

Computer School

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Project 0

Old-fashioned multiplication

Technical Report

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1 Introduction

The search of this report is to make an analysis on the data gathered in tests of how long does it take for a computer to do different multiplication versions by a certain number of times, always with the same random numbers. There will be three different multiplications versions used for this report:

- 1. Empty version, the function only return zero.
- 2. Standard version, the function uses the language usual multiplication method (i.e. A * B).
- 3. Ancient version, this algorithm uses an old algorithm explained in the project specification.

To make a more interesting analysis, the three methods showed before will be made in two different languages. First in a high level language as C, and then in a low level language as assembler, in this case NASM.

2 Program Structure

2.1 Basics

As explained before, the idea is to repeat each method of multiplication a number N times and get the time it took for each method to finish the process. Before starting is necessary to know and understand the global variables that will be used in the program. Now the first step is to get the random generated numbers that will be used over and over for every multiplication method, and save them in a variable. The second step is to print for the user the tables that will be used for the program first in the form A*B and later in the form B*A. Next step is do each multiplication step N times and show the results and times in screen for A*B and vice versa. And last but not least wait for the user to press a key to see the next method. For an example run of the program, please check the annexes 1 through 7

2.2 Variables

The program uses the following global variables:

- clock_t tInicio: Used to take the time when the function starts.
- clock t tFin: Used to take the time when the function finishes.
- clock_t tDecorrido: Will have the time between tInicio and tFin.
- int num aleatorio[7]: Is a list with the 8 random generated numbers.
- int mat[4][4]: Is a matrix of 4 by 4 that saves the result of the operation.
- int n=10000000: Is the n with the amount of times the user wants to repeat each operation.

```
///Variables
clock_t tInicio, tFin, tDecorrido;
int num aleatorio[7], mat[4][4], n=10000000;
```

2.3 Multiplication Functions

The functions for multiplication made in C where the following in the order empty, standard and ancient:

```
///Funciones para multiplicar
int f_vacia(int A, int B){
    return 0;
}
int f_estandar(int A,int B){
    return A*B;
int f antigua(int A,int B){
    int P = 0;
    while(B != 0){
         if((B%2) != 0){
              P += A;
         A += A;
         B = B >> 1;
    return P;
}
extern int f_vaciaE(int, int);
extern int f_estandarE(int, int);
extern int f antiguaE(int, int);
```

Notice that the last three are just defining that those functions will exist, the extern implies those three are actually programmed in assembler as follows:

```
global f_vaciaE
global f_estandarE
global f_antiguaE
section .text
f_vaciaE:
      xor eax,eax ret
                              ; deja en cero eax
f_estandarE:
     mov eax, edi
mul esi
ret
                             ; edi = A, esi = B
; multiplica A por B
f_antiguaE:
      mov edx, edi
mov ebx, esi
xor eax, eax
                             ; edx = A
; ebx = B
; eax = P
      BNoEsCero:
                              ; mientras B no sea cero
      cmp ebx, 0
      je BEsCero
                              ; si B es 0 sale del while
            test ebx, 1 ; Test hace un and con edx con 1 jnz esImpar ; Si es impar va a quedar prendido el bit 1 jmp esPar
            esImpar:
            esimpar:
add eax, edx
esPar:
add edx, edx
shr ebx, 1
                                          ; P = P + A
            jmp BNoEsCero
BEsCero:
      ret
```

2.4 Writing Functions

To make sure all the times and random generated numbers were properly saved in a text file, there are a few functions dedicated only to the matter. This helped a lot in the process of analysing the data. They were the following:

2.5 Interface functions

One of the most important parts of the program is to show the data obtained to the user. So that's why there are a lot function which purpose is to show the interface in a nice and clear way. The most important interface functions will be explained.

• escribir_base: It writes the two matrices used, not with the results but with the operands used, in both A*B and B*A.

• escribir: In this case it presents both matrices results, plus the time it took for it to finish.

• escribir_num_aleatorio: This one just prints the 8 random numbers used.

