WE PRODUCE A SYSTEM OF EQUATIONS THAT PRODUCE A TRANSITION GEOMETRY BETWEEN ANTI DE-SITTER AND LIFSHITZ SPACE.

HERE WE DEFINE THE NEAR-HORIZON LIFSHITZ GEOMETRY AND SHOOT INTO R->INFINITY. OUR CODE CREATES A FOR LOOP THAT COMPUTES DIFFERENT SOLUTIONS UNTIL WE OBTAIN ONE THAT HAS THE CORRECT ADS ASYMPTOTICS. WE ALSO NOTE THAT GETTING THE ADS ASYMPTOTIC IS FACILITATED BY CHOOSING OUR GAUGE CONDITIONS APPROPRIATELY.

```
Clear[slnhold2]
slnhold2[c0h_, rh_, d_, α_, Qh_, facD0h_, RmaxFact_] :=
 Module[{E1, E2, E3, E4, E41, eAseed, eCseed, eGseed, IePhseed, ePhiseed, c0,
   D0h, a0, a1, c1, g0, g1, rIni, eOm, bc, eq, rmax, a, sol, e2aPhi, Qhlocal},
  (* EoM *)
  E1 =
   D[eA1[r], r] - eC1[r] * N[(D0h + 2 * d * eG1[r] * α * Qhlocal) / (d * r^2 (r^d) * α), 50];
  E2 = D[eC1[r], r] + N[eC1[r]^2 *
      (2 * Qhlocal * eG1[r] * eA1[r] * r^d * r * α * d - 2 * eC1[r] * Qhlocal^2 * eG1[r]^2 *
         α^3 + D0h * eA1[r] * r^d * r) / (r^3 * (r^d)^2 * eA1[r]^2 * d * α), 50];

  E3 = D[eG1[r], r] - N[1 / (2) * 1 / (r^2 * eC1[r] * eA1[r] * Qhlocal * α * r^d) *
      (d * (r^(2 * d)) * r^2 * α * (eC1[r]^2 - 1) * (d + 1) * eA1[r]^2 -
        2 * eC1[r] * r^(d + 1) * (D0h + 2 * d * eG1[r] * α * Qhlocal) * eA1[r] +
        2 * eC1[r]^2 * eG1[r]^2 * α^3 * Qhlocal^2), 50];
  (*
  E3 = D[eG1[r],r]-
    N[d*r^(2*d+2)/(Qhlocal*2*r^(d+2))*((d+1)*(eA1[r]*eC1[r]-eA1[r]/eC1[r])-
       2*r^(d+1)*(D0h+2*d*α*Qhlocal*eG1[r])+
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        2*α*eC1[r]/eA1[r]*α^2*Qhlocal^2*eG1[r]^2), 50];
*)
E4 = D[ePhi[r], r] -
  N[Simplify[D[1 / (2 * r^2 * Qhlocal^2 * α * eA2[r] * eC2[r]) * (d * r^(2 * d + 2) * α *
              (d + 1) * (eA2[r] * eC2[r] - eA2[r] / eC2[r]) - 2 * r^(d + 1) * D0h -
             4 * d * r^(d + 1) * eG2[r] * α * Qhlocal + 2 * α * eC2[r] / eA2[r] * α^2 *
              Qhlocal^2 * eG2[r]^2) /. eA2 → eA1 /. eC2 → eC1 /. eG2 → eG1 , r] /.
       eA1'[r] → eC1[r] * (D0h + 2 * d * eG1[r] * α * Qhlocal) / (d * r^2 (r^d) * α) /.
      eC1'[r] → -eC1[r]^2 * (2 * Qhlocal * eG1[r] * eA1[r] * r^d * r * α * d -
          2 * eC1[r] * Qhlocal^2 * eG1[r]^2 * α^3 + D0h * eA1[r] * r^d * r) /
         (r^3 * (r^d)^2 * eA1[r]^2 * d * α) /.
     eG1'[r] → 1 / (2) * 1 / (r^2 * eC1[r] * eA1[r] * Qhlocal * α * r^d) *
       (d * (r^(2 * d)) * r^2 * α * (eC1[r]^2 - 1) * (d + 1) * eA1[r]^2 -
         2 * eC1[r] * r^(d + 1) * (D0h + 2 * d * eG1[r] * α * Qhlocal) * eA1[r] +
         2 * eC1[r]^2 * eG1[r]^2 * α^3 * Qhlocal^2)], 50];

E41 = D[IePh[r], r] -
  N[2 * r^2 / (α * d * r^(2 * d + 2) * eA1[r] * eA1[r] * (eC1[r] - 1) * (eC1[r] + 1) * (d + 1) -
       2 * r^(d + 1) * eC1[r] * eA1[r] * (D0h + 2 * d * eG1[r] * α * Qhlocal) + 2 * eC1[r]^2 *
        eG1[r]^2 * α^3 * Qhlocal^2) * eA1[r]^2 * eC1[r]^2 * Qhlocal^2 * α, 50];

(* e^(2*α*ϕ[r]) as defined by 76 *)
(* WE WILL SEE LATER WHY WE DEFINED eA2 instead of eA1 and so on *)

(* e2aPhi[r_]:= 2*r^2/(α*d*r^(2*d+2)*eA2[r]^2*(eC2[r]-1)*(eC2[r]+1)*(d+1)-
      2*r^(d+1)*eC2[r]*eA2[r]*(D0h+2*d*eG2[r]*α*Qhlocal)+
      2*eC2[r]^2*eG2[r]^2*α^3*Qhlocal^2)*eA2[r]^2*eC2[r]^2*Qhlocal^2*α;
*)
(* Seed Functions *)
eAseed[r_] := N[a0 * (Sqrt[r - rh] + a1 * (r - rh)^(3 / 2)), 50];
eCseed[r_] := N[c0 / (Sqrt[r - rh]) + c1 * Sqrt[r - rh], 50];
eGseed[r_] := N[a0 * (g0 * (r - rh) + g1 * (r - rh)^2), 50];
(* E^(-2*α*ϕ[r])=ePh[r]; *)
IePhseed[r_] := N[1, 50];
ePhiseed[r_] := 1 / (2 * r^2 * Qh^2 * α * eAseed[r] * eCseed[r]) *
  (d * r^(2 * d + 2) * α * (d + 1) * (eAseed[r] * eCseed[r] - eAseed[r] / eCseed[r]) -
    2 * r^(d + 1) * D0h - 4 * d * r^(d + 1) * eGseed[r] * α * Qh +
    2 * α * eCseed[r] / eAseed[r] * α^2 * Qh^2 * eGseed[r]^2);
(* Coefficient of the seed functions *)
a0 = 2 * rh^(-2 - d) * c0 * D0h / (d * α);
a1 = (α^2 * d * (d + 1)^2 * c0^4 - 2 * d * rh * (α^2 - 2) * (d + 1) * c0^2 +
    rh^2 * α^2 * d - 4 * rh^2 - 6 * d * rh^2) / (8 * rh^3);
c1 = c0 * (3 * α^2 * d * (d + 1)^2 * c0^4 - 2 * d * rh * (3 * α^2 + 2) * (d + 1) * c0^2 +
     3 * rh^2 * α^2 * d + 4 * rh^2 + 6 * d * rh^2) / (8 * rh^3);
```

```
  g0 = d * (c0^2 * d + c0^2 - rh) / (2 * rh^(-d) * c0 * Qhlocal);
  g1 = d^2 * (rh^(-2 + d) * α^2 * (d + 1)^3 * c0^6 -
        rh^(d - 1) * α^2 * (d + 1)^2 * c0^4 - rh^(d) * (α^2 + 2) * (d + 1) * c0^2 +
        rh^(d + 1) * α^2 + 2 * rh^(d + 1)) / (8 * Qhlocal * rh * c0);
  (* Some conditions on Qh and c0, D0h *)
  c0 = N[(c0h * ((Sqrt[α^2 + 2] - α) / (α)) * Sqrt[rh] + Sqrt[rh]) / (Sqrt[d + 1]), 50];
  D0h = N[1 / 2 * rh^(d + 1) * (d + 1) * d * α * facD0h, 50];
  Qhlocal = N[Qh, 50];
  rIni = N[rh + rh * 1*^-10, 50];
  (* NSolve *)
  eOm = {E1 == 0, E2 == 0, E3 == 0, (*E41==0,*) E4 == 0};
  bc =
   N[{eA1[rIni] == eAseed[rIni], eC1[rIni] == eCseed[rIni], eG1[rIni] == eGseed[rIni],
     (*IePh[rIni]== IePhseed[rIni],*)ePhi[rIni] == ePhiseed[rIni] }, 50];
  eq = Flatten[{eOm, bc}];
  a = (12 / (d + 1));
  rmax = 10^a * rh * RmaxFact;
  sol = NDSolve[eq, {eA1[r], eC1[r], eG1[r], ePhi[r]}, {r, rh + rh * 1*^-10, rmax},
    MaxSteps → 900 000, WorkingPrecision → MachinePrecision ];
  (* Give back eA1, eC1, eG1 and e^(2*α*ϕ[r]) *)
  List[sol⟦1, 1, 2, 0⟧, sol⟦1, 2, 2, 0⟧, sol⟦1, 3, 2, 0⟧, sol⟦1, 4, 2, 0⟧]
 ]
slngen2[c0h_, rh_, d_, α_, Qh_, facD0h_, RmaxFac_] :=
  Module[{sln, rmax, eA1val, eC1val, eG1val, ePval,
    mu, T, c0, D0h, Qhlocal, e2aPhi, a, scaleRinit},
   scaleRinit = 10;
   sln = slnhold2[c0h, rh, d, α, Qh, facD0h, RmaxFac];
   rmax =  sln⟦1⟧["Domain"]⟦1, 2⟧;
   (*
   (* Make the rInit larger until you get rmax = 10^(12/(d+1)) --> *)
   While[ rmax<10^(12/(d+1))&& scaleRinit<1000,
    {scaleRinit,sln =slnhold2[c0h, rh, d,α,Qh,facD0h, scaleRinit],
     rmax =  sln⟦1⟧["Domain"]⟦1,2⟧}; scaleRinit =  scaleRinit*10];
   rmax =   sln⟦1⟧["Domain"]⟦1,2⟧;(* 10^(12/(d+1)); *)
   *)
   eA1val = sln⟦1⟧[rmax];
   eC1val = sln⟦2⟧[rmax];
   eG1val = sln⟦3⟧[rmax / 2];
   ePval = sln⟦4⟧[rmax];
   (* Value of the dilaton. Note
    we need to eA2 to express as a function of eA1 *)
   mu = eG1val / (eA1val * Sqrt[ePval]);

   (* Some conditions on Qh and c0, D0h *)
```

```
c0 = (c0h * ((Sqrt[α^2 + 2] - α) / (α)) * Sqrt[rh] + Sqrt[rh]) / (Sqrt[d + 1]);
D0h = 1 / 2 * rh^ (d + 1) * (d + 1) * d * α * facD0h;
Qhlocal = N[Qh, 50];
T = 1 / 2 * D0h / (α * d * (rh^d) * Pi * eA1val);
List[N[eA1val, 10], N[eC1val, 10], N[eG1val, 10],
 N[ePval, 10], N[mu, 10], N[T, 10], N[mu, 10] / N[T, 10] , rmax]
];
```

Out[ ]=

```
0.419273
```

WE EVALUATE THE LARGE R LIMIT OF THE FUNCTION. WE SEE THAT THEY HAVE THE CORRESPONDING ADS LIMIT. HOWEVER, DUE TO INSTABILITIES, THEY START TO DIVERGE BEYOND A CERTAIN VALUE OF R. THIS IS ENTIRELY DUE TO OUR INTERPOLATION BREAKING DOWN IN THIS REGIME.

IT GIVES US A NICE DEFINITION FOR OUR INTERPOLATION.

In[ ]:=

```
Clear[BCval, sln]
d = 3; α = 2; rh = 1; c0 = 0.25;
BCval1 = slngen2[c0, rh, d, α, 1, 1, 1]
BCval2 = slngen2[c0, rh, d, α, Sqrt[BCval1⟦4⟧], 1 / BCval1⟦1⟧, 100 000]
sln = slnhold2[c0, rh, d, α, Sqrt[BCval1⟦4⟧], 1 / BCval1⟦1⟧, 100 000]
L = 1;
Legended[LogLogPlot[{sln⟦1⟧[r], sln⟦2⟧[r], sln⟦3⟧[r], sln⟦4⟧[r]},
  {r, rh, BCval2⟦-1⟧}, PlotRange → {{rh, BCval2⟦-1⟧}, {0.02, 3}},
  PlotStyle → {Red, Orange, Green, Blue}, Frame → True , FrameTicks → Automatic,
  FrameLabel → {Style["r", Black, 25], None(*Style[ "Field",Black, 25]*)},
  FrameTicksStyle → Directive[Black, 20], RotateLabel → {False, True},
  Axes → False, Mesh → False, ImageSize → 700,
  Epilog → {Black, Dashed, Line[{{0, Log[1]}, {1000, Log[1]}}]}(* ,
  GridLines→Automatic,GridLinesStyle→Directive[Gray, Dashed]*)],
 Placed[SwatchLegend[{Red, Orange, Green, Blue},
   {"e^A1 (r)", "e^C1 (r)", "e^G1 (r)" , "e^2 αϕ (r)"}, LegendLayout → "Column",
   LegendMarkerSize → 25, LabelStyle → {Black, FontSize → 25, Font → "Arial"},
   LegendFunction → Framed], {0.87, 0.25}]]
```

Out[ ]=

```
{1.13126, 1., 0.470227, 0.981598, 0.419547, 0.281377, 1.49105, 1000.}
```

••• NDSolve : Maximum number of 900000 steps reached at the point r == 1.0456132602430481` *^7.

Out[ ]=

$\{1., 1., 6.36308 \times 10^{14}, 5.05628 \times 10^{18}, 282977., 0.281377, 1.00569 \times 10^{6}, 1.04561 \times 10^{7}\}$

••• NDSolve : Maximum number of 900000 steps reached at the point r == 1.0456132602430481` *^7.

Out[ ]=

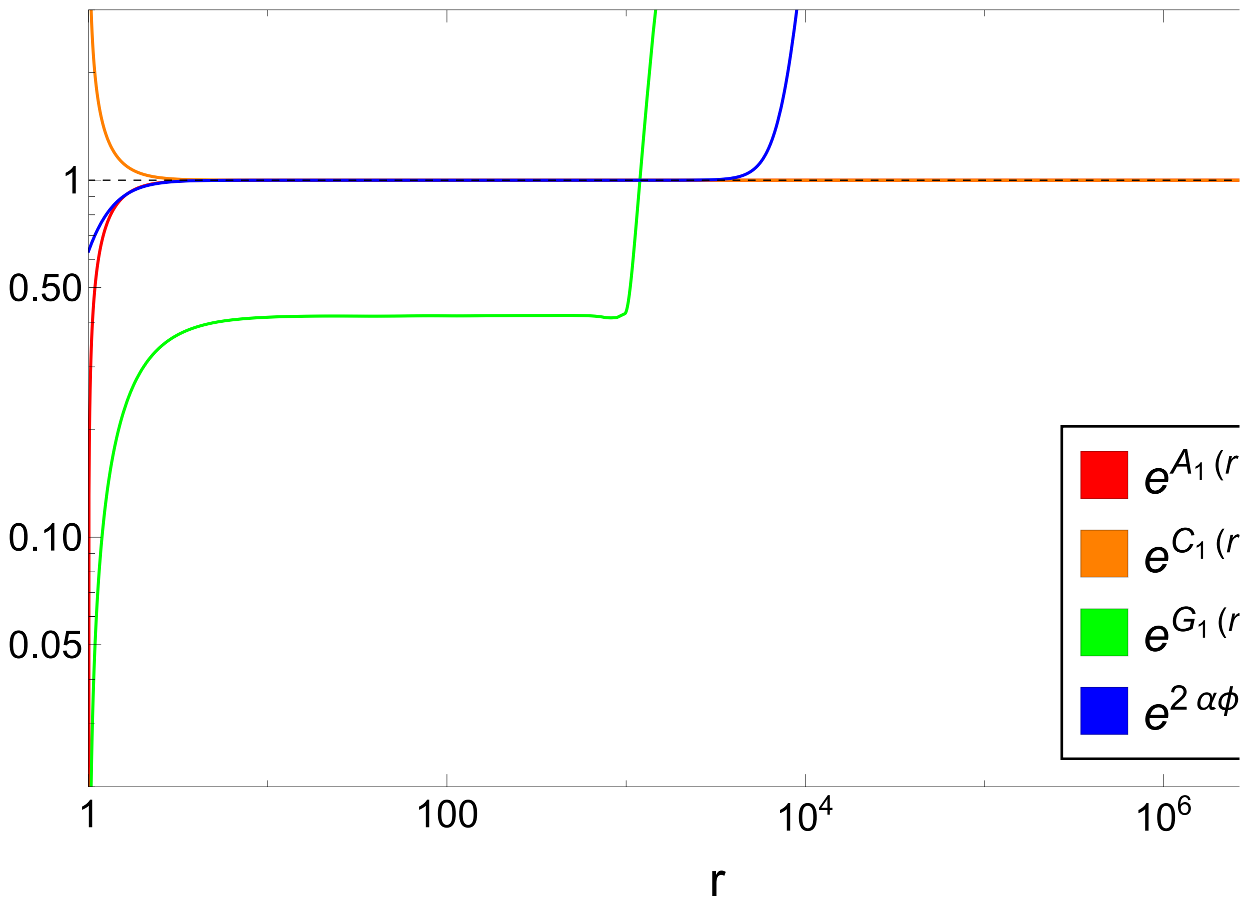
{InterpolatingFunction[Domain: $\{\{1., 1.05 \times 10^{7}\}\}$ Output: scalar
Data not saved. Save now],

InterpolatingFunction[Domain: $\{\{1., 1.05 \times 10^{7}\}\}$ Output: scalar
Data not saved. Save now],

InterpolatingFunction[Domain: $\{\{1., 1.05 \times 10^{7}\}\}$ Output: scalar
Data not saved. Save now],

InterpolatingFunction[Domain: $\{\{1., 1.05 \times 10^{7}\}\}$ Output: scalar
Data not saved. Save now]}

Out[ ]=



FROM NOW ON, WE EXPLORE THE EFFECTS OF CHOOSING DIFFERENT CHOICES FOR OUR PARAMETERS (d, \alpha, r_h, c_0). THEY CONTROL THE BOUNDARY CONDITIONS OF THE NEAR-HORIZON AS WELL AS WHERE THE SYMMETRY BREAKING WILL OCCUR AND “HOW STRONG” IT WILL BE.

