Created on Sat Nov 06 16:00:00 2021. @author: Cedric Yu

Climate Change: Global Temperature

We study the global temperatures dataset available on:

https://www.kaggle.com/berkeleyearth/climate-change-earth-surface-temperature-data

We focus on the time series 'LandAverageTemperature', which is the monthly global average land temperature in Celsius.

We first explore the given relative absolute uncertainty of the time series, and find that it is large during the early record 1750-1849. Because of this, we decide to take the yearly averages and only use the data from 1850-2015.

By eyeballing and using the augmented Dickey-Fuller test, we see a clear increasing trend (hence global 'warming'), which is rendered stationary by taking the first difference.

Next, we study the auto-correlation function (ACF) and partial auto-correlation function (PACF), we find that the first difference can be described by an AR(3) model. We fit AR(3) models on the series, using fixed partitioning and rolling forecast, and compare the model performance.

Preamble

In [1]:

```
import pandas as pd
# Make the output look better
pd.set_option('display.max_rows', 500)
pd.set_option('display.max_columns', 500)
# pd.set_option('display.width', 1000)
pd.options.mode.chained_assignment = None  # default='warn' # ignores warning about dropping columns inplace


import numpy as np
import seaborn as sn
import matplotlib.pyplot as plt

import os
os.chdir(r'C:\Users\Cedric Yu\Desktop\Works\13_time_series_global_temperatures')
```

load dataset and parse datetime

In [2]:

```
"""# import GlobalTemperatures dataset and parse datetime column 'dt'"""

from datetime import datetime
def parser(x):
        return datetime.strptime(x, '%Y-%m-%d')

global_temperatures = pd.read_csv('GlobalTemperatures.csv', header = [0], parse_dates= [0], date_parser=parser)
```

Preliminary observations

In [4]:

```
global_temperatures.tail()
```

Out[4]:

	dt	LandAverageTemperature	LandAverageTemperatureUncertainty	LandMaxTemperature	LandMaxTemperatureUncertainty	La
3187	2015-08-01	14.755	0.072	20.699	0.110	
3188	2015-09-01	12.999	0.079	18.845	0.088	
3189	2015-10-01	10.801	0.102	16.450	0.059	
3190	2015-11-01	7.433	0.119	12.892	0.093	

	dt	LandAverageTemperature	LandAverageTemperatureUncertainty	LandMaxTemperature	LandMaxTemperatureUncertainty	La
3191	2015-12-01	5.518	0.100	10.725	0.154	

In [5]:

```
"""# dtype of column 'dt' is datetime64[ns]"""
global_temperatures['dt']
```

Out[5]:

```
0       1750-01-01
1       1750-02-01
2       1750-03-01
3       1750-04-01
4       1750-05-01
           ...
3187    2015-08-01
3188    2015-09-01
3189    2015-10-01
3190    2015-11-01
3191    2015-12-01
Name: dt, Length: 3192, dtype: datetime64[ns]
```

In [6]:

```
"""# missing values"""
global_temperatures.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 3192 entries, 0 to 3191
Data columns (total 9 columns):
 #   Column                                     Non-Null Count  Dtype
---  ------                                     --------------  -----
 0   dt                                         3192 non-null   datetime64[ns]
 1   LandAverageTemperature                     3180 non-null   float64
 2   LandAverageTemperatureUncertainty          3180 non-null   float64
 3   LandMaxTemperature                         1992 non-null   float64
 4   LandMaxTemperatureUncertainty              1992 non-null   float64
 5   LandMinTemperature                         1992 non-null   float64
 6   LandMinTemperatureUncertainty              1992 non-null   float64
 7   LandAndOceanAverageTemperature             1992 non-null   float64
 8   LandAndOceanAverageTemperatureUncertainty  1992 non-null   float64
dtypes: datetime64[ns](1), float64(8)
memory usage: 224.6 KB
```

In [7]:

```
"""# 'LandAverageTemperature' and 'LandAverageTemperatureUnvertainty' have no NaN after 1752-12-31"""
global_temperatures[global_temperatures[['LandAverageTemperature']].isna().any(axis=1)]['dt']
```

Out[7]:

```
10    1750-11-01
16    1751-05-01
18    1751-07-01
21    1751-10-01
22    1751-11-01
23    1751-12-01
25    1752-02-01
28    1752-05-01
29    1752-06-01
30    1752-07-01
31    1752-08-01
32    1752-09-01
Name: dt, dtype: datetime64[ns]
```

In [8]:

```
# 'LandAverageTemperature' and 'LandAverageTemperatureUnvertainty' have no NaN after 1752-12-31
global_temperatures[global_temperatures['dt'] > pd.Timestamp(1752,12,31)].info()
```

```
<class 'pandas.core.frame.DataFrame'>
Int64Index: 3156 entries, 36 to 3191
Data columns (total 9 columns):
 #   Column                                     Non-Null Count  Dtype
---  ------                                     --------------  -----
 0   dt                                         3156 non-null   datetime64[ns]
 1   LandAverageTemperature                     3156 non-null   float64
 2   LandAverageTemperatureUncertainty          3156 non-null   float64
 3   LandMaxTemperature                         1992 non-null   float64
 4   LandMaxTemperatureUncertainty              1992 non-null   float64
 5   LandMinTemperature                         1992 non-null   float64
 6   LandMinTemperatureUncertainty              1992 non-null   float64
```

```
 7   LandAndOceanAverageTemperature             1992 non-null   float64
 8   LandAndOceanAverageTemperatureUncertainty  1992 non-null   float64
dtypes: datetime64[ns](1), float64(8)
memory usage: 246.6 KB
```

In [9]:

```
"""# No NaN at all after 1849-12-31"""
global_temperatures[global_temperatures[['LandMaxTemperature']].isna().any(axis=1)]['dt']
```

Out[9]:

```
0      1750-01-01
1      1750-02-01
2      1750-03-01
3      1750-04-01
4      1750-05-01
          ...
1195   1849-08-01
1196   1849-09-01
1197   1849-10-01
1198   1849-11-01
1199   1849-12-01
Name: dt, Length: 1200, dtype: datetime64[ns]
```