• siguiente: This one is really important, it waits for the user to press enter so the program can continue.

```
void siguiente(char* version){
    printf("Precione enter para %s\n", version);
    getchar();
    system("clear");
}
```

• versionVaciaC: For this functions where NNNN is the versions name and L the language. This are the most important because they are ones that actually call the multiplication functions and store the result in the matrix. Only one will be shown because all of them are very similar.

```
void versionVaciaC(){
    printf("\n\n \t\t VERSIÓN VACIA\n");
    printf("-----
    printf(" \n \t\t PROGRAMADO EN C\n\n");
    printf("---
    printf("\t TABLA BASE \n\n");
     /********* TABLA BASE **********/
    tInicio = clock();
    for (int r = 0; r < n; r++) {
    for (int i = 0; i < 4; i++) {
             for (int j = 0; j < 4; j++){
    mat[i][j] = f_vacia(num_aleatorio[i],num_aleatorio[j+4]);</pre>
         }
    tFin = clock();
   tFin = clock();
terminarBase(tInicio, tFin, tDecorrido);
                **** FORMA B X A ***
    tInicio = clock();
    for (int r = 0; r < n; r++) {
         for (int i = 0; i < 4; i++) {
             for (int j = 0; j < 4; j++){
    mat[i][j] = f_vacia(num_aleatorio[j+4],num_aleatorio[i]);</pre>
         }
    tFin = clock();
    terminarAlterna(tInicio, tFin, tDecorrido);
```

2.6 Others

There were other important functions used.

• terminar: this functions finished the run of one of the methods and is where the time is calculated.

```
void terminarBase(){
    tDecorrido = ((tFin - tInicio) / (CLOCKS_PER_SEC / 1000));
    guardarTiempo(tDecorrido);
    escribir(tDecorrido, 1);
}

void terminarAlterna(){
    tDecorrido = ((tFin - tInicio) / (CLOCKS_PER_SEC / 1000));
    guardarTiempo(tDecorrido);
    escribir(tDecorrido, 2);
    escribir_num_aleatorio();
}
```

• main: This is the principal function, that starts the program and call the rest of the functions

```
int main(void){
    //for(int repet=0; repet<75; repet++){</pre>
    srand(time(NULL));
    for( int i= 0; i<8; i++){</pre>
        num_aleatorio[i] = rand() % 9999;
    }
    /*printf("Digite el numero N: ");
    scanf("%d",&n);
    getchar();
    system("clear");*/
    system("clear");
    escribir_base();
    siguiente("ver la solución vacía");
   escribir_base();
    versionVaciaC();
    siguiente("ver solucion en ensamblador");
    escribir_base();
    versionVaciaASM();
    siguiente("continuar con la solucion en la versión estándar");
    escribir_base();
   versionEstandarC();
    siguiente("ver solucion en ensamblador");
    escribir_base();
    versionEstandarASM();
    siguiente("continuar con la solucion en la versión antigua");
    escribir_base();
    versionAntiguaC();
    siguiente("ver solucion en ensamblador");
    escribir_base();
   versionAntiguaASM();
    siguiente("finalizar");
    nuevaLinea();
    guardarAleatorios();
    //}
    return 0;
```

3 Data Analysis

Now is presented three table with interesting data obtained for the program after the use of a N number of one million and 100 tests. There's data such as the highest time, the lowest time, the average time, the difference between the highest and lowest, the most repeated time and how many times it repeated. Also there will be a deeper analysis from the data obtained for each version of the program. For the full table, check in annexes for table 8.1, 8.2 and 8.3.

3.1 Considerations

For this test, the specifications and operating system of the computer in which the tests were carried out are taken into account, which are:

• OS: Ubuntu 16.04.03

• RAM Memory: 12056MB

• Processor: Intel Core i7-6500U CPU @ 2.50GHz

3.1.1 The N number choice

Before starting all the tests, there was the idea of trying the program with 500 thousand, 1 million or 10 million times to see which one could give significant data without during an eternity. The discarded ones were 500 thousand because the information it gave wasn't significant enough and 10 million because it was too slow, and for the amount of test that were planned, it wasn't good enough. So at the end the test with a N of 1 million, were really fast and give some interesting information that could be work on.

3.1.2 The amount of tests

To have a significant amount of data there was needed to run the program a lot of times, and after a little deliberation between the members of the project it was defined that 100 tests would give enough information for this analysis.

