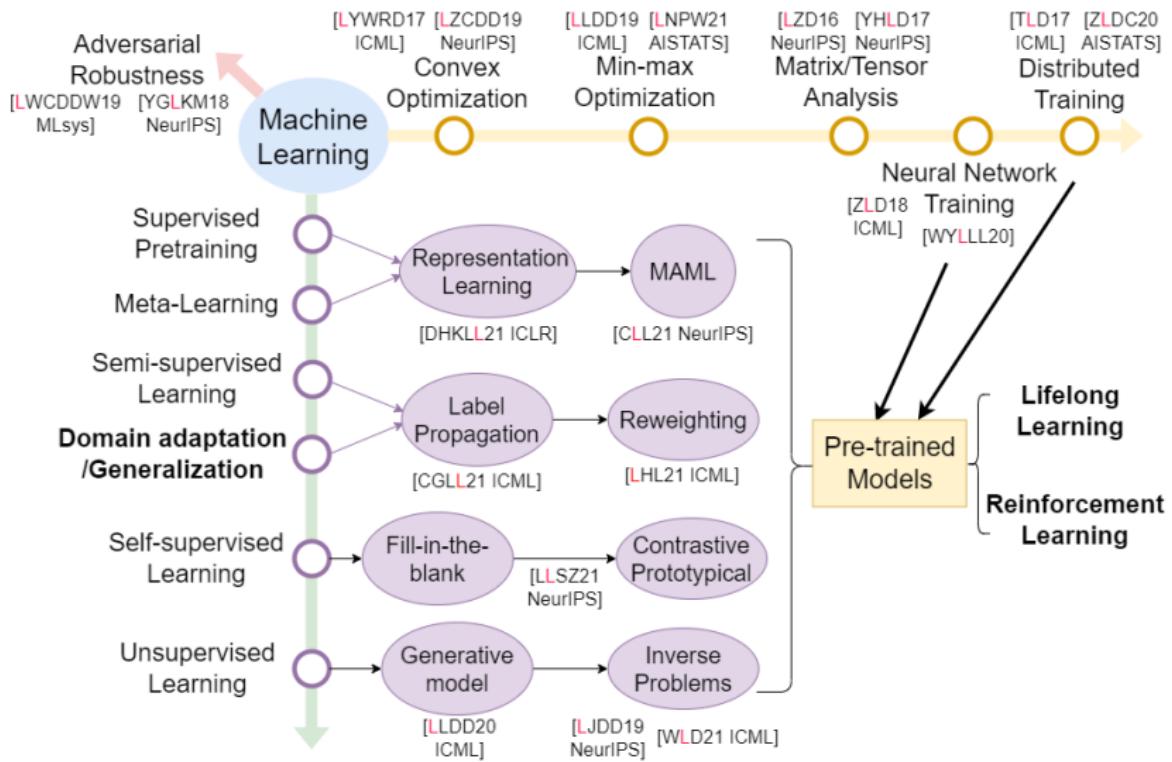
3 Ongoing and Future Work

- Domain Adaptation
- Lifelong Learning
- Meta Reinforcement Learning

Future Work



Future Work



We investigate sufficient conditions that guarantee:

- Deep networks **learn a good pre-trained model.**
- drastically **reduce sample complexity.**
- be **learned on unlabeled data.**
- **transfer to other tasks/domains with covariate shift.**

We investigate sufficient conditions that guarantee:

- Deep networks **learn a good pre-trained model.**
- drastically **reduce sample complexity.**
- be **learned on unlabeled data.**
- **transfer to other tasks/domains with covariate shift.**

Future Work

Explore the following directions centered at pre-trained models:

- Domain Adaptation/Generalization
- Continual (Lifelong) Learning
- Meta Reinforcement Learning
- Optimization: Training and Testing Time Speed-ups

Thank you!

Thank you!

