Different linearization control techniques for a quadrotor system

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Different Linearization Control Techniques for a Quadrotor System

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***Abstract*— In this paper, we propose different linearization control algorithms to solve the stabilization problem of the quadrotor. First we introduce the nonlinear model of the quadrotor. Then using tangent linearization method, a linear model is generated of the system where decentralized and centralized LQR control methods are applied. The second strategy is based on exact feedback linearization of the nonlinear model of the quadrotor. The comparison between these methods is highlighted by simulations to show effectiveness of the proposed methods.**

***Index Terms*— Quadrotor, Stability, Feedback linearization, LQR.**

I. Introduction

Unmanned Aerial Vehicles (UAVs) control problem attracts many researchers in the world due to the advantages that they provide in many civil and military applications like rescue and research, remote inspection, surveillance, therefore saving human pilots from dangerous flight conditions [1].

The miniature rotor craft flight formation control involves the integration of different domains such as, rotor craft control, coordination control among others. The work reported in the literature is by now quite vast and addresses different approaches for miniature rotor craft stabilization including linear control [5], robust control [8], nonlinear control [3], [7] among others. Many researchers have proposed different control algorithms based on nonlinear model of the quadrotor but these algorithms work based on simplifying the nonlinear model and supposing some decoupling conditions [4]. In this paper we are interested in developing different linearization control algorithms of the quadrotor that solves the problem of tracking of the quadrotor based on a nonlinear model of the quadrotor. The control strategy can be further robustified using other robust nonlinear control techniques [2] to overcome the problem of neglected aerodynamic perturbations and uncertainty in the model.

II. The Quadrotor Modeling

A. Description and Mechanical dynamics

A quadrotor helicopter consists of four rotors that are mounted at the end of two perpendicular axes as depicted in Fig 1 the quadrotor motion is controlled by the aerodynamic forces generated by the rotations of the propellers controlled by dc motors [1].

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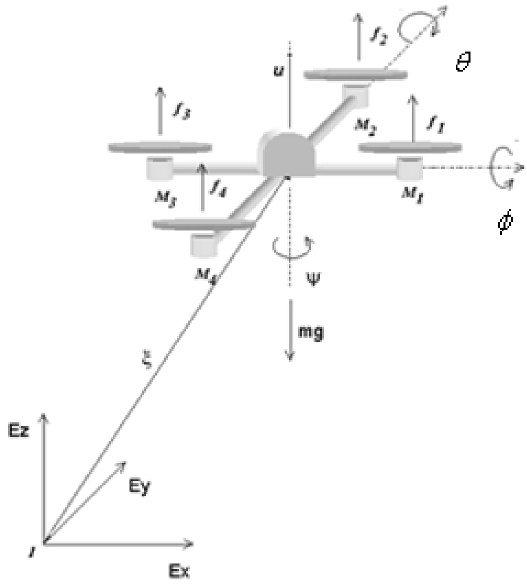

Fig. 1. Quadrotor system

The generalized coordinates for the quadrotor are

$$q = (x, y, z, \phi, \theta, \psi) \in R^6$$

where $\xi = (x, y, z) \in R^3$ denotes the position of the center of mass of the quadrotor relative to the earth fixed frame, and $\eta = (\phi, \theta, \psi) \in R^3$ are the Euler angles (yaw, pitch and roll) that give the orientation of the quadrotor in the body fixed frame.

The total kinetic energy of the quadrotor is

$$T = \frac{m}{2}\dot{\xi}^T\dot{\xi}^T + \frac{1}{2}\dot{\eta}^T I\dot{\eta}^T$$

where m is the mass of the quadrotor and I is the inertia matrix. The potential energy is given by $U = mgz$

Using the Lagrangian

$$L(q, \dot{q}) = T - U = \frac{m}{2}\dot{\xi}^T\dot{\xi}^T + \frac{1}{2}\dot{\eta}^T I\dot{\eta}^T - mgz$$

one can derives the equations of motion of the quadrotor

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = F(q) \tag{1}$$

where $F(q) = (F_\xi, \tau_\eta)$ is composed of the translational force vector F_ξ applied to the quadrotor and τ_η is the generalized moments. The forces generated by the dc motors are supposed to be linear with square of the propellers angular speed that drive the quadrotor Ω according to these relations:

$$\left\{ \begin{array}{l} F_i = k_i \Omega_i^2 \\ \tau_i = k_d \Omega_i^2 \end{array} \right. for \quad i = 1,..4 \tag{2}$$

where k_i, k_d are coefficients depend on the rotor blades geometry and air density.

