

Cambridge International Examinations

Cambridge International Advanced Level

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	
FURTHER MATHEMA	ATICS		9231/23
Paper 2		Oct	tober/November 2017
			3 hours
Candidates answer on	the Question Paper.		
Additional Materials:	List of Formulae (MF10)		

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO NOT WRITE IN ANY BARCODES.

Answer all the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Where a numerical value is necessary, take the acceleration due to gravity to be $10 \,\mathrm{m}\,\mathrm{s}^{-2}$.

The use of a calculator is expected, where appropriate.

Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.



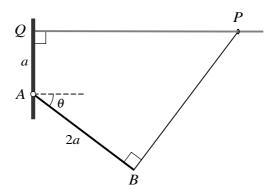
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	below its highest level, the rate of change of its height is $\frac{1}{4}\pi$ metres per second. F and period of the motion.	ind the ampl
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the coefficient of restitution between any pair of the sphere B , sphere B collides with sphere C .	res is $\frac{1}{2}$. After sphere <i>A</i> has collide
(i) Find an inequality satisfied by k .	

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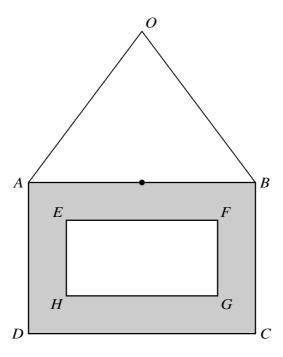
4



A small ring P of weight W is free to slide on a rough horizontal wire, one end of which is attached to a vertical wall at Q. The end A of a thin uniform rod AB of length 2a and weight $\frac{5}{2}W$ is freely hinged to the wall at the point A which is a distance a vertically below Q. A light elastic string of natural length 2a has one end attached to the ring P and the other end attached to the rod at B. The string is at right angles to the rod and A, B, P and Q lie in a vertical plane. The system is in limiting equilibrium with AB making an angle θ with the horizontal, where $\sin \theta = \frac{3}{5}$ (see diagram).

(i)	Find the tension in the string in terms of W .	[2]
(ii)	Find the coefficient of friction between the ring and the wire.	[2]
(iii)	Find the magnitude of the resultant force on the rod at the hinge in terms of W .	[3]

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(iv)	Find the modulus of elasticity of the string in terms of W .	[3]
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A uniform picture frame of mass m is made by removing a rectangular lamina EFGH in which EF = 4a and FG = 2a from a larger rectangular lamina ABCD in which AB = 6a and BC = 4a. The side EF is parallel to the side AB. The point of intersection of the diagonals AC and BD coincides with the point of intersection of the diagonals EG and EG and EG and EG and EG and EG are the mid-point of the string. A small object of mass $\frac{11}{12}m$ is fixed to the mid-point of EG (see diagram).

(i)	Show that the moment of inertia of the system, consisting of frame and small object, about ar axis through O perpendicular to the plane of the frame, is $\frac{169}{3}ma^2$.					

(ii)	Show that small oscillations of the system about this axis are approximately simple harmonic and state their period. [5]

6

A pair of fair dice is thrown repeatedly until a pair of sixes is obtained. The number of throws taken

(i)	Find the mean value of X .	[
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(ii)	Find the probability that exactly 12 throws are required to obtain a pair of sixes.	[
(II <i>)</i>	Thid the probability that exactly 12 throws are required to obtain a pair of sixes.	l
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(iii)	Find the probability that more than 12 throws are required to obtain a pair of sixes.	[
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The	random variable X has I			
		$f(x) = \begin{cases} 0.2e^{-0.2x} \\ 0 \end{cases}$	$x \geqslant 0$,	
		$\Gamma(x) = \bigcup_{i=1}^{n}$	otherwise.	
(i)	Find the distribution fu	nction of X .		
(1)	Time the distribution is	neuon of 11.		
(ii)	Find $P(X > 2)$.			
(iii)	Find the median of X .			

8 Members of a Statistics club are voting to elect a new president of the club. Members must choose to vote either by post or by text or by email. The method of voting chosen by a random sample of 60 male members and 40 female members is given in the following table.

