1)

```
NEWTONS METHOD:

newton's method with initial guess at [1, 1]
tolerance @ 1e-12

Found solution after 7 iterations.
the approximate root is [-1.81626407  0.8373678 ]
F(root) = [3.81028542e-13 2.55351296e-14]


newton's method with initial guess at [1, -1]
tolerance @ 1e-12

Found solution after 5 iterations.
the approximate root is [ 1.00416874 -1.72963729]
F(root) = [0. 0.]


newton's method with initial guess at [0, 0]
tolerance @ 1e-12

Jacobian at iteration 0 is
 [[0. 0.]
 [1. 1.]]
which is singular. unable to converge.
```

```
BROYDEN'S METHOD:

broyden's method with initial guess at [1, 1]
tolerance @ 1e-12

converged after 13 iterations
the approximate root is [-1.81626407  0.8373678 ]
F(root) = [1.42108547e-14 4.44089210e-15]


broyden's method with initial guess at [1, -1]
tolerance @ 1e-12

converged after 7 iterations
the approximate root is [ 1.00416874 -1.72963729]
F(root) = [-8.88178420e-16  2.22044605e-16]


broyden's method with initial guess at [0, 0]
tolerance @ 1e-12

Solution not found, Jacobian is singular at iteration 0
```

```
LAZY NEWTON'S METHOD:

lazy newton's method with initial guess at [1, 1]
tolerance @ 1e-12

/Users/cedergrund/Documents/fall-2023/APPM4600/homework/hw6/py_files/prob1.py:123: RuntimeWarning: overflow encountered in exp
  F[1] = np.exp(x[0]) + x[1] - 1
Could not converge after 100 iterations.
Solution not found.


lazy newton's method with initial guess at [1, -1]
tolerance @ 1e-12

converged after 44 iterations
the approximate root is [ 1.00416874 -1.72963729]
F(root) = [8.52651283e-13 2.44249065e-15]


lazy newton's method with initial guess at [0, 0]
tolerance @ 1e-12

Jacobian matrix at initial point is singular.
Solution not found.
```

2)

```
NEWTONS METHOD:

newton's method with initial guess at [0, 0, 0]
tolerance @ 1e-06

Found solution after 3 iterations.
the approximate root is [0.         0.10005001 1.00100113]
F(root) = [0.0, 3.128011183406443e-09, 1.285092361413831e-07]



STEEPEST DESCENT:
steepest descent method with initial guess at [0, 0, 0]
tolerance @ 1e-06

Found solution after 5 iterations.
the approximate root is [-6.27724957e-05  9.99685854e-02  9.99984493e-01]
F(root) = [-6.277251540975914e-05, -1.4946447643549021e-05, -0.0010148828015164035]



HYBRID APPROACH:
steepest descent method with initial guess at [0, 0, 0]
tolerance @ 0.05

Found solution after 1 iterations.

now finishing iteration through newtons.
initial guess @ [-0.02001828  0.09005646  0.99526475]
tolerance @ 1e-06

Found solution after 2 iterations.
the approximate root is [-7.67191304e-15  1.00049982e-01  1.00100039e+00]
F(root) = [-7.66053886991358e-15, 1.3029467949010609e-08, -6.118529712884069e-07]
```