3.2 Empty Version

Table 1: Empty Version Data

	Empty						
	(\mathbb{C}	AS	SM			
	AXB	BXA	AXB	BXA			
Higher	98	67	82	52			
Average	83.50	65.37	68.54	49.11			
Lower	64	62	53	48			
Lower-Higher Difference	34	5	29	4			
Most repeated	89	66	77	49			
Times repeated	16	40	17	67			

It might be expected that this version will be basically immediate, however, it averages a time of 83.5ms for A*B and 65.3ms for the operations written in Language C, this operation is the result of what the program takes to communicate with the operating system and create a space in memory that returns a 0, what we can observe is the response time of the memory and as though it does not perform any significant function it shows at the time that we are limited by access to memory. At the same time we have the comparison with the assembly language which averages 68.5ms for A*B and 49.1 for B*A, which shows the limitation of the time of access to memory, but still shows a better time in a low level language in comparison to the high level counterpart.

3.2.1 What is the empty version of the algorithms for?

It doesn't really serves a purpose, because as it has been shown it always return 0 so it never does anything. But as the testing has shown it does takes time to do, and that's why the empty function is good it show how long does it takes for the computer to access memory and return a value, in this case 0.

3.3 Standard Version

Table 2: Standard Version Data

	Standard						
	(C	ASM				
	AXB	BXA	AXB	BXA			
Higher	107	75	85	53			
Average	90.16	71.86	73.26	52.45			
Lower	74	71	56	52			
Lower-Higher Difference	33	4	29	1			
Most repeated	78	72	82	52			
Times repeated	15	73	15	55			

We observe an average a little bit higher to the one in the empty function (between 5 and 10 ms slower). For the standard multiplication operation in C language, we have an average of 90.1ms in A*B and 71.8ms for B*A which shows the access time to memory plus the calculation of Multiplication processing, using the algorithm imposed by the compiler. Now if we focus on the assembly counterpart we observe a really good result which shows an average of 73.2ms for A*B and 52.45 for B*A, this can be deduced to the memory access time and that working in the registers themselves is a lot faster in assembler which reduces use other system resources to perform the calculation, making it faster.

3.4 Ancient Version

Table 3: Ancient Version Data

	Ancient						
	(C	AS	SM			
	AXB	BXA	AXB	BXA			
Higher	691	665	478	482			
Average	639.27	608.59	422.88	408.68			
Lower	549	528	366	340			
Lower-Higher Difference	142	137	112	142			
Most repeated	622	623	416	428			
Times repeated	3	8	5	5			

This function shown by the J.Nyberg paper shows that the average C language time is 639.2ms for A*B and 608.5ms for B*A, far exceeding the result of the multiplication algorithm of the system, this because these instructions require the interpretation and storage of more variables in memory at the same time that requires the use of processing and access to memory, which undoubtedly makes it an inefficient algorithm for multiplication. For assembly language we have an average of 422.88ms for A*B and 408.6 in B*A which leads us to think that said language, again by being low level and working directly in the

For assembly language we have an average of 422.88ms for A*B and 408.6 in B*A which leads us to think that said language, again by being low level and working directly in the registers, requires a greater amount of resources than the standard multiplication method, but makes it faster than the on programmed in C.

3.4.1 Is the ancient algorithm good? How much?

After all the tests done, it's safe to say that the ancient algorithm is not good for computers. Actually it's way worse than the standard version, in the case of C is close to 10 times worse. This doesn't mean that this version is completely bad, is just that in the specific case for computers is not good enough, for example, in some cases it might be faster than the standard version for some humans without a calculator.

3.5 Other Information

3.5.1 Is there gain in assembler versions?

Yeah, there is and is actually quite significant This is a list with how much percent faster is the assembler version against the C version:

- Empty Version (A*B): 17.91% faster
- Empty Version (B*A): 24.87% faster
- Standard Version (A*B): 18.74% faster
- Standard Version (B*A): 27.01% faster
- Ancient Version (A*B): 33.85% faster
- Ancient Version (B*A): 32.85% faster

This shows perfectly that the assembler version is way faster than his counterpart versions in C

3.5.2 Does the order of the arguments affect? There are some rule?

Unexpectedly enough B*A is a lot faster than A*B in all six versions, either NASM or C.

