

Cambridge International AS & A Level

CANDIDATE NAME						
CENTRE NUMBER				CANDIDATE NUMBER		

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FURTHER MATHEMATICS

9231/22

Paper 2 Further Pure Mathematics 2

May/June 2020

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has 16 pages. Blank pages are indicated.

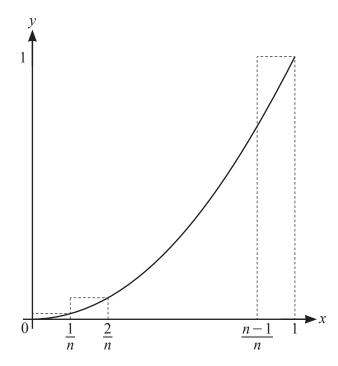
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1	Find	the	solution	of the	differential	equation

$\frac{\mathrm{d}y}{\mathrm{d}x} + 5$	$iy = e^{-7x}$
for which $y = 0$ when $x = 0$. Give your answer in t	the form $y = f(x)$. [6]

(a)	By differentiating $\ln y$ with respect to x, show that $\frac{dy}{dx} = 2^x \ln 2$.	[3]
(b)	Write down $\frac{d^2y}{dx^2}$.	[1]
(c)	Hence find the first three terms in the Maclaurin's series for 2^x .	[3]

	$\leq \theta < 2\pi$.					
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<i>v</i> =	$=z_1^{3k}+z_2^{3k}+z_3^{3k}$, who	ere k is a positive	e integer and z_1	z_1, z_2, z_3 are the re	oots of $z^3 = -1$	-i.
	press w in the form	$Re^{i\alpha}$, where R	> 0, giving R a	nd α in terms of	`k.	
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The diagram shows the curve with equation $y = x^2$ for $0 \le x \le 1$, together with a set of n rectangles of width $\frac{1}{n}$.

(a) By considering the sum of the areas of these rectangles, show that

$\int_0^1 x^2 \mathrm{d}x < \frac{2n^2 + 3n + 1}{6n^2}.$	[4]

(b)	Use a similar method to find, in terms of n , a lower bound for $\int_0^1 x^2 dx$. [4]

Find the exact value of a , giving your answer in logarithmic form.	
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considering $\frac{d}{dx}\left(x(1-x^2)^{-\frac{1}{2}n}\right)$, or otherwise, show that	
$nI_{n+2} = 2^{n-1}3^{-\frac{1}{2}n} + (n-1)I_n.$	[5]
	considering $\frac{1}{dx} \left(X(1-X) \right)$, of otherwise, show that $nI_{n+2} = 2^{n-1} 3^{-\frac{1}{2}n} + (n-1)I_n.$

determined.	[

7 It is given that $x = t^3 y$ and

$$t^{3} \frac{d^{2} y}{dt^{2}} + (4t^{3} + 6t^{2}) \frac{dy}{dt} + (13t^{3} + 12t^{2} + 6t)y = 61e^{\frac{1}{2}t}.$$

	di .
(a)	Show that
	$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 13x = 61e^{\frac{1}{2}t}.$ [4]

	3x + y + z = 0,	
	ax + 6y - z = 0,	
	ay - 2z = 0,	
	does not have a unique solution.	I
he	matrix A is given by	
	$\mathbf{A} = \begin{pmatrix} 3 & 1 & 1 \\ 0 & 6 & -1 \\ 0 & 0 & -2 \end{pmatrix}.$	
	$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & -2 \end{pmatrix}$	
o)	Use the characteristic equation of A to find the inverse of A^2 .	
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Find a matrix P and a diagonal matrix D such that $\mathbf{A}^5 = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}$.	[7]
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Additional Page

If you use the following fined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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