The thrust force vector F is expressed by the superposition of the command force

$$\left\{ \begin{array}{l} F_x = 0 \\ F_y = 0 \\ F_z = -F_1 - F_2 - F_3 - F_4 \end{array} \right. \tag{3}$$

.

The yaw, pitch and roll generated moments are given by :

$$\left\{ \begin{array}{l} \tau_\phi = l\left(F_4 - F_2\right) \\ \tau_\theta = l\left(F_1 - F_3\right) \\ \tau_\psi = \tau_1 - \tau_2 + \tau_3 - \tau_4 \end{array} \right. \tag{4}$$

where l is the distance between the mass of center and the motors. Using the transformation matrix representing the orientation of the quadrotor,

$$R = \left[\begin{array}{ccc} c_\theta c_\psi & s_\phi s_\theta c_\psi - c_\phi s_\psi & c_\phi s_\theta c_\psi + s_\phi s_\psi \\ c_\theta s_\psi & s_\phi s_\theta s_\psi + c_\phi s_\psi & c_\phi s_\theta c_\psi - s_\phi s_\psi \\ -s_\theta & s_\phi c_\theta & c_\phi c_\theta \end{array} \right]$$

$F_\xi = RF$.

According to the lagrangian equation and based on the assumption that the quadrotor is made of a rigid symmetrical structure with a diagonal inertia matrix I and in the absence of aerodynamic external perturbations, the equations that give the dynamics of the quadrotor can be written as:

$$m\ddot{\xi} = F_\xi - mg \left(\begin{array}{c} 0 \\ 0 \\ 1 \end{array} \right) \tag{5}$$

$$I\ddot{\eta} + \dot{I}\dot{\eta} - \frac{1}{2}\frac{\partial}{\partial \eta}\left(\dot{\eta}^T I \dot{\eta}\right) = \tau \tag{6}$$

Defining the Coriolis/Centripetal vector

$$\bar{C}(\eta, \dot{\eta}) = C(\eta, \dot{\eta})\dot{\eta} = \dot{I}\dot{\eta} - \frac{1}{2}\frac{\partial}{\partial \eta}\left(\dot{\eta}^T I \dot{\eta}\right) \tag{7}$$

we may write

$$I\ddot{\eta} + \bar{C}(\eta, \dot{\eta}) = \tau \tag{8}$$

B. Quadrotor State Space Model

In the literature, many studies deal with dynamic modeling and control for quadrotor [5], [10]. There are complex and nonlinear systems. The complete models are difficult to use in control applications. The most part of applications deal with simplified and partial models [3]. In this paper the derived state space model based on the mechanical equations and the chosen state vector will be useful for simplifying controller synthesis using either linear or nonlinear control theory methods.

Using the state vector composed of the following elements: $\chi^T = [x, y, z, \dot{x}, \dot{y}, \dot{z}, \phi, \theta, \psi, \dot{\phi}, \dot{\theta}, \dot{\psi}]$, and the input vector is composed of the four control command signals: $u^T = [F_z, \tau_\phi, \tau_\theta, \tau_\psi]$

Partitioning the state space vector as follows

$$\left\{ \begin{array}{l} x_1 = x \\ x_2 = \dot{x} \end{array} \right., \left\{ \begin{array}{l} y_1 = y \\ y_2 = \dot{y} \end{array} \right., \left\{ \begin{array}{l} z_1 = z \\ z_2 = \dot{z} \end{array} \right.$$

$$\left\{ \begin{array}{l} \phi_1 = \phi \\ \phi_2 = \dot{\phi} \end{array} \right., \left\{ \begin{array}{l} \theta_1 = \theta \\ \theta_2 = \dot{\theta} \end{array} \right., \left\{ \begin{array}{l} \psi_1 = \psi \\ \psi_2 = \dot{\psi} \end{array} \right.$$

and considering the dynamic equations (5-8), the state space model of the quadrotor will be a multi-variable system composed of six 2^{nd} order nonlinear subsystems and it takes this form:

