

Cambridge International AS & A Level

CANDIDATE NAME					
CENTRE NUMBER			CANDIDATE NUMBER		

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FURTHER MATHEMATICS

9231/22

Paper 2 Further Pure Mathematics 2

May/June 2022

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has 16 pages.

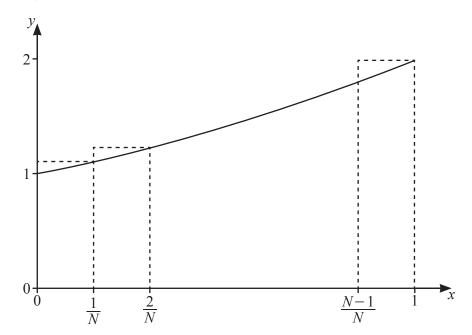
Given that the length of C is s , find α in terms of s .	[:

(a)	Starting from the definitions of cosn and sinn in terms of exponentials, prove that	
	$\cosh 2x = 2\sinh^2 x + 1.$	[3]
(b)	Find the set of values of k for which $\cosh 2x = k \sinh x$ has two distinct real roots.	[5]

		$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + \frac{\mathrm{d}x}{\mathrm{d}t} + x = t^2 + 1.$	
(a)	Find the general solution for x is	in terms of t .	[6
			• • • • • • • •

			•••••
1. N	Deduce an approximate value of	f $\frac{d^2x}{dt^2}$ for large positive values of t.	[2]
D)	Beaute an approximate variety	1.2 for large positive values of i.	
D)	Deduce an approximate value of	dt^2	
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4 The diagram shows the curve with equation $y = 2^x$ for $0 \le x \le 1$, together with a set of N rectangles each of width $\frac{1}{N}$.



(a) By considering the sum of the areas of these rectangles, show that $\int_0^1 2^x dx < U_N$, where

1	J 0	
$2\frac{1}{N}$		
$II = Z^N$		Γ <i>1</i> 1
$U_N = \frac{2}{N\left(2^{\frac{1}{N}} - 1\right)}.$		141
$N = \frac{1}{\sqrt{2}}$		
N(2N-1)		
1 · _ 1/		

Find the least value of N such that $U_N - L_N < 10^{-4}$.	

	$(x+1)y + (x+y+1)^3 = 1.$	
(a)	Show that $\frac{dy}{dx} = -\frac{3}{4}$ when $x = 0$.	[:
		•••••
		•••••
<i>a</i> .		
(b)	Find the Maclaurin's series for y up to and including the term in x^2 .	[
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Use the substitution y = vx to find the solution of the differential equation

6

		[10
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	12			
7	(a)	Use de Moivre's theorem to show that		
		$\csc 7\theta = \frac{\csc^7 \theta}{7 \csc^6 \theta - 56 \csc^4 \theta + 112 \csc^2 \theta - 64}.$		
		cosec /θ =	$\frac{1}{7\operatorname{cosec}^6\theta - 56\operatorname{cosec}^4\theta + 112\operatorname{cosec}^2\theta - 64}$	[6

(b) Hence obtain the roots of the equation

7	$14x^6 +$	1124	224-2	1 120	_ 0
x' —	$14x^{\circ} +$	112x ⁻ -	- 224x²	+128	= ()

in the form $\csc q\pi$, where q is rational.	[5]

a)	Find the value of a for which the system of equations	
	3x + ay = 0,	
	5x-y = 0,	
	x + 3y + 2z = 0,	
	does not have a unique solution.	
		•••••
Γhe	matrix A is given by	
	$\mathbf{A} = \begin{pmatrix} 3 & 0 & 0 \\ 5 & -1 & 0 \\ 1 & 3 & 2 \end{pmatrix}.$	
	\1 3 2/	
(b)	$1 3 2$ Find a matrix P and a diagonal matrix D such that $\mathbf{A}^2 = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}$.	
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Additional page

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