WE SEE THAT THE ENTANGLEMENT ENTROPY MOVES FROM AN AREA LAW (LOG) TO A POWER LAW (VOLUME) AROUND THE PHASE TRANSITION (GREEN LINE), WHICH IS WHAT WE EXPECT TO HAPPEN.

In[ ]:=
```
Clear[BCval, sln]
d = 3; α = 2; rh = 1; c0 = 0.25;
BCval1 = slngen2[c0, rh, d, α, 1, 1, 1]
BCval2 = slngen2[c0, rh, d, α, Sqrt[BCval1⟦4⟧], 1 / BCval1⟦1⟧, 1]
```

```
sln = slnhold2[c0, rh, d, α, Sqrt[BCval1[[4]]], 1 / BCval1[[1]], 1]
L = 1;
Legended[LogLogPlot[{sln[[1]][r], sln[[2]][r], sln[[3]][r], sln[[4]][r]},
  {r, rh, BCval2[[-1]]}, PlotRange → {{rh, BCval2[[-1]]}, {0.02, 3}},
  PlotStyle → {Red, Orange, Green, Blue}, Frame → True , FrameTicks → Automatic,
  FrameLabel → {Style["r", Black, 25], None(*Style[ "Field",Black, 25]*)},
  FrameTicksStyle → Directive[Black, 20], RotateLabel → {False, True},
  Axes → False, Mesh → False, ImageSize → 700,
  Epilog → {Black, Dashed, Line[{{0, Log[1]}, {1000, Log[1]}}]}(* ,
  GridLines→Automatic,GridLinesStyle→Directive[Gray, Dashed]*)],
 Placed[SwatchLegend[{Red, Orange, Green, Blue},
   {"e^(A1 (r))", "e^(C1 (r))", "e^(G1 (r))" , "e^(2 αϕ (r))"}, LegendLayout → "Column",
   LegendMarkerSize → 25, LabelStyle → {Black, FontSize → 25, Font → "Arial"},
   LegendFunction → Framed], {0.87, 0.25}]]
Clear[Lslab, ΔSIBB, ΔSIBBr, xIBB, xIBBr, Lslab, Lslabr]


ΔSIBB[us_, d_ ] := Re[NIntegrate[
   1 / u^3 * ( sln[[2]][1 / u] - 1) / (Sqrt[1 - (u / us) ^ (2 * d)]), {u, 1 / BCval2[[-1]], us}]]
ΔSIBBr[rs_, d_ ] := Re[
  NIntegrate[-r * ( sln[[2]][r] - 1) / (Sqrt[1 - (rs / r) ^ (2 * d)]), {r, BCval2[[-1]], rs}]]


xIBB[U_, us_, d_] :=
 Re[ L^2 * NIntegrate[-( sln[[2]][1 / u]) / (Sqrt[(us / u) ^ (2 * d) - 1]), {u, us, U}]]
xIBBr[U_, rs_, d_] :=
 Re[L^2 * NIntegrate[ 1 / r^2 * sln[[2]][r] / Sqrt[(r / rs) ^ (2 * d) - 1], {r, rs, U}]]

Lslab[us_, d_ ] := Re[-L^2 * NIntegrate[
    -( sln[[2]][1 / u]) / (Sqrt[(us / u) ^ (2 * d) - 1]), {u, 1 / BCval2[[-1]], us}]]
Lslabr[rs_, d_] :=
 Re[ L^2 * NIntegrate[1 / r^2 * 1 / Sqrt[(r / rs) ^ (2 * d) - 1], {r, rs, BCval2[[-1]]}]]


ΔSIBB[us_, d_ ] := Re[NIntegrate[
   1 / u^3 * ( sln[[2]][1 / u] - 1) / (Sqrt[1 - (u / us) ^ (2 * d)]), {u, 1 / BCval2[[-1]], us}]]
ΔSIBBr[rs_, d_ ] := Re[
  NIntegrate[-r * ( sln[[2]][r] - 1) / (Sqrt[1 - (rs / r) ^ (2 * d)]), {r, BCval2[[-1]], rs}]]

RSmax = 100; (* BCval1[[-1]];*)

If[BCval1[[5]] < rh, transPoint = BCval1[[6]], transPoint = BCval1[[5]]];
(*
```

```
LogLogPlot[(Abs[ΔSIBB[rs, d]]) ,
 {rs,1/RSmax, 1/rh}, AspectRatio→1/GoldenRatio, Epilog→
  {Green,Thick, Dashed, Line[{{Log[transPoint],-100},{Log[transPoint],100}}]}
  (*,{Black, Dashed, Line[{{Log[1],-100},{Log[1],100}}]}}*),PlotStyle→{Blue},
 PlotRange→{{1/(1.1*RSmax),1.3},{-3, 3}}, Frame→True ,FrameTicks→All,
 FrameLabel→{Style["z_t",Black, 25],Style[ "4G_NΔS_IBB",Black, 25]},
 FrameTicksStyle→Directive[Black,20],RotateLabel→{False,True},
 GridLines→Automatic,GridLinesStyle→Directive[Gray, Dashed],
 Axes→False, Mesh→False,ImageSize→700]
*)
Lslab[us_, d_ ] := Re[-L^2 * NIntegrate[
    -( sln⟦2⟧[1 / u]) / (Sqrt[(us / u) ^ (2 * d) - 1]), {u, 1 / BCval2⟦-1⟧, us}]]
LogLogPlot[Lslab[rs, d] , {rs, 1 / RSmax, 1 / rh},
 AspectRatio → 1 / GoldenRatio, Epilog →
  {Green, Thick, Dashed, Line[{{Log[transPoint], -100}, {Log[transPoint], 100}}]}
  (*,{Black, Dashed, Line[{{Log[1],-100},{Log[1],100}}]}}*), PlotStyle → {Blue}
 (*, PlotRange→{{Log[1/(1.01*RSmax)],1.1},{Log[0.0001], 3}}*),
 Frame → True , FrameTicks → All,
 FrameLabel → {Style["Log[u_t]", Black, 25], Style[ "Log[L_slab]", Black, 25]},
 FrameTicksStyle → Directive[Black, 20], (*RotateLabel→{False,True},*)
 GridLines → Automatic, GridLinesStyle → Directive[Gray, Dashed],
 Axes → False, Mesh → False, ImageSize → 700]
(*
ΔSIBB[us_, d_ ]:= Re[NIntegrate[
     1/u^3*( sln⟦2⟧[1/u]-1)/(Sqrt[1-(u/us)^(2*d)]), {u, 1/BCval2⟦-1⟧, us}]]
   SIBB[us_, d_ ]:= Re[NIntegrate[
      1/u^3*( sln⟦2⟧[1/u])/(Sqrt[1-(u/us)^(2*d)]), {u, 1/BCval2⟦-1⟧, us}]]
    Lslab[us_, d_ ]:= Re[-L^2*
      NIntegrate[-( sln⟦2⟧[1/u])/(Sqrt[(us/u)^(2*d)-1]), {u, 1/BCval2⟦-1⟧, us}]]
    ParametricPlot[{Lslab[us,d],ΔSIBB[us, d]},{us, 1/RSmax, 1/rh},
     ScalingFunctions→{"Log","Log"},Epilog→{Red,Thick, Dashed,
       Line[{{Log[transPoint],-900000},{Log[transPoint],100}}]},
     PlotStyle→{Thick, Blue}, (*PlotRange→{{Log[1/(1.01*RSmax)],Log[4]},
       {Log[0.0001], 3}},*) Frame→True ,FrameTicks→All,
     FrameLabel→{Style["L_slab",Black, 25],Style[ "4G_N ΔS_IBB",Black, 25]},
     FrameTicksStyle→Directive[Black,20],RotateLabel→{False,True},
     Axes→False,GridLines→Automatic,GridLinesStyle→Directive[Gray, Dashed],
     Mesh→False,AspectRatio→1/GoldenRatio,ImageSize→700]
*)
Aads[r_, rh_, d_] := (L * r * Sqrt[1 - (rh / r) ^ (d + 1)]) ^ 2
Cads[r_, rh_, d_] := (L / (r * Sqrt[1 - (rh / r) ^ (d + 1)])) ^ 2
BCval1 = slngen[c0, 1, d, α, 1, 1];
ThAds[rh_, d_] := Sqrt[
   (D[Aads[r, rh1, d], r] /. r → rh1) * (D[1 / Cads[r, rh1, d], r] /. r → rh1)] / (4 * Pi)
```

```
rHAds = N[Solve[ThAds[rh, 3] == BCval1[[6]], rh1][[2, 1, 2]]]
C1AdS[r_, d_, rh1_] := 1 / Sqrt[1 - (rh1 / r) ^ (d + 1)]
ΔSAdS[us_, d_ ] :=
 Re[NIntegrate[1 / u ^ 3 * (C1AdS[1 / u, d, rHAds] - 1) / (Sqrt[1 - (u / us) ^ (2 * d)]),
   {u, 1 / BCval2[[-1]], us}]]
ΔSAdS1[us_, d_ ] :=
 Re[NIntegrate[1 / u ^ 3 * (C1AdS[1 / u, d, rh] - 1) / (Sqrt[1 - (u / us) ^ (2 * d)]),
   {u, 1 / BCval2[[-1]], us}]]
(*
(* TEMPERATURE OF THE Lifshitz BRANES *)
z[d_]:=2*d/(α^2)+1;
Llif[d_]:= L*Sqrt[((z[d]+d)*(z[d]+d-1))/(d*(d+1))];
aL[rh_,d_]:=
 Sqrt[2]*(1/2*rh^(d+1)*(d+1)*d*α*1/BCval1[[1]])/(rh^(d+z[d])*α*d*(d+z[d]));
Alif[r_, rh_, d_]:= (Llif[d]*aL[rh,d]*r^(z[d])*Sqrt[1-(rh/r)^(d+z[d])])^2;
Clif[r_,rh_,d_]:=(Llif[d]/(r*Sqrt[1-(rh/r)^(d+z[d])]))^2;
Blif[r_]:= (L*r)^2;
Clear[Thlif]
  Thlif[rh_,d_]:=
 Sqrt[(D[Alif[r,rh1,d],r]/.r→rh1)*(D[1/Clif[r,rh1,d],r]/.r→rh1)]/(4*Pi);
rHlif =N[Solve[Thlif[rh, 3]==BCval1[[6]],rh1][[2,1,2]]];
C1Lif[r_,d_]:=(Llif[d]/L)*1/Sqrt[1-(rh/r)^(d+z[d])];
ΔSLif[us_, d_ ]:= Re[NIntegrate[
   1/u^3*(C1Lif[1/u,d]-1)/(Sqrt[1-(u/us)^(2*d)]), {u, 1/BCval2[[-1]], us}]];
*)
If[rHAds < rh, rIntAdS = rh, rIntAdS = rHAds];
Legended[Show[(*ParametricPlot[{Lslab[us,d],ΔSLif[us, d]},
   {us, 1/RSmax, 1/rh},ScalingFunctions→{"Log","Log"},Epilog→
    {{Red,Thick, Dashed, Line[{{Log[BCval1[[5]]],-100},{Log[BCval1[[5]]],100}}]},
     {Orange, Thick,Dashed, Line[{{Log[ BCval1[[6]]],-100},
        { Log[BCval1[[6]]],100}}]}},PlotStyle→{Dashed, Black},
   (*PlotRange→{{Log[1/(1.01*RSmax)],Log[4]},{Log[0.0001], 3}},*)
   Frame→True ,FrameTicks→All,
   FrameLabel→{Style["Lslab",Black, 25],Style[ "4GN ΔSIBB",Black, 25]},
   FrameTicksStyle→Directive[Black,20],RotateLabel→{False,True},
   Axes→False,GridLines→Automatic,GridLinesStyle→Directive[Gray, Dashed],
   Mesh→False,AspectRatio→1/GoldenRatio,ImageSize→700],*)
  ParametricPlot[{Lslab[us, d], ΔSAdS[us, d]}, {us, 1 / RSmax, 1 / rIntAdS },
   ScalingFunctions → {"Log", "Log"}, Epilog → {{Green, Thick, Dashed,
      Line[{{Log[BCval1[[5]]], -100}, {Log[BCval1[[5]]], 100}}]}, {Orange, Thick,
      Dashed, Line[{{Log[ BCval1[[6]]], -100}, { Log[BCval1[[6]]], 100}}]}},
   PlotStyle → {Dashed, Black}, (*PlotRange→{{Log[1/(1.01*RSmax)],Log[4]},
     {Log[0.0001], 3}},*) Frame → True , FrameTicks → All,
   FrameLabel → {Style["Log[Lslab]", Black, 25], Style[ "Log[4GNΔSIBB]", Black, 25]},
```

```
    FrameTicksStyle → Directive[Black, 20] (*,RotateLabel→{False,True}*),
    Axes → False, Mesh → False, AspectRatio → 1 / GoldenRatio],
   ParametricPlot[{Lslab[us, d], ΔSAdS1[us, d]}, {us, 1 / RSmax, 1 / rh },
    ScalingFunctions → {"Log", "Log"}, Epilog → {{Green, Thick, Dashed,
       Line[{{Log[BCval1⟦5⟧], -100}, {Log[BCval1⟦5⟧], 100}}]}, {Orange, Thick,
       Dashed, Line[{{Log[ BCval1⟦6⟧], -100}, { Log[BCval1⟦6⟧], 100}}]}},
    PlotStyle → {Dashed, Red}, (*PlotRange→{{Log[1/(1.01*RSmax)],Log[4]},
      {Log[0.0001], 3}},*) Frame → True , FrameTicks → All,
    FrameLabel → {Style["Lslab", Black, 25], Style[ "4GN ΔS", Black, 25]},
    FrameTicksStyle → Directive[Black, 20], RotateLabel → {False, True},
    Axes → False, Mesh → False, AspectRatio → 1 / GoldenRatio],
   ParametricPlot[{Lslab[us, d], ΔSIBB[us, d]}, {us, 1 / RSmax, 1 / rh},
    ScalingFunctions → {"Log", "Log"}, Epilog → {{Red, Thick, Dashed,
       Line[{{Log[BCval1⟦5⟧], -100}, {Log[BCval1⟦5⟧], 100}}]}, {Orange, Thick,
       Dashed, Line[{{Log[ BCval1⟦6⟧], -100}, { Log[BCval1⟦6⟧], 100}}]}},
    PlotStyle → {Thick, Blue}, (*PlotRange→{{Log[1/(1.01*RSmax)],Log[4]},
      {Log[0.0001], 3}}, *)Frame → True , FrameTicks → All,
    FrameLabel → {Style["Log[Lslab]", Black, 25], Style[ "Log[4GNΔSIBB]", Black, 25]},
    FrameTicksStyle → Directive[Black, 20], (*RotateLabel→{False,True},*)
    Axes → False, Mesh → False, AspectRatio → 1 / GoldenRatio],
   PlotRange → All, ImageSize → 1000], Placed[LineLegend[
    {Blue, {Dashed, Black}, {Dashed, Red}(*,{Dashed,Orange}, {Dashed,Red}*)},
    {"IBB", "AdS (same T)", "AdS (same rh)"(*, "l = T", "l = μ"*)},
    LegendLayout → "Column", LegendMarkerSize → 25,
    LabelStyle → {Black, FontSize → 25, Font → "Arial"},
    LegendFunction → Framed], {0.17, 0.82}]]
 (* ---------------- *)
```

Out[ ]=

```
{1.13126, 1., 0.470227, 0.981598, 0.419547, 0.281377, 1.49105, 1000.}
```

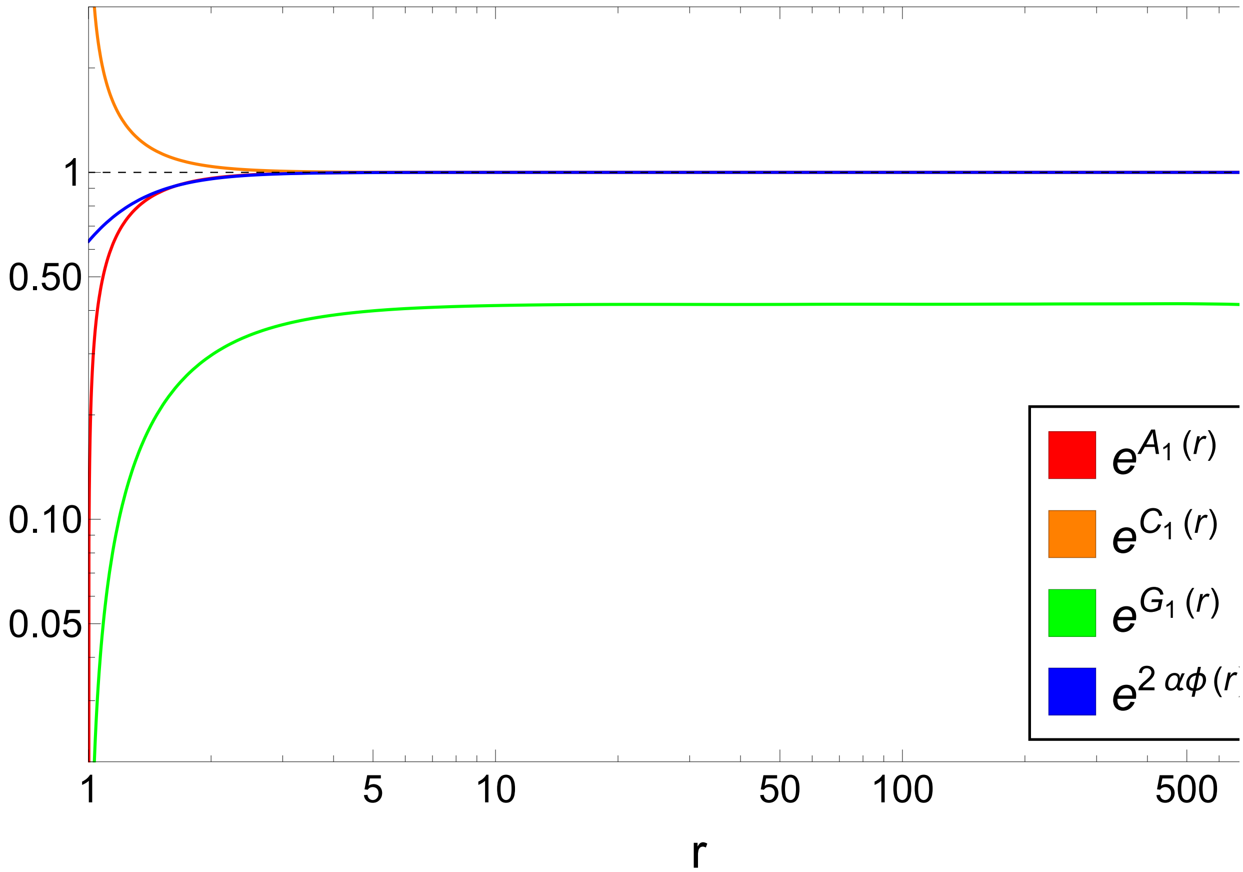
Out[ ]=

```
{1., 1., 0.418181, 0.999952, 0.418191, 0.281377, 1.48623, 1000.}
```

