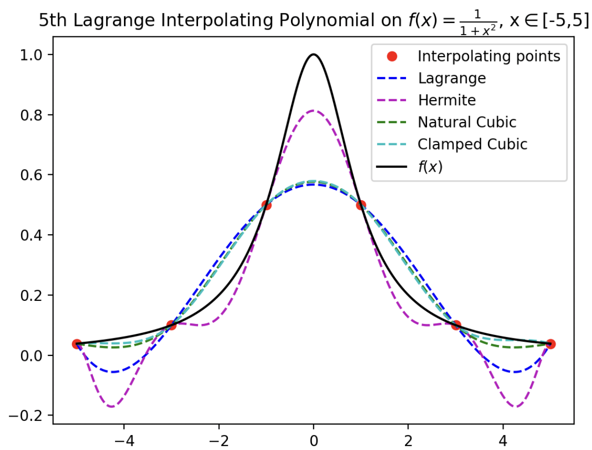
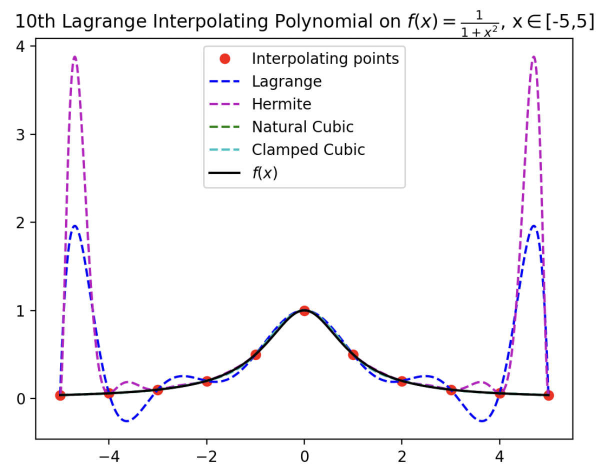
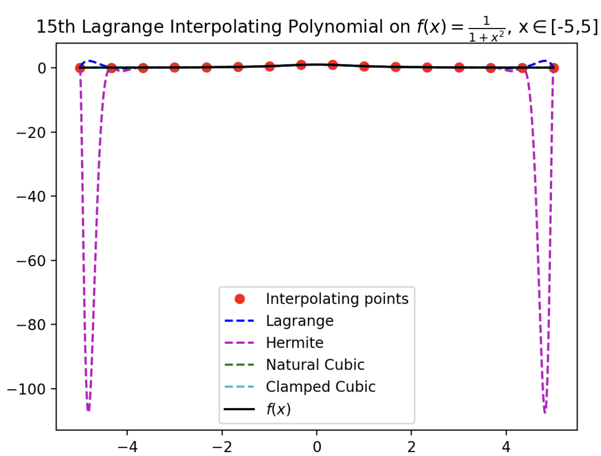
APPM 4600 HW8 - Gustav Cedergrund

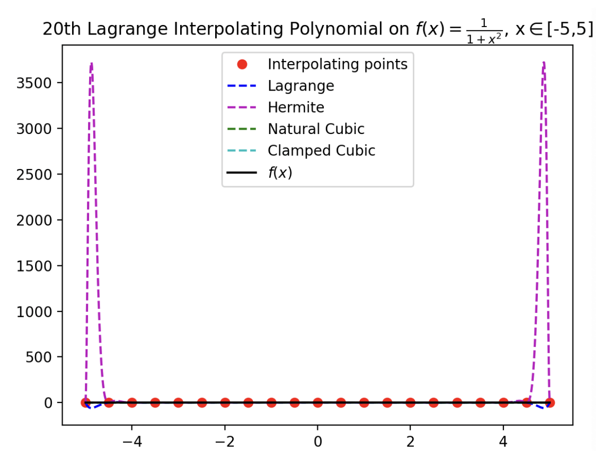
1)

Below is the output for interpolation of f(x) on the interval [-5,5] for the various methods…

