	APPM 4600 - HW #4 Gostav Cedeg and
	1) Assure temp. in (T(x,t) at districe x meters below surface "t' seconds
	aller cold snap salokes
	aller cold Snap salshes $\frac{T(x,t)-Ts}{T:-Ts}=erf\left(\frac{x}{2\sqrt{aE}}\right)$
	Let Ti=20°C, Ts=-15°C, a= 0.138.10-6 m 2/s
	a) How steep should whe man be burned so only tocere after 60 days?
	The , we mat T(x, t) = 0°C at t=60.24.602=5184000.
-	1 x 1 x = 0 at t=5184000
	f(x)=1T(x, b)=1(T;-Ts) or t(2/00) + Ts = 0 at t=5184000
	(1) - O a washing a red-fallow and on
	st f(x)=0 is simply a roof-hally problem of
	1. 1/(V)= (T:-TS). E. 8/20081 " TET
	(1'(x) - (T;-Ts)) Tate e 40t Plot for f(x) affailed
	b) Using bisection if study values as = 0 and bo = x (x chosen at 1)
	b) Using sisection of strong vaccy as =0 and 50 = x [x emert at]
	ne get not approximed at x=0.676.9618544819309 m
	(Fill offput attached)
	c) Umy rentry method il startay vales x = 0.01m and x = x = 2m,
	ve get an approximate depth of
	x=0.6769618544819365 (9366 to- x)
	Bith converge after 4 storatory, any saylor than that we
	breeton nethod, elen though it produces a smaller error.
	(Full output attacked)

Problem 1:

$$f(x) = (T_i - T_s) \cdot erf\left(\frac{x}{2\sqrt{\alpha \cdot t}}\right) + T_s$$

5

-10

-15

0.0

0.2

0.4

0.6

0.8

1.0

```
| bisection method:
| iteration: 1 | curr_root = 0.75 |
| iteration: 2 | curr_root = 0.625 |
| iteration: 3 | curr_root = 0.625 |
| iteration: 4 | curr_root = 0.6875 |
| iteration: 5 | curr_root = 0.6796875 |
| iteration: 6 | curr_root = 0.6796875 |
| iteration: 7 | curr_root = 0.67978125 |
| iteration: 8 | curr_root = 0.677734375 |
| iteration: 9 | curr_root = 0.677734375 |
| iteration: 10 | curr_root = 0.677734375 |
| iteration: 11 | curr_root = 0.677734578 |
| iteration: 12 | curr_root = 0.67704693375 |
| iteration: 13 | curr_root = 0.676978828125 |
| iteration: 14 | curr_root = 0.67694891796875 |
| iteration: 15 | curr_root = 0.676961767578125 |
| iteration: 16 | curr_root = 0.676961767578125 |
| iteration: 15 | curr_root = 0.676951435546875 |
| iteration: 16 | curr_root = 0.6769516767578125 |
| iteration: 17 | curr_root = 0.67695161523438 |
| iteration: 18 | curr_root = 0.6769618988837109 |
| iteration: 19 | curr_root = 0.676961888838139 |
| iteration: 20 | curr_root = 0.6769618988837109 |
| iteration: 21 | curr_root = 0.6769618598185131 |
| iteration: 22 | curr_root = 0.6769618598185131 |
| iteration: 23 | curr_root = 0.6769618598185180 |
| iteration: 24 | curr_root = 0.676961859808274 |
| iteration: 25 | curr_root = 0.676961859808274 |
| iteration: 26 | curr_root = 0.6769618578255177 |
| iteration: 27 | curr_root = 0.6769618578255177 |
| iteration: 28 | curr_root = 0.676961854318493 |
| iteration: 30 | curr_root = 0.6769618544893125 |
| iteration: 31 | curr_root = 0.6769618544893125 |
| iteration: 32 | curr_root = 0.6769618544893125 |
| iteration: 33 | curr_root = 0.6769618544893125 |
| iteration: 34 | curr_root = 0.6769618544893125 |
| iteration: 35 | curr_root = 0.6769618544893125 |
| iteration: 36 | curr_root = 0.6769618544893125 |
| iteration: 37 | curr_root = 0.6769618544893125 |
| iteration: 38 | curr_root = 0.6769618544893125 |
| iteration: 40 | curr_root = 0.6769618544893125 |
| iteration: 41 | curr_root = 0.6769618544819309 |
| iteration: 42 | curr_root = 0.6769618544819309 |
| iteration: 43 | c
```

