

# Graph Denoising for Molecular Imaging

Cédric Mendelin <cedric.mendelin@stud.unibas.ch>

Department of Mathematics and Computer Science, University of Basel

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#### Outline

- Molecular Imaging Methods
- Graphs & Manifolds
- 3 GAT-Denoiser
- 4 Results on LoDoPaB-CT dataset
- 5 Summary & Future Work
- 6 Questions

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  - > During freezing, molecules rotate randomly
- > Observations can be reconstructed to a 3D model

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Only single particle cryo-EM is considered.

## Cryo-Electron Microscopy (Cryo-EM) - Illustration

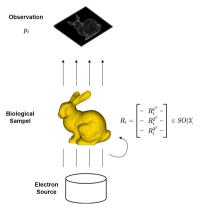


Figure: Cryo-EM overview

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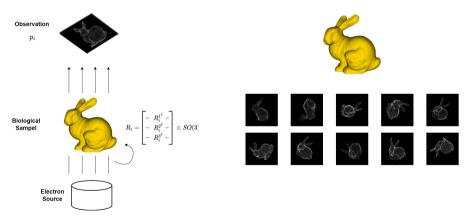
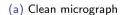


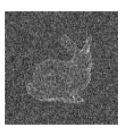
Figure: Cryo-EM overview

# Cryo-Electron Microscopy (Cryo-EM) - Challenges

- > High level of noise
- > Unknown rotation during freezing
- > (Structural variety of observations)







(b) Noisy micrograph

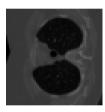
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(a) Biological sample



(b) Clean observation (sinogram)

#### Observation

$$y = p + \eta \tag{1}$$

- > y: noisy observation
- > p: noiseless observation
- $>\eta$ : noise, assumed  $\eta_i \sim \mathcal{N}(0,\sigma^2)$

#### Observation

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$$y_i[j] = p_i[j] + \eta_i[j] \quad \text{with } 1 \le i \le N, 1 \le j \le M$$
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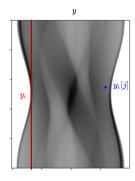
$$y_i = A(x, \theta_i) + \eta_i$$

$$(1)$$

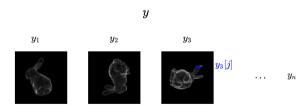
- > v: noisy observation
- p: noiseless observation
- $\eta$ : noise, assumed  $\eta_i \sim \mathcal{N}(0, \sigma^2)$
- > x: biological sample

- > N: number of observations
- > M: observation dimension
- $A: x \mapsto A(x; \theta_i) \in \mathbb{R}^M$ : a non-linear operator
- $> \theta_i$ : observation angle

### Observation - Illustration







(b) Cryo-EM Observation - micrographs

### Reconstruction

### Reconstruction

#### Reconstruction

$$Recon: \mathbb{R}^{M \times N} \to \mathbb{R}^{M \times M} \quad y \mapsto Recon(y; \theta)$$
 (2)

- > SNR is a measure, which compares the power of an input signal to the power of the undesired noise
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 $SNR_{y}$  is used to define the level of noise in an observation.

SNR is used as a metric for the quality of reconstructions.

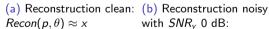
## Reconstruction - Computed Tomography

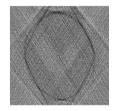
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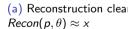


with SNR<sub>v</sub> 0 dB:  $Recon(y, \theta) \not\approx x$ 

## Reconstruction - Computed Tomography

- Filter Backprojection (FBP)
  - Can be considered historical approach
  - Enables reconstruction for moderate noise
- Neural Network Approaches
  - Today state-of-the art
  - Using result of FBP and further denoise
  - U-Net Ronneberger, Fischer, and Brox 2015







(a) Reconstruction clean: (b) Reconstruction noisy with SNR<sub>v</sub> 0 dB:  $Recon(y, \theta) \not\approx x$ 

### Problem and Goal

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#### Goal

denoiser : 
$$y_i = (p_i + \eta) \mapsto p_i^* \approx p_i$$
  
 $Recon(denoiser(y; \theta)) \approx x$ 

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# Graph - Definitions

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### **Graph Definition**

A graph is defined as  $G = \langle V, E \rangle$ , where V is a set of nodes and E is a set of edges.

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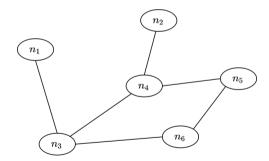


Figure: Sample graph

Molecular Imaging Methods Graphs & Manifolds GAT-Denoiser Results on LoDoPaB-CT dataset Summary & Future Work Questions Reference

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#### **Nodes**

 $(n_1, n_2, \dots) \in \mathbb{R}^F$ , with F as node feature dimensions

### Edges

Edges are defined as a set of tuples (i, j), where i and j determine the index of the nodes.