In [10]:

```
global_temperatures[global_temperatures['dt'] > pd.Timestamp(1849,12,31)].info()
```

```
<class 'pandas.core.frame.DataFrame'>
Int64Index: 1992 entries, 1200 to 3191
Data columns (total 9 columns):
 #   Column                                     Non-Null Count  Dtype
---  ------                                     --------------  -----
 0   dt                                         1992 non-null   datetime64[ns]
 1   LandAverageTemperature                     1992 non-null   float64
 2   LandAverageTemperatureUncertainty          1992 non-null   float64
 3   LandMaxTemperature                         1992 non-null   float64
 4   LandMaxTemperatureUncertainty              1992 non-null   float64
 5   LandMinTemperature                         1992 non-null   float64
 6   LandMinTemperatureUncertainty              1992 non-null   float64
 7   LandAndOceanAverageTemperature             1992 non-null   float64
 8   LandAndOceanAverageTemperatureUncertainty  1992 non-null   float64
dtypes: datetime64[ns](1), float64(8)
memory usage: 155.6 KB
```

Looking at the uncertainties

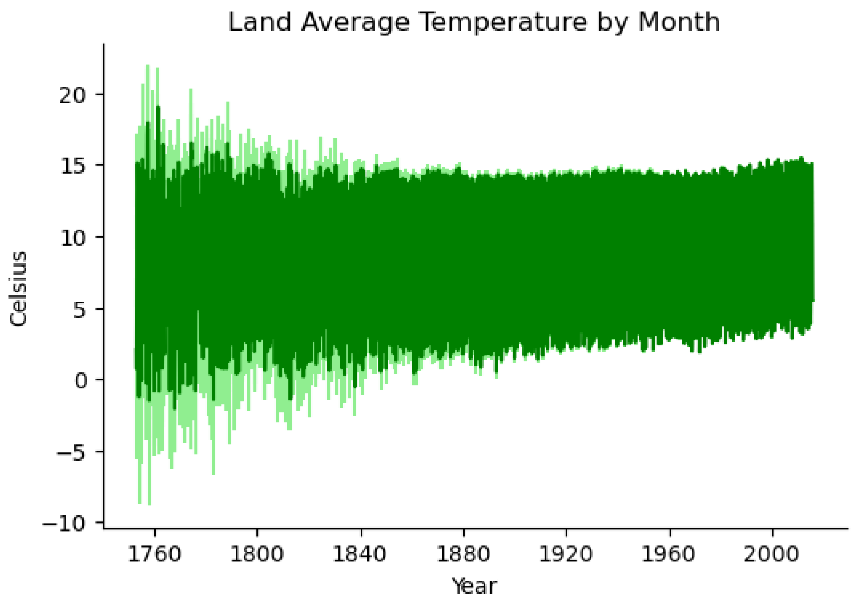
LandAverageTemperature: plots by month

In [3]:

```
temperatures_1753_avg = global_temperatures[global_temperatures['dt'] > pd.Timestamp(1752,12,31)][['dt', 'LandAve
temperatures_1850_all = global_temperatures[global_temperatures['dt'] > pd.Timestamp(1849,12,31)]
```

In [15]:

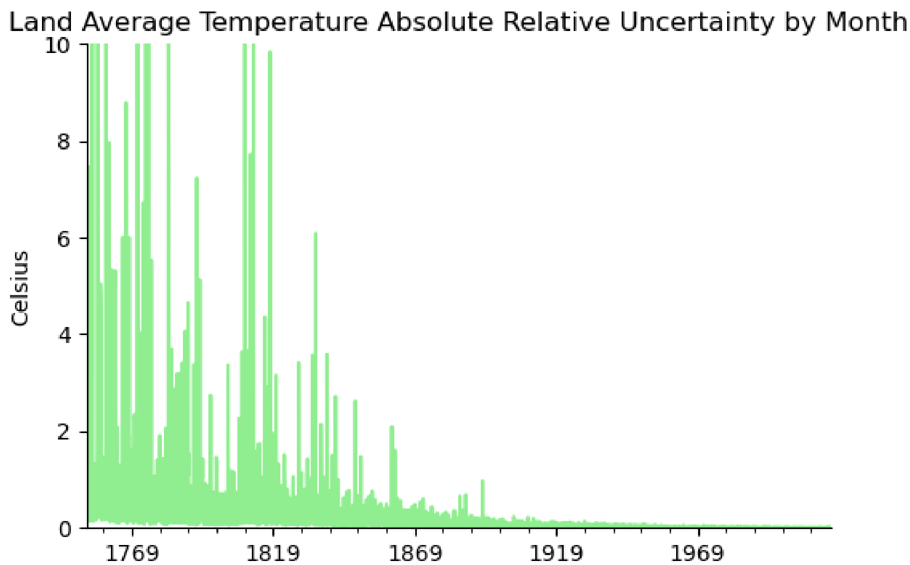
```
"""# plot LandAverageTemperature (green) and error bar 'LandAverageTemperatureUncertainty' (light green)"""

plt.figure(dpi=100)
plt.errorbar('dt', 'LandAverageTemperature', yerr='LandAverageTemperatureUncertainty', data=temperatures_1753_avg
ax1 = plt.gca()
ax1.set_title('Land Average Temperature by Month')
ax1.set_xlabel('Year')
ax1.set_ylabel('Celsius')
ax1.spines['top'].set_visible(False)
ax1.spines['right'].set_visible(False)
```

