Statistical approach to muography as a non-destructive testing technique for industry problem solving

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Muons systems

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Outline

- Introduction
- Muons and muography
- Statistical basis of the algorithm
  - Probability density functions
  - Kernel density estimation
  - Monte-Carlo simulations
  - Likelihood minimization
- The algorithm
- Results obtained
- Conclusions

Section I
General introduction

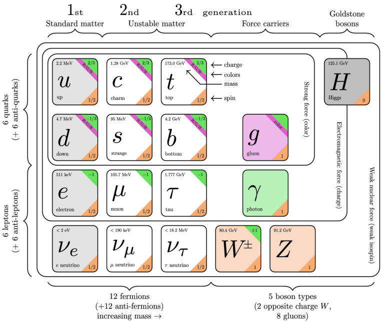
Main goal of this work

Develop a new framework allowing to perform a muography experiment to characterize the inner properties of physical objects using data science and advanced statistical models.

Particle physics and muons

The Standard Model **describes the fundamental particles** existing and their interactions:

- Introduced in the 1970s and still considered to be valid, but probably incomplete
- Simple in concept but extremely precise
- Lots of successful predictions made over the years, such as the existence of the top quark and the Higgs boson



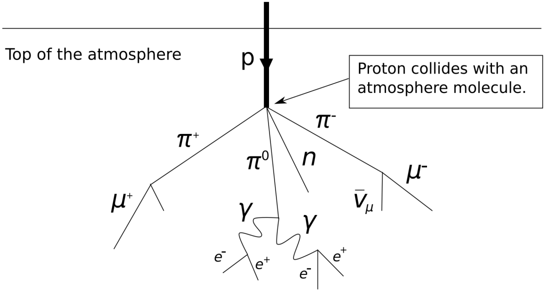
Muons

- Muons μ^- are one of the 12 fundamental particles existing
- They have a relatively small interaction cross-section with ordinary matter, allowing them to cross material without being stopped, making them interesting.

Cosmic rays

Comic rays are a **constant flux of high energy particles** reaching the Earth:

- Mostly made out of protons and atomic nuclei
- Trigger a decay chain by interacting with the atmosphere, producing muons
- Muons are not stable ($\tau \simeq 2.2\mu$s) but relativity can make them live long enough to reach the ground $\rightarrow$ 10.000 cosmic muons are observed per m^2 and per minute at sea level.



Interaction with matter

Muons interact with matter through two main processes:

Ionization

Ionization, when the incident muon gives some of its energy to the electrons of the absorber, but quite small for MIPs such as cosmic muons.

→ Basis for the **absorption muography**.

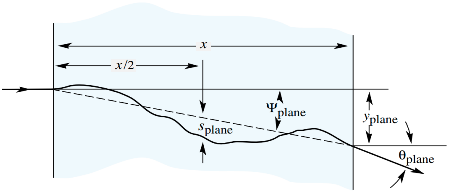
Multiple scattering

Multiple Coulomb scattering inducing a **stochastic deviation** whose central angular deviation can be described by a Gaussian of width θ_0.

$$\theta_0 = \frac{13.6\text{ MeV}}{\beta c p}\sqrt{\frac{x}{X_0}}\left[1 + 0.038\ln\left(\frac{x}{X_0\beta^2}\right)\right]$$

This deviation depends on the number of radiation lengths X_0 and therefore on the medium crossed.

→ Basis for the **scattering muography**.

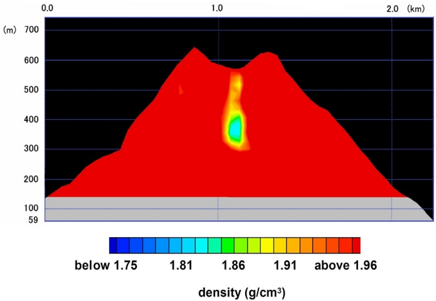


Instead of *calculating* the deviation expected for a cosmic muon, we can *measure* the positional and angular deviation suffered **to estimate the properties of the medium crossed**.
→ Main idea behind the principle of **muon tomography**, or **muography**.

