KATHMANDU UNIVERSITY

Dhulikhel, Kavre

DEPARTMENT OF GEOMATICS ENGINEERING

ASSIGNMENT-2

PHYSICAL GEODESY

SUBMITTED BY :

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Roll 10, 3rd year

SUBMITTED TO :

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1.Define:

Geodesy

Geodesy is the branch of applied mathematics concerned with the exact measurements of position of points, the figures, and areas of large proportions. It also deals with study of shape, size of the earth and the variations of terrestrial gravity.

Geometrical Geodesy

Geometrical Geodesy is a branch of geodesy that deals with describing locations in terms of geometry. Coordinate systems are one of the important of geometrical geodesy.

Physical geodesy

Physical geodesy is the study of the physical properties of the gravity field of the Earth, its geo- potential, with a view to their application in field of geodesy.

Geodetic Astronomy

Geodetic Astronomy is branch of practical astronomy which determine the latitude and longitude of a place as well as azimuth of direction of terrestrial object and local sidereal time from astronomical observation. It also determine the changing position of star and other celestial objects

Importance of Geodesy

a. Geodesy plays important role in the establishment and maintenance of national and global three dimensional geodetic networks
b. Geodesy is also applicable in the measurement and analyses of geodynamic phenomena like earth rotation, earth tides, crustal movements, etc.)
c. To determine the exact positions of points on the earth’s surface.
d. To determine the figures and areas of large portions of the earth’s surface
e. To determine the shape and size of the earth
f. Geodesy is also applicable determine the variations of terrestrial gravity.
g. Geodesy is also widely applied in ocean navigation

Geoid and Ellipsoid:

The geoid may be described as a surface coinciding with mean sea -level in oceans, and lying under the land at the level to which the sea would reach if admitted by small frictionless channels. It is that equipotential surface of the Earth's attraction and rotation which, on average, coincides with mean sea -level in the open ocean.

A simplified mathematical surface model used to measure the Earth is ellipsoid. The ellipsoid exists only in theory and not in real life .It is completely smooth and does not take any irregularities in terrain into account, it is only a model for estimation.

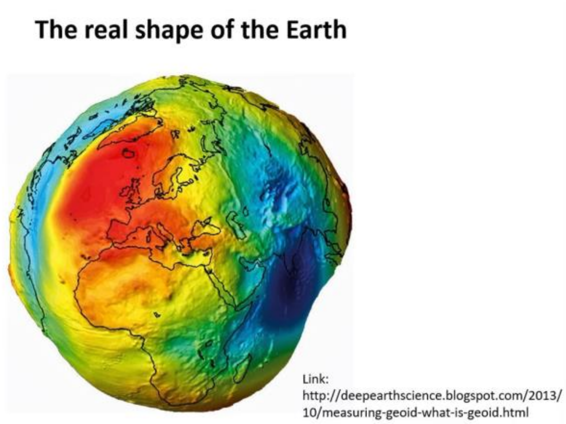


Figure: Geoid

Celestial horizon

Celestial horizon is great circle traced upon the celestial sphere by a plane passing through center of the Earth & perpendicular to the zenith nadir line.

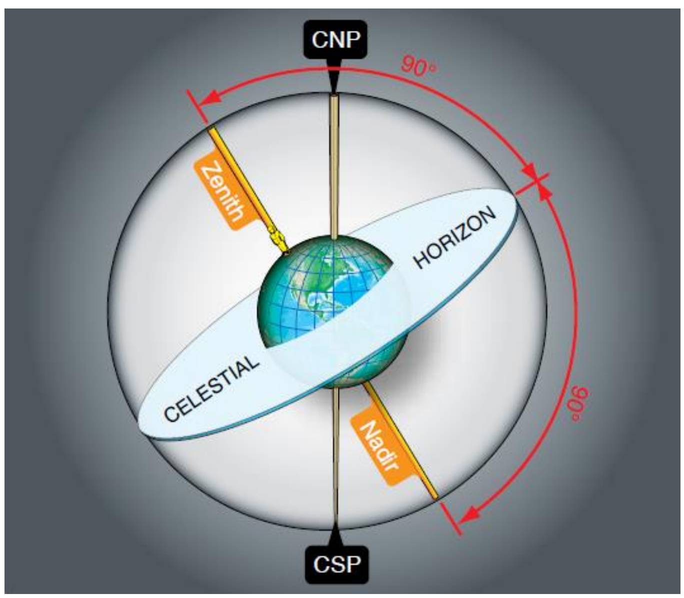


Figure: Celestial Horizon

Visible Horizon

Visible Horizon is small circle of Earth obtained by passing visual rays through the point of observation.