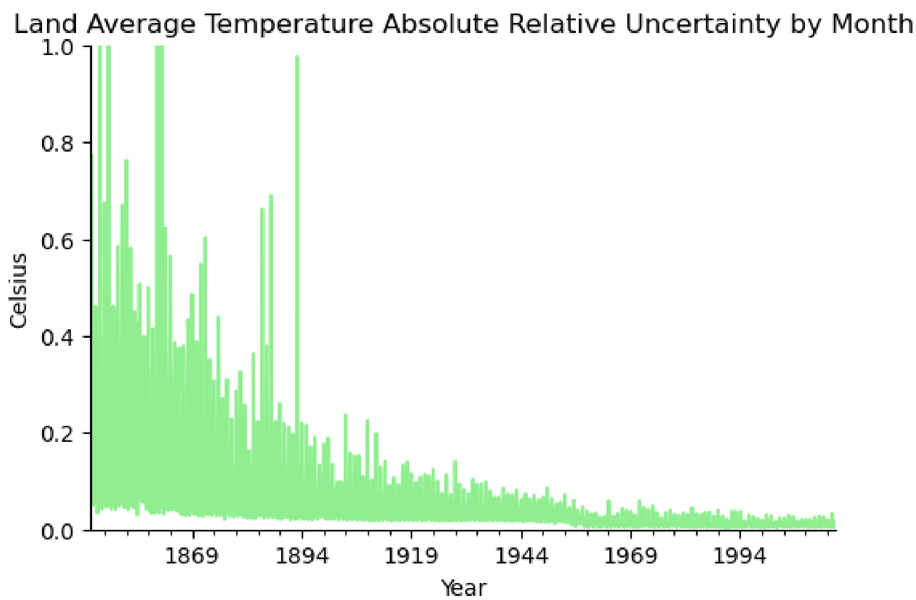


Absolute Relative Uncertainty

In [35]:

```
"""relative uncertainty"""
temperatures_1753_avg['LandAverageTemperatureRelativeUncertainty'] = (temperatures_1753_avg['LandAverageTemperatu

"""# large relative error in earlier observations"""
plt.figure(dpi=100)
temperatures_1753_avg.plot(x = 'dt', y='LandAverageTemperatureRelativeUncertainty', color='lightgreen', ax = plt.
ax1 = plt.gca()
ax1.set_title('Land Average Temperature Absolute Relative Uncertainty by Month')
ax1.set_xlabel('Year')
ax1.set_ylabel('Celsius')
ax1.set_ylim(0,10)
ax1.set_xlabel(None)
ax1.spines['top'].set_visible(False)
ax1.spines['right'].set_visible(False)
ax1.get_legend().remove()
# plt.savefig(r'plots/LandAverageTemperatureRelativeUncertainty_month1', dpi=150)

plt.figure(dpi=100)
temperatures_1753_avg.plot(x = 'dt', y='LandAverageTemperatureRelativeUncertainty', color='lightgreen', ax = plt.
ax1 = plt.gca()
ax1.set_ylim(0,1)
ax1.set_title('Land Average Temperature Absolute Relative Uncertainty by Month')
ax1.set_xlabel('Year')
ax1.set_ylabel('Celsius')
ax1.set_xlim(pd.Timestamp(1845, 12, 31))
ax1.spines['top'].set_visible(False)
ax1.spines['right'].set_visible(False)
ax1.get_legend().remove()
# plt.savefig(r'plots/LandAverageTemperatureRelativeUncertainty_month2', dpi=150)
```





LandAverageTemperature: Yearly mean temperature and uncertainty

In [4]:

```
def yearly_uncertainty(col):
    return np.sqrt(np.sum(col ** 2)) / len(col)

temperatures_1753_avg_yearly = temperatures_1753_avg[['dt', 'LandAverageTemperature']].groupby(pd.Grouper(key='dt
temperatures_1753_avg_yearly['LandAverageTemperatureUncertainty'] = temperatures_1753_avg.groupby(pd.Grouper(key=
temperatures_1753_avg_yearly['LandAverageTemperatureRelativeUncertainty'] = (temperatures_1753_avg_yearly['LandAv

temperatures_1753_avg_yearly.head()
```

```
C:\Users\CEDRIC~1\AppData\Local\Temp/ipykernel_16880/2418994677.py:5: FutureWarning: Indexing with multiple keys
(implicitly converted to a tuple of keys) will be deprecated, use a list instead.
  temperatures_1753_avg_yearly['LandAverageTemperatureUncertainty'] = temperatures_1753_avg.groupby(pd.Grouper(ke
y='dt', freq='1Y'))['dt', 'LandAverageTemperatureUncertainty'].agg(yearly_uncertainty)
C:\ProgramData\Anaconda3\lib\site-packages\pandas\core\groupby\generic.py:303: FutureWarning: Dropping invalid co
lumns in SeriesGroupBy.agg is deprecated. In a future version, a TypeError will be raised. Before calling .agg, s
elect only columns which should be valid for the aggregating function.
  results[key] = self.aggregate(func)
```

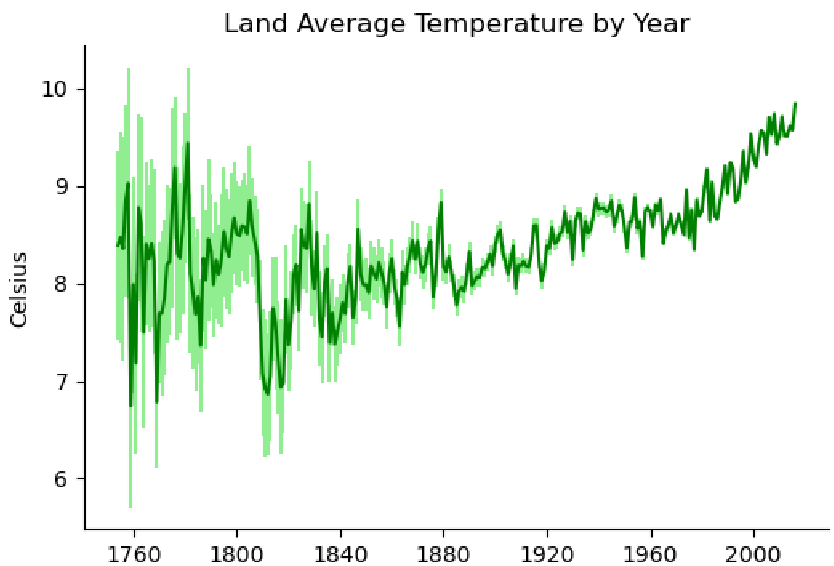
Out[4]:

	LandAverageTemperature	LandAverageTemperatureUncertainty	LandAverageTemperatureRelativeUncertainty
dt			
1753-12-31	8.388083	0.970134	0.115656
1754-12-31	8.469333	1.091063	0.128825

dt	LandAverageTemperature	LandAverageTemperatureUncertainty	LandAverageTemperatureRelativeUncertainty
1755-12-31	8.355583	1.153430	0.138043
1756-12-31	8.849583	0.981928	0.110958
1757-12-31	9.022000	1.180627	0.130861

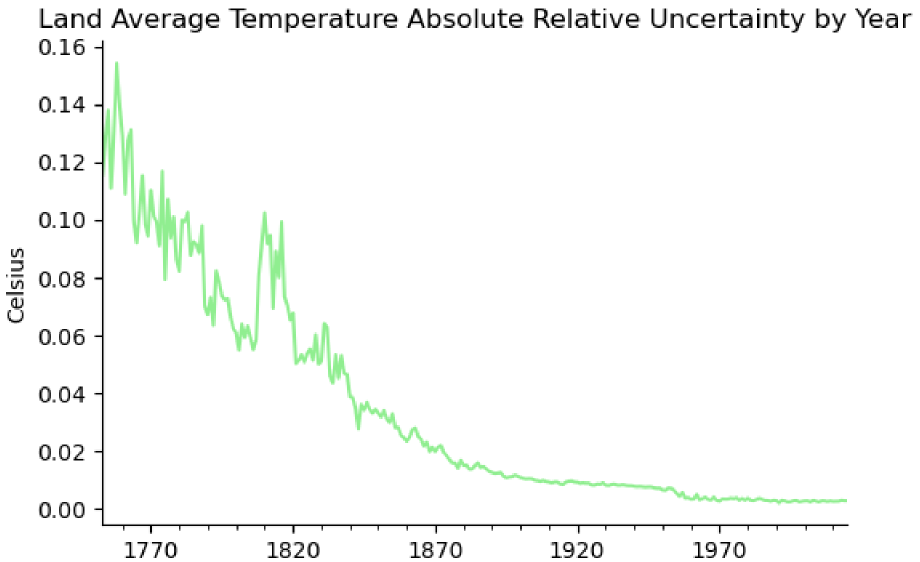
In [27]:

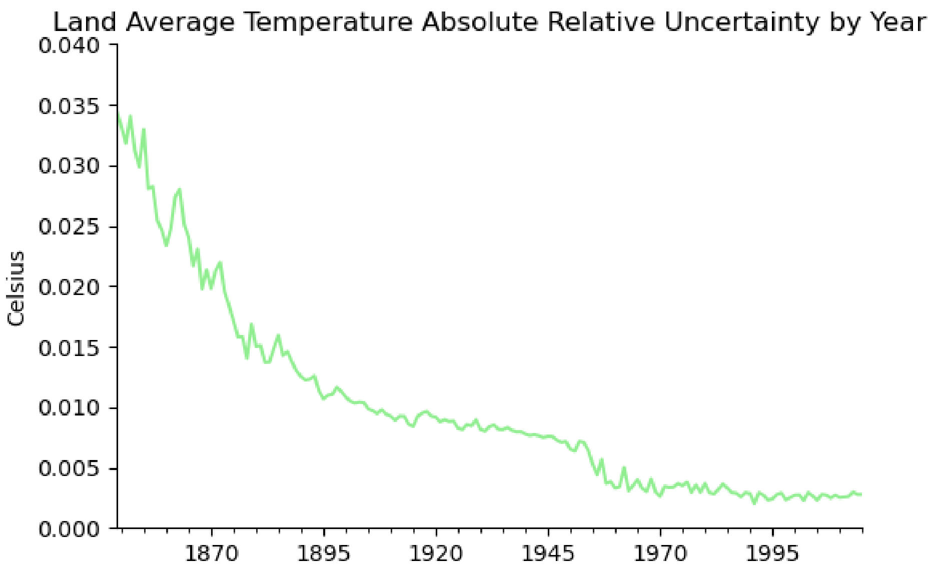
```
"""# plot yearly-averaged LandAverageTemperature and error bar 'LandAverageTemperatureUncertainty'"""
""" relative uncertainty becomes smaller after ca. 1850"""

plt.figure(dpi=100)
plt.errorbar(temperatures_1753_avg_yearly.index, 'LandAverageTemperature', yerr='LandAverageTemperatureUncertaint
ax1 = plt.gca()
ax1.set_title('Land Average Temperature by Year')
ax1.set_xlabel('Year')
ax1.set_ylabel('Celsius')
# ax1.set_ylim(0,10)
ax1.set_xlabel(None)
ax1.spines['top'].set_visible(False)
ax1.spines['right'].set_visible(False)
```



In [34]:

```
plt.figure(dpi=100)
temperatures_1753_avg_yearly.plot(y = 'LandAverageTemperatureRelativeUncertainty', color = 'lightgreen', ax = plt
ax1 = plt.gca()
ax1.set_title('Land Average Temperature Absolute Relative Uncertainty by Year')
ax1.set_xlabel('Year')
ax1.set_ylabel('Celsius')
# ax1.set_ylim(0,10)
ax1.set_xlabel(None)
ax1.spines['top'].set_visible(False)
ax1.spines['right'].set_visible(False)
ax1.get_legend().remove()

plt.figure(dpi=100)
temperatures_1753_avg_yearly.plot(y = 'LandAverageTemperatureRelativeUncertainty', color = 'lightgreen', ax = plt
ax1 = plt.gca()
ax1.set_title('Land Average Temperature Absolute Relative Uncertainty by Year')
ax1.set_xlabel('Year')
ax1.set_ylabel('Celsius')
ax1.set_ylim(0,0.04)
ax1.set_xlim(pd.Timestamp(1849, 12, 31))
ax1.set_xlabel(None)
ax1.spines['top'].set_visible(False)
ax1.spines['right'].set_visible(False)
ax1.get_legend().remove()
```





Land Average/Max/Min Temperature: Yearly mean temperature and uncertainty

In [5]:

```
temperatures_1850_all.columns
```

Out[5]:

```
Index(['dt', 'LandAverageTemperature', 'LandAverageTemperatureUncertainty',
       'LandMaxTemperature', 'LandMaxTemperatureUncertainty',
       'LandMinTemperature', 'LandMinTemperatureUncertainty',
       'LandAndOceanAverageTemperature',
       'LandAndOceanAverageTemperatureUncertainty'],
      dtype='object')
```

