Lab



Future directions

Datatype notations (1.1)

↑

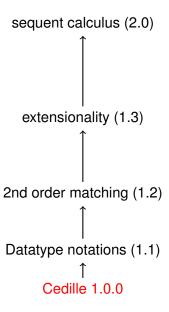
Cedille 1.0.0

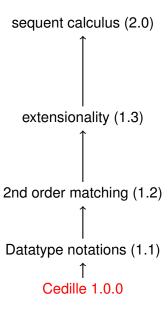
```
2nd order matching (1.2)

Datatype notations (1.1)

Cedille 1.0.0
```

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extensionality (1.3)
2nd order matching (1.2)
Datatype notations (1.1)
      Cedille 1.0.0
```





Datatype notations (Cedille 1.1)

- High-level notation (like Coq/Agda) for
 - Declaring datatypes
 - Pattern-matching recursion
- Elaboration to pure lambda calculus!
- The theory we have supports
 - Provably monotone datatypes
 - Histomorphic recursion

Cedilleum ⊆ Cedille, for Victor Maia of Ethereum Foundation

Seeking extensionality

$$\forall x : A. \{f \ x \simeq g \ x\}$$

$$\downarrow \{f \simeq g\}$$

In extensional MLTT

$$\forall x : A. \{f \ x \simeq_B g \ x\}$$

$$\downarrow \qquad \qquad \downarrow$$

$$x : A \vdash \{f \ x \simeq_B g \ x\}$$

$$\downarrow \qquad \qquad \downarrow$$

$$\{\lambda x. f \ x \simeq_{A \to B} \lambda x. g \ x\}$$

$$\downarrow \qquad \qquad \downarrow$$

$$\{f \simeq_{A \to B} g\}$$

In extensional MLTT

$$\forall x : A. \{f \ x \simeq_B g \ x\}$$

$$\downarrow \qquad \qquad \downarrow$$

$$x : A \vdash \{f \ x \simeq_B g \ x\}$$

$$\downarrow \qquad \qquad \downarrow$$

$$\{\lambda x. f \ x \simeq_{A \to B} \lambda x. g \ x\}$$

$$\downarrow \qquad \qquad \downarrow$$

$$\{f \simeq_{A \to B} g\}$$

Seems to require typed equality!

Extending Cedille's equality

Currently:

Proposed:

Terms equal at *T* iff indistinguishable by *T*-contexts

This is not ETT

In ETT:

$$\frac{\Gamma \vdash t : A \qquad \Gamma \vdash t' : A}{\Gamma \vdash t =_A t' : \star}$$

In proposed extension:

$$\frac{\Gamma \vdash T : \star \qquad FV(t \ t') \subseteq dom(\Gamma)}{\Gamma \vdash \{t \simeq t' \ @ \ T\} : \star}$$

(Strange) Example

True =
$$\forall X : \star . X \rightarrow X$$

The following are equal at type $True \rightarrow True$:

$$\lambda s. \lambda z. s (s (z \lambda q. q))$$

$$\lambda s. \lambda z. s (z \lambda q. q)$$

- Neither term has type True → True
- ▶ Applied to $\lambda x. x$, they are β -equal

Why?

General considerations, and

Certain examples, including higher-order abstract syntax

$$LamTrm: \star = \forall X: \star .((X \to X) \to X) \to (X \to X \to X) \to X$$

Sequent calculus?

- Good formalism for duality
- Under CH, supports control
- Challenge: retain canonicity
 - BiInt adds dual to subtraction, loses disjunction property
 - With T. Cantor: logic 2Int^x
- Motivation: coinduction dual to induction

Recap



⊢ CeDiLIE

cedille

 \rightarrow cedille_{core}

c d 11



Motivation and background for Cedille

Syntax and semantics

Tooling: emacs frontend ↔ backend

Elaboration to Cedille Core

Spine-local type inference

Future directions

Thanks

Rest of Cedille dev:
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