Wage Contracts and Financial Frictions

Cedomir Malgieri
Stanford University

Luca Citino
Bank of Italy

Job Market Paper

November 7, 2024

Click here for the latest version

Abstract

Financial crises often lead to drastic reductions in firms' access to credit, impacting significantly their ability to finance their operations. This paper shows that firms can partly offset the effects of these shocks by optimally adjusting their wage bills. We augment a baseline financial frictions model to account for two well-documented features of the labor market: wages are set at the firm level and within long-term employment relationships. Because of these features, wage dynamics depend on the financial conditions of firms, reflecting a trade-off between smoothing wages of risk-averse workers and investing in capital. We validate the model predictions on wage dynamics using matched employer-employee data from Italy. We find that more constrained firms adjust wages more in response to idiosyncratic shocks. In addition, firms that suffer the most during recessions back-load wages by offering steeper wage-tenure profiles to their workers. When matching these statistics with our general equilibrium model, we find that these wage adjustments reduce the sensitivity of output to financial shocks by 20%: wage back-loading enhances investment and job creation while improving allocative efficiency. We conclude by studying policies aimed at reducing inputs cost during recessions. Our findings show that these wage adjustments diminish the effectiveness of temporary payroll subsidies while enhancing the effectiveness of temporary investment subsidies in stimulating output.

We are sincerely grateful to Adrien Auclert, Luigi Bocola, Patrick Kehoe, Elena Pastorino and Christopher Tonetti for their advice and invaluable support. We would also like to thank Rafael Berriel, Nick Cao, Daniele Caratelli, Federico Cingano, Sebastian Di Tella, John Grigsby, Pete Klenow, Virgiliu Midrigan, Luigi Pistaferri, Enrico Sette and seminar participants at Stanford University for feedbacks and comments. We are grateful to Francesco Beraldi for insightful discussions. This project was supported by the Gale and Steve Kohlhagen Fellowship in Economics through a grant to the Stanford Institute for Economic Policy Research. The opinions expressed in this paper are those of the authors alone and do not necessarily reflect the views of the Bank of Italy. All remaining errors are our own.

1 Introduction

After the Great Recession there has been a wide consensus that financial shocks played an important role in recent cycles. The central framework for understanding the macroeconomic effects of financial shocks hinges on the role of firms' credit, building on the work of Kiyotaki and Moore (1997) and Bernanke, Gertler, and Gilchrist (1999). The main mechanism is based on the idea that a reduction in the availability of credit forces employers to cut investment and hiring because of the shortage of funds, leading to large macroeconomic effects of financial shocks.

The standard mechanism implicitly relies on firms purchasing inputs in spot markets. In other words, whenever firms experience a financial shock the burden of adjustment must fall on quantities. This assumption is quantitatively important in explaining the large macroeconomic effects of financial shocks, especially when applied to the labor market, as the wage bill constitutes a significant share of firms' costs.

However, this assumption conflicts with two well-documented characteristics of the labor market: a significant portion of wages is determined at the firm level (Card, Heining, and Kline, 2013; Card et al., 2018), and wages are set within long-term employment relationships. We argue these features are crucial as the effects of financial shocks are heterogeneous and persistent across firms. Moreover, these features are consistent with evidence that firms adjust wages over time and in response to shocks (Guiso, Pistaferri, and Schivardi, 2005, 2012), suggesting that firms can adjust wages rather than quantities when access to credit is limited.[1] How do firms adjust wages during periods of credit tightening? What are the macroeconomic implications of these wage adjustments? And how effective are stabilization policies aimed at reducing the cost of inputs during financial crises?

This paper proposes answers to these questions with four contributions. First, we propose a general equilibrium model of frictional financial and labor markets, where wages are set at the firm-level as part of long-term employment relationships, and firms face occasionally binding financial constraints. Second, we provide novel empirical evidence using matched employer-employee data that supports model predictions on wage dynamics: firms adjust wages based on their financial conditions and to ease the effects of credit constraints. Third, we show that these wage adjustments are quantitatively important during business cycle, as they significantly mitigate the macroeconomic impacts of a credit tightening. Fourth, we study the effects of temporary payroll and investment subsidies, commonly used stabilization policy to reduce the cost of inputs. We find that optimal wage adjustments over long-term employment relationships act as substitute to

[1] Guiso, Pistaferri, and Schivardi (2005) finds that firms adjust wages in response to shocks in a way to partially insure workers' earnings from idiosyncratic fluctuations in firms' productivity. Guiso, Pistaferri, and Schivardi (2012) finds that firms in less financially developed regions pay steeper wage-tenure profile, thus adjusting wage payments over time.

payroll subsidies and as complement to investment subsidies.

We build on a canonical model of firms' dynamics under financial frictions (Moll, 2014; Khan and Thomas, 2013). In our economy, firms face idiosyncratic productivity shocks and issue debt in order to finance their operations subject to a potentially binding financial constraint. As in Jermann and Quadrini (2012), we model an aggregate financial shock as a tightening of this financial constraint. Importantly, we introduce search frictions in the labor market and long-term wage contracts between firms and their workers in the tradition of Thomas and Worrall (1988).

Credit market frictions generates a trade-off between providing insurance to risk-averse workers and investing in capital. To understand the mechanism, consider a firm that currently operates below "optimal scale" due to a binding borrowing constraints. Over time, as this constraint is gradually eased, the firm's output will increase.[2] Risk-averse workers would like to receive a constant wage throughout this transition, but the firm would like to pay lower wages and invest more in capital when borrowing constraints are binding. More generally, wages adjust over time and in response to shocks depending on firm-specific financial conditions.

In the model we illustrate two key findings. First, firms significantly affected by financial frictions adjust wages more in response to idiosyncratic shocks, as it is more costly for them to hedge workers against these shocks. Second, firms back-load wage payments when they require more credit, resulting in a steeper wage-tenure profile. This means that during recessions, firms more impacted by a credit tightening offer lower wages so to invest more in productive capital and ease the credit constraint, with the implicit promise of future wage increases.

We propose new empirical evidence supporting these two key predictions of the model on the dynamics of wages. We use matched employer-employee data from Italy, including administrative data on workers' compensation and firms' balance sheets. First, using a model-consistent indicator, we show that in the cross-section firms that are more financially constrained adjust wages significantly more in response to shocks. Specifically, we estimate the "pass-through" of value-added per worker to wages, a commonly used statistic that measures the extent to which workers are subject to firm-specific shock. We find that this pass-through coefficient is 1.5 times larger for more financially constrained firms. Second, we show that in recession more financially constrained firms back-load wages by offering a steeper wage tenure profile. We compare the wage-tenure profile of workers hired during the Great Recession by constrained and unconstrained firms, and we find that wages grow 3 percentage points more over the first four years of tenure at constrained firms.

The estimated model is consistent both with the empirical evidence on the heteroge-

[2] This dynamics is standard and common to several models of financial frictions, as in Moll (2014) and Midrigan and Xu (2014).

neous wage dynamics across firms and stylized facts on aggregate wage dynamics. Despite firms adjust wages to offset the effects of a credit tightening, the average wage remains relatively stable in line with evidence that it moves little during recessions (Grigsby, 2022). However, the modest cyclicality of the average wage masks substantial heterogeneity in the cross-section, where financially constrained firms contract and substantially reduce wage payments, while unconstrained firms expand and moderately increase wages. In the aggregate, this firm-level heterogeneity in wage adjustments implies that the skewness of the wage adjustment distribution is lower in recessions than in booms, consistent with evidence documented by Adamopoulou et al. (2016).

We use our model to understand how these wage adjustments affect the transmission of financial shocks. When firms back-load wage payments during recessions, this frees resources for current *and* future investment. A temporary lower wages boosts current investment *and* leads to more output next period. Because the effects of financial frictions are persistent, as in Moll (2014), an increase in output and retained earnings next period further enhances future investment. This virtuous cycle propagates forward up to a time when the effects of borrowing constraints are eased, and an increase in retained earnings has no effects on investment. The effect of wage back-loading on current *and* future investment enhances job creation, by substantially increasing the present discounted value of output over the length of the employment relationship. Additionally, the ability to make state-contingent wage adjustments lowers the expected cost of hiring a worker by minimizing the expected present discounted value of all future wage payments, discounted using the firm-specific stochastic discount factor.

In the quantitative analysis we show that firm-specific wage adjustments substantially mitigate the effects of an aggregate financial shock. Toward that purpose, we construct an alternative economy in which firms cannot commit to future wages, a restriction that prevents firms from adjusting wages over time and in response to shocks. This second economy works as the canonical model of financial frictions, so the comparison with the baseline will help understand how far dynamic wage contracts go into smoothing the effects of shocks. By comparing the impulse response functions to an aggregate financial shock in the two economies, we find that output drops by 20% less in our model.

The differential response of output to an aggregate financial shock is primarily driven by differences in aggregate employment and productivity. Employment falls less in the baseline economy, as dynamic wage contracts enhance job creation. Investment to also falls less in the baseline economy, since the newly matched entrepreneurs invest in capital and wage back-loading frees resources for current investment. However, general equilibrium effects render the difference in aggregate investment between the two economies small. This is due to investment in newly created matches with high marginal product of capital "crowding-out" investment in existing matches with low marginal product of capital. In other words, despite aggregate investment follows a similar path in the two economies, in the baseline model capital is re-allocated towards more productive firms. Consequently, the drop in endogenous total factor productivity, a measure of allocative efficiency, is less pronounced in the economy with dynamic wage contracts.

Finally, we illustrate that incorporating the dynamic structure of wage contracts during financial crises has important policy implications. We study the impact of policies aimed at reducing inputs cost during recessions, such as temporary investment and payroll subsidies.[3] We find that a payroll subsidy on new hires is not as effective as a standard model with a spot labor market would suggest because firms' optimal wage adjustments and payroll subsidies act as subsidies, meaning they are both aimed at reducing the cost of labor. As a result, payroll subsidies are less effective at stimulating output during recessions when firms optimally backload wages to reduce the cost of labor. On the other hand, we find that an investment subsidy is more effective than in a standard model with a spot labor market because firms' optimal wage adjustments and investment subsidies act as complements. Indeed, when an investment subsidies transfer resources to firms and make investment opportunities more attractive, financially constrained entrepreneurs will backload wages even more during recessions as to free additional resources for investment, thus amplifying the stimulative effect of the policy. We illustrate these results quantitatively by simulating in our model investment and payroll subsidies similar to those implemented in the United States after the Great Recession.

Related Literature

This paper relates to a large literature studying the role of financial frictions during recessions. Early research showed that financial imperfections can amplify shocks originated outside the financial sectors, as in the work of Kiyotaki and Moore (1997), Bernanke, Gertler, and Gilchrist (1999), Gertler and Karadi (2011), Arellano, Bai, and Kehoe (2019). Subsequent studies showed that shocks originated in the financial sector can propagate to the rest of the economy leading to financial recessions, as in the work of Jermann and Quadrini (2012), Khan and Thomas (2013), Buera and Moll (2015).

Assessing the quantitative importance of these mechanisms has been a large and an ongoing area of research, as several forces can either dampen or amplify their effects. For instance, Chari (2012) and Moll (2014) pointed out that non-financial firms might be able to self-finance themselves. More closely related to our paper, Di Tella (2017), Carlstrom, Fuerst, and Paustian (2016), Dmitriev and Hoddenbagh (2017) show that financial contracts that are state contingent with respect to aggregate shocks can mitigate financial amplification, while Bocola and Bornstein (2023) study study how trade credit within long-term supplier relationships amplifies the effects of financial shocks.[4]

[3] These policies have been implemented by several OECD countries, often as a temporarily accelerated tax depreciation and temporary payroll tax cuts on new hires.

[4] Other related papers introducing optimal contracts in business cycle models are Boldrin and Horvath (1995), Kehoe and Perri (2002), Cooley, Marimon, and Quadrini (2004).

Our paper differs from these studies by highlighting a new channel that mitigate the effects of financial shocks. In practice, we focus on wage contracts rather than financial contracts, and we provide rich empirical evidence supporting the main mechanism at the micro level. Conceptually, we emphasize the importance of contracts that are long-term in nature and state-contingent on idiosyncratic characteristics –and not only on aggregates.

This paper also connects to the literature on optimal wage contracts within long-term employment relationships. Building on the seminal work of Thomas and Worrall (1988) and Harris and Holmstrom (1982), optimal wage contracts have been studied in the context of rich search models of the labor market by Fukui (2020), Balke and Lamadon (2022), Souchier (2023), with an emphasis on the role of on-the-job search. However, this literature abstracts from firms' investment decisions and financial market imperfections limiting firms' access to credit. We see our paper as complementary to their work, as we study the role of financial frictions and firms' investment decision in affecting optimal wage contracts. Related to the seminal studies by Michelacci and Quadrini (2009) and Berk, Stanton, and Zechner (2010), which explore properties of optimal wage contracts with firms' financial constraints, we develop a business cycle model where wages respond to both idiosyncratic and aggregate shocks, provide empirical evidence supporting the model's predictions and assess quantitatively their macroeconomic implications for business cycle and stabilization policies.

This paper also addresses the recent macroeconomic literature exploring the link between wage rigidity and financial frictions. The work of Favilukis, Lin, and Zhao (2020), Schoefer (2021), Wang (2022), Donangelo et al. (2019), and Acabbi, Panetti, and Sforza (2020), builds on the idea of "labor as leverage", where wage rigidity increases firms' leverage, as poorly flexible payrolls act like predetermined debt obligations. Our research adds to this by considering wage rigidity arising endogenously from the optimal contract between firms and risk-averse workers. While nesting the idea of "labor as leverage" in our framework, we demonstrate through both modeling and data that financially constrained firms tend to adjust wages more following a shock. Crucially, we highlight the role of wage back-loading arising within long-term employment relationship in easing financial constraints. Thus, we show that the view of long-term employment relationship as simply a source of wage rigidity misses important patterns in the data that have large quantitative implications over the business cycle.

Finally, this paper relates to several studies documenting the effectiveness of payroll and investment subsidies. Cahuc, Carcillo, and Le Barbanchon (2018) and House and Shapiro (2008) found that the stimulus of these policies is substantial, while Neumark and Grijalva (2017) and Zwick and Mahon (2017) highlighted their increased effectiveness during the Great Recession.[5] Using a general equilibrium model, we document rich

[5] Saez, Schoefer, and Seim (2019) also documented larger impacts of payroll subsidies on financially constrained firms

interactions between these policies and how firms set wages over long-term employment relationships during a financial crises. We show both conceptually and quantitatively that optimal wage adjustments implied by the dynamic nature of wage contracts make temporary payroll subsidies less effective at stimulating output while making temporary investment subsidies more effective.

The paper is organized as follows. Section 2 describes the model. Section 3 illustrates the model mechanism: it characterizes properties of dynamic wage contracts, it shows how wages vary with firms' financial conditions, and illustrates how hiring and investment decisions depend on the structure of wage contracts. Section 4 illustrates novel empirical evidence on wage dynamics that validate predictions of the model. Section 5 presents the main quantitative results, discussing the macroeconomics implications of dynamic wage contracts for business cycle and stabilization policies. Section 6 concludes.

2 Model

We consider an economy populated by a continuum of entrepreneurs and workers. Entrepreneurs are heterogeneous in their productivity and produce output using capital and labor. There are financial frictions in the form of a collateral constraint, that is the borrowing capacity of entrepreneurs is limited by a fraction of the value of their capital stock, that serves as collateral. We model aggregate financial shocks as a decrease in the collateral value of capital. These features of our environment are common to several business cycle models with financial frictions and heterogeneity, as Khan and Thomas (2013), Buera and Moll (2015), Kiyotaki and Moore (2019).

Workers and entrepreneurs meet in a frictional labor market and engage in long-term employment relationships. As a result workers can be employed or non-employed, and entrepreneurs can be either matched with workers or vacant. Before matching with a worker, entrepreneurs offer wage contracts that specify the path of wages for any possible history of future shocks, as in Thomas and Worrall (1988). We describe the environment in detail in Section 2.1, we illustrate the decision problems and value functions of entrepreneurs and workers in Section 2.2, we define macroeconomic aggregates in Section 2.3, we define the equilibrium in Section 2.4, and we conclude by discussing some of the model assumptions in 2.5

2.1 Environment

Time is discrete and indexed by $t = 0, 1, \ldots$. There is a continuum of entrepreneurs with measure 1, indexed by $j \in [0, 1]$, and a continuum of workers with measure M, indexed by $i \in [0, M]$. All agents in the economy have time-separable preferences with discount factor β, but entrepreneurs and workers differ in their utility functions. Entrepreneurs have utility

$$E_0 \sum_{t=0}^{\infty} \beta^t v\left(c_{jt}\right), \quad v(c) = \frac{c^{1-\sigma_E}}{1-\sigma_E}$$

and workers have utility

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_{it}\right), \quad u(c) = \frac{c^{1-\sigma_W}}{1-\sigma_W}$$

Production technology

If entrepreneurs are not matched with a worker, they produce the value of home production $\bar{b}$. Similarly, non-employed workers also produce $\bar{b}$. Matched entrepreneurs produce output using a standard constant returns to scale production function $f(k, \ell)$, and they are heterogeneous in their idiosyncratic productivity z, so that output y_{jt} is given by

$$y_{jt} = z_{jt} \times f(k_{jt}, \ell_{jt})$$

Idiosyncratic productivity z follows a discrete Markov process, taking values $z \in \{z_1, \ldots, z_{N_z}\}$, with transition matrix Π_z. We assume that realizations of productivity shocks are independent across entrepreneurs and also independent over time. These assumptions imply a law of large numbers so the share of entrepreneurs experiencing any particular sequence of shocks is deterministic. As we assume that entrepreneurs can hire only one worker, we write the production function more compactly as $f(k) = f(k, 1)$. Entrepreneurs have access to a technology that can convert final good into physical capital one for one. In what follows we drop the j subscript whenever it is not needed for clarity.

Financial markets

Entrepreneurs are the only agents in the economy that have access to financial markets. They can borrow or save using uncontingent risk-free bonds that are in zero net-supply. Borrowing is subject to a standard collateral constraint, as entrepreneurs can borrow an amount b_{t+1} that must be less or equal to a share ξ_t of their capital stock k_{t+1}, that is

$$b_{t+1} \leq \xi_t k_{t+1}$$

While the collateral constraint is assumed to be exogenous here, it can be obtained as an endogenous outcome in an environment with limited enforcement of debt contract, where entrepreneurs can decide to not repay their debt and steal profits and share $(1 - \xi_t)$ of their capital stock.[6]

The collateral value of capital ξ_t is common to all entrepreneurs and it follows a discrete Markov Process. It can take two values $\xi \in \{\xi_L, \xi_H\}$ with transition matrix Π_ξ. We interpret ξ_H as the collateral value of capital in normal times, and ξ_L as the collateral value in recession. The stochastic behaviour of the collateral value of capital allows us to study the macroeconomics implications of aggregate financial shocks, as in Jermann and Quadrini (2012), Khan and Thomas (2013), Buera and Moll (2015), Bocola and Bornstein (2023). We refer to a change from ξ_H to ξ_L as a financial shock, as when $\xi = \xi_L$ entrepreneurs face more limited access to credit.

We assume that workers do not have access to financial markets, that is they are hand-to-mouth. This is a common assumption in models of dynamic wage contracts as it simplifies the contracting problem, and is also in line with the view that firms have better access to financial markets than workers. We see this as a conservative assumption, as it greatly limits the ability of firms to adjust wages.

Labor market

Entrepreneurs can be matched to a worker or they can be vacant. Workers can be employed or not employed. Matched entrepreneurs and employed workers separate with probability ϕ. Non-employed workers and vacant entrepreneurs meet in a frictional labor market with directed search, as in Moen (1997). In the economy there is a continuum of sub-markets with a constant returns to scale matching function $m(v, s)$, where v is the measure of vacant entrepreneurs and s is the measure of workers searching in a given sub-market.

Each vacant entrepreneur can open a vacancy in one sub-market. When an entrepreneur opens a vacancy he commits to a wage contract $\mathcal{C} = \{w_\tau(z^\tau, \xi^\tau)\}_{\tau=t}^{\infty}$ that specifies wages contingent on all future histories of idiosyncratic shocks z^τ and aggregate shocks ξ^τ. We assume that workers can also commit to a wage contract upon matching with an entrepreneur. Each non employed worker can search for a job in one sub-market. We use $\theta = v/s$ to denote the labor market tightness of each sub-market. Given the matching function, one can define the job finding probability $\lambda_w(\theta)$ and the probability of filling a vacancy $\lambda_f(\theta)$ as

$$\lambda_w(\theta) = \frac{m(v, s)}{s}, \qquad \lambda_f(\theta) = \frac{m(v, s)}{v}$$

Each sub-market is indexed by the tuple (θ, W), where W is the expected utility of a worker conditional on finding a job in that sub-market. When entrepreneurs open a vacancy in a sub-market indexed by (θ, W), they commit to a wage contract that will

[6] See Bocola and Lorenzoni (2023) for an example of the limited enforcement problem with a collateral constraint.

deliver the worker an expected utility equal to W. As each entrepreneur can open only one vacancy, we assume that there are no vacancy posting costs. A non employed worker can search for a job, but search is costly and it implies a disutility cost, as in models of non-participation similar to Krusell et al. (2017). We assume that non-employed workers who search for a job have to forgo a share x of the value of home production, in line with empirical evidence that non employed workers have to spend time searching for a job. Therefore, the flow utility of a non employed worker earning the flow output of home production $\bar{b}$ and searching for a job is equal to $u((1-x)\bar{b})$.

Timing

At the beginning of each period idiosyncratic and aggregate shocks z_{jt}, ξ_t are realized. Each period can be divided into two stages, that we label as morning (or *before* matching and separation) and afternoon (or *after* matching and separation).

In the morning, matched entrepreneurs produce output and pay wages to the employed workers. Vacant entrepreneurs and non-employed workers produce $\bar{b}$. Then, vacant entrepreneurs post vacancies and non-employed workers search for jobs. Vacant entrepreneurs decide whether they want to open a vacancy or not, and if they do so they decide in which market indexed by (θ, W). Some entrepreneurs may not find it profitable to open a vacancy, and if so they will stay vacant until the beginning of the subsequent period. Similarly, non employed workers decide whether to search for a job or not. Conditional on searching, non employed workers choose a market indexed by (θ, W) where to locate.

