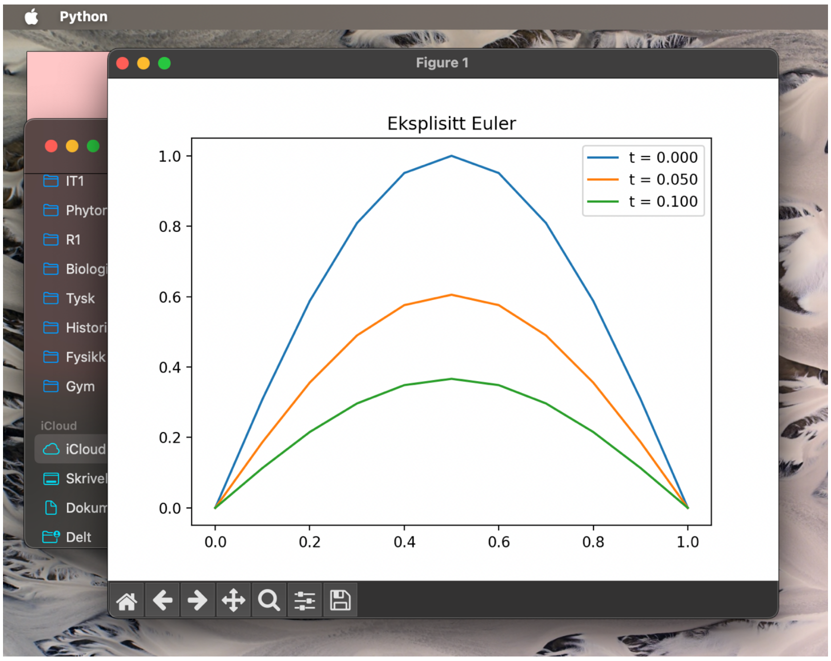
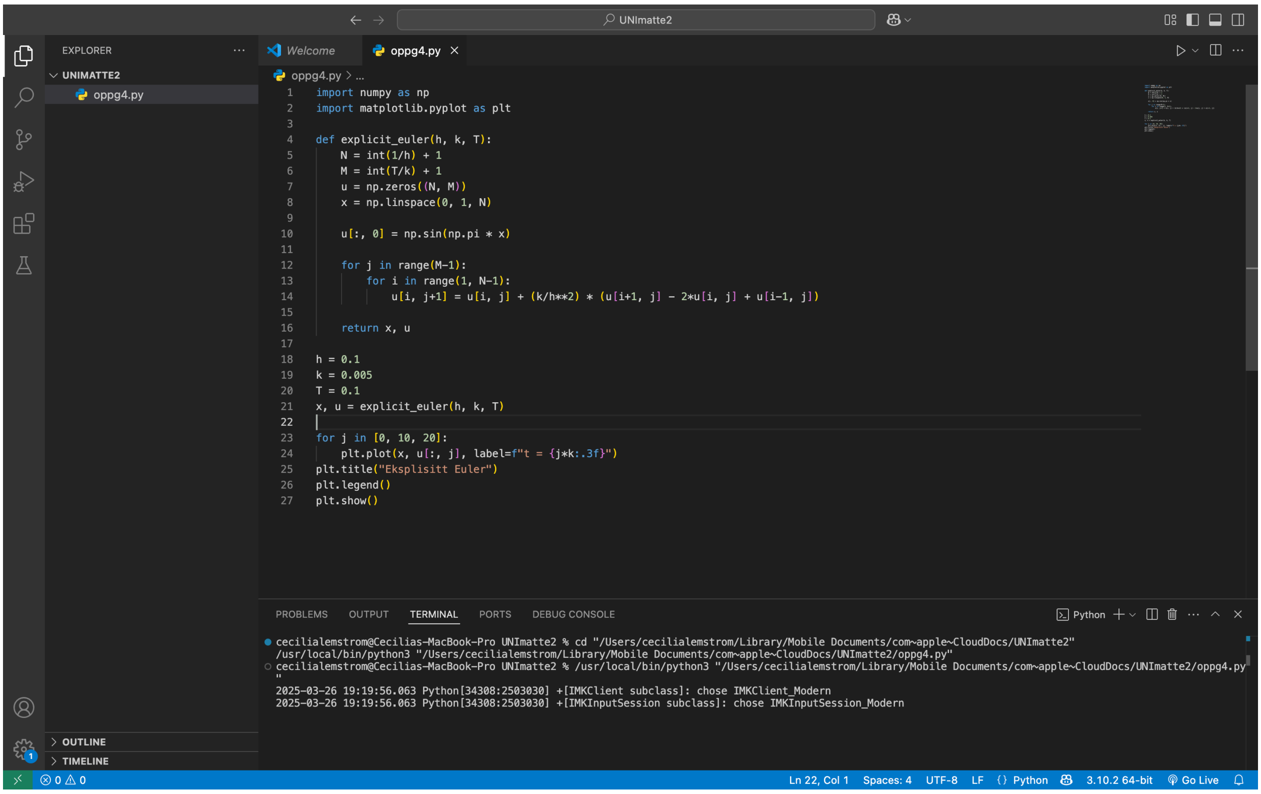
