Lab 5 Prelab - Chart for Root Finding

Method	Input	Iteration	Idea behind	Required for convergence	Pros	Cons
Bisection	f, a, b, tolerance	Check midpoint of a, b to bisect interval	Two points on f either side of x-axis have root b/w them	$f(a) \cdot f(b) < 0$	simple, effective	slow, doesn't work for even multiplicity
Fixed Point	f, $x0$, tolerance	$x_{n+1} = f(x_n)$	Fixed point iteration converges to fixed point given conditions are satisfied.	For fixed point p of f, $f(p) = p$. $f \in C[a,b]$ $0 < f'(x) < 1 \quad \forall x \in (a,b)$	accurate, low computational cost	slow, requires $f'(x) < 1$ around root
Newton's method	f, f', $x0$, tolerance	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$	Derived from Taylor expansion around root α s.t. $f'(x_n) \neq 0$ and $\lvert \alpha - x_n \rvert$ is small.	$f \in C^2[a,b]$ $\exists p \in (a,b)$ s.t. $f(p)=0$, $f'(p) \neq 0$	fast convergence	computationally expensive, need derivative, slower w/ multiplicities
Secant method	f, a, b, tolerance	$x_n = x_{n-1} - \frac{f(x_{n-1})(x_{n-1} - x_{n-2})}{f(x_{n-1}) - f(x_{n-2})}$	f' is expensive, so we can instead use secant lines b/w last 2 iterations in some fashion	$f'(x) \neq 0$ on interval, a, b need to be close enough to root	faster than linear but not like Newton's - computationally inexpensive	might not converge and has no error bounds