

Cambridge International AS & A Level

CANDIDATE NAME					
CENTRE NUMBER			CANDIDATE NUMBER		

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FURTHER MATHEMATICS

9231/23

Paper 2 Further Pure Mathematics 2

October/November 2020

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has 16 pages. Blank pages are indicated.

	By differentiating e^{-x^2} , find the Maclaurin's series for e^{-x^2} up to and including the term in
	1
b)	Deduce an approximation to $\int_0^{\frac{1}{5}} e^{-x^2} dx$, giving your answer as a rational fraction in its low terms.

2	The variables x	and y are i	related by the	differential	equation
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by the differential equation
$$9\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + y = 3x^2 + 30x.$$

(a)	Find the general solution for y in terms of x .	[6]
(b)	State an approximate solution for large positive values of x .	[1]

[2]

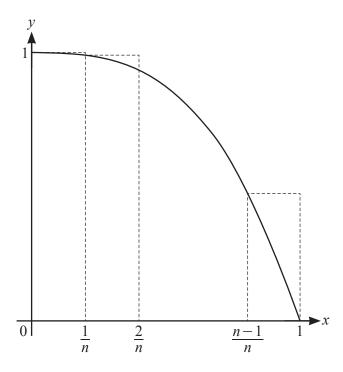
3 (a)	Show	that	the	system	of o	equations
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	x-2y-4z=1,	
	x - 2y + kz = 1,	
	-x+2y+2z=1,	
	where k is a constant, does not have a unique solution.	[2]
(b)	Given that $k=-4$, show that the system of equations in part (a) is consistent. Interpret t situation geometrically.	this [3]

Given that $k=-4$, show that the system of equations in part (a) is consistent. Interpret this situation geometrically.

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	where $k \neq 1$ is situation			w that the s	ystem of equ	ations in pa	art (a) is in	onsi
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The diagram shows the curve with equation $y = 1 - x^3$ for $0 \le x \le 1$, together with a set of n rectangles of width $\frac{1}{n}$.

(a) By considering the sum of the areas of the rectangles, show that

$\int_0^1 (1 - x^3) \mathrm{d}x \le \frac{3n^2 + 2n - 1}{4n^2}.$	[4]

(b) Use a similar method to find, in terms of n , a lower bound for $\int_0^1 (1-x^3) dx$. [4]		
	(b)	Use a similar method to find, in terms of n , a lower bound for $\int_0^1 (1-x^3) dx$. [4]

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$$x = \sinh^{-1}t, \quad y = \cos^{-1}t,$$

where -1 < t < 1.

that $\frac{\mathrm{d}y}{\mathrm{d}t} = -\frac{1}{\sqrt{1-t^2}}$.	$\frac{d}{dt}$	spect to t , sho	ung cosy with i	by differentiating

Find $\frac{d^2y}{dx^2}$ in terms of t, simplifying your answer.	

Use de Moivre's theorem to show that $\sin^4 \theta = \frac{1}{8}(\cos 4\theta - 4\cos 2\theta + 3)$.	

Find the solution of the differential equation				
	$\frac{\mathrm{d}y}{\mathrm{d}\theta} + y\cot\theta =$	$=\sin^3\theta$		
for which $y = 0$ when θ	$=\frac{1}{2}\pi$.		[6	

7 The matrix **P** is given by

$$\mathbf{P} = \begin{pmatrix} 1 & 4 & 2 \\ 0 & -1 & 1 \\ 0 & 0 & 2 \end{pmatrix}.$$

(a)	State the eigenvalues of P .	[1]
		•••••
(b)	Use the characteristic equation of \mathbf{P} to find \mathbf{P}^{-1} .	[4]

The 3×3 matrix **A** has distinct eigenvalues b, -1, 1 with corresponding eigenvectors

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 4 \\ -1 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix},$$

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spectively.			
) Find \mathbf{A} in terms of b .			[4]
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()	Sketch the graph of $y = \coth x$ for $x > 0$ and state the equations of the asymptotes.	
(b)	Starting from the definitions of coth and cosech in terms of exponentials, prove that	
	$\coth^2 x - \operatorname{cosech}^2 x = 1.$	
		•••••
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The	e curve C has equation $y = \ln \coth(\frac{1}{2}x)$ for $x > 0$.	
(c)	Show that $\frac{dy}{dx} = -\operatorname{cosech} x$.	[3]
(d)	It is given that the arc length of C from $x = a$ to $x = 2a$ is $\ln 4$, where a is a positive	e constant.
	Show that $\cosh a = 2$ and find, in logarithmic form, the exact value of a .	[7]

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