APPM 4600 — HOMEWORK # 10

For all homeworks, you should use Python. **Do not use** symbolic software such as Maple or Mathematica.

1. For the function $f(x) = \sin(x)$. Determine the Padé approximations of degree 6 with

   (a) Both the numerator and denominator are cubic

   (b) The numerator is quadratic and the denominator is a fourth degree polynomial.

   (c) The numerator is a fourth degree polynomial and the denominator is quadratic.

   Compare the accuracy of these approximations with the sixth order Maclaurin polynomial by ploting the error over the interval $[0, 5]$.

2. Find the constants x_0, x_1 and c_1 so that the quadrature formula

$$\int_0^1 f(x)dx = \frac{1}{2}f(x_0) + c_1 f(x_1)$$

   has the highest possible degree of precision.

3. (a) Write a code to approximate $\int_{-5}^{5} \frac{1}{1+s^2} ds$ using a composite Trapezoidal rule. To do this, partition the interval $[-5, 5]$ into equally spaced points $t_0, t_1, \ldots, t_n$.
   Write another code to approximate $\int_{-5}^{5} \frac{1}{1+s^2} ds$ using a composite Simpson's rule. To do this, partition the interval $[-5, 5]$ into equally spaced points $t_0, t_1, \ldots, t_n$ where $n = 2k$ is even. The even indexed points should be the endpoints of your subintervals.
   You may combine the two into one code that selects the desired method if you wish.
   Turn in a listing of your code(s).

   b) Use the error estimates derived in class to choose n so that

$$\left| \int_{-5}^{5} \frac{1}{1+s^2} ds - T_n \right| < 10^{-4} \quad \text{and} \quad \left| \int_{-5}^{5} \frac{1}{1+s^2} ds - S_n \right| < 10^{-4},$$

   where T_n is the result of the composite Trapezoidal rule and where S_n is the result of the composite Simpson's rule. Be sure to explain your reasoning for choosing n in both cases (these n values will be different in the two cases).

   c) Run your code with the predicted values of n and compare your computed values S_n and T_n with that of `SCIPY`'s `quad` routine on the same problem. Run the built in quadrature twice, once with the default tolerance of 10^{-6} and another time with the set tolerance of 10^{-4}. Report the number of function evaluations required in both cases and compare these to the number of function values your codes (both S_n and T_n) required to meet the tolerance
   Turn in your codes and the results of this test.