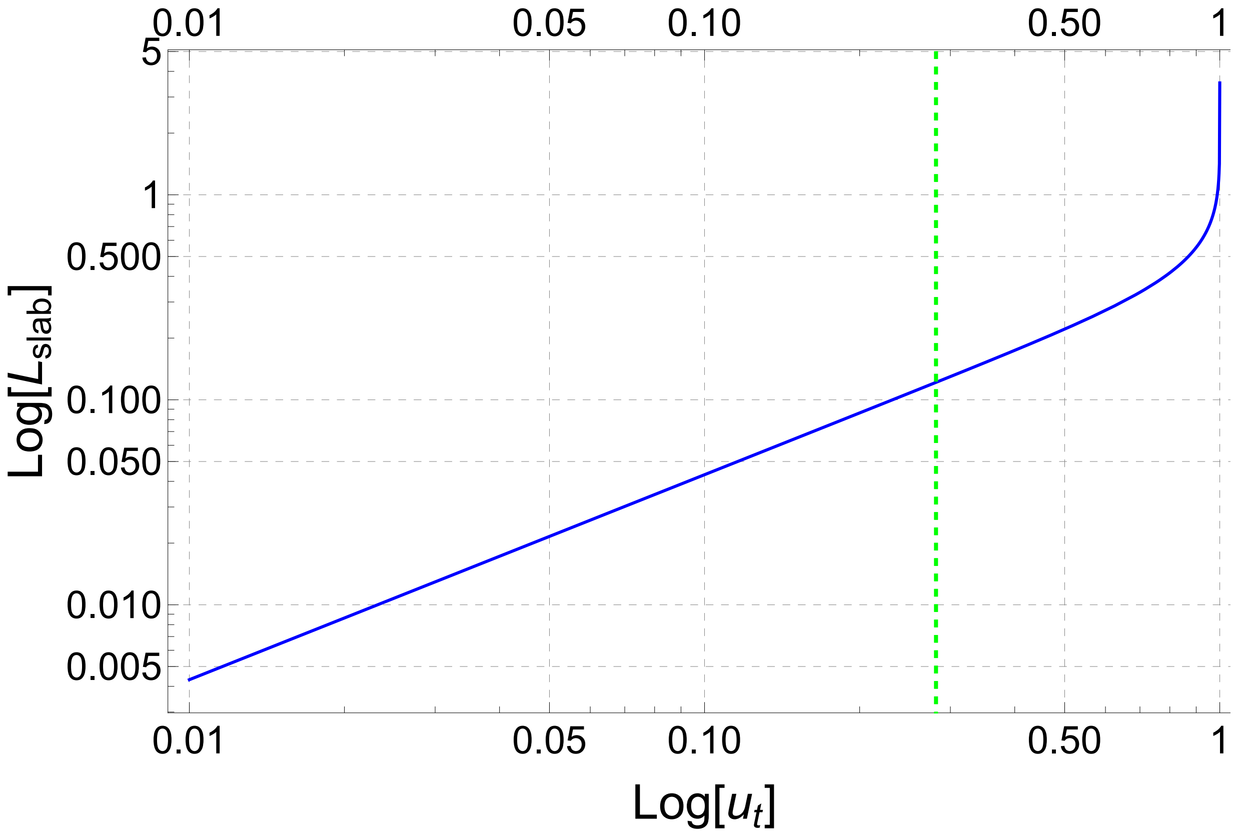
Out[ ]=

```
{InterpolatingFunction[ Domain: {{1., 1.00 ×10^3}} Output: scalar ],
 InterpolatingFunction[ Domain: {{1., 1.00 ×10^3}} Output: scalar ],
 InterpolatingFunction[ Domain: {{1., 1.00 ×10^3}} Output: scalar ],
 InterpolatingFunction[ Domain: {{1., 1.00 ×10^3}} Output: scalar ]}
```

Out[ ]=



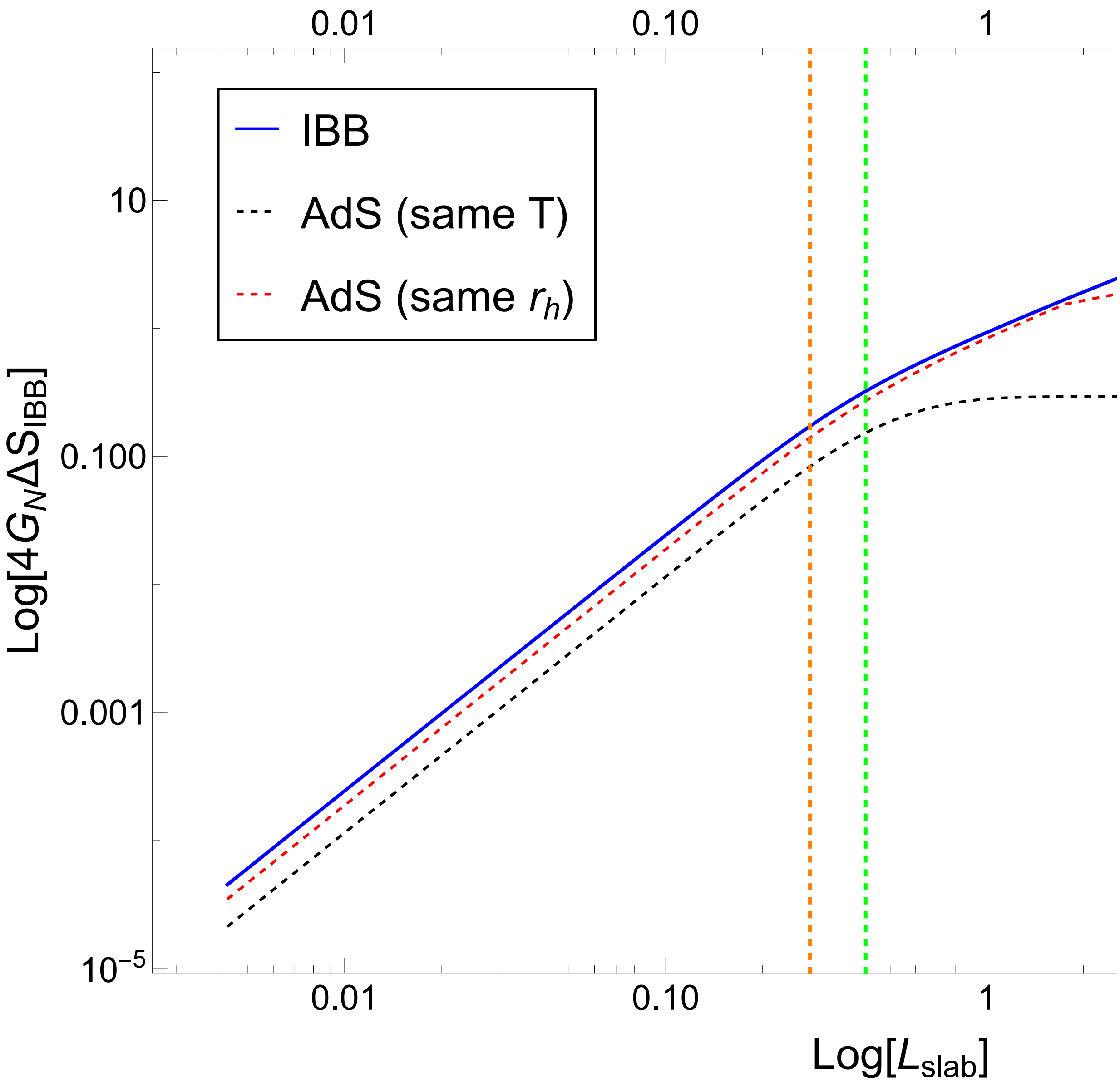
Out[ ]=



Out[ ]=

0.883973

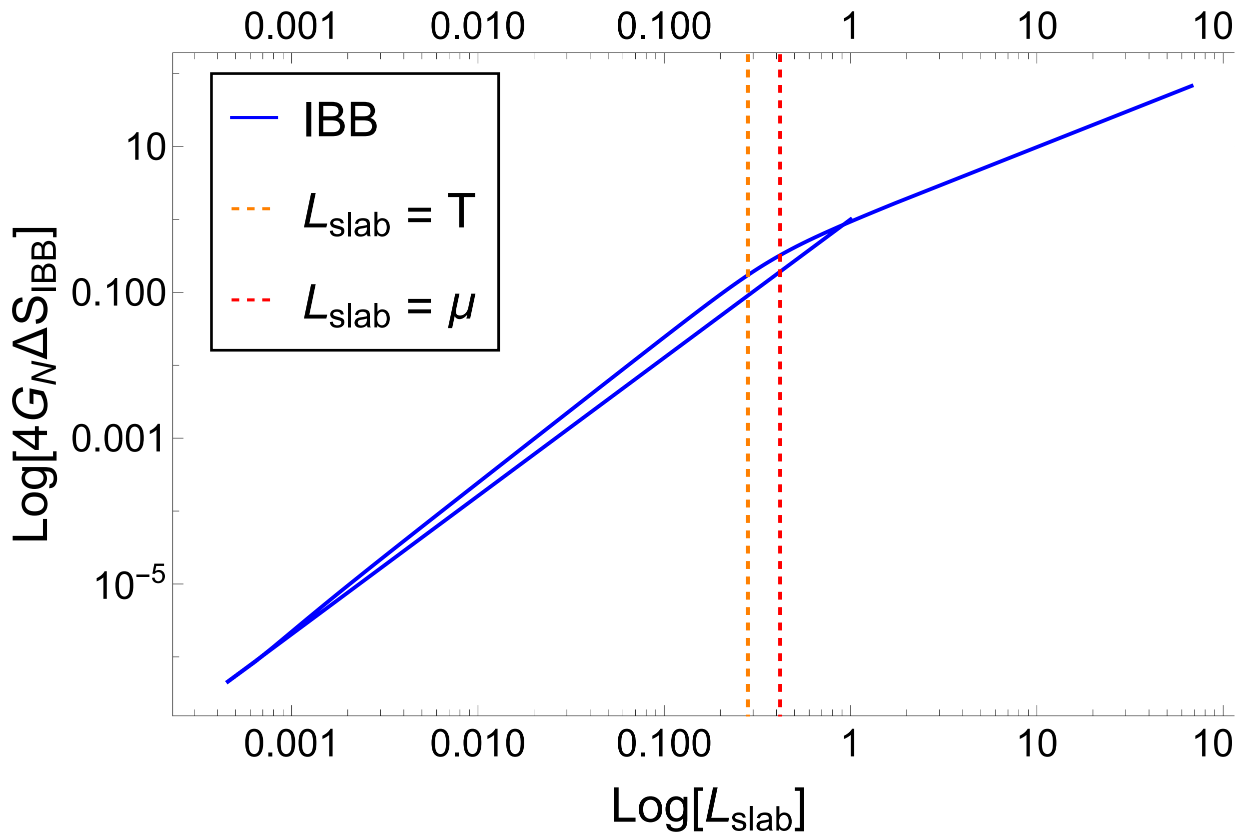
Out[ ]=


0.01
0.10
1
10
0.100
0.001
10^{-5}
IBB
AdS (same T)
AdS (same r_h)
Log[$4G_N\Delta S_{\mathrm{IBB}}$]
Log[L_{slab}]

In[•]:=
```
RSmax = 1000;
ΔSIBB[us_, d_] := Re[NIntegrate[
   1 / u^3 * (sln⟦2⟧[1 / u] - 1) / (Sqrt[1 - (u / us)^(2 * d)]), {u, 1 / BCval2⟦-1⟧, us}]]
Lslab[us_, d_] := Re[-L^2 * NIntegrate[
    - (sln⟦2⟧[1 / u]) / (Sqrt[(us / u)^(2 * d) - 1]), {u, 1 / BCval2⟦-1⟧, us}]]
ParametricPlot[{Lslab[us, d], ΔSIBB[us, d]},
 {us, 1 / RSmax, 1 / rh}, ScalingFunctions → {"Log", "Log"}, Epilog →
  {{Red, Thick, Dashed, Line[{{Log[BCval1⟦5⟧], -100}, {Log[BCval1⟦5⟧], 100}}]},
   {Orange, Thick, Dashed,
    Line[{{Log[ BCval1⟦6⟧], -100}, { Log[BCval1⟦6⟧], 100}}]}},
 PlotStyle → {Thick, Blue}, PlotRange → Full, Frame → True , FrameTicks → All,
 FrameLabel → {Style["Log[L_slab]", Black, 25], Style[ "Log[4G_NΔS_IBB]", Black, 25]},
 FrameTicksStyle → Directive[Black, 20],
 (*RotateLabel→{False,True},*)Axes → False,
 Mesh → False, AspectRatio → 1 / GoldenRatio, ImageSize → 700,
 PlotLegends → Placed[LineLegend[{Blue, {Dashed, Orange}, {Dashed, Red}},
    {"IBB", "L_slab = T", "L_slab = μ"}, LegendLayout → "Column",
    LegendMarkerSize → 25, LabelStyle → {Black, FontSize → 25, Font → "Arial"},
    LegendFunction → Framed], {0.17, 0.76}]]
```

••• NIntegrate : NIntegrate failed to converge to prescribed accuracy after 9 recursive bisections in u near {u} = {0.00102024674558859695644751445500145875522335359164571855217218399048}. NIntegrate obtained $8.06656 \times 10^{-8} - 1.34468 \times 10^{-19}\,i$ and 2.2401417180333675`*^-13 for the integral and error estimates.

Out[ ]=




```
xIBB[U_, us_, d_] :=
 Re[ L^2 * NIntegrate[-( sln⟦2⟧[1 / u]) / (Sqrt[(us / u) ^ (2 * d) - 1]), {u, us, U}]]
xIBBr[U_, rs_, d_] :=
 Re[L^2 * NIntegrate[ 1 / r^2 * sln⟦2⟧[r] / Sqrt[(r / rs) ^ (2 * d) - 1], {r, rs, U}]]
Plot[{-xIBB[U, BCval1⟦5⟧, d], xIBB[U, BCval1⟦5⟧, d]},
 {U, BCval1⟦5⟧, 1 / (BCval2⟦-1⟧ / 10)}, Epilog →
  {Black, Thick, Line[{{ 1 / (BCval2⟦-1⟧ / 10), -100}, {1 / (BCval2⟦-1⟧ / 10), 100}}]},
 PlotStyle → {Thick, Blue}, PlotRange → {{Log[1 / 100], Log[4]}, {Log[0.0001], 3}},
 Frame → True , FrameTicks → All,
 FrameLabel → {Style["L_slab", Black, 25], Style[ "4G_N ΔS_IBB", Black, 25]},
 FrameTicksStyle → Directive[Black, 20], RotateLabel → {False, True},
 Axes → False, Mesh → False, AspectRatio → 1 / GoldenRatio, ImageSize → 700]
```