(c):

newton's method:

initial guess at x=0.01 meters Found solution after 4 iterations. the approximate root is 0.6769618544819365 f(root) = -5.329070518200751e-15

initial guess at x=1 meters
Found solution after 4 iterations.
the approximate root is 0.6769618544819366
f(root) = 0.0

2) a) root a has multiplicity in at weeker f(x) great that Alx ca be unsu flx = lx- glx) b) For flx) w/ m/tp/resty m at root a, role thete

(a) = f'(a) = - = pm-1(a) = 0 , f(a) = 0 is Taylor by t(x) at root d

Lo f(x) = 0+ ... + 0+ fm(x) (x-a) + (m+1)! (x-a) m+1

(x-a) m+1 Sit (x) = f (x) m (x-a) 1 1 1 (mr) (x-2) enor lem XILAI = XIL - F(XIL) => XILI-d = XIL-d - F(XIL) cons from Xxx-d= (xx-a) Kurl-2 = (ku=d) (1-m) XILTI = m-1 <1 : Iner conserve c) It we insted use fixed port of g(x) = x -m f(x) $Xu_1 = g(xu) = Xu - m \frac{f(xu)}{f'(xu)} => Xu_1 - a = Xu - a - m \frac{f(xu)}{f'(xu)}$ plugging on the same Taylor expansions from (b), he have

XXLI-d= XX-d-m (+1)(x-a)m+ fmile(En)(x+1)(x-a)m

XXLI-d= XX-d-m (+1)(x-a)m+ fmile(En)(x+1)(x-a)m

(x-a)m+ fmile(x-a)m+ fmile(x-a)m+ fmile(x-a)m

(x-a)m+ fmile(x-a)m+ fmile(x-a)m+ fmile(x-a)m

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(x-a)m+ fmile(x-a)m+ fmile(x-a)m+ fmile(x-a)m

(x-a)m+ fmile(x-a)m+ fmil XW1-4 = X 11-4 = X (X/11-4) = X/11-X/1+4-4 = 0 \$1 } not men greete the her guardes second-orde convince. d) part (1) provides a mediked method of Neuton's that tueps arele of convergence as gradulic even it the original Lucken his a multiplicity.

3) let {xx} = be a seque that convoyed to d By defution, [XII] > x II order p given 3 x III Ochcl sit Then when he is sullarally large, Xxxx - a) = 1 | Xx - a|P log (|xxx - a|) = log () |xx-n|p) = log () + log (|xx-a|p) log(ken-al) = log(1) + plog(lxn - al)

relating this to slope equation y= mx+b,

we see that the relationship blue log(1xn+1-al) and log (|xn-al) is mean w/ order of convergece o bury the slope and log(1) as y-mbrupt when h is Whiely loge. 4) Usmy Hx)= e3x-27x4+27x4ex-9x2e2x w/ the blue rethors we can see dilivet order of consporte. I modelled conspect using the literary methods through coding (code + ortest attacked) and saw that (1) coneges thereby write (11) and (111) conege closer to smelmetically stightly father for (21).

I preter nethod (111) ble you don't need to calculate f'(x) and lawyere is still very guick. 5) a) Plotting error by stention (output attached) we see that error does decrese as expected, just a lot shows ut second melled then w/ Neuto's b) As was shown in (3), the slopes of these lines are the order of conveyace p'. Newbord nettod has a steeper plot than secont nellal, showing Eight arts of convergue.