At the end of each morning matching and separation take place. Matched entrepreneurs can become vacant with probability ϕ, while vacant entrepreneurs who opened a vacancy in a market with tightness θ will be matched to a worker with probability $\lambda_f(\theta)$. Similarly, non-employed workers who search for a job in a market with tightness θ will be matched to an entrepreneur with probability $\lambda_w(\theta)$.

In the afternoon all agents consume. Workers consume the income earned in the morning, before matching and separation: if they were employed they consume the wage they earned, if they were not employed they consume $\bar{b}$ or $(1-x)\bar{b}$, depending on whether they searched for a job. All entrepreneurs solve a consumption/saving problem. We assume that vacant entrepreneurs cannot hold capital, so they save using risk-free bonds.[7] Matched entrepreneurs decide on how much to borrow or save in the risk-free bonds, and how much capital stock to hold next period.[8]

[7]Note that vacant entrepreneurs would never choose to hold physical capital as long as the interest rate is not less than minus the depreciation rate, as capital depreciates without producing any output when entrepreneurs are not matched. As a result, vacant entrepreneurs cannot borrow and they save using the risk-free bonds.

[8]Capital is predetermined, as in standard business cycle models. Note that this implies the investment decision of entrepreneurs is risky, as they choose the capital stock before observing realizations of idiosyncratic productivity shocks, as in Angeletos (2007) and David, Schmid, and Zeke (2022).

2.2 Value functions and wage determination

We describe the problem of each agent recursively. First we discuss the recursive state space, and then we describe in details the maximization problem of each agent.

Recursive state space

We characterize the optimal contract recursively. We define a recursive contract as wages and promised utility $w'(z',\xi'), W'(z',\xi')$ that depends on current state variables and are contingent only on the realizations of shocks next period (z',ξ'). This formulation requires to include the utility promised to the worker W as a state variable in the problem of matched entrepreneurs. In other words, in each period, matched entrepreneurs choose state-contingent wages and promised utility for the next period, subject to a promise-keeping constraint where the expected utility of the worker must equal the utility W promised in the previous period.

Heterogeneity across matched entrepreneurs can be summarized by the exogenous state variable z and two endogenous state variables (m, W), where W is the utility promised to the worker and m is net worth, or cash-on hand, that is equal to the sum of output and undepreciated capital stock, minus wage payments and the repayment of outstanding debt, according to the law of motion:

$$m'(z',\xi') \leq z' f(k') + (1-\delta)k' - w'(z',\xi') - b' \tag{1}$$

Similarly, heterogeneity across vacant entrepreneurs can be summarized by the exogenous state variable z and the endogenous state variable m. Heterogeneity across employed workers is fully summarized by their expected utility W, and there is no heterogeneity across non-employed workers as they are hand-to-mouth and there is no ex-ante heterogeneity. The aggregate state of the economy, that is denoted by S is summarized by the realization of the aggregate shock ξ, and distribution of matched and vacant entrepreneurs over their states, that we denote by $\Lambda^m(m, W, z)$ and $\Lambda^v(m, z)$.

Matched entrepreneurs

At the core of our model there is the decision problem of matched entrepreneurs, whose solution characterizes the optimal wage contract. We denote by $J(m, W, z, S)$ their value function after matching and separation, according to equation (2). This depends on net worth m, idiosyncratic productivity z, utility promised to the worker W and the aggregate state of the economy S.

These entrepreneurs choose how much to consume this period, how much to borrow or save b' –where $b' > 0$ means they borrow–, and the capital stock that will be productive next period k'. They also choose how to fulfill their promise to the worker, meaning they decide how to deliver the utility W with state contingent wages and continuation values

$w'(z',\xi'), W'(z',\xi')$.

The budget constraint of matched entrepreneurs at the end of period implies that the sum of consumption and physical capital has to be equal to the sum of net worth m and net borrowing qb'. The law of motion of net worth is given by the sum of output and the undepreciated capital stock minus wages paid to the worker and the repayment of outstanding debt. Borrowing is limited by a collateral constraint, so that firms can borrow up to a share ξ of their future capital stock. The promise keeping constraint makes sure the expected utility of the worker is at least equal to the promised utility W.

The value of being matched with a worker at the end of period is equal to the flow utility of consumption plus the expected continuation values of the entrepreneur. With probability $(1-\phi)$ the match will survive until the end of next period, while with probability ϕ the match will separate and the entrepreneur will get the continuation value V of being vacant at the end of next period. We define the value V in (4).

$$J(m,W,z,S) = \max_{\substack{c^e,b',k',m'(z',\xi'),\\ w'(z',\xi'),W'(z',\xi')}} \Bigg\{ v(c^e) + \beta(1-\phi)\underbrace{\mathbb{E}\left[J(m'(z',\xi'),W'(z',\xi'),z',S')|z,S\right]}_{\text{not separate}} + \beta\phi\underbrace{\mathbb{E}\left[V(m'(z',\xi'),z',S')|z,S\right]}_{\text{separate}}\Bigg\} \tag{2}$$

$$\begin{aligned}
&(\text{Budget constraint}: \lambda^e) && c^e + k' \leq m + qb' \\
&(\text{Net worth}: \eta(z',\xi')) && m'(z',\xi') \leq z'f(k') + (1-\delta)k' - w'(z',\xi') - b' \\
&(\text{Collateral constraint}: \mu) && b' \leq \xi k' \\
&(\text{Promise keeping}: \gamma) && W \leq \mathbb{E}\Big[u(w'(z',\xi')) + \beta(1-\phi)W'(z',\xi') + \beta\phi\mathcal{U}(S'')|z,S\Big]
\end{aligned}$$

Vacant entrepreneurs

Vacant entrepreneurs face two decision problems: before matching and separation they can choose to open a vacancy to become matched by the end of the period, and after matching and separation they face a consumption/savings problem.

First, there is a discrete choice problem between posting a vacancy or not. Entrepreneurs that decide to open a vacancy have to choose a sub-market (θ, W) where to open it. With probability $\lambda_f(\theta)$ the entrepreneur is matched to a worker, where $J(m,W,z,S)$ denotes the value of a matched entrepreneur with utility W promised to the worker. With probability $1-\lambda_f(\theta)$ the entrepreneur remains vacant, where $V(m,z,S)$ denotes the value of a vacant entrepreneur in the afternoon, after matching and separation. The entrepreneur opens a vacancy only if the expected continuation value from doing so is greater than the value $V(m, z, S)$ of being vacant at the end of period.

$$\widehat{V}(m, z, S) = \max\left(\max_{(\theta,W)}\Big\{ [\lambda_f(\theta)J(m, W, z, S) + (1 - \lambda_f(\theta))V(m, z, S)] \Big\}, V(m, z, S)\right) \tag{3}$$

After matching and separation, vacant entrepreneurs decide how much to consume and how much to save, according to (4). The value of being vacant at the end of period is equal to the flow utility of consumption plus the expected continuation value of being vacant next period, before matching and separation. At that stage, net worth will be equal to the returns on savings plus the flow value of home production $\bar{b}$.

$$V(m, z, S) = \max_{a', c^e, m'} \left\{ v(c^e) + \beta \mathbb{E}\left[\widehat{V}(m', z', S') \mid z, S\right]\right\} \tag{4}$$

$$\text{(Budget constraint)}: \quad c^e + qa' \leq m$$

$$\text{(Net worth)}: \quad m' \leq a' + \bar{b}$$

Workers

Workers decide whether to search for a job, and if they search they choose a sub-market (θ, W) where to locate.

The value of a non employed worker before matching and separation, that we denote by $\mathcal{U}$, is defined in equation (5). First, they face a discrete choice problem between searching and not searching.

$$\mathcal{U}(S) = \max\Big(\underbrace{u(\bar{b}) + \beta\mathbb{E}\left[\mathcal{U}(S') \mid \mathcal{S}\right]}_{\text{if not search}}, \underbrace{u(\bar{b}(1-x)) + \beta\mathcal{W}(S)}_{\text{if search}}\Big) \tag{5}$$

The value of a non employed worker who does not search is equal to the flow utility of home production and the expected continuation value of being not employed next period. The value of a non employed worker who does search is given by the flow utility of home production, adjusted for the disutility cost of searching, and the expected continuation value of a worker who search, denoted by $\mathcal{W}(S)$ and defined in equation (6).

$$\mathcal{W}(S) = \max_{(\theta,W)} \left\{\lambda_w(\theta)W + [1 - \lambda_w(\theta)]\,\mathbb{E}\left[\mathcal{U}(S') \mid S\right]\right\} \tag{6}$$

A non employed worker who searches in sub-market (θ, W) finds a job with probability $\lambda_w(\theta)$, and receives expected utility W next period conditional on finding a job. We say that a sub-market is active if there are at last some workers and some entrepreneurs searching in that sub-market. The problem of a worker who searches, as defined by equation (6), implies that workers search in a given sub-market $(\tilde{\theta}, \tilde{W})$ if and only if it is weakly better than searching in any other sub-market, that is:

$$\lambda_w(\tilde{\theta})\tilde{W} + \left[1 - \lambda_w(\tilde{\theta})\right] \mathbb{E}\left[\mathcal{U}\left(S'\right) \mid S\right] \geq \max_{(\theta,W)} \left\{\lambda_w(\theta)W + \left[1 - \lambda_w(\theta)\right] \mathbb{E}\left[\mathcal{U}\left(S'\right) \mid S\right]\right\}$$

As all non-employed workers are homogeneous, the expected continuation value of a worker who searches $\mathcal{W}(S)$ must be equalized across all the active sub-markets.

2.3 Aggregation

We define aggregate output Y_t, capital K_t, employment N_t, and investment I_t as follows.[9]

$$Y_t = \int_0^1 y_{jt} \tag{7}$$

$$K_t = \int_0^1 k_{jt} \tag{8}$$

$$N_t = \int d\Lambda^m_{t-1}(m, W, z) \tag{9}$$

$$I_t = K_t - (1 - \delta)K_{t-1} \tag{10}$$

We define aggregate debt B_t as the sum of gross debt of matched entrepreneurs:

$$B_t = \int \max\left(b(m, W, z, S_{t-1}), 0\right) d\Lambda^m_{t-1}(m, W, z)$$

Finally, we define aggregate productivity A_t such that:

$$Y_t = A_t K_t^\alpha L_t^{1-\alpha}$$

We can characterize aggregate productivity as a function of three terms, according to equation (11), where the expectation and covariance operators are taken with respect to the distribution of matched entrepreneurs Λ^m_{t-1} -normalized so that it adds up to one. The first term shows that A_t is proportional to the average productivity of active entrepreneurs $E[z_t]$. The second term, that is the ratio between $E[k^\alpha]$ and $E[k]^\alpha$, shows that aggregate productivity is lower when the cross-sectional dispersion in capital is higher. Finally, the covariance term implies that A_t is larger when more productive entrepreneurs produce a

[9]Recall that $\Lambda^m_t(m, W, z)$ and $\Lambda^v_t(m, z)$ are the distributions of matched and vacant entrepreneurs at the end of period t, after matching and separation occurred.

larger share of aggregate output.

$$A_t = E[z_t]\left[\frac{E[k^\alpha]}{E[k]^\alpha}\left(1+\frac{Cov(z,k^\alpha)}{E[z]E[k^\alpha]}\right)\right] \tag{11}$$

More precisely, let us denote by $\tilde{\Lambda}^m_{t-1}$ the normalized distribution of matched entrepreneurs. We have that:

$$A_t = \int \sum_{z'} z'\Pi(z'|z)d\tilde{\Lambda}^m_{t-1}(m,W,z)\left[\frac{\left(\int k(m,W,z)^\alpha d\tilde{\Lambda}^m_{t-1}(m,W,z)\right)}{\left(\int k(m,W,z)d\tilde{\Lambda}^m_{t-1}(m,W,z)\right)^\alpha}\left(1+\frac{Cov(z,k^\alpha)}{E[z]E[k^\alpha]}\right)\right] \tag{12}$$

2.4 Competitive equilibrium

Definition 1. *A recursive competitive equilibrium is defined as : i) a law of motion Γ for the aggregate state S, ii) entrepreneurs' policy functions and value functions, iii) workers' policy functions and value functions, iv) distributions of matched and vacant entrepreneurs $\Lambda^m(m,W,z), \Lambda^v(m,z)$ v) price q and market tightness $\{\theta\}$ in active sub-markets, such that:*

- *non-employed workers solve their problem given Γ, $\{(\theta, W)\}$*
- *entrepreneurs solve their problem given ,$\mathcal{W}, q, \Gamma$*
- *law of motion Γ is consistent with the policy functions and the value functions*
- *the measure of workers who search is consistent with $\{\theta\}$*
- *q is such that the bond market clears*

$$\int b'(m,W,z,S)d\Lambda^m(m,W,z) = \int a'(m,z,S)d\Lambda^v(m,z)$$

In Appendix A we show that the definition of equilibrium implies the resource constraint from Walras' Law, and we define formally the law of motion Γ for the aggregate state.

Solving this model poses some challenges, as the decision problems of matched and vacant entrepreneurs depend on $q, \mathcal{W}(S), \mathcal{U}(S)$, that are all endogenous objects, that depend on the exogenous aggregate state ξ as well as the endogenous state S. In Proposition 1, we show that the values $\mathcal{W}, \mathcal{U}$ do not depend on the aggregate state of the economy S.[10] This result substantially simplifies the analysis we conduct in Section 5, as the problem of entrepreneurs depends on S only through (ξ, q).

[10]For this step it is crucial to show that the values $\mathcal{W}, \mathcal{U}$ do not depend on any endogenous aggregate state. Indeed, if these values were functions of exogenous aggregate states only, they would be easy to forecast and they would not pose any computational challenge to solving the model.

Proposition 1. *If the measure of workers M is large, in equilibrium these properties hold: (i) a positive measure of workers does not search (ii) non employed workers are indifferent between searching and not searching (iii) the values $\mathcal{W}(S)$ and $\mathcal{U}(S)$ do not depend on the aggregate state S, (iv) the value functions of entrepreneurs depend on S only through (ξ, q).*

Proof: See Appendix A

2.5 Discussion

Before moving on, let us discuss some of the assumptions we made.

First, we assume that financial frictions takes the form of a collateral constraint, consistent with a large literature that built on Kiyotaki and Moore (1997). We could have alternatively modeled financial frictions as a working capital constraint, in the spirit of Jermann and Quadrini (2012) or Bocola and Lorenzoni (2023), where firms raise funds with intraperiod loans to purchase inputs. This alternative modeling assumption would amplify the effects of wage back-loading on investment: lower wages would not only free up resources for investment by increasing net worth but also enhance borrowing capacity, as entrepreneurs would need to borrow less to pay the wage bill.

Second, we assume that entrepreneurs -firms' owners- have a concave utility function and cannot issue equity. While this assumption is fairly common in models of financial frictions (e.g. Moll (2014), Kiyotaki and Moore (2019)), our mechanism would extend to an economy with risk-neutral entrepreneurs -e.g. Khan and Thomas (2013)- that can issue equity subject to some adjustment costs, as in Jermann and Quadrini (2012). Wage contracts would still solve a risk-sharing problem, where wages co-move with the marginal value of a dollar for the entrepreneur.[11]

Third, we assume that workers and firms can commit to a wage contract. Firms' commitment is a common assumption in the literature studying dynamic wage contracts, as in Harris and Holmstrom (1982) and Balke and Lamadon (2022), and it is often motivated by firms' reputational concerns. Here we assume that workers can also commit to a contract, as in Boldrin and Horvath (1995), because we abstract from on-the-job search in our model. Indeed, the reason why workers usually cannot commit to a wage contract is they keep searching for better opportunities while currently employed. Interestingly, workers' commitment is not key to one of the main mechanism of the model, that is firms back-load wage payments when they need more credit. Indeed, if workers could

[11]In this class firms don't distribute dividends and don't consume, as long as there is a positive probability of facing a binding borrowing constraint at any time in the future.

not commit to a wage contract firms would *further* tilt the wage-tenure profile of their workers using wage back-loading as a tool to retain workers (e.g. Balke and Lamadon (2022), Souchier (2023)).

Fourth, we assume that each entrepreneur can hire only one worker. Despite this assumption is common to several search models of the labor market, it is not without loss of generality in a model with investment and search frictions. For instance, firms' ability to adjust wages of incumbent workers might affect the decision of hiring new employees within the same multi-worker firms. While this would be an interesting extension, the implied contracting problem would be intractable with an almost infinite dimensional state variable, since one would have to keep track of the promised utility offered to each worker.[12]

Fifth, we assume that workers are hand-to-mouth. This assumption is common to several search models of the labor market and models of dynamic wage contracts, and consistent with the view that firms have better access to financial markets than workers. Note that if workers had unrestricted access to financial markets, then entrepreneurs would implicitly borrow from their employees as to completely offset the effects of financial frictions. It would be interesting to consider an intermediate case, where workers could save and borrow subject to some borrowing constraint (Souchier, 2024). In this setting firms could implicitly borrow from their employees even more then they do under our stark assumption of hand-to-mouth workers, potentially giving firms more room to back-load wages after a credit tightening.

3 Model mechanism

This section characterizes properties of the optimal wage contract and illustrates the main model mechanism.

In Section 3.1 we consider a special case of our model that is analytically tractable. We show that the optimal wage contract takes a simple form: wages are a constant share of entrepreneurs' net worth over the length of an employment relationship. This tractability allows to explicitly characterize how wages adjust over time and in response to shocks as a function of entrepreneurs' leverage, that is b/k. We focus on leverage as in our model firms with higher leverage are more likely to be financially constrained. We show that entrepreneurs with high leverage adjust wages more in response to idiosyncratic shocks, and increase wages more over time.

In Section 3.2 we turn to the general case. We use optimality conditions to illustrate

[12]Workers hired in different years will have different promised utilities, so one would have at least one state variable for each cohort of incumbent workers. The problem can be made tractable with some specific assumptions, as in Michelacci and Quadrini (2009), that consider a deterministic environment, but the general case is not.

the determinants of the investment, consumption, and savings decision. Then, we explain the main trade-off in wage setting, highlighting similarities with the special case. Both cases share the same underlying logic, which accounts for the heterogeneity in wage dynamics across firms. We illustrate that in the general case entrepreneurs that are more financially constrained *i)* back-load wage payments (i.e. pay steeper wage-tenure profile), *ii)* back-load wage payments even more during recessions, and *iii)* adjust wages more in response to idiosyncratic shocks.

Finally, Section 3.3 highlights the link between the structure of wage contracts, investment, and job creation, by considering the problem of vacant entrepreneurs.

3.1 Special case: analytical results

In this section we consider a special case of the problem of matched entrepreneurs that is analytically tractable. This special case allows us to solve analytically for the policy functions of consumption and wages, and to characterize how wage dynamics vary with entrepreneurs' financial conditions. We consider an economy where both entrepreneurs and workers have log utility, the production technology is linear in capital, and there is no separation.

$$v(c_t^e) = \log(c_t^e), \qquad u(w_t) = \log(w_t), \qquad f(k_t) = k_t, \qquad \phi = 0$$

Combining the optimality conditions for state-contingent wages and workers' continuation values we obtain a risk-sharing condition, according to equation (13). Indeed, because both entrepreneurs and workers can commit to future wages, a wage contract offers a full set of state-contingent claims within the employment relationship that implies perfect risk-sharing. Since we assume that entrepreneurs and workers have the same preferences, this risk-sharing problem implies that wages move one-for-one with the entrepreneur's consumption.

$$\frac{c^e\left(z^{t+s}, \xi^{t+s}\right)}{w\left(z^{t+s}, \xi^{t+s}\right)} = \frac{c^e\left(z^{t+k}, \xi^{t+k}\right)}{w\left(z^{t+k}, \xi^{t+k}\right)} \quad \forall z^{t+s}, z^{t+k}, \xi^{t+s}, \xi^{t+k} \tag{13}$$

The next proposition characterizes policy functions for consumption and wages.

Proposition 2. *Entrepreneur's consumption and worker's wage are linear in net worth:*

$$c_t = (1-x)m_t, \qquad w_t = \gamma(1-x)m_t$$
$$\textit{with} \quad 1 - x = \frac{1-\beta}{1+\beta\gamma}$$

where γ is the multiplier on the promise keeping constraint, that is constant over time.

Proof. See Appendix A

Proposition 2 shows that wages are a constant share of entrepreneurs' net worth over the length of an employment relationship. This constant share depends on the discount factor β and the multiplier on the promise keeping constraint γ. If entrepreneurs initially promised a higher utility W to the worker, the multiplier γ will be higher. Intuitively, this result implies that entrepreneurs with low net worth pay lower wages, and wages are expected to grow and adjust with net worth.

An immediate corollary of this result is that wages increase when idiosyncratic productivity increases, as that leads to an increase in future net worth of the entrepreneur.

Corollary 1. *State contingent wages $w(z_{t+s}|z^{t+s-1})$ are increasing in productivity z_{t+s}.*

We now turn to study how wage dynamics depend on entrepreneurs' financial conditions. To this end, we define leverage as the ratio between entrepreneurs' debt and capital, that is b/k. Because of the collateral constraint, leverage must be below the collateral value of capital ξ, and entrepreneurs with higher leverage are more financially constrained. The next proposition characterizes how wages change both in response to idiosyncratic shocks and over time depending on the leverage of the entrepreneur.

Proposition 3. *For each time t, the pass-through of idiosyncratic productivity shocks to wages, defined in (14), and the expected growth rate of wages, defined in (15), are increasing in entrepreneurs' leverage b_{t+1}/k_{t+1}.*

$$\frac{\partial[\log(w_{t+1}) - \log(w_t)]}{\partial[\log(z_{t+1}) - \log(z_t)]} \tag{14}$$

$$\mathbb{E}[\log(w_{t+1}) - \log(w_t)] \tag{15}$$

Proof. See Appendix A

Proposition 3 implies that highly levered entrepreneurs adjust wages more in response to idiosyncratic shocks, as for the same change in productivity they adjust wages more. Intuitively, this result is implied by the law of motion of net worth and Proposition 2. Because entrepreneurs borrow using uncontingent debt, a higher leverage makes their net worth in $t+1$ more sensitive to fluctuations in future productivity at z_{t+1}. As their net worth fluctuates more, so must do the wage according to Proposition 2.

