Spine-local Type Inference in Cedille

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Outline

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1. Background and Motivation
   - Local Type Inference
   - Spine-local Type Inference

2. Detailed Example

3. Annotation Requirements

What is “Local Type Inference”?

- Uses two main techniques
  - *Bidirectional typing rules:*
  - *Local type-argument inference:*

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Synthesis mode:	$\lambda\, x : Nat.\, x$	$\Uparrow$	$Nat \to Nat$
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  - *Local type-argument inference:*

Let	$id : \forall\, X.\, X \to X$	
Type	$\boxed{id\ 0}$	$\Uparrow Nat$
Infer	$X = Nat$	from 0

$\boxed{\text{Local}}$ and Synthetic

Why use local type inference?

- It is a method of *partial* type inference
  - *Complete* type inference: no annotations **ever** (e.g. *Damas-Hindley-Milner* and ML)
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- It is user-friendly
  - Infers many type annotations
  - Predictable annotation requirements
  - Better-quality error messages
- It is implementer-friendly
  - Relatively simple implementation
  - *Extensible*: new features added without threatening decidability

Local Type Inference in Cedille

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Cedille also has a novel inference system: *spine-local type inference*

Limitations of other LTI Systems

Why a novel system? Other local type inference systems can sometimes still require "silly" type annotations...

Assume	*mkpair*	:	$\forall X\ Y.\, X \rightarrow Y \rightarrow Pair\ \cdot X\ \cdot Y$
Type	$mkpair\ (\lambda\, x.\, x)\ 0$		

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- We do not expect to locally synthesize a type
- ... but we would expect to check it against a type
  - We could call this "contextual" type-argument inference.
- Unfortunately, not done in the two major published LTI systems
  - Popular "unofficial" extension (used in e.g. Scala, Rust)

Limitations of other LTI Systems (cont.)

- Usually uses "fully-uncurried" function applications

$$f\,(t_1, .., t_n)$$

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- Usually uses "fully-uncurried" function applications

$$f(t_1, .., t_n)$$

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- Usually without partial type application ("all-or-nothing")

$$f[T_1, ..., T_m](t_1, ..., t_n)$$

Our Contributions

- Type inference for some expressions not typed by other variants of local type inference, by using *contextual* type-argument inference
- Precise, specificational account of this technique
- Better support function currying and partial type applications by being "spine-local."

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$$\boxed{f[S, T, V](t_1, t_2, t_3)}$$

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$$\boxed{f\ \cdot S \qquad\qquad t_1\ \ t_2}$$

Room for Improvement

Type inference in Cedille will continue to be improved upon. Some things we want to address:

- Higher-order type arguments must be provided explicitly:
  $(\forall\, F : \star \rightarrow \star.\,)$

- Type arguments inferred *only* between applications:
  $nil \Downarrow List\ \ \cdot Nat$

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  - **Coming soon:** polymorphic subsumption
    $\forall A. List \cdot A \ \leq\ List \cdot Nat$

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- *Application head*: variable or abstraction

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$$t_1 \text{ in } t_1 \;\; t_2$$

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- *Maximal application:* spine that is not an applicand

$$\begin{array}{ll} \text{Not max} & \underline{x \;\; t_1 \;\; t_2} \;\; t_3 \\ \text{Max} & \underline{x \;\; t_1 \;\; t_2 \;\; t_3} \end{array}$$

Example – High Level Goals

Assume	*const*	:	$\forall X\ Y.\, X \rightarrow Y \rightarrow X$
Assume	*zero*	:	*Nat*
Type	*const* $(\lambda\, x.\, x)$ *zero*	$\Downarrow$	$Nat \rightarrow Nat$

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- We fill in missing types to "elaborate" to $const\ \cdot (Nat \to Nat)\ \cdot Nat\ (\lambda\, x : Nat.\, x)\ zero$

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  - $?Y : \star = \mathit{Nat}$

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  In Cedille: $\boxed{\mathit{const}\ (\lambda\, x.\, x)\ \mathit{zero}}$
  - $?X : \star\ \triangleleft\ \mathit{Nat} \to \mathit{Nat}$
  - $?Y : \star\ = \mathit{Nat}$
- And term argument $\lambda\, x.\, x$ *checked* against $\mathit{Nat} \to \mathit{Nat}$

Spine Local

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  - $\boxed{\text{cage meta-variables}}$ here
    never "escape" the spine or "descend" into the arguments

$$\boxed{f}\ \overset{X\quad Y\quad Z}{t_1 \ldots t_n}$$

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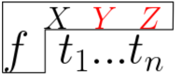
    Using the expected result type



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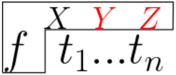
    Using the expected result type
- **Consequence:** you know where to look when type-argument inference fails!



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Synthesize type for head

Example - Details

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Assume	$zero$	$:$	Nat
Type	$const\ (\lambda\, x.\, x)\ zero$	$\Downarrow$	$Nat \to Nat$
Match	X	$\lhd_X$	$Nat \to Nat$

Match head return with expected return type

Example - Details

$$
\begin{array}{llcl}
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\text{Match} & X & \lhd_X & \mathit{Nat} \to \mathit{Nat}
\end{array}
$$

Get a contextual *instantiation*

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Type first argument

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Match	X	$\lhd_X$	$\mathit{Nat} \to \mathit{Nat}$
Type	$\lambda\, x.\, x$	$\Downarrow$	$[\mathit{Nat} \to \mathit{Nat}/X]X$

Type first argument in *checking mode*

Example - Details

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Type second argument

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Type	$const\ (\lambda\, x.\, x)\ zero$	$\Downarrow$	$Nat \to Nat\ = [Nat \to Nat/X]X$
Match	X	$\lhd_X$	$Nat \to Nat$
Type	$\lambda\, x.\, x$	$\Downarrow$	$[Nat \to Nat/X]X$
Type	$zero$	$\Uparrow$	$[Nat/Y]Y$

Type second argument in *synthesis mode*

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Type	$const\ (\lambda x. x)\ zero$	$\Downarrow$	$Nat \to Nat\ = [Nat \to Nat/X]X$
Match	X	$\lhd_X$	$Nat \to Nat$
Type	$\lambda x. x$	$\Downarrow$	$[Nat \to Nat/X]X$
Type	$zero$	$\Uparrow$	$[Nat/Y]Y$

Conclude the spine has the expected type!

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- **But:** when is a term's type checked? In a spine?
  - when the checked type of the spine
  - and the synthesized type of the head
  - and any synthesized types from earlier args
  - tells us the *complete* type to check

$$f^{\Uparrow}\ t_1^{\Uparrow}\ t_2^{\Downarrow}\ t_3 \Downarrow T$$

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  - Synthesized type for args: $\mathit{zero} \Uparrow [\mathit{Nat}/Y]Y$
- **Example:** $\forall\, Y\ X.\, X \to X$
  Y doesn't occur in result or arg position
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- **Advantage:** meta-variables are *one-to-one* with quantified type variables in functions!
  It's always easy to understand why a meta-var was introduced!

Thanks!

- `language-overview/type-inference.ced`
  For practical examples
- Paper can be found on arXiv
  (To appear in proceedings of IFL 2018)