Out[ ]=

```
$Aborted
```

HERE WE LOOK AT THE CASE WHERE μ = T. IT TRANSLATES INTO CHOOSING c_0 = 0.5

```
(*---------------------------------------- *)
```

```
(* ---------------- LOOK AT μ >> T *)

In[∘]:= rh = 1; c0 = 0.50; d = 3; α = 2;
BCval1 = slngen2[c0, rh, d, α, 1, 1, 1]
BCval2 = slngen2[c0, rh, d, α, Sqrt[BCval1⟦4⟧], 1 / BCval1⟦1⟧, 1]
sln = slnhold2[c0, rh, d, α, Sqrt[BCval1⟦4⟧], 1 / BCval1⟦1⟧, 1]
L = 1;

Clear[Lslab, ΔSIBB, ΔSIBBr, xIBB, xIBBr, Lslab, Lslabr]


ΔSIBB[us_, d_] := Re[NIntegrate[
   1 / u^3 * (sln⟦2⟧[1 / u] - 1) / (Sqrt[1 - (u / us)^(2 * d)]), {u, 1 / BCval2⟦-1⟧, us}]]
ΔSIBBr[rs_, d_] := Re[
  NIntegrate[-r * (sln⟦2⟧[r] - 1) / (Sqrt[1 - (rs / r)^(2 * d)]), {r, BCval2⟦-1⟧, rs}]]


xIBB[U_, us_, d_] :=
 Re[L^2 * NIntegrate[-(sln⟦2⟧[1 / u]) / (Sqrt[(us / u)^(2 * d) - 1]), {u, us, U}]]
xIBBr[U_, rs_, d_] :=
 Re[L^2 * NIntegrate[1 / r^2 * sln⟦2⟧[r] / Sqrt[(r / rs)^(2 * d) - 1], {r, rs, U}]]

Lslab[us_, d_] := Re[-L^2 * NIntegrate[
    -(sln⟦2⟧[1 / u]) / (Sqrt[(us / u)^(2 * d) - 1]), {u, 1 / BCval2⟦-1⟧, us}]]
Lslabr[rs_, d_] :=
 Re[L^2 * NIntegrate[1 / r^2 * 1 / Sqrt[(r / rs)^(2 * d) - 1], {r, rs, BCval2⟦-1⟧}]]


ΔSIBB[us_, d_] := Re[NIntegrate[
   1 / u^3 * (sln⟦2⟧[1 / u] - 1) / (Sqrt[1 - (u / us)^(2 * d)]), {u, 1 / BCval2⟦-1⟧, us}]]
ΔSIBBr[rs_, d_] := Re[
  NIntegrate[-r * (sln⟦2⟧[r] - 1) / (Sqrt[1 - (rs / r)^(2 * d)]), {r, BCval2⟦-1⟧, rs}]]

RSmax = 100; (* BCval1⟦-1⟧;*)

If[BCval1⟦5⟧ < rh, transPoint = BCval1⟦6⟧, transPoint = BCval1⟦5⟧];

Aads[r_, rh_, d_] := (L * r * Sqrt[1 - (rh / r)^(d + 1)])^2
Cads[r_, rh_, d_] := (L / (r * Sqrt[1 - (rh / r)^(d + 1)]))^2
BCval1 = slngen[c0, 1, d, α, 1, 1];
ThAds[rh_, d_] := Sqrt[
   (D[Aads[r, rh1, d], r] /. r → rh1) * (D[1 / Cads[r, rh1, d], r] /. r → rh1)] / (4 * Pi)
rHAds = N[Solve[ThAds[rh, 3] == BCval1⟦6⟧, rh1]⟦2, 1, 2⟧]
C1AdS[r_, d_, rh1_] := 1 / Sqrt[1 - (rh1 / r)^(d + 1)]
```

```
ΔSAdS[us_, d_] :=
 Re[NIntegrate[1 / u^3 * (C1AdS[1 / u, d, rHAds] - 1) / (Sqrt[1 - (u / us) ^ (2 * d)]),
   {u, 1 / BCval2⟦-1⟧, us}]]
ΔSAdS1[us_, d_] :=
 Re[NIntegrate[1 / u^3 * (C1AdS[1 / u, d, rh] - 1) / (Sqrt[1 - (u / us) ^ (2 * d)]),
   {u, 1 / BCval2⟦-1⟧, us}]]

If[rHAds < rh, rIntAdS = rh, rIntAdS = rHAds];
Legended[Show[(*ParametricPlot[{Lslab[us,d],ΔSLif[us, d]},
   {us, 1/RSmax, 1/rh},ScalingFunctions→{"Log","Log"},Epilog→
    {{Red,Thick, Dashed, Line[{{Log[BCval1⟦5⟧],-100},{Log[BCval1⟦5⟧],100}}]},
     {Orange, Thick,Dashed, Line[{{Log[ BCval1⟦6⟧],-100},
        { Log[BCval1⟦6⟧],100}}]}},PlotStyle→{Dashed, Black},
   (*PlotRange→{{Log[1/(1.01*RSmax)],Log[4]},{Log[0.0001], 3}},*)
   Frame→True ,FrameTicks→All,
   FrameLabel→{Style["L_slab",Black, 25],Style[ "4G_N ΔS_IBB",Black, 25]},
   FrameTicksStyle→Directive[Black,20],RotateLabel→{False,True},
   Axes→False,GridLines→Automatic,GridLinesStyle→Directive[Gray, Dashed],
   Mesh→False,AspectRatio→1/GoldenRatio,ImageSize→700],*)
  ParametricPlot[{Lslab[us, d], ΔSAdS[us, d]}, {us, 1 / RSmax, 1 / rIntAdS },
   ScalingFunctions → {"Log", "Log"}, Epilog → {{Green, Thick, Dashed,
      Line[{{Log[BCval1⟦5⟧], -100}, {Log[BCval1⟦5⟧], 100}}]}, {Orange, Thick,
      Dashed, Line[{{Log[ BCval1⟦6⟧], -100}, { Log[BCval1⟦6⟧], 100}}]}},
   PlotStyle → {Dashed, Black}, (*PlotRange→{{Log[1/(1.01*RSmax)],Log[4]},
     {Log[0.0001], 3}},*) Frame → True , FrameTicks → All,
   FrameLabel → {Style["Log[L_slab]", Black, 25], Style[ "Log[4G_NΔS_IBB]", Black, 25]},
   FrameTicksStyle → Directive[Black, 20](*,RotateLabel→{False,True}*),
   Axes → False, Mesh → False, AspectRatio → 1 / GoldenRatio],
  ParametricPlot[{Lslab[us, d], ΔSAdS1[us, d]}, {us, 1 / RSmax, 1 / rh },
   ScalingFunctions → {"Log", "Log"}, Epilog → {{Green, Thick, Dashed,
      Line[{{Log[BCval1⟦5⟧], -100}, {Log[BCval1⟦5⟧], 100}}]}, {Orange, Thick,
      Dashed, Line[{{Log[ BCval1⟦6⟧], -100}, { Log[BCval1⟦6⟧], 100}}]}},
   PlotStyle → {Dashed, Red}, (*PlotRange→{{Log[1/(1.01*RSmax)],Log[4]},
     {Log[0.0001], 3}},*) Frame → True , FrameTicks → All,
   FrameLabel → {Style["L_slab", Black, 25], Style[ "4G_N ΔS", Black, 25]},
   FrameTicksStyle → Directive[Black, 20], RotateLabel → {False, True},
   Axes → False, Mesh → False, AspectRatio → 1 / GoldenRatio],
  ParametricPlot[{Lslab[us, d], ΔSIBB[us, d]}, {us, 1 / RSmax, 1 / rh},
   ScalingFunctions → {"Log", "Log"}, Epilog → {{Red, Thick, Dashed,
      Line[{{Log[BCval1⟦5⟧], -100}, {Log[BCval1⟦5⟧], 100}}]}, {Orange, Thick,
      Dashed, Line[{{Log[ BCval1⟦6⟧], -100}, { Log[BCval1⟦6⟧], 100}}]}},
   PlotStyle → {Thick, Blue}, (*PlotRange→{{Log[1/(1.01*RSmax)],Log[4]},
     {Log[0.0001], 3}}, *)Frame → True , FrameTicks → All,
   FrameLabel → {Style["Log[L_slab]", Black, 25], Style[ "Log[4G_NΔS_IBB]", Black, 25]},
```

```
    FrameTicksStyle → Directive[Black, 20], (*RotateLabel→{False,True},*)
    Axes → False, Mesh → False, AspectRatio → 1 / GoldenRatio],
   PlotRange → All, ImageSize → 1000], Placed[LineLegend[
    {Blue, {Dashed, Black}, {Dashed, Red}(*,{Dashed,Orange}, {Dashed,Red}*)},
    {"IBB", "AdS (same T)", "AdS (same r_h)"(*, "l = T", "l = μ"*)},
    LegendLayout → "Column", LegendMarkerSize → 25,
    LabelStyle → {Black, FontSize → 25, Font → "Arial"},
    LegendFunction → Framed], {0.17, 0.82}]]
```

Out[◦]=

```
{1.32594, 1., 1.49548, 3.30495, 0.620402, 0.240063, 2.58432, 1000.}
```

Out[◦]=

```
{1., 1., 0.619345, 0.999953, 0.61936, 0.240063, 2.57998, 1000.}
```

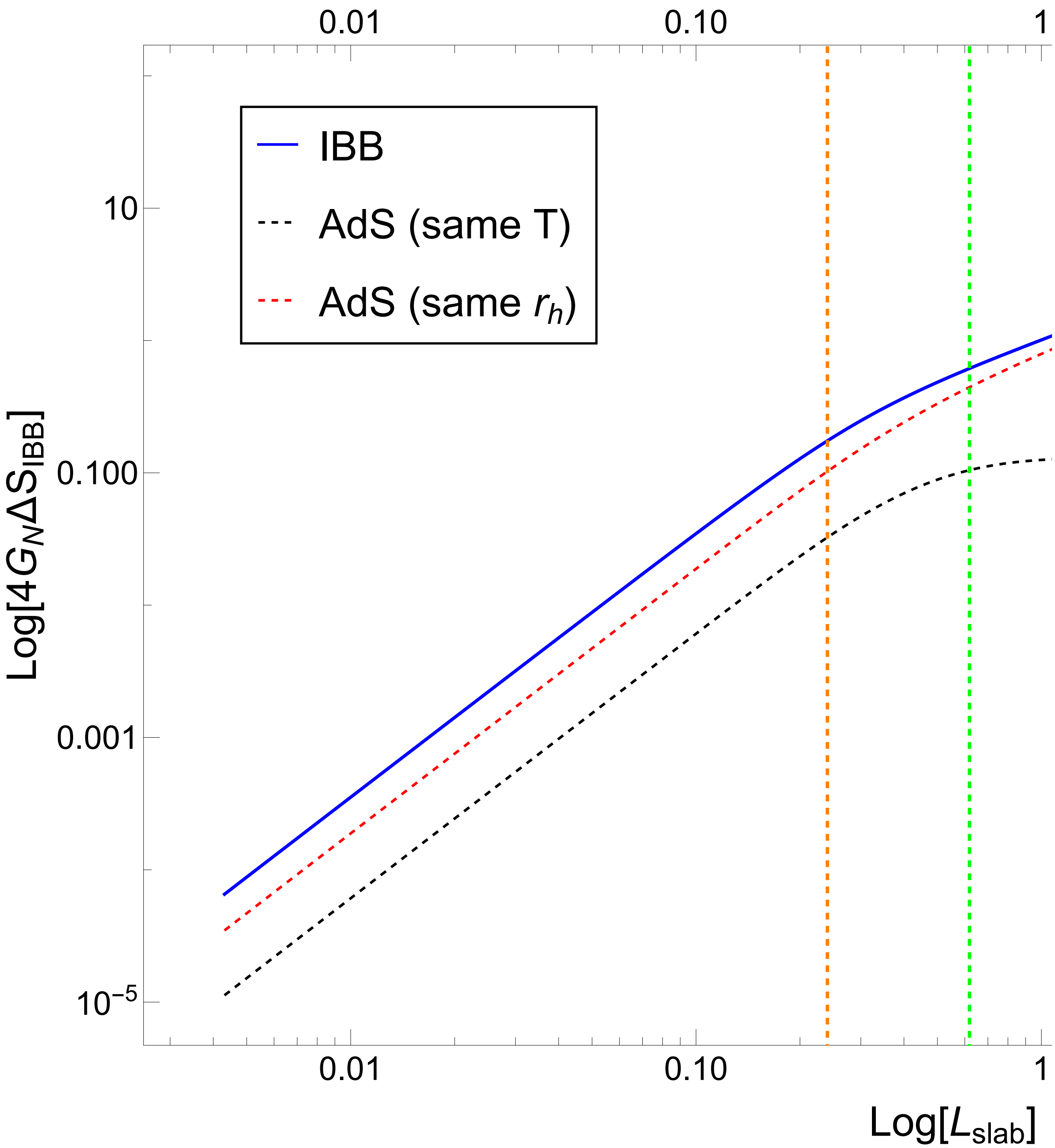
Out[◦]=

```
{InterpolatingFunction[ Domain: {{1., 1.00 ×10^3}} Output: scalar ],
 InterpolatingFunction[ Domain: {{1., 1.00 ×10^3}} Output: scalar ],
 InterpolatingFunction[ Domain: {{1., 1.00 ×10^3}} Output: scalar ],
 InterpolatingFunction[ Domain: {{1., 1.00 ×10^3}} Output: scalar ]}
```

Out[◦]=

```
0.754181
```

Out[ ]=



HERE WE LOOK AT THE CASE WHERE μ > T, WHICH TRANSLATE INTO CHOOSING c_0 > 0.5. WE START TO SEE A CHANGE IN CURVATURE WHERE THE TRANSITION HAPPENS, WHICH IS ROUGHLY AROUND THE GREEN LINE. THIS SUGGESTS SOMETHING NOVEL AND A POTENTIAL VIOLATIONS OF C-THEOREMS.

In[ ]:=
```
rh = 1; c0 = 0.85; d = 3; α = 2;
BCval1 = slngen2[c0, rh, d, α, 1, 1, 1]
```

```
BCval2 = slngen2[c0, rh, d, α, Sqrt[BCval1[[4]]], 1 / BCval1[[1]], 1]
sln = slnhold2[c0, rh, d, α, Sqrt[BCval1[[4]]], 1 / BCval1[[1]], 1]
L = 1;

Clear[Lslab, ΔSIBB, ΔSIBBr, xIBB, xIBBr, Lslab, Lslabr]


ΔSIBB[us_, d_] := Re[NIntegrate[
   1 / u^3 * (sln[[2]][1 / u] - 1) / (Sqrt[1 - (u / us)^(2 * d)]), {u, 1 / BCval2[[-1]], us}]]
ΔSIBBr[rs_, d_] := Re[
  NIntegrate[-r * (sln[[2]][r] - 1) / (Sqrt[1 - (rs / r)^(2 * d)]), {r, BCval2[[-1]], rs}]]


xIBB[U_, us_, d_] :=
 Re[L^2 * NIntegrate[-(sln[[2]][1 / u]) / (Sqrt[(us / u)^(2 * d) - 1]), {u, us, U}]]
xIBBr[U_, rs_, d_] :=
 Re[L^2 * NIntegrate[1 / r^2 * sln[[2]][r] / Sqrt[(r / rs)^(2 * d) - 1], {r, rs, U}]]

Lslab[us_, d_] := Re[-L^2 * NIntegrate[
    -(sln[[2]][1 / u]) / (Sqrt[(us / u)^(2 * d) - 1]), {u, 1 / BCval2[[-1]], us}]]
Lslabr[rs_, d_] :=
 Re[L^2 * NIntegrate[1 / r^2 * 1 / Sqrt[(r / rs)^(2 * d) - 1], {r, rs, BCval2[[-1]]}]]


ΔSIBB[us_, d_] := Re[NIntegrate[
   1 / u^3 * (sln[[2]][1 / u] - 1) / (Sqrt[1 - (u / us)^(2 * d)]), {u, 1 / BCval2[[-1]], us}]]
ΔSIBBr[rs_, d_] := Re[
  NIntegrate[-r * (sln[[2]][r] - 1) / (Sqrt[1 - (rs / r)^(2 * d)]), {r, BCval2[[-1]], rs}]]

RSmax = 100; (* BCval1[[-1]];*)

If[BCval1[[5]] < rh, transPoint = BCval1[[6]], transPoint = BCval1[[5]]];

Aads[r_, rh_, d_] := (L * r * Sqrt[1 - (rh / r)^(d + 1)])^2
Cads[r_, rh_, d_] := (L / (r * Sqrt[1 - (rh / r)^(d + 1)]))^2
BCval1 = slngen[c0, 1, d, α, 1, 1];
ThAds[rh_, d_] := Sqrt[
   (D[Aads[r, rh1, d], r] /. r → rh1) * (D[1 / Cads[r, rh1, d], r] /. r → rh1)] / (4 * Pi)
rHAds = N[Solve[ThAds[rh, 3] == BCval1[[6]], rh1][[2, 1, 2]]]
C1AdS[r_, d_, rh1_] := 1 / Sqrt[1 - (rh1 / r)^(d + 1)]
ΔSAdS[us_, d_] :=
 Re[NIntegrate[1 / u^3 * (C1AdS[1 / u, d, rHAds] - 1) / (Sqrt[1 - (u / us)^(2 * d)]),
   {u, 1 / BCval2[[-1]], us}]]
ΔSAdS1[us_, d_] :=
```

```
 Re[NIntegrate[1 / u^3 * (C1AdS[1 / u, d, rh] - 1) / (Sqrt[1 - (u / us) ^ (2 * d)]),
   {u, 1 / BCval2⟦-1⟧, us}]]

If[rHAds < rh, rIntAdS = rh, rIntAdS = rHAds];
Legended[Show[(*ParametricPlot[{Lslab[us,d],ΔSLif[us, d]},
   {us, 1/RSmax, 1/rh},ScalingFunctions→{"Log","Log"},Epilog→
    {{Red,Thick, Dashed, Line[{{Log[BCval1⟦5⟧],-100},{Log[BCval1⟦5⟧],100}}]},
     {Orange, Thick,Dashed, Line[{{Log[ BCval1⟦6⟧],-100},
        { Log[BCval1⟦6⟧],100}}]}},PlotStyle→{Dashed, Black},
   (*PlotRange→{{Log[1/(1.01*RSmax)],Log[4]},{Log[0.0001], 3}},*)
   Frame→True ,FrameTicks→All,
   FrameLabel→{Style["Lslab",Black, 25],Style[ "4GN ΔSIBB",Black, 25]},
   FrameTicksStyle→Directive[Black,20],RotateLabel→{False,True},
   Axes→False,GridLines→Automatic,GridLinesStyle→Directive[Gray, Dashed],
   Mesh→False,AspectRatio→1/GoldenRatio,ImageSize→700],*)
  ParametricPlot[{Lslab[us, d], ΔSAdS[us, d]}, {us, 1 / RSmax, 1 / rIntAdS },
   ScalingFunctions → {"Log", "Log"}, Epilog → {{Green, Thick, Dashed,
      Line[{{Log[BCval1⟦5⟧], -100}, {Log[BCval1⟦5⟧], 100}}]}, {Orange, Thick,
      Dashed, Line[{{Log[ BCval1⟦6⟧], -100}, { Log[BCval1⟦6⟧], 100}}]}},
   PlotStyle → {Dashed, Black}, (*PlotRange→{{Log[1/(1.01*RSmax)],Log[4]},
     {Log[0.0001], 3}},*) Frame → True , FrameTicks → All,
   FrameLabel → {Style["Log[Lslab]", Black, 25], Style[ "Log[4GNΔSIBB]", Black, 25]},
   FrameTicksStyle → Directive[Black, 20](*,RotateLabel→{False,True}*),
   Axes → False, Mesh → False, AspectRatio → 1 / GoldenRatio],
  ParametricPlot[{Lslab[us, d], ΔSAdS1[us, d]}, {us, 1 / RSmax, 1 / rh },
   ScalingFunctions → {"Log", "Log"}, Epilog → {{Green, Thick, Dashed,
      Line[{{Log[BCval1⟦5⟧], -100}, {Log[BCval1⟦5⟧], 100}}]}, {Orange, Thick,
      Dashed, Line[{{Log[ BCval1⟦6⟧], -100}, { Log[BCval1⟦6⟧], 100}}]}},
   PlotStyle → {Dashed, Red}, (*PlotRange→{{Log[1/(1.01*RSmax)],Log[4]},
     {Log[0.0001], 3}},*) Frame → True , FrameTicks → All,
   FrameLabel → {Style["Lslab", Black, 25], Style[ "4GN ΔS", Black, 25]},
   FrameTicksStyle → Directive[Black, 20], RotateLabel → {False, True},
   Axes → False, Mesh → False, AspectRatio → 1 / GoldenRatio],
  ParametricPlot[{Lslab[us, d], ΔSIBB[us, d]}, {us, 1 / RSmax, 1 / rh},
   ScalingFunctions → {"Log", "Log"}, Epilog → {{Red, Thick, Dashed,
      Line[{{Log[BCval1⟦5⟧], -100}, {Log[BCval1⟦5⟧], 100}}]}, {Orange, Thick,
      Dashed, Line[{{Log[ BCval1⟦6⟧], -100}, { Log[BCval1⟦6⟧], 100}}]}},
   PlotStyle → {Thick, Blue}, (*PlotRange→{{Log[1/(1.01*RSmax)],Log[4]},
     {Log[0.0001], 3}}, *)Frame → True , FrameTicks → All,
   FrameLabel → {Style["Log[Lslab]", Black, 25], Style[ "Log[4GNΔSIBB]", Black, 25]},
   FrameTicksStyle → Directive[Black, 20], (*RotateLabel→{False,True},*)
   Axes → False, Mesh → False, AspectRatio → 1 / GoldenRatio],
  PlotRange → All, ImageSize → 1000], Placed[LineLegend[
   {Blue, {Dashed, Black}, {Dashed, Red}(*,{Dashed,Orange}, {Dashed,Red}*)},
```

```
    {"IBB", "AdS (same T)", "AdS (same r_h)"(*, "l = T", "l = μ"*)},
    LegendLayout → "Column", LegendMarkerSize → 25,
    LabelStyle → {Black, FontSize → 25, Font → "Arial"},
    LegendFunction → Framed], {0.17, 0.82}]]
```