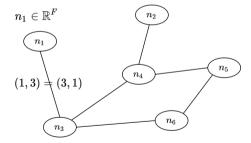


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### Adjacency Matrix

The binary adjacency matrix of graph  $G = \langle V, E \rangle$  is defined as:

$$A_{ij} = \begin{cases} 1 & \text{if } (i,j) \in E \\ 0, & \text{otherwise} \end{cases}$$
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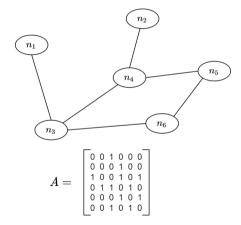


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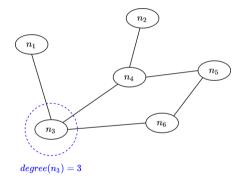


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#### Degree Matrix of Graph G

Is a diagonal matrix with degree of nodes as entries.

$$D_{ii} = degree(n_i)$$

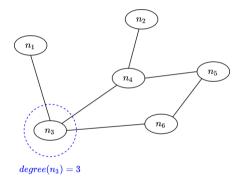


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How to construct a graph for molecular imaging?

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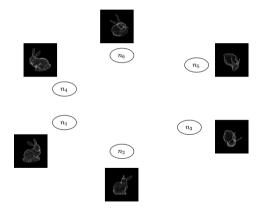


Figure: Sample graph for cryo-EM observation

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- > Nodes: Single observation  $y_i$
- Edges: Use k-nearest neighbours (k-NN) to construct a graph

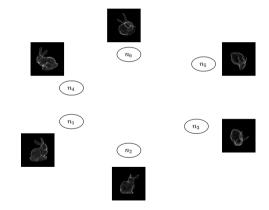


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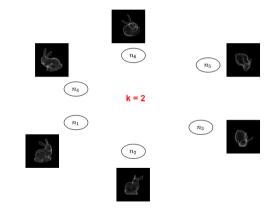


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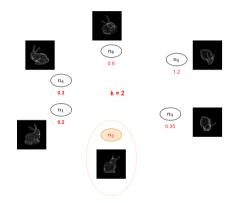


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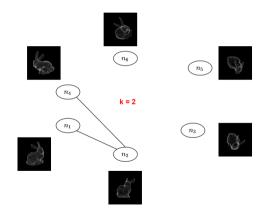


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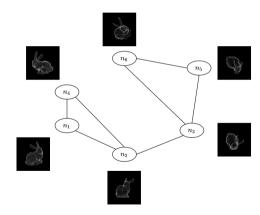


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What happens with our noisy observations?

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With noise, graph will capture neighborhood inaccurately.

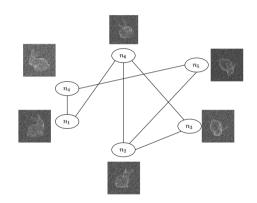


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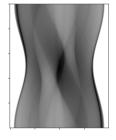
- 1. Construct a k-NN graph from observations.
- 2. Calculate L = D A
- 3. Get 2nd and 3rd smallest eigenvalue with corresponding eigenvectors.

#### Low-dimensional Embedding for Computed Tomography

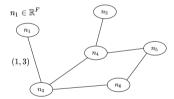


(a) Clean CT observation

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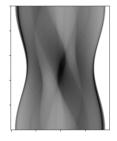


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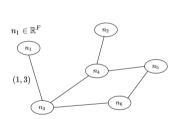


(b) Building k-NN graph with k = 2

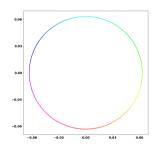
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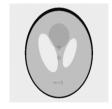
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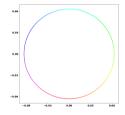
(c)  $2_{nd}$  and  $3_{rd}$  smallest eigenvectors of L = D - A



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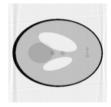
(b) GL-Embedding from k = 2



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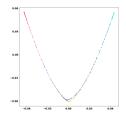
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GL-Embedding estimates observation angles

What happens in the noisy case?