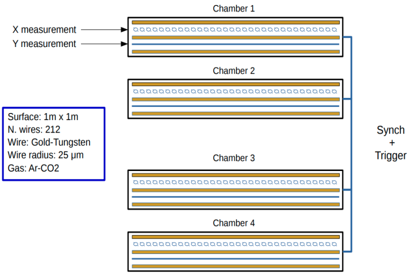
This method prevents several advantages over other imaging techniques:

- Non-destructive technique
- High penetrating capabilities allowing to probe large and dense objects
- Completely safe, by using natural cosmic rays for the measurement.

Muography can be used in many different fields, for example to find hidden rooms in Pyramids or in volcanology to know whether a pocket is empty or full of lava [1].



Experimental setup



Quite simple experimental setup:

- Two $1m^2$ detectors placed below and above the object under investigation
- Two chambers in each detector, to measure the position and direction of muons along the x and y axes
- Each chamber is filled with a mixture of Argon and CO_2
- More than 200 wires separated by 4mm make up each chamber

The data is collected from a USB stick and goes through a complete **reconstruction process** before being available in a rootfile.

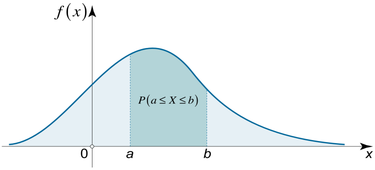
Section II
Statistical basis

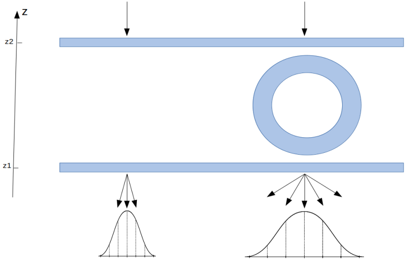
The algorithm developed heavily relies on several important statistical concepts that we can now define.

Probability density functions

PDFs are mathematical expressions defining probability distributions **which represent the likelihood of any given outcome**.

The area below the PDF in an interval can be interpreted as the value of the probability of a random variable occurring.



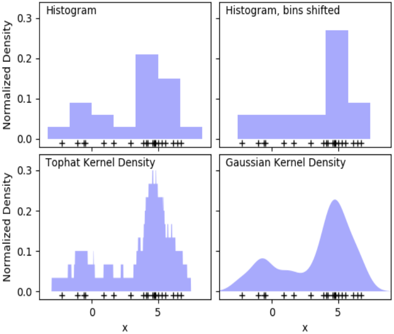


The multiple scattering is a stochastic process, making PDFs extremely important.

$\rightarrow$ A thicker and denser object results in a higher expected deviation and a larger standard deviation σ of the Gaussian PDF.

Kernel density estimation

This method allows us to estimate the shape of an unknown PDF f of a random variable X from a set of N observations.



The usual way to proceed is to simply put the observations in an histogram, but this results in a non continous function with possible gaps.

We then define $\hat{f}_h(x)$, an estimator of f defined as the sum of continous functions instead.

$$\hat{f}_h(x) = \frac{1}{Nh} \sum_{i=1}^{N} K\left(\frac{x - x_i}{h}\right)$$

Two important parameters

- The **kernel** K, chosen as Gaussian functions in this case
- And the **bandwidth** h, a smoothing parameter.

Monte-Carlo simulations

Monte-Carlo simulations are obtained from algorithms developed to **compute approximate numerical values to stochastic problems** using random processes and probabilistic techniques.

Instead of relying on actual data collected from the experiment, we can:

- Use CRY, a cosmic ray generator to simulate thousands of incident muons
- Make these muons go through a complete simulation of the detector done with Geant4
- Go through the same post-processing process as actual data, **simulating thousands of experiments** without the need to perform them
- Build the PDF for a given experiment from these simulations

Simulating an experiment is cheaper and faster than running it and allows to compare results obtained from both channels. In this work, the dependance on Geant4 will be removed as well, to make this process even faster.

Maximum likelihood estimation

We have a way to simulate experiments, but we still need a method allowing us to reverse this process in order to **reverse this process and estimate the geometry of the object from a given measurement**.

The likelihood measures the goodness of a fit with respect to a sample of data for one or several unknown parameters. If $\boldsymbol{\theta}$ are the parameters of the model and x is the measurement of a random variable X defined from a PDF f, then:

$$\mathcal{L}(\boldsymbol{\theta}|x) = f_{\boldsymbol{\theta}}(x) = P(X = x|\boldsymbol{\theta})$$

The likelihood can be described as an hypersurface whose peak gives the optimal set of parameters maximizing the probability of drawing the actual sample measured.