Out[ ]=

```
{2.01841, 1., 10.0432, 25.6717, 0.98206, 0.157704, 6.22725, 1000.}
```

Out[ ]=

```
{1., 1., 0.982705, 0.999994, 0.982708, 0.157704, 6.23136, 1000.}
```

Out[ ]=

```
{InterpolatingFunction[ Domain: {{1., 1.00 ×10^3}} Output: scalar ],
 InterpolatingFunction[ Domain: {{1., 1.00 ×10^3}} Output: scalar ],
 InterpolatingFunction[ Domain: {{1., 1.00 ×10^3}} Output: scalar ],
 InterpolatingFunction[ Domain: {{1., 1.00 ×10^3}} Output: scalar ]}
```

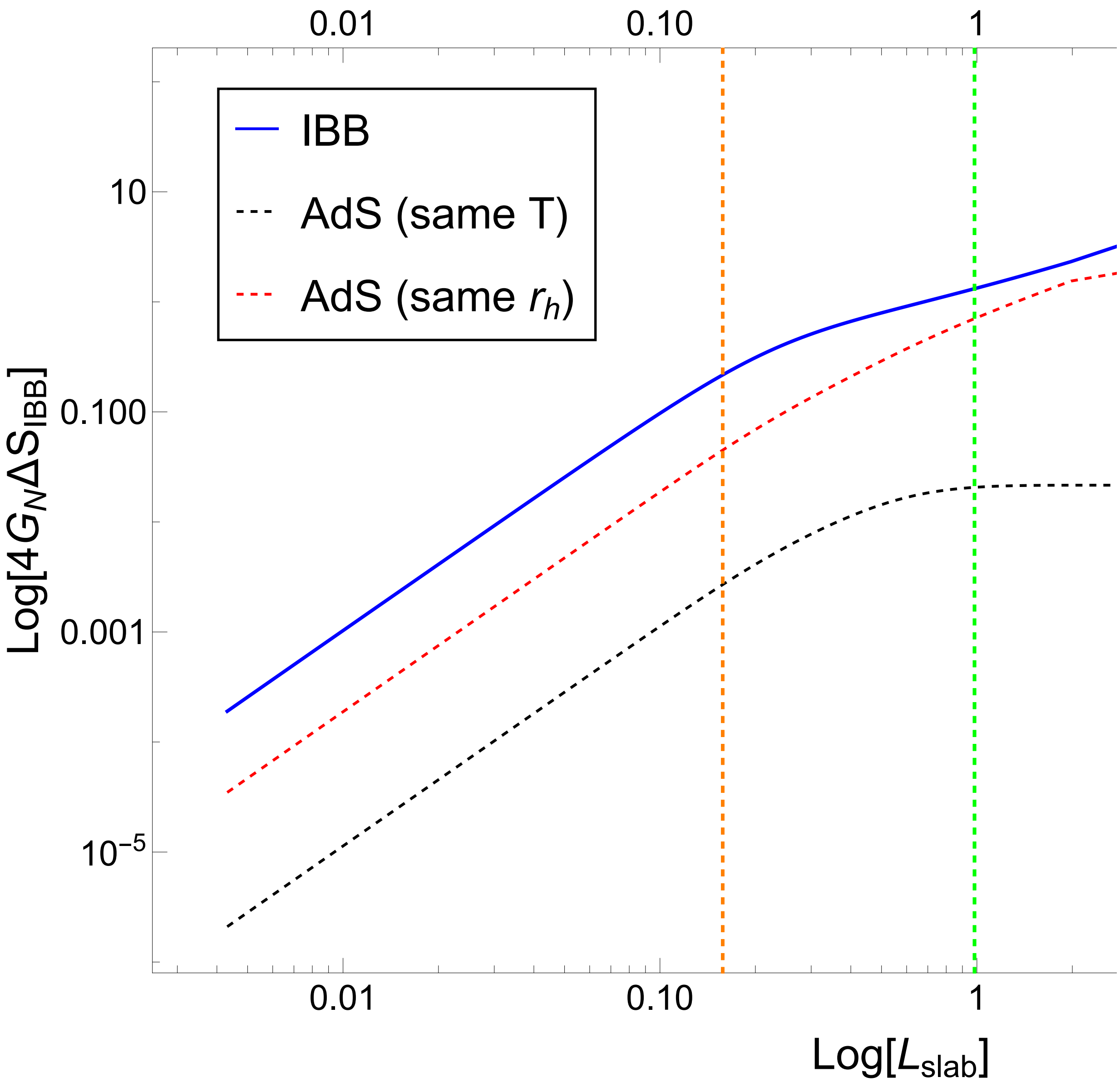
Out[ ]=

```
0.495438
```

NIntegrate : Numerical integration converging too slowly; suspect one of the following: singularity, value of the integration is 0, highly oscillatory integrand, or WorkingPrecision too small.

NIntegrate : NIntegrate failed to converge to prescribed accuracy after 9 recursive bisections in u near {u} = {0.0010437068766009455846087775054975566035864176228642463684082031250O}. NIntegrate obtained 2.1055513154500188`*^-6 and 3.755081109160993`*^-12 for the integral and error estimates.

Out[ ]=



HERE WE LOOK AT THE CASE WHERE μ >> T, WHICH TRANSLATE INTO CHOOSING c_0 ~ 1. THE CHANGE IN CURVATURE BECOMES MUCH MORE PRONOUNCED. WE TEST THE EFFECTS OF INCREASING THE DIMENSIONS (d), THE STRENGTH OF THE GAUGE COUPLING (α) AND THE RADIUS OF THE HORIZON r_h TO EXPLORE THE EFFECTS THAT IT HAS ON THE CURVATURE SHIFTS.

In[ ]:=
```
rh = 1; c0 = 0.95; d = 3; α = 2;
BCval1 = slngen2[c0, rh, d, α, 1, 1, 1]
```

```
BCval2 = slngen2[c0, rh, d, α, Sqrt[BCval1⟦4⟧], 1 / BCval1⟦1⟧, 1]
sln = slnhold2[c0, rh, d, α, Sqrt[BCval1⟦4⟧], 1 / BCval1⟦1⟧, 1]
L = 1;

Clear[Lslab, ΔSIBB, ΔSIBBr, xIBB, xIBBr, Lslab, Lslabr]


ΔSIBB[us_, d_] := Re[NIntegrate[
   1 / u^3 * (sln⟦2⟧[1 / u] - 1) / (Sqrt[1 - (u / us) ^ (2 * d)]), {u, 1 / BCval2⟦-1⟧, us}]]
ΔSIBBr[rs_, d_] := Re[
  NIntegrate[-r * (sln⟦2⟧[r] - 1) / (Sqrt[1 - (rs / r) ^ (2 * d)]), {r, BCval2⟦-1⟧, rs}]]


xIBB[U_, us_, d_] :=
 Re[ L^2 * NIntegrate[-(sln⟦2⟧[1 / u]) / (Sqrt[(us / u) ^ (2 * d) - 1]), {u, us, U}]]
xIBBr[U_, rs_, d_] :=
 Re[L^2 * NIntegrate[ 1 / r^2 * sln⟦2⟧[r] / Sqrt[(r / rs) ^ (2 * d) - 1], {r, rs, U}]]

Lslab[us_, d_] := Re[-L^2 * NIntegrate[
    -(sln⟦2⟧[1 / u]) / (Sqrt[(us / u) ^ (2 * d) - 1]), {u, 1 / BCval2⟦-1⟧, us}]]
Lslabr[rs_, d_] :=
 Re[ L^2 * NIntegrate[1 / r^2 * 1 / Sqrt[(r / rs) ^ (2 * d) - 1], {r, rs, BCval2⟦-1⟧}]]


ΔSIBB[us_, d_] := Re[NIntegrate[
   1 / u^3 * (sln⟦2⟧[1 / u] - 1) / (Sqrt[1 - (u / us) ^ (2 * d)]), {u, 1 / BCval2⟦-1⟧, us}]]
ΔSIBBr[rs_, d_] := Re[
  NIntegrate[-r * (sln⟦2⟧[r] - 1) / (Sqrt[1 - (rs / r) ^ (2 * d)]), {r, BCval2⟦-1⟧, rs}]]

RSmax = 100; (* BCval1⟦-1⟧;*)

If[BCval1⟦5⟧ < rh, transPoint = BCval1⟦6⟧, transPoint = BCval1⟦5⟧];

Aads[r_, rh_, d_] := (L * r * Sqrt[1 - (rh / r) ^ (d + 1)]) ^2
Cads[r_, rh_, d_] := (L / (r * Sqrt[1 - (rh / r) ^ (d + 1)])) ^2
BCval1 = slngen[c0, 1, d, α, 1, 1];
ThAds[rh_, d_] := Sqrt[
   (D[Aads[r, rh1, d], r] /. r → rh1) * (D[1 / Cads[r, rh1, d], r] /. r → rh1)] / (4 * Pi)
rHAds = N[Solve[ThAds[rh, 3] == BCval1⟦6⟧, rh1]⟦2, 1, 2⟧]
C1AdS[r_, d_, rh1_] := 1 / Sqrt[1 - (rh1 / r) ^ (d + 1)]
ΔSAdS[us_, d_] :=
 Re[NIntegrate[1 / u^3 * (C1AdS[1 / u, d, rHAds] - 1) / (Sqrt[1 - (u / us) ^ (2 * d)]),
   {u, 1 / BCval2⟦-1⟧, us}]]
ΔSAdS1[us_, d_] :=
```

```
 Re[NIntegrate[1 / u^3 * (C1AdS[1 / u, d, rh] - 1) / (Sqrt[1 - (u / us) ^ (2 * d)]),
   {u, 1 / BCval2[[-1]], us}]]

If[rHAds < rh, rIntAdS = rh, rIntAdS = rHAds];
Legended[Show[(*ParametricPlot[{Lslab[us,d],ΔSLif[us, d]},
   {us, 1/RSmax, 1/rh},ScalingFunctions→{"Log","Log"},Epilog→
    {{Red,Thick, Dashed, Line[{{Log[BCval1[[5]],-100},{Log[BCval1[[5]],100}}]},
     {Orange, Thick,Dashed, Line[{{Log[ BCval1[[6]],-100},
        { Log[BCval1[[6]],100}}]}},PlotStyle→{Dashed, Black},
   (*PlotRange→{{Log[1/(1.01*RSmax)],Log[4]},{Log[0.0001], 3}},*)
   Frame→True ,FrameTicks→All,
   FrameLabel→{Style["Lslab",Black, 25],Style[ "4GN ΔSIBB",Black, 25]},
   FrameTicksStyle→Directive[Black,20],RotateLabel→{False,True},
   Axes→False,GridLines→Automatic,GridLinesStyle→Directive[Gray, Dashed],
   Mesh→False,AspectRatio→1/GoldenRatio,ImageSize→700],*)
  ParametricPlot[{Lslab[us, d], ΔSAdS[us, d]}, {us, 1 / RSmax, 1 / rIntAdS },
   ScalingFunctions → {"Log", "Log"}, Epilog → {{Green, Thick, Dashed,
      Line[{{Log[BCval1[[5]], -100}, {Log[BCval1[[5]], 100}}]}, {Orange, Thick,
      Dashed, Line[{{Log[ BCval1[[6]], -100}, { Log[BCval1[[6]], 100}}]}},
   PlotStyle → {Dashed, Black}, (*PlotRange→{{Log[1/(1.01*RSmax)],Log[4]},
     {Log[0.0001], 3}},*) Frame → True , FrameTicks → All,
   FrameLabel → {Style["Log[Lslab]", Black, 25], Style[ "Log[4GNΔSIBB]", Black, 25]},
   FrameTicksStyle → Directive[Black, 20](*,RotateLabel→{False,True}*),
   Axes → False, Mesh → False, AspectRatio → 1 / GoldenRatio],
  ParametricPlot[{Lslab[us, d], ΔSAdS1[us, d]}, {us, 1 / RSmax, 1 / rh },
   ScalingFunctions → {"Log", "Log"}, Epilog → {{Green, Thick, Dashed,
      Line[{{Log[BCval1[[5]], -100}, {Log[BCval1[[5]], 100}}]}, {Orange, Thick,
      Dashed, Line[{{Log[ BCval1[[6]], -100}, { Log[BCval1[[6]], 100}}]}},
   PlotStyle → {Dashed, Red}, (*PlotRange→{{Log[1/(1.01*RSmax)],Log[4]},
     {Log[0.0001], 3}},*) Frame → True , FrameTicks → All,
   FrameLabel → {Style["Lslab", Black, 25], Style[ "4GN ΔS", Black, 25]},
   FrameTicksStyle → Directive[Black, 20], RotateLabel → {False, True},
   Axes → False, Mesh → False, AspectRatio → 1 / GoldenRatio],
  ParametricPlot[{Lslab[us, d], ΔSIBB[us, d]}, {us, 1 / RSmax, 1 / rh},
   ScalingFunctions → {"Log", "Log"}, Epilog → {{Red, Thick, Dashed,
      Line[{{Log[BCval1[[5]], -100}, {Log[BCval1[[5]], 100}}]}, {Orange, Thick,
      Dashed, Line[{{Log[ BCval1[[6]], -100}, { Log[BCval1[[6]], 100}}]}},
   PlotStyle → {Thick, Blue}, (*PlotRange→{{Log[1/(1.01*RSmax)],Log[4]},
     {Log[0.0001], 3}}, *)Frame → True , FrameTicks → All,
   FrameLabel → {Style["Log[Lslab]", Black, 25], Style[ "Log[4GNΔSIBB]", Black, 25]},
   FrameTicksStyle → Directive[Black, 20], (*RotateLabel→{False,True},*)
   Axes → False, Mesh → False, AspectRatio → 1 / GoldenRatio],
  PlotRange → All, ImageSize → 1000], Placed[LineLegend[
   {Blue, {Dashed, Black}, {Dashed, Red}(*,{Dashed,Orange}, {Dashed,Red}*)},
```

```
{"IBB", "AdS (same T)", "AdS (same r_h)"(*, "l = T", "l = μ"*)},
LegendLayout → "Column", LegendMarkerSize → 25,
LabelStyle → {Black, FontSize → 25, Font → "Arial"},
LegendFunction → Framed], {0.17, 0.82}]]
```

Out[ ]=

```
{2.88215, 1., 39.373, 113.439, 1.28263, 0.110442, 11.6136, 1000.}
```

Out[ ]=

```
{1., 1., 1.28301, 1.00001, 1.283, 0.110442, 11.6169, 1000.}
```

Out[ ]=

```
{InterpolatingFunction[ Domain: {{1., 1.00 ×10^3}} Output: scalar ],
 InterpolatingFunction[ Domain: {{1., 1.00 ×10^3}} Output: scalar ],
 InterpolatingFunction[ Domain: {{1., 1.00 ×10^3}} Output: scalar ],
 InterpolatingFunction[ Domain: {{1., 1.00 ×10^3}} Output: scalar ]}
```

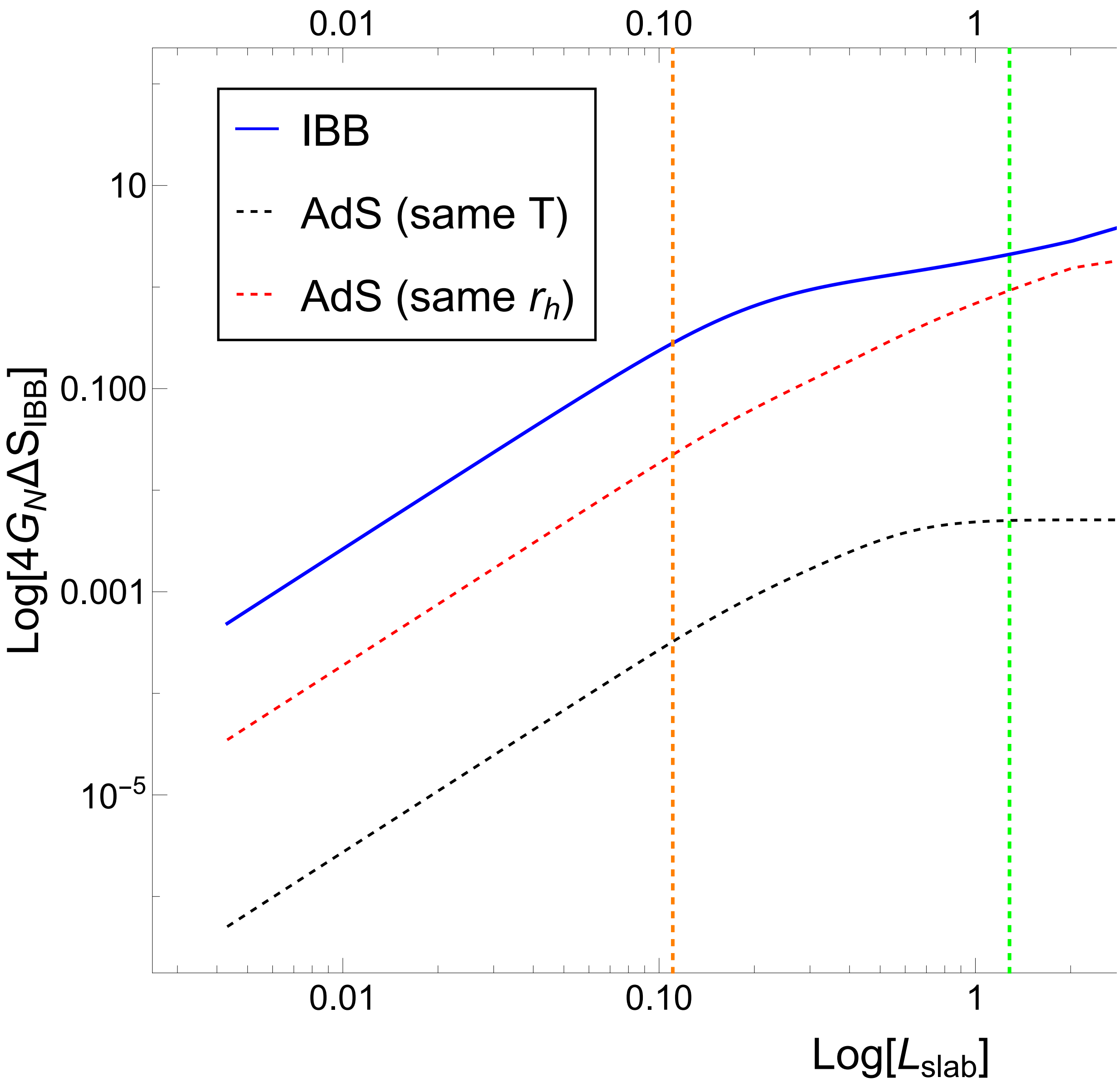
Out[ ]=

```
0.346963
```

NIntegrate : Numerical integration converging too slowly; suspect one of the following: singularity, value of the integration is 0, highly oscillatory integrand, or WorkingPrecision too small.

