

Future directions

Datatype notations (1.1)

↑

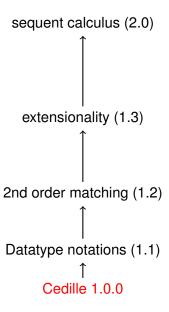
Cedille 1.0.0

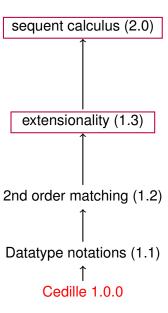
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2nd order matching (1.2)

Datatype notations (1.1)

Cedille 1.0.0
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extensionality (1.3)
2nd order matching (1.2)
Datatype notations (1.1)
      Cedille 1.0.0
```





Extensionality?

$$\forall x : A. \{ f x \simeq g x \}$$

$$\downarrow \{ f \simeq g \}$$

In extensional MLTT

$$\forall x : A. \{f \ x \simeq_B g \ x\}$$

$$\downarrow \qquad \qquad \downarrow$$

$$x : A \vdash \{f \ x \simeq_B g \ x\}$$

$$\downarrow \qquad \qquad \downarrow$$

$$\{\lambda x. f \ x \simeq_{A \to B} \lambda x. g \ x\}$$

$$\downarrow \qquad \qquad \downarrow$$

$$\{f \simeq_{A \to B} g\}$$

In extensional MLTT

$$\forall x : A. \{f \ x \simeq_B g \ x\}$$

$$\downarrow \downarrow$$

$$x : A \vdash \{f \ x \simeq_B g \ x\}$$

$$\downarrow \downarrow$$

$$\{\lambda x. f \ x \simeq_{A \to B} \lambda x. g \ x\}$$

$$\downarrow \downarrow$$

$$\{f \simeq_{A \to B} g\}$$

Seems to require typed equality!

Extending Cedille's equality

Currently:

$$\left\{t\simeq t'\ @\ T\right\}$$

$$\left[\!\left\{t\simeq t'\right\}\right]\!\right]\ =\ \left\{\hat{t}\mid \left[\!\left[t\right]\!\right]=_{\beta\eta}\left[\!\left[t'\right]\!\right]\right\}$$

Proposed:

Terms equal at *T* iff indistinguishable by *T*-contexts

This is not ETT

In ETT:

$$\frac{\Gamma \vdash t : A \qquad \Gamma \vdash t' : A}{\Gamma \vdash t =_A t' : \star}$$

In proposed extension:

$$\frac{\Gamma \vdash T : \star \qquad FV(t \ t') \subseteq dom(\Gamma)}{\Gamma \vdash \{t \simeq t' \ @ \ T\} : \star}$$

(Strange) Example

True =
$$\forall X : \star . X \rightarrow X$$

The following are Equal at type $True \rightarrow True$:

$$\lambda s. \lambda z. s (s (z \lambda q. q))$$

$$\lambda s. \lambda z. s (z \lambda q. q)$$

- Neither term has type True → True
- ▶ Applied to $\lambda x. x$, they are β -equal

Why?

General considerations, and

Certain examples, including higher-order abstract syntax

$$LamTrm: \star = \forall X: \star .((X \to X) \to X) \to (X \to X \to X) \to X$$

Thanks

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Funders: NSF, AFOSR(MURI)

Recap



⊢ CeDiLIE

cedille

 \rightarrow cedille_{core}

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Motivation and background for Cedille

Syntax and semantics

Tooling: emacs frontend ↔ backend

Elaboration to Cedille Core

Spine-local type inference

Future directions