$\rightarrow$ The objective is then to **find the set of parameters minimizing the log-likelihood** $l(\boldsymbol{\theta}|x) = -2\log(\mathcal{L}(\boldsymbol{\theta}|x))$. In this work, the parameter to be optimized will be the thickness of a steel pipe placed between the detectors.

Section III
The algorithm

Text goes here.

MuonState

Surfaces and Volumes

Propagator

Likelihood

Section IV
Results obtained

Text goes here.

Generator validation

Pipes geometries

Kernel density functions

Likelihood curves

Conclusions

Future improvements

Thank you
for your attention!

Any questions?

Ionization

Ionization happens when the incident muon gives some of its energy to the electrons of the absorber, as described by the Bethe-bloch formula.

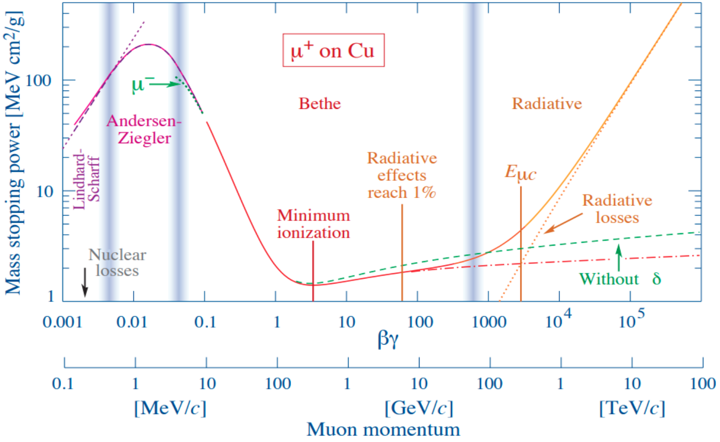
$$-\left\langle \frac{dE}{dx} \right\rangle = Kz^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \left(\frac{2m_e c^2 \beta^2 \gamma^2 W_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right) \right]$$

The **mass stopping power** of material depends on:

- The charge number of incident particle z
- The atomic mass and charge of absorber A and Z
- The relativistic factors β and γ
- The maximum possible energy transfer to an electron in a single collision $W_{\max}$
- And the mean excitation energy I.

Ionization

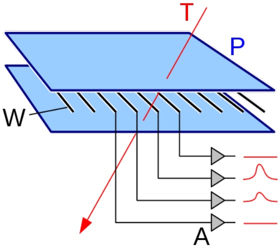
Cosmic muons have an energy of the order of the GeV and are therefore referred to as minimum ionizing particles, so ionization is not considered in this work.



Muon detectors

Multiwire proportional chambers use an array of high-voltage wires, placed within a chamber filled with a gas, in which an electric field is created.

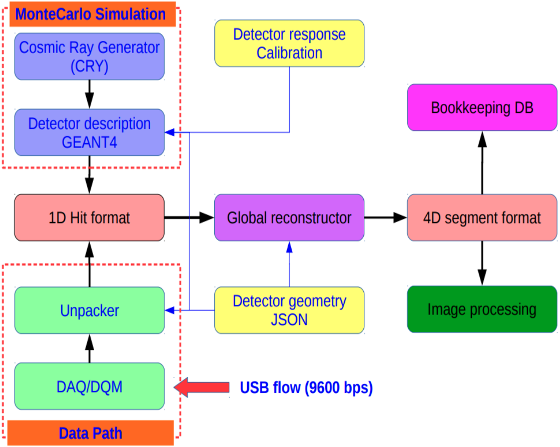
A muon crosses the detector leaves small electric charges behind, collected by the wires while leaving a signal. The combination of the signals on the different wires give us information regarding the muon.



Most important parameters of a muon detector:

- The **spatial resolution**, ideally as small as possible
- The **acceptance**, related to the size of the detector
- And the **efficiency**, which should be as high as possible to make the measurement reliable and fast.

Data flow


MonteCarlo Simulation
Cosmic Ray Generator (CRY)
Detector description GEANT4
Detector response Calibration
Bookkeeping DB
1D Hit format
Global reconstructor
4D segment format
Unpacker
Detector geometry JSON
Image processing
DAQ/DQM
USB flow (9600 bps)
Data Path

Additional references

[1] "A window into the Earth's interior", Earthquake Research Institute,2014