

# Cambridge International AS & A Level

CANDIDATE NAME					
CENTRE NUMBER			CANDIDATE NUMBER		

# 7729284444

### **FURTHER MATHEMATICS**

9231/22

Paper 2 Further Pure Mathematics 2

October/November 2020

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

### **INSTRUCTIONS**

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

### **INFORMATION**

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has 16 pages. Blank pages are indicated.

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Find, in terms	of $\pi$ and e, the	$ \cosh x $ , for $0 \le $ the area of the s	$x \le \frac{1}{2}$ . Surface generate	d when the curv	ve is rotated thr	ough 2π ra
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dy	. 2	_	<sub>2</sub> X
$x \frac{d}{dx}$	+2y	=	e

for which $y = 3$ when $x = 1$ . Give your answer in the form $y = f(x)$ .	[8]

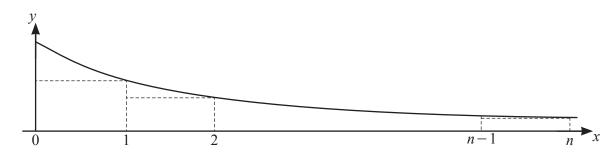
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5	The	curve C has equation	
		$y^2 + (xy+1)^2 = 5.$	
	(a)	Show that, at the point (1,1) on C, $\frac{dy}{dx} = -\frac{2}{3}$ .	[3
			•••••
	(b)	Find the value of $\frac{d^2y}{dx^2}$ at the point (1,1).	[5

	$\frac{d^2x}{dt^2} + 8\frac{dx}{dt} + 15x = 102\cos 3t,$
given that, when $t = 0$ , $x = 1$ and	$\frac{\mathrm{d}x}{\mathrm{d}t} = 0.$


Show that $\sum_{r=1}^{\infty} z^{2r} = \frac{z^{2n+1} - z}{z - z^{-1}}$ , for $z \neq 0, 1, -1$ .	

$1 + 2\sum_{r=1}^{n} \cos\left(2r\theta\right)$	$=\frac{\sin(2n+1)\theta}{\sin\theta}.$	

8



The diagram shows the curve  $y = \frac{1}{\sqrt{x^2 + x + 1}}$  for  $x \ge 0$ , together with a set of *n* rectangles of unit width. By considering the sum of the areas of these rectangles, show that

$\sum_{r=1}^{n} \frac{1}{\sqrt{r^2 + r + 1}} < \ln\left(\frac{1}{3} + \frac{2}{3}n + \frac{2}{3}\sqrt{n^2 + n + 1}\right).$	[10]
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- 9 It is given that a is a positive constant.
  - (a) Show that the system of equations

$$ax + (2a+5)y + (a+1)z = 1,$$
  
 $-4y = 2,$   
 $3y-z = 3,$ 

has a unique solution and interpret this situation geometrically.	[3]

The matrix  $\mathbf{A}$  is given by

$$\mathbf{A} = \begin{pmatrix} a & 2a+5 & a+1 \\ 0 & -4 & 0 \\ 0 & 3 & -1 \end{pmatrix}.$$

<b>(b)</b>	Show that the eigenvalues of <b>A</b> are $a$ , $-1$ and $-4$ .	[2]
(c)	Find a matrix <b>P</b> such that	
	$\begin{pmatrix} a & 0 & 0 \end{pmatrix}$	
	$\mathbf{A} = \mathbf{P} \begin{pmatrix} a & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -4 \end{pmatrix} \mathbf{P}^{-1}.$	[5]
	( <b>V V I</b> )	
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## **Additional Page**

must be	clearly showr	1.			(s), the question	
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