Link back to:

METAREP

FINETUNEDREP

SSL

Back-up slides:

FINETUNEDREP

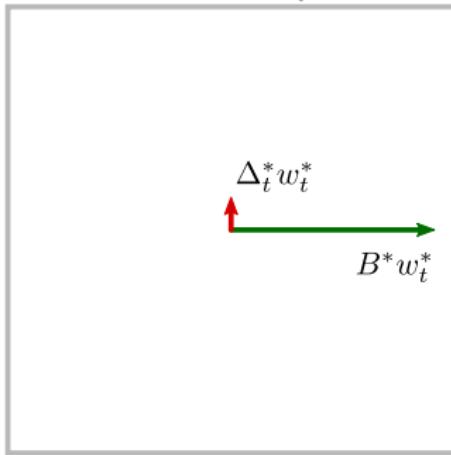
SSL Example

Experiments

METAREP (Fixed feature) has $\Omega\left(\frac{d}{n_T}\right)$ minimax rate on
the target task

METAREP chooses representation based on prediction space norm, not parameter space norm!

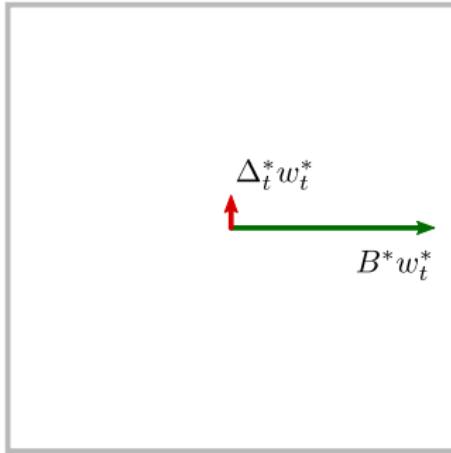
Parameter space (L_2 -norm)



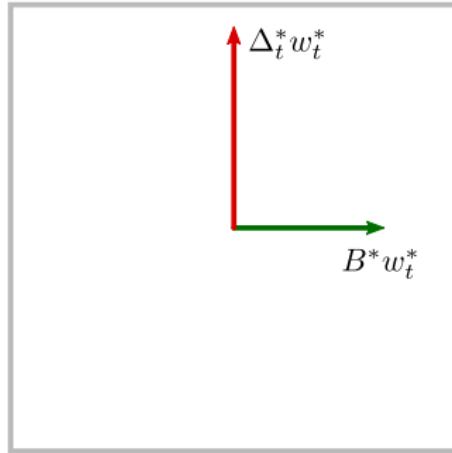
Failure with Previous Algorithm: METAREP

METAREP chooses representation based on prediction space norm, not parameter space norm!

Parameter space (L_2 -norm)



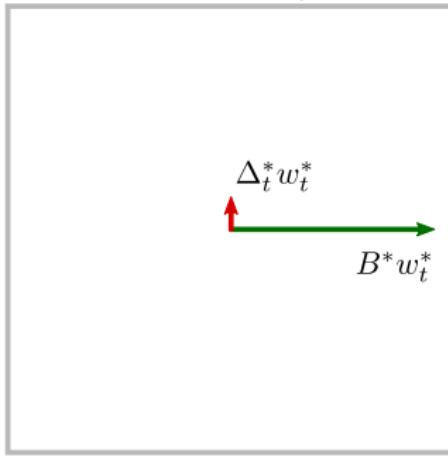
Prediction space (Σ -norm)



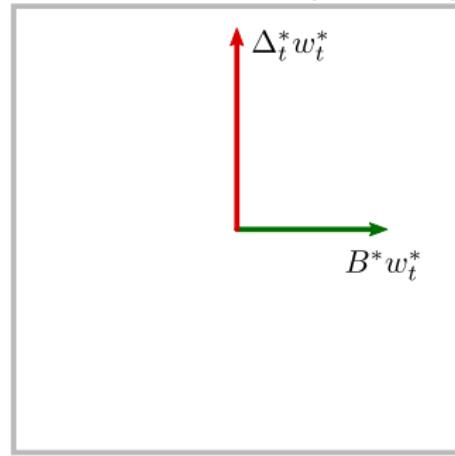
Failure with Previous Algorithm: METAREP

METAREP *chooses representation based on prediction space norm, not parameter space norm!*

Parameter space (L_2 -norm)



Prediction space (Σ -norm)



Intuition: Trick METAREP into learning subspace of δ_t^* 's, and pay cost of having to fine-tune to learn large-norm $B^*w_{test}^*$.