NIntegrate : NIntegrate failed to converge to prescribed accuracy after 9 recursive bisections in u near {u} = {0.00191597 }. NIntegrate obtained 5.064942805158671` *^-7 and 1.5053725736422425` *^-12 for the integral and error estimates.

Out[ ]=



In[ ]:=
```
rh = 2; c0 = 0.95; d = 3; α = 2;
BCval1 = slngen2[c0, rh, d, α, 1, 1, 1]
BCval2 = slngen2[c0, rh, d, α, Sqrt[BCval1⟦4⟧], 1 / BCval1⟦1⟧, 1]
sln = slnhold2[c0, rh, d, α, Sqrt[BCval1⟦4⟧], 1 / BCval1⟦1⟧, 1]
L = 1;

Clear[Lslab, ΔSIBB, ΔSIBBr, xIBB, xIBBr, Lslab, Lslabr]


ΔSIBB[us_, d_] := Re[NIntegrate[
   1 / u^3 * (sln⟦2⟧[1 / u] - 1) / (Sqrt[1 - (u / us) ^ (2 * d)]), {u, 1 / BCval2⟦-1⟧, us}]]
ΔSIBBr[rs_, d_] := Re[
```

```
   NIntegrate[-r * ( sln⟦2⟧[r] - 1) / (Sqrt[1 - (rs / r) ^ (2 * d)]), {r, BCval2⟦-1⟧, rs}]]


xIBB[U_, us_, d_] :=
 Re[ L^2 * NIntegrate[- ( sln⟦2⟧[1 / u]) / (Sqrt[(us / u) ^ (2 * d) - 1]), {u, us, U}]]
xIBBr[U_, rs_, d_] :=
 Re[L^2 * NIntegrate[ 1 / r^2 * sln⟦2⟧[r] / Sqrt[(r / rs) ^ (2 * d) - 1], {r, rs, U}]]

Lslab[us_, d_ ] := Re[-L^2 * NIntegrate[
    - ( sln⟦2⟧[1 / u]) / (Sqrt[(us / u) ^ (2 * d) - 1]), {u, 1 / BCval2⟦-1⟧, us}]]
Lslabr[rs_, d_] :=
 Re[ L^2 * NIntegrate[1 / r^2 * 1 / Sqrt[(r / rs) ^ (2 * d) - 1], {r, rs, BCval2⟦-1⟧}]]



ΔSIBB[us_, d_ ] := Re[NIntegrate[
   1 / u^3 * ( sln⟦2⟧[1 / u] - 1) / (Sqrt[1 - (u / us) ^ (2 * d)]), {u, 1 / BCval2⟦-1⟧, us}]]
ΔSIBBr[rs_, d_ ] := Re[
  NIntegrate[-r * ( sln⟦2⟧[r] - 1) / (Sqrt[1 - (rs / r) ^ (2 * d)]), {r, BCval2⟦-1⟧, rs}]]

RSmax = 100; (* BCval1⟦-1⟧;*)

If[BCval1⟦5⟧ < rh, transPoint = BCval1⟦6⟧, transPoint = BCval1⟦5⟧];

Aads[r_, rh_, d_] := (L * r * Sqrt[1 - (rh / r) ^ (d + 1)]) ^2
Cads[r_, rh_, d_] := (L / (r * Sqrt[1 - (rh / r) ^ (d + 1)])) ^2
BCval1 = slngen[c0, 1, d, α, 1, 1];
ThAds[rh_, d_] := Sqrt[
   (D[Aads[r, rh1, d], r] /. r → rh1) * (D[1 / Cads[r, rh1, d], r] /. r → rh1)] / (4 * Pi)
rHAds = N[Solve[ThAds[rh, 3] == BCval1⟦6⟧, rh1]⟦2, 1, 2⟧]
C1AdS[r_, d_, rh1_] := 1 / Sqrt[1 - (rh1 / r) ^ (d + 1)]
ΔSAdS[us_, d_ ] :=
 Re[NIntegrate[1 / u^3 * (C1AdS[1 / u, d, rHAds] - 1) / (Sqrt[1 - (u / us) ^ (2 * d)]),
   {u, 1 / BCval2⟦-1⟧, us}]]
ΔSAdS1[us_, d_ ] :=
 Re[NIntegrate[1 / u^3 * (C1AdS[1 / u, d, rh] - 1) / (Sqrt[1 - (u / us) ^ (2 * d)]),
   {u, 1 / BCval2⟦-1⟧, us}]]

If[rHAds < rh, rIntAdS = rh, rIntAdS = rHAds];
Legended[Show[(*ParametricPlot[{Lslab[us,d],ΔSLif[us, d]},
   {us, 1/RSmax, 1/rh},ScalingFunctions→{"Log","Log"},Epilog→
    {{Red,Thick, Dashed, Line[{{Log[BCval1⟦5⟧],-100},{Log[BCval1⟦5⟧],100}}]},
     {Orange, Thick,Dashed, Line[{{Log[ BCval1⟦6⟧],-100},
        { Log[BCval1⟦6⟧],100}}]}},PlotStyle→{Dashed, Black},
   (*PlotRange→{{Log[1/(1.01*RSmax)],Log[4]},{Log[0.0001], 3}},*)
```

```
   Frame→True ,FrameTicks→All,
   FrameLabel→{Style["Lslab",Black, 25],Style[ "4GN ΔSIBB",Black, 25]},
   FrameTicksStyle→Directive[Black,20],RotateLabel→{False,True},
   Axes→False,GridLines→Automatic,GridLinesStyle→Directive[Gray, Dashed],
   Mesh→False,AspectRatio→1/GoldenRatio,ImageSize→700],*)
  ParametricPlot[{Lslab[us, d], ΔSAdS[us, d]}, {us, 1 / RSmax, 1 / rIntAdS },
   ScalingFunctions → {"Log", "Log"}, Epilog → {{Green, Thick, Dashed,
      Line[{{Log[BCval1[[5]]], -100}, {Log[BCval1[[5]]], 100}}]}, {Orange, Thick,
      Dashed, Line[{{Log[ BCval1[[6]]], -100}, { Log[BCval1[[6]]], 100}}]}},
   PlotStyle → {Dashed, Black}, (*PlotRange→{{Log[1/(1.01*RSmax)],Log[4]},
     {Log[0.0001], 3}},*) Frame → True , FrameTicks → All,
   FrameLabel → {Style["Log[Lslab]", Black, 25], Style[ "Log[4GNΔSIBB]", Black, 25]},
   FrameTicksStyle → Directive[Black, 20](*,RotateLabel→{False,True}*),
   Axes → False, Mesh → False, AspectRatio → 1 / GoldenRatio],
  ParametricPlot[{Lslab[us, d], ΔSAdS1[us, d]}, {us, 1 / RSmax, 1 / rh },
   ScalingFunctions → {"Log", "Log"}, Epilog → {{Green, Thick, Dashed,
      Line[{{Log[BCval1[[5]]], -100}, {Log[BCval1[[5]]], 100}}]}, {Orange, Thick,
      Dashed, Line[{{Log[ BCval1[[6]]], -100}, { Log[BCval1[[6]]], 100}}]}},
   PlotStyle → {Dashed, Red}, (*PlotRange→{{Log[1/(1.01*RSmax)],Log[4]},
     {Log[0.0001], 3}},*) Frame → True , FrameTicks → All,
   FrameLabel → {Style["Lslab", Black, 25], Style[ "4GN ΔS", Black, 25]},
   FrameTicksStyle → Directive[Black, 20], RotateLabel → {False, True},
   Axes → False, Mesh → False, AspectRatio → 1 / GoldenRatio],
  ParametricPlot[{Lslab[us, d], ΔSIBB[us, d]}, {us, 1 / RSmax, 1 / rh},
   ScalingFunctions → {"Log", "Log"}, Epilog → {{Red, Thick, Dashed,
      Line[{{Log[BCval1[[5]]], -100}, {Log[BCval1[[5]]], 100}}]}, {Orange, Thick,
      Dashed, Line[{{Log[ BCval1[[6]]], -100}, { Log[BCval1[[6]]], 100}}]}},
   PlotStyle → {Thick, Blue}, (*PlotRange→{{Log[1/(1.01*RSmax)],Log[4]},
     {Log[0.0001], 3}}, *)Frame → True , FrameTicks → All,
   FrameLabel → {Style["Log[Lslab]", Black, 25], Style[ "Log[4GNΔSIBB]", Black, 25]},
   FrameTicksStyle → Directive[Black, 20], (*RotateLabel→{False,True},*)
   Axes → False, Mesh → False, AspectRatio → 1 / GoldenRatio],
  PlotRange → All, ImageSize → 1000], Placed[LineLegend[
   {Blue, {Dashed, Black}, {Dashed, Red}(*,{Dashed,Orange}, {Dashed,Red}*)},
   {"IBB", "AdS (same T)", "AdS (same rh)"(*, "l = T", "l = μ"*)},
   LegendLayout → "Column", LegendMarkerSize → 25,
   LabelStyle → {Black, FontSize → 25, Font → "Arial"},
   LegendFunction → Framed], {0.17, 0.82}]]
```

Out[ ]=

```
{2.88215, 1., 629.98, 7260.09, 2.5653, 0.220884, 11.6138, 2000.}
```

Out[ ]=

```
{0.999999, 1., 2.56308, 0.999982, 2.5631, 0.220884, 11.6039, 2000.}
```

Out[ ]=

{InterpolatingFunction[Domain: {{2., 2.00 ×10^3}} Output: scalar],

InterpolatingFunction[Domain: {{2., 2.00 ×10^3}} Output: scalar],

InterpolatingFunction[Domain: {{2., 2.00 ×10^3}} Output: scalar],

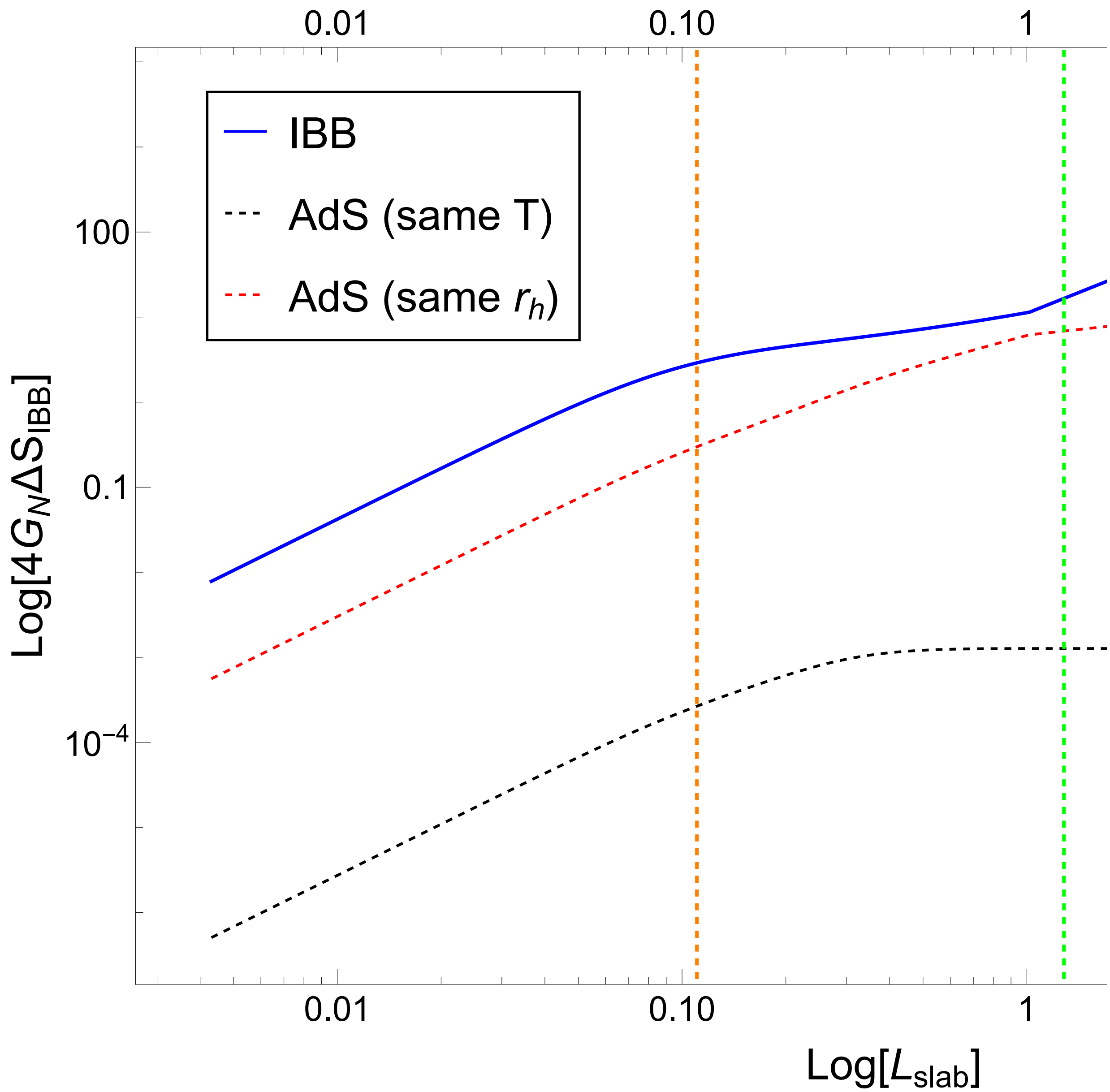
InterpolatingFunction[Domain: {{2., 2.00 ×10^3}} Output: scalar]}

Out[ ]=

0.346963

NIntegrate : NIntegrate failed to converge to prescribed accuracy after 9 recursive bisections in u near {u} = {0.000599876927610635019981651094855834571717423386871814727783203125000}. NIntegrate obtained 5.082856174725574` *^-7 and 7.17129358957050 44` *^-12 for the integral and error estimates.