Sensible horizon

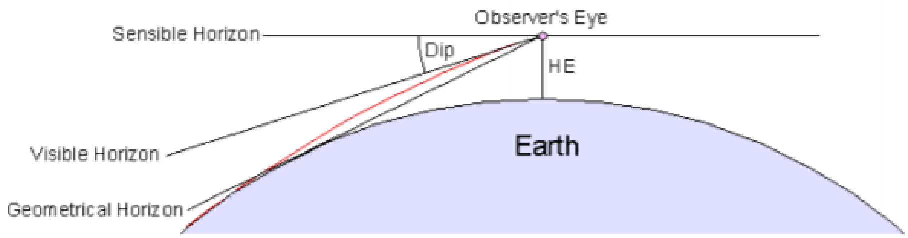


Figure showing sensible horizon and visible horizon

Sensible horizon is Circle obtained by the intersection of the celestial sphere with the plane passing through observer's station and perpendicular to the zenith nadir line at the point of intersection.

Declination and Azimuth

The azimuth is the angle formed between a reference direction (in this example north) and a line from the observer to a point of interest projected on the same plane as the reference direction orthogonal to the zenith.

Declination of a celestial body is the angular distance from the plane of equator measured along the declination circle to the celestial body. Varies from 0 to 90 degree and is measured +ve or –ve as the body is N or S of equator.

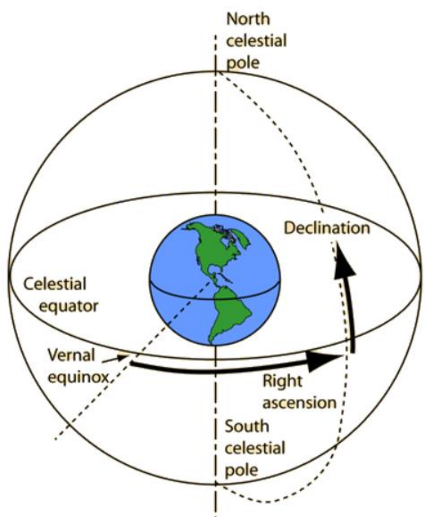


Figure showing Declination

Figure showing sensible horizon and visible horizon

Ecliptic and Equinoctial points

The points where ecliptic crosses the celestial equator are called equinoxes or equinoctial points. It is also the point in a year where day and night time are in equal length. The Sun has 0 degree declination at these points. There are two such points they are Vernal Equinox and Autumnal Equinox.

Gravitational Potential

It is defined as the amount of work done in bringing a unit mass from infinity to that point.

V=-(GM)/r

Here – ve sign is introduced to equate the mathematical concepts of potential with physical concept of potential energy.

Laplace equation & Harmonic Function:

A Laplace function is a second order partial differential equation in the n dimensional euclidean space.

$$\nabla^2\phi = 0$$

$$\frac{\partial^2 \emptyset}{\partial x^2} + \frac{\partial^2 \emptyset}{\partial y^2} + \frac{\partial^2 \emptyset}{\partial z^2} = 0$$

Where ∇^2 is a Laplace operator and Ø is scalar function

The solution of Laplace equation is **harmonic function**.

$$\partial^2 V = 0$$

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

3. Define Newton's law of Motion and Newton's law of Gravitation?

a. Newton's First law

Every object in universe remains continuously in it's state of rest or motion in a straight line unless it is acted upon by an external force. This law gives the definition of force and is also known as law of motion.

b. Newton's Second Law

It state that, "The rate of change of linear momentum of body is directly proportional to force applied and change takes place along the direction of force".

i.e. F$\propto \partial P \; \partial t$

, P= linear momentum of body and F= force on body

c. Newton's Third Law

It state that," In every action there is no equal and opposite reaction."

d. Newton's Law of Gravitation

It states that," Everybody in this universe attracts another body with force which is directly proportional to the product of their masses and inversely proportional to the square of distance between their center.

