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Contents

Gratier, Favreau, and Renard [2]

1 Physics & Equations

1.1 Basic equations

1.1.1 Stokes flow

$$\vec{\nabla}\cdot\boldsymbol{\sigma}+\rho\vec{g} = \vec{0} \tag{1}$$
$$\vec{\nabla}\cdot\vec{\text{v}} = 0 \tag{2}$$

$$\boldsymbol{\sigma} = -p\mathbf{1}+\boldsymbol{\tau}$$
$$\boldsymbol{\tau} = 2\eta\dot{\boldsymbol{\varepsilon}}(\vec{\text{v}})$$

$$-\vec{\nabla}p+\vec{\nabla}\cdot(2\eta\dot{\boldsymbol{\varepsilon}}(\vec{\text{v}}))+\rho\vec{g} = \vec{0} \tag{3}$$
$$\vec{\nabla}\cdot\vec{\text{v}} = 0 \tag{4}$$

$$\eta_{dsl} = \frac{1}{2}A^{-1/n}\dot{\varepsilon}_e^{1/n-1}\exp\frac{Q}{nRT}$$

where $\dot{\varepsilon}_e$ is the effective strain rate defined as

$$\dot{\varepsilon}_e = \sqrt{\frac{1}{2}(\dot{\varepsilon}_{xx}^2+\dot{\varepsilon}_{yy}^2)+\dot{\varepsilon}_{xy}^2)}$$

describe plasticity

$$\sigma_y = p\sin\phi + c\cos\phi$$

1.1.2 Darcy flow

Following chapter 10 of Guy Simpson's book Simpson [3] the equations governing the evolution of the fluid pressure and fluid flow velocities are[1]

$$\varphi\beta\frac{\partial p_f}{\partial t} = -\vec{\nabla}\cdot\vec{\text{v}}+H-\frac{\partial\varphi}{\partial t} \tag{5}$$
$$\vec{\text{v}} = -\frac{K}{\eta_f}\vec{\nabla}p_f \tag{6}$$

is this the equation we want to use ? other ref ? what are other ppl using ?

where

- p_f is the pore fluid pressure,
- $\vec{\text{v}}$ is the fluid velocity vector,
- φ is the porosity[2] (-). Wikipedia: "Porosity or void fraction is a measure of the void (i.e. "empty") spaces in a material, and is a fraction of the volume of voids over the total volume, between 0 and 1, or as a percentage between 0% and 100%".
- β is the bulk compressibility(Pa^{-1}), typical va for rocks?

  we find online multiple compressbilities in the context of rocks. Which are we to use?
- K is the permeability[3] (m^2) Wikipedia: " Permeability is a property of porous materials that is an indication of the ability for fluids (gas or liquid) to flow through them. Fluids can more easily flow through a material with high permeability than one with low permeability. The permeability of a medium is related to the porosity, but also to the shapes of the pores in the medium and their level of connectedness. Fluid flows can also be influenced in different lithological settings by brittle deformation of rocks in fault zones; the mechanisms by

[1] I have slightly changed the notations.
[2] https://en.wikipedia.org/wiki/Porosity
[3] https://en.wikipedia.org/wiki/Permeability_(Earth_sciences)

which this occurs are the subject of fault zone hydrogeology. Permeability is also affected by the pressure inside a material."

In the code it is assumed to be given by Skarbek and Rempel [4] and Bernaudin and Gueydan [1]

$$K = K_0 \left(\frac{\phi}{\phi_0}\right)^3$$

where K_0 is the permeability at a reference porosity ϕ_0.

- η_f is the fluid viscosity (Pa s), typically water[4]
- ρ_f is the water density ($\mathrm{kg\,m^{-3}}$),
- g is acceleration due to gravity ($\mathrm{m\,s^{-2}}$),
- $\vec{e}$ is a unit vector oriented in the vertical direction,
- H accounts for any fluid pressure sources or sinks (e.g., due to devolatilization reactions)

substituting (6) into (5) lead to a single parabolic equation for the excess fluid pressure as follows:

$$\varphi\beta \frac{\partial p_f}{\partial t} = \vec{\nabla}\cdot\left(\frac{K}{\eta_f}\vec{\nabla}p_f\right) + H - \frac{\partial \phi}{\partial t} \tag{7}$$

Note that once the excess pressure is computed one can recover the fluid velocity via $\vec{\mathrm{v}} = -\frac{K}{\eta_f}\vec{\nabla}p_f$. Note that $H = 0$ in experiments 1,2,3.

