University of Basel

Status Meeting - Application of Graph Learning to inverse problems

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Problem and Goal

- Denoise observations
- Motivation from imaging (computed tomography, cryo-EM)

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Problem

$\mathcal{Y}$: observation (where $y_i \in \mathbb{R}^M$ with M as observation dimension.)
y_i not observable directly only $y_i + \eta$ is observable, where $\eta \sim \mathcal{N}(0, \sigma^2)$.

Goal

$$\textit{denoiser} : (y_i + \eta) \mapsto y_i^* \approx y_i$$

Sinogram Denoiser

Forward and backward domain

forward : radon transform

backward : filter back projection

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Sinogram Denoiser

$$\textit{Goal} : \textit{denoiser} : (y_i + \eta) \mapsto y_i^* \approx y_i$$

$$\textit{denoiser} : \textit{forward}(x_i) + \eta \mapsto \textit{forward}(x_i)^* \approx \textit{forward}(x_i)$$

Where $x \in \mathbb{R}^N$ is original object with N as original object dimension.

GAT Denoiser

GAT - Graph Attention Network

$$GAT(A, \text{GAT args}) :\rightarrow \text{fixed angles} \rightarrow \text{k-nn circle graph}$$

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Input Graph

- Learning does work with circle graph
- Learning does not work with random (Erdős–Rényi) graph

GAT Denoiser - 2

GAT - Graph Attention Network

$$GAT(A, \text{GAT args}) :\rightarrow \text{fixed angles} \rightarrow \text{k-nn circle graph}$$

Train it with class of input images, enables denoising of class of images.

GAT Loss

$$\mathcal{L}_1 = ||forward(x_i) - denoiser(forward(x_i) + \eta)||_2$$

$$\mathcal{L}_1 = ||y_i - y_i^*||_2$$

$$\mathcal{L}_2 = ||x_i - backward(denoiser(forward(x_i) + \eta))||_2$$

$$\mathcal{L}_2 = ||x_i - backward(y_i^*)||_2$$

Current experiments - Toy images

- Input images uniformly position shapes on image.
- Mostly fixed SNR, validation with same SNR
- Graph size $= 1024$ nodes, k-nn $= 8$
- Results with $\mathcal{L}_1$ available, few with $\mathcal{L}_2$.

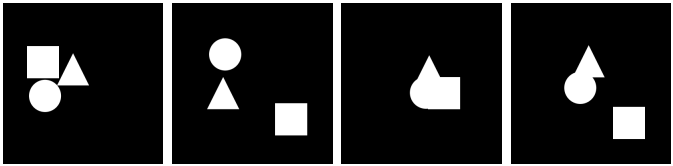

Figure: Example Toy images

Current Results - $\mathcal{L}_1$ vs $\mathcal{L}_2$

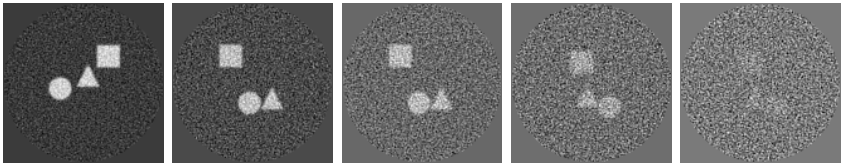

Figure: Noisy fbp with snr [5, 0, -5, -10 ,-20]

Figure: Denoised fbp with snr [5, 0, -5, -10 ,-20]

Current Results - $\mathcal{L}_1$ vs $\mathcal{L}_2$ vs BM3D

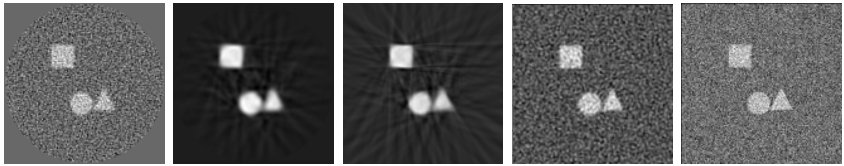

Figure: SNR -5 : noisy, denoised model $\mathcal{L}_1$, denoised model $\mathcal{L}_2$, BM3d noisy sinogram, BM3D noisy input

Some wandb reports

- end-to-end: https://wandb.ai/cedric-mendelin/end-to-enduniformgeneratedtoyimagesscicore?workspace=user-cedric-mendelin
- K-nn: https://wandb.ai/cedric-mendelin/autogeneratedtoyimagesscicoreknn/reports/Auto-toyimages-knn--VmlldzoxOTA4MjM2
- 3d-Model: https://wandb.ai/cedric-mendelin/bunniesscicore?workspace=user-cedric-mendelin
- GAT parameters: https://wandb.ai/cedric-mendelin/toy-images-input-size/reports/Toy-Images-Overview-Advanced-GAT--VmlldzoxNzczNDc0

Final steps

- Gather final results
- Write report
- Comparing with BM3D