

## **Cambridge International Examinations**

Cambridge International Advanced Level

CANDIDATE NAME												
CENTRE NUMBER							CANDID NUMBE					
FURTHER MAT	HEMAT	ics									92	31/22
Paper 2								0	ctobe	r/Nov	embe	r 2018
											3	hours
Candidates ansv	wer on th	ne Questi	on Pa	per.								
Additional Mater	ials:	List of F	ormul	ae (MF10	0)							

#### **READ THESE INSTRUCTIONS FIRST**

Write your Centre number, candidate number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO NOT WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Where a numerical value is necessary, take the acceleration due to gravity to be  $10 \text{ m s}^{-2}$ .

The use of a calculator is expected, where appropriate.

Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

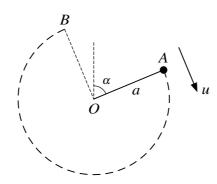
The number of marks is given in brackets [] at the end of each question or part question.



	e $O$ . The kinetic energy of $P$ when it is at $A$ is twice its kinetic energy itude of the motion.	e harmonic motion when it is at <i>B</i> . Fin
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Γωο ι	uniform small smooth spheres $A$ and $R$ have equal radii and masses	2m and m respecti
Spher B whi	uniform small smooth spheres $A$ and $B$ have equal radii and masses re $A$ is moving with speed $u$ on a smooth horizontal surface when it coll ich is at rest. The coefficient of restitution between the spheres is $\frac{2}{3}$ . Find, in terms of $u$ , the speeds of $A$ and $B$ after this collision.	
Sphero B whi	re <i>A</i> is moving with speed <i>u</i> on a smooth horizontal surface when it collich is at rest. The coefficient of restitution between the spheres is $\frac{2}{3}$ .	
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Sphere B is initially at a distance d from a fixed smooth vertical wall which is perpendicular to the direction of motion of A. The coefficient of restitution between B and the wall is  $\frac{1}{2}$ .

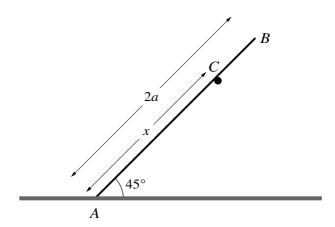
A and $B$ .	[5]



A particle of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The point A is such that OA = a and OA makes an angle  $\alpha$  with the upward vertical, where  $\tan \alpha = \frac{12}{5}$ . The particle is projected downwards from A with speed u perpendicular to the string and moves in a vertical plane (see diagram). The string becomes slack after the string has rotated through  $270^{\circ}$  from its initial position, with the particle now at the point B.

(i)	Show that $u^2 = 2ag$ .	[5]
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A uniform rod AB of length 2a and weight W rests against a smooth horizontal peg at a point C on the rod, where AC = x. The lower end A of the rod rests on rough horizontal ground. The rod is in equilibrium inclined at an angle of  $45^{\circ}$  to the horizontal (see diagram). The coefficient of friction between the rod and the ground is  $\mu$ . The rod is about to slip at A.

(i)	Find an expression for $x$ in terms of $a$ and $\mu$ .	[5]
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(ii)	Hence show that $\mu \ge \frac{1}{3}$ .
(iii)	Given that $x = \frac{3}{2}a$ , find the value of $\mu$ and the magnitude of the resultant force on the rod at $A$ .

of le s rig	belief belief by the solution of the object, passes through $A$ and is perpendicular to $AB$ , and a uniform rod $AB$ , and $B$ and mass $B$ , where $B$ is a constant. The centre of the disc is $B$ . The end $B$ of the rod gidly joined to a point on the circumference of the disc so that $B$ is a straight line. The fixed zontal axis $B$ is in the plane of the object, passes through $B$ and is perpendicular to $B$ .
(i)	Show that the moment of inertia of the object about the axis $l$ is $3Ma^2(26 + k)$ . [5]

The object is free to rotate about l.

the period	t small oscillated of these osci	llations is $4\pi$	$\sqrt{\left(\frac{a}{g}\right)}$ , find	the value of	k.	
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	14.2	11.3	10.8	8.4	12.8	11.5	12.1	9.2		
Assuming					normally	distribu	ited, cal	culate a	95% conf	ìd
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7	The continuous	random	variable X	has	distribution	function	given	by
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$$F(x) = \begin{cases} 0 & x < 0, \\ \frac{1}{90}(x^2 + x^4) & 0 \le x \le 3, \\ 1 & x > 3. \end{cases}$$

The random variable Y is defined by  $Y = X^2$ .