$$\left\{ \begin{array}{l} \dot{x_1} = x_2 \\ \dot{x_2} = \frac{1}{m}\left(\cos\phi_1 \sin\theta_1 + \sin\phi_1 \sin\psi_1\right) U_1 \\ \dot{y_1} = y_2 \\ \dot{y_2} = \frac{1}{m}\left(\cos\phi_1 \sin\theta_1 \sin\psi_1 - \sin\phi_1 \cos\psi_1\right) U_1 \\ \dot{z_1} = z_2 \\ \dot{z_2} = -g + \frac{1}{m} cos\phi_1 cos\theta_1 U_1 \\ \dot{\phi_1} = \phi_2 \\ \dot{\phi_2} = \frac{I_y - I_z}{I_x}\theta_2\psi_2 + \frac{J_{TP}}{I_y}\theta_2\Omega + \frac{1}{I_x}U_2 \\ \dot{\theta_1} = \theta_2 \\ \dot{\theta_2} = \frac{I_z - I_x}{I_y}\phi_2\psi_2 + \frac{J_{TP}}{I_y}\phi_2\Omega + \frac{1}{I_y}U_3 \\ \dot{\psi_1} = \psi_2 \\ \dot{\psi_2} = \frac{I_x - I_y}{I_z}\phi_2\theta_2 + \frac{1}{I_z}U_4 \end{array} \right. \tag{9}$$

C. Control strategy

We are interested in stabilizing the quadrotor in hover, so our controller synthesis procedure aims to provide a control law that makes the quadrotor follows a given trajectory (x^d, y^d, z^d, ψ^d) and maintaining the two other angles bounded and small enough to be near the linearization trajectory.

III. Linearized system and LQR control

A. Centralized Control

Using the state vector χ and defining the following vector fields:

$$F(\chi) = \begin{pmatrix} x_2 \\ 0 \\ y_2 \\ 0 \\ z_2 \\ -g \\ \phi_2 \\ \frac{I_y - I_z}{I_x}\theta_2\psi_2 + \frac{J_{TP}}{I_y}\theta_2\Omega \\ \theta_2 \\ \frac{I_z - I_x}{I_y}\phi_2\psi_2 + \frac{J_{TP}}{I_y}\phi_2\Omega \\ \psi_2 \\ \frac{I_x - I_y}{I_z}\phi_2\theta_2 \end{pmatrix}$$

$$G(\chi) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ \frac{1}{m}\left(c\phi_1 s\theta_1 + s\phi_1 s\psi_1\right) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{m}\left(c\phi_1 s\theta_1 s\psi_1 - s\phi_1 c\psi_1\right) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{m}c\phi_1 c\theta_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{1}{I_x} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{I_y} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{I_z} \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

The output vector to be controlled is

$$Y = (x_1, y_1, z_1, \psi_1)$$

so the dynamics of the whole system in vector form is

$$\begin{array}{l} \dot{\chi} = F(\chi) + G(\chi, \bar{\Omega})U \\ Y = C\chi \end{array} \tag{10}$$

This system is linearized at its equilibrium point $(\bar{\chi}, \bar{U})$ given by solving

$$\dot{\chi} = 0 \Rightarrow F(\bar{\chi}) + G(\bar{\chi}, \bar{\Omega})\bar{U} = 0 \tag{11}$$

which gives $\bar{U}_2 = \bar{U}_3 = \bar{U}_4 = 0$ and $\phi_2 = \theta_2 = \psi_2 = 0$ Generally the objective of control is to make the Quadrotor follows a given trajectory (x^d, y^d, z^d) with a ϕ_d orientation angle about the vertical axis z. So for a given trajectory we have to find $\bar{U}_1$ that satisfies the equations

$$\begin{cases} \cos\bar{\phi}_1 \sin\bar{\theta}_1 + \sin\bar{\phi}_1 \sin\bar{\psi}_1 = 0 \\ \cos\bar{\phi}_1 \sin\bar{\theta}_1 \sin\bar{\psi}_1 - \sin\bar{\phi}_1 \cos\bar{\psi}_1 = 0 \\ \cos\bar{\phi}_1 \cos\bar{\theta}_1 \bar{U}_1 = 0 \end{cases} \tag{12}$$

and

$$\begin{cases} \bar{U}_2 = lb(\Omega_4^2 - \Omega_2^2) = 0 \\ \bar{U}_3 = lb(\Omega_3^2 - \Omega_1^2) = 0 \\ \bar{U}_4 = d(\Omega_2^2 + \Omega_4^2 - \Omega_3^2 - \Omega_1^2) = 0 \end{cases} \tag{13}$$

.