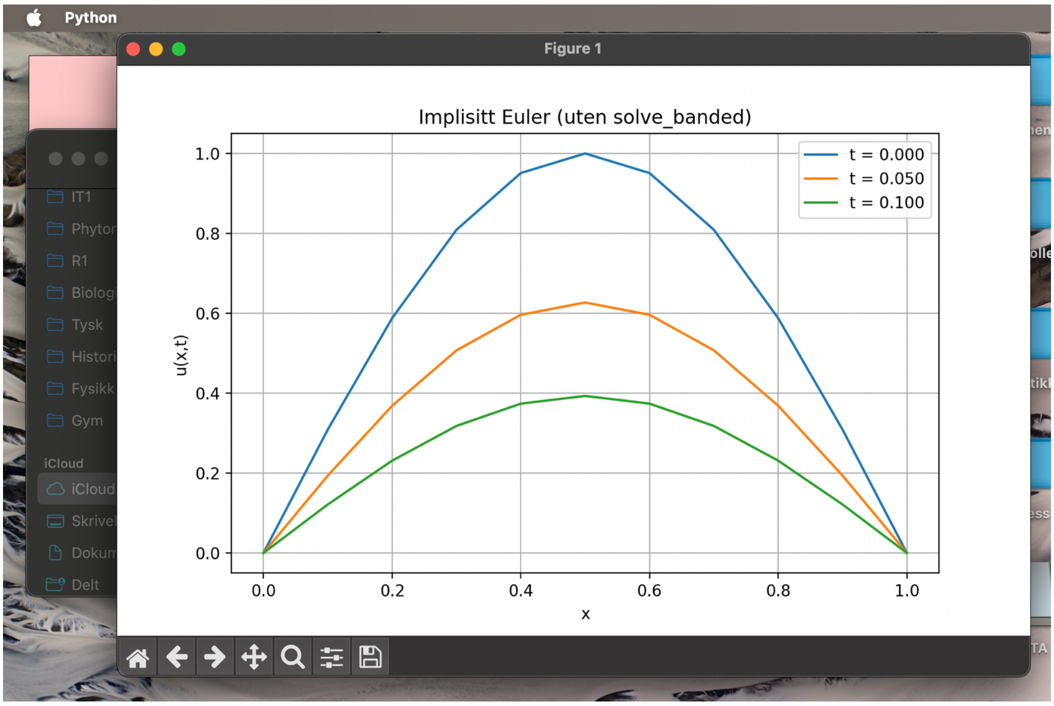
Oblig Matte 2

