Project Proposal - Iterative Solvers for Linear Systems
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Introduction:
Solving a linear system in the form of Ax=b is a standard yet computationally intensive step in many algorithms. Specifically, if A is an n×n matrix, solving by direct inversion results in a computational cost of $O(n^3)$, drastically increasing the resources needed for the algorithm to run. As a common practical work-around, it is quite useful to employ iterative solvers to generate an approximate solution to the system, as this process is much less computationally expensive. In this project, we will aim to investigate a few of those iterative solvers with broad applications, discussing their inherent potentials and limitations.

Methods of Presentation & Collaboration:
On a high level, the project can be split into two parts. First, in the 'introductory material' portion, we will focus on two specific iterative solvers. Next, in the 'independent study' portion, we diverge from the pre-written material to explore related topics that we find interesting. For the introductory material, we will collaborate on code and derivations using a GitHub repository (link - https://github.com/cedergrund/4600IterativeSolvers). We will also include in the same repository our work for the independent study portion of the project; thus, all of the work/results will be consolidated in one place when moving on to the writing of the report. The draft and final project reports will be written in LaTeX and include all results from the GitHub Repo for the introductory material and independent study. We list more specifics of the two parts in the sections below.

In addition to using text messages to communicate, we will also collaborate through the use of group meetings. We will meet once weekly for at least 30 minutes to review project updates and future plans. These meetings may also include working through some more challenging portions of the project together. Furthermore, we will plan meetings before each deadline listed in the 'timelines for milestones' table below to reflect and address project advancements. The "navigational milestone" meetings with a TA/Instructor will also be included as directed.

Introductory Material:
The introductory material, phrased as 'Questions to Investigate' on the project write-up, investigates Richardson's Iteration and Generalized Minimal Residual (GMRES) iteration. We will split the work of this material evenly. Both members will do parts of each iteration technique in order to get a holistic understanding of the introductory material before continuing to the independent portion. However, to ensure that both parties have an input in the final product, we say that Gustav will oversee work for Richardson's Iteration, and Kevin will oversee work for the GMRES iteration. Using Git, we will both be able to work on the same code and generate the same output. All work for this portion of the project will be included in the 'introductory_material' folder in the GitHub repository.

Independent Extension:
The independent extension of the project requires project members to investigate deeper another topic related to iterative solvers. For our project, the group will focus on iterative techniques for approximating eigenvalues. Additionally, we will discuss some of the real-world applications of

these iterative techniques. Below, we specify the direction for each group member regarding the independent extension.

Gustav will be investigating and reporting on two iterative methods for approximating eigenvalues - the Power Method and the QR iteration. The Power Method is intrinsic to many eigenvalue iteration techniques and is useful for approximating a single eigenvalue. This method can also be extended simply to derive more powerful iterations, such as the Rayleigh-Ritz and the Shifted Inverse Power methods. The QR Iteration is a more complex extension of the Power Method, and it is able to approximate all eigenvalues of the matrix instead of just one. Gustav will compare the performance and applicability of the two methods.

Kevin will be investigating and reporting on some of the popular applications of eigenvalue iteration techniques. For example, one interesting application of eigenvalues methods is the PageRank algorithm from the Google search engine. While this algorithm is not currently used by Google, at the time of its implementation, it was the state-of-the-art technique for generating search results quickly. Eigenvalues can also be used for data compression; popular methods include the truncated singular value decomposition, where the square root of the largest eigenvalues is used to represent the most important elements of a matrix in a fraction of the original matrix's size.

All work for this portion of the project will be included in the 'independent_extention' folder in the GitHub repository.

Timelines for Milestones:

Project Proposal Due Date	**Fri., October 29, 2023**
Richardson's Iteration 'Questions to Investigate' Complete (Gustav)	Sun., November 5, 2023
Project Proposal Revision Due Date	**Fri., November 10, 2023**
GMRES 'Questions to Investigate' Complete (Kevin)	Fri., November 17, 2023
Independent Ext.: Methods Code + derivation Complete (Gustav)	Fri., November 17, 2023
Finished rough draft writing on Latex document for 'Introductory Material' (mathematical formulation + numerical results) and 'Abstract' and 'Introduction' sections	Sun., November 26, 2023
'Mathematical Formulation for Independent Extension' + 'Discussion' Section on Latex document complete	Fri., December 1, 2023
'Rough Draft' Due Date	**Fri., December 1, 2023**
Group Meeting to discuss polishing of report	After feedback for Rough Draft

‘Mathematical Formulation for Independent Extension’ Section and ‘Conclusion’ section on Latex document complete	Sun., December 10, 2023
Meet with instructor to inspect report	Mon., December 11, 2023
Final report Due Date	**Tues., December 19, 2023**

***Notes**: Bold milestones/dates are official due dates. Dates and milestones may be subject to change or grow as we work through the project. This timeline will be included in the repository and will continuously be updated with completed and current objectives - specified as such.*