- Empty Version (C): 21.71% faster
- Empty Version (NASM): 28.35% faster
- Standard Version (C): 20.30% faster
- Standard Version (NASM): 28.41% faster
- Ancient Version (C): 4.80% faster
- Ancient Version (NASM): 3.36% faster

Even in the ancient version is still faster, not by a great margin though. But in the cases of the empty and the standard version, the difference is significant. It's even more in the NASM versions of the previously said ones.

3.5.3 Interesting Data

From all the data there are some interesting occurrences, that will be shown now:

• The Standard, NASM, in both versions is so fast that, is even faster than Empty, C, in both versions.

- The most consistent version: This achievement goes to the Standard, C, B*A version. In this version the most repeated time is 72ms and is repeated a whopping 73 times in 100 tests.
- The least variable version: The Standard, NASM, B*A version. In this version, in all the 100 tests, there were only two different times 52ms and 53ms, no more nor less.
- The most variable version: The Ancient, C, A*B version, which is also the slowest, is the one with most variables. It's also the least consistent version, the most repeated time was 622ms with only 3 times from the 100 tests.

4 Annexes

3887	X	3790	3887	Χ	5731	3887 X	3947	3887	Х	4775		
9413	Х	3790	9413	Χ	5731	9413 X	3947	9413	Х	4775		
9743	Х	3790	9743	Χ	5731	9743 X	3947	9743	Х	4775		
5274	Х	3790	5274	Χ	5731	5274 X	3947	5274	Х	4775		
										_		
	PRESENTACION DE LA TABLA ALTERNA:											
3790	Χ	3887	3790	Х	9413	3790 X	9743	3790	Х	5274		
5731	Х	3887	5731	Χ	9413	5731 X	9743	5731	Х	5274		
3947	Х	3887	3947	Х	9413	3947 X	9743	3947	Х	5274		
4775	Х	3887	4775	Χ	9413	4775 X	9743	4775	Х	5274		
										_		
Preci	.or	ne enter p	oara y	vei	r la s	solución va	cía					

Figure 1: Presentation of the numbers generated that will be multiplied

```
PRESENTACION DE LA TABLA BASE:
                                 3887 X 3947
3887 X 3790
                 3887 X 5731
                                                  3887 X 4775
                9413 X 5731 9413 X 3947
9743 X 5731 9743 X 3947
9413 X 3790
                                                   9413 X 4775
9743 X 3790
                                                   9743 X 4775
5274 X 3790
                5274 X 5731
                                  5274 X 3947
                                                    5274 X 4775
        PRESENTACION DE LA TABLA ALTERNA:
               3790 X 9413 3790 X 9743 3790 X 5274
5731 X 9413 5731 X 9743 5731 X 5274
3947 X 9413 3947 X 9743 3947 X 5274
4775 X 9413 4775 X 9743 4735 X 5274
3790 X 3887
5731 X 3887
3947 X 3887
4775 X 3887
                  VERSIÓN VACIA
                 PROGRAMADO EN C
        TABLA BASE
       0
                                         0
                                                            0
                         0
                                        0
                                                           0
                        0
       0
       0
                        0
                                         0
       0
                         0
Tiempo de ejecución para la tabla base: 63 ms
         TABLA ALTERNA
                                       0
0
0
       0
                        0
       0
                       0
       0
                       0
                        0
Tiempo de ejecución para la tabla alterna: 54 ms
         NUMEROS ALEATORIOS USADOS:
          0 -> 3887
          1 -> 9413
          2 -> 9743
          3 -> 5274
          14
             -> 3790
          15
            -> 5731
          16
            -> 3947
          |7 -> 4775|
Precione enter para ver solucion en ensamblador
```