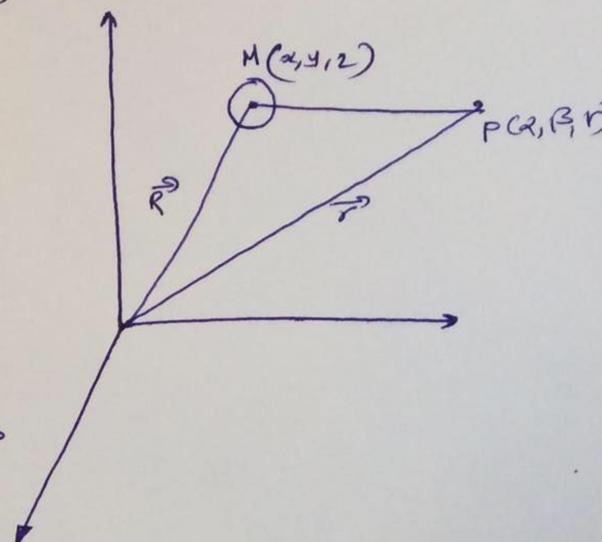
$F = GM1M2$ /r^2

where G=Gravitational constant= 6.67 × 10-11 Nm2 kg-2 in SI units

m1,m2 are masses of two such bodies and r is distance between their centers

2. Prove that Gravitational Potential is Harmonic function ?

Lets consider a mass 'M' with centre of mass situated at coordinates (x, y, z) in an arbitary fram of reference and point mass at $P(\alpha, \beta, \gamma)$



$$\vec{r} = \alpha i + \beta j + \gamma k$$

$$\vec{R} = x i + y j + z k$$

from Δ law of vector

$$\vec{s} = \vec{R} - \vec{r} \quad [\because \vec{R} = \vec{r} + \vec{s}]$$

$$\vec{s} = (x-\alpha)\vec{i} + (y-\beta)\vec{j} + (z-\gamma)\vec{k}$$

$$|\vec{s}| = \sqrt{(x-\alpha)^2 + (y-\beta)^2 + (z-\gamma)^2}$$

$$= \sqrt{\Sigma (x-\alpha)^2}$$

We know,
The gravitational potential is given as, $V = -\frac{GM}{r}$

$$V = \frac{-GM}{\sqrt{\Sigma (x-\alpha)^2}}$$

Differentiation w.r.t 'x'

$$\frac{\partial V}{\partial x} = -GM \frac{\partial}{\partial x}\left(\frac{1}{\sqrt{(x-\alpha)^2 + (y-\beta)^2 + (z-\gamma)^2}}\right)$$

$$= \frac{1}{2} GM \left\{(x-\alpha)^2 + (y-\beta)^2 + (z-\gamma)^2\right\}^{-3/2} 2(x-\alpha)$$

$$= \frac{GM(x-\alpha)}{\left((x-\alpha)^2 + (y-\beta)^2 + (z-\gamma)^2\right)^{3/2}}$$

$\mu(t) = GM = 398600.4405 \ km^3 s^{-3}$

$$= \frac{\mu(t)\ (x-\alpha)}{[(x-\alpha)^2+(y-\beta)^2+(z-\gamma)^2]^{3/2}}$$

differentiation again w.r.t 'x'

$$\frac{\partial^2 V}{\partial x^2} = \frac{\mu(t)\{(x-\alpha)^2+(y-\beta)^2+(z-x)^2]^{3/2} \cdot \frac{\partial}{\partial x}\{(x-\alpha)^2+(y-\beta)^2+(z-\gamma^2)]^{3/2}}{[(x-\alpha)^2+(y-\beta)^2+(z-x)^2]^2}$$

$$= \frac{\mu(t)[\Sigma(x-\alpha)^2]^{3/2} - (x-\alpha)\cdot\frac{3}{2}[\{\Sigma(x-\alpha)^2\}^{3/2-1}\cdot 2(x-\alpha)}{[[\Sigma(x-\alpha)^2]^{3/2}]^2}$$

$$= \mu(t)[\{\Sigma(x-\alpha)\}^{3/2} - 3(x-\alpha)^2\{\Sigma(x-2)^2\}^{1/2}]$$

$$\Rightarrow \frac{\partial^2 V}{\partial x^2} = \frac{\mu(t)\{s^3 - 3(x-\alpha)^2\cdot s\}}{s^6} \quad \text{where } s = \sqrt{\Sigma(x-\alpha)^2}$$

$$= \frac{\mu(t)\{s^2 - 3(x-\alpha)^2\}}{s^5}$$

also $\frac{\partial^2 V}{\partial y^2} = \frac{\mu(t)\{s^2 - 3(y-\beta)^2\}}{s^5}$

$$\frac{\partial^2 V}{\partial z^2} = \frac{\mu(t)\{s^2 - 3(y-\alpha)^2\}}{s^5}$$

Finally, $\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = \mu(t)\{s^3 - 3(x-\alpha)^2\cdot s/s^5$

$+ \mu(t)\{s^3 - 3(y-\beta)^2\}/s^5 + \mu(t)\{s^3 - 3(z-\gamma)^2\}/s^5$

$= \frac{\mu(t)}{s^5}\{3s^2 - 3s^2\} = 0$ $\boxed{\therefore \nabla^2 V = 0, \text{ gravitational Potential is harmonic function.}}$