In [6]:

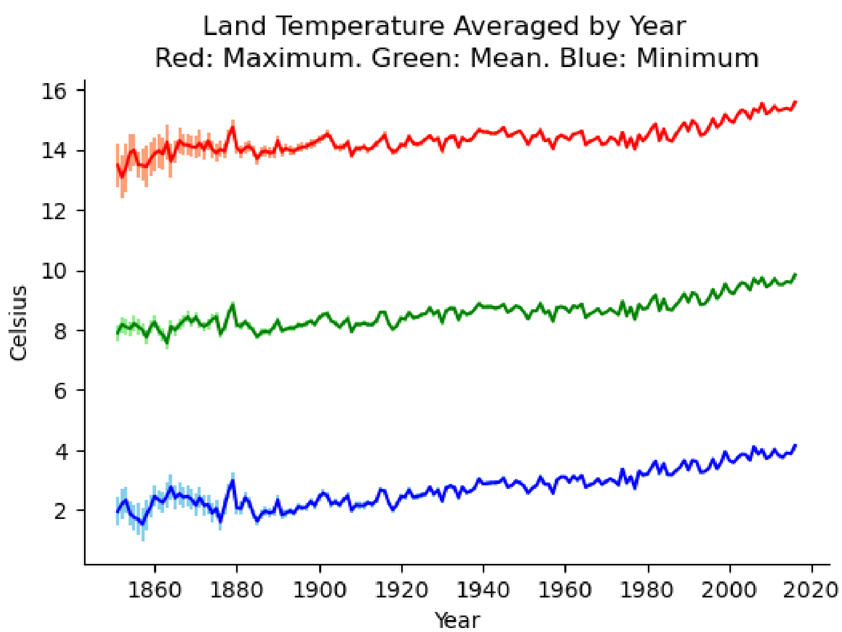
```
# annual mean temperature and uncertainty
def yearly_uncertainty(col):
    return np.sqrt(np.sum(col ** 2)) / len(col)
temperatures_1850_all_yearly = temperatures_1850_all[['dt', 'LandAverageTemperature',
                       'LandMaxTemperature', 'LandMinTemperature',
                       'LandAndOceanAverageTemperature']].groupby(pd.Grouper(key='dt', freq='1Y')).mean()
temperatures_1850_all_yearly['LandAverageTemperatureUncertainty'] = temperatures_1850_all.groupby(pd.Grouper(key=
temperatures_1850_all_yearly['LandMaxTemperatureUncertainty'] = temperatures_1850_all.groupby(pd.Grouper(key='dt'
temperatures_1850_all_yearly['LandMinTemperatureUncertainty'] = temperatures_1850_all.groupby(pd.Grouper(key='dt'
temperatures_1850_all_yearly['LandAndOceanAverageTemperatureUncertainty'] = temperatures_1850_all.groupby(pd.Grou
```

```
C:\Users\CEDRIC~1\AppData\Local\Temp/ipykernel_16880/177883175.py:7: FutureWarning: Indexing with multiple keys
(implicitly converted to a tuple of keys) will be deprecated, use a list instead.
  temperatures_1850_all_yearly['LandAverageTemperatureUncertainty'] = temperatures_1850_all.groupby(pd.Grouper(ke
y='dt', freq='1Y'))['dt', 'LandAverageTemperatureUncertainty'].agg(yearly_uncertainty)
C:\ProgramData\Anaconda3\lib\site-packages\pandas\core\groupby\generic.py:303: FutureWarning: Dropping invalid co
```

```
lumns in SeriesGroupBy.agg is deprecated. In a future version, a TypeError will be raised. Before calling .agg, s
elect only columns which should be valid for the aggregating function.
  results[key] = self.aggregate(func)
C:\Users\CEDRIC~1\AppData\Local\Temp/ipykernel_16880/177883175.py:8: FutureWarning: Indexing with multiple keys
(implicitly converted to a tuple of keys) will be deprecated, use a list instead.
  temperatures_1850_all_yearly['LandMaxTemperatureUncertainty'] = temperatures_1850_all.groupby(pd.Grouper(key='d
t', freq='1Y'))['dt', 'LandMaxTemperatureUncertainty'].agg(yearly_uncertainty)
C:\ProgramData\Anaconda3\lib\site-packages\pandas\core\groupby\generic.py:303: FutureWarning: Dropping invalid co
lumns in SeriesGroupBy.agg is deprecated. In a future version, a TypeError will be raised. Before calling .agg, s
elect only columns which should be valid for the aggregating function.
  results[key] = self.aggregate(func)
C:\Users\CEDRIC~1\AppData\Local\Temp/ipykernel_16880/177883175.py:9: FutureWarning: Indexing with multiple keys
(implicitly converted to a tuple of keys) will be deprecated, use a list instead.
  temperatures_1850_all_yearly['LandMinTemperatureUncertainty'] = temperatures_1850_all.groupby(pd.Grouper(key='d
t', freq='1Y'))['dt', 'LandMinTemperatureUncertainty'].agg(yearly_uncertainty)
C:\ProgramData\Anaconda3\lib\site-packages\pandas\core\groupby\generic.py:303: FutureWarning: Dropping invalid co
lumns in SeriesGroupBy.agg is deprecated. In a future version, a TypeError will be raised. Before calling .agg, s
elect only columns which should be valid for the aggregating function.
  results[key] = self.aggregate(func)
C:\Users\CEDRIC~1\AppData\Local\Temp/ipykernel_16880/177883175.py:10: FutureWarning: Indexing with multiple keys
(implicitly converted to a tuple of keys) will be deprecated, use a list instead.
  temperatures_1850_all_yearly['LandAndOceanAverageTemperatureUncertainty'] = temperatures_1850_all.groupby(pd.Gr
ouper(key='dt', freq='1Y'))['dt', 'LandAndOceanAverageTemperatureUncertainty'].agg(yearly_uncertainty)
C:\ProgramData\Anaconda3\lib\site-packages\pandas\core\groupby\generic.py:303: FutureWarning: Dropping invalid co
lumns in SeriesGroupBy.agg is deprecated. In a future version, a TypeError will be raised. Before calling .agg, s
elect only columns which should be valid for the aggregating function.
  results[key] = self.aggregate(func)
```