Figure 2: Empty function programmed in C language

```
PRESENTACION DE LA TABLA BASE:
                  3887 X 5731
3887 X 3790
                                    3887 X 3947
                                                     3887 X 4775
                                   9413 X 3947
9743 X 3947
5274 X 3947
                 9413 X 5731
                                                     9413 X 4775
9413 X 3790
9743 X 3790
                 9743 X 5731
                                                     9743 X 4775
5274 X 3790
                 5274 X 5731
                                                     5274 X 4775
         PRESENTACION DE LA TABLA ALTERNA:
3790 X 3887
                 3790 X 9413
                                   3790 X 9743
                                                     3790 X 5274
                5731 X 9413
3947 X 9413
                                   5731 X 9743
3947 X 9743
4775 X 9743
5731 X 3887
3947 X 3887
4775 X 3887
                                                     5731 X 5274
3947 X 5274
                 4775 X 9413
                                                     4775 X 5274
                  VERSIÓN VACIA
                   PROGRAMADO EN ENSAMBLADOR
         TABLA BASE
       0
                         0
                                           0
       0
                                           0
                                           0
       0
                         0
                                                             0
                         0
Tiempo de ejecución para la tabla base: 76 ms
        TABLA ALTERNA
       0
                         0
                                           0
                                                              0
       0
                         0
                                           0
                                                              0
                                           0
                                                             0
       0
                         0
        0
                         0
                                           0
Tiempo de ejecución para la tabla alterna: 42 ms
          NUMEROS ALEATORIOS USADOS:
          0 -> 3887
          1 -> 9413
          2 -> 9743
          3 -> 5274
          4 -> 3790
          5 -> 5731
          6 -> 3947
          |7 -> 4775|
Precione enter para continuar con la solucion en la versión estándar
```

Figure 3: Empty function programmed in Assembler Language

```
PRESENTACION DE LA TABLA BASE:
                3887 X 5731 3887 X 3947 3887 X 4775
9413 X 5731 9413 X 3947 9413 X 4775
9743 X 5731 9743 X 3947 9743 X 4775
3887 X 3790
9413 X 3790
                                                     9743 X 4775
9743 X 3790
5274 X 3790
                 5274 X 5731
                                    5274 X 3947
                                                       5274 X 4775
         PRESENTACION DE LA TABLA ALTERNA:
              3790 X 9413 3790 X 9743 3790 X 5274
5731 X 9413 5731 X 9743 5731 X 5274
3947 X 9413 3947 X 9743 3947 X 5274
4775 X 9413 4775 X 9743 4775 X 5274
3790 X 3887
5731 X 3887
3947 X 3887
4775 X 3887
                   VERSIÓN ESTANDAR
                   PROGRAMADO EN C
         TABLA BASE
                 22276397 15341989
53945903 37153111
55837133 38455621
                                                     18560425
44947075
14731730
35675270
36925970
                                                      46522825
                 30225294
                                   20816478
19988460
                                                      25183350
Tiempo de ejecución para la tabla base: 67 ms
         TABLA ALTERNA
                 35675270
53945903
                                   36925970 19988460
55837133 30225294
14731730
22276397
                                                      20816478
15341989
                 37153111
                                    38455621
18560425
                 44947075
                                    46522825
                                                       25183350
Tiempo de ejecución para la tabla alterna: 64 ms
          NUMEROS ALEATORIOS USADOS:
          0 -> 3887
           1 -> 9413
           2 -> 9743
           3 -> 5274
          4 -> 3790
          5 -> 5731
           i6 -> 3947i
          |7 -> 4775|
Precione enter para ver solucion en ensamblador
```