Proposition 3 also implies that the expected growth rate of wages is higher at highly levered entrepreneurs. The fact that leverage is endogenous means that highly levered entrepreneurs are choosing to borrow more and invest more in capital –relative to their net worth–. This means that highly levered entrepreneurs must expect higher returns from their investment, and thus they promise higher returns to their workers in the form of higher wage growth.

Note that the results from Proposition 3 apply to a cross-section of firm at any given point in time. We want to stress this feature of our results with respect to growth rate of wages. This means that in normal times wages grow faster at more levered entrepreneurs, but also that during recessions wages grow faster at more levered entrepreneurs.

3.2 General case

We now turn to the problem of a matched entrepreneur in the general case. First, we discuss the optimal choice of investment, debt and consumption using the optimality conditions. Then, we illustrate how wage dynamics depends on entrepreneurs' financial conditions, emphasizing how the general case shares the same underlying logic underneath the analytical result from Section 3.1.

3.2.1 Investment, debt, and consumption

The recursive problem of matched entrepreneurs defined in (2) implies fairly common optimality conditions for debt and capital:

$$v'(c^e) = \frac{\mu}{q} + \frac{1}{q}\mathbb{E}\left[\eta\left(z', \xi'\right)\ |z, \xi\right] \tag{16}$$

$$v'(c^e) = \mu\xi + \mathbb{E}\left[\eta\left(z', \xi'\right)\left[z' f'(k') + 1 - \delta\right]\ |z, \xi\right] \tag{17}$$

where μ denotes the multiplier on the collateral constraint, $\eta(z', \xi')$ denotes the multiplier on the law of motion of net worth that is the marginal value of a dollar for the entrepreneur in state (z', ξ').

In the general case we have $f(k) = k^{\alpha}$. Compared to the special case, the marginal product of capital is decreasing in k. This implies there is always a positive and fined first-best level of capital, according to equation (18), that we denote by k^{FB}.[13]

$$\frac{1}{q} - (1-\delta) = f'(k^{FB})\mathbb{E}[z'|z] \tag{18}$$

Because of financial frictions, capital is always below the first best level, that is $k' < k^{FB}$. Indeed, distortions appear in the optimality condition for capital, that can be re-arranged to obtain (19) by combining (16) and (17). The presence of borrowing constraint on the left hand-side of (19) reduces entrepreneurs' capital whenever the constraint is binding, that is $\mu > 0$. The presence of uninsurable idiosyncratic risk lowers the returns to capital on the right hand-side of (19), which are equal to the product of the risk-neutral marginal product of capital and a risk adjustment term. This is because, as in

[13]The first-best level of capital is the optimal choice of entrepreneurs in an economy with complete markets. This means there are no borrowing constraints, entrepreneurs can issue state-contingent claims to perfectly insure against idiosyncratic shocks. Note that in the first-best case there are no aggregate financial shocks, as borrowing is unconstrained.

the seminal work of Guiso and Parigi (1999), investing in capital is risky as entrepreneurs' consumption is correlated with their idiosyncratic productivity. This risk premium term reduces returns to capital because the marginal value of a dollar for the entrepreneur, $\eta(z', \xi')$, is negatively correlated with future productivity z'.

$$\underbrace{\frac{\mu}{\mathbb{E}\left[\eta\left(z', \xi'\right)|z, \xi\right]}\left(\frac{1}{q} - \xi\right)}_{\text{borrowing constraint}} + \frac{1}{q} - [1 - \delta] = f'\left(k'\right)\mathbb{E}\left[z'|z\right]\underbrace{\left[1 - \left|\frac{\text{Cov}\left(\mathbb{E}\left[\eta\left(z', \xi'\right)|z', \xi\right], z'|z\right)}{\mathbb{E}\left[\eta\left(z', \xi'\right)|z, S\right]\mathbb{E}\left[z'|z\right]}\right|\right]}_{\text{idiosyncratic risk}} \tag{19}$$

Entrepreneurs with low values of net worth face a binding collateral constraint. Indeed, they are less able to self-finance investment in capital and *ceteris paribus* they need to borrow more. For higher level of net worth the collateral constraint will not be binding, and entrepreneurs will approach the first-best level of capital. As the collateral constraint is eased, leverage of entrepreneurs decreases, and entrepreneurs with higher net worth build a sufficiently high buffer stock of savings to insure against idiosyncratic shocks. As a result, the covariance term on the right-hand side of (19) decreases with net worth, making capital further close to k^{FB}.

3.2.2 Risk sharing

Equation (20) is the optimality conditions for state-contingents wages $w'(z', \xi')$. On the left-hand side of (20), the marginal cost of increasing $w'(z', \xi')$ is equal to the marginal value of a dollar for the entrepreneur in that state of the world. Intuitively, the marginal value of a dollar for the entrepreneur is high when net worth is low, that is when the entrepreneurs' level of capital is likely below the first best level. On the right-hand side, the benefit of increasing wages is large in states when workers' marginal utility is low, and proportional to the multiplier γ on the promise keeping constraint.

$$\eta(z', \xi') = \gamma u'(w'(z', \xi')) \tag{20}$$

$$\gamma = \gamma(z', \xi') \tag{21}$$

Crucially, (20) implies a trade-off underneath wage setting between smoothing earnings of risk-averse workers –from the right hand-side– and smoothing output to relax the effects of financial frictions –from the left hand-side–. If the collateral constraint binds, smoothing output means invest more in capital in the short run to ease the constraint. If the collateral constraint does not bind, smoothing output means building a buffer stock of savings to avoid binding constraints in the future.

As in the special case, the optimal wage contracts between entrepreneurs and workers can be described as the solution to a risk-sharing problem. The ratio between the marginal utility of the worker and the marginal value of a dollar for the entrepreneur is equal to γ, which does not depend on future states (z', ξ'). The optimality conditions for state-contingent continuation values $W'(z', \xi')$ implies that γ has to be constant over time, according to equation (21). Therefore, as long as the entrepreneur and the worker do not separate, the optimal wage contract implies perfect risk-sharing with respect to idiosyncratic productivity shocks and aggregate financial shocks.[14] As consumption of entrepreneurs depends on their net worth, wages also depend on net worth because of optimal risk sharing, in a way that reminds the special case.

Differently from the special case, the degree of risk-sharing underneath wage contracts depend on the relative risk-aversion coefficients of entrepreneurs and workers, σ_E and σ_W. Under the assumption of CRRA preferences, σ_W captures both the relative risk-aversion coefficient and the inverse elasticity of inter-temporal substitution. In Appendix D we characterize the solution of the risk-sharing problem under the assumption that workers have Epstein-Zin preferences (Epstein and Zin, 1989), and we highlight the role played by risk-aversion and inter-temporal substitution. Interestingly, the ability of firms to adjust wages over time depends both on the elasticity of inter-temporal substitution and the degree of risk-aversion of workers. Indeed, future earnings are uncertain due to the risk of separation, and wage dynamics depend on workers' preference towards persistence risk, that is a function of both risk-aversion and inter-temporal substitution.

3.2.3 Wage dynamics and firms' financial conditions

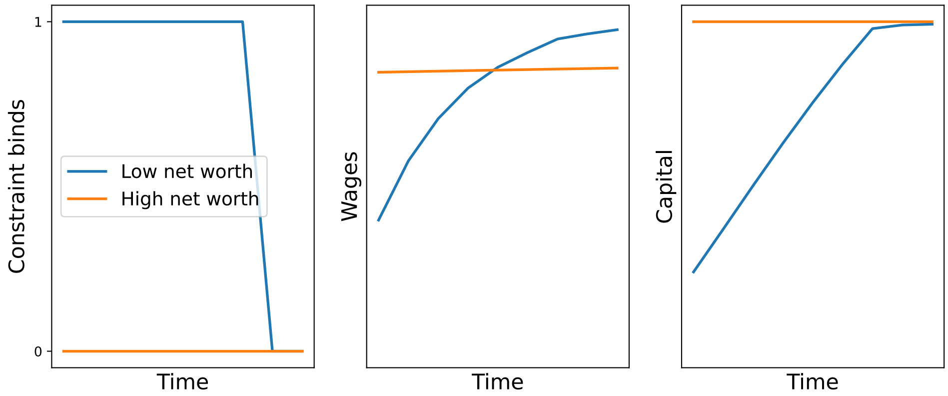
In the special case presented in Section 3.1 wage dynamics depend on financial frictions: more constrained entrepreneurs adjust wages more in response to idiosyncratic shocks and pay steeper wage-tenure profiles. We know illustrate the mechanism in more details for the general case, where entrepreneurs with low net worth are *ceteris paribus* financially constrained, as they are not able to self-finance investment.[15]

To illustrate the mechanism consider two entrepreneurs that start at time $t = 0$ with different net worth m_0, but same promised utility to the worker and same productivity (W_0, z_0) .Figure 1 plots whether the constraints binds, wages, and capital as a function of time for these two entrepreneurs.

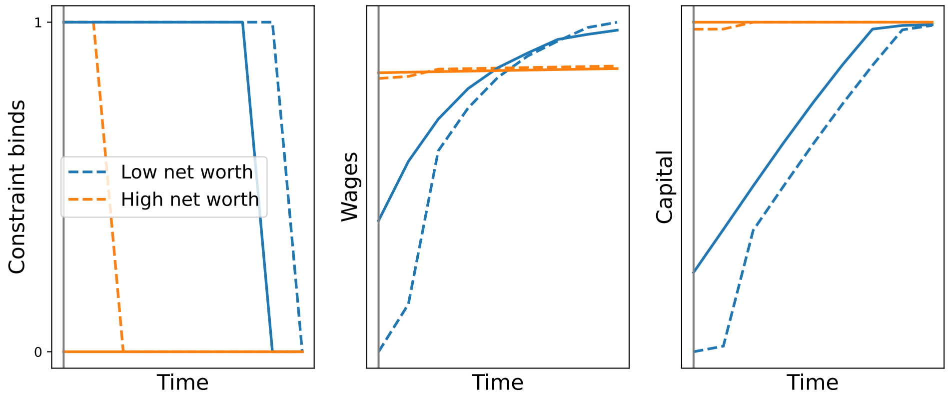
Panel (a) considers a case where no shocks a realized. In this case, the entrepreneur that starts with high net worth is unconstrained, and both capital and wages are almost flat over time. On the other hand, the entrepreneur that starts with lower net worth is financially constrained and offers a steep wage tenure profile to the worker. This means wages are lower initially, but they increase more over time as the entrepreneur accumulates net worth and approaches the first best level of capital. As in this example

[14]Since wages cannot be paid after separation, entrepreneurs and workers are still exposed to the idiosyncratic risk of separation.

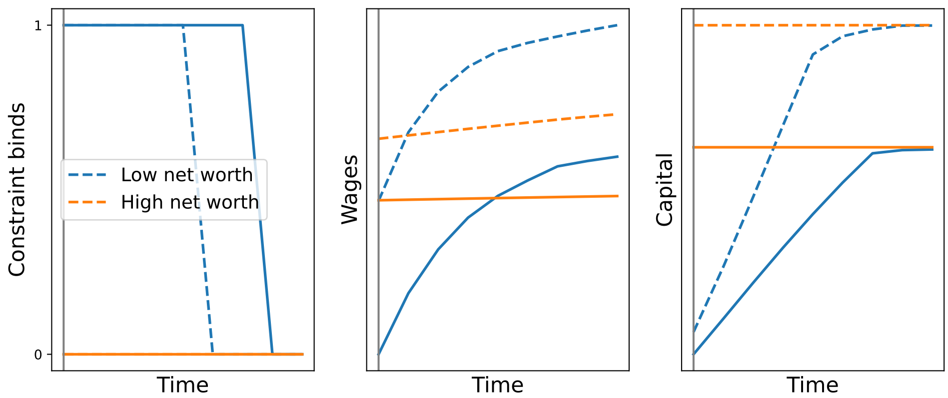
[15]This means that in the entrepreneurs with low net worth have higher leverage, so that by emphasizing the link between wage dynamics and net worth we also illustrate how the findings from the special case extend to the general case.



(a) No shocks



(b) Aggregate financial shock: $\xi \downarrow$



(c) Idiosyncratic productivity shock: $z \uparrow$

Figure 1: Wage-tenure profile, capital and net worth dynamics for entrepreneurs with same (W, z) but different net worth m in the first period. Wages, capital and net worth are plotted an entrepreneur with low net worth (blue line) and high net worth (orange line).

both workers receive the same expected utility W_0, but the timing of wage payments differ, we say that the entrepreneur with low net worth back-loads wages. This property of wage contracts aligns with the findings of Michelacci and Quadrini (2009), that studies optimal wage contract when firms are subject to financial constraints in a deterministic environment.

Consider now an aggregate financial shock. Panel (b) plots whether the constraint binds, wages and capital as a function of time, when there is a one-time financial shock at $t = 0$ (dashed line). The entrepreneur with high net worth is only mildly affected by the shock, that leads to small changes in wages and capital. The entrepreneurs with low net worth is greatly affected by the shock, leading to a substantial contraction in current and future investment. In response to the shock, the entrepreneur with low net worth makes the wage-tenure profile of the worker steeper, thus back-loading wages more after an aggregate financial shock.

The dashed lines in Panel (c) plot whether the constraint binds, wages and capital as a function of time when there is a positive idiosyncratic productivity shock at time $t = 0$.[16] The mechanism is the same underneath Corollary 1 and Proposition 3. An increase in productivity leads to higher net worth and higher wages, because of optimal risk-sharing. The entrepreneur with low net worth is more sensitive to changes in idiosyncratic productivity, as higher leverage makes his returns more correlated with fluctuations in output. Consequently, entrepreneurs with low net worth adjust wages more in response to idiosyncratic shocks.

3.3 Job creation, investment, and wages

The dynamic structure of wage contract, namely the fact that entrepreneurs can adjust wages both over time and in response to shocks, affect both investment and job creation. First, we discuss the implications for investment, relating to the problem of matched entrepreneurs. Then, we characterize the optimal job creation decision of vacant entrepreneurs and we explain how it depends on the dynamic structure of wage contracts.

3.3.1 Investment and wages

The ability of firms to adjust wages both over time and in response to shocks enhances investment. First, the ability to back-load wages over time allow entrepreneurs that are currently financially constrained to pay lower wages in the short term so to have more resources to invest in capital. Additionally, as illustrated in (20), wage contracts determine the degree of risk-sharing between the entrepreneur and the worker. When an

[16] We consider a permanent increase in productivity that occurs at period $t = 0$, so that productivity increases and stays higher in future periods.

entrepreneur shares some idiosyncratic risk with the worker, all else equal, it decreases his exposure to idiosyncratic risk, thus making investment less risky and more attractive.

To understand these effects, consider an alternative economy where wages are determined on a spot labor market. Spot market wages would not be contingent on firms' idiosyncratic productivity simply because of the law of one price. As a result, the entrepreneurs would bear entirely the investment risk, making capital less attractive. Similarly, if wages would be determined on a spot labor market, entrepreneurs would not adjust the wage-tenure profile of workers according to their financial conditions.

Importantly, even a small change in the current wage can have large effects on the entire dynamics of investment. Consider an entrepreneur with a binding constraint that offer a lower wage at time t. Current investment mechanically increases as more resources become available. If the collateral constraint binds also in future periods, future investment also increases. Indeed, higher investment in t increases net worth in $t+1$, which further enhances investment in $t+1$, thus increasing net worth in $t+2$, and so on.[17] As highlighted in Khan and Thomas (2013) and Moll (2014), the effects of financial frictions are persistent both in recession and in the cross-section, suggesting that a small change in wages can have persistent effects on investment. This dynamic effect of current wages on future investment has implications for job creation, that we illustrate in the next section.

3.3.2 Job creation

Job creation is determined by the solution of the problem of vacant entrepreneurs, according to (3), who decide whether to open a vacancy and a sub-market in which to post. The latter decision implies the job creation equation (22) that mimics a standard surplus sharing rule common to a large class of search models.[18] On the left-hand side of (22) there is the ratio between the surplus of the entrepreneur, that is $J - V$, and the surplus of the worker, that is $W - \mathcal{U}$. On the right-hand side there is the product between a standard term in search models that reminds of the Hosios' condition[19] and the multiplier on the promise keeping constraint γ, that intuitively captures how costly it is for the entrepreneur to deliver the promised utility W.

In the standard competitive search model the path of wages is not uniquely pinned down, as standard assumptions on risk-sharing and complete financial markets make it irrelevant in terms of the equilibrium allocation. In this model, because of incomplete markets, the specific path of wages is allocative for job creation and thus affects the equilibrium allocation. We emphasize two ways in which the dynamic structure of wage

[17] Note that if an entrepreneur becomes unconstrained in $t+1$, higher earnings in $t+1$ will have little effects on investment in $t+1$ as the entrepreneur would be able to self-finance himself.

[18] Appendix A derives equation (22) from the problem of vacant entrepreneurs using Proposition 1.

[19] In the efficient allocation of a standard search model, the ratio between firms' surplus and workers' surplus is equal to the ratio $(1-\eta)/\eta$. See Hosios (1990).

contracts affect job creation.

$$\frac{J(m,W,z,S)-V(m,z,S)}{W-\mathcal{U}} = \underbrace{\left(-\frac{\partial J(m,W,z,S)}{\partial W}\right)}_{\gamma(m,W,z,S)}\left(\frac{1-\eta}{\eta}\right) \tag{22}$$

First, the structure of wage contracts affects job creation through the surplus $(J-V)$. For instance, when an entrepreneur with low net worth back-load wages to enhance investment, he will make the worker more productive over the length of a match increasing the surplus $(J-V)$. In the previous section we emphasized that even a small change in current wage can have large effects on the dynamics of investment when credit constraints bind for several periods. Crucially, job creation that depends on the present discounted value of a match and thus internalizes these dynamics effects of wages on investment.

Second, the entrepreneur optimally set wages in order to minimize the present discounted value of all future wage payments made to the worker. Intuitively, the ability of making state-dependent wage adjustments in the future decreases the cost of hiring a worker in present discounted value terms today, thus fostering job creation. The following lemma formalizes this result.

Lemma 1. *The optimal wage contract that solves* (2) *is also a solution to:*

$$\min_{\{w_{s+1}(z^{s+1},\xi^{s+1})\}_{s=t}^{\infty}} \mathbb{E}\left[\sum_{s=t}^{\infty} w_{s+1}(z^{s+1},\xi^{s+1}) \times [\beta(1-\phi)]^{(s-t)}\frac{\eta_{s+1}(z^{s+1},\xi^{s+1})}{v'(c_t^e)}\right] \tag{23}$$

$$\textit{subject to:} \quad W \leq \mathbb{E}\left[\sum_{s=t}^{\infty}[\beta(1-\phi)]^{(s-t)}u(w_{s+1}(z^{s+1},\xi^{s+1})) \; + \; [\beta(1-\phi)]^{(s-t)}\beta\phi\mathcal{U}\right]$$

Proof: See Appendix A

4 Empirical evidence

We use matched employer-employee data from Italy to provide empirical evidence supporting model's prediction on wage dynamics.

In Section 3.1 we showed that financially constrained firms –those with higher leverage– adjust wages more in response to idiosyncratic shocks and offer wages that are expected to grow faster. In Section 3.2 we illustrated the broad implications of these results for how wages adjust with tenure at different firms. Financially constrained firms offer their workers a steeper wage-tenure profile, and thus increasing wages over time as they accumulate more capital and net worth. This heterogeneity in wage-tenure profiles across firms is particularly pronounced after an aggregate financial shock, where firms with high leverage make the wage-tenure profiles of their workers even steeper in response to the shock.

In the empirical analysis, we validate two key predictions of the model: financially constrained firms adjust wages more in response to idiosyncratic shocks and offer steeper wage-tenure profiles during recessions. Testing these predictions is crucial for two reasons. First, the differential wage response to idiosyncratic shocks provides evidence for the *risk-sharing* mechanism in wage-setting. In practice, we estimate that financially constrained firms display higher pass-through of value-added per worker to wages than unconstrained firms. We focus on pass-through coefficients as a measure of risk-sharing between firms and workers, building on the work of (Guiso, Pistaferri, and Schivardi, 2005; Guiso and Pistaferri, 2020). Second, the steeper wage-tenure profiles offered by constrained firms during recessions highlight the *dynamic* nature of wage adjustments over the length of employment relationships, at the same time emphasizing how firms optimally adjust wages to ease the effects of an aggregate tightening. In the data, we show that during the Great Recession financially constrained firms offered steeper wage-tenure profiles to their workers than the unconstrained ones.

We consider firms with higher leverage to be more financially constrained, in line with our model and several studies that use leverage as a proxy for the strength of financial frictions across firms (Gopinath et al., 2017; Ottonello and Winberry, 2020). Leverage is a particularly suitable proxy in countries like Italy, where firms rely heavily on bank financing and where bank credit is largely collateralized (Garrido, Kopp, and Weber, 2016; Affinito, Sabatini, and Stacchini, 2021). Furthermore, evidence shows that firms with higher leverage were more significantly impacted by the Great Recession and the EU Sovereign Debt crises (Arellano, Bai, and Bocola, 2017; Kalemli-Özcan, Laeven, and Moreno, 2022).

The section is organized as follows: first we describe the data and the institutional background, then we present the empirical strategy and the main results.

4.1 Matched employer-employee data

The analysis relies on matched employer-employee data sourced from official social security records maintained by the Italian Social Security Institute (Istituto Nazionale Previdenza Sociale, INPS). This comprehensive dataset, spanning from 2005 to 2019, includes the entire employment history of all private-sector employees who were at any time employed by firms participating in the Survey of Industrial and Service Firms (INVIND survey) conducted by the Bank of Italy. The relevant population for the INVIND survey includes firms in manufacturing and services, with at least 20 employees. Known as the INPS-INVIND dataset, it integrates employer and employee data, covering approximately

10 million workers and 10 thousand firms.[20]

For each job spell in every year, we observe worker and firm identifiers, along with gross earnings, the number of weeks worked in full time equivalent units, part-time status and a coarse occupational code (apprentice, blue collar, high-skilled blue collar, white collar, middle manager or manager). For each worker we also observe a series of basic demographic characteristics such as gender and year of birth. We complement the INPS-INVIND data by matching it to balance sheet and income statement information from CERVED.[21] For each firm in the sample we retrieve firms' total assets, value added and various measures of firm debt, including debt with banks, suppliers, or other intermediaries. These variables are particularly important as they allow us to construct different measures of leverage. We describe in more details the data, the cleaning procedure, and the sample construction in Appendix B.

