

Cambridge International AS & A Level

CANDIDATE NAME					
CENTRE NUMBER			CANDIDATE NUMBER		

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FURTHER MATHEMATICS

9231/22

Paper 2 Further Pure Mathematics 2

May/June 2023

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has 16 pages. Any blank pages are indicated.

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1 ((a)	Show	that t	he s	ystem	of ed	quations
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$$x+2y+3z = 1,$$

 $4x+5y+6z = 1,$
 $7x+8y+9z = 1,$

loes not have a unique solution.	[2
Show that the system of equations	s in part (a) is consistent. Interpret this situation geometricall

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,	Lice the cubefifution	r = r + 1	tind the	collition	Of the	differential	equiation
4	Use the substitution	12-x+v to	i iiiiu uic	SOLULION	or uic	unitorential	Cuuanon

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1+3x+3y}{3x+3y-1}$$

for which $y = 0$ when $x = 1$. Give your answer in the form $a \ln(x+y) + b(x-y) + c = 0$, who and c are constants to be determined.	ere <i>a</i> , <i>b</i> [7]
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Use the substitution $x = \sin \theta$	to find the exact va	alue of $\int_0^{\frac{1}{2}} (1-x^2)^{\frac{3}{2}} dx$	r.
Use the substitution $x = \sin \theta$	to find the exact va	alue of $\int_0^{\frac{1}{2}} (1-x^2)^{\frac{3}{2}} dx$	ς.
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Use the substitution $x = \sin \theta$	to find the exact va	alue of $\int_0^{\frac{1}{2}} (1-x^2)^{\frac{3}{2}} dx$	ν.
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Jse the substitution $x = \sin \theta$	to find the exact va	alue of $\int_0^{\frac{1}{2}} (1-x^2)^{\frac{3}{2}} dx$	v.
Use the substitution $x = \sin \theta$			

- 4 The integral I_n is defined by $I_n = \int_0^1 (1+x^5)^n dx$.
 - (a) By considering $\frac{d}{dx}(x(1+x^5)^n)$, or otherwise, show that

$(5n+1)I_n = 2^n + 5nI_{n-1}.$	[5]

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5	1 ne	matrix	A	1S	given	υV

$$\mathbf{A} = \begin{pmatrix} 18 & 5 & -11 \\ 8 & 6 & -4 \\ 32 & 10 & -20 \end{pmatrix}.$$

Show that the characteristic equation of A is $\lambda^3 - 4\lambda^2 - 20\lambda + 48 = 0$ and hence fix eigenvalues of A .	.110	(
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	10	
6	Find the particular solution of the differential equation	
	$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} - 12\frac{\mathrm{d}x}{\mathrm{d}t} + 36x = 37\sin t,$	
	$\mathrm{d}t^2$ $\mathrm{d}t$	
	given that, when $t = 0$, $x = \frac{dx}{dt} = 0$.	[11]
		•••••
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7

(a)	Use the substitution $u = x^2 - 1$ to find $\int \frac{x}{\sqrt{x^2 - 1}} dx$.	[3]
		-1 N x

The diagram shows the curve with equation $y = \cosh^{-1}x$ together with a set of (N-1) rectangles of unit width.

(b) By considering the sum of the areas of these rectangles, show that

$\sum_{r=2}^{N} \ln\left(r + \sqrt{r^2 - 1}\right) > N$	$I\ln\left(N+\sqrt{N^2-1}\right)-\sqrt{N}$	$\overline{V^2-1}$.	[5]

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		• • •
(a)	Use a similar method to find in terms of N on upon hound for $\sum_{i=1}^{N} \ln(x_i + \sqrt{x_i^2 + 1})$	27
(c)	Use a similar method to find, in terms of N , an upper bound for $\sum_{r=2}^{N} \ln(r + \sqrt{r^2 - 1})$.	3]
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(c)		

	$1 - \mathrm{sech}^2 t = \tanh^2 t.$	[3
 Γhe cu	rve C has parametric equations	
	$x = \frac{1}{2}\tanh^2 t + \ln \operatorname{sech} t, \qquad y = 1 + \tanh^4 t, \qquad \text{for } t > 0.$	
(b) Sł	now that $\frac{dy}{dx} = -4 \operatorname{sech}^2 t$.	[5

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Additional page

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