Parameter Estimation

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Methods in Modern Modeling

Introducing Data for Parameters

When building a model for a current world problem, we want to capture dynamics as closely as possible.

Sometimes we can build off preexisting work and find parameters in the literature.

Other times we have data related to our model and need to infer parameter values from the data.

- Least-Squares Minimization
- Markov Chain Monte Carlo method
- Particle Swarm Optimization (PSO)
- many more!

MATLAB: Using FMINCON and MULTISTART

MATLAB's "**fmincon**" function will estimate the unknown parameters in the differential equation by minimizing the distance between the solution and the actual data points[1].

$$\underset{\text{Minimize}}{J} = ||Data - Model||_2,$$

MATLAB's "**MultiStart**" object repeatedly runs a local solver to generate a global max/min, and the solver attempts to find multiple local solutions to a problem by starting from various points [2].

We can parallelize the process using Mutlistart to efficiently search for parameters.

SIR and Data Model for LA County COVID-19

$$S \xrightarrow{\beta} I \xrightarrow{\gamma} R$$
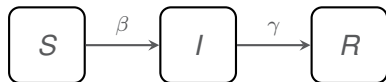
4/20/2020 through 5/16/2020

- Confirmed Cases
- Deaths due to Disease
- Hospitalizations

$$S' = -\beta SI$$
$$I' = \beta SI - \gamma I$$
$$R' = \gamma I$$

$$CI = \beta SI$$

Based off a more involved model for LA County (SEIRMD) and then the Claremont Colleges (SEIRMDHH) - To Open or Not to Open

Set-Up for Parameter estimation

We made assumptions based on data and knowledge at the time for initial conditions - S0, I0, R0, CI0

Also, $\gamma = 1/6$ based on the time frame.

We need to estimate β, we used bounds of $[10^{-9}, 10^{-7}]$

$$\underset{\text{Minimize}}{J} = ||ConfirmedCases_{Data} - ConfirmedCases_{Model}||_2,$$

Code Overview - Github

SIR_MultiStart.m

- Runs the Parameter Estimation - requires upper and lower bounds for selected parameters along with a start value.
- Calls fmincon and MultiStart using parallel processing - multiple cores on your computer
- Parameter Estimation Calls SIR_RUN_ODE45 in the SIR_MultiStart.m files which runs solver ode45 for the give parameter being swept and returns J
- SIR_RUN_ODE45 calls SIR_Model.m, a separate file

SIR_Model.m

- Includes the differential equation model you want to use
- Called during the Parameter Estimation and then when you have the "best fit" to plot
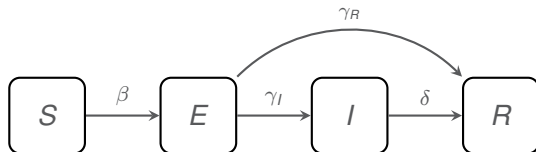
Parameters: fitting to LA County Data

We fit the model to data for specific dates from the Los Angeles times GitHub site

```
https:
//github.com/datadesk/california-coronavirus-data/
```

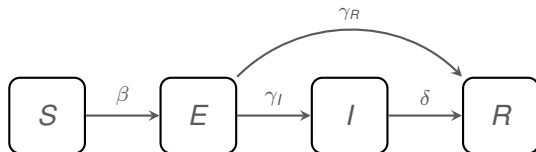Try a different range of data dates. [3]

Adding in Exposed

$$\frac{dS}{dt} = -\beta S(I + \alpha E)$$

$$\frac{dE}{dt} = \beta S(I + \alpha E) - (\gamma_I + \gamma_R)E$$

$$\frac{dI}{dt} = \gamma_I E - \delta I$$

$$\frac{dR}{dt} = \delta I + \gamma_R E \tag{1}$$

$$\tag{2}$$

Activity

Update the Code to reflect the new model

- You will need to change the ODE model

- Set-Up to approximate a new parameters

- Change the plotting output
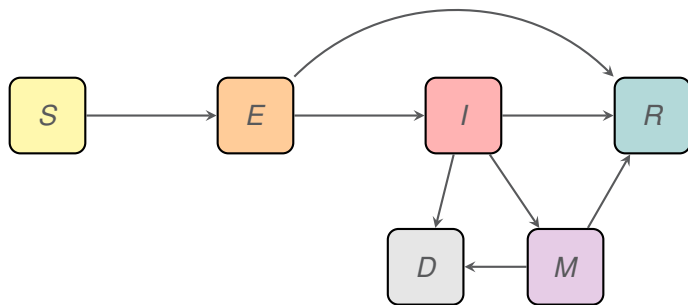
Adding in Disease-Induced Death

