Application of Graph Learning to inverse problems

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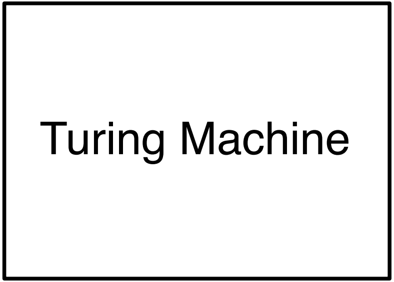
Date

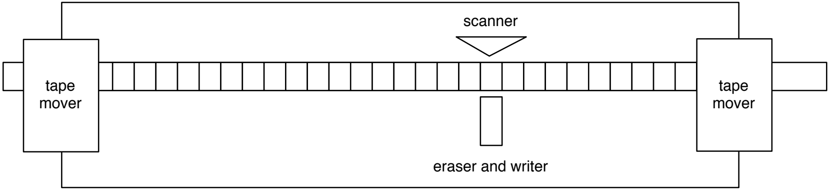
Overview

Some Images



Turing machine Coifman and Lafon 2006

Some Images



A Turing Machine.

Some Equations

Now we introduce an equation.

Theorem

A Turing Machine is a 7-Tuple:

$$M = \langle Q, \Gamma, b, \Sigma, \delta, q_0, F \rangle \tag{1}$$

A Turing Machine is a 7-Tuple even if defined in the text, as in $M = \langle Q, \Gamma, b, \Sigma, \delta, q_0, F \rangle$.

Graph construction

Let Ω be a dataset $\Omega \subset \mathbb{R}^M$, a graph can be constructed like:

- V: each node associated with $x \in \Omega$
- E: Calculate adjacency matrix with similarity measure d and some threshold τ

Definition

$$A_{ij} = \begin{cases} 1 & \text{if } d(x_i, x_j) < \tau \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

Graph construction

Let Ω be a dataset $\Omega \subset \mathbb{R}^M$, but only observation of $y = x + \eta$ (with η drawn from $\mathcal{N} \sim (0, \sigma^2)$), a graph can be constructed like:

- V: each node associated with $y \in \Omega$
- E: Calculate adjacency matrix with similarity measure d and some threshold τ

Definition

$$A_{ij} = \begin{cases} 1 & \text{if } d(x_i, x_j) < \tau \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

Definition

$$A_{0_{ij}} = \begin{cases} 1 & \text{if } d(y_i, y_j) < \tau \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

Noisy graph

Theorem

For every noisy graph $G_0 = \langle V, E_0 \rangle$, there exists a noiseless graph $G = \langle V, E \rangle$. Both graphs consists of the same set of vertices V but different edges. E_0 is defined as follows:

$$E_0 = E \setminus E_0^- \cup E_0^+,$$

where $E_0^- \subseteq E$ and $E_0^- \subseteq U$ with U the set of all possible edges and $E_0^+ \cap E = \emptyset$

Graph Denoising

Graph denoising is the task, to estimate a denoised graph $\tilde{G}$ from a given noisy graph G_O, with underlying original graph G:

Definition

$$GD : G_0 \mapsto \tilde{G} \approx G,$$

where G_0, $\tilde{G}$, G denotes noisy, estimated denoised and original graph respectively.

Definition

$$GD : A_0 \mapsto \tilde{A} \approx A,$$

where A_0, $\tilde{A}$, A denotes adjacency matrix from G_0, $\tilde{G}$ and G respectively.

Items and Numbers

- one
- two
- three

1. first
2. second
3. third

Questions?