Theoretical results:

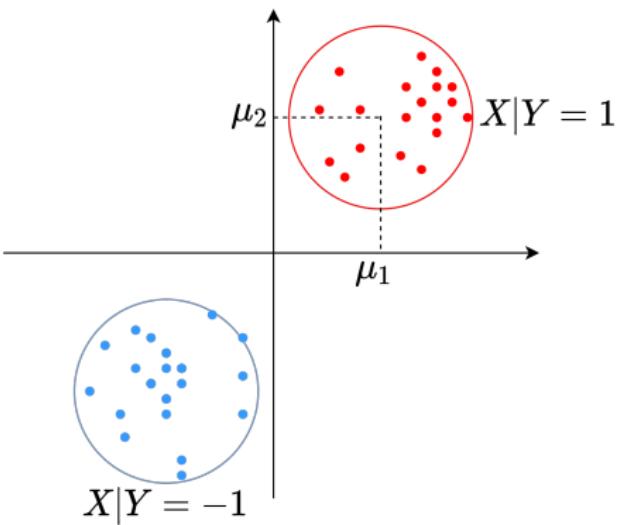
- ① If $X_1 \perp X_2 | Y$,

$$f^* = W^* \phi(X_1).$$

- ② Only need k (dimension of Y) samples.

How can $\phi(X_1)$ be better than X_2 to predict Y ?