Problem 4:

```
lambda x: np.exp(3 * x)
        + 27 * x**4 * np.exp(x)
        - 9 * x**2 * np.exp(2 * x)
        lambda x: 3 * np.exp(3 * x)
        + (-18 * x**2 - 18 * x) * np.exp(2 * x)
        + (27 * x**4 + 108 * x**3) * np.exp(x)
         - 162 * x**5
 DDf = (
      lambda x: 9 * np.exp(3 * x)
+ (-36 * x**2 - 72 * x - 18) * np.exp(2 * x)
+ (27 * x**4 + 216 * x**3 + 324 * x**2) * np.exp(x)
        - 810 * x**4
tol = 1e-13
x0 = 3
 m = 3
                                                                                                                                                                       (i):
newton's method with initial guess at x=3
print("(i): \newton's method with initial guess at x=3\n")
[astar1, iter1] = newton(f, Df, x0, tol, max_iter=1000)
print("the approximate root is", astar1)
print("f(root) =", f(astar1))
                                                                                                                                                                       Found solution after 39 iterations. the approximate root is 3.7330596890832597 f(root) = 0.0  
Order of convergence evaluated with alpha = 1: [0.50757867 0.74893505 0.73290845 0.71833404 0.70574058 0.69540162 0.6873103 0.65124106 0.67685341 0.67378749 0.67172564 0.67042018 0.66969935 0.66946391 0.66946278 0.670429345 0.67170752 0.67382757 0.67074891 0.68190559 0.68898273 0.08555116 0.71340202 0.7266279 0.73656509 0.80890729 0.94654482 0.72319919 0.57718158 45.44962427 0.65812179 0.65390651 0.64732316 0.63656239 0.61948291 0.58897791 0.53735942 0.47459956 0.
print("order of convergence evaluated with alpha = 1:")
print(orderOfConvergence(astar1, iter1, 1))
print("(ii):\nmodified newton's method from class x0=3:\n")
 \begin{array}{l} mu = lambda \ x: \ f(x) \ / \ Df(x) \\ Dmu = lambda \ x: \ (Df(x) \ * \ Df(x) \ - \ f(x) \ * \ DDf(x)) \ / \ (Df(x) \ ** \ 2) \\ \end{array} 
[astar2, iter2] = newton(mu, Dmu, x0, tol, max_iter=1000) print("the approximate root is", astar2)
                                                                                                                                                                        (ii):
modified newton's method from class x0=3:
print("f(root) =", f(astar2))
print("order of convergence evaluated with alpha = 2:")
print(orderOfConvergence(astar2, iter2, 2))
                                                                                                                                                                        Found solution after 6 iterations.
the approximate root is 3.733078868957922
f(root) = 0.0
order of convergence evaluated with alpha = 2:
[1.10037411 1.07697453 1.02581073 0.96211941 0.92750855 0.
print("\n")
\label{lem:print("(iii):\nmodified newton's (fixed point) method from (2)\nx0=3 and m=3:\n")} \\
g = lambda x: x - m * f(x) / Df(x)
[astar3, _, iter3] = fixedpt(g, x0, tol, Nmax=1000)
print("Found solution after", len(iter3), "iterations.")
print("the approximate root is", astar3)
                                                                                                                                                                       (iii): modified newton's (fixed point) method from (2) x0=3 and m=3:
                                                                                                                                                                       Found solution after 10 iterations. the approximate root is 3.7330791332651536 f(root) = 0.0 order of convergence evaluated with alpha = 2: [4.88523330 . 26108624 0.33601366 0.45114718 0.61916366 0.80643188 0.90761802 0.
 print("f(root) =", f(astar3))
 print("order of convergence evaluated with alpha = 2:")
 print(orderOfConvergence(astar3, iter3[:-1], 2))
print("\n")
```

Problem 5:

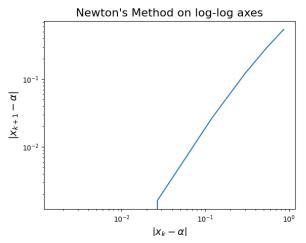
Note: using tolerance 10^-3 # of iterations too large otherwise

newton's method with initial guess at x=2:

Found solution after 6 iterations.
the approximate root is 1.134730528343629 f(root) = 6.573836771295305e-05

secant method with x0=2 and x1=1:
root found after 48 iterations.
the approximate root is 1.1346359946857905

Error with newtons method: 0		
0 0.865269 0.118507 1 0.545898 0.103961 2 0.296008 0.090919 3 0.120240 0.079289 4 0.026808 0.068969 5 0.001623 0.059853 6 0.000000 0.051833 7 0.000000 0.034805 8 0.000000 0.033317 10 0.000000 0.033317 10 0.000000 0.033317 10 0.000000 0.028671 11 0.000000 0.024645 12 0.000000 0.021162 13 0.000000 0.021162 14 0.000000 0.011561 15 0.000000 0.01328 16 0.000000 0.013407 17 0.000000 0.013407 18 0.000000 0.00321 20 0.000000 0.007125 20 0.000000 0.007125 21 0.000000 <td>table with errors:</td> <td></td>	table with errors:	
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f(root) = -0.0009065966333561271