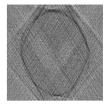
(a) Reconstruction known angles  $SNR_v$ : 10 dB



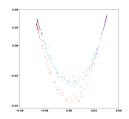
(b) GL-Embedding from k = 6 and  $SNR_v : 10 \text{ dB}$ 



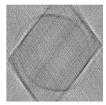
(c) Reconstruction unknown angles  $SNR_v$ : 10 dB



(a) Reconstruction known angles  $SNR_v$ : 0 dB



(b) GL-Embedding from k = 6 and  $SNR_v : 0$  dB



(c) Reconstruction unknown angles  $SNR_v$ : 0 dB

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- > Use existing denoising algorithms
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  - No graph as data structure
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Exploit graph as a data structure and the GL-Embedding

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#### **GAT-Denoiser**

- > GAT-Denoiser is a graph neural network (GNN) to denoise observations.
- > Consists of three components:
  - Convolution
  - > Graph Attention Network (GAT) Veličković et al. 2017
  - End-to-End Learning

**GAT-Denoiser Pipeline** 

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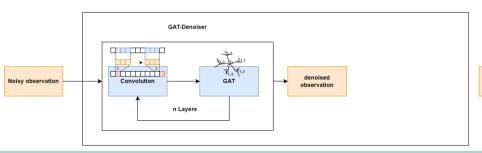


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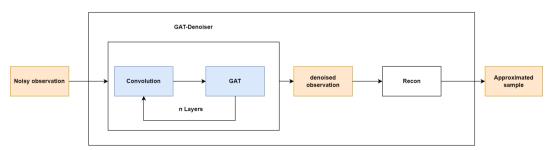


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# Graph Attention Network - GAT

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- Extends Graph Convolution Network with attention (weights)
- Computes new node features
- > Averages graph over neighborhood
- Multi-head available, motivated by Vaswani et al. 2017

$$h_{1,2}$$
 $h_{1,3}$ 
 $h_{1,4}$ 
 $h_{1,5}$ 

# Graph Attention Network - GAT

- Extends Graph Convolution Network with attention (weights)
- Computes new node features
- Averages graph over neighborhood
- Multi-head available, motivated by Vaswani et al. 2017
- $\sigma$ : activation function (Exponential Linear Unit)
- W: learnable weight matrix
- $\geq \alpha$ : normalized attention coefficients

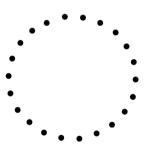
$$h_{1,2}$$
 $h_{1,3}$ 
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 $h_{1,5}$ 

$$h_1' = \sigma \left( \sum_{i=1}^5 \alpha_i W h_{1,i} \right)$$

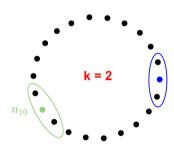
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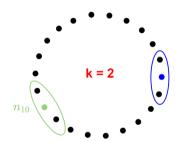


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### Input Graph

- Exploit information from GL
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- Construct graph from observation angles
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  - > Use k-NN with great-circle distance



Observation angles  $\theta$  are assumed to be equally spaced.

# GAT-Denoiser Implementation for Computed Tomography

- Use U-Net for reconstruction
- > During Trainig, U-Net might be trained jointly

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### LoDoPaB-CT dataset

- > Dataset for low-dose Computed Tomography
- > 35'820 train samples
- > 3'553 test samples
- > BM3D as baseline algorithm
- Resolution 64 x 64









Figure: Some samples from the LoDoPaB-CT dataset.

#### **Evaluation**

- > Small Scale Experiments
  - > 1024 train samples
  - > 100 test samples
  - > 200 epochs
- Large Scale Experiments
  - > Complete LoDoPaB-CT dataset
  - > 20 40 epochs

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- > Small Scale Experiments
  - > 1024 train samples
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  - > 200 epochs
  - > Goal: Find most promising architecture
- Large Scale Experiments
  - Complete LoDoPaB-CT dataset
  - > 20 40 epochs
  - > Goal: Find best model

# **Training**

- U-Net used for reconstruction
  - $\rightarrow$  Pre-trained with complete dataset and  $SNR_y$  in [-10, 0] for 200 epochs
- > Mini-batch gradient descent with batch size 64
- > Adam optimizer
- > Joint U-Net training possible

# Small Scale Experiments - Overall Results

- Learning fails with random graph
- Learning succeeds with defined input graph



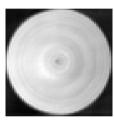
(a) Reconstruction with random input graph



(b) Reconstruction with k-NN input graph k = 10

# Small Scale Experiments - Overall Results

- Learning fails with random graph
- Learning succeeds with defined input graph
- Components contribute to success of GAT-Denoiser
- > Best model with joint U-Net training

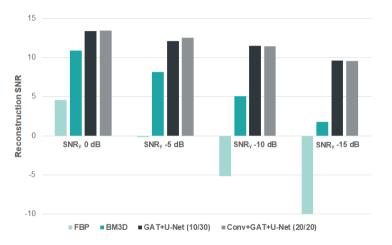


(a) Reconstruction with random input graph



(b) Reconstruction with k-NN input graph k = 10

# Large Scale Experiment - SNR Results



### Large Scale Experiment - Visual Results - SNR<sub>v</sub> 0 dB

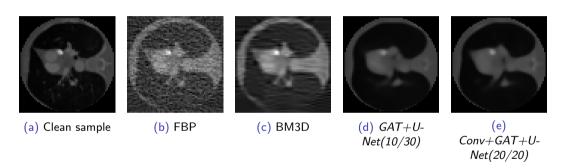


Figure: Large Scale Experiment: Visual results for SNR<sub>v</sub> 0 dB.