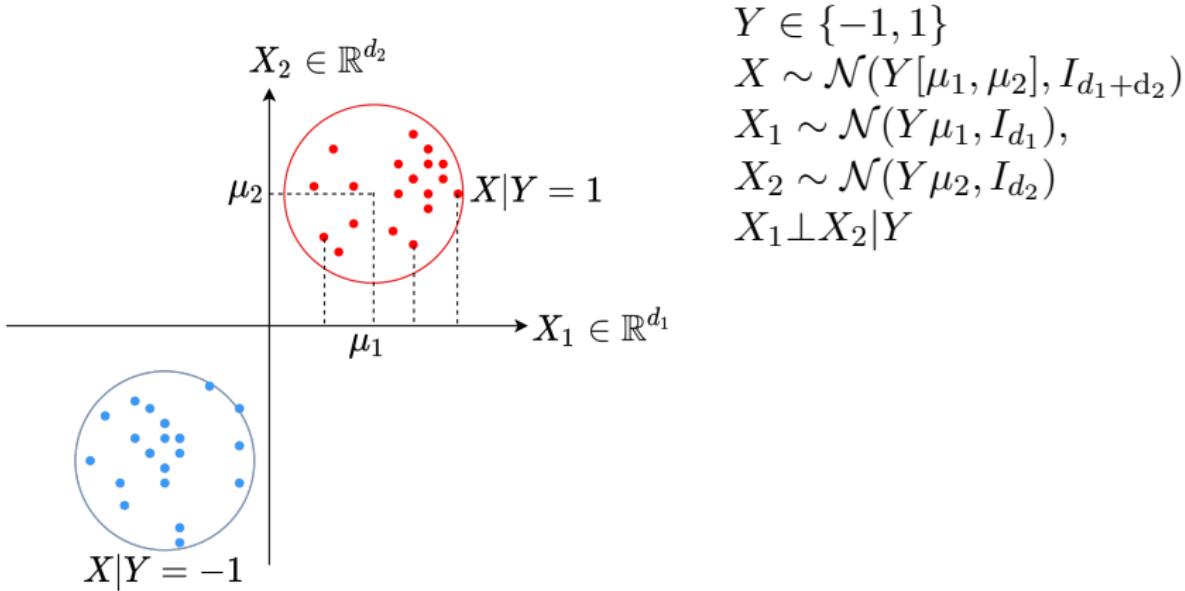
Example: Gaussian Mixture



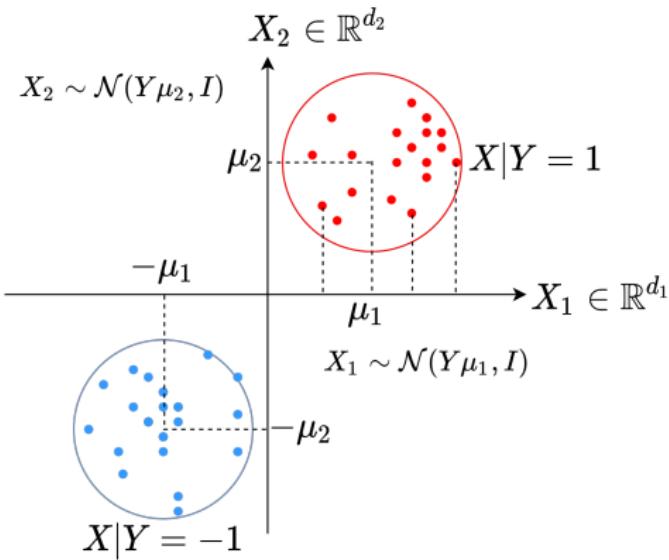
$$Y \in \{-1, 1\}$$

$$X \sim \mathcal{N}(Y[\mu_1, \mu_2], I_{d_1+d_2})$$

Example: Gaussian Mixture

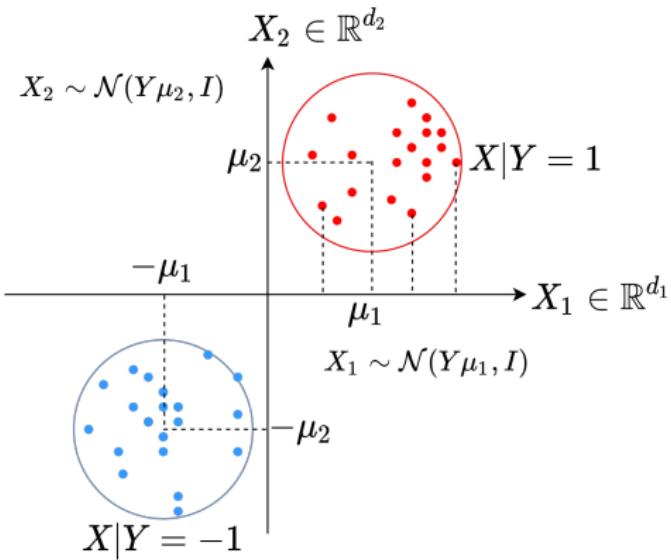


Example: Gaussian Mixture



$Y \in \{-1, 1\}$
 $X \sim \mathcal{N}(Y[\mu_1, \mu_2], I_{d_1+d_2})$
 $X_1 \sim \mathcal{N}(Y\mu_1, I_{d_1}),$
 $X_2 \sim \mathcal{N}(Y\mu_2, I_{d_2})$
 $X_1 \perp X_2 | Y$
 $\mathbb{E}[Y|X_2]$ is not linear, but
 $\mathbb{E}[Y|\phi]$ is linear:

Example: Gaussian Mixture



$Y \in \{-1, 1\}$
 $X \sim \mathcal{N}(Y[\mu_1, \mu_2], I_{d_1+d_2})$
 $X_1 \sim \mathcal{N}(Y\mu_1, I_{d_1}),$
 $X_2 \sim \mathcal{N}(Y\mu_2, I_{d_2})$
 $X_1 \perp X_2 | Y$
 $\mathbb{E}[Y|X_2]$ is not linear, but
 $\mathbb{E}[Y|\phi]$ is linear:

- $\phi(x_1) = \mathbb{E}[X_2 | X_1 = x_1] = p_1(x_1)\mu_2 + p_{-1}(x_1)(-\mu_2)$
- $\mathbb{E}[Y|X_1] = p_1(x_1) - p_{-1}(x_1) = \mu_2^\top \phi(X_1) / \|\mu_2\|^2.$