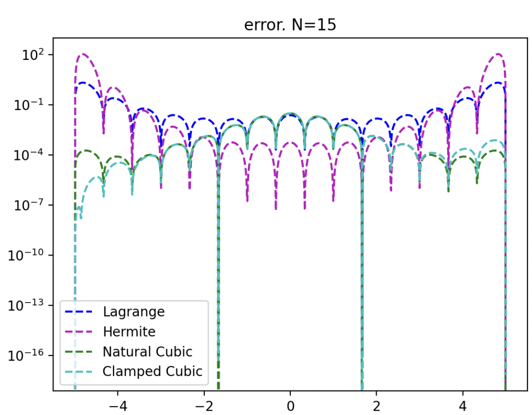


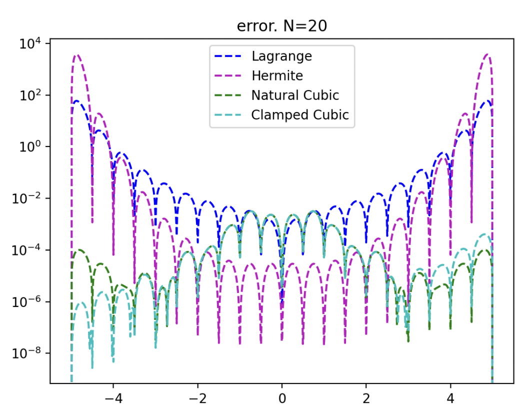






And error plots for N=15 and N=20 as well



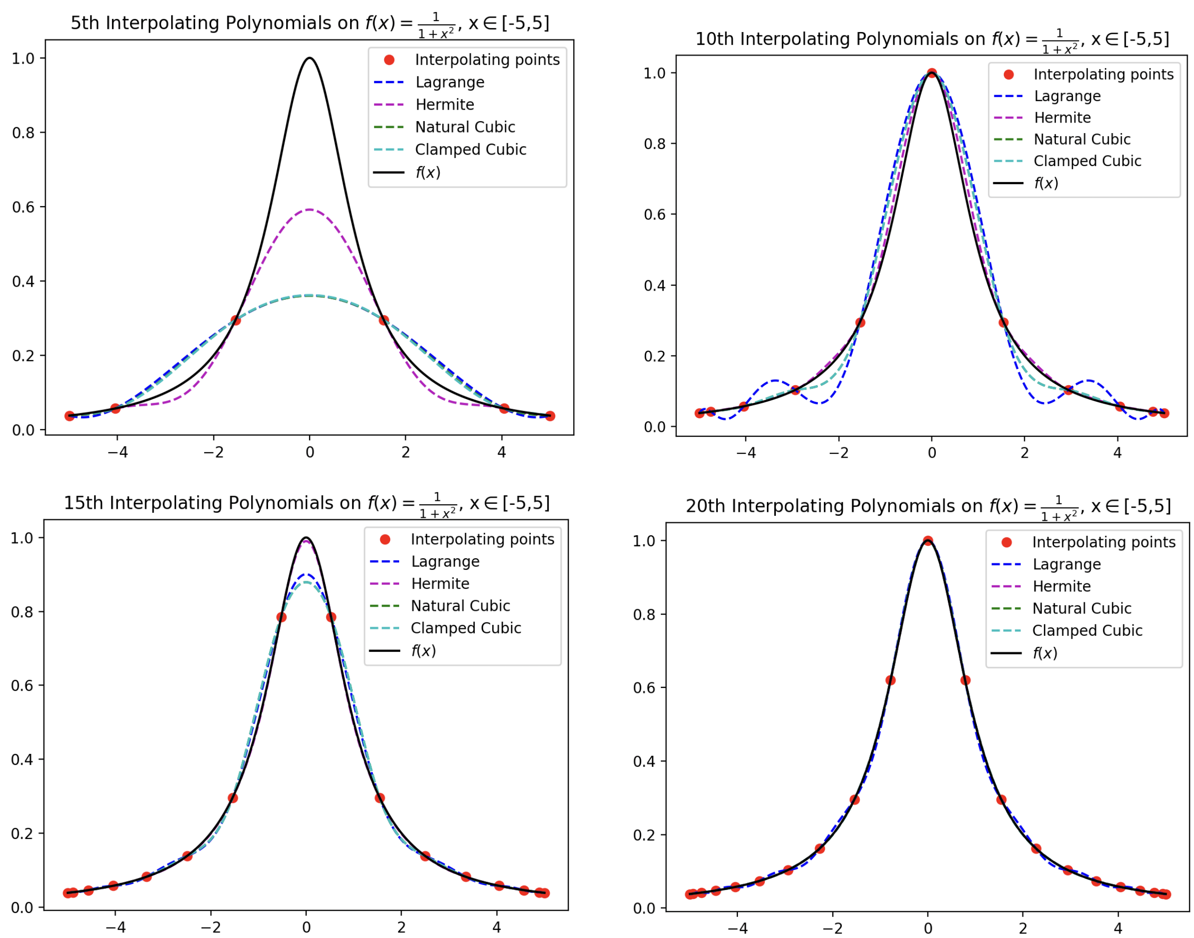


As shown in the plots above, we see that hermite generally performs the best within the center of the interval, followed by clamped and natural cubic, and lastly lagrange. This makes sense as this mostly follows the “amount of information” provided to each method; hermite has the derivative information at all points, clamped cubic spline has derivative information at just the endpoints, while the natural cubic and lagrange just have the y values at each point.

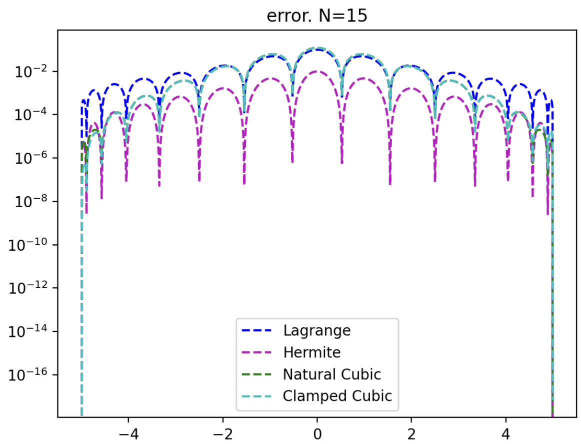
However, while it performs the best closer to the center, we can also see that hermite experiences strange behavior near the endpoints, even more so than lagrange. This may be due to the large amount of information that Hermite has to fit to its polynomial at each point, which increases instability near endpoints as the created interpolating polynomial is of very high order. That is why the cubic splines are nice as only having a cubic polynomial to fit to the data allows for low instability all throughout the interval, even with high N.