		Met	hod of w	oting
		Post	Text	Email
Gender	Male	10	12	38
Gender	Female	5	21	14

Test, at the 1% significance level, whether there is an association between method of voting and gender. [8]

9 The land areas *x* (in suitable units) and populations *y* (in millions) for a sample of 8 randomly chosen cities are given in the following table.

Land area (x)	1.0	4.5	2.4	1.6	3.8	8.6	7.5	6.5
Population (y)	0.8	8.4	4.2	1.6	2.2	10.2	4.2	5.2

$$[\Sigma x = 35.9, \ \Sigma x^2 = 216.47, \ \Sigma y = 36.8, \ \Sigma y^2 = 244.96, \ \Sigma xy = 212.62.]$$

(i)	Find, showing all necessary working, the value of the product moment correlation coefficient for this sample. [3]
(ii)	Using a 1% significance level, test whether there is positive correlation between land area and population of cities. [4]

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oro	nd areas and populations for another randomly chosen sample of cities, this time of size n , uct moment correlation coefficient of 0.651. Using a test at the 1% significance level, the	
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10	A factory produces bottles of an energy juice. Two different machines are used to fill empty bottles
	with the juice. The manager chooses a random sample of 50 bottles filled by machine X and a random
	sample of 60 bottles filled by machine Y . The volumes of juice, x and y respectively, measured in
	appropriate units, are summarised by

$$\Sigma x = 45.5$$
, $\Sigma (x - \bar{x})^2 = 19.56$, $\Sigma y = 72.3$, $\Sigma (y - \bar{y})^2 = 30.25$,

where \bar{x} and \bar{y} are the sample means of the volume of juice in the bottles filled by X and Y respectively.

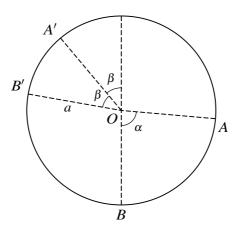
illied by ma	achine <i>X</i> and the	e mean volur	ne of juice	in bottles fil	led by machin	ie Y.	
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A test at the $\alpha\%$ significance level does not provide evidence that there is any difference in the means of the volume of juice in bottles filled by machine X and the volume of juice in bottles filled by machine Y.

i)	Find the set of possible values of α .	[6]
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11 Answer only **one** of the following two alternatives.

EITHER



A particle P of mass m is free to move on the smooth inner surface of a fixed hollow sphere of radius a. The centre of the sphere is O. The points A and A' are on the inner surface of the sphere, on opposite sides of the vertical through O; the radius OA makes an angle α with the downward vertical and the radius OA' makes an angle β with the upward vertical. The point B is on the inner surface of the sphere, vertically below O. The point B' is on the inner surface of the sphere and such that OB' makes an angle 2β with the upward vertical through O (see diagram). It is given that $\cos \alpha = \frac{1}{16}$.

(i)	P is projected from A with speed u along the surface of the sphere downwards towards B .
	Subsequently it loses contact with the sphere at A'. Show that $u^2 = \frac{1}{8}ag(1 + 24\cos\beta)$. [5]

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OR

A large number of people attended a course to improve the speed of their logical thinking. The times taken to complete a particular type of logic puzzle at the beginning of the course and at the end of the course are recorded for each person. The time taken, in minutes, at the beginning of the course is denoted by x and the time taken, in minutes, at the end of the course is denoted by y. For a random sample of 9 people, the results are summarised as follows.

$$\Sigma x = 45.3$$
 $\Sigma x^2 = 245.59$ $\Sigma y = 40.5$ $\Sigma y^2 = 195.11$ $\Sigma xy = 218.72$

Ken attended the course, but his time to complete the puzzle at the beginning of the course was not recorded. His time to complete the puzzle at the end of the course was 4.2 minutes.

(i)	By finding, showing all necessary working, the equation of a suitable regression line, find an estimate for the time that Ken would have taken to complete the puzzle at the beginning of the course. [5]

The values of x - y for the sample of 9 people are as follows.

0.2 0.8 0.5 1.0 0.2 0.6 0.2 0.5 0.8

The organiser of the course believes that, on average, the time taken to complete the puzzle decreases between the beginning and the end of the course by more than 0.3 minutes.

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