4.2 Institutional background

The relevant tiers for wage formation in the Italian labor market are at the industry and company levels (Guiso, Pistaferri, and Schivardi, 2005). The first pillar consists of sectoral bargaining agreements at the industry level that establish minimum wages (contractual minimums, or *minimi tabellari*) for different occupational classes. The second pillar consists of company-specific supplementary components of the compensation package, where the firm can decide on some components of the compensation unilaterally and can also agree to a company contract with the unions. The most important share of the workers' compensation established at the company level is a company-level wage increment (*superminimum*), added to the contractual minimum on a permanent basis (in nominal terms); the increment has a firm-level and a worker-level component. Other forms of firm-specific compensations are transitory production bonuses (*premi di produzione*), and a contingent component (*retribuzione variabile*).

A substantial share of workers' compensation package is determined at the firm level. Using data from 1975 to 2000, Card, Devicienti, and Maida (2014) show that nearly all employees earn some premium above the contractual minimum, with the median premium being 24%. Guiso, Pistaferri, and Schivardi (2005) reports that in 1994 the average wage component due to firm-specific pay policies was around 23%. The latter grew to around 30% in 2009 according to the same data source (Daruich, Di Addario, and Saggio, 2023). Since then, the Italian labor market has witnessed a gradual erosion of centralized bargaining agreements (D'Amuri and Giorgiantonio, 2015). This process

[20]The INPS-INVIND data have been used in a number of recent studies, including (Macis and Schivardi, 2016; Daruich, Di Addario, and Saggio, 2023; Di Addario et al., 2023) among others.

[21]CERVED is a leading Italian data provider, offering detailed balance sheet data and comprehensive business information.

culminated in the Inter-sectoral Agreement of June 2011 which broadened the scope of decentralized wage-bargaining and defined the procedures for its activation. Meanwhile Law 148/2011 introduced the possibility of signing firm and local-level agreements in derogation of the law and of the national collective agreements.

4.3 Estimates of pass-through coefficients

We estimate the pass-through coefficients of value-added per worker to wages, following a well established methodology that built on Guiso, Pistaferri, and Schivardi (2005). Estimates of pass-through coefficients in the range between 5% and 15% have often been interpreted as evidence of some form of risk-sharing, or partial insurance, within long-term employment relationships.

In our empirical analysis we isolate idiosyncratic changes in firms' value added per worker from fixed heterogeneity (i.e. firm level fixed effects) and changes common to all firms in the same sector. To this end, we construct residuals of log of value added per worker against firm-specific fixed effects and 2-digit NACE sector-year fixed effects. Then, we take first differences of the residuals, that we denote by ε_{jt} and winsorize these changes at the 1st and 99th percentile to remove for possibly extraordinary events that are not present in our model. We measure monthly wages as workers' annual earnings at the same employer divided by the number of weeks worked. We focus on a sample of *stayers*, who work full-time and for 52 weeks in the firm for at least two consecutive years. We isolate idiosyncratic changes in workers' earnings from changes that can be attributed to demographics characteristics (gender-age-occupation fixed effects) or aggregate trends (year fixed effects). Then, we take first differences of the residuals, that we denote by ω_{jt} and winsorize these changes at the 1st and 99th percentile to remove for possibly extraordinary events that are not present in our model.

Once we have constructed measures of earnings growth and growth in value added per worker, we estimate the pass-through of value added per worker to wages using standard techniques from Guiso, Pistaferri, and Schivardi (2005); Guiso and Pistaferri (2020). We define the pass-through coefficient as the following moment of the data:

$$\mathcal{P} = \frac{Cov\left(\Delta w_{ijt}, \sum_{s=-1}^{1} \Delta\varepsilon_{jt+s}\right)}{Cov\left(\Delta\varepsilon_{jt}, \sum_{s=-1}^{1} \Delta\varepsilon_{jt+s}\right)} \tag{24}$$

which corresponds to the instrumental variable regression of Δw_{ijt} on $\Delta\varepsilon_{jt}$, using $\Delta\varepsilon_{jt-1} + \Delta\varepsilon_{jt} + \Delta\varepsilon_{jt+1}$ as an instrument. The instrumental variable strategy filters out the effect of purely transitory shocks, which we interpret as measurement error. In a model with permanent productivity shocks and static pass-through as Guiso, Pistaferri, and

Estimates of pass-through coefficients		
	(1)	(2)
$\Delta\varepsilon_{jt}$	0.0612***	
	(0.001)	
$\Delta\varepsilon_{jt} \times$ 1(Below median leverage)		0.0474***
		(0.001)
$\Delta\varepsilon_{jt} \times$ 1(Above median leverage)		0.0680***
		(0.001)

Robust standard errors in parentheses.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 1: Estimates of the pass-through coefficient of firms' value-added per worker to wages, obtained using the instrumental variable estimator defined in (24). The first column reports the average pass-through. In the second column we split the sample and report estimates for firms with leverage above and below median.

Schivardi (2005), this estimator recovers the true pass-through of shocks to the persistent component of idiosyncratic productivity to wages.

In our model the pass-through coefficient is not static –as wages adjust dynamically adjust in response to shocks– and productivity shocks are not permanent, so the pass-through coefficient of value-added per worker to wages is different from the pass-through of idiosyncratic productivity shocks to wages. We regard these pass-through coefficients as crucial moments of the data that measure the degree of risk-sharing between firms and workers, which we use to discipline and validate our model. We estimate an average pass-through coefficient of 6.1%, as reported in Table 1, in line with the existing literature. Using Italian data from 1982 to 1994 Guiso, Pistaferri, and Schivardi (2005) finds an average pass-through coefficient of 6.8%.

We proceed by estimating how the pass-through coefficient varies with firms' leverage. We split firms according to whether at $t-1$ their leverage was above or below the median in the leverage distribution of that year, and estimate the pass-through coefficient for each sub-sample. Our baseline measure of leverage is the ratio between firms' debt, measured as the sum of all financial debt and debt towards suppliers, divided by the firm's total assets. Results are reported in Table 1.[22] Firms with leverage above median have a pass-through coefficient of 6.8%, that is almost 1.5 times larger than the estimated pass-through for firms having leverage below median.

4.4 Wage-backloading in the Great Recession

We provide empirical evidence that firms with high leverage offered steeper wage-tenure profiles during the Great Recession. To this end, we estimate the difference in the wage-

[22]Results are robust to considering alternative measures of leverage.

tenure profiles between firms with high leverage and low leverage for workers hired during the Great Recession.

We estimate a wage-tenure profile over the first four years of tenure for workers hired during the Great Recession, according to equation (25). We restrict our attention to workers whose first year of year-round employment at the firm was in $t_0 = 2009$. This means that we are restricting our sample to workers hired during 2008 or at the very beginning of 2009. In this way we are comparing wage-tenure profiles of workers hired when the drop in firms' credit was most pronounced.[23] As usual, we focus on full-time workers employed year-round as to abstract form changes in earnings related to an imprecise measure of the effective days worked in a given year and from changes in hours worked.

$$\begin{aligned}\log w_{ij(t_0)t_0+s} = \beta_0 + \beta_s T_{ij(t_0)t_0+s} + \gamma_s T_{ij(t_0)t_0+s} 1(lev_{jt_0} > \text{median}) \\ + \delta_s X'_{ijt_0} T_{ij(t_0)t_0+s} + u_{ij(t_0)t_0+s}, \quad \text{for } s = 0, 2, \ldots \end{aligned} \tag{25}$$

On the left-hand side, $w_{ij(t_0)t_0+s}$ are earnings of worker i, hired by firm j in period t_0, observed at tenure s, namely in period $t_0 + s$. The variable $T_{ij(t_0)t_0+s}$ is a dummy equal to one if tenure at $t_0 + s$ is equal to s, and zero otherwise. The coefficient β_s for $s = 0, 1, \ldots$ estimates a non-parametric wage-tenure profile. We allow this wage-tenure profile to differ for firms with leverage above and below median at the time of hire, that is t_0. In other words, the coefficient γ_s measures the difference in log-wages at tenure s for workers hired in t_0 by firms with leverage above median compared to those hired in t_0 by firms with leverage below median. We allow the tenure profile to depend on certain observable characteristics of firms and workers at the time of hire, included in the control vector X_{ijt_0}.

Simply comparing workers with different tenure levels at firms with different leverages would induce bias, as better workers or higher-quality matches may correlate positively with lower quit rates (e.g. Altonji and Shakotko (1987)). By exploiting the nature of our data, we focus on a sample of workers hired in a given year (Topel, 1991) who then remain with the same firm for at least five years, in the spirit of controlling for completed tenure as in Abraham and Farber (1987).[24] For this sub-sample of *stayers*, we only consider the first five years in the firm.

As we aim to measure how wage differences across firms with different leverage evolve over time, our focus is on the coefficient $(\gamma_s - \gamma_0)$. Fixed heterogeneity in match-specific quality that affects earnings independently of tenure is not a concern, as it would not influence $(\gamma_s - \gamma_0)$.

[23]Total corporate debt of non-financial firms in Italy dropped substantially in 2008 and started to recover in 2009.

[24]We find similar results using a sample of workers that stayed at the firm for seven years.

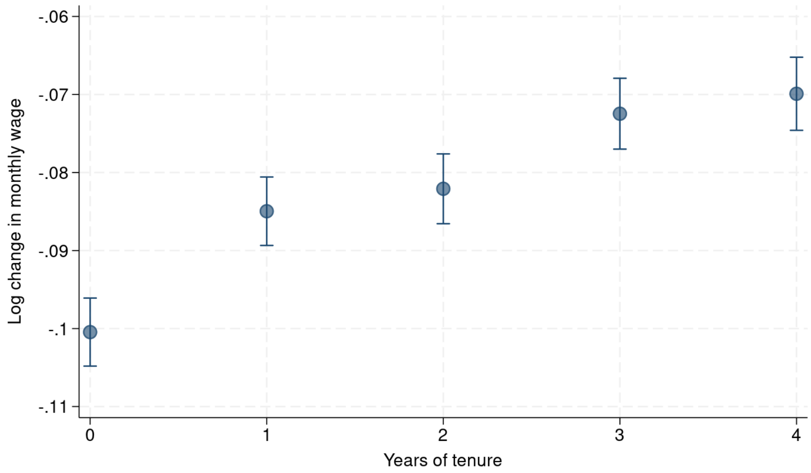


Figure 2: We plot estimates for the coefficients $\{\gamma_s\}_{s=0}^4$ in equation (25), for $t_0 = 2009$.

One remaining concern is that leverage at t_0 may be correlated with other firms' characteristics at t_0 that have an effect on the wage-tenure profile independently from leverage. For this reason, we allow the tenure profiles to depend on firms' total assets and firms' value-added per worker at the time of hire, by including them in X_{ijt_0}. A similar concern is that highly levered firms can persistently hire different types of workers, whose earnings are expected to differently vary over time. For this reason, we control for workers' occupation, gender and age specific tenure profile, including occupation, gender and date of birth in the vector of controls X_{ijt_0}, thus isolating the differential tenure effect from being employed by a highly levered firms for observationally similar workers.

We plot estimates of γ_s in Figure 2, considering workers hired in 2009, that is in the midst of the Great Recession. The estimate of the coefficient γ_0 shows that firms with leverage above median offered to their newly hired workers a wage that is 10% lower compared to similar workers hired by firms with leverage below median.[25] Estimates of $(\gamma_s - \gamma_0)$ are informative on how wage differences across firms with different leverage evolve over time. We find that over time, wages grow faster for workers employed by highly levered firms: in the first four years of tenure, wages grow 3 percentage points more. These results provide evidence for the heterogeneity in wage-tenure profile illustrated in Figure 1: firms with higher leverage pay offer wage tenure profiles during an aggregate credit tightening.

4.5 Wage back-loading and investment

This section provides descriptive evidence supporting the model mechanism by exploiting differences in wage flexibility across firms. We show that firms with more flexibility in

[25] Michaels, Beau Page, and Whited (2019) show that highly levered firms pay lower wages, something which we also find (see Pagano et al. (2020) for a recent survey).

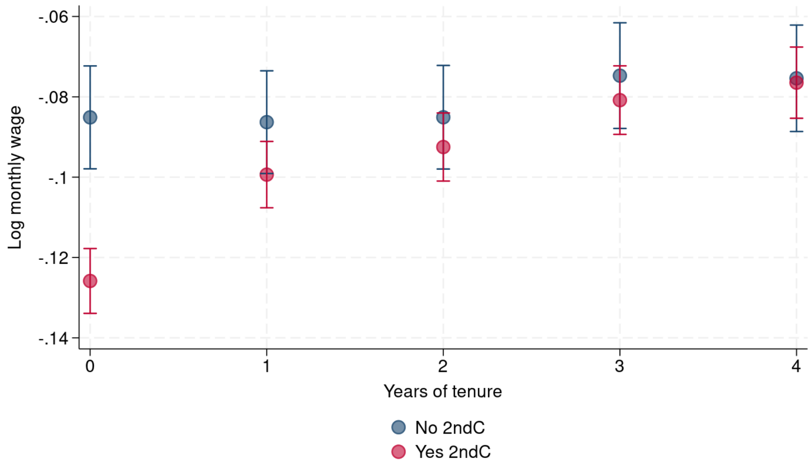


Figure 3: We plot estimates for the coefficients $\{\gamma_s\}_{s=0}^{4}$ in equation (25), setting $t_0 =$ 2009 for firms that had "second level contract" (red) and firms who did not (in blue).

wage setting back-loaded wages more during the Great Recession while experiencing a lower drop in investment.

The INVIND survey provides information on whether firms in our sample signed any "second level contract". In the Italian labor market, "second-level contracts" (also known as decentralized bargaining agreements) refer to collective agreements negotiated at a level below the national sectoral agreements –typically at the company or regional level. These contracts complement the national collective agreements (known as first-level contracts), which set the baseline conditions for wages, hours, and benefits across an entire sector. These contracts enhance wage flexibility by allowing firms to adjust wages based on specific performance or productivity metrics, as well as regional economic conditions. First, we estimate equation (25) separately for firms that did and did not have any "second level contract" in place during 2008. We plot estimates of γ_s in Figure 3 for these two groups of firms. The sample differs from the one used in Section 4.4 as the information on second level contracts is not available for all firms in our sample. Estimates from (25) show that firms with high leverage and a second level contract during 2008 back-loaded wages of newly hired workers substantially more than highly levered firms without a second level contract during 2008. In other words, financially constrained firms with greater flexibility in wage setting offered steeper wage-tenure profiles to workers hired during the Great Recession.

Then, we turn to study the investment dynamics of these different groups of firms during the Great Recession. We estimate a triple-difference specification, as described in equation (26), to measure the differential response of investment during the Great

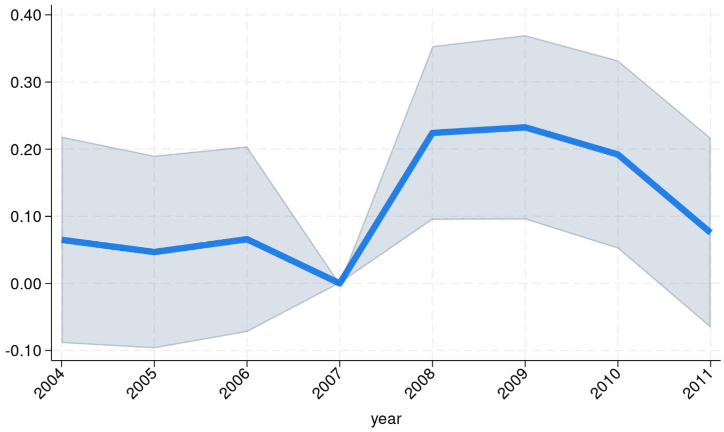


Figure 4: The blue line interpolates estimates for the coefficients $\{\pi_s\}_{s=0}^{4}$ in equation (26), setting $t_0 = 2009$. The gray shaded area corresponds to the 95% confidence intervals.

Recession for financially constrained firms with and without second level contracts.

$$\log(i/k)_{jt} = \beta_0 + \sum_s \alpha_s year_s 1(lev_{jt_0} > \text{median}) + \sum_s \beta_s year_s 1(2ndC_{jt_0}) \\ \sum_s \pi_s year_s 1(2ndC_{jt_0})) 1(lev_{jt_0} > \text{median}) + \sum_s \delta_s X_j' + u_{jt} \quad (26)$$

On the left-hand side of (26) there is the logarithm of investment rate of firm j at time t. On the left-hand side, $2ndC_{jt_0}$ is a dummy equal to one if firm j has a second level contract at time t_0. The coefficient π_s measures the difference in $\log(i/k)$ between firms with leverage above median with a second level contract and firms with leverage above median without a second level contract. We control for sector-year fixed effects by including firms' sectors in X_j', as the dynamics of investment rates over time can vary substantially across sectors.

Estimates of the coefficients π_s are plotted in Figure 4. While there is no significative difference between investment rates of firms with and without second level contracts before the Great Recession, estimates of the coefficients π_s during the Great Recession are positive and statistically significant. These estimates imply that that highly levered firms with second level contracts experienced a less pronounced drop in investment during the Great Recession compared to highly levered firms without second level contracts.

This descriptive evidence provides additional empirical support for the model mechanism, as firms with greater flexibility in wage setting back-loaded wages more during the Great Recession and experienced a less pronounced drop in investment. These results are consistent with the model mechanism illustrated in Section 3, namely that wage back-loading frees resources for investment. In the next section we use our quantitative model to study the macro-economic implications of dynamic wage contracts during financial crises.

5 Quantitative analysis

We present the quantitative model starting from the calibration of model's parameters. Then, we assess the ability of the quantitative model to reproduce salient features of the data in Section 5.2. Sections 5.3 and 5.4 present the main quantitative results aimed at quantifying the importance of dynamic wage contracts for the propagation of financial shocks and for the effects of stabilization policies. We solve the model using techniques from Krusell and Smith (1997), including the price q as a state variable in the entrepreneurs' problems and approximating a forecasting rule for future prices. Details are described in Appendix C.

5.1 Calibration

We begin by describing how we choose parameters for our quantitative analysis. We interpret one period in the model as one quarter. The firm level production function and the matching technology are both Cobb-Douglas and described in equations (27), (28), where B is a constant that measures matching efficiency, η is the matching function elasticity and α is the production function elasticity.

$$f(k,\ell) = k^{\alpha}\ell^{1-\alpha} \tag{27}$$

$$m(v,s) = Bv^{1-\eta}s^{\eta} \tag{28}$$

Nine parameters are assigned and listed in Table 2. We set the common discount factor β equal to 0.99, the depreciation of capital equal to 0.025, and the elasticity of the Cobb-Douglas production function equal to 0.3, as these are standard values in macroeconomics. We follow Ljungqvist and Sargent (2017) in setting the matching function elasticity η equal to 0.5, based on estimate for several EU countries reviewed in Petrongolo and Pissarides (2001). We assume that entrepreneurs have log-utility, that is $\sigma_E = 1$, following a large body of work that studied the effects of financial frictions in models with heterogeneity, as Midrigan and Xu (2014), Moll (2014), Kiyotaki and Moore (2019).[26] We set $P(\xi_H|\xi_H) = 0.99$ to be consistent with the notion that financial crises are rare events in advanced economies.We set $P(\xi_L|\xi_L) = 0.8$, meaning the average duration of a recession in the model is five quarters, that corresponds to the length of the Great Recession in Italy according to the OECD based recession indicators. Finally, we set the probability of separation in the model equal to 0.024, as to match the separation rate measured in D'Amuri et al. (2022) for Italy.[27] We set the disutility cost of searching for a job in line

[26]This assumption is not far from empirical evidence. Estimates from Herranz, Krasa, and Villamil (2015) imply a median relative risk aversion coefficient of entrepreneurs of 1.5, obtained using a sample of small firms in the US.

[27]Using microdata from the Labour Force Survey, D'Amuri et al. (2022) find a quarterly EU rate (employment to unemployment) of 1.2% and a quarterly EN rate (employment to out of labor force) of

with evidence on the amount of time spent searching for a job (Manning, 2011; Krueger and Mueller, 2010). In practice, we set x equal to 0.05, meaning that agents have to forego 5% of the value of home production when they search for a job.

The remaining parameters are estimated in two separate exercises. We recover the parameters that discipline the stochastic process for firms' idiosyncratic productivity, the persistence ρ and the standard deviation $\sigma(\varepsilon)$, by estimating a stochastic process for firms' productivity from balance-sheets data. We use a GMM estimator that filters out fixed heterogeneity across firms as well as purely transitory productivity shocks, as none of them are present in the model. Details are discussed in Appendix B.3. The remaining five parameters $(\xi_H, \xi_L, B, \bar{b}, \sigma_W)$ are chosen simultaneously so that the model matches a set of moments of interest. Below, we describe the targeted moments and discuss heuristically which model parameters they help us discipline.

We pin down the collateral value of capital ξ_L during financial crises by targeting the observed 14% drop in the aggregate debt of the non-financial sector in 2008[28], following the same calibration strategy proposed in Khan and Thomas (2013). We set the collateral value of capital during normal times ξ_H, that is the maximum leverage b/k that a firm can have, as to match the 99-th percentile of firms' leverage distribution. Motivated by the analytical results illustrated in Section 3, that is wages co-move with entrepreneurs' consumption according to the ratio of the two relative risk aversion coefficients, we set the relative risk aversion coefficient of workers in order to match the average pass-through of value-added per worker to wages that we estimated in Section 4. We estimate the value of home production $\bar{b}$ and the efficiency of the matching function B following a calibration strategy similar to Shimer (2005). We target an average unemployment rate of 8.1%, that was the average unemployment rate in Italy for the years before 2008, and an average job finding rate equal to 30%, consistent with empirical evidence from D'Amuri et al. (2022), Cingano and Rosolia (2012).

1.2% for men aged 35-55, that implies a quarterly separation rate equal to 2.4%. We focus on males between 35-55 as the impact of fertility, schooling, and retirement decision on the EN rate is negligible, as these features are not present in the model.

[28]The outstanding amount of total debt securities in non-financial corporations sector was 100 billions in Q1 of 2008 and 87 billions in Q4 of 2008.