- $p_y(x_1) := P(Y = y | X_1 = x_1)$

Connection to SimSiam method

- Before, we learn $\phi(x_1) = \mathbb{E}[X_2|X_1 = x_1]$, which naturally requires X_2 and Y to be linearly correlated
- We can actually predict any $g(X_2)|X_1$, or even $p(X_2|X_1)$

ACE and nonlinear CCA

- Alternating conditional expectation (ACE):

$$\min_{\phi, \eta} L_{ACE}(\phi, \eta) = \mathbb{E}_{X_1, X_2} \left[\|\phi(X_1) - \eta(X_2)\|^2 \right],$$

s.t. $\Sigma_{\phi, \phi} = \Sigma_{\eta, \eta} = I_k.$

- This is equivalent to the following canonical correlation analysis (CCA):

$$\max_{\phi, \eta} L_{CCA}(\phi, \eta) = \mathbb{E}_{X_1, X_2} \left[\phi(X_1)^\top \eta(X_2) \right],$$

s.t. $\Sigma_{\phi, \phi} = \Sigma_{\eta, \eta} = I_k.$

Algorithm (SimSiam):

$$\max_{\phi, \eta, \text{normalized}} \mathbb{E}[\phi(X_1)^\top \eta(X_2)]$$

New measure of conditional independence:

$$\epsilon_{CI} := \max_{\|g\|_{L^2(X_2)} = 1} \mathbb{E}_{X_1} (\mathbb{E}[g(X_2)|X_1] - \mathbb{E}[\mathbb{E}[g(X_2)|Y]|X_1])^2.$$

