APPM 4600 - HW #6   Gustav Cedergrund

1) Solving nonlinear system:

$$\begin{cases} f(x,y) = x^2+y^2-4=0 \\ g(x,y) = e^x+y-1=0 \end{cases}$$

for initial guesses: i) (1,1) ii) (1,-1) iii) (0,0)

Attached is iteration results for 2 quasi-newton methods, as well as, Newton for comparison.

Performance is worse than Newtons, which makes sense as Quasi-Newton methods are, by definition, approximations of Newton's Method that are less computationally expensive.

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2) Consider nonlinear system:

$$x+\cos(xyz)-1=0$$
$$(1-x)^{1/4}+y+0.05z^2-0.15z-1=0$$
$$-x^2-0.1y^2+0.01y+z-1=0$$

Attached is output for approximating solution w/ 3 methods outlined.

Testing various initial guesses, I have seen Newton's method converge fastest generally, but Steepest → Newton's doesn't take many more iterations and sometimes converges as quickly as Newton. Again, this makes sense as Newton is very fast, but expensive at each iteration. Thus, using steepest descent, or running steepest descent and then Newtons, is a bit slower, but is also less expensive in computational cost. So the trade-off is worthwhile. For a larger system, I really enjoy the idea of using steepest descent first into Newton at the end to get more accuracy near the root.

1)

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NEWTONS METHOD:

newton's method with initial guess at [1, 1]
tolerance @ 1e-12

Found solution after 7 iterations.
the approximate root is [-1.81626407  0.8373678 ]
F(root) = [3.81028542e-13 2.55351296e-14]


newton's method with initial guess at [1, -1]
tolerance @ 1e-12

Found solution after 5 iterations.
the approximate root is [ 1.00416874 -1.72963729]
F(root) = [0. 0.]


newton's method with initial guess at [0, 0]
tolerance @ 1e-12

Jacobian at iteration 0 is
 [[0. 0.]
 [1. 1.]]
which is singular. unable to converge.
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BROYDEN'S METHOD:

broyden's method with initial guess at [1, 1]
tolerance @ 1e-12

converged after 13 iterations
the approximate root is [-1.81626407  0.8373678 ]
F(root) = [1.42108547e-14 4.44089210e-15]


broyden's method with initial guess at [1, -1]
tolerance @ 1e-12

converged after 7 iterations
the approximate root is [ 1.00416874 -1.72963729]
F(root) = [-8.88178420e-16  2.22044605e-16]


broyden's method with initial guess at [0, 0]
tolerance @ 1e-12

Solution not found, Jacobian is singular at iteration 0
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LAZY NEWTON'S METHOD:

lazy newton's method with initial guess at [1, 1]
tolerance @ 1e-12

/Users/cedergrund/Documents/fall-2023/APPM4600/homework/hw6/py_files/prob1.py:123: RuntimeWarning: overflow encountered in exp
  F[1] = np.exp(x[0]) + x[1] - 1
Could not converge after 100 iterations.
Solution not found.


lazy newton's method with initial guess at [1, -1]
tolerance @ 1e-12

converged after 44 iterations
the approximate root is [ 1.00416874 -1.72963729]
F(root) = [8.52651283e-13 2.44249065e-15]


lazy newton's method with initial guess at [0, 0]
tolerance @ 1e-12

Jacobian matrix at initial point is singular.
Solution not found.
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2)

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NEWTONS METHOD:

newton's method with initial guess at [0, 0, 0]
tolerance @ 1e-06

Found solution after 3 iterations.
the approximate root is [0.         0.10005001 1.00100113]
F(root) = [0.0, 3.128011183406443e-09, 1.285092361413831e-07]



STEEPEST DESCENT:
steepest descent method with initial guess at [0, 0, 0]
tolerance @ 1e-06

Found solution after 5 iterations.
the approximate root is [-6.27724957e-05  9.99685854e-02  9.99984493e-01]
F(root) = [-6.277251540975914e-05, -1.4946447643549021e-05, -0.0010148828015164035]



HYBRID APPROACH:
steepest descent method with initial guess at [0, 0, 0]
tolerance @ 0.05

Found solution after 1 iterations.

now finishing iteration through newtons.
initial guess @ [-0.02001828  0.09005646  0.99526475]
tolerance @ 1e-06

Found solution after 2 iterations.
the approximate root is [-7.67191304e-15  1.00049982e-01  1.00100039e+00]
F(root) = [-7.66053886991358e-15, 1.3029467949010609e-08, -6.118529712884069e-07]
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