/	Find the probability density function of $Y$ .	[4]
:/	Find the mean value of <i>Y</i> .	[2
.,	This the mean value of T.	[-

Lan starts a new job on Monday. He will catch the bus to work every day from Monday to Friday

wee]	bability that Lan will not get a seat on the Monday, Tuesday, Wednesday or Thursday of k is 0.4096.	
(i)	Show that $p = 0.2$ .	1
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(ii)	Find the probability that Lan first gets a seat on Monday of the second week in his new j	job.
(ii)	Find the probability that Lan first gets a seat on Monday of the second week in his new j	
(ii)		

correspond	ls to this va	alue of $N$ .						
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For a random sample of 5 observations of pairs of values (x, y), the equation of the regression line of y

	Test at the 5% significance level whether there is evidence of non-zero correlation betwee variables.
(ii)	Find the values of $c$ and $d$ .
(ii)	Find the values of $c$ and $d$ .
(ii)	Find the values of c and d.
(ii)	Find the values of c and d.
(ii)	Find the values of c and d.
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(ii)	Find the values of c and d.
(ii)	Find the values of c and d.

(iii)	Use an appropriate regression line to estimate the value of $x$ when $y = 3.5$ , and comment on the
	reliability of your estimate. [2]

10 The number of accidents, *x*, that occur each day on a motorway are recorded over a period of 40 days. The results are shown in the following table.

Number of accidents	0	1	2	3	4	5	6	<b>≥</b> 7
Observed frequency	3	5	8	10	5	7	2	0

sample. Explain why					.95 and distribut			
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uencies, correct to 2 de	ecimal pl	aces. Th	e results	are show	n in the f	Collowing	table.	- 1
Poisson distribution with quencies, correct to 2 de Number of accidents  Observed frequency	ecimal pl	aces. Th	e results	are show	n in the f	following 5	table.	≥7
uencies, correct to 2 de	ecimal pl	aces. Th	e results	are show	n in the f	Collowing	table.	- 1
Number of accidents Observed frequency Expected frequency	0 3 2.09	1 5 6.18	2 8 9.11	3 10 8.96	4 5 6.61	following 5	table.  6 2	≥7 0
Number of accidents Observed frequency Expected frequency	0 3 2.09	1 5 6.18	2 8 9.11	3 10 8.96	4 5 6.61	following 5	table.  6 2	≥7 0
Number of accidents Observed frequency Expected frequency	0 3 2.09	1 5 6.18	2 8 9.11	3 10 8.96	4 5 6.61	following 5	table.  6 2	≥7 0
Number of accidents Observed frequency Expected frequency	0 3 2.09	1 5 6.18	2 8 9.11	3 10 8.96	4 5 6.61	following 5	table.  6 2	≥7 0
Number of accidents Observed frequency Expected frequency	0 3 2.09	1 5 6.18	2 8 9.11	3 10 8.96	4 5 6.61	following 5	table.  6 2	≥7 0
Number of accidents  Observed frequency	0 3 2.09	1 5 6.18	2 8 9.11	3 10 8.96	4 5 6.61	following 5	table.  6 2	≥7 0

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11 Answer only **one** of the following two alternatives.

### **EITHER**

One end of a light elastic spring, of natural length  $0.8 \,\mathrm{m}$  and modulus of elasticity  $40 \,\mathrm{N}$ , is attached to a fixed point O. The spring hangs vertically, at rest, with particles of masses  $2 \,\mathrm{kg}$  and  $M \,\mathrm{kg}$  attached to its free end. The  $M \,\mathrm{kg}$  particle becomes detached from the spring, and as a result the  $2 \,\mathrm{kg}$  particle begins to move upwards.

(i)	Show that the 2 kg particle performs simple harmonic motion about its equilibrium position with period $\frac{2}{5}\pi$ s. State the distance below $O$ of the centre of the oscillations. [7]

The speed of the 2 kg particle is  $0.4\,\mathrm{m\,s^{-1}}$  when its displacement from the centre of oscillation is  $0.06 \, \text{m}$ . (ii) Find the amplitude of the motion. [3] (iii) Deduce the value of M. [4]

#### OR

In a particular country, large numbers of ducks live on lakes A and B. The mass, in kg, of a duck on lake A is denoted by x and the mass, in kg, of a duck on lake B is denoted by y. A random sample of 8 ducks is taken from lake A and a random sample of 10 ducks is taken from lake B. Their masses are summarised as follows.

$$\Sigma x = 10.56$$
  $\Sigma x^2 = 14.1775$   $\Sigma y = 12.39$   $\Sigma y^2 = 15.894$ 

A scientist claims that ducks on lake A are heavier on average than ducks on lake B.

(i)	Test, at the 10% significance level, whether the scientist's claim is justified. You should assume that both distributions are normal and that their variances are equal. [9]

A se	second random sample of 8 ducks is taken from lake $A$ and their masses are summarised as
	$\Sigma x = 10.24$ and $\Sigma (x - \bar{x})^2 = 0.294$ ,
lake	re $\bar{x}$ is the sample mean. The scientist now claims that the population mean mass of ducks on $A$ is greater than $p$ kg. A test of this claim is carried out at the 10% significance level, using only second sample from lake $A$ . This test supports the scientist's claim.
(ii)	Find the greatest possible value of $p$ . [5]

# **Additional Page**

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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