In what follows we assume the buoyancy forces are negligible, i.e. the term $\rho\vec{g}$ is neglected.

1.1.3 Coupling

Remark1: when using tectonic dt (which is $\mathcal{O}(10^5)$ year, we find that $\frac{\partial\phi}{\partial t} \simeq \frac{\phi^n - \phi^{n-1}}{\delta t} \to 0$ so that the contribution of this term to the equation is virtually inexistant.

Remark2: Likewise the characteristic Darcy time is probably about $\mathcal{O}(10^0)$ year so that when carrying out tectonic time steps the Darcy diffusion process has the time to reach steady state. Since we are currently not relying on an operator splitting approach, we might as well directly solve the steady state Darcy equation, i.e. $\partial p_f/\partial t \to 0$.

[4] https://en.wikipedia.org/wiki/Water

2 Numerical aspects

2D
    Cartesian domain
    Finite elements
    quadrilateral elements Q2Q1
    solver

2.1 FE formulation of the equations

2.1.1 Stokes equations

2.1.2 Darcy equations

Applying standard FE methodology to what is essentially a diffusion equation, we arrive at:

$$\mathbb{M} \cdot \dot{\vec{\mathcal{P}}}_e + \mathbb{K}_d \cdot \vec{\mathcal{P}}_e = rhs$$

with

$$\mathbb{M} = \int_\Omega \varphi \beta \vec{\mathcal{N}}^T \vec{\mathcal{N}}$$

$$\mathbb{K}_d = \int_\Omega \boldsymbol{B}^T \boldsymbol{B} \frac{K}{\eta_f}$$

$$rhs = \int_\Omega \vec{\mathcal{N}}^T H$$

Using a simple first order time discretisation yields

$$(\mathbb{M} + \mathbb{K}_d \delta t) \cdot \vec{\mathcal{P}}_e^n = \mathbb{M} \cdot \vec{\mathcal{P}}_e^{n-1} + rhs\delta t$$

2.2 Specific algorithms

2.2.1 computing time step

CFL dt

2.2.2 Generating weak seeds

Poisson disc distribution seeds generated in square. Notes that seeds which are kept are those inside circle or size ainclusion-w

For each marker im we test whether it is at a distance of w or less of seed is and the prescribed strain is then parameterised as follows

$$A\frac{1}{2}\left(\cos(\pi\frac{\sqrt{(x_{im}-x_{is})^2+(y_{im}-y_{is})^2}}{w})+1\right)$$

where w is the radius of the seed, and A its maximum amplitude

include image examples

2.2.3 Time dependent b.c.

At the moment these are parameterised by t_1, t_2 and velofact.

```
       +------------------+ -- velofact
       |                  |
-------|------------------|------> time
       t1                 t2
```

2.2.4 Nonlinear residual

2.3 Numerical parameter values and meaning

2.3.1 Marker in cell technique

projection/averaging , advection, periodic pc, painting

    ...

3 Benchmarks

3.1 Pure shear

3.2 Simple shear

3.3 SolVi

3.4 Poiseuille nonlinear

3.5 (E)VP experiment gerya 3 mats

3.6 darcy? simpson ?

3.7 gaussian diffusion in time

References

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[3] Guy Simpson. *Practical Finite Element Modelin in Earth Science Using Matlab*. Wiley-Blackwell, 2017. ISBN: 978-1-119-24862-0.

[4] Rob M Skarbek and Alan W Rempel. "Dehydration-induced porosity waves and episodic tremor and slip". In: *Geochemistry, Geophysics, Geosystems* 17.2 (2016), pp. 442–469. DOI: 10.1002/2015GC006155.