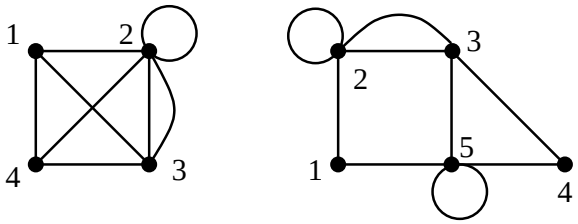
Exercises 1

18 January

Hand in: 4,8. Due: Monday 25 January

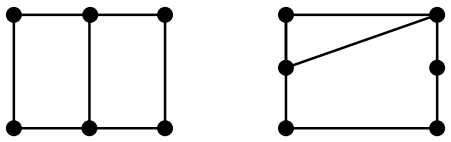
1. Give the adjacency matrices and the adjacency list representations of the graphs below:
Convention: A loop contributes *twice* to the corresponding diagonal entry in the adjacency matrix. This is so that every arc contributes twice to the matrix, and the sum of the matrix entries is twice the number of arcs (and the sum of each row is the degree of that node).
Loops are only recorded *once* in adjacency lists.



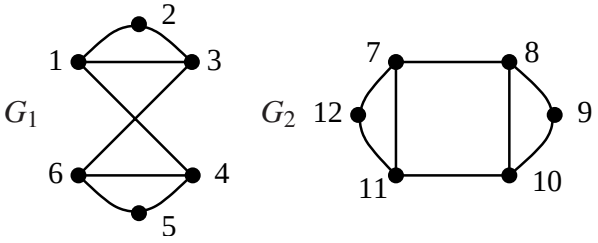
2. Draw the graphs corresponding to these adjacency matrices:

$$\text{(a)}\begin{pmatrix} 2 & 1 & 0 & 1 \\ 1 & 0 & 2 & 0 \\ 0 & 2 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix} \qquad \text{(b)}\begin{pmatrix} 0 & 1 & 0 & 1 & 2 \\ 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 2 \\ 1 & 0 & 0 & 2 & 1 \\ 2 & 0 & 2 & 1 & 0 \end{pmatrix}$$

3. (Gersting) Explain why the accompanying two graphs are not isomorphic.

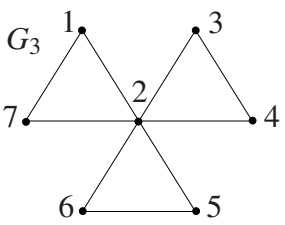


4. This question concerns two isomorphic graphs G_1 and G_2 shown in the diagram.
(a) [1 mark] State an isomorphism from G_1 to G_2 (as a mapping from nodes to nodes, though of course arcs have to be mapped to arcs as well).
(b) [2 marks] How many isomorphisms are there from G_1 to G_2? Explain your answer.
(c) [1 mark] An *automorphism* on a graph G is an isomorphism from G to itself. How many automorphisms are there on G_1?



(d) [2 marks] The identity map *id* is always an automorphism for any graph. Give an example of a graph G with four nodes, such that G has no automorphism apart from id_G. [Hint: consider the degrees of the nodes]

5. A graph is *simple* if it has no loops or parallel arcs. Give *six* simple connected non-isomorphic graphs, each with four nodes. [To clarify, none of your graphs should be isomorphic to each other.]



6. [From 2011 exam] Graph G_3 is as shown in the diagram. How many isomorphisms are there from G_3 to itself (including the identity)? Justify your answer briefly.

7. [From 2000 exam] Construct a graph with exactly three automorphisms (including the identity).

8. [From 2006 exam] The *complement* $\overline{G}$ of an undirected graph G is defined to be the simple graph which has the same nodes as G and where the arcs of $\overline{G}$ are those arcs obtained by joining pairs of distinct nodes precisely if they are not adjacent in G.
(a) [2 marks] Give example graphs G_4 and G_5, with four and five nodes respectively, such that $\overline{G_4}$ is isomorphic to G_4, and similarly for G_5.
(b) [2 marks] Explain why there can be no graph with six nodes which is isomorphic to its complement.