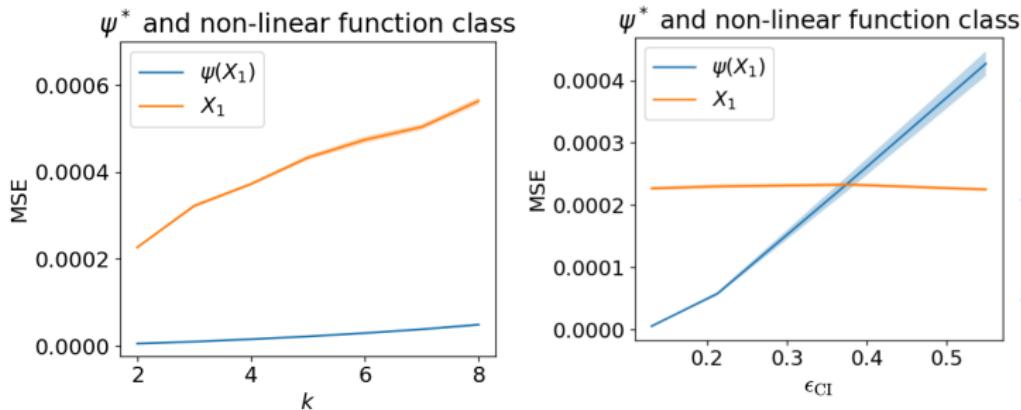
- Extension of previous result:

$$\text{test error} \lesssim \epsilon_{CI} + \frac{k}{n_L}.$$

- If both X_2 and X_1 can well predict Y , i.e., $P_{X_1,Y}(g^*(x_1) \neq y) \leq \alpha$ (same for X_2), we have:

$$\text{test error} \lesssim \frac{\alpha}{1-\epsilon_{CI}} + \frac{k}{n_L}.$$

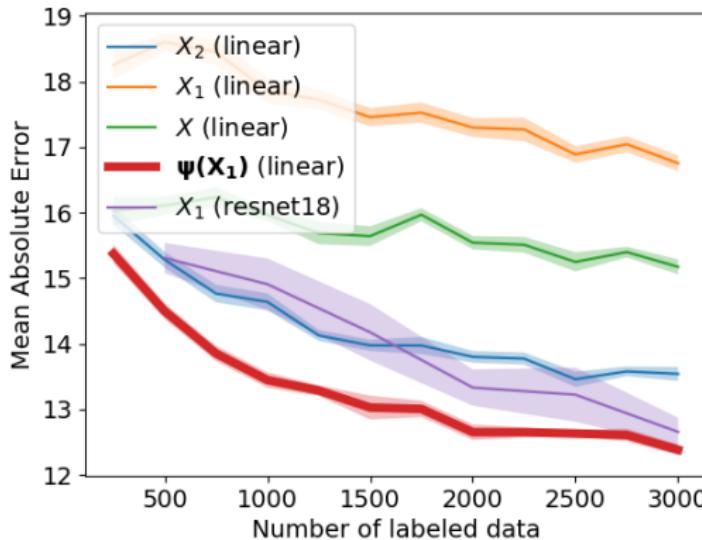
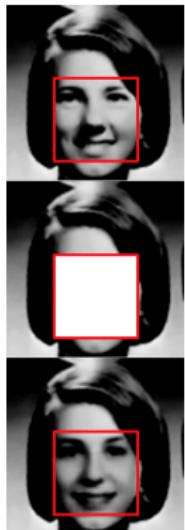
Simulations: Both Terms Tight in $\frac{k}{n_L} + \epsilon_{\text{CI}}$



Left: Class Conditional Gaussian $X \sim \mathcal{N}(\mu_Y, I)$, $\mu_Y \in \mathbb{R}^{90}$, $Y \in \{1, 2, \dots, k\}$, $X_1 = X_{1:50}$, $X_2 = X_{51:90}$. $X_1 \perp X_2 | Y$

Right: Similar mixture of Gaussian: $X \sim \mathcal{N}(\mu_Y, \Sigma_{\epsilon_{\text{CI}}})$, $\alpha \propto \epsilon_{\text{CI}}$ controls the dependence of X_1 and X_2 : $\epsilon_{\text{CI}} = 0 \Rightarrow$ exact CI, and $\epsilon_{\text{CI}} = 1 \Rightarrow X_2$ fully depends on X_1 .