Solving equations (12, and 13) and using $\bar{U}_1 = mg = b(\Omega_2^2 + \Omega_4^2 + \Omega_3^2 + \Omega_1^2)$ we obtain

$$\begin{cases} \bar{\phi}_1 = 0 \\ \bar{\theta}_1 = 0 \\ \bar{U}_1 = mg \\ \bar{\Omega}_1 = \bar{\Omega}_2 = \bar{\Omega}_3 = \bar{\Omega}_4 = \bar{\Omega} = \frac{1}{2}\sqrt{\frac{mg}{b}} \end{cases}$$

.

Therefore, the equilibrium point is

$$\bar{\chi} = \begin{pmatrix} \bar{x}_1 & 0 & \bar{y}_1 & 0 & \bar{z}_1 & 0 & 0 & 0 & 0 & 0 & \bar{\psi}_1 & 0 \end{pmatrix},$$

$\bar{\Omega} = \frac{1}{2}\sqrt{\frac{mg}{b}}$, and $\bar{U} = \begin{pmatrix} \bar{U}_1 & 0 & 0 & 0 \end{pmatrix}$

By defining the new error vectors $\tilde{\chi} = \chi - \bar{\chi}, \tilde{U} = U - \bar{U}$, and $\tilde{Y} = Y - \bar{Y}$, the linearized error dynamics become

$$\begin{cases} \dot{\tilde{\chi}} = A\tilde{\chi} + B\tilde{U} \\ \tilde{Y} = C\tilde{\chi} \end{cases}$$

.

with $A = \frac{\partial F}{\partial \eta}|_{\eta=\bar{\eta}}, B = \frac{\partial GU}{\partial \eta}|_{\eta=\bar{\eta}} = G(\bar{\chi})$ substituting for the equilibrium points we get:

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & g & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -g & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

and

$$B = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{m} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{1}{I_x} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{I_y} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{I_z} \end{pmatrix}$$

The objective of LQR control is to find a state feedback law $\tilde{U} = -K\tilde{\chi}$ that minimizes the objective function $J = \frac{1}{2}\int_{t=0}^{\infty}\left(\tilde{\chi}^T Q\tilde{\chi} + \tilde{U}^T R\tilde{U}\right)$, with a right choice of the weighting matrices Q and R in a manner that the rate of convergence of the error on the orientation angles is faster than that of the position vector which makes the linearized dynamics approximate the quadrotor dynamics. The optimal control problem reduces to solve the following linear algebraic Riccati equation for a given Q and R. and the optimal control law is

$$U^* = -K^*\tilde{\eta} = -R^{-1}B^T\tilde{\eta} \tag{14}$$

.

B. Decentralized Control

The linearized multi-variable system can be rewritten in decentralized form as follows:

$$\begin{pmatrix} \dot{\tilde{x}}_1 \\ \dot{\tilde{x}}_2 \\ \dot{\tilde{\theta}}_1 \\ \dot{\tilde{\theta}}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & g & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{\theta}_1 \\ \tilde{\theta}_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{I_x} \end{pmatrix} \tilde{U}_2 \tag{15}$$

$$\begin{pmatrix} \dot{\tilde{y}}_1 \\ \dot{\tilde{y}}_2 \\ \dot{\tilde{\phi}}_1 \\ \dot{\tilde{\phi}}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -g & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ \tilde{\phi}_1 \\ \tilde{\phi}_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{I_y} \end{pmatrix} \tilde{U}_3 \tag{16}$$

$$\begin{pmatrix} \dot{\tilde{z}}_1 \\ \dot{\tilde{z}}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \tilde{z}_1 \\ \tilde{z}_2 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{m} \end{pmatrix} \tilde{U}_1 \tag{17}$$

$$\begin{pmatrix} \dot{\tilde{\psi}}_1 \\ \dot{\tilde{\psi}}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \tilde{\psi}_1 \\ \tilde{\psi}_2 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{I_z} \end{pmatrix} \tilde{U}_4 \tag{18}$$

The obtained four subsystems are adequate for linear control design thanks to their controller canonical form for each with single input and single output which makes the synthesis of linear controllers for tracking easier and straightforward.

