APPM 4600 Lab 3
Playing with the bisection and fixed point

1 Overview

In this lab, you will play with some root finding algorithms from class. You will explore the limitations and capabilities of the different methods.

2 Before lab

There is not much for you to do before this lab. You should rename your repo files so that they are named APPM 4600 and you have both a homework and lab folder. These should have subfolders for each of the labs and homeworks. Then you should have the bisection and fixed point example codes from lecture downloaded so you are ready to make the class at the beginning of the lab session.

3 Lab day: Exploring the capabilities of the root finding algorithm

To start lab you will begin by refreshing your memory on the different algorithms.

For the bisection method

- What does the method do?
- What is required for the method to work?

For the fixed point iteration

- What problem is the fixed point iteration trying to solve?

4 Exercises

1. Consider the function $f(x) = x^2(x-1)$ and use bisection with the following starting intervals.
   (a) $(a, b) = (0.5, 2)$
   (b) $(a, b) = (-1, 0.5)$
   (c) $(a, b) = (-1, 2)$

   What happens for each choice of interval? If the method is not successful, explain why. Is it possible for bisection to find the root $x = 0$?

2. Apply bisection to some functions listed below. You should set your desired accuracy to $\epsilon = 10^{-5}$.
   (a) $f(x) = (x-1)(x-3)(x-5)$ with $a = 0$ and $b = 2.4$.
   (b) $f(x) = (x-1)^2(x-3)$ with $a = 0$ and $b = 2$.
   (c) $f(x) = \sin(x)$ with $a = 0$, $b = 0.1$. What about $a = 0.5$ and $b = \frac{3\pi}{4}$

   Is the behavior what you expect? Was your code successful? Did it achieve at least the desired accuracy?

3. Consider the following functions

   (a) $f(x) = x\left(1 + \frac{7-x^5}{x^2}\right)^3$

   (b) $f(x) = x - \frac{x^5-7}{x^2}$

   (c) $f(x) = x - \frac{x^5-7}{5x^4}$

   (d) $f(x) = x - \frac{x^5-7}{12}$

   - Verify that $x = 7^{1/5}$ is a fixed point for these functions.
   - Apply the fixed point iteration with $x_0 = 1$ with a tolerance of 10^{-10}. For which functions does the fixed point iteration converge? For which functions does it not converge? Try to come up with an explanation of why the fixed point iteration performs the way it does for each of the functions.

5 Deliverables

To receive full credit for this lab, you need to commit and push your lab codes to git.