# Large Scale Experiment - Visual Results - SNR<sub>y</sub> 0 dB

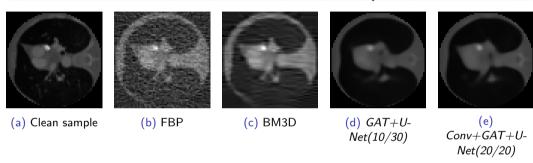


Figure: Large Scale Experiment: Visual results for SNR<sub>v</sub> 0 dB.

GAT-Denoiser improves BM3D for SNR<sub>v</sub> 0 dB by 27.6%.

### Large Scale Experiment - Visual Results - SNR<sub>v</sub> -10 dB

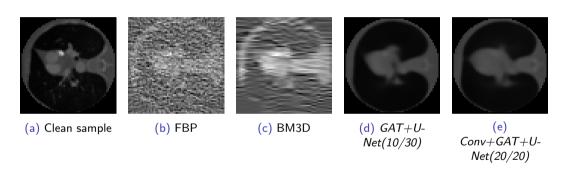


Figure: Large Scale Experiment: Visual results for SNR<sub>v</sub> -10 dB.

### Large Scale Experiment - Visual Results - SNR<sub>v</sub> -10 dB

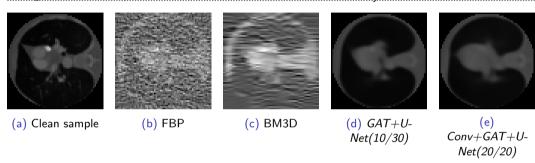


Figure: Large Scale Experiment: Visual results for SNR<sub>v</sub> -10 dB.

GAT-Denoiser improves BM3D for SNR<sub>v</sub> 0 dB by 126.0%.

### Large Scale Experiment - Visual Results - $SNR_v$ -15 dB

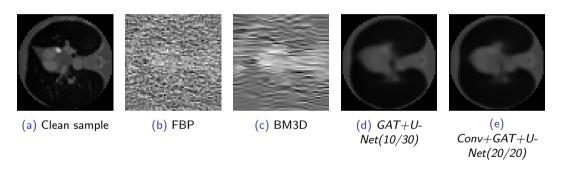


Figure: Large Scale Experiment: Visual results for SNR<sub>v</sub> -15 dB.

### Large Scale Experiment - Visual Results - SNR<sub>v</sub> -15 dB

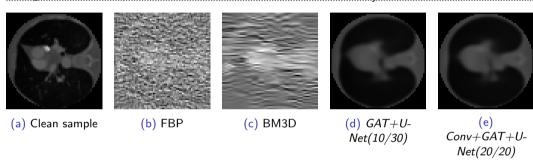


Figure: Large Scale Experiment: Visual results for SNR<sub>v</sub> -15 dB.

GAT-Denoiser improves BM3D for  $SNR_{\nu}$  0 dB by 379.9%.

#### Outline

- Molecular Imaging Methods
- Graphs & Manifolds
- GAT-Denoiser
- 4 Results on LoDoPaB-CT dataset
- 5 Summary & Future Work
- 6 Questions

### Summary

- > GAT-Denoiser enables denoising of observations
  - > Convolution
  - GAT
  - End-To-End Learning
  - > Joint U-Net training boost performance

### Summary

- GAT-Denoiser enables denoising of observations
  - > Convolution
  - GAT
  - End-To-End Learning
  - > Joint U-Net training boost performance
- Evaluated on LoDoPaB-CT dataset
  - > Outperformed baseline BM3D by up to 379.9 %

#### Future Work

- > Improve current GAT-Denoiser
- > Derive GAT-Denoiser for 3D
- > Make it work for unknown angles

#### Future Work

- > Improve current GAT-Denoiser
- > Derive GAT-Denoiser for 3D
- Make it work for unknown angles
- Cryo-EM
  - Known angles
  - > Unknown angles
  - > Work with structural variety in observations

#### Outline

- Molecular Imaging Methods
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# Questions



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