Assigned parameters				
Parameter	Intuition		Value	
β	Discount factor		0.99	
δ	Depreciation rate		0.025	
α	Capital share		0.3	
η	Matching function elasticity		0.5	
σ_E	RA coefficient of entrepreneurs		1	
$P(\xi_L\|\xi_H)$	Probability of financial recession		0.01	
$P(\xi_L\|\xi_L)$	Persistence of financial recession		0.8	
ϕ	Separation probability		0.024	
x	Share of time spent searching for a job		0.05	
Externally calibrated parameters				
Parameter	Intuition		Estimated value	
ρ	Persistence of idiosyncratic productivity z		0.96	
$\sigma(\varepsilon)$	Std. deviation of innovations in z		0.07	
Internally calibrated parameters				
Parameter	Value	Moment	Model	Target
ξ_L	0.33	Drop of corporate debt in 2008	-14%	-14%
ξ_H	0.87	99p of leverage distribution	0.87	0.87
σ_W	11	Pass-through of VA/worker to wages	0.06	0.06
B	0.5	Job finding rate	0.30	0.30
$\bar{b}$	0.15	Unemployment rate	8.1%	8.1%

Table 2: Values for all model's parameters.

5.2 Model vs Data

The quantitative model can match several untargeted moments describing wage dynamics, leverage distribution and investment dynamics. The first panel of Table 3 shows that the model can quantitatively replicates the heterogeneity in wage dynamics across leverage that we documented in Section 4. As in the data, the pass-through of value-added worker to wages is higher for firms with leverage above median, and the magnitudes are comparable. Also, we estimate the wage tenure-profile for workers hired in recession by firms with leverage above and below median. We find that wages of workers hired by highly levered firms in recession grow 3 percentage points more over the first 4 years of tenure, consistently with the empirical evidence presented in Figure 2.

The model generates a cross-sectional distribution of firms' leverage similar to the data. Panel B in Table 3 reports the three quartiles of the leverage distribution, that are remarkably similar between the model and the data, despite targeting only the 99th percentile. Note that in normal times only few firms face a binding collateral constraint, as the 75-th percentile of the leverage distribution is substantially below ξ_H. We measure the average standard deviation of investment rates (i/k) in the data using a balanced panel of firms, finding the same value documented in Cooper and Haltiwanger (2006),

Panel A: Wage dynamics				
	Pass-through coefficients × 100		Additional wage growth at levered firms	
	Leverage < median	Leverage > median	Cum. growth, 4y horizon	
Model	4.1	7.1	3%	
Data	4.7	6.8	3%	
Panel B: Investment and leverage				
	p25 leverage	p50 leverage	p75 leverage	Investment volatility
Model	0.20	0.46	0.63	0.30
Data	0.25	0.43	0.70	0.33
Panel C: Macroeconomic effects of a financial shock				
	Drop in aggregate variables, first year of recession			
	$\Delta\log(Y_t)$	$\Delta\log(N_t)$	$\Delta\log(I_t)$	$\Delta\log(A_t)$
Model	-4.7 %	-2.9%	-24 %	-3.6 %
Data	-7.1 %	-3.4%	-12 %	.
Panel D: Distribution of wage adjustments				
	Skewness		Kelley's skewness	
	Recession	Normal times	Recession	Normal times
Model	1.1	2.5	-0.47	- 0.02
Data	0.20	0.71	0.01	0.20

Table 3: Panel A reports moments for the dynamics of wages in the model and empirical findings from Section 4. Panel B reports moments for the distribution of leverage and investment dynamics in the model and in the data. Panel C reports the drop of macroeconomics aggregates from peak to through in the first year of the recession using data and the impulse response functions of the model. Panel D reports the skewness of the wage adjustment distribution in the data (Adamopoulou et al., 2016) and in the model.

which is very close to the value implied by the model.[29]

We inspect the ability of our model to reproduce the dynamics of macroeconomic aggregates during the 2008 recession in Italy. Panel C in Table 3 reports the drop in aggregate output, employment, total factor productivity, and investment from peak to trough during the first year of recession. The model has been calibrated to match a drop in aggregate debt of 14%, and it accounts for a substantial share of the observed drop in aggregate output, consistent with the view that financial shocks played a key role in the Great Recession. The model also implies a substantial drop in aggregate employment, investment and total factor productivity. The dynamics of employment and capital depend on a direct effect and a general equilibrium effect.

A drop in the collateral value of capital ξ leads to a contraction in debt and investment for entrepreneurs with low net-worth that are not able to self-finance themselves. The contraction in aggregate debt generates excess savings in the economy that increases the

[29] An alternative calibration strategy for the productivity process would have been to estimate the standard deviation of innovation to productivity by targeting the standard deviation of investment rates, as in Khan and Thomas (2013). We rather estimate a productivity process externally and we keep this moment for the validation.

price of bonds q. At the same time, an increase in q induces wealthy unconstrained entrepreneurs to substitute away from risk free bonds and to invest more in physical capital.[30]

The response of aggregate employment is disciplined by the job creation decision of vacant entrepreneurs. When ξ drops, the surplus of a match $J - V$ falls for entrepreneurs with low net worth. As a result, they either post a lower promised value W – with a drop in the vacancy filling rate $\lambda_f(\theta)$– or they decide not to open a vacancy at all, leading to a drop in aggregate employment. In general equilibrium the price of bonds increases fostering job creation of unconstrained entrepreneurs. Indeed, entrepreneurs with high net worth experience an increase in the match surplus $J - V$, as investment in capital becomes relatively more attractive than saving in risk-free bonds.

While the direct effect dominates, the coexistence of these two countervailing channels affects allocative efficiency. Because of the direct effect, constrained entrepreneurs with high marginal product of capital are forced to reduce their investment, while wealthy entrepreneurs with lower marginal product of capital invest more. This reallocation of capital towards unconstrained entrepreneurs leads to a drop in aggregate productivity A, as defined in equation (11).

The model is not only consistent with evidence on the heterogeneous wage dynamics presented in Section 4, but also with stylized facts on aggregate wage dynamics. The model implies a modest cyclicality of the average wage in recession, that drops by 0.3%. The modest decline in the average wage masks substantial heterogeneity in the cross-section: firms strongly impacted by the credit tightening cut their wages substantially, while unconstrained firms expand and pay higher wages. In the aggregate, this firm-level heterogeneity in wage adjustments implies that the skewness of the wage adjustment distribution is lower in recessions than in booms. This feature of our model is consistent with recent evidence that the skewness of the wage adjustment distribution changes during recession (Adamopoulou et al., 2016), as reported in Panel D of Table 3. The model is also consistent with evidence from Gertler, Huckfeldt, and Trigari (2020), Grigsby, Hurst, and Yildirmaz (2021) that wages of new hires are as cyclical as wages of incumbents, and with evidence from Kudlyak (2014) and Basu and House (2016) that the average user cost of labor is substantially more cyclical than the average wage.

5.3 The macroeconomic implications of dynamic wage contracts

Dynamic wage contracts, meaning the ability of firms to adjust wages over time and in response to shocks, play an important role when the economy is hit by an aggregate financial shock. In order to quantitatively evaluate this channel we compare our model

[30] This heterogeneous effects of an aggregate financial shock on firms' investment decision is similar to the one illustrated by Khan and Thomas (2013).

to a counterfactual economy where firms cannot optimally adjust wages.

There are two fundamental assumptions in the model that allow firms to adjust wages over time and in response to shocks: employment relationships are long-term in nature and firms can commit to future wages. Indeed, for firms to be able to back-load wage payments they must stay matched with the same workers for more than one period, and they must be able to implicitly promise higher future wages to their employees. To quantitatively assess the importance of dynamic wage contracts during financial crises, we relax the former assumption on firms' commitment, as this exercise can be done without changing the economic environment.[31]

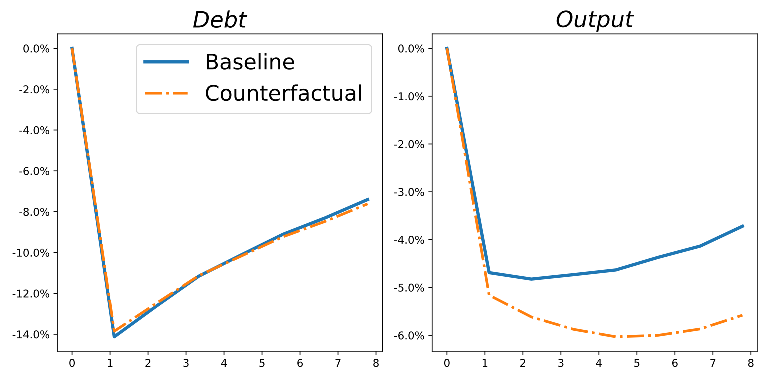
We propose a counterfactual economy that has two features of a spot labor market: firms cannot adjust timing of wage payments over the length of an employment relationship, and the allocative wage for job creation is only the current wage w_t. These features are obrained by assuming entrepreneurs can commit only to a wage for the first period after matching with a worker. More precisely, if an entrepreneur hires a worker in period t, we assume that the entrepreneur can commit only to a wage w_{t+1}, that is for the first period in which the match is productive. Consistently with the assumption of no commitment on the firms' side, we also assume that workers cannot commit to an employment relationship with a given entrepreneur. As a result, entrepreneurs would pay workers their outside option $\bar{b}$ for any period $s > t + 1$, as this is a dominant strategy compared to any wages above $\bar{b}$. As in the baseline model, firms compete for workers in the expected utility W of a match, but they to do so by choosing only w_t. Therefore, the main difference from the baseline model is that firms cannot choose how to deliver the utility W over time and in response to shocks by adjusting wages.

Figure 5 plots the impulse response function of macroeconomic aggregates to an aggregate financial shock in the baseline model and in the counterfactual economy with no commitment. The two economies display the same drop in aggregate debt.[32] However, in the counterfactual economy output is more than one percentage point lower compared to the baseline economy, meaning that dynamic wage contracts substantially mitigate the effects of an aggregate financial shock on output.

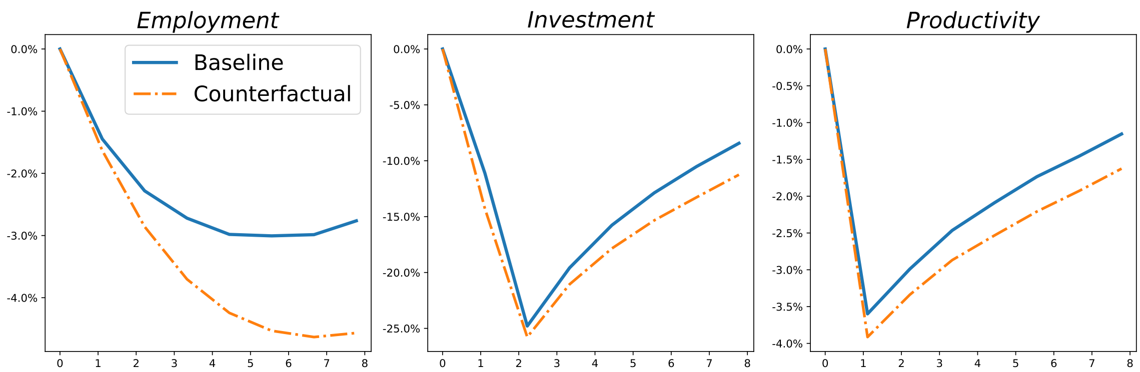
The differential response of output is primarily driven by differences in aggregate employment and productivity. In the baseline economy firms optimally adjust wages to ease the effects of the credit tightening. As a result, aggregate employment and investment fall less in the baseline economy, as illustrated in Figure 5. In other words, in the baseline economy financially constrained entrepreneurs invest more than in the counterfactual

[31]On the other hand, relaxing the assumption of long-term employment relationships would substantially change the economic environment, thus limiting the extent of the quantitative exercise to capture only the role of dynamic wage contracts, which is the main channel we aim to isolate.

[32]We re-calibrated the value of ξ_L in the counterfactual economy as to obtain the same drop in aggregate debt, as we interpret a drop in aggregate debt, rather than a lower value of ξ_L *per se*, is the primitive shock propelling a financial recession.



(a) Aggregate debt and output



(b) Aggregate employment, investment, and productivity

Figure 5: Impulse response functions for aggregate debt (D), output (Y), employment (N), investment (I) and productivity (A) in response to an aggregate financial shock. The solid line plots impulse response functions in the baseline economy, and the dashed line plots impulse response functions in the counterfactual economy with no commitment. We compute $2 \times M$ simulations of length T. We draw M sequences of uniform random numbers that we use to simulate realizations of ξ. In the first M simulations we set $\xi = \xi_L$ at $T - 10$. The IRFs are computed taking the difference in logs between the first and second set of simulations from $T - 10$ to T, averaging across M.

economy. This implies that in general equilibrium unconstrained entrepreneurs invest *less* in the baseline economy, as the opportunity cost of capital q is greater. As a result, the impulse response functions of aggregate investment do not differ substantially between the two economies. However, this general equilibrium effect implies that in the baseline economy aggregate productivity is higher, as capital is reallocated from unconstrained entrepreneurs with low marginal product of capital to constrained entrepreneurs with high marginal product of capital. This reallocation implies that the drop in aggregate productivity is less pronounced in the baseline economy, as shown in Figure 5.

In Appendix C we consider a comparative static exercise where we increase the relative risk aversion coefficient of workers, thus decreasing firms' ability to adjust wages, and we find results similar to Figure 5.

5.4 Wage adjustments and stabilization policies

We now turn to study how effective are stabilization policies aimed at reducing input costs, in light of our findings that firms can optimally adjust the timing of wage payments over long-term employment relationships to reduce the cost of labor. We consider two broad types of policies: payroll subsidies and investment subsidies.

Payroll subsidies have been implemented by several OECD countries after the Great Recession in the form of payroll tax cuts. In some cases the tax cut was small and applied to all employees, but more often it has been generous and it applied only to new hires. In the US the social security contribution for workers hired from unemployment has been set to zero as part of the HIRE Act, leading to a 6.2 percentage points cut. The contribution has been even more generous in some European countries, such as Ireland and Portugal, where the cost of employment contribution has been set to zero in 2010, leading to a 10 percentage points cut.[33] These policies are often motivated by the presence of wage rigidity (Bils and Klenow, 2009) and targeted towards new hires based on the idea that the cost of incumbent workers is infra-marginal and thus not allocative. We focus on payroll subsidies for new hires, as these policies have been more used in practice and there is a large consensus on their potential positive effects.[34]

Investment subsidies have often been implemented to foster recoveries in downturns. In practice, these subsidies are often implemented using accelerated depreciation schemes, that allow firms to deduct a large share of their investment from taxes immediately. Both in 2003 and in 2008 the United States introduced a 50 percent bonus depreciation, giving firms the possibility to immediately deduct 50 percent of investment purchases and then depreciate the remaining 50 percent under standard depreciation schedules. According to House and Shapiro (2008) this policy was equivalent to an investment subsidy ranging between 0.5% and 4.5%, depending on the recovery period of the investment good and the nominal interest rate. To facilitate the comparison between payroll subsidies and investment subsidies, we focus on investment subsidies targeted to newly created matches.

How effective are these policies when firms can optimally adjust the timing of wage payments over long-term employment relationships? To answer these questions we compare the effects of these policies between our model and the counterfactual economy described in Section 5.3 where firms cannot adjust wages over time. We show that a payroll subsidy on new hires is not as effective as it is the counterfactual economy because it crowds out the firms incentives to backload wages and its effectiveness is lower when firms already pay lower wages during recessions. On the other hand, an investment

[33] See OECD (2010) for a detailed discussion of payroll tax cuts and hiring credit measures after 2008.

[34] As illustrated by Schoefer (2021), in a model with financial constraints broad-based subsidies applied to incumbent workers too would increase hiring and investment because the cost of incumbent workers is not infra-marginal. Another rational for broad-based subsidies is to prevent firms from firing workers during downturns and prevent a spike in unemployment. As in our model separation is exogenous and matches are one-to-one, we focus on policies targeted to newly created matches.

subsidy makes firms want to invest more, which spur firms' incentives to backload wages as the current marginal value of a dollar in the firm increases relative to the future. As a results, firms make wage tenure profiles steeper in response to the policy making more resources available for investment and amplifying the initial stimulus.

5.4.1 Modeling payroll and investment subsidies

We introduce payroll and investment subsidies in the baseline model. The government sets payroll subsidies, investment subsidies, lump sum taxes, and government debt. The subsidies $\tau_N(\xi), \tau_I(\xi)$ depend on the realization of the aggregate shock, with $\tau_N(\xi_L) > 0, \tau_I(\xi_L) > 0$ and $\tau_N(\xi_L) = 0, \tau_I(\xi_L) = 0$, meaning that there is a temporary subsidy in recession and no taxes or subsidies in normal times.[35] While the government is restricted to running a balanced-budget fiscal policy in the long run, we allow for short-run debt-financed government expenditure. This means the government can raise funds using public debt and lump-sum taxes. Denote by G government expenditure on payroll and investment subsidies. The budget constraint of the government reads according to (29).[36] We parameterize the persistence of government debt by ρ_B. Given the law of motion of government debt (30), lump-sum taxes are set period by period to satisfy the budget constraint (29). We assume that lump-sum taxes are levied on entrepreneurs and employed workers.

$$qB' = G + T + B \tag{29}$$

$$B' = \rho_B(B + G) \tag{30}$$

The problem of matched entrepreneurs in an economy with subsidies is defined in equation (31). Because of investment subsidies the value of a matched entrepreneur now depends explicitly on the current capital stock k. Since we focus on policies targeted to newly created matches, the value of a matched entrepreneur depends on whether he is eligible for the subsidy ($e = 1$) or not ($e = 0$). Newly created matches are always eligible for the subsidy, and an entrepreneur remains eligible if $\xi' = \xi_L$. The budget constraint and the law of motion of net worth are modified as to account for the subsidies and for lump-sum transfers T. Both policies transfer resources to possibly constrained entrepreneurs by reducing the cost if inputs, and at the same time provide incentives for job creation and investment. More details on the economy with subsidies are provided in

[35]An alternative way to perform this exercise is to assume there are payroll taxes in normal times, that is $\tau_N(\xi_H) < 0$, and a payroll tax cut, rather than a payroll subsidy, in recession. This exercise would be country-specific. We choose a more stylized approach to make a broad argument on the effects of such policies. A logic applies to investment subsidies.

[36]We allow the government to issue public debt for two reasons. First, an increase in government expenditure during economic downturns is often funded with government debt and not by a contemporaneous increase in taxes. Second, the use of government debt to foster an economic recovery can play an important role during a credit tightening, when private credit markets are disrupted

Appendix C.2.

$$J(m,W,z,k,e,S) = \max_{\substack{c^e,b',k',i,m'(z',\xi') \\ w'(z',\xi'),W'(z',\xi')}} \Big\{ v(c^e) + \beta(1-\phi)\mathbb{E}\left[J(m'(z',\xi'),W'(z',\xi'),z',k',e',S')|z,S\right] + \beta\phi\mathbb{E}\left[V(m'(z',\xi'),z',S')|z,S\right] \Big\} \quad (31)$$

$$\text{(Budget constraint} : \lambda^e) \qquad c^e + i[1-e\tau_I] \leq m - k(1-\delta) + q(s_0)b'$$

$$\text{(Capital} : I) \qquad k' \leq i + (1-\delta)k$$

$$\text{(Net worth} : \eta(z',\xi')) \qquad m'(z',\xi') \leq z'f(k') + (1-\delta)k' - w'(z',\xi')[1-e\tau_N] - b' + T'(\xi',S')$$

$$\text{(Collateral constraint} : \mu) \qquad b' \leq \xi k'$$

$$\text{(Promise keeping} : \gamma) \qquad W \leq \mathbb{E}\Big[u(w'(z',\xi') + T'(\xi',S')) + \beta(1-\phi)W'(z',\xi') + \beta\phi\mathcal{U}(S'')|z,S\Big]$$

5.4.2 The effects of payroll subsidies

We consider first the effects of payroll subsidies. When the HIRE act was passed in the United States, the former Chief Economist at the Treasury Alan Krueger said that the HIRE act "provides an incentive for private-sector employers to hire new workers sooner than they otherwise would". The ability of these policies to stimulate employment depends crucially on how they lower the cost of labor. The cost of labor in present discounted value terms, accounting for payroll subsidies is:

$$PDV_t = \mathbb{E}\left[\sum_{s=t}^{\infty} \underbrace{[1-\tau(\xi_s)] \times w_{s+1}(z_{s+1},\xi_{s+1})}_{\text{flow cost of labor}} \times \underbrace{[\beta(1-\phi)]^{(s-t)}\frac{\eta_{s+1}(z_{s+1},\xi_{s+1})}{v'(c^e_t)}}_{\text{SDF of entrepr.}}\right] \quad (32)$$

Crucially, temporary payroll subsidies lower the cost of labor for new hires according to the share of PDV_t that is paid during recession, that is as long as the subsidy is in place. As the stochastic discount factor of entrepreneurs increases substantially during a credit tightening, the share of PDV_t paid in recession is large. This implies that temporary payroll subsidies can have large effects on employment during a financial crises.[37] On the other hand, the share of the flow cost of labor that is paid in recession depends on

[37] This is in line with evidence from Saez, Schoefer, and Seim (2019), finding that payroll subsidies are more effective on financially constrained firms.

the solution of the dynamic contracting problem. To highlight this channel we study the effects of payroll subsidies in the baseline economy and in the counterfactual economy with no commitment that we presented in Section 5.3.