IV. Feedback Linearization Control

Feedback linearization, known also by nonlinear decoupling, control is a nonlinear control strategy aims to decouple and linearize the system dynamics without approximation and using nonlinear transformation. Then linear control methods can be used for the linearized model and the nonlinear control law will be found by an inverse transformation [9], [6]. Considering the quadrotor model composed of six subsystems of 2nd order defined by equation (9), and using the following transformation that relates the actual command vector U with a fictitious control vector ν :

$$\begin{cases} \nu_x = (c\phi_1 s\theta_1 + s\phi_1 s\psi_1)\frac{1}{m}U_1 \\ \nu_y = (c\phi_1 s\theta_1 s\psi_1 - s\phi_1 c\psi_1)\frac{1}{m}U_1 \\ \nu_z = -g + c\phi_1 c\theta_1 \frac{1}{m}U_1 \\ \nu_\phi = \frac{I_y - I_z}{I_x}\theta_2\psi_2 + \frac{J_{TP}}{I_x}\theta_2\Omega + \frac{1}{I_x}U_2 \\ \nu_\theta = \frac{I_z - I_x}{I_y}\phi_2\psi_2 + \frac{J_{TP}}{I_y}\phi_2\Omega + \frac{1}{I_y}U_3 \\ \nu_\psi = \frac{I_x - I_y}{I_z}\phi_2\theta_2 + \frac{1}{I_z}U_4 \end{cases} \tag{19}$$

Defining the errors between the measured states and their desired values, as

$$\tilde{x}_1 = x_1 - \bar{x}_1, \tilde{y}_1 = y_1 - \bar{y}_1, \tilde{z}_1 = z_1 - \bar{z}_1,$$

$$\tilde{x}_2 = x_2 - \bar{x}_2, \tilde{y}_2 = y_2 - \bar{y}_2, \tilde{z}_2 = z_2 - \bar{z}_2,$$

$$\tilde{\phi}_1 = \phi_1 - \bar{\phi}_1, \tilde{\theta}_1 = \theta_1 - \bar{\theta}_1, \tilde{\psi}_1 = \psi_1 - \bar{\psi}_1,$$

$$\tilde{\phi}_2 = \phi_2 - \bar{\phi}_2, \tilde{\theta}_2 = \theta_2 - \bar{\theta}_2, \tilde{\psi}_2 = \psi_2 - \bar{\psi}_2,$$

So the error dynamics of the system are linearized and reduced to a system composed of six identical 2nd order linear systems as follows in the new error space with the fictitious input:

$$\begin{cases} \dot{\tilde{x}}_1 = \tilde{x}_2 \\ \dot{\tilde{x}}_2 = \nu_x \end{cases} \quad \begin{cases} \dot{\tilde{y}}_1 = \tilde{y}_2 \\ \dot{\tilde{y}}_2 = \nu_y \end{cases} \quad \begin{cases} \dot{\tilde{z}}_1 = \tilde{z}_2 \\ \dot{\tilde{z}}_2 = \nu_z \end{cases} \tag{20}$$

.

$$\begin{cases} \dot{\tilde{\phi}}_1 = \tilde{\phi}_2 \\ \dot{\tilde{\phi}}_2 = \nu_\phi \end{cases} \quad \begin{cases} \dot{\tilde{\theta}}_1 = \tilde{\theta}_2 \\ \dot{\tilde{\theta}}_2 = \nu_\theta \end{cases} \quad \begin{cases} \dot{\tilde{\psi}}_1 = \tilde{\psi}_2 \\ \dot{\tilde{\psi}}_2 = \nu_\psi \end{cases} \tag{21}$$

