University of Basel

Status Meeting - Application of Graph Learning to inverse problems

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05.05.2022

Problem and Goal

- Denoise observations
- Motivation from imaging (computed tomography, cryo-EM)

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- Motivation from imaging (computed tomography, cryo-EM)

Input

$\mathcal{I}$: class of images (where $i \in \mathbb{R}^M$ with M as image dimension.)

Output

$$\textit{denoiser} : (i + \eta) \mapsto i^* \approx i$$

Where $i \in \mathcal{I}$ and $\eta \sim \mathcal{N}(0, \sigma^2) \in \mathbb{R}$.

Sinogram Denoiser

forward : radon transform

backward : filter back projection

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$$\textit{forward} : \text{radon transform}$$

$$\textit{backward} : \text{filter back projection}$$

Sinogram Denoiser

$$\textit{denoiser}_{\textit{sino}} : \textit{forward}(i) + \eta \mapsto \textit{forward}(i)^* \approx \textit{forward}(i)$$

$$\textit{denoiser} = \textit{backward}(\textit{denoiser}_{\textit{sino}}(\textit{forward}(i) + \eta))$$

Where $i \in \mathcal{I}$ and $\eta \sim \mathcal{N}(0, \sigma^2) \in \mathbb{R}$.

GAT Denoiser

GAT - Graph Attention Network

$$GAT(A, \text{GAT args}) :\rightarrow \text{fixed angles} \rightarrow \text{k-nn circle graph}$$

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GAT Loss

$$\mathcal{L}_1 = ||forward(i) - denoiser_{sino}(forward(i) + \eta)||_2$$

$$\mathcal{L}_2 = ||i - backward(denoiser_{sino}(forward(i) + \eta))||_2$$

Where $i \in \mathcal{I}$ and $\eta \sim \mathcal{N}(0, \sigma^2) \in \mathbb{R}$.

GAT Denoiser - 2

GAT - Graph Attention Network

$$GAT(A, \text{GAT args}) :\rightarrow \text{fixed angles} \rightarrow \text{k-nn circle graph}$$

Input Graph

- Learning does work with circle graph
- Learning does not work with random (Erdős–Rényi) graph

Toy images

- Uniformly places shapes on image

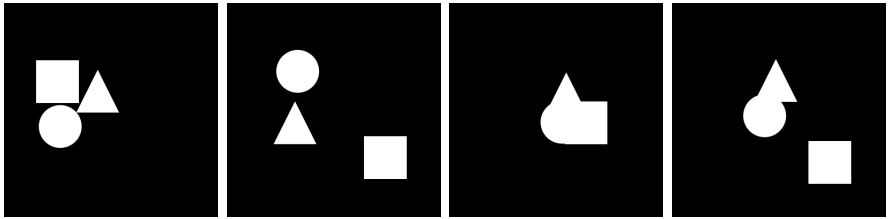

Figure: Example toy images.

Current Results

- Results with $\mathcal{L}_1$ available.
- Results with $\mathcal{L}_2$ calculating.
- If time, add some comparison with BM3d

Final steps

- Gather final results
- Write report
- Comparing with BM3D