In our model firms back-load wages of new hires after an aggregate financial shock. When wages are back-loaded the share of PDV_t that is paid in recession, that is as long as the subsidy is in place, is lower because the flow cost of labor is lower. Consequently, one should expect temporary payroll subsidies to be less effective in lowering the present discounted value of wage payments in the baseline model when compared to a counterfactual economy with no dynamic wage contracts.[38]

Temporary payroll subsidies have also an effect on wages, as subsidies distort the risk-sharing condition between entrepreneurs and workers, as illustrated in equation (33). Indeed, the policy provides incentives to pay higher wages in recession because it is less costly to deliver utility to the workers when wages are subsidized. In this sense, in our model entrepreneurs adjust wages in response to a payroll subsidy to increase the flow cost of labor and reducing resources available for investment.

$$\eta(z', \xi')[1 - \tau(\xi')] = \gamma u'\left(w'\left(z', \xi'\right)\right) \tag{33}$$

Table 4 reports the effects of a payroll subsidy on new hires equal to 6 percentage points, as the one introduced in 2010 in the United States as part of the HIRE Act. The effect on aggregate output is substantially lower in the economy with dynamic wage contracts, both on impact and cumulatively one year after the beginning of the recession. Payroll subsidies on new hires are not as effective as they would be in a model where firms cannot adjust wages over long-term employment relationships. Intuitively firms' optimal wage adjustments and payroll subsidies are substitute to each other, as they both aim to reduce the cost of labor.

[38]Similarly, a consequence of wage back-loading is also that the increase of the stochastic discount factor of entrepreneurs during recession is less pronounced.

	Panel A: Payroll subsidies					
	Effect on impact: $\Delta \log X_t$			Cumulative effect: $\sum_{s=0}^{3} \Delta \log X_{t+s}$		
	Y	N	I	Y	N	I
Baseline model	+0.1 %	+0.1%	+1.6 %	+1.1 %	+1.3%	+5.9 %
Counterfactual	+0.2 %	+0.2%	+3.3 %	+1.4 %	+1.3%	+11.2 %
	Panel B: Investment subsidies					
	Effect on impact: $\Delta \log X_t$			Cumulative effect: $\sum_{s=0}^{3} \Delta \log X_{t+s}$		
	Y	N	I	Y	N	I
Baseline model	+1.3 %	+1.5%	+10 %	+6.3 %	+7.4%	+32 %
Counterfactual	+0.8 %	+0.8%	+4.2 %	+5.1 %	+5.7%	+17 %

Table 4: Macroeconomic effects of a temporary payroll and investment subsidy on newly created matches. Panel A reports results for an economy with only payroll subsidies $\tau_L(\xi_L) = 0.06$, while Panel B reports results for an economy with only investment subsidies $\tau_I(\xi_L) = 0.016$. The table reports differences in output, employment and investment between the economy with subsidies and the economy without. Differences are reported on impact and cumulatively at one year horizon. The first line of each panel reports results for the baseline model, while the second line for the counterfactual model. We set $\rho_B = 0.9$.

5.4.3 The effects of investment subsidies

We now turn to studying the effects of investment subsidies. Evidence from House and Shapiro (2008), Zwick and Mahon (2017) show that firms respond strongly to these policies and that financial frictions can amplify the investment response. Intuitively, constrained firms value future cash flows with high effective discount rates, which amplify the perceived value of bonus incentives because the difference in todays tax benefits dwarfs the present value comparison that matters in frictionless models. We illustrate the differential effects of these policies on constrained and unconstrained firms using the optimality conditions implied by (31). To understand this heterogeneous response we focus on whether it affects the intertemporal consumption margin, and thus wages through the risk sharing condition.

When the constraint does not bind the intertemporal consumption decision is unaffected by the investment subsidy, and is characterized by the Euler equation for bonds in equation (34). An investment subsidy makes capital more attractive, and this affects the optimal allocation of resources between debt and capital for unconstrained entrepreneurs. However, as long as entrepreneurs are not financially constrained the subsidy does not affect the intertemporal consumption decision and the Euler equation for bonds (34) is identical to the in the model without taxes. Through the risk-sharing condition this means that also the intertemporal path of wages is not affected by the subsidy.

$$qv'(c^e) = \mathbb{E}[\eta(z', \xi')] \tag{34}$$

However, when the constraint binds the investment subsidy affects the intertemporal consumption decision. By combining the optimality conditions for bonds and capital we obtain equation (35). An investment subsidy makes capital more attractive, but since constrained entrepreneurs cannot substitute away from bonds to invest more in capital, as the choice of bonds is not interior, the only way they can invest more in capital is by reducing their current consumption. As a results, the investment subsidy affects the intertemporal consumption decision for constrained firms by increasing the current marginal value of a dollar relative to the future. As entrepreneurs make their own consumption profile steeper in response to the subsidy, the risk sharing condition implies that they also make the wage tenure profile of their workers steeper. In this sense, in our model entrepreneurs adjust wages in response to an investment subsidy to free more resources for investment thus amplifying the initial stimulus.

$$v'(c^e)[1 - q\xi] < \mathbb{E}\Big[\eta(z', \xi')(z'f(k') + 1 - \delta - \xi)\Big] + \underbrace{\Big(\tau v'(c^e) - (1-\delta)\mathbb{E}[\eta(z', \xi')\tau(\xi')]\Big)}_{>0} \tag{35}$$

Table 4 reports the effects of an investment subsidy equal to 1.6%, that according to House and Shapiro (2008) replicates the effects of a bonus depreciation of 50%.[39] The effect on aggregate output is substantially larger in the economy with dynamic wage contracts. Investment subsidies are more effective than they would be in a model where firms cannot adjust wages over long-term employment relationships. Intuitively firms' optimal wage adjustments and investment subsidies are complement to each other: when the policy reduce the cost of investment by making capital more attractive, firms respond to the policy by backloading wages in a way to free more resources to invest in capital thus amplifying the effects of the stimulus.[40]

6 Conclusion

This paper explored the macroeconomic effects of financial shocks through a novel lens, highlighting the role of dynamic wage contracts within firms facing credit constraints. By

[39]Calculations from House and Shapiro (2008) show that a 50% bonus depreciation implies a 1.6% investment subsidy for investment goods with a recovery period of 10 years given a nominal interest rate of 3%.

[40]This mechanism is consistent with evidence from Garrett, Ohrn, and Surez Serrato (2020) that did not find a positive effect of investment subsidies on earnings per-worker, while documenting positive effects on employment.

integrating a simple dynamic contracting problem in a general equilibrium model with financial frictions and aggregate shocks, we illustrated how firms adjust wages over time and in response to shocks, depending on their financial conditions. We illustrated the mechanism within a theoretical framework and we provided empirical evidence on wage dynamics using matched employer-employee data from Italy.

Our findings show that financially constrained firms tend to back-load wages and exhibit higher wage adjustments in response to shocks, thereby enhancing liquidity for investment and job creation. The quantitative analysis revealed that dynamic wage contracts can substantially mitigate the adverse effects of financial shocks on aggregate output, employment, and allocative efficiency. Quantitatively, our model replicates the observed cross-sectional heterogeneity in wage dynamics, and it is at the same time consistent with evidence that the average wage is relatively stable over the cycle. We highlighted the policy implications of our findings, particularly regarding the effectiveness of payroll and investment subsidies during financial crises. Our analysis suggests that incorporating the dynamic structure of wage contracts dampen the effects of payroll subsidies while enhancing the effects of investment subsidies.

References

Abraham, Katharine and Henry S Farber. 1987. "Job Duration, Seniority, and Earnings." *American Economic Review* 77 (3):278–97.

Acabbi, Edoardo M., Ettore Panetti, and Alessandro Sforza. 2020. "The Financial Channels of Labor Rigidities: Evidence from Portugal." Working paper.

Adamopoulou, Effrosyni, Emmanuele Bobbio, Marta De Philippis, and Federico Giorgi. 2016. "Wage rigidities and business cycle fluctuations: A linked employer-employee analysis." Questioni di Economia e Finanza (Occasional Papers) 338, Bank of Italy, Economic Research and International Relations Area.

Affinito, Massimiliano, Fabiana Sabatini, and Massimiliano Stacchini. 2021. "Collateral in bank lending during the financial crises:a borrower and a lender story." Temi di discussione (Economic working papers) 1352, Bank of Italy, Economic Research and International Relations Area.

Altonji, Joseph G and Robert A Shakotko. 1987. "Do wages rise with job seniority?" *The Review of Economic Studies* 54 (3):437–459.

Angeletos, George-Marios. 2007. "Uninsured idiosyncratic investment risk and aggregate saving." *Review of Economic Dynamics* 10 (1):1–30.

Arellano, Cristina, Yan Bai, and Luigi Bocola. 2017. "Sovereign Default Risk and Firm Heterogeneity." Working Paper 23314, National Bureau of Economic Research.

Arellano, Cristina, Yan Bai, and Patrick J. Kehoe. 2019. "Financial Frictions and Fluctuations in Volatility." *Journal of Political Economy* 127 (5):2049–2103.

Balke, Neele and Thibaut Lamadon. 2022. "Productivity Shocks, Long-Term Contracts, and Earnings Dynamics." *American Economic Review* 112 (7):2139–77.

Basu, S. and C.L. House. 2016. "Chapter 6 - Allocative and Remitted Wages: New Facts and Challenges for Keynesian Models." Elsevier, 297–354.

Berk, Jonathan B., Richard Stanton, and Josef Zechner. 2010. "Human Capital, Bankruptcy, and Capital Structure." *The Journal of Finance* 65 (3):891–926.

Bernanke, Ben S., Mark Gertler, and Simon Gilchrist. 1999. "Chapter 21 The financial accelerator in a quantitative business cycle framework." Elsevier, 1341–1393.

Bils, Mark and Pete Klenow. 2009. "Fiscal Stimulus in the Form of Lower Payroll Taxes." *SIEPR Policy Brief* .

Bocola, Luigi and Gideon Bornstein. 2023. "The Macroeconomics of Trade Credit." Working Paper 31026, National Bureau of Economic Research.

Bocola, Luigi and Guido Lorenzoni. 2023. "Risk-Sharing Externalities." *Journal of Political Economy* 131 (3):595–632.

Boldrin, Michele and Michael Horvath. 1995. "Labor Contracts and Business Cycles." *Journal of Political Economy* 103 (5):972–1004.

Buera, Francisco J. and Benjamin Moll. 2015. "Aggregate Implications of a Credit Crunch: The Importance of Heterogeneity." *American Economic Journal: Macroeconomics* 7 (3):142.

Cahuc, Pierre, Stphane Carcillo, and Thomas Le Barbanchon. 2018. "The Effectiveness of Hiring Credits." *The Review of Economic Studies* 86 (2):593–626.

Card, David, Ana Rute Cardoso, Joerg Heining, and Patrick Kline. 2018. "Firms and labor market inequality: Evidence and some theory." *Journal of Labor Economics* 36 (S1):S13–S70.

Card, David, Francesco Devicienti, and Agata Maida. 2014. "Rent-sharing, Holdup, and Wages: Evidence from Matched Panel Data." *The Review of Economic Studies* 81 (1 (286)):84–111.

Card, David, Jörg Heining, and Patrick Kline. 2013. "Workplace heterogeneity and the rise of West German wage inequality." *The Quarterly journal of economics* 128 (3):967–1015.

Carlstrom, Charles T., Timothy S. Fuerst, and Matthias Paustian. 2016. "Optimal Contracts, Aggregate Risk, and the Financial Accelerator." *American Economic Journal: Macroeconomics* 8 (1):11947.

Chari, V. V. 2012. *A Macroeconomist's Wish List of Financial Data*. University of Chicago Press, 215–232.

Cingano, Federico and Alfonso Rosolia. 2012. "People I Know: Job Search and Social Networks." *Journal of Labor Economics* 30 (2):291–332.

Cooley, Thomas, Ramon Marimon, and Vincenzo Quadrini. 2004. "Aggregate Consequences of Limited Contract Enforceability." *Journal of Political Economy* 112 (4):817–847.

Cooper, Russell W. and John Haltiwanger. 2006. "On the Nature of Capital Adjustment Costs." *The Review of Economic Studies* 73 (3):611–633.

D'Amuri, Francesco, Marta De Philippis, Elisa Guglielminetti, and Salvatore Lo Bello. 2022. "Slack and prices during Covid-19: Accounting for labor market participation." *Labour Economics* 75:102129.

D'Amuri, Francesco and Cristina Giorgiantonio. 2015. "The Institutional and Economic Limits to Bargaining Decentralization in Italy." IZA Policy Papers 98, Institute of Labor Economics (IZA).

Daruich, Diego, Sabrina Di Addario, and Raffaele Saggio. 2023. "The effects of partial employment protection reforms: Evidence from Italy." *Review of Economic Studies* 90 (6):2880–2942.

David, Joel, Lukas Schmid, and David Zeke. 2022. "Risk-adjusted capital allocation and misallocation." *Journal of Financial Economics* 145 (3):684–705.

den Haan, Wouter J., Garey Ramey, and Joel Watson. 2000. "Job Destruction and Propagation of Shocks." *American Economic Review* 90 (3):482498.

Di Addario, Sabrina, Patrick Kline, Raffaele Saggio, and Mikkel Sølvsten. 2023. "It aint where youre from, its where youre at: hiring origins, firm heterogeneity, and wages." *Journal of Econometrics* 233 (2):340–374.

Di Tella, Sebastian. 2017. "Uncertainty Shocks and Balance Sheet Recessions." *Journal of Political Economy* 125 (6):2038–2081.

Dmitriev, Mikhail and Jonathan Hoddenbagh. 2017. "The financial accelerator and the optimal state-dependent contract." *Review of Economic Dynamics* 24:43–65.

Donangelo, Andres, Franois Gourio, Matthias Kehrig, and Miguel Palacios. 2019. "The cross-section of labor leverage and equity returns." *Journal of Financial Economics* 132 (2):497–518.

Epstein, Larry G. and Stanley E. Zin. 1989. "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework." *Econometrica* 57 (4):937–969.

Favilukis, Jack, Xiaoji Lin, and Xiaofei Zhao. 2020. "The Elephant in the Room: The Impact of Labor Obligations on Credit Markets." *American Economic Review* 110 (6):1673–1712.

Fukui, Masao. 2020. "A Theory of Wage Rigidity and Unemployment Fluctuations with On-the-Job Search." *Working Paper* .

Garrett, Daniel G., Eric Ohrn, and Juan Carlos Surez Serrato. 2020. "Tax Policy and Local Labor Market Behavior." *American Economic Review: Insights* 2 (1):83100.

Garrido, Jose M, Emanuel Kopp, and Miss Anke Weber. 2016. "Cleaning-up Bank Balance Sheets: Economic, Legal, and Supervisory Measures for Italy." IMF Working Papers 2016/135, International Monetary Fund.

Gertler, Mark, Christopher Huckfeldt, and Antonella Trigari. 2020. "Unemployment Fluctuations, Match Quality, and the Wage Cyclicality of New Hires." *The Review of Economic Studies* 87 (4):1876–1914.

Gertler, Mark and Peter Karadi. 2011. "A model of unconventional monetary policy." *Journal of Monetary Economics* 58 (1):17–34. Carnegie-Rochester Conference Series on Public Policy: The Future of Central Banking April 16-17, 2010.

Gopinath, Gita, ebnem Kalemli-zcan, Loukas Karabarbounis, and Carolina Villegas-Sanchez. 2017. "Capital Allocation and Productivity in South Europe*." *The Quarterly Journal of Economics* 132 (4):1915–1967.

Grigsby, John, Erik Hurst, and Ahu Yildirmaz. 2021. "Aggregate Nominal Wage Adjustments: New Evidence from Administrative Payroll Data." *American Economic Review* 111 (2):42871.

Grigsby, John R. 2022. "Skill Heterogeneity and Aggregate Labor Market Dynamics." Working Paper 30052, National Bureau of Economic Research.

Guiso, Luigi and Giuseppe Parigi. 1999. "Investment and Demand Uncertainty." *The Quarterly Journal of Economics* 114 (1):185–227.

Guiso, Luigi and Luigi Pistaferri. 2020. "The insurance role of the firm." *The Geneva Risk and Insurance Review* 45:1–23.

Guiso, Luigi, Luigi Pistaferri, and Fabiano Schivardi. 2005. "Insurance within the firm." *Journal of Political Economy* 113 (5):1054–1087.

———. 2012. "Credit within the Firm." *The Review of Economic Studies* 80 (1):211–247.

Harris, Milton and Bengt Holmstrom. 1982. "A Theory of Wage Dynamics." *The Review of Economic Studies* 49 (3):315–333.

Herranz, Neus, Stefan Krasa, and Anne P. Villamil. 2015. "Entrepreneurs, Risk Aversion, and Dynamic Firms." *Journal of Political Economy* 123 (5):1133–1176.

Hosios, Arthur. 1990. "On The Efficiency of Matching and Related Models of Search and Unemployment." *The Review of Economic Studies* 57 (2):279–298.

House, Christopher L. and Matthew D. Shapiro. 2008. "Temporary Investment Tax Incentives: Theory with Evidence from Bonus Depreciation." *American Economic Review* 98 (3):73768.

Jermann, Urban and Vincenzo Quadrini. 2012. "Macroeconomic Effects of Financial Shocks." *American Economic Review* 102 (1):238–71.

Kalemli-Özcan, Şebnem, Luc Laeven, and David Moreno. 2022. "Debt Overhang, Rollover Risk, and Corporate Investment: Evidence from the European Crisis." *Journal of the European Economic Association* 20 (6):2353–2395.

Kehoe, Patrick J. and Fabrizio Perri. 2002. "International Business Cycles with Endogenous Incomplete Markets." *Econometrica* 70 (3):907–928.

Khan, Aubhik and Julia K. Thomas. 2013. "Credit Shocks and Aggregate Fluctuations in an Economy with Production Heterogeneity." *Journal of Political Economy* 121 (6):1055–1107.

Kiyotaki, Nobuhiro and John Moore. 1997. "Credit Cycles." *Journal of Political Economy* 105 (2):211–248.

———. 2019. "Liquidity, Business Cycles, and Monetary Policy." *Journal of Political Economy* 127 (6):2926–2966.

Krueger, Alan B. and Andreas Mueller. 2010. "Job search and unemployment insurance: New evidence from time use data." *Journal of Public Economics* 94 (3):298–307.

Krusell, Per, Toshihiko Mukoyama, Richard Rogerson, and Ayegl ahin. 2017. "Gross Worker Flows over the Business Cycle." *The American Economic Review* 107 (11):3447–3476.

Krusell, Per and Anthony A. Smith. 1997. "Income and Wealth Heterogeneity, Portfolio Choice, and Equilibrium Asset Returns." *Macroeconomic Dynamics* 1 (2):387422.

Krusell, Per and Anthony A. Smith, Jr. 1998. "Income and Wealth Heterogeneity in the Macroeconomy." *Journal of Political Economy* 106 (5):867–896.

Kudlyak, Marianna. 2014. "The cyclicality of the user cost of labor." *Journal of Monetary Economics* 68:53–67.

Ljungqvist, Lars and Thomas J. Sargent. 2017. "The Fundamental Surplus." *American Economic Review* 107 (9):263065.

Macis, Mario and Fabiano Schivardi. 2016. "Exports and wages: Rent sharing, workforce composition, or returns to skills?" *Journal of Labor Economics* 34 (4):945–978.

Manning, Alan. 2011. "Chapter 11 - Imperfect Competition in the Labor Market." Elsevier, 973–1041.

Menzio, Guido and Shouyong Shi. 2011. “Efficient Search on the Job and the Business Cycle.” *Journal of Political Economy* 119 (3):468–510.

Michaels, Ryan, T Beau Page, and Toni M Whited. 2019. “Labor and capital dynamics under financing frictions.” *Review of Finance* 23 (2):279–323.

Michelacci, Claudio and Vincenzo Quadrini. 2009. “Financial Markets and Wages.” *The Review of Economic Studies* 76 (2):795–827.

Midrigan, Virgiliu and Daniel Yi Xu. 2014. “Finance and Misallocation: Evidence from Plant-Level Data.” *American Economic Review* 104 (2):422–58.

Moen, Espen R. 1997. “Competitive Search Equilibrium.” *Journal of Political Economy* 105 (2):385–411.

Moll, Benjamin. 2014. “Productivity Losses from Financial Frictions: Can Self-Financing Undo Capital Misallocation?” *American Economic Review* 104 (10):3186–3221.

Neumark, Davis and Diego Grijalva. 2017. “THE EMPLOYMENT EFFECTS OF STATE HIRING CREDITS.” *ILR Review* 70 (5):1111–1145.

OECD. 2010. *OECD Employment Outlook 2010*. Paris: OECD Publishing.

Ottonello, Pablo and Thomas Winberry. 2020. “Financial Heterogeneity and the Investment Channel of Monetary Policy.” *Econometrica* 88 (6):2473–2502.

Pagano, Marco et al. 2020. “Risk sharing within the firm: A primer.” *Foundations and Trends® in Finance* 12 (2):117–198.

Petrongolo, Barbara and Christopher A. Pissarides. 2001. “Looking into the Black Box: A Survey of the Matching Function.” *Journal of Economic Literature* 39 (2):390431.

Saez, Emmanuel, Benjamin Schoefer, and David Seim. 2019. “Payroll Taxes, Firm Behavior, and Rent Sharing: Evidence from a Young Workers’ Tax Cut in Sweden.” *American Economic Review* 109 (5):171763.

Schoefer, Benjamin. 2021. “The Financial Channel of Wage Rigidity.” Working Paper 29201, National Bureau of Economic Research.

Shimer, Robert. 2005. “The Cyclical Behavior of Equilibrium Unemployment and Vacancies.” *American Economic Review* 95 (1):2549.

Souchier, Martin. 2023. “The Pass-through of Productivity Shocks to Wages and the Cyclical Competition for Workers.” *Working Paper* .

———. 2024. “Insurance Inside and Outside the Firm.” *Working Paper* .

Thomas, Jonathan and Tim Worrall. 1988. "Self-Enforcing Wage Contracts." *The Review of Economic Studies* 55 (4):541–553.

Topel, Robert. 1991. "Specific Capital, Mobility, and Wages: Wages Rise with Job Seniority." *Journal of Political Economy* 99 (1):145–176.

Wang, Yajie. 2022. "Uncertainty and Unemployment Revisited: The Consequences of Financial and Labor Contracting Frictions." *Available at SSRN: https://ssrn.com/abstract=4110474* .

Zwick, Eric and James Mahon. 2017. "Tax Policy and Heterogeneous Investment Behavior." *American Economic Review* 107 (1):21748.

Appendix

A Model Appendix and Proofs

A.1 Proof of Proposition 1

First, we show that the measure of workers who search for a job is bounded from above, and that this bound does not depend on the measure of workers M. Then, we will take M such that there must be a positive measure of workers who do not search. Once we show that there is positive measure of workers who do not search, the other results will follow.