The linearized subsystems are identical and control synthesis is simplified and reduces to finding a decentralized state feedback law that ensures the exponential convergence of the error dynamics in (20,21) between the nominal values and the measured values. The error convergence rate is controlled by the right choice of fictitious control gains:

$$\begin{cases} \nu_x = -k_1^x\tilde{x}_1 - k_2^x\tilde{x}_2 \\ \nu_y = -k_1^y\tilde{y}_1 - k_2^y\tilde{y}_2 \\ \nu_z = -k_1^z\tilde{z}_1 - k_2^z\tilde{z}_2 \end{cases} \quad \begin{cases} \nu_\phi = -k_1^\phi\tilde{\phi}_1 - k_2^\phi\tilde{\phi}_2 \\ \nu_\theta = -k_1^y\tilde{\theta}_1 - k_2^\theta\tilde{\theta}_2 \\ \nu_\psi = -k_1^z\tilde{\psi}_1 - k_2^\psi\tilde{\psi}_2 \end{cases} \tag{22}$$

.

After finding the fictitious control law ν using pole placement or LQR control, we get the actual control law U by inverse transformation of equation 19 as follows:

$$\begin{cases} \bar{\theta}_1 = \arctan\frac{\nu_x}{\nu_z + g} \\ \bar{\phi}_1 = \arctan\frac{\nu_x}{\nu_z + g} \end{cases} \tag{23}$$

.

$$\begin{cases} U_1 = \frac{m\nu_z + g}{\cos\theta_1 \cos\phi_1} \\ U_2 = I_x\nu_\phi - (I_y - I_z)\theta_2\psi_2 - J_{TP}\theta_2\Omega \\ U_3 = I_y\nu_\theta - (I_z - I_x)\phi_2\psi_2 - J_{TP}\phi_2\Omega \\ U_4 = I_z\nu_\psi - (I_z - I_y)\phi_2\theta_2 \end{cases} \tag{24}$$

. The control strategy highlighted in section III can be explained well by the diagram shown in figure 2

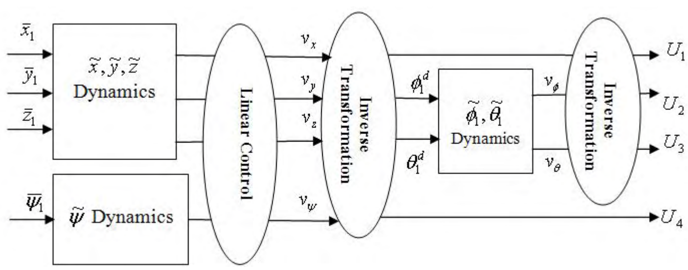

Fig. 2. Feedback Linearization Control Scheme

and then the command vector composed of the angular speeds Ω_i that are sent to the dc motor controller which is the real input vector is calculated by another inversion based on the following equations

$$\begin{pmatrix} \Omega_1^2 \\ \Omega_2^2 \\ \Omega_3^2 \\ \Omega_4^2 \end{pmatrix} = \begin{pmatrix} b & b & b & b \\ 0 & -lb & 0 & lb \\ -lb & 0 & lb & 0 \\ -d & d & -d & d \end{pmatrix}^{-1} \begin{pmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{pmatrix} \quad (25)$$

.

V. Simulation Results

we have built the nonlinear model of the quadrotor on Matlab file to be used as an s-function in Simulink to test the control algorithms proposed in sections (III,IV) for the quadrotor.

The linear controller then is synthesized using control toolbox to calculate the LQR gains using different weighting Q and R matrices.

TABLE I
Quadrotor Simulation parameters

Parameter	Description
$m = 1kg$	The weight of the quadrotor
$I_x = 8.1 \times 10^{-3}$	Moment of inertia along x
$I_y = 8.1 \times 10^{-3}$	Moment of inertia along y
$I_z = 14.2 \times 10^{-3}$	Moment of inertia along z
$b = 54.2 \times 10^{-6}$	Lift coefficient
$d = 4.1 \times 10^{-6}$	Drag coefficient
$l = 0.24m$	Distance

The tracking ability of the proposed linearisation algorithms is tested based on the objectif to follow a given trajectory in specified in cartesian coordinates x, y, z . The quadroror starts at point $(X_0 = 0, Y_0 = 0, Z_0 = 0)$at $t_0 = 0$ then reaches $(X_1 = 0.5m, Y_1 = 1m, Z_1 = 1m)$ at $t = 10s$. Finally it arrives back to $(X_0 = 0, Y_0 = 0, Z_0 = 1m)$ at $t = 20s$.