Out[ ]=



In[ ]:=
```
rh = 1; c0 = 0.95; d = 6; α = 2;
BCval1 = slngen2[c0, rh, d, α, 1, 1, 1]
BCval2 = slngen2[c0, rh, d, α, Sqrt[BCval1⟦4⟧], 1 / BCval1⟦1⟧, 1]
sln = slnhold2[c0, rh, d, α, Sqrt[BCval1⟦4⟧], 1 / BCval1⟦1⟧, 1]
L = 1;

Clear[Lslab, ΔSIBB, ΔSIBBr, xIBB, xIBBr, Lslab, Lslabr]


ΔSIBB[us_, d_] := Re[NIntegrate[
   1 / u^3 * (sln⟦2⟧[1 / u] - 1) / (Sqrt[1 - (u / us) ^ (2 * d)]), {u, 1 / BCval2⟦-1⟧, us}]]
```

```
ΔSIBBr[rs_, d_ ] := Re[
  NIntegrate[-r * ( sln⟦2⟧[r] - 1) / (Sqrt[1 - (rs / r) ^ (2 * d)]), {r, BCval2⟦-1⟧, rs}]]


xIBB[U_, us_, d_] :=
 Re[ L ^ 2 * NIntegrate[- ( sln⟦2⟧[1 / u]) / (Sqrt[(us / u) ^ (2 * d) - 1]), {u, us, U}]]
xIBBr[U_, rs_, d_] :=
 Re[L ^ 2 * NIntegrate[ 1 / r ^ 2 * sln⟦2⟧[r] / Sqrt[(r / rs) ^ (2 * d) - 1], {r, rs, U}]]

Lslab[us_, d_ ] := Re[-L ^ 2 * NIntegrate[
    - ( sln⟦2⟧[1 / u]) / (Sqrt[(us / u) ^ (2 * d) - 1]), {u, 1 / BCval2⟦-1⟧, us}]]
Lslabr[rs_, d_] :=
 Re[ L ^ 2 * NIntegrate[1 / r ^ 2 * 1 / Sqrt[(r / rs) ^ (2 * d) - 1], {r, rs, BCval2⟦-1⟧}]]


ΔSIBB[us_, d_ ] := Re[NIntegrate[
   1 / u ^ 3 * ( sln⟦2⟧[1 / u] - 1) / (Sqrt[1 - (u / us) ^ (2 * d)]), {u, 1 / BCval2⟦-1⟧, us}]]
ΔSIBBr[rs_, d_ ] := Re[
  NIntegrate[-r * ( sln⟦2⟧[r] - 1) / (Sqrt[1 - (rs / r) ^ (2 * d)]), {r, BCval2⟦-1⟧, rs}]]

RSmax = BCval1⟦-1⟧ / 2; (* BCval1⟦-1⟧;*)

If[BCval1⟦5⟧ < rh, transPoint = BCval1⟦6⟧, transPoint = BCval1⟦5⟧];

Aads[r_, rh_, d_] := (L * r * Sqrt[1 - (rh / r) ^ (d + 1)]) ^ 2
Cads[r_, rh_, d_] := (L / (r * Sqrt[1 - (rh / r) ^ (d + 1)])) ^ 2
BCval1 = slngen[c0, 1, d, α, 1, 1];
ThAds[rh_, d_] := Sqrt[
   (D[Aads[r, rh1, d], r] /. r → rh1) * (D[1 / Cads[r, rh1, d], r] /. r → rh1)] / (4 * Pi)
rHAds = N[Solve[ThAds[rh, 3] == BCval1⟦6⟧, rh1]⟦2, 1, 2⟧]
C1AdS[r_, d_, rh1_] := 1 / Sqrt[1 - (rh1 / r) ^ (d + 1)]
ΔSAdS[us_, d_ ] :=
 Re[NIntegrate[1 / u ^ 3 * (C1AdS[1 / u, d, rHAds] - 1) / (Sqrt[1 - (u / us) ^ (2 * d)]),
   {u, 1 / BCval2⟦-1⟧, us}]]
ΔSAdS1[us_, d_ ] :=
 Re[NIntegrate[1 / u ^ 3 * (C1AdS[1 / u, d, rh] - 1) / (Sqrt[1 - (u / us) ^ (2 * d)]),
   {u, 1 / BCval2⟦-1⟧, us}]]

If[rHAds < rh, rIntAdS = rh, rIntAdS = rHAds];
Legended[Show[(*ParametricPlot[{Lslab[us,d],ΔSLif[us, d]},
   {us, 1/RSmax, 1/rh},ScalingFunctions→{"Log","Log"},Epilog→
    {{Red,Thick, Dashed, Line[{{Log[BCval1⟦5⟧],-100},{Log[BCval1⟦5⟧],100}}]},
     {Orange, Thick,Dashed, Line[{{Log[ BCval1⟦6⟧],-100},
        { Log[BCval1⟦6⟧],100}}]}},PlotStyle→{Dashed, Black},
```

```
  (*PlotRange→{{Log[1/(1.01*RSmax)],Log[4]},{Log[0.0001], 3}},*)
  Frame→True ,FrameTicks→All,
  FrameLabel→{Style["Lslab",Black, 25],Style[ "4GN ΔSIBB",Black, 25]},
  FrameTicksStyle→Directive[Black,20],RotateLabel→{False,True},
  Axes→False,GridLines→Automatic,GridLinesStyle→Directive[Gray, Dashed],
  Mesh→False,AspectRatio→1/GoldenRatio,ImageSize→700],*)
 ParametricPlot[{Lslab[us, d], ΔSAdS[us, d]}, {us, 1 / RSmax, 1 / rIntAdS },
  ScalingFunctions → {"Log", "Log"}, Epilog → {{Green, Thick, Dashed,
     Line[{{Log[BCval1[[5]]], -100}, {Log[BCval1[[5]]], 100}}]}, {Orange, Thick,
     Dashed, Line[{{Log[ BCval1[[6]]], -100}, { Log[BCval1[[6]]], 100}}]}},
  PlotStyle → {Dashed, Black}, (*PlotRange→{{Log[1/(1.01*RSmax)],Log[4]},
    {Log[0.0001], 3}},*) Frame → True , FrameTicks → All,
  FrameLabel → {Style["Log[Lslab]", Black, 25], Style[ "Log[4GNΔSIBB]", Black, 25]},
  FrameTicksStyle → Directive[Black, 20](*,RotateLabel→{False,True}*),
  Axes → False, Mesh → False, AspectRatio → 1 / GoldenRatio],
 ParametricPlot[{Lslab[us, d], ΔSAdS1[us, d]}, {us, 1 / RSmax, 1 / rh },
  ScalingFunctions → {"Log", "Log"}, Epilog → {{Green, Thick, Dashed,
     Line[{{Log[BCval1[[5]]], -100}, {Log[BCval1[[5]]], 100}}]}, {Orange, Thick,
     Dashed, Line[{{Log[ BCval1[[6]]], -100}, { Log[BCval1[[6]]], 100}}]}},
  PlotStyle → {Dashed, Red}, (*PlotRange→{{Log[1/(1.01*RSmax)],Log[4]},
    {Log[0.0001], 3}},*) Frame → True , FrameTicks → All,
  FrameLabel → {Style["Lslab", Black, 25], Style[ "4GN ΔS", Black, 25]},
  FrameTicksStyle → Directive[Black, 20], RotateLabel → {False, True},
  Axes → False, Mesh → False, AspectRatio → 1 / GoldenRatio],
 ParametricPlot[{Lslab[us, d], ΔSIBB[us, d]}, {us, 1 / RSmax, 1 / rh},
  ScalingFunctions → {"Log", "Log"}, Epilog → {{Red, Thick, Dashed,
     Line[{{Log[BCval1[[5]]], -100}, {Log[BCval1[[5]]], 100}}]}, {Orange, Thick,
     Dashed, Line[{{Log[ BCval1[[6]]], -100}, { Log[BCval1[[6]]], 100}}]}},
  PlotStyle → {Thick, Blue}, (*PlotRange→{{Log[1/(1.01*RSmax)],Log[4]},
    {Log[0.0001], 3}}, *)Frame → True , FrameTicks → All,
  FrameLabel → {Style["Log[Lslab]", Black, 25], Style[ "Log[4GNΔSIBB]", Black, 25]},
  FrameTicksStyle → Directive[Black, 20], (*RotateLabel→{False,True},*)
  Axes → False, Mesh → False, AspectRatio → 1 / GoldenRatio],
 PlotRange → All, ImageSize → 1000], Placed[LineLegend[
  {Blue, {Dashed, Black}, {Dashed, Red}(*,{Dashed,Orange}, {Dashed,Red}*)},
  {"IBB", "AdS (same T)", "AdS (same rh)"(*, "l = T", "l = μ"*)},
  LegendLayout → "Column", LegendMarkerSize → 25,
  LabelStyle → {Black, FontSize → 25, Font → "Arial"},
  LegendFunction → Framed], {0.17, 0.82}]]
```

Out[ ]=

```
{2.9343, 1., 44.4851, 457.49, 0.708795, 0.189839, 3.73367, 51.7947}
```

Out[ ]=

```
{1., 1., 0.708695, 0.999963, 0.708709, 0.189839, 3.73322, 51.7947}
```

Out[ ]=

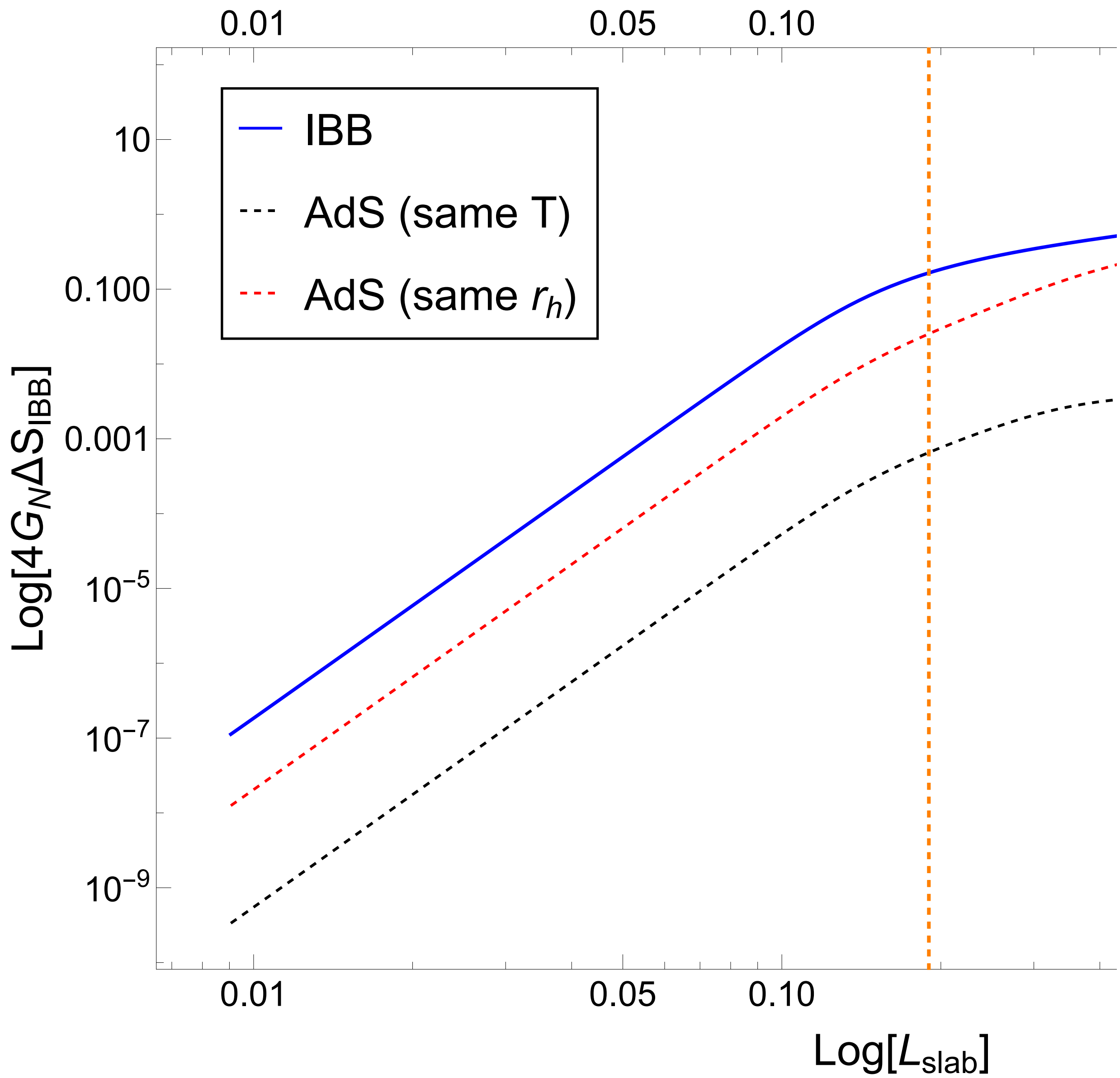
```
{InterpolatingFunction[ Domain: {{1., 51.8 }} Output: scalar ],
 InterpolatingFunction[ Domain: {{1., 51.8 }} Output: scalar ],
 InterpolatingFunction[ Domain: {{1., 51.8 }} Output: scalar ],
 InterpolatingFunction[ Domain: {{1., 51.8 }} Output: scalar ]}
```

Out[ ]=

```
0.596395
```

NIntegrate : Numerical integration converging too slowly; suspect one of the following: singularity, value of the integration is 0, highly oscillatory integrand, or WorkingPrecision too small.

NIntegrate : NIntegrate failed to converge to prescribed accuracy after 9 recursive bisections in u near {u} = {0.0253085 }. NIntegrate obtained $3.3671 \times 10^{-10} - 2.00617 \times 10^{-19}\, i$ and 6.390180737112419`*^-15 for the integral and error estimates.



In[∘]:=
```
rh = 1; c0 = 0.95; d = 2; α = 4;
BCval1 = slngen2[c0, rh, d, α, 1, 1, 1]
BCval2 = slngen2[c0, rh, d, α, Sqrt[BCval1⟦4⟧], 1 / BCval1⟦1⟧, 1]
sln = slnhold2[c0, rh, d, α, Sqrt[BCval1⟦4⟧], 1 / BCval1⟦1⟧, 1]
L = 1;

Clear[Lslab, ΔSIBB, ΔSIBBr, xIBB, xIBBr, Lslab, Lslabr]


ΔSIBB[us_, d_] := Re[NIntegrate[
   1 / u^3 * (sln⟦2⟧[1 / u] - 1) / (Sqrt[1 - (u / us) ^ (2 * d)]), {u, 1 / BCval2⟦-1⟧, us}]]
ΔSIBBr[rs_, d_] := Re[
```

```
  NIntegrate[-r * ( sln⟦2⟧[r] - 1) / (Sqrt[1 - (rs / r) ^ (2 * d)]), {r, BCval2⟦-1⟧, rs}]]


xIBB[U_, us_, d_] :=
 Re[ L ^ 2 * NIntegrate[-( sln⟦2⟧[1 / u]) / (Sqrt[(us / u) ^ (2 * d) - 1]), {u, us, U}]]
xIBBr[U_, rs_, d_] :=
 Re[L ^ 2 * NIntegrate[ 1 / r ^ 2 * sln⟦2⟧[r] / Sqrt[(r / rs) ^ (2 * d) - 1], {r, rs, U}]]

Lslab[us_, d_ ] := Re[-L ^ 2 * NIntegrate[
    - ( sln⟦2⟧[1 / u]) / (Sqrt[(us / u) ^ (2 * d) - 1]), {u, 1 / BCval2⟦-1⟧, us}]]
Lslabr[rs_, d_] :=
 Re[ L ^ 2 * NIntegrate[1 / r ^ 2 * 1 / Sqrt[(r / rs) ^ (2 * d) - 1], {r, rs, BCval2⟦-1⟧}]]



ΔSIBB[us_, d_ ] := Re[NIntegrate[
   1 / u ^ 3 * ( sln⟦2⟧[1 / u] - 1) / (Sqrt[1 - (u / us) ^ (2 * d)]), {u, 1 / BCval2⟦-1⟧, us}]]
ΔSIBBr[rs_, d_ ] := Re[
  NIntegrate[-r * ( sln⟦2⟧[r] - 1) / (Sqrt[1 - (rs / r) ^ (2 * d)]), {r, BCval2⟦-1⟧, rs}]]

RSmax = 100; (* BCval1⟦-1⟧;*)

If[BCval1⟦5⟧ < rh, transPoint = BCval1⟦6⟧, transPoint = BCval1⟦5⟧];

Aads[r_, rh_, d_] := (L * r * Sqrt[1 - (rh / r) ^ (d + 1)]) ^ 2
Cads[r_, rh_, d_] := (L / (r * Sqrt[1 - (rh / r) ^ (d + 1)])) ^ 2
BCval1 = slngen[c0, 1, d, α, 1, 1];
ThAds[rh_, d_] := Sqrt[
   (D[Aads[r, rh1, d], r] /. r → rh1) * (D[1 / Cads[r, rh1, d], r] /. r → rh1)] / (4 * Pi)
rHAds = N[Solve[ThAds[rh, 3] == BCval1⟦6⟧, rh1]⟦2, 1, 2⟧]
C1AdS[r_, d_, rh1_] := 1 / Sqrt[1 - (rh1 / r) ^ (d + 1)]
ΔSAdS[us_, d_ ] :=
 Re[NIntegrate[1 / u ^ 3 * (C1AdS[1 / u, d, rHAds] - 1) / (Sqrt[1 - (u / us) ^ (2 * d)]),
   {u, 1 / BCval2⟦-1⟧, us}]]
ΔSAdS1[us_, d_ ] :=
 Re[NIntegrate[1 / u ^ 3 * (C1AdS[1 / u, d, rh] - 1) / (Sqrt[1 - (u / us) ^ (2 * d)]),
   {u, 1 / BCval2⟦-1⟧, us}]]

If[rHAds < rh, rIntAdS = rh, rIntAdS = rHAds];
Legended[Show[(*ParametricPlot[{Lslab[us,d],ΔSLif[us, d]},
   {us, 1/RSmax, 1/rh},ScalingFunctions→{"Log","Log"},Epilog→
    {{Red,Thick, Dashed, Line[{{Log[BCval1⟦5⟧],-100},{Log[BCval1⟦5⟧],100}}]},
     {Orange, Thick,Dashed, Line[{{Log[ BCval1⟦6⟧],-100},
        { Log[BCval1⟦6⟧],100}}]}},PlotStyle→{Dashed, Black},
   (*PlotRange→{{Log[1/(1.01*RSmax)],Log[4]},{Log[0.0001], 3}},*)
```

```
  Frame→True ,FrameTicks→All,
  FrameLabel→{Style["Lslab",Black, 25],Style[ "4GN ΔSIBB",Black, 25]},
  FrameTicksStyle→Directive[Black,20],RotateLabel→{False,True},
  Axes→False,GridLines→Automatic,GridLinesStyle→Directive[Gray, Dashed],
  Mesh→False,AspectRatio→1/GoldenRatio,ImageSize→700],*)
 ParametricPlot[{Lslab[us, d], ΔSAdS[us, d]}, {us, 1 / RSmax, 1 / rIntAdS },
  ScalingFunctions → {"Log", "Log"}, Epilog → {{Green, Thick, Dashed,
     Line[{{Log[BCval1[[5]]], -100}, {Log[BCval1[[5]]], 100}}]}, {Orange, Thick,
     Dashed, Line[{{Log[ BCval1[[6]]], -100}, { Log[BCval1[[6]]], 100}}]}},
  PlotStyle → {Dashed, Black}, (*PlotRange→{{Log[1/(1.01*RSmax)],Log[4]},
    {Log[0.0001], 3}},*) Frame → True , FrameTicks → All,
  FrameLabel → {Style["Log[Lslab]", Black, 25], Style[ "Log[4GNΔSIBB]", Black, 25]},
  FrameTicksStyle → Directive[Black, 20](*,RotateLabel→{False,True}*),
  Axes → False, Mesh → False, AspectRatio → 1 / GoldenRatio],
 ParametricPlot[{Lslab[us, d], ΔSAdS1[us, d]}, {us, 1 / RSmax, 1 / rh },
  ScalingFunctions → {"Log", "Log"}, Epilog → {{Green, Thick, Dashed,
     Line[{{Log[BCval1[[5]]], -100}, {Log[BCval1[[5]]], 100}}]}, {Orange, Thick,
     Dashed, Line[{{Log[ BCval1[[6]]], -100}, { Log[BCval1[[6]]], 100}}]}},
  PlotStyle → {Dashed, Red}, (*PlotRange→{{Log[1/(1.01*RSmax)],Log[4]},
    {Log[0.0001], 3}},*) Frame → True , FrameTicks → All,
  FrameLabel → {Style["Lslab", Black, 25], Style[ "4GN ΔS", Black, 25]},
  FrameTicksStyle → Directive[Black, 20], RotateLabel → {False, True},
  Axes → False, Mesh → False, AspectRatio → 1 / GoldenRatio],
 ParametricPlot[{Lslab[us, d], ΔSIBB[us, d]}, {us, 1 / RSmax, 1 / rh},
  ScalingFunctions → {"Log", "Log"}, Epilog → {{Red, Thick, Dashed,
     Line[{{Log[BCval1[[5]]], -100}, {Log[BCval1[[5]]], 100}}]}, {Orange, Thick,
     Dashed, Line[{{Log[ BCval1[[6]]], -100}, { Log[BCval1[[6]]], 100}}]}},
  PlotStyle → {Thick, Blue}, (*PlotRange→{{Log[1/(1.01*RSmax)],Log[4]},
    {Log[0.0001], 3}}, *)Frame → True , FrameTicks → All,
  FrameLabel → {Style["Log[Lslab]", Black, 25], Style[ "Log[4GNΔSIBB]", Black, 25]},
  FrameTicksStyle → Directive[Black, 20], (*RotateLabel→{False,True},*)
  Axes → False, Mesh → False, AspectRatio → 1 / GoldenRatio],
 PlotRange → All, ImageSize → 1000], Placed[LineLegend[
  {Blue, {Dashed, Black}, {Dashed, Red}(*,{Dashed,Orange}, {Dashed,Red}*)},
  {"IBB", "AdS (same T)", "AdS (same rh)"(*, "l = T", "l = μ"*)},
  LegendLayout → "Column", LegendMarkerSize → 25,
  LabelStyle → {Black, FontSize → 25, Font → "Arial"},
  LegendFunction → Framed], {0.17, 0.82}]]
```