Figure 4: Standard function programmed in Assembler Language

```
PRESENTACION DE LA TABLA BASE:
3887 X 3790
                       3887 X 5731
                                               3887 X 3947
                                                                  3887 X 4775
9413 X 4775
9743 X 4775
5274 X 4775
                                                                      3887 X 4775
                     9413 X 5731 9413 X 3947
9743 X 5731 9743 X 3947
9413 X 3790
9743 X 3790
                     5274 X 5731 5274 X 3947
5274 X 3790
           PRESENTACION DE LA TABLA ALTERNA:
                                                                 3790 X 5274
5731 X 5274
3947 X 5274
4775 X 5274
3790 X 3887
5731 X 3887
3947 X 3887
                     3790 X 9413 3790 X 9743
5731 X 9413 5731 X 9743
3947 X 9413 3947 X 9743
4775 X 9413 4775 X 9743
4775 X 3887
                        VERSIÓN ESTANDAR
                        PROGRAMADO EN ENSAMBLADOR
           TABLA BASE
                     22276397 15341989
53945903 37153111
55837133 38455621
30225294 20816478
                                                                  18560425
44947075
46522825
14731730
35675270
36925970
19988460
                                                                     25183350
Tiempo de ejecución para la tabla base: 73 ms
      TABLA ALTERNA
                                                             19988460
30225294
20816478

      35675270
      36925970

      53945903
      55837133

      37153111
      38455621

      44947075
      46522825

14731730
22276397
15341989
                                                                     25183350
18560425
Tiempo de ejecución para la tabla alterna: 45 ms
             NUMEROS ALEATORIOS USADOS:
             0 -> 3887
             |1 -> 9413|
              2 -> 9743
              3 -> 5274
              4 -> 3790
              5 -> 5731
                 -> 3947
             7 -> 4775
Precione enter para continuar con la solucion en la versión antigua
```

Figure 5: Standard function programmed in C language

DDECE	NTACION DE LA TA	ARIA RASE.	
PRESE	INTACION DE LA TA	ADLA DASE:	
3887 X 3790	3887 X 5731	3887 X 3947	3887 X 4775
		9413 X 3947	
		9743 X 3947	
5274 X 3790	5274 X 5731	5274 X 3947	5274 X 4775
DDECE	NTACION DE LA TA	ARIA ALTEDNA•	
FRESE	INTACTON DE LA TA	ADLA ALTERNA.	
3790 X 3887	3790 X 9413	3790 X 9743	3790 X 5274
5731 X 3887	5731 X 9413	5731 X 9743	5731 X 5274
3947 X 3887	3947 X 9413	3947 X 9743	3947 X 5274
4775 X 3887	4775 X 9413	4775 X 9743	4775 X 5274
	VERSIÓN ANTIC	GUA	
	DDOCDAMADO EN	1.0	
	PROGRAMADO EN	· C	
TABLA	BASE		
14731730	22276397	15341989 37153111	18560425
35675270 36925970		37153111	
	55837133 30225294	38455621	46522825
19988460	30225294	20810478	25183350
Tiempo de ejec	ución para la ta	abla base: 534 ms	5
TARI A	ALTERNA		
moer	, neremn		
14731730	35675270	36925970	
22276397	53945903	55837133	30225294
15341989	37153111	38455621	20816478
18560425	44947075	46522825	25183350
Tiempo de ejec	ución para la ta	abla alterna: 537	7 ms
recipo de ejec	decon para ca co	55.0	
MUMES	AL FATORIAGE		
NUMER	OS ALEATORIOS US	SADUS:	
10 ->	3887		
	9413		
	9743		
	5274		
	3790		
	5731		
	3947		
7 ->	4775		
Descions		op	
Prectone enter	para ver soluci	lon en ensamblado	Л

Figure 6: Ancient function programmed in C Language

	PRESENTACIO	N DE LA	TARLA RAS	EF+									
3887 X 379	3887	7 X 5731	3887	X 3947	3887 X	4775							
9413 X 379	90 9413	3 X 5731	9413	X 3947	9413 X	4775							
					9743 X								
52/4 X 3/9	52/2	1 X 5/31	5274	X 3947	5274 X	4//5							
	PRESENTACIO	ON DE LA	TABLA AL1	ERNA:									
2700 V 200	27 270	V 0443	2700	V 0743	2700 V	5274							
					3790 X								
					5731 X								
					3947 X 4775 X								
4113 X 300	or 4 112) V 3413	4113	A 3143	4//3 X	3214							
	VERSIÓN ANTIGUA												
	PRO	OGRAMADO	EN ENSAME	BLADOR									
1	TABLA BASE												
14731730	2227	76397	15341	1989	1856042	25							
35675270	5394	15903	37153	3111	4494707	75							
35675270 36925970	5583	15903 37133	38455	621	4652282	25							
19988460	3022	25294	20816	5478	251833	50							
Tiempo de	ejecución	para la	tabla bas	se: 379	ms								
1	TABLA ALTER	RNA											
					1998846	50							
22276397	5394	15903	55837	7133	3022529								
					2081647								
18560425	4494	17075	46522	2825	2518335	50							
Tiempo de	ejecución	para la	tabla alt	terna: 3	76 ms								
1	NUMEROS ALE	EATORIOS	USADOS:										
	0 -> 3887												
	1 -> 9413												
· ·	2 -> 9743												
	3 -> 5274												
	4 -> 3790												
	5 -> 5731												
	6 -> 3947												
	7 -> 4775												
Precione 6	enter para	finaliza	ar			-							

