Project Proposal - Iterative Solvers for Linear Systems

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Introduction:

Solving a linear system in the form of Ax=b is a standard yet computationally intensive step in many algorithms. Specifically, if A is an n×n matrix, solving by direct inversion results in a computational cost of $O(n^3)$, drastically increasing the resources needed for the algorithm to run. As a common practical work-around, it is quite useful to employ iterative solvers to generate an approximate solution to the system, as this process is much less computationally expensive. In this project, we will aim to investigate a few of those iterative solvers with broad applications, discussing their inherent potentials and limitations.

Methods of Presentation & Collaboration:

On a high level, the project can be split into two parts. First, in the 'introductory material' portion, we will focus on two specific iterative solvers. Next, in the 'independent study' portion, we diverge from the pre-written material to explore related topics that we find interesting. For the introductory material, we will collaborate on both code and derivations through the use of a GitHub repository (link - https://github.com/cedergrund/4600IterativeSolvers). We will also include in the same repository our work for the independent study portion of the project; thus, all of the work/results will be consolidated in one place when moving on to the writing of the report. The draft and final project reports will be written in LaTeX and include all results from the GitHub Repo for the introductory material and independent study. We list more specifics of the two parts in the sections below.

In addition to using text messages to communicate, we will also collaborate through the use of group meetings. We will meet once weekly for at least 30 minutes to review project updates and future plans. These meetings may also include working through some more challenging portions of the project together. Furthermore, we will plan meetings before each deadline listed in the 'timelines for milestones' table below to reflect and address project advancements. The "navigational milestone" meetings with a TA/Instructor will also be included as directed.

Introductory Material:

The introductory material, phrased as 'Questions to Investigate' on the project write-up, investigates Richardson's Iteration and GMRES iteration. We will split the work of this material evenly. Both members will do parts of each iteration technique in order to get a holistic understanding of the introductory material before splitting off into the independent portion. However, to ensure that both parties have an input in the final product, we say that Gustav will oversee work for Richardson's Iteration, and Kevin will oversee work for the GMRES iteration. Using Git, we will both be able to work on the same code and generate the same output. All work for this portion of the project will be included in the 'introductory_material' folder in the GitHub repository.

Independent Extension:

The Independent extension of the project requires each member to investigate their own topic related to iterative solvers. Both project members are interested in the possibility of working on a practical connection to iterative solvers' usage in applications; however, before committing to that direction, it would be helpful to have more information about such applications from the professor. For now, Gustav will be investigating eigenvalues, and Kevin will be investigating additional iterative solvers. Below, we specify the direction for each group member regarding the independent extension.

Gustav will be investigating and reporting on different iterative methods for approximating eigenvalues. The methods that will be considered are the power method and its applications. The Power Method is intrinsic to many eigenvalue iteration techniques. Thus, Gustav will extend this technique to derive more powerful and complex iterations such as the Rayleigh-Ritz (Rayleigh Quotient) method and Lanczos algorithm, comparing their performance and applicability. Also being considered, perhaps in addition to the previous methods, is the QR Iteration/algorithm. This is again an extension of the Power Method, but it can be improved on through the implementation of pre-conditioners as explored in the introductory material. Gustav would appreciate some direction regarding the "entertainment factor" of the previously listed methods, as determined by instructor discretion. He will make due even if the entertainment factor is low, but in numerical analysis, of course, such an occurrence is an impossibility.

Kevin will be investigating and reporting on the Biconjugate Stabilized Gradient (BiCSTAB) method. BiCSTAB is a general iterative method that can be used to solve linear systems. Its development was inspired by an improvement on the Biconjugate method, which is also a method that uses Krylov subspaces. We are especially interested in seeing how this method compares to methods previously implemented in the introductory portion of this project. Further, seeing how the author's version of the software stacks up against popular numerical Python methods will be quite enlightening.

All work for this portion of the project will be included in the 'independent_extention' folder in the GitHub repository.

Timelines for Milestones:

Project Proposal	Sunday, Oct. 29, 2023
Work and Writing Complete for 'Introductory Material'	Wed, Nov. 8, 2023
Elementary Results for Independent Project + Rough Draft	Sat., Nov. 18, 2023
Final report	Week before Final

***Note**: Dates and milestones may be subject to change or grow as we work through the project. This timeline will be included in the repository and will continuously be updated with completed and current objectives - specified as such.*