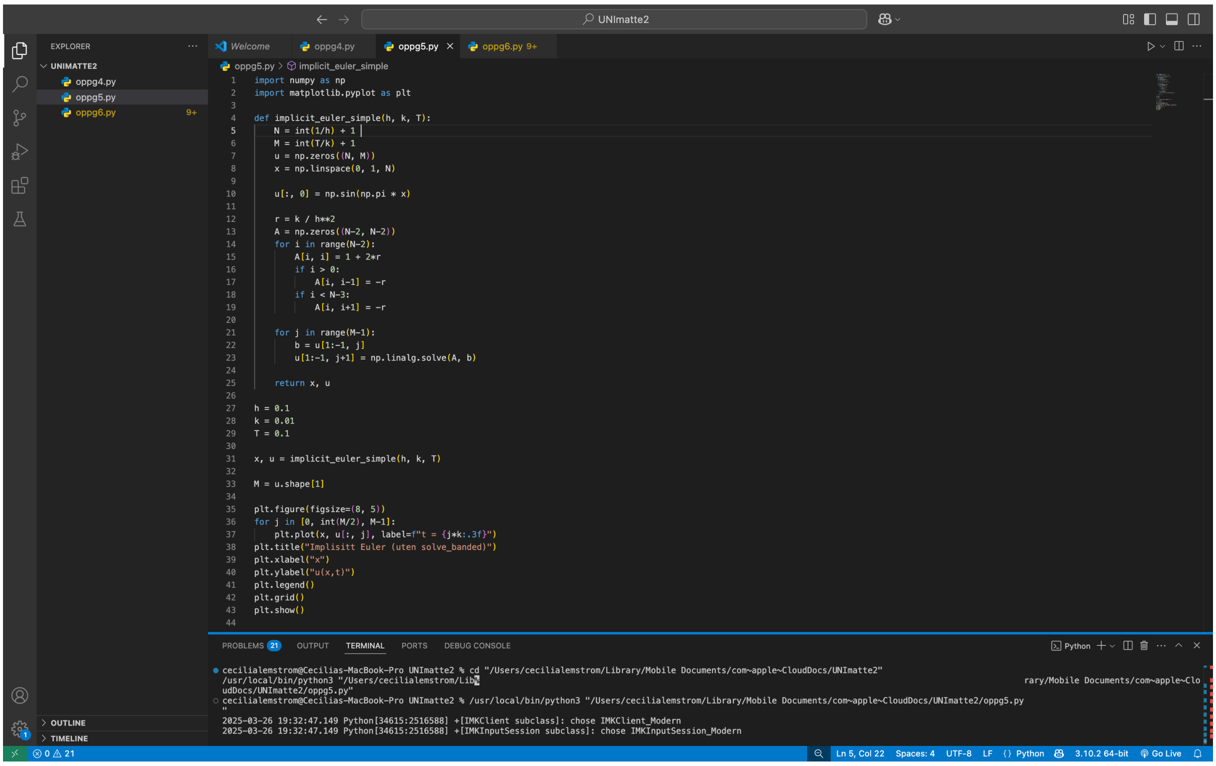
Eksplisitt Euler



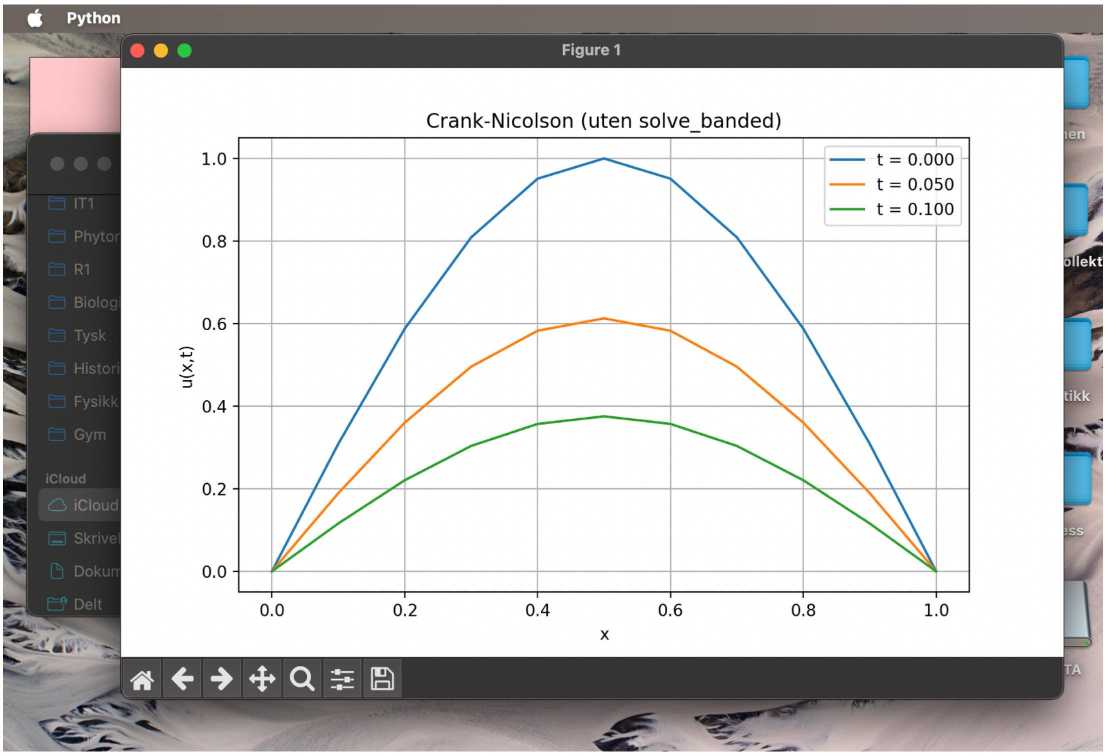


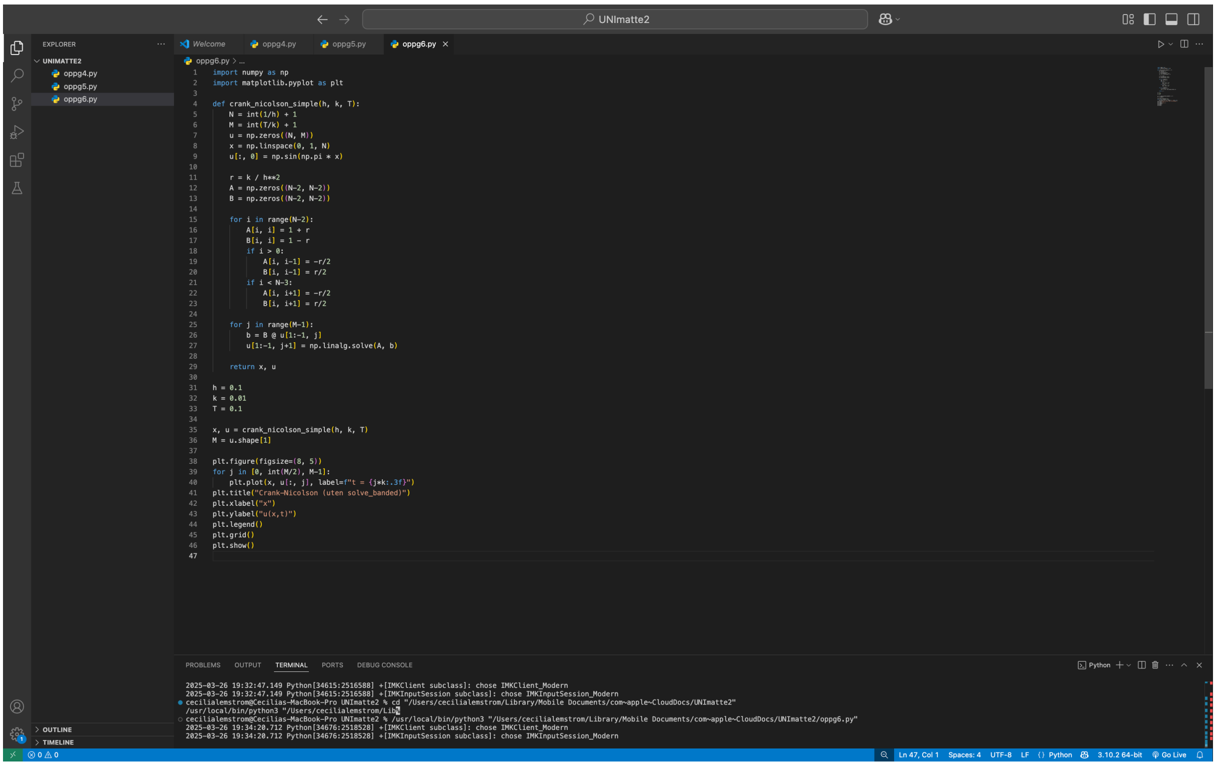
Implisitt Euler





Crank Nicolson





Øving matte 2

1. $f'(x) = \dfrac{f(x+h) - f(x)}{h}$ for $f(x) = e^x$

og $x = 1,5$

h	tilnærmet $f'(1,5)$	eksakt $f'(1,5) = e^{1,5}$	feil
0,1	4,7134	4,4817	0,2317
0,01	4,4984		0,0167
0,001	4,4835		0,0018
10^{-4}	4,4820		0,0003
10^{-5}	4,4818		0,0001
10^{-6}	4,4817		0

for $h < 10^{-6}$ begynner avrundingsfeil å dominere pga flyttallsrepresentasjon

2. $f'(x) = \dfrac{f(x+h) - f(x-h)}{2h}$

h	tilnærmet $f'(1,5)$	feil
0,1	4,8 19	0,0002
0,01	4,4817	0
0,001	4,4817	0

forklaring med Taylorrekker:

$$f(x+h) = f(x) + f'(x)h + \frac{f''(x)}{2} h_2 + O(h^3)$$

$$f(x-h) = f(x) - f'(x)h + \frac{f''(x)}{2} h^2 + O(h^3)$$

$$f(x+h) - f(x-h) = 2f'(x)h + O(h^3)$$

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h} + O(h^2)$$

feilen er nå proporsjonal med h^2 dermed mye mer nøyaktig

3. $f'(x) = \dfrac{f(x-2h) - 8f(x-h) + 8f(x+h) - f(x+2h)}{12h}$

sin feil er orden $O(h^4)$, enda mer nøyaktig

4,5,6 $\dot{u}(x,t) = u''(x,t)$

romlig diskretisering:

$$u''(x_i,t) = \frac{u_{i+1}(t) - 2u_i(t) + u_{i-1}(t)}{h^2}$$

tidsdiskretisering:

- eksplisitt Euler:

$$\frac{u_{i,j+1} - u_{i,j}}{k} = \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2}$$

$$u_{i,j+1} = u_{i,j} + \frac{k}{h^2}\left(u_{i+1,j} - 2u_{i,j} + u_{i-1,j}\right)$$

stabilitetskrav: $\frac{k}{h^2} \leq \frac{1}{2}$

- implisitt Euler

$$\frac{u_{i,j+1} - u_{i,j}}{k} = \frac{u_{i+1,j+1} - 2u_{i,j+1} + u_{i-1,j+1}}{h^2}$$

- Crank-Nicolson

$$\frac{u_{i,j+1} - u_{i,j}}{k} = \frac{1}{2}\left(\frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2} + \frac{u_{i+1,j+1} - 2u_{i,j+1} + u_{i-1,j+1}}{h^2}\right)$$

- eksplisitt Euler: løsningen er stabil hvis $k \leq h^2/2$
- implisitt Euler: fungerer for alle $k > 0$
- Crank-Nicolson: fungerer for alle $k > 0$

metode	stabilitet	nøyaktighet	beregningskostnad
eksplisitt	$k \leq h^2/2$	$O(h^2 + k)$	lav
implisitt	alle $k > 0$	$O(h^2 + k)$	høy
Crank	alle $k > 0$	$O(h^2 + k^2)$	høy