The problem of a vacant entrepreneur before matching and separation, defined in (3), can be written as

$$\widehat{V}(m,z,S) = \max\left(\max_{\theta,W}\left\{[\lambda_f(\theta)J(m,W,z,S) + (1-\lambda_f(\theta))V(m,z,S)]\right\}, V(m,z,S)\right) \tag{36}$$

$$\text{s.t.} \qquad \mathcal{W}(S) \leq \lambda_w(\theta)W + [1-\lambda_w(\theta)]\,\mathbb{E}\left[\mathcal{U}\left(S'\right) \mid S\right]$$

where the constraint can be re-arranged as

$$\frac{\mathcal{W}(S) - \mathbb{E}\left[\mathcal{U}\left(S'\right) \mid S\right]}{\lambda_w(\theta)} \leq W - \mathbb{E}\left[\mathcal{U}\left(S'\right) \mid S\right] \tag{37}$$

The optimality conditions of this problem imply

$$\theta) \qquad \lambda_f'(\theta)(J(m,W,z,S) - V(m,z,S)) + \nu\frac{\mathcal{W}(S) - \mathbb{E}\left[\mathcal{U}\left(S'\right) \mid S\right]}{\lambda_w^2(\theta)}\lambda_w'(\theta) = 0$$

$$W) \qquad \lambda_f(\theta)J_W'(m,W,z,S) + \nu = 0$$

Combining the two FOCs, one obtains

$$\theta) \qquad J(m,W,z,S) - V(m,z,S) = -J_W'(m,W,z,S)\frac{\mathcal{W}(S) - \mathbb{E}\left[\mathcal{U}\left(S'\right) \mid S\right]}{\lambda_w}\left(\frac{1-\eta}{\eta}\right)$$

where we have used properties of a Cobb-Douglas matching function[41].

This optimality condition and the constraint jointly determine (W,θ) given workers' values. Market tightness in sub-market (W,θ) is implicitly characterized by

[41] If $\lambda_f = B\theta^{-\eta}$, then $\lambda_f' = -B\eta\theta^{-\eta-1} = -\eta B\lambda_f/\theta < 0$, and $\lambda_w = B\theta^{1-\eta}$ so $\lambda_w' = B(1-\eta)\theta^{-\eta} = B(1-\eta)\lambda_w/\theta$

$$\lambda_w(\theta) = \left(\frac{1-\eta}{\eta}\right) \frac{\mathcal{W}(S) - \mathbb{E}\left[\mathcal{U}\left(S'\right) \mid S\right]}{J(m, W, z, S) - V(m, z, S)} \gamma(m, W, z, S)$$

Then, it must be that the total measure of workers who search, denoted by $s(S)$, is

$$s(S) = \int s(\theta, W)$$

$$s(S) = \int \underbrace{\left(\frac{1}{B}\right)^{-\frac{1}{1-\eta}} \left[\left(\frac{1-\eta}{\eta}\right) \frac{\mathcal{W}(S) - \mathbb{E}\left[\mathcal{U}\left(S'\right) \mid S\right]}{J(m, W, z, S) - V(m, z, S)} \gamma(m, W, z, S)\right]^{-\frac{1}{1-\eta}}}_{\theta^{-1}} \mathbb{1}(m, W, z, S) d\Lambda^m(m, z)$$

where B denotes matching efficiency, and $\mathbb{1}(m, W, z, S)$ is an indicator function equal to one if the entrepreneur opens a vacancy and zero otherwise.

Alternatively, one can write $s(S)$ using the participation constraint to obtain

$$s(S) = \int \underbrace{\left(\frac{1}{B}\right)^{-\frac{1}{1-\eta}} \left[\frac{\mathcal{W}(S) - \mathbb{E}\left[\mathcal{U}\left(S'\right) \mid S\right]}{W - \mathbb{E}\left[\mathcal{U}\left(S'\right) \mid S\right]}\right]^{-\frac{1}{1-\eta}}}_{\theta^{-1}} \mathbb{1}(m, W, z, S) d\Lambda^m(m, z) \tag{38}$$

First, note that if the ratio

$$\frac{W - \mathbb{E}\left[\mathcal{U}\left(S'\right) \mid S\right]}{\mathcal{W}(S) - \mathbb{E}\left[\mathcal{U}\left(S'\right) \mid S\right]} \tag{39}$$

is bounded in all sub-markets, than $s(S)$ must be bounded, as the measure of workers is 1.

We now show that the ratio (39) is bounded. If there is a positive measure of workers who search, we must have

$$\mathcal{W}(S) - \mathbb{E}\left[\mathcal{U}\left(S'\right) \mid S\right] \geq u(\bar{b}) - u(\bar{b}(1-x))$$

so that the denominator of (39) is positive and bounded from below. Note that if there is not a positive measure of workers who search we simply have that $s(S) = 0$, and therefore $s(S)$ is still bounded. Moreover, since the productivity process for productivity follows a discrete Markov process with upper bound $\bar{z}$, it must be that there exists $\bar{W}$ such that no entrepreneurs would ever find it profitable to open a vacancy with $W > \bar{W}$. Therefore, the numerator of (39) is bounded from above:

$$W - \mathbb{E}\left[\mathcal{U}\left(S'\right) \mid S\right] \leq \bar{W} - \mathbb{E}\left[\mathcal{U}\left(S'\right) \mid S\right] \leq \bar{W} - \frac{u(\bar{b})}{1-\beta}$$

where the last inequality follows from the fact that at least a positive measure of workers

is searching.
As we assumed that the production function is bounded from above by $\bar{y}$, for large values of capital, we have that $\bar{W}$ is bounded from above

$$\bar{W} \leq \frac{u(\bar{y})}{1-\beta}$$

as no entrepreneurs will be willing to offer a promised utility greater than $\bar{W}$. Note that in one could prove that the ergodic distribution of entrepreneurs' net worth is bounded from above, then W would be bounded without assuming that the production function is bounded.

As a result, the ratio in equation (39) is bounded, as the numerator is bounded from above and the denominator is bounded from below, and both must be positive. Note that the measure of employed workers is bounded above by one, that is the measure of entrepreneurs. Therefore, the measure of workers who search s is bounded as

$$s \leq \underbrace{\left(\frac{1}{B}\right)^{-\frac{1}{1-\eta}} \left[\frac{u(\bar{y})}{(1-\beta)h}\right]^{-\frac{1}{1-\eta}}}$$

which is a function of primitives $\bar{y}, \beta, h, B$. Then there exists a finite measure of workers M that satisfies

$$M > 1 + \underbrace{\left(\frac{1}{B}\right)^{-\frac{1}{1-\eta}} \left[\frac{u(\bar{y})}{(1-\beta)h}\right]^{-\frac{1}{1-\eta}}}_{s}$$

such that a positive measure of workers is not searching. If a positive measure of workers is not searching, this means that in equilibrium workers must be indifferent between searching and not searching. This implies

$$\mathcal{U}(S) = u(\bar{b}) + \beta\mathbb{E}\left[\mathcal{U}\left(S'\right) \mid \mathcal{S}\right]$$

from which it follows that the value $\mathcal{U}$ does not depend on S and it solves

$$\mathcal{U} = \frac{u(\bar{b})}{1-\beta} \tag{40}$$

Moreover, workers being indifferent between searching and not searching also implies

$$u(\bar{b}) + \beta\mathbb{E}\left[\mathcal{U}\left(S'\right) \mid \mathcal{S}\right] = u(\bar{b}(1-x)) + \beta\mathcal{W}(S)$$

and by combining it with equation (40) we get

$$\mathcal{W} = \mathcal{U} + \frac{u(\bar{b}) - u(\bar{b}(1-x))}{\beta}$$

Finally, note that the constraint in (46) does not depend on S anymore, and it simplifies to

$$\frac{u(\bar{b}) - u(\bar{b}(1-x))}{\lambda_w(\theta)\beta} \leq W - \mathcal{U}$$

A.2 Proof of Proposition 2

We guess that the consumption function takes the form

$$c_t = (1-x)m_t \tag{41}$$

with

$$x = \frac{\beta + \beta\gamma}{1 + \beta\gamma}$$

Combining (41) with the risk-sharing condition we obtain

$$w_t = \gamma(1-x)m_t \tag{42}$$

Combining (41), (42) with the law of motion of net worth we obtain

$$m_{t+1} = \frac{1}{1+\gamma(1-x)}\left[k_{t+1}[z_{t+1} + 1 - \delta] - b_{t+1}\right]$$

We consider two cases: the collateral constraint does not bind, or the collateral constraint binds. We will show that in both cases the guess (41) is verified.

- Case 1: the collateral constraint does not bind.
  Here we first guess and verify that the policy function for capital and debt take the form

  $$k_{t+1} = \phi(z_t, \xi_t, q_t)xm_t, \qquad b_{t+1} = -(1 - \phi(z_t, \xi_t, q_t))xm_t$$

  where the function $\phi(z_t, \xi_t, q_t)$ crucially does not depend on net worth m_t.
  Combining this guess with the law of motion of net worth we obtain

  $$m_{t+1} = m_t \frac{x}{1+\gamma(1-x)}\left[\phi(z_t, \xi_t, q_t)[z_{t+1} + 1 - \delta] + (1 + r_t)[1 - \phi(z_t, \xi_t, q_t)]\right] \tag{43}$$

where we denote $1 + r_t = \frac{1}{q_t}$.
We combine (43) with the Euler equation for bonds and capital and obtain

$$1 = \frac{\beta(1+r_t)}{x}\mathbb{E}\left[\frac{1+\gamma(1-x)}{[\phi(z_t,\xi_t,q_t)[z_{t+1}+1-\delta]+(1+r_t)[1-\phi(z_t,\xi_t,q_t)]]}\right]$$
$$1 = \frac{\beta}{x}\mathbb{E}\left[\frac{[1+\gamma(1-x)]\,(z_{t+1}+1-\delta)}{[\phi(z_t,\xi_t,q_t)[z_{t+1}+1-\delta]+(1+r_t)[1-\phi(z_t,\xi_t,q_t)]]}\right]$$

Combining the two Euler equation we are left with

$$\mathbb{E}\left[\frac{z_{t+1}-\delta-r_t}{\phi(z_t,\xi_t,q_t)[z_{t+1}-\delta-r_t]+1+r_t}\right] = 0$$

that provides an equation that implicitly characterizes $\phi(z_t,\xi_t,q_t)$ and also verifies the guess for the functional form of k_{t+1}, b_{t+1}.
Then, take a weighted average of the Euler equations for bonds and capital, with weights $\phi(z_t,\xi_t,q_t)$ and $1-\phi(z_t,\xi_t,q_t)$, to obtain, after some manipulation:

$$1 = \beta\mathbb{E}\left[\frac{1+\gamma(1-x)}{x}\right]$$

which implies an equation for x:

$$x = \frac{\beta+\beta\gamma}{1+\beta\gamma}$$

that does not depend on net worth.
Next we move to the case when the collateral constraint is binding, and we show that we obtain the same equation for x, that verifies our initial guess (41).

- Case 2: the collateral constraint binds.
  In this case, the law of motion for net worth is

$$m_{t+1} = m_t\frac{1}{1-q_t\xi_t}\frac{x}{1+\gamma(1-x)}\left[z_{t+1}+1-\delta-\xi_t\right] \tag{44}$$

Combining the Euler equation for bonds and capital to substitute for the multiplier on the collateral constraint we obtain

$$v'(c_t^e)[1-q_t\xi_t] = \beta\mathbb{E}\left[v'(c_{t+1}^e)[z_{t+1}+1-\delta\xi]\right] \tag{45}$$

Combining (44) with (45) we obtain:

$$1 = \beta\mathbb{E}\left[\frac{1+\gamma(1-x)}{x}\right]$$

which implies an equation for x:

$$x = \frac{\beta + \beta\gamma}{1 + \beta\gamma}$$

which verifies the initial guess from (41)

A.3 Proof of Proposition 3

Using Proposition 2, entrepreneurs leverage can be expressed as:

$$\frac{b_t}{k_t} = 1 - \frac{1}{\phi(z_t, \xi_t, q_t)}$$

Using Proposition 2 the pass-through of idiosyncratic productivity shocks to wages can be expressed as

$$\frac{\partial \left[\log\left(w_{t+1}\right) - \log\left(w_t\right)\right]}{\partial \left[\log\left(z_{t+1}\right) - \log\left(z_t\right)\right]} = \frac{\phi\left(z_t, \xi_t, q_t\right) z_{t+1}}{\phi\left(z_t, \xi_t, q_t\right)\left[z_{t+1} - \delta\right] + \left(1 - \phi\left(z_t, \xi_t, q_t z_{t+1}\right)\right) \frac{1}{q_t}}$$

from which we find that in the cross-section the pass-through of idiosyncratic shocks to wages is increasing in ϕ. As leverage is increasing in ϕ, then the pass-through is increasing in leverage.

Using Proposition 2 the average growth rate of wages can be expressed as:

$$\mathbb{E}[\log(w_{t+1}) - \log(w_t)] = \mathbb{E}\left[\frac{x}{1 + \gamma(1-x)}\phi(z_t, \xi_t, q_t)[z_{t+1} + 1 - \delta] + [1 - \phi(z_t, \xi_t, q_t)]\frac{1}{q_t}\right]$$

From which we obtain, after some manipulation, that

$$\frac{\partial \mathbb{E}[\log(w_{t+1}) - \log(w_t)]}{\partial \phi} = \log\left(\frac{1}{\beta}\right) > 0$$

that implies the average growth rate of wages is increasing in entrepreneurs' leverage.

A.4 Proof of Lemma 1

Let denote by γ_0 the multiplier on the constraint defined in (23). The first order conditions of the problem defined in (23) imply:

$$\eta_{s+1}(z^{s+1}, \xi^{s+1}) = \frac{\gamma_0}{v'(c_t^e)} u'(w(z^{s+1}, \xi^{s+1})), \quad \forall s, z^{s+1}, \xi^{s+1}$$

Taking the ratio of the optimality conditions for two different histories we obtain a risk-sharing condition similar to the optimality condition of the recursive problem defined in

(2).

$$\frac{\eta_{s+1}(z^{s+1},\xi^{s+1})}{\eta_{p+1}(z^{p+1},\xi^{p+1})} = \frac{u'(w(z^{s+1},\xi^{s+1}))}{u'(w(z^{p+1},\xi^{p+1}))}, \qquad \forall s,p,z^{s+1},\xi^{s+1},z^{p+1},\xi^{p+1}$$

Given the same promised utility W, this implies that the optimal contract that solves (2) is also a solution to the problem defined in (23). To see that, start from $s = t$: as we have $z^{t+1} = z_{t+1}$, $\xi^{t+1} = \xi_{t+1}$ given z_t, ξ_t, the optimal wage contract implied by (2) trivially satisfies the first order conditions of (23). Similarly, one can use the intertemporal dimension of the risk-sharing condition to check that the optimal wage contract implied by (2) satisfies the optimality condition of (23) for $s > t$.

A.5 Optimal job creation of vacant entrepreneurs

Using Proposition 1, the problem of a vacant entrepreneur before matching and separation, defined in (3), can be written as

$$\widehat{V}(m,z,S) = \max\left(\max_{\theta,W}\left\{[\lambda_f(\theta)J(m,W,z,S) + (1-\lambda_f(\theta))V(m,z,S)]\right\}, V(m,z,S)\right) \tag{46}$$

$$\text{s.t.} \qquad \frac{u(\bar{b}) - u(\bar{b}(1-x))}{\beta\lambda_w(\theta)} = W - \mathcal{U}$$

The first order conditions for an interior solution to this problem imply

$$\theta) \qquad \lambda_f'(\theta)(J(m,W,z,S) - V(m,z,S)) + \nu\frac{u(\bar{b}) - u(\bar{b}(1-x))}{\beta\lambda_w^2(\theta)}\lambda_w'(\theta) = 0$$

$$W) \qquad \lambda_f(\theta)J_W'(m,W,z,S) + \nu = 0$$

where ν denotes the multiplier on the constraint. Combining the two first order conditions, and replacing the constraint in the first order condition for θ, we obtain:

$$\theta) \qquad J(m,W,z,S) - V(m,z,S) = -J_W'(m,W,z,S)\,[W-\mathcal{U}]\left(\frac{1-\eta}{\eta}\right)$$

where we have used properties of a Cobb-Douglas matching function[42].

[42] If $\lambda_f = B\theta^{-\eta}$, then $\lambda_f' = -B\eta\theta^{-\eta-1} = -\eta B\lambda_f/\theta < 0$, and $\lambda_w = B\theta^{1-\eta}$ so $\lambda_w' = B(1-\eta)\theta^{-\eta} = B(1-\eta)\lambda_w/\theta$

A.6 Walras' Law

If the market for risk-free bonds clears, then the resource constraint holds. First, note that the budget constraint of each matched entrepreneur j can be written as:

$$c_j^e + k_j' \leq y_j + (1-\delta)k_j - w_j - b_j + qb_j'$$

The budget constraint of each vacant entrepreneur i can be written as:

$$c_i^e \leq -b_i + qb_i'$$

We can integrate the two budget constraints over the measure of matched entrepreneurs j and vacant entrepreneurs i; and sum the integrated budget constraint to obtain:

$$C^e + K' + W \leq Y + (1-\delta)K - B + qB'$$

where W denotes the sum of all wages paid to workers. Using the market clearing condition for risk-free bonds, that is $B = 0, B' = 0$, and the identity $W = C^w$ as workers are hand-to-mouth, to obtain

$$C^e + K' + C^w \leq Y + (1-\delta)K$$

that is, the resource constraint holds

A.7 Law of motion Γ

The law of motion Γ for the aggregate state S is made of

- An exogenous law of motion for the aggregate shock ξ
- An endogenous law of motion H^m for the distribution $\Lambda^m(m, W, z)$:

$$H^m(\Lambda^m)(\mathcal{M}, \mathcal{W}, \Xi) = \int Q_{\Lambda^m}((m, W, z), \mathcal{M}, \mathcal{W}, \Xi) d\Lambda^m(m, W, z)$$

$$Q_{\Lambda^m}((m, W, z)\mathcal{M}, \mathcal{W}, \Xi) = \sum_{z' \in \Xi} \begin{cases} \pi(z' \mid z) & \text{if } m'(m, W, z; S) \in \mathcal{M}, W'(m, W, z; S) \in \mathcal{W} \\ 0 & \text{otherwise} \end{cases}$$

- An endogenous law of motion H^v for the distribution $\Lambda^v(m, z)$:

$$H^v(\Lambda^v)(\mathcal{M}, \Xi) = \int Q_{\Lambda^v}((m, z), \mathcal{M}, \Xi) d\Lambda^v(m, z)$$

$$Q_{\Lambda^v}((m, z)\mathcal{M}, \Xi) = \sum_{z' \in \Xi} \begin{cases} \pi(z' \mid z) & \text{if } m'(m, z; S) \in \mathcal{M} \\ 0 & \text{otherwise} \end{cases}$$

A.8 Average Job Finding Rate

The average job finding rate in the economy is

$$Average(\lambda_w(\theta)) = \frac{\int \lambda_w(\theta_j) s_j dj}{\int s_j dj}$$

$$Average(\lambda_w(\theta)) = \frac{\int \lambda_w(\theta)\ \theta(m, W, z, S)^{-1} \underbrace{\mathbb{1}(m, W, z, S) d\Lambda^v(m, z)}_{v}}{\int \theta(m, W, z, S)^{-1} \underbrace{\mathbb{1}(m, W, z, S) d\Lambda^v(m, z)}_{v}}$$

The model solution algorithm easily returns $\lambda_f(\theta) = B\theta^{-\eta}$, from which we can get

$$\theta = \left(\frac{\lambda_f(\theta)}{B}\right)^{-\frac{1}{\eta}}, \qquad \lambda_w(\theta) = B\theta^{1-\eta}$$

$$Average(\lambda_w(\theta)) = \frac{\int \lambda_f(\theta) \underbrace{\mathbb{1}(m, W, z, S) d\Lambda^v(m, z)}_{v}}{\int \theta(m, W, z, S)^{-1} \underbrace{\mathbb{1}(m, W, z, S) d\Lambda^v(m, z)}_{v}}$$

B Data Appendix

B.1 Matched employer-employee data

In this section, we provide further details on the matched employer-employee data used for our analysis. We restrict our analysis to firms that participated in the Bank of Italy's annual Survey of Industrial and Service Firms (INVIND). Each year the survey gathers information on investments, gross sales, workforce and other economic variables relating to Italian industrial and service firms with 20 or more employees. More precisely, the survey cover firms operating in the following industries: "Food and beverages", "Textiles and apparel", "Chemical, pharmaceutical, rubber", "Non-metallic minerals", "Metalworking industry", "Wood, paper, furniture", "Water and waste", "Wholesale/retail trade", "Hotels and restaurants", "Transportation and telecommunication", "Other (real estate etc.)".

The National Social Security Institute (*Istituto Nazionale di Predivenza Sociale*, INPS) was asked to provide the complete works histories of all workers that ever transited in an INVIND firm. While the data included spells of employment in which workers were employed at firms not listed in the INVIND survey (e.g. they were employed at an INVIND firm in 2010, but then they changed job in 2012), we restrict our attention to INVIND firms for which we have information on the entire population of employees.

B.1.1 Cleaning procedure and sample construction

We start from the balance sheets data of all firms available in Cerved from 2005 to 2019. These are 1,526,216 firms in total. We restrict our sample to these firms that took part of the INVIND survey between 2005 and 2019 (i.e. that have been surveyed by the Bank of Italy and for which we have access to the entire work histories of all their employees). This step restricts our sample to 9,698 firms.

Then, we perform some cleaning of balance-sheets data, as to remove extreme values and clearly implausible entries. We drop firms that have negative entries for value added in at least one year between 2005 and 2019. These are 1,949. We are left with 7,749 firms. We exclude 484 firms with intermittent participation in Cerved and drop 233 firms that appear in the data for less than 3 years (notice that the IV estimation of pass-through coefficients requires at least 3 years of data). We are left with 7032 firms.

