

Two-phase model

The two-dimensional two-phase model of water and air is given by

$$\partial_t u + \partial_x uu + \partial_z wu - fv = -(\partial_x p)/\rho \quad (1)$$

$$\partial_t v + \partial_x uv + \partial_z wv + fu = 0 \quad (2)$$

$$\partial_t w + \partial_x uw + \partial_z ww = -(\partial_z p)/\rho - g \quad (3)$$

$$\partial_x u + \partial_z w = 0 \quad (4)$$

which holds both in water with $\rho = \rho_o$ and air with $\rho = \rho_a$, and $\rho = c\rho_o + (1 - c)\rho_a$.

Solve this with two level discrete time step. Knowing u^n and w^n from previous time step, first calculate intermediate solution u^* and w^* from

$$\frac{u^* - u^n}{\Delta t} = -(\partial_x uu + \partial_z wu)_d, \quad \frac{w^* - w^n}{\Delta t} = -(\partial_x uw + \partial_z ww)_d - g, \quad (5)$$

Then take divergence and calculate pressure p such that

$$\partial_z u^* + \partial_z w^* = -\Delta t(\partial_x(\partial_x p)/\rho + \partial_z(\partial_z p)/\rho) \quad (6)$$

This is solved for p with conjugate gradient solver with preconditioner, and so

$$u^{n+1} = u^* - \Delta t(\partial_x p)/\rho, \quad w^{n+1} = w^* - \Delta t(\partial_z p)/\rho \quad (7)$$

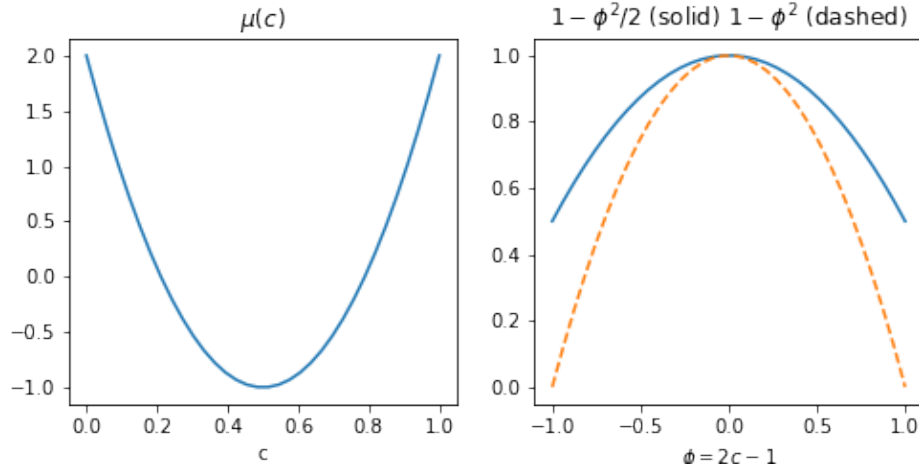
such that next time step u^{n+1} and w^{n+1} is also free of divergence.

Concentration c with $c = 1$ in water and $c = 0$ in air given by

$$\partial_t c + \partial_x uc + \partial_z wc = \partial_z M(c) \partial_z \mu, \quad 0 < c < 1 \quad (8)$$

with chemical potential μ and mobility parameter M given by

$$\mu = 12c^2 - 12c + 2, \quad M_c(1 - \gamma\phi^2), \quad \phi = 2c - 1, \quad -1 < \phi < 1 \quad (9)$$



and $M_c = 10^{-4}$

Discretisation

Discretisation is on a C-grid.

Options

The configuration is controlled in the template subroutines `SET_PARAMETER` and `INITIAL_CONDITIONS`. One example is provided.

A number of switches can be set to either true or false.

name	default	meaning
<code>enable_upwind3_advection</code>	false	
<code>enable_dst3_advection</code>	false	
<code>enable_superbee_advection</code>	false	
<code>enable_multidim_advection</code>	false	
<code>enable_AB3_time_stepping</code>	false	
<code>enable_particles</code>	false	
<code>enable_v_velocity</code>	false	