Out[ ]=

```
{1.37888, 1., 15.3783, 41.3404, 1.73457, 0.173135, 10.0186, 10000.}
```

Out[ ]=

```
{0.999998, 1., 1.73324, 0.999955, 1.73328, 0.173135, 10.0111, 10000.}
```

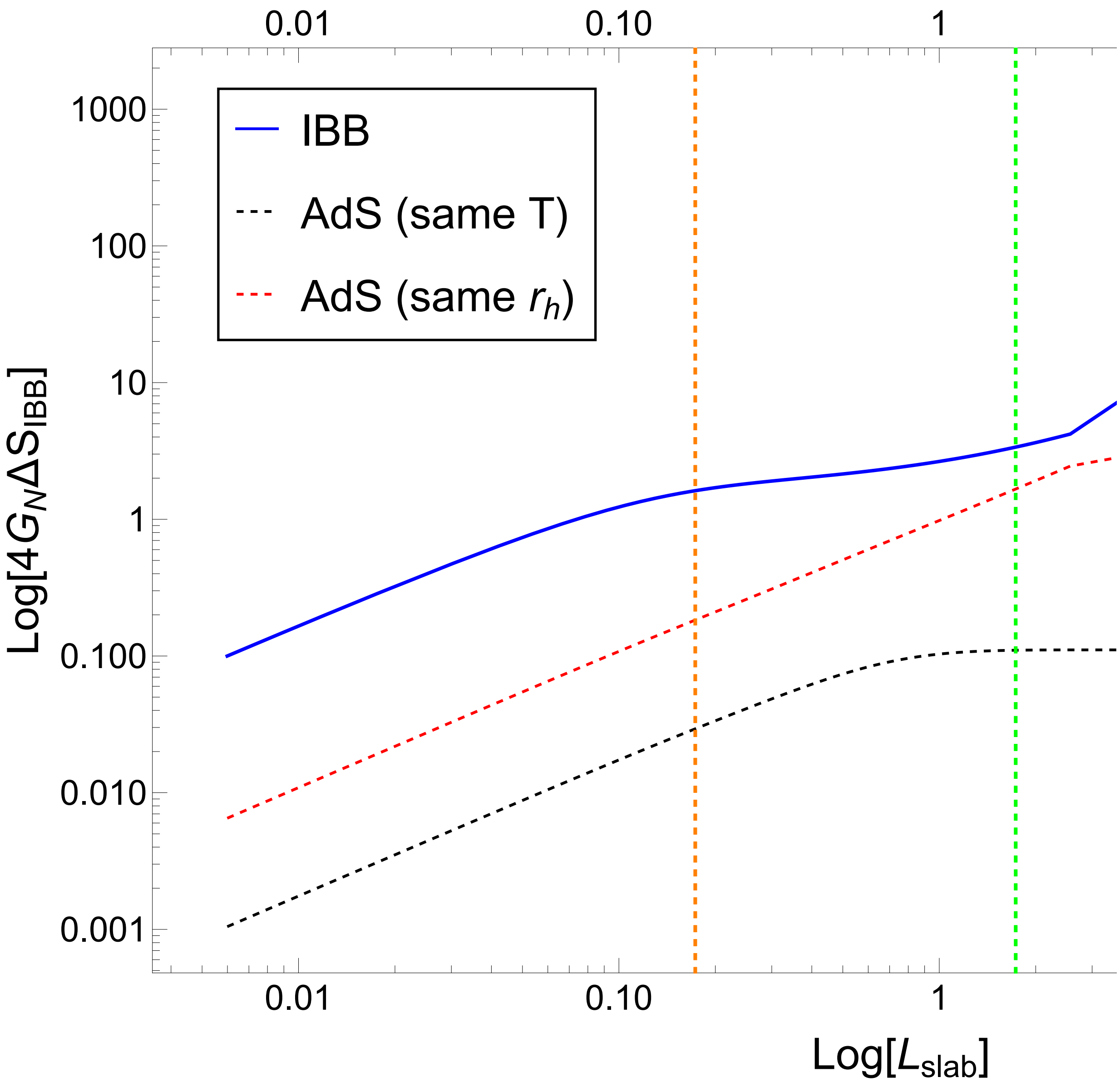
Out[ ]=

```
{InterpolatingFunction[ Domain: {{1., 1.00 ×10^4}}  Output: scalar ],
 InterpolatingFunction[ Domain: {{1., 1.00 ×10^4}}  Output: scalar ],
 InterpolatingFunction[ Domain: {{1., 1.00 ×10^4}}  Output: scalar ],
 InterpolatingFunction[ Domain: {{1., 1.00 ×10^4}}  Output: scalar ]}
```

Out[ ]=

```
0.543918
```

Out[ ]=



In[ ]:=
```
rh = 1; c0 = 0.05; d = 3; α = 2;
BCval1 = slngen2[c0, rh, d, α, 1, 1, 1]
BCval2 = slngen2[c0, rh, d, α, Sqrt[BCval1⟦4⟧], 1 / BCval1⟦1⟧, 1]
sln = slnhold2[c0, rh, d, α, Sqrt[BCval1⟦4⟧], 1 / BCval1⟦1⟧, 1]
L = 1;

Clear[Lslab, ΔSIBB, ΔSIBBr, xIBB, xIBBr, Lslab, Lslabr]


ΔSIBB[us_, d_] := Re[NIntegrate[
   1 / u^3 * (sln⟦2⟧[1 / u] - 1) / (Sqrt[1 - (u / us)^(2 * d)]), {u, 1 / BCval2⟦-1⟧, us}]]
ΔSIBBr[rs_, d_] := Re[
```

```
  NIntegrate[-r * ( sln⟦2⟧[r] - 1) / (Sqrt[1 - (rs / r) ^ (2 * d)]), {r, BCval2⟦-1⟧, rs}]]


xIBB[U_, us_, d_] :=
 Re[ L ^ 2 * NIntegrate[- ( sln⟦2⟧[1 / u]) / (Sqrt[(us / u) ^ (2 * d) - 1]), {u, us, U}]]
xIBBr[U_, rs_, d_] :=
 Re[L ^ 2 * NIntegrate[ 1 / r ^ 2 * sln⟦2⟧[r] / Sqrt[(r / rs) ^ (2 * d) - 1], {r, rs, U}]]

Lslab[us_, d_ ] := Re[-L ^ 2 * NIntegrate[
    - ( sln⟦2⟧[1 / u]) / (Sqrt[(us / u) ^ (2 * d) - 1]), {u, 1 / BCval2⟦-1⟧, us}]]
Lslabr[rs_, d_] :=
 Re[ L ^ 2 * NIntegrate[1 / r ^ 2 * 1 / Sqrt[(r / rs) ^ (2 * d) - 1], {r, rs, BCval2⟦-1⟧}]]



ΔSIBB[us_, d_ ] := Re[NIntegrate[
   1 / u ^ 3 * ( sln⟦2⟧[1 / u] - 1) / (Sqrt[1 - (u / us) ^ (2 * d)]), {u, 1 / BCval2⟦-1⟧, us}]]
ΔSIBBr[rs_, d_ ] := Re[
  NIntegrate[-r * ( sln⟦2⟧[r] - 1) / (Sqrt[1 - (rs / r) ^ (2 * d)]), {r, BCval2⟦-1⟧, rs}]]

RSmax = 100; (* BCval1⟦-1⟧;*)

If[BCval1⟦5⟧ < rh, transPoint = BCval1⟦6⟧, transPoint = BCval1⟦5⟧];

Aads[r_, rh_, d_] := (L * r * Sqrt[1 - (rh / r) ^ (d + 1)]) ^ 2
Cads[r_, rh_, d_] := (L / (r * Sqrt[1 - (rh / r) ^ (d + 1)])) ^ 2
BCval1 = slngen[c0, 1, d, α, 1, 1];
ThAds[rh_, d_] := Sqrt[
   (D[Aads[r, rh1, d], r] /. r → rh1) * (D[1 / Cads[r, rh1, d], r] /. r → rh1)] / (4 * Pi)
rHAds = N[Solve[ThAds[rh, 3] == BCval1⟦6⟧, rh1]⟦2, 1, 2⟧]
C1AdS[r_, d_, rh1_] := 1 / Sqrt[1 - (rh1 / r) ^ (d + 1)]
ΔSAdS[us_, d_ ] :=
 Re[NIntegrate[1 / u ^ 3 * (C1AdS[1 / u, d, rHAds] - 1) / (Sqrt[1 - (u / us) ^ (2 * d)]),
   {u, 1 / BCval2⟦-1⟧, us}]]
ΔSAdS1[us_, d_ ] :=
 Re[NIntegrate[1 / u ^ 3 * (C1AdS[1 / u, d, rh] - 1) / (Sqrt[1 - (u / us) ^ (2 * d)]),
   {u, 1 / BCval2⟦-1⟧, us}]]

If[rHAds < rh, rIntAdS = rh, rIntAdS = rHAds];
Legended[Show[(*ParametricPlot[{Lslab[us,d],ΔSLif[us, d]},
   {us, 1/RSmax, 1/rh},ScalingFunctions→{"Log","Log"},Epilog→
    {{Red,Thick, Dashed, Line[{{Log[BCval1⟦5⟧],-100},{Log[BCval1⟦5⟧],100}}]},
     {Orange, Thick,Dashed, Line[{{Log[ BCval1⟦6⟧],-100},
         { Log[BCval1⟦6⟧],100}}]}},PlotStyle→{Dashed, Black},
   (*PlotRange→{{Log[1/(1.01*RSmax)],Log[4]},{Log[0.0001], 3}},*)
```

```
 Frame→True ,FrameTicks→All,
 FrameLabel→{Style["Lslab",Black, 25],Style[ "4GN ΔSIBB",Black, 25]},
 FrameTicksStyle→Directive[Black,20],RotateLabel→{False,True},
 Axes→False,GridLines→Automatic,GridLinesStyle→Directive[Gray, Dashed],
 Mesh→False,AspectRatio→1/GoldenRatio,ImageSize→700],*)
ParametricPlot[{Lslab[us, d], ΔSAdS[us, d]}, {us, 1 / RSmax, 1 / rIntAdS },
 ScalingFunctions → {"Log", "Log"}, Epilog → {{Green, Thick, Dashed,
    Line[{{Log[BCval1[[5]]], -100}, {Log[BCval1[[5]]], 100}}]}, {Orange, Thick,
    Dashed, Line[{{Log[ BCval1[[6]]], -100}, { Log[BCval1[[6]]], 100}}]}},
 PlotStyle → {Dashed, Black}, (*PlotRange→{{Log[1/(1.01*RSmax)],Log[4]},
   {Log[0.0001], 3}},*) Frame → True , FrameTicks → All,
 FrameLabel → {Style["Log[Lslab]", Black, 25], Style[ "Log[4GNΔSIBB]", Black, 25]},
 FrameTicksStyle → Directive[Black, 20](*,RotateLabel→{False,True}*),
 Axes → False, Mesh → False, AspectRatio → 1 / GoldenRatio],
ParametricPlot[{Lslab[us, d], ΔSAdS1[us, d]}, {us, 1 / RSmax, 1 / rh },
 ScalingFunctions → {"Log", "Log"}, Epilog → {{Green, Thick, Dashed,
    Line[{{Log[BCval1[[5]]], -100}, {Log[BCval1[[5]]], 100}}]}, {Orange, Thick,
    Dashed, Line[{{Log[ BCval1[[6]]], -100}, { Log[BCval1[[6]]], 100}}]}},
 PlotStyle → {Dashed, Red}, (*PlotRange→{{Log[1/(1.01*RSmax)],Log[4]},
   {Log[0.0001], 3}},*) Frame → True , FrameTicks → All,
 FrameLabel → {Style["Lslab", Black, 25], Style[ "4GN ΔS", Black, 25]},
 FrameTicksStyle → Directive[Black, 20], RotateLabel → {False, True},
 Axes → False, Mesh → False, AspectRatio → 1 / GoldenRatio],
ParametricPlot[{Lslab[us, d], ΔSIBB[us, d]}, {us, 1 / RSmax, 1 / rh},
 ScalingFunctions → {"Log", "Log"}, Epilog → {{Red, Thick, Dashed,
    Line[{{Log[BCval1[[5]]], -100}, {Log[BCval1[[5]]], 100}}]}, {Orange, Thick,
    Dashed, Line[{{Log[ BCval1[[6]]], -100}, { Log[BCval1[[6]]], 100}}]}},
 PlotStyle → {Thick, Blue}, (*PlotRange→{{Log[1/(1.01*RSmax)],Log[4]},
   {Log[0.0001], 3}}, *)Frame → True , FrameTicks → All,
 FrameLabel → {Style["Log[Lslab]", Black, 25], Style[ "Log[4GNΔSIBB]", Black, 25]},
 FrameTicksStyle → Directive[Black, 20], (*RotateLabel→{False,True},*)
 Axes → False, Mesh → False, AspectRatio → 1 / GoldenRatio],
PlotRange → All, ImageSize → 1000], Placed[LineLegend[
 {Blue, {Dashed, Black}, {Dashed, Red}(*,{Dashed,Orange}, {Dashed,Red}*)},
 {"IBB", "AdS (same T)", "AdS (same rh)"(*, "l = T", "l = μ"*)},
 LegendLayout → "Column", LegendMarkerSize → 25,
 LabelStyle → {Black, FontSize → 25, Font → "Arial"},
 LegendFunction → Framed], {0.17, 0.82}]]
```

Out[ ]=

```
{1.02311, 1., 0.0725783, 0.144251, 0.186779, 0.311121, 0.60034, 1000.}
```

Out[ ]=

```
{1., 1., 0.185603, 1.00001, 0.185602, 0.311121, 0.596558, 1000.}
```

Out[ ]=

{InterpolatingFunction[Domain: {{1., 1.00 ×10^3}} Output: scalar],
Data not saved. Save now

InterpolatingFunction[Domain: {{1., 1.00 ×10^3}} Output: scalar],
Data not saved. Save now

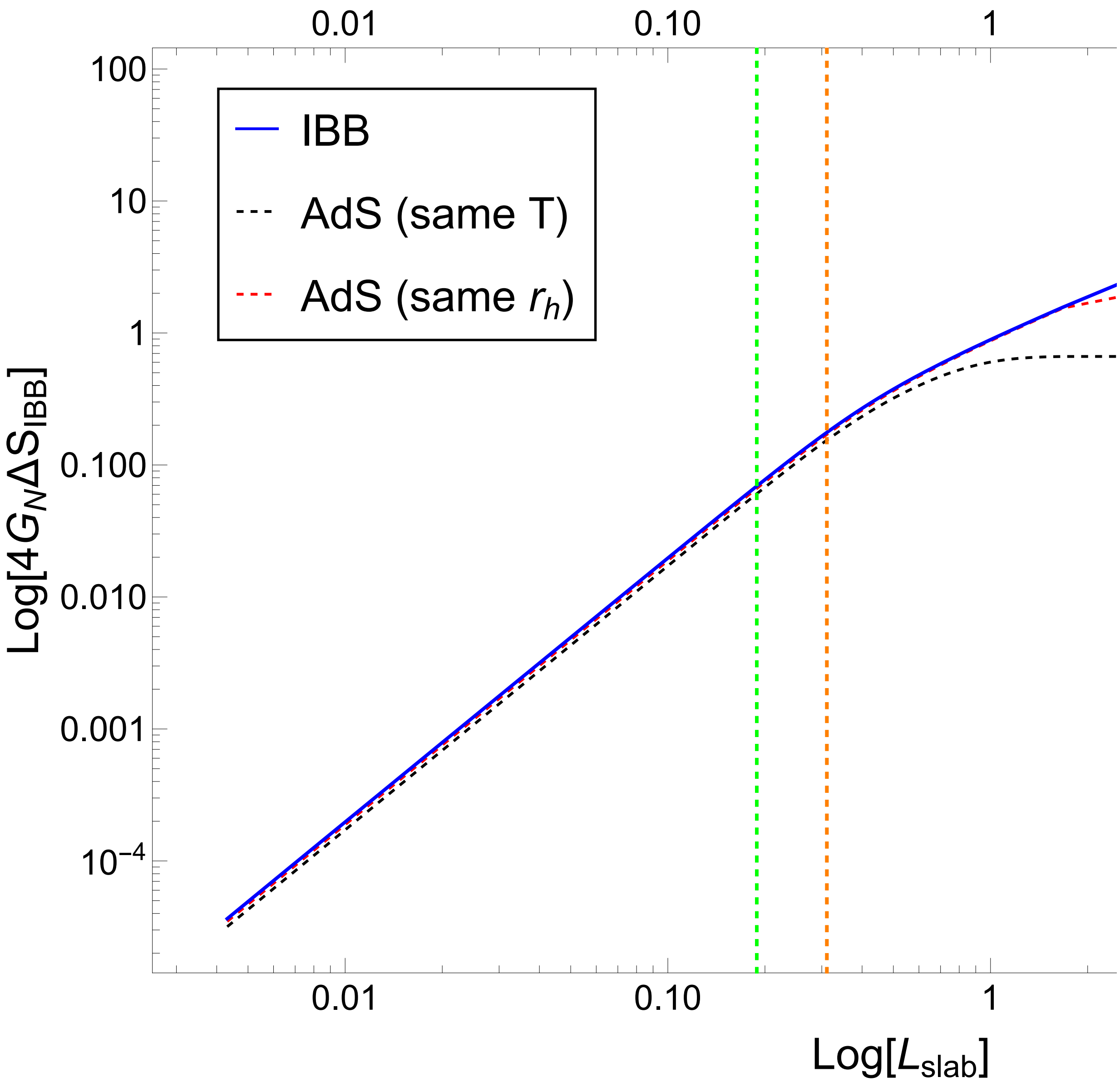
InterpolatingFunction[Domain: {{1., 1.00 ×10^3}} Output: scalar],
Data not saved. Save now

InterpolatingFunction[Domain: {{1., 1.00 ×10^3}} Output: scalar]}
Data not saved. Save now

Out[ ]=

0.977416

Out[ ]=



SCRAP FILES

Scrap!

Circular Photon Orbits:

Get c0h:

r’’(x) and r(x) analysis at rt = 5.1 rH