The simulation results based on the proposed design algorithms for a quadrotor with the parameters of table I

$$Q = 10^{-1} \times I_{12}, R = \begin{pmatrix} 10^{-1} & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 10^2 & 0 \\ 0 & 0 & 0 & 10 \end{pmatrix}$$

are shown in Figures 3,4.

The results show that the LQR controller ensures the tracking of the reference signals and the error dynamics converge for both the decentralized and centralized cases. Feedback linearization control algorithms is implemented in Simulink block diagram as an s-function. The simulation is performed using a Matlab script which provided the model parameters and those of controller of table II, and starts the simulation to test the performance of this method of control to follow the above mentioned trajectory in $3d$.

Simulations results in figures 6,7,8,9 show the effectiveness of the proposed algorithm which can be further made robust for parametric uncertainties.

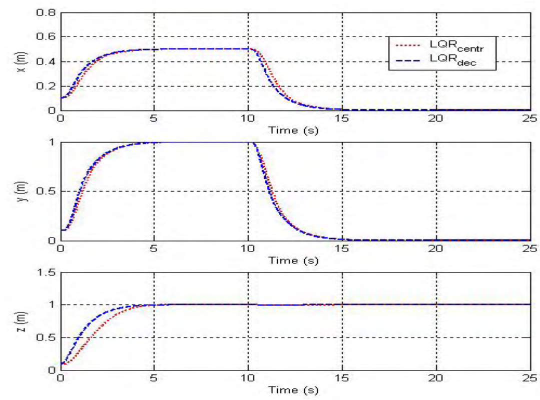

Fig. 3. Simulation results of (x, y, z) tracking

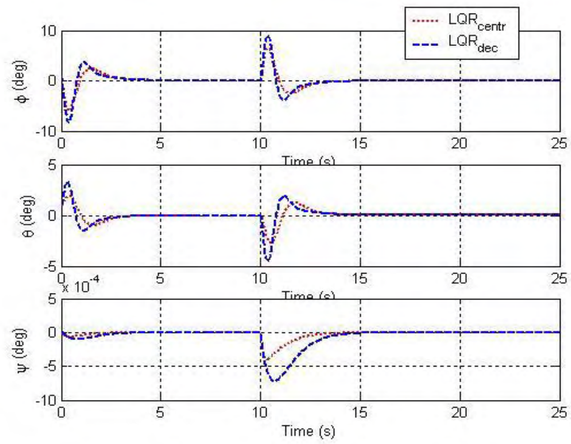

Fig. 4. Simulation results of (ϕ, θ, ψ) tracking

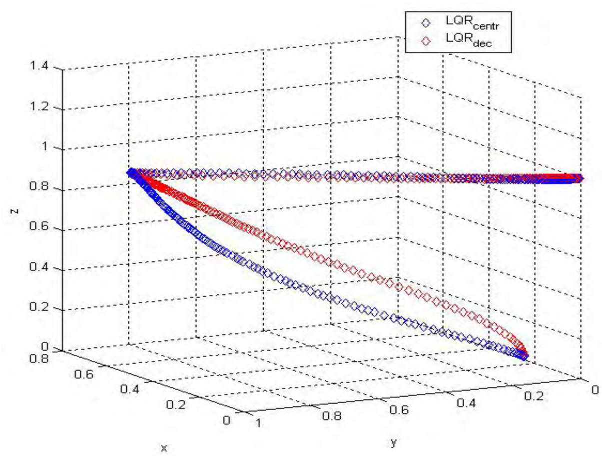

Fig. 5. Trajectory in 3-d for (x, y, z) tracking

TABLE II
Feedback linearization Regulator parameters

Feedback gain	closed loop poles
$K_\psi = K_\phi = K_\theta = \begin{pmatrix} 10 & 10.95 \end{pmatrix}$	$\lambda_1 = -10, \lambda_2 = -9.95$
$K_x = K_y = K_z = \begin{pmatrix} 3.16 & 4.04 \end{pmatrix}$	$\lambda_1 = -1, \lambda_2 = -2.98$