In [46]:

```
"""# plot yearly-averaged LandAverageTemperature and error bar 'LandAverageTemperatureUncertainty'"""

plt.figure(dpi=100)
plt.suptitle('Land Temperature Averaged by Year')
plt.errorbar(temperatures_1850_all_yearly.index, 'LandAverageTemperature', yerr='LandAverageTemperatureUncertaint
plt.errorbar(temperatures_1850_all_yearly.index, 'LandMaxTemperature', yerr='LandMaxTemperatureUncertainty', data
plt.errorbar(temperatures_1850_all_yearly.index, 'LandMinTemperature', yerr='LandMinTemperatureUncertainty', data
ax1 = plt.gca()
ax1.set_title('Red: Maximum. Green: Mean. Blue: Minimum')
ax1.set_xlabel('Year')
ax1.set_ylabel('Celsius')
# ax1.set_ylim(0,10)
ax1.spines['top'].set_visible(False)
ax1.spines['right'].set_visible(False)
# plt.legend()
```



Correlations between the temperatures

In [58]:

```
"""highly correlated at lag = 0"""

from scipy.stats import pearsonr
```

```
print(pearsonr(temperatures_yearly_first_diff['LandAverageTemperature'], temperatures_yearly_first_diff['LandMaxT
# (0.7041473540151104, 5.074132061908521e-26)

print(pearsonr(temperatures_yearly_first_diff['LandAverageTemperature'], temperatures_yearly_first_diff['LandMinT
# (0.8674877678792309, 2.675622965207922e-51)

print(pearsonr(temperatures_yearly_first_diff['LandMaxTemperature'], temperatures_yearly_first_diff['LandMinTempe
# (0.7165142434197547, 2.7704818163204947e-27)
```

```
(0.7041473540151104, 5.074132061908521e-26)
(0.8674877678792309, 2.675622965207922e-51)
(0.7165142434197547, 2.7704818163204947e-27)
```

So it looks like the three time series behave very similarly. For our purpose, we just study the 'LandAverageTemperature'. Moreover, we use data starting from 1850 from now on, due to the smaller uncertainty.

Augmented Dickey-Fuller test for non-stationarity

Non-stationarity is the null hypothesis; p-value > 0.05 infers non-stationary.

In [47]:
```
from statsmodels.tsa.stattools import adfuller
```

In [48]:
```
"""# p-values > 0.05: non-stationary"""
print(adfuller(temperatures_1850_all_yearly['LandAverageTemperature'])[1])
print(adfuller(temperatures_1850_all_yearly['LandMaxTemperature'])[1])
print(adfuller(temperatures_1850_all_yearly['LandMinTemperature'])[1])
```

```
0.9505464412068493
0.8231808221341292
0.9814993947787101
```

In [9]:
```
"""# first differences"""
temperatures_yearly_first_diff = temperatures_1850_all_yearly[['LandAverageTemperature', 'LandMaxTemperature', 'L
# drop year 1850 (first row; NaN after first difference)
```

In [51]:
```
temperatures_yearly_first_diff.head()
```

Out[51]:

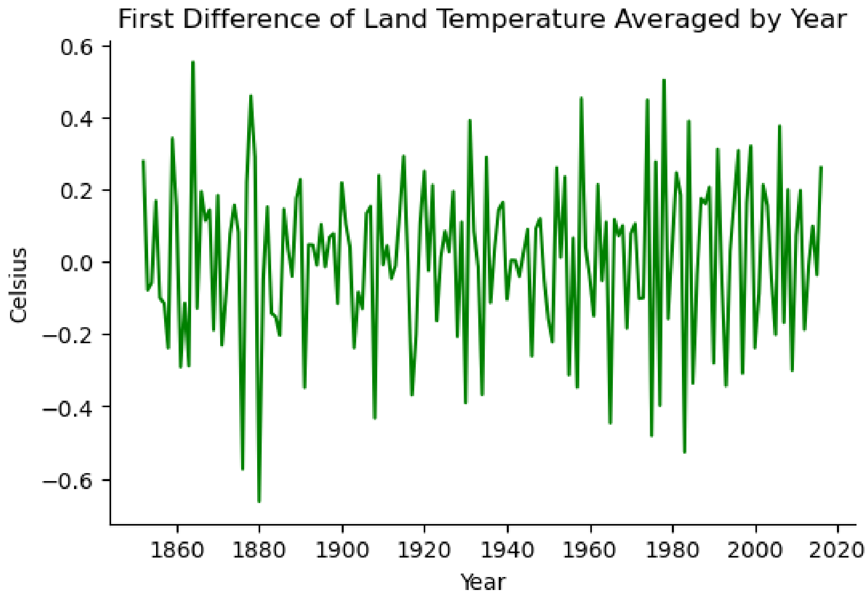
dt	LandAverageTemperature	LandMaxTemperature	LandMinTemperature
1851-12-31	0.277917	-0.395667	0.239583
1852-12-31	-0.078417	0.316333	0.133083
1853-12-31	-0.058333	0.489250	-0.444500
1854-12-31	0.168667	0.090833	-0.130333
1855-12-31	-0.099750	-0.484250	-0.059333

In [50]:
```
"""# p-values for first differences << 0.05: stationary"""
print(adfuller(temperatures_yearly_first_diff['LandAverageTemperature'])[1])
print(adfuller(temperatures_yearly_first_diff['LandMaxTemperature'])[1])
print(adfuller(temperatures_yearly_first_diff['LandMinTemperature'])[1])
```

```
2.625626865464363e-25
6.030368572920849e-24
1.3653286686687314e-08
```

