

Cambridge International AS & A Level

CANDIDATE NAME					
CENTRE NUMBER			CANDIDATE NUMBER		

4671471129

FURTHER MATHEMATICS

9231/21

Paper 2 Further Pure Mathematics 2

May/June 2021

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has 16 pages.

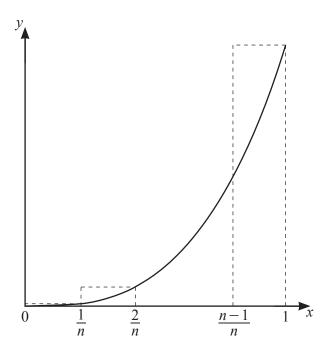
ax + 3y + z = 14, 2x + y + 3z = 0, -x + 2y - 5z = 17,	
has a unique solution and interpret this situation geometrically.	[4
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Find the value of a for which $x = 1$, $y = 4$, $z = -2$ is the solution to the system of equation part (a).	s i [1
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The variables x and y are related by the differential equation

2

Fi	nd the general solution for y in terms of x .	
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St	ate an approximate solution for large positive values of x .	

3



The diagram shows the curve with equation $y = x^3$ for $0 \le x \le 1$, together with a set of *n* rectangles of width $\frac{1}{n}$.

(a) By considering the sum of the areas of these rectangles, show that $\int_0^1 x^3 dx < U_n$, where

$\left(\frac{1}{n}\right)^2$. [4]	$U_n = \left(\frac{n+1}{2n}\right)^2$

	of n , a lower bound L_n for $\int_0^1 x^3 dx$.	
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Find the least value of n such that U_n –	$-L_n < 10^{-3}$.	
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	$-L_n < 10^{-3}$.	

4 Find the solution of the differential equation

$\sin\theta \frac{\mathrm{d}y}{\mathrm{d}\theta}$	+y =	$\tan \frac{1}{2}\theta$
Ab · ····	. ,	2 ,

where $0 < \theta < \pi$, given that $y = 1$ when $\theta = \frac{1}{2}\pi$. Give your answer in the form $y = f(\theta)$. [You may use without proof the result that $\int \csc\theta d\theta = \ln \tan \frac{1}{2}\theta$.]				

(a)	State the sum of the series $z+z^2+z^3++z^n$, for $z \neq 1$.	[1
(b)	Given that z is an nth root of unity and $z \neq 1$, deduce that $1+z+z^2++z^{n-1}=0$.	[2
(c)	Given instead that $z = \frac{1}{3}(\cos\theta + i\sin\theta)$, use de Moivre's theorem to show that	
	$\sum_{m=1}^{\infty} 3^{-m} \cos m\theta = \frac{3\cos\theta - 1}{10 - 6\cos\theta}.$	[7

6 The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} 5 & -\frac{22}{3} & 8 \\ 0 & -6 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Find a matrix P and a diagonal matrix D such that $A^2 = PDP^{-1}$.	
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(b)	Use the characteristic equation of \mathbf{A} to find \mathbf{A}^3 .	[4]

(a)	It is given that $y = \operatorname{sech}^{-1}\left(x + \frac{1}{2}\right)$.	
	Express cosh y in terms of x and hence show that sinh $y \frac{dy}{dx} = -\frac{1}{\left(x + \frac{1}{2}\right)^2}$.	[3]
(b)) Find the first three terms in the Maclaurin's series for $\operatorname{sech}^{-1}\left(x+\frac{1}{2}\right)$ in the form	
(b)) Find the first three terms in the Maclaurin's series for sech $^{-1}\left(x+\frac{1}{2}\right)$ in the form $\ln a + bx + cx^2$,	
(b)		[7]
(b)	$\ln a + bx + cx^2,$	[7]
(b)	$\ln a + bx + cx^2,$	
(b)	$\ln a + bx + cx^2$, where a , b and c are constants to be determined.	
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8	The curve	Chas	narametric	equations
o	THE CUIVE	C mas	parametric	equations

$$x = 2 \cosh t$$
, $y = \frac{3}{2}t - \frac{1}{4}\sinh 2t$, for $0 \le t \le 1$.

(a) Fin	$\frac{dx}{dt}$ and show that $\frac{dy}{dt} = 1 - \sinh^2 t$.	[3]
	a of the surface generated when C is rotated through 2π radians about t	he x-axis is denoted by A
(b) (i)	Show that $A = \pi \int_0^1 \left(\frac{3}{2}t - \frac{1}{4}\sinh 2t \right) (1 + \cosh 2t) dt$.	[4]

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Additional Page

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