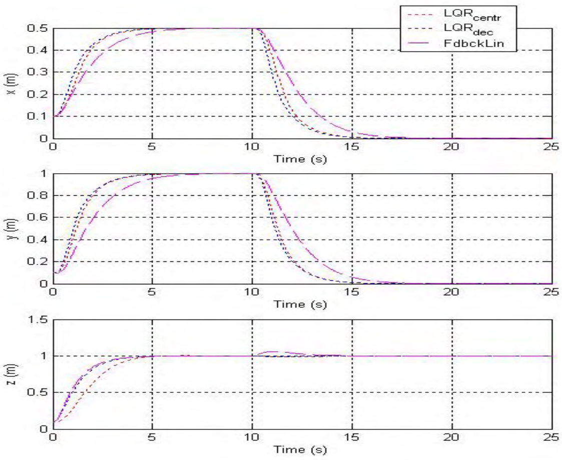

Fig. 6. Simulation results of (x, y, z) tracking

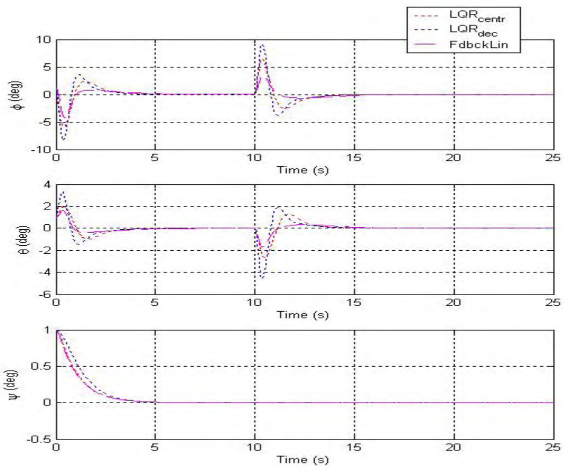

Fig. 7. Simulation results of (ϕ, θ, ψ) tracking

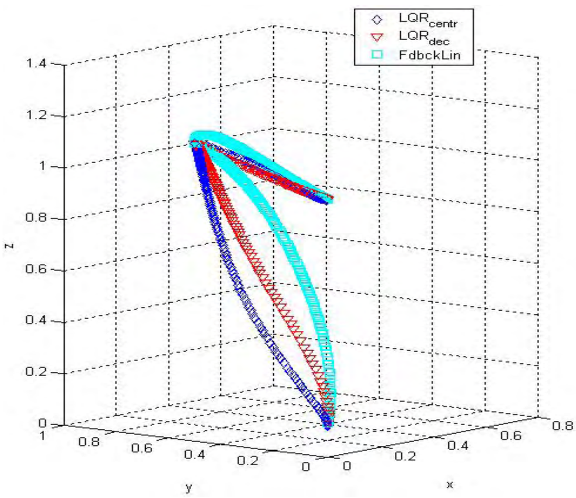

Fig. 8. Trajectory in 3-d for (x, y, z) tracking

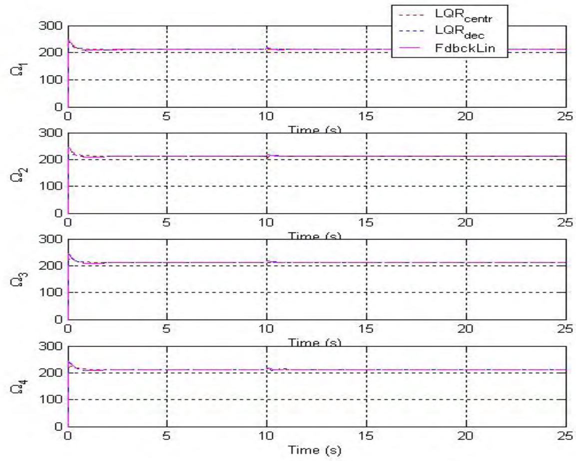

Fig. 9. Control signals $(\Omega_i, i = 1..4)$

VI. Conclusion

This paper proposes different linearization control techniques for the tracking and stabilization of a quadrotor system. The nonlinear model is first obtained and then represented by a state space model. We have designed a state feedback controller using centralized , decentralized LQR and Feedback linearization. Simulation results are carried out to show that the designed controllers ensure the global convergence of the error dynamics in closed-loop and the tracking of reference trajectory is well achieved. Some simulations in the presence of model parametric uncertainty and some external perturbation forces will be included to complete this study.

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