In [10]:
```
"""# plot first difference of yearly-averaged LandAverageTemperature and error bar 'LandAverageTemperatureUncerta

plt.figure(dpi=100)
plt.errorbar(temperatures_yearly_first_diff.index, 'LandAverageTemperature', data=temperatures_yearly_first_diff,
# plt.errorbar(temperatures_yearly_first_diff.index, 'LandMaxTemperature', data=temperatures_yearly_first_diff, c
# plt.errorbar(temperatures_yearly_first_diff.index, 'LandMinTemperature', data=temperatures_yearly_first_diff, c
ax1 = plt.gca()
# ax1.set_ylim(0,10)
```

```
plt.title('First Difference of Land Temperature Averaged by Year')
ax1.set_xlabel('Year')
ax1.set_ylabel('Celsius')
ax1.spines['top'].set_visible(False)
ax1.spines['right'].set_visible(False)
# ax1.get_legend().remove()
```



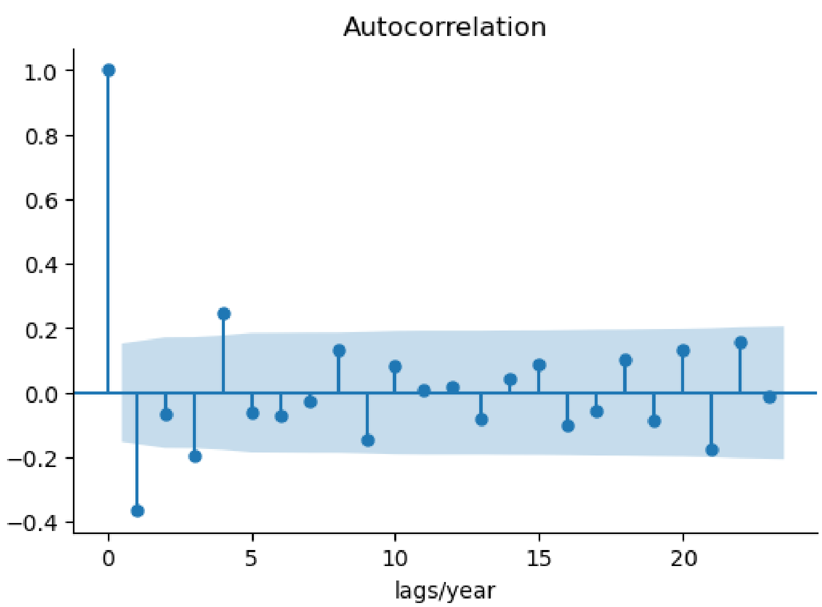
Auto-correlation function (ACF) and partial auto-correlation function (PACF)

In [54]:

```
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
```

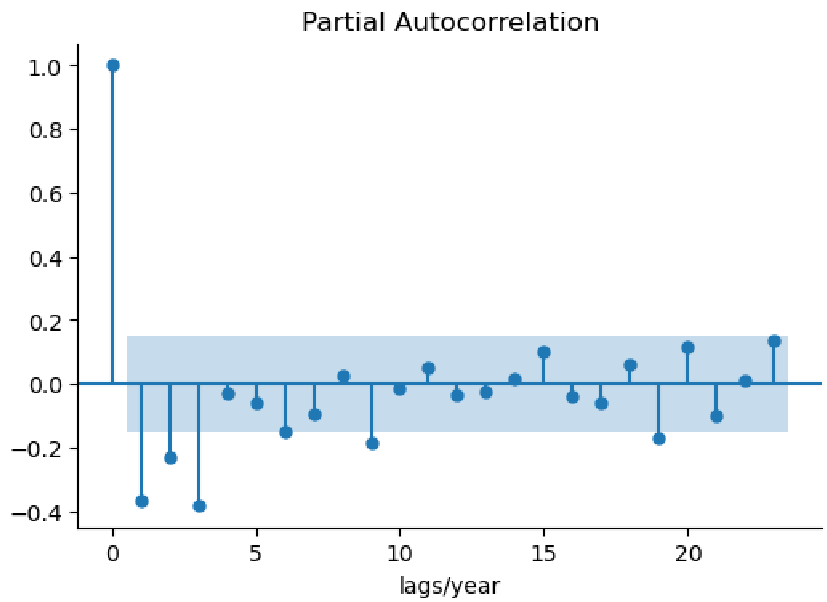
In [56]:

```
"""# Auto-correlations"""
plt.figure(dpi=100)
plot_acf(temperatures_yearly_first_diff['LandAverageTemperature'], ax = plt.gca());
ax1 = plt.gca()
ax1.set_xlabel('lags/year')
ax1.spines['top'].set_visible(False)
ax1.spines['right'].set_visible(False)
# plt.savefig(r'plots/LandAverageTemperature_year_first_diff_ACF', dpi=150)
```



In [57]:

```
"""# Partial auto-correlations"""
plt.figure(dpi=100)
plot_pacf(temperatures_yearly_first_diff['LandAverageTemperature'], ax = plt.gca());
ax1 = plt.gca()
ax1.set_xlabel('lags/year')
ax1.spines['top'].set_visible(False)
ax1.spines['right'].set_visible(False)
# plt.savefig(r'plots/LandAverageTemperature_year_first_diff_PACF', dpi=150)
```



The decay in ACF, and the PACF, suggest AR(3).

Train-validation (initial) split

In [20]:
```
# temperatures_yearly_first_diff.index[0]
# Timestamp('1851-12-31 00:00:00', freq='A-DEC')
# temperatures_yearly_first_diff.index[-1]
# Timestamp('2015-12-31 00:00:00', freq='A-DEC')

"""split at 1986; 1851-1985 is training set, 1986-2015 is validation set"""
split_time = pd.Timestamp(1986,6,30)
# temperatures_yearly_first_diff.index[136]
# Timestamp('1986-12-31 00:00:00', freq='A-DEC')
# 136


temperatures_1850_all_yearly_train = temperatures_1850_all_yearly[temperatures_1850_all_yearly.index < split_time
temperatures_yearly_first_diff_train = temperatures_yearly_first_diff[temperatures_yearly_first_diff.index < spli

temperatures_1850_all_yearly_valid = temperatures_1850_all_yearly[temperatures_1850_all_yearly.index > split_time
temperatures_yearly_first_diff_valid = temperatures_yearly_first_diff[temperatures_yearly_first_diff.index > spli
```

