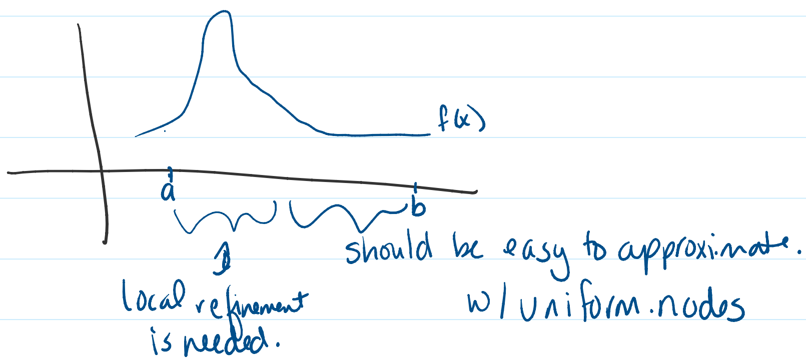
Adaptive quadrature

Friday, October 29, 2021 2:23 PM

What do we do when we need to integrate something complicated?



For a general function, it can be difficult to determine where you need to refine. & how much refinement is needed.

We are going to build a way of determining where to refine.

Let Quad denote the quad I am going to use in each interval. $Q(f(x), I_i)$

We need to determine $I_i = [x_i, x_{i+1}]$ for $i = 1, \dots n-1$

st

$$\left| \int_{-1}^{1} f(x)\,dx - \sum_{i=1}^{n-1} \text{Quad}(f(x), I_i) \right| < \varepsilon = \text{desired accuracy.}$$

Basic idea

let Q_0 denote the approximation of $\int_{I^0} f(x)\,dx$, $I^0 = [a,b]$

We break I^0 into 2 intervals $I^1 = [a, x_{mid}]$ $I^2 = [x_{mid}, b]$

let Q_1 denote the approx on I^1
Q_2 " " I^2

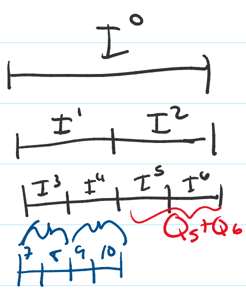
look at $|Q_0 - (Q_1 + Q_2)|$ if it is not less than ε
we need to subdivide I^1 & I^2

We will continue only subividing intervals where
the refinement is needed.

This results in a tree structure for keeping track
of intervals.

Ex of tree

level 0
level 1
level 2



Quad applied on I_2 Quad applied on I^5 & I^6

suppose $|Q_2 - (Q_5 + Q_6)| < \varepsilon$

$\Rightarrow$ No refinement needed
for I^5 & I^6

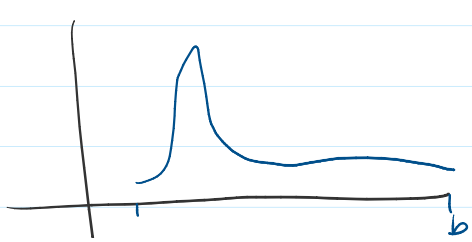
$\rightarrow$ Check if our approximation
is converging.

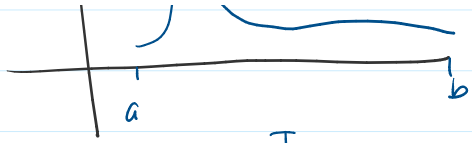
Now look at
① $|Q_3 - (Q_7 + Q_8)|$
& ② $|Q_4 - (Q_9 + Q_{10})|$

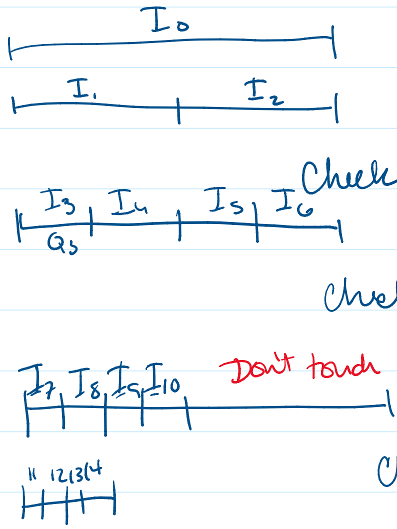
If ① $> \varepsilon$, I^7 & I^8
need to be refined
If ② $> \varepsilon$, I^9 & I^{10}
need to be refined.
etc.

$$\boxed{\text{Relative error option} \quad \frac{|Q_2 - (Q_5 + Q_6)|}{|Q_5 + Q_6|} < \varepsilon}$$

Warm-up: What is adaptive quadrature?







Q_0 = approx w/ quad on I_0

$Q_1 + Q_2$ = appro w/ quad on I_1 & I_2

Check $|Q_0 - (Q_1 + Q_2)| < \varepsilon$ Not good enoug

Check $|Q_1 - (Q_3 + Q_4)| < \varepsilon$ still not good enough.

& $|Q_2 - (Q_5 + Q_6)| < \varepsilon$ ✓ Good.

Check $|Q_3 - (Q_7 + Q_8)| < \varepsilon$ still more refinement necessary.

$|Q_4 - (Q_9 + Q_{10})| < \varepsilon$ (Maybe good

Pretend no more refinement needed ✓

Approx. $I_5 + I_6 + I_9 + I_{10} + \sum_{i=11}^{14} I_i$