Figure 7: Ancient function programmed in Assembler Language

Data obtained 8.1

					Γ	ime pe	r versio	n					
			pty			Standard				Ancient			
#	(SM		C	1	SM		C		SM	
	AXB	BXA	AXB	BXA	AXB	BXA	AXB	BXA	AXB	BXA	AXB	BXA	
1	93	63	63	50	101	72	81	52	666	592	421	421	
2	70	62	76	50	81	74	83	52	627	624	436	418	
3	87	63	60	49	99	72	81	53	622	567	402	365	
4	87	63	80	49	97	72	84	53	663	623	422	428	
5	90	63	76	49	103	73	82	52	659	621	402	424	
6	86	64	74	49	77	72	79	53	610	598	413	388	
7	91	63	77	49	98	72	77	52	678	528	431	341	
8	89	64	73	50	98	72	81	53	620	582	409	398	
9	92	63	77	49	102	72	83	52	593	561	416	370	
10	89	65	76	49	98	71	79	52	668	594	419	393	
11	88	63	78	49	102	72	81	53	658	628	433	406	
12	91	64	78	49	99	72	83	53	689	628	446	441	
13	69	65	80	49	80	73	82	53	650	632	425	442	
14	67	64	77	49	103	72	82	53	639	620	445	399	
15	85	62	75	50	97	72	79	53	633	579	406	393	
16	89	65	77	49	101	72	80	53	649	582	409	358	
17	87	67	57	49	77	72	80	52	667	620	444	396	
18	86	66	73	49	74	73	79	53	585	636	406	456	
19	86	66	78	50	100	72	82	52	667	615	434	415	
20	89	65	76	49	101	72	63	53	614	614	422	401	
21	86	65	78	50	80	72	81	53	645	575	435	377	
22	93	66	74	49	101	72	78	53	582	616	409	433	
23	92	66	78	50	97	72	82	52	672	595	435	369	
24	65	66	53	49	99	72	80	53	645	637	448	468	
25	70	66	79	50	99	72	82	52	669	623	470	395	
26	90	66	78	49	102	72	79	53	683	591	420	396	
27	86	65	59	50	98	72	58	52	581	601	416	423	
28	64	65	77	52	80	71	59	53	633	625	414	419	
29	90	67	57	49	99	72	82	53	589	582	377	373	
30	88	65	57	49	82	72	60	52	615	591	392	405	
31	71	65	60	49	96	71	65	52	669	623	423	417	
32	89	66	55	48	99	72	57	53	663	640	425	382	
33	86	65	74	49	77	72	60	53	617	619	433	395	
34	65	67	77	49	102	71	56	53	663	600	420	398	
35	69	66	64	49	100	71	61	52	600	603	381	404	
36	86	65	77	50	98	71	79	53	640	575	413	363	
37	91	65	77	49	78	71	61	53	601	641	404	461	
38	69	66	75	49	98	72	75	53	636	636	433	403	
39	92	65	77	49	100	72	80	53	628	623	416	409	