Naive forecase (lag 1)

In [22]:
```
temperature_yearly_naive = [np.nan] + temperatures_1850_all_yearly['LandAverageTemperature'].to_list()[:-1]
```

In [16]:
```
from sklearn.metrics import mean_absolute_error
```

In [23]:
```
mean_absolute_error(temperatures_1850_all_yearly_valid['LandAverageTemperature'], pd.Series(temperature_yearly_na
```

Out[23]:
```
0.18930555555555575
```

Auto-regressive (AR) models

In [60]:
```
from sklearn.metrics import mean_absolute_error
# ARIMA models
from statsmodels.tsa.arima_model import ARIMA
```

In [61]:
```
"""fit model of the initial fixed partition"""
model1 = ARIMA(temperatures_yearly_first_diff_train['LandAverageTemperature'], order=(3,0,0))
model1_fit = model1.fit()

# summary of the model
print(model1_fit.summary())
```

```
                              ARMA Model Results
==============================================================================
Dep. Variable:     LandAverageTemperature   No. Observations:                  135
Model:                         ARMA(3, 0)   Log Likelihood                  34.518
Method:                           css-mle   S.D. of innovations              0.187
Date:                    Sun, 07 Nov 2021   AIC                            -59.035
Time:                            00:01:57   BIC                            -44.509
Sample:                        12-31-1851   HQIC                           -53.132
                             - 12-31-1985
==================================================================================================
                                   coef    std err          z      P>|z|      [0.025      0.975]
--------------------------------------------------------------------------------------------------
const                            0.0053      0.007      0.749      0.454      -0.009       0.019
ar.L1.LandAverageTemperature    -0.5168      0.079     -6.577      0.000      -0.671      -0.363
ar.L2.LandAverageTemperature    -0.3708      0.085     -4.382      0.000      -0.537      -0.205
ar.L3.LandAverageTemperature    -0.4115      0.079     -5.217      0.000      -0.566      -0.257
                                    Roots
=============================================================================
                  Real          Imaginary           Modulus         Frequency
-----------------------------------------------------------------------------
AR.1           -1.3303           -0.0000j            1.3303           -0.5000
AR.2            0.2146           -1.3345j            1.3516           -0.2246
AR.3            0.2146           +1.3345j            1.3516            0.2246
-----------------------------------------------------------------------------
```

```
C:\ProgramData\Anaconda3\lib\site-packages\statsmodels\tsa\arima_model.py:472: FutureWarning:
statsmodels.tsa.arima_model.ARMA and statsmodels.tsa.arima_model.ARIMA have
been deprecated in favor of statsmodels.tsa.arima.model.ARIMA (note the .
between arima and model) and
statsmodels.tsa.SARIMAX. These will be removed after the 0.12 release.

statsmodels.tsa.arima.model.ARIMA makes use of the statespace framework and
is both well tested and maintained.

To silence this warning and continue using ARMA and ARIMA until they are
removed, use:

import warnings
warnings.filterwarnings('ignore', 'statsmodels.tsa.arima_model.ARMA',
                        FutureWarning)
warnings.filterwarnings('ignore', 'statsmodels.tsa.arima_model.ARIMA',
                        FutureWarning)

  warnings.warn(ARIMA_DEPRECATION_WARN, FutureWarning)
```

Indeed, p-values are small (zero) at lag = 1, 2, 3.

[Fixed window] predictions/forecast

In [63]:
```
"""# in-sample (training set) prediction of first difference """
pred_diff_train1 = model1_fit.predict(start=temperatures_yearly_first_diff_train.index[0], end=temperatures_yearl

# add back the trend
predictions_train1 = temperatures_1850_all_yearly_train['LandAverageTemperature'].shift() + pred_diff_train1
```

```
C:\ProgramData\Anaconda3\lib\site-packages\statsmodels\tsa\base\tsa_model.py:132: FutureWarning: The 'freq' argum
ent in Timestamp is deprecated and will be removed in a future version.
  date_key = Timestamp(key, freq=base_index.freq)
```

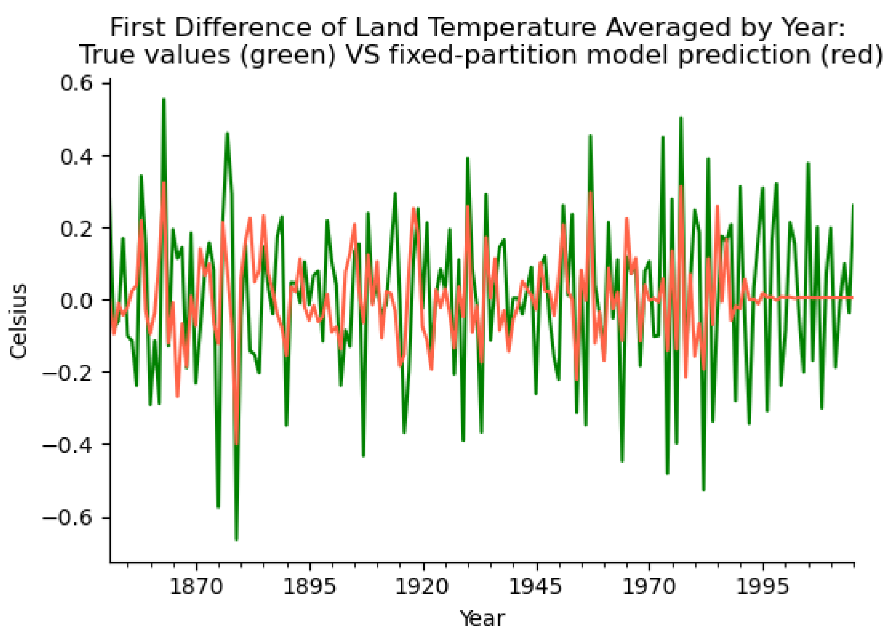
In [64]:
```
"""# mean_absolute_error of the training set """
mean_absolute_error(temperatures_1850_all_yearly_train['LandAverageTemperature'].iloc[1:],predictions_train1.iloc
```