Using this sample we run a simple regression to remove persistent heterogeneity across firms, as well as aggregate and sector-specific trends:

$$\log(VA_{it}/L_{it}) = \alpha_i + \text{2digsector-timeFE} + \varepsilon_{it} \tag{47}$$

and get the first difference of residuals. We further exclude 601 firms with residual changes in VA per worker greater than 1 and 661 with changes smaller than -1. This leaves us with 6129 firms. Then, we drop firms for which Cerved declares that debt-related information is not reliable. Cerved classify some firms with non-reliable debt information whenever these firms are not required by the law to report detailed information on their debt in their balance sheets. This step leads us to remove 756 firms. Finally, we also clean from outliers in leverage, as there are firms with values greater than 100 or negative leverage. This leaves us with 5107 firms.

Then, we move to worker-level data. For each worker that has ever transited by an INVIND firm, we have access to their entire work history. We select a 25% random sample of workers from the sample. These are 2,521,206 workers. By merging the balance sheet data and the worker-level data, we find that basically all firms in our sample have at least one worker in the 25% sample. Once we focus on the sub-sample of firms that we obtained from the cleaning procedure we are left with 1,153,746 workers. We exclude 208 workers who have some duplicate record in the dataset. A duplicate record is defined by the combination of earnings × type of contract × number of weeks × which months she worked at the same firm in the same year. This leaves us with 1,153,538 As our empirical analysis is focused on stayers, we only keep workers who in a given year have only one employer, and work 52 weeks. By considering workers who only have one employer in one year, we drop 97,782 workers. This leaves us with 1,055,756 workers. We drop workers who only have part-time contracts during our observation window. This is because this workers may be more affected by an hours response during a value added shock. This leads us to drop 252,719 workers. This leaves us with 803,037 workers.

B.2 Aggregate data

In Section 5 we use time-series data for several macro-economic aggregates. The data come mainly from OECD (variable codes in parenthesis), and we retrieved them from FRED, Federal Reserve Bank of St. Louis.

- Corporate debt: we measure corporate debt as the amount outstanding of total debt securities issued by corporations in the non-financial sector, including all maturities, such that the residence of the issuer is in Italy ('TDSAMRIAONCIT'). Data are available at quarterly frequency.

- Output: we measure aggregate output as real gross domestic product ('CLVMNAC-SCAB1GQIT') at quarterly frequency.

- Unemployment: we measure it as the unemployment rate ('LRUN64TTITQ156S') for people aged 15-64. Data are available at quarterly frequency.

- Employment: we measure the number of employed people aged 15-64 ('LFEM64TTITQ647S'). Data are available at quarterly frequency.

- Hours: we measure hours as the average annual hours worked by employed persons ('AVHWPEITA065NRUG'). Data are available at annual frequency, and we use linear interpolation to construct a measure of hours at quarterly frequency.

- Labor: we measure total labor inputs as the product of employment and hours.

- Investment: we measure investment as grossed fixed capital formation ('ITAGFCFQDSNAQ'). Data are available at quarterly frequency.

B.3 Estimation of productivity process

Let consider the following Error Correction Model (ECM)

$$\begin{aligned} y_{jt} &= B_j + z_{jt} + \nu_{jt} \\ z_{jt} &= \rho z_{jt-1} + \varepsilon_{jt} \\ v \sim (0, \sigma_v)\,, \quad \varepsilon &\sim (0, \sigma_\varepsilon)\,, \quad B \sim (0, \sigma_B) \end{aligned} \tag{48}$$

where log-productivity y_{jt} is the sum of a firm-specific component B_j, a persistent component z_{jt} and a purely idiosyncratic component ν_{jt}. The goal of this exercise is to recover estimates of ρ and σ_ε, that are the parameters disciplining the stochastic process for idiosyncratic productivity in our model. At the same time, this exercise allows us to isolate variation in productivity driven by a persistent components from cross-sectional variation in firms' productivity driven by fixed heterogeneity and purely idiosyncratic shocks, as these features are not part of our model. Once we have a proxy for firms' log-productivity y_{jt}, the stochastic process specified in equation (48) is identified from panel data and can be estimated using a generalized method of moments estimator.

We use data for a balanced panel of firms between 2007 and 2018. Consistently with the production function in our model we measure firms' productivity as

$$y_{jt} = \log(\text{VA/worker})_{jt} - \alpha \log(k)_{jt}$$

We use fixed assets at book value to measure k_{jt}. We report also estimates obtained using total assets at book value to measure k_{jt}, as well estimates in which we simply measure y_{jt} as the log of value-added per worker. Estimates are reported in Table 5. In all these cases we obtain similar estimates. We obtain estimates of the parameters disciplining the stochastic process at quarterly frequencies as:

$$\begin{aligned} \sigma^2_{\varepsilon,\text{quarter}} &= \frac{\sigma^2_{\varepsilon,\text{annual}}}{4} \\ \rho_{\text{quarter}} &= \rho^{1/4}_{\text{annual}} \end{aligned}$$

	(1)	(2)	(3)
	Fixed assets	Total assets	VA/worker
ρ	0.86	0.86	0.88
σ_ε	0.16	0.13	0.13
	Quarterly frequencies		
	Fixed assets	Total assets	VA/worker
ρ	0.96	0.96	0.96
σ_ε	0.08	0.07	0.07

Table 5: Estimates of ρ, σ_ε obtained using different specifications. The first panel reports estimates at annual frequency, as balance-sheets data. The second panel reports the implied estimates at quarterly frequency.

C Quantitative details and additional results

C.1 Numerical Algorithm

We solve the model using standard projection methods and we approximate the law of motion of the aggregate state S following Krusell and Smith (1998). Because of Proposition 1, the problem of entrepreneurs depends on the aggregate state S only through the realization of the aggregate shock ξ and the price of bonds q. While the law of motion for ξ is exogenous, we rely on the following approximation to characterize the law of motion of q as:

$$q_{t+1} = \beta_0(\xi_t, \xi_{t+1}) + \beta_1(\xi_t) q_t \tag{49}$$

that is defined by the six coefficients: $\beta_0(\xi_L, \xi_L), \beta_0(\xi_L, \xi_H), \beta_0(\xi_H, \xi_L), \beta_0(\xi_H, \xi_H), \beta_1(\xi_H), \beta(\xi_L)$. In this sense, we summarize the aggregate state in period $t+1$ as (ξ_{t+1}, ξ_t, q_t), where q_t depends on the history of previous shocks ξ according to equation (49). We start the algorithm with an initial guess for the coefficients in (49). As there is not an explicit characterization for q as a function of a one-dimensional aggregate state variable, we follow Krusell and Smith (1997) and we include the price of bonds q as a state variable in the entrepreneurs' decision problem.

We solve the problem of matched entrepreneurs defined in (2) using the multiplier γ rather than the promised utility W as a state variable, taking advantage of the fact that the multiplier γ has to be constant throughout the length of a match. This way, one can solve for the value functions without explicitly characterizing the state-contingent future promised values $W'(z', \xi')$. More formally, one can define the Pareto problem $P(m, \gamma, z, S)$ as

$$P(m, \gamma, z) = \max_W \Big[J(m, W, z) + \gamma W \Big]$$

and one can easily show that the policy functions that solve the program defined in (50) are a solution to (2).

$$\begin{aligned} P(m, \gamma, z, S) = \max_{\substack{c^m, b', k', \\ m'(z', \xi'), w(z', \xi')}} \Bigg\{ & v(c^m) + \beta\phi \mathbb{E}\left[V(m'(z', \xi'), z', S') | z, S \right] + \gamma \mathbb{E}[u(w'(z', \xi'))] \\ & + \beta(1-\phi) \Big\{ \mathbb{E}\Big[P(m', \gamma, z', S') | z, S \Big] \Big\} \Bigg\} \end{aligned} \tag{50}$$

$$\text{(Budget constraint : } \lambda^e) \qquad c^m + k' \leq m + qb'$$

$$\text{(Net worth : } \eta(z', \xi')) \qquad m'(z', \xi') \leq z' f(k') + (1-\delta)k' - w'(z', \xi') - b'$$

$$\text{(Collateral constraint : } \mu) \quad b' \leq \xi k'$$

We solve the problem of matched entrepreneurs using grids for the state variables (m, γ, z, ξ, q), and the problem of vacant entrepreneurs with respect on grids for the states (m, z, ξ, q). We use a GPU to iterate over policy functions and value functions, given the relatively large number of states. Similarly to Menzio and Shi (2011), we impose $\lambda_f(\theta) = \min(1, B\theta^{-\eta})$. Note that the participation constraint of workers searching in sub-market (θ, W) always implies $\lambda_w(\theta) \in (0, 1)$[43]. Once we have solved for the policy functions and the value functions we simulate the model by approximating the distribution of idiosyncratic states $\Lambda^m(m, W, z), \Lambda^v(m, z)$ on a grid. We solve for the market clearing price of risk-free bonds q period-by-period during simulation, that is we solve for q such that

$$\sum_{m,W,z} b'(m, W, z, S, q) \times \Lambda^m(m, W, z) = \sum_{m,z} m'(m, z, S, q) \times \Lambda^v(m, z)$$

We simulate the economy for T periods, we drop the first T_0 observations, and we use the simulated series of prices q to update the coefficients of the forecasting rule (49). We keep iterating until the root mean squared error obtained from using the initial guess for (49) to predict q is small enough, that is when the agents forecasting rule is accurate and consistent with the time series of prices. We stop the algorithm when the R^2 from the forecasting regression (49) on newly simulated data is greater than 0.999.

When we solve the model with payroll subsidies we also include government debt as an aggregate state variable in agents' decision problem. Entrepreneurs also need to forecast future values of public debt. Assuming that government debt follows an AR(1) with persistency ρ_B greatly simplifies the forecasting problem.

C.2 Model with subsidies: details

The problem of matched entrepreneurs in the economy with payroll and investment subsidies have been described in (31). Compared to the problem of matched entrepreneur in the baseline model, there are two additional idiosyncratic sate variables (k, e). The state variable e is equal to one if the entrepreneur is eligible for the subsidies, and zero

[43] We calibrated and solved an alternative version of our model using the matching function proposed by den Haan, Ramey, and Watson (2000), that implies job finding rates and vacancy filling rates always below one. We find very similar results.

otherwise. Its law of motion is given by

$$e' = e \quad \text{if } \xi' = \xi_L$$
$$e' = 0 \quad \text{if } \xi' = \xi_H$$

meaning that eligible entrepreneurs remain eligible during recessions, and all matched entrepreneurs become ineligible at the end of a recession.

Vacant entrepreneurs face a discrete choice problem between posting a vacancy or not. Entrepreneurs that decide to open a vacancy have to choose a sub-market (θ, W) where to open it. Their problem is described in (51). If a vacant entrepreneur is matched to a worker, he obtains the continuation value $J(m, W, z, 0, 1, S)$, that is the value of being a matched entrepreneur with 0 capital and eligible for subsidies ($e = 1$).

$$\widehat{V}(m, z, S) = \max\left(\max_{(\theta,W)}\left\{ [\lambda_f(\theta)J(m, W, z, S) + (1-\lambda_f(\theta))V(m, z, S)] \right\}, V(m, z, S)\right) \tag{51}$$

After matching and separation, vacant entrepreneurs decide how much to consume and how much to save, according to (52). The main difference compared to the model with no subsidies is that lump sum transfers enter the law of motion of net worth.

$$V(m, z, S) = \max_{a', c^e, m'} \left\{ v\left(c^e\right) + \beta \mathbb{E}\left[\widehat{V}\left(m', z', S'\right) \mid z, S\right] \right\} \tag{52}$$

$$\text{(Budget constraint)}: \quad c^e + qa' \leq m$$

$$\text{(Net worth)}: \quad m' \leq a' + \bar{b} + T'$$

The decision problem of workers is unchanged from the baseline model described in Section 2.

The market clearing condition for risk-free bonds is now:

$$\int b'(m, W, z, k, e, S) d\Lambda^m(m, W, z, k, e) + B' = \int a'(m, z, S) d\Lambda^v(m, z)$$

Government expenditure G on payroll subsidies and investment subsidies is equal to

$$G_t = \int \mathbb{E}\left[w\left(m_{t-1}, W_{t-1}, z_{t-1}, k_{t-1}, e_{t-1}, S_{t-1}, z_t, S_t\right) \mid z_{t-1}\right] \times \tau_N\left(\xi_t\right) d\Lambda^m_{t-1}\left(m_{t-1}, W_{t-1}, z_{t-1}, k_{t-1}, 1\right)$$
$$+ \int i\left(m_t, W_t, z_t, k_t, e_t, S_t\right) \times \tau_I\left(\xi_t\right) d\Lambda^m_t\left(m_t, W_t, z_t, k_t, 1\right)$$

C.3 Additional counterfactual: changing workers' risk aversion

Wage contracts in this model solve a risk-sharing problem between entrepreneurs and workers. Section 3 highlighted that the magnitude of wage adjustments depend on the ratio between the risk-aversion coefficient of workers and entrepreneurs σ_W/σ_E. When this ratio is high, wage adjustments must be low and entrepreneurs have to bear more risk. The structure of wage contract affects entrepreneurs' investment and hiring decisions, as illustrated in Section 3, and the magnitudes are quantitatively relevant, as shown in Section 5. Here we use the quantitative model to further illustrate the role played by wage adjustments in shaping the dynamics of investment, employment and output after an aggregate financial shock. We compare the calibrated model to an economy where workers have a higher coefficient of relative risk aversion. Figure 6 plots the impulse response functions to a drop in ξ in the baseline model ($\sigma_W = 11$, solid line) and in a model with a higher value for the workers' relative risk aversion coefficient ($\sigma_W = 22$, dashed line). We re-calibrated the value of ξ_L in the counterfactual economy with higher σ_W as to obtain the same drop in aggregate debt. While the two economies experience the same drop in aggregate debt, output falls more in the economy with higher σ_W, that is when firms adjust wages less. The results are qualitatively similar to those presented in Figure 5 from Section 5: output falls more in response to an aggregate financial shock when we constrain firms' ability to back-load wage payments and to adjust wages in response to shocks.

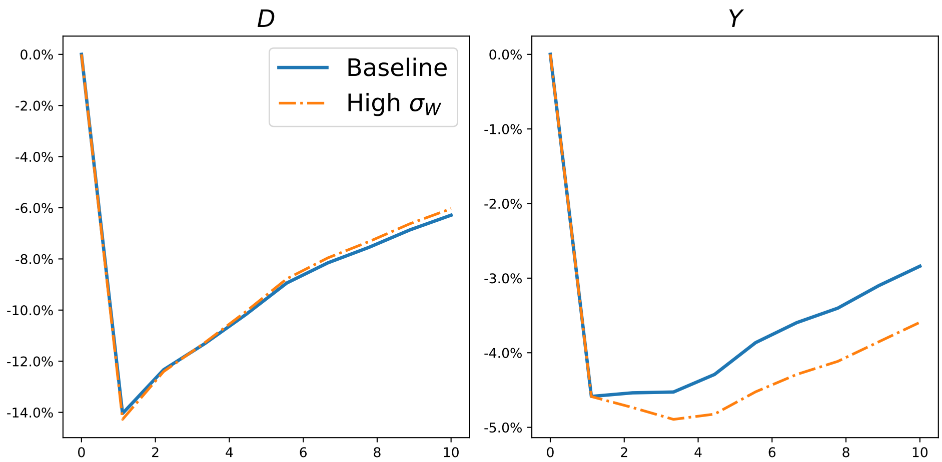


Figure 6: We compute $2 \times M$ simulations of length T. We draw M sequences of uniform random numbers that we use to simulate realizations of ξ. In the first M simulations we set $\xi = \xi_L$ at $T - 10$. The IRFs are computed taking the difference in logs between the first and second set of simulations from $T - 10$ to T, averaging across M.

D Model extensions

D.1 Model with Epstein-Zin Preferences

The baseline model assumes that workers have standard CRRA preferences, and in Section 3 we show that the key properties of the optimal wage contracts between entrepreneurs and workers depend on the ratio between their relative risk aversion coefficients. As it is well known, with CRRA preferences the relative risk aversion coefficient is also the inverse of the elasticity of intertemporal substitution (EIS). In principle, both the risk-aversion coefficient and the EIS of workers should matter. If workers have a large coefficient of RRA, we should expect them to be less inclined to accept wage contracts that are sensitive to idiosyncratic shocks. On the other hand, if workers have small EIS, we should expect them to be less inclined to accept wage payments that vary over time. In this section we derive a version of the baseline model where workers have Epstein-Zin preferences, that we use to better highlight the role played by risk and intertemporal substitution. In order to simplify the exposition we illustrate the model when Proposition 1 holds.

The value of a matched worker with promised utility W at the end of period (i.e. in the afternoon) is defined recursively as

$$W \leq E\Big[\Big\{(1-\beta)w'(z',\xi')^{1-\rho} + \beta\left[(1-\phi)W'(z',\xi') + \phi\mathcal{U}^{1-RRA}\right]^{\frac{1-\rho}{1-RRA}}\Big\}^{\frac{1-RRA}{1-\rho}}\Big]$$

Note that this definition is equivalent to define the value of a matched worker with promised utility W in the morning, *before* wages are paid, as

$$W \leq \left\{(1-\beta)w^{1-\rho} + \beta E_t\left[(1-\phi)W'\left(z',S'\right)^{1-RRA} + \phi\mathcal{U}^{1-RRA}\right]^{\frac{1-\rho}{1-RRA}}\right\}^{\frac{1}{1-\rho}}$$

that is more similar to the standard timing used with these preferences.
The problem of matched entrepreneurs in the afternoon is identical to (2), but with a modified version of the promise keeping constraint.

$$\begin{aligned} J(m,W,z,S) = \max_{\substack{c^m,b',k',m'(z',\xi'),\\ w'(z',\xi'),W'(z',\xi')}} \Bigg\{ & v(c^m) + \beta(1-\phi)\underbrace{\mathbb{E}\left[J(m'(z',\xi'),W'(z',\xi'),z',S')|z,S\right]}_{\text{not separate}} \\ & + \beta\phi\underbrace{\mathbb{E}\left[V(m'(z',\xi'),z',S')|z,S\right]}_{\text{separate}}\Bigg\} \end{aligned}$$

$$\text{(Budget constraint : }\lambda^e) \qquad c^m + k' \leq m + qb'$$

$$\text{(Net worth : }\eta(z',\xi')) \qquad m'(z',\xi') \leq y(k',z') + (1-\delta)k' - w'(z',\xi') - b'$$

$$\text{(Collateral constraint : }\mu) \quad b' \leq \xi(S)k'$$

$$\text{(Promise keeping : }\gamma) \qquad W \leq E\Big[\Big\{(1-\beta)w'(z',\xi')^{1-\rho} + \beta\left[(1-\phi)W'(z',\xi') + \phi\mathcal{U}^{1-RRA}\right]^{\frac{1-\rho}{1-RRA}}\Big\}^{\frac{1-RRA}{1-\rho}}\Big]$$

The optimality conditions for state-contingent wages and promised values are

$$\begin{aligned}
\eta(z',\xi') =& \gamma\Big\{(1-\beta)w'(z',\xi')^{1-\rho} + \beta\left[(1-\phi)W'(z',\xi') + \phi\mathcal{U}^{1-RRA}\right]^{\frac{1-\rho}{1-RRA}}\Big\}^{\frac{\rho-RRA}{1-\rho}} \\
&(1-RRA)(1-\beta)w'(z',\xi')^{-\rho} \\
\gamma' =& \gamma\Big\{(1-\beta)w'(z',\xi')^{1-\rho} + \beta\left[(1-\phi)W'(z',\xi') + \phi\mathcal{U}^{1-RRA}\right]^{\frac{1-\rho}{1-RRA}}\Big\}^{\frac{\rho-RRA}{1-\rho}} \\
&\times \left[(1-\phi)W'(z',\xi') + \phi\mathcal{U}^{1-RRA}\right]^{\frac{RRA-\rho}{1-RRA}}
\end{aligned}$$

Conceptually these optimality conditions are similar to the simple risk-sharing condition from Section 3, where now the marginal value of a dollar for a worker earning $w'(z',\xi')$ does not depend only on the current wage payment, but also on future promised utilities. In order to highlight the role played here by the EIS and the RRA coefficient, it is useful to consider three illustrative examples.

Example 1: static problem. In order to highlight the role of the RRA coefficient, consider a two period economy where agents contract in the first period the wages will be paid in the second period. We have that for any pair (z'_1,ξ'_1), (z'_2,ξ'_2):

$$\frac{\eta(z'_1,\xi'_1)}{\eta(z'_2,\xi'_2)} = \frac{w'(z'_1,\xi'_1)^{-RRA}}{w'(z'_2,\xi'_2)^{-RRA}}$$

In this setting wages solve a pure infra-temporal risk-sharing problem, as there is no dynamic, that depends solely on the risk aversion coefficient of workers RRA, and on the risk aversion of entrepreneurs through the usual multiplier η.

Example 2: deterministic problem with no separation. In order to highlight the role of the EIS, consider an economy with no idiosyncratic and aggregate risk, so that z and S are constant and $\phi = 1$. We can re-arrange the optimality conditions to obtain

$$\frac{\eta_t}{\eta_{t+1}} = \frac{w_t^{-\rho}}{w_{t+1}^{-\rho}}$$

In this setting wages solve a pure inter-temporal problem, as there are no shocks, that depends solely on the EIS coefficient of workers ρ, and on the EIS of entrepreneurs through the usual multiplier η.

Example 3: deterministic problem with separation. In order to highlight the role of separation, consider an economy where z and S are constant, but matches are subject to the idiosyncratic risk of separation. The optimality conditions for wages read:

$$\frac{\eta_t}{\eta_{t+1}} = \left[\frac{(1-\phi)W' + \phi\mathcal{U}^{1-RRA}}{W'}\right]^{\frac{\rho-RRA}{1-RRA}} \frac{w_t^{-\rho}}{w_{t+1}^{-\rho}}$$

Note that whenever workers would prefer to be employed, we have that $W' < \mathcal{U}^{1-RRA}$. Then we have that

$$\frac{\eta_t}{\eta_{t+1}} > \frac{w_t^{-\rho}}{w_{t+1}^{-\rho}} \quad \Leftrightarrow \quad RRA > \rho$$

which means workers wages are less back-loaded than they would in the CRRA case if $RRA > \rho$. Intuitively, when $RRA > \rho$ workers have a preference for early resolution of uncertainty, where here uncertainty comes from separation occurring with probability ϕ. Thus, workers are less willing to accept back-loaded wages as there is a positive probability ϕ that they won't get the future promised utility W'.