$$\frac{dS}{dt} = -\beta S(I + \alpha E)$$

$$\frac{dE}{dt} = \beta S(I + \alpha E) - (\gamma_I + \gamma_R)E$$

$$\frac{dI}{dt} = \gamma_I E - (\delta + \mu_I)I$$

$$\frac{dR}{dt} = \delta I + \gamma_R E \tag{3}$$

$$\tag{4}$$

Activity

Update the Code to reflect the new model

- You will need to change the ODE model

- Set-Up to approximate a new parameters

- Set-Up to use number of disease-induced deaths

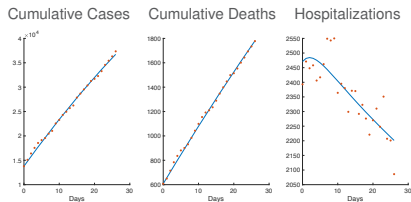- Change the plotting output

Updated Basic COVID-19 Model
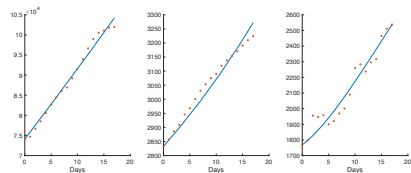

[4]

Updated Basic COVID-19 Model

$$\frac{dS}{dt} = -\beta S(g + I + \alpha E)$$

$$\frac{dE}{dt} = \beta S(g + I + \alpha E) - (\gamma_I + \gamma_R)E$$

$$\frac{dI}{dt} = \gamma_I E - (\delta_I + \mu_I)I - \omega_I e^{-(M/M_{max})^2}I$$

$$\frac{dR}{dt} = \delta_I I + \delta_M M + \gamma_R E \tag{5}$$

$$\frac{dM}{dt} = \omega_I e^{-(M/M_{max})^2}I - (\delta_M + \mu_M)M$$

$$\frac{dD}{dt} = \mu_I I + \mu_M M$$

[4]

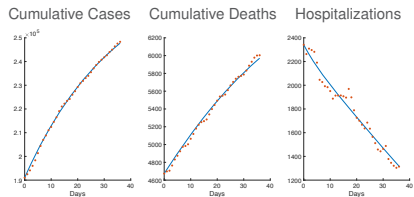# Results - Plots

Cumulative Cases   Cumulative Deaths   Hospitalizations

4/20 - 5/16, 2020
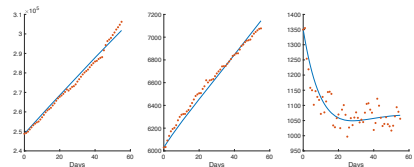
Cumulative Cases   Cumulative Deaths   Hospitalizations

8/1 - 9/6, 2020

6/13 - 6/30, 2020

9/7 - 11/1, 2020

[4]

Results - Parameter Values

Symbol	4/20-5/16/20	6/13-6/30/20	8/1-9/06/20	9/7-11/1/20
α	0.0389	2.2096	0.2231	0.6358
β	4.9486×10^{-8}	2.787×10^{-9}	5.0163×10^{-9}	6.4247×10^{-9}
δ_I	0.2669	0.046	0.0887	0.0562
δ_M	0.0129	0.0129	0.038	0.038
γ_I	0.0092	0.01332	0.0048	0.0049
γ_R	0.0378	0.0013	0.0375	0.0212
M_{max}	2548	5694	10,000	10,000
μ_I	3.4126×10^{-4}	1.0016×10^{-8}	6.2976×10^{-4}	6.7116×10^{-5}
μ_M	0.0186	0.0144	0.0128	0.0177
ω_I	0.0433	0.006	0.0031	0.0039

[4]

To Do Items

- Read one of the suggested or a related article on a parameter estimation technique. Summarize the technique, and ensure you give insight into the method for a novice. (This should be around 5-8 sentences and can also include images for explanation aid.)

  - https://www.projectrhea.org/rhea/index.php/Bayes_Parameter_Estimation
  - https://towardsdatascience.com/probability-concepts-explained-bayesian-inference-for-p
  - https://mjlaine.github.io/mcmcstat/#orgb3e1f3a
  - https://towardsdatascience.com/particle-swarm-optimization-visually-explained-46289eeb2
    https://www.mathworks.com/help/gads/particleswarm.html

- Expand the base parameter estimation code from an SIR model to an SEIR model as specified in the class slides. Ensure you record initial set-up values, and then the results (values for parameters, fit error, and plot here). Discuss and display your findings, then compare with the SIR model fit.

📄 Mathworks fmincon.
https:
//www.mathworks.com/help/optim/ug/fmincon.html.
Accessed: 2021-09-27.

📄 Mathworks multistart.
https:
//www.mathworks.com/help/gads/multistart.html.
Accessed: 2021-09-27.

📄 Christina Edholm.
SIR Parameter Estimation in MATLAB with Fmincon MultiStart
Parallelization, February 2024.

📄 Christina Joy Edholm, Maryann Hohn, Nicole Lee Falicov,
Emily Lee, Lily Natasha Wartman, and Ami Radunskaya.
To open or not to open: Developing a covid-19 model specific
to small residential campuses.
CODEE Journal, 17(1):1, 2024.