Experiments



- Yearbook: portraits date from 1905 to 2013.

Ongoing Work: Distribution Shift



Entity30 - Passerine

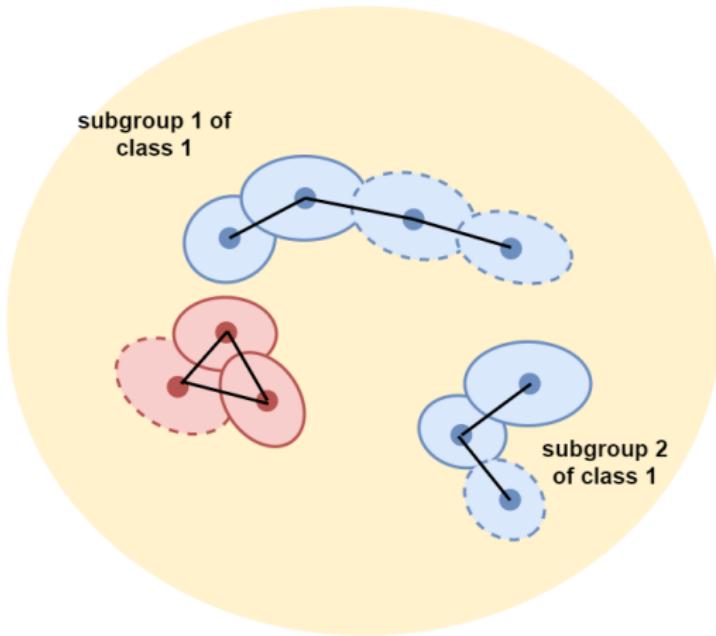
Entity30 - Tableware

Our New Framework: Subpopulation Shift

	Class 1	Class -1
Source		
Target		

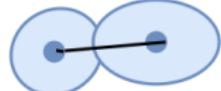


Components connected
through data augmentation

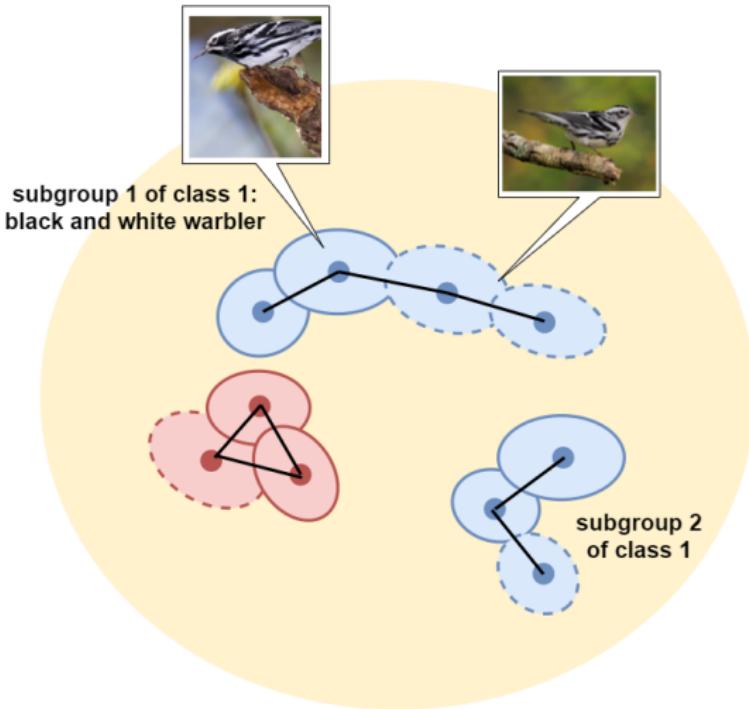


Our New Framework: Subpopulation Shift

	Class 1	Class -1
Source		
Target		



Components connected
through data augmentation

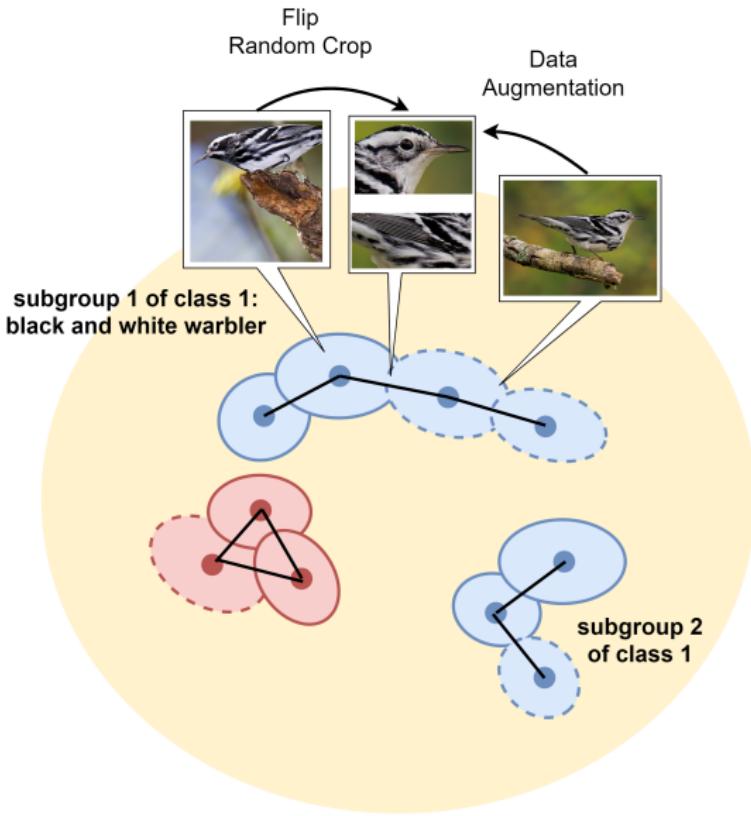


Our New Framework: Subpopulation Shift

	Class 1	Class -1
Source		
Target		



Components connected
through data augmentation



Experiments

Method	A → W	D → W	W → D	A → D	D → A	W → A	Average
MDD	94.97±0.70	98.78±0.07	100±0	92.77±0.72	75.64±1.53	72.82±0.52	89.16
Ours	95.47±0.95	98.32±0.19	100±0	93.71±0.23	76.64±1.91	74.93±1.15	89.84

Performance of MDD¹ and our method on Office-31 dataset.

Method	Ar → Cl	Ar → Pr	Ar → Rw	Cl → Ar	Cl → Pr	Cl → Rw	Pr → Ar
MDD	54.9±0.7	74.0±0.3	77.7±0.3	60.6±0.4	70.9±0.7	72.1±0.6	60.7±0.8
Ours	55.1±0.9	74.7±0.8	78.7±0.5	63.2±1.3	74.1±1.8	75.3±0.1	63.0±0.6
Method	Pr → Cl	Pr → Rw	Rw → Ar	Rw → Cl	Rw → Pr	Average	
MDD	53.0±1.0	78.0±0.2	71.8±0.4	59.6±0.4	82.9±0.3	68.0	
Ours	53.0±0.6	80.8±0.4	73.4±0.1	59.4±0.7	84.0±0.5	69.6	

Performance of MDD and our method on Office-Home dataset.

¹MDD: (Zhang et al. 2019)

Experiments: Subpopulation Shift Dataset

- ENTITY-30 task from BREEDS tasks.
- We use FixMatch, an existing consistency regularization method. We also leverage SwAV, an existing unsupervised representation learned from ImageNet, where there can be a better structure of subpopulation shift. We compare with popular distribution matching methods like DANN and MDD.

Method	Source Acc	Target Acc
Train on Source	91.91 ± 0.23	56.73 ± 0.32
DANN (Ganin et al., 2016)	92.81 ± 0.50	61.03 ± 4.63
MDD (Zhang et al., 2019)	92.67 ± 0.54	63.95 ± 0.28
FixMatch (Sohn et al., 2020)	90.87 ± 0.15	72.60 ± 0.51