Data obtained 8.2

	Time per version												
		Em	pty			Standard				Ancient			
#	(C	AS	\overline{SM}	(C	AS	SM	(C	AS	$_{\rm SM}$	
	AXB	BXA	AXB	BXA	AXB	BXA	AXB	BXA	AXB	BXA	AXB	BXA	
40	87	67	78	50	78	72	58	52	627	634	407	454	
41	89	65	73	48	100	72	58	53	588	625	396	415	
42	91	66	78	48	102	72	76	52	680	609	448	410	
43	89	67	54	48	78	72	82	52	654	592	445	373	
44	87	65	57	49	100	72	81	52	676	612	428	406	
45	87	64	75	49	75	71	85	53	642	600	425	422	
46	87	66	64	49	96	72	82	53	631	636	441	452	
47	90	66	57	49	103	72	60	52	622	616	420	423	
48	69	66	54	49	99	72	84	52	643	608	403	392	
49	68	66	73	49	97	72	65	53	654	631	430	405	
50	71	65	55	49	98	71	58	52	661	601	431	403	
51	90	65	76	49	95	72	75	53	616	630	450	440	
52	84	65	77	49	83	71	78	52	629	641	421	431	
53	80	66	61	48	99	72	63	52	613	531	415	340	
54	87	67	74	49	78	72	58	52	615	604	411	411	
55	89	67	60	48	91	75	81	52	618	577	376	400	
56	87	66	56	49	82	72	82	52	679	597	476	374	
57	89	65	55	49	82	72	82	52	672	626	422	405	
58	90	67	71	49	77	71	80	52	623	546	416	355	
59	87	65	79	49	98	72	58	52	680	599	429	364	
60	89	67	58	49	78	72	82	52	652	612	409	426	
61	88	64	55	49	78	72	63	52	691	560	424	357	
62	86	66	55	49	79	72	78	53	620	623	431	422	
63	87	66	77	49	97	71	58	53	677	623	421	405	
64	89	66	56	49	80	71	80	52	661	631	433	428	
65	92	66	77	50	85	72	80	53	653	566	430	375	
66	89	66	62	49	81	71	77	52	661	622	455	437	
67	88	66	73	50	101	71	83	53	688	651	459	461	
68	70	66	77	50	98	72	84	53	656	565	407	401	
69	88	65	61	49	102	72	60	52	595	578	399	392	
70	92	65	59	49	101	71	60	52	637	559	427	366	
71	88	65	76	49	96	72	82	52	639	618	417	428	
72	89	66	76	48	80	72	60	53	637	596	386	411	
73	89	66	56	49	79	72	84	52	608	576	427	361	
74	98	65	78	49	102	72	78	52	657	588	452	377	
75	68	66	77	50	82	72	63	52	604	646	398	452	
76	90	65	77	48	78	71	62	52	622	620	416	419	
77	72	64	76	49	97	72	84	53	683	600	478	392	
78	90	65	58	49	80	72	59	52	628	608	428	401	
79	88	64	75	49	80	72	79	52	602	623	413	444	

Data obtained 8.3

					Г	Time per version						
#		Em	pty			Stan	dard		Ancient			
#	С		ASM		С		ASM		С		ASM	
	AXB	BXA	AXB	BXA	AXB	BXA	AXB	BXA	AXB	BXA	AXB	BXA
80	68	65	82	48	83	71	61	52	648	613	420	419
81	89	66	55	49	77	72	78	52	595	619	366	414
82	67	66	56	48	92	72	61	52	649	634	419	401
83	73	66	76	49	101	72	61	52	658	665	448	482
84	88	66	63	50	85	72	79	52	657	608	415	417
85	68	66	56	49	78	71	81	52	631	660	413	386
86	70	65	76	49	85	72	60	53	657	627	460	406
87	76	67	57	49	82	72	63	52	665	553	430	362
88	87	66	57	50	78	72	63	53	662	617	447	441
89	71	67	57	50	76	71	82	53	643	636	425	412
90	87	66	79	49	78	72	79	52	593	614	404	428
91	69	66	55	49	78	72	59	52	676	609	414	392
92	89	66	62	49	99	72	65	52	627	623	421	426
93	88	66	56	49	85	72	63	53	604	641	421	462
94	68	64	77	50	82	71	80	53	549	603	376	443
95	91	65	56	49	107	72	83	52	635	621	423	414
96	88	66	57	49	78	72	84	53	599	631	422	428
97	87	67	55	49	77	71	61	52	612	607	408	431
98	72	66	77	48	78	72	81	53	664	653	463	479
99	88	66	79	50	101	72	82	52	639	613	435	422
100	85	67	73	48	78	72	62	53	682	592	453	430