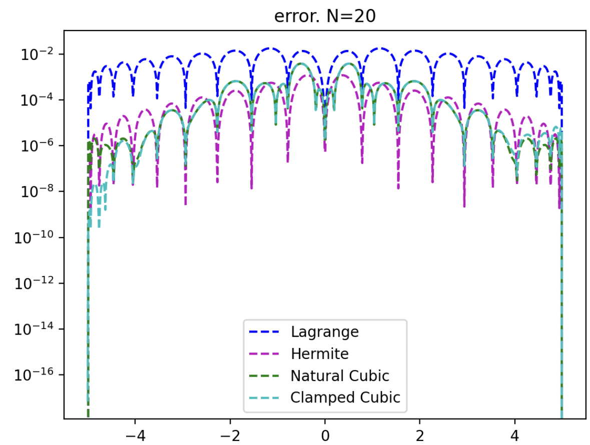
2)

Below is the output for interpolation of f(x) on the interval [-5,5] with Chebychev nodes…



As seen above, this dramatically reduces the variability at the endpoints of hermite and lagrange interpolating polynomials, and overall improves the error for any x inside the interval. This can be seen in the error graphs for N=15 and N=20 shown below





3)

While approximating the periodic function such as f(x) = sin(10x) on [0, 2π] using the cubic spline, to make the spline naturally periodic one must make the endpoint conditions the same on both sides. Specifically, as f'(x) =10cos(10x) such that f'(0)=f'(2π)=10, creating a clamped spline with boundary conditions such that the cubic spline has a derivative of 10 at both endpoints, the spline will be naturally periodic as there already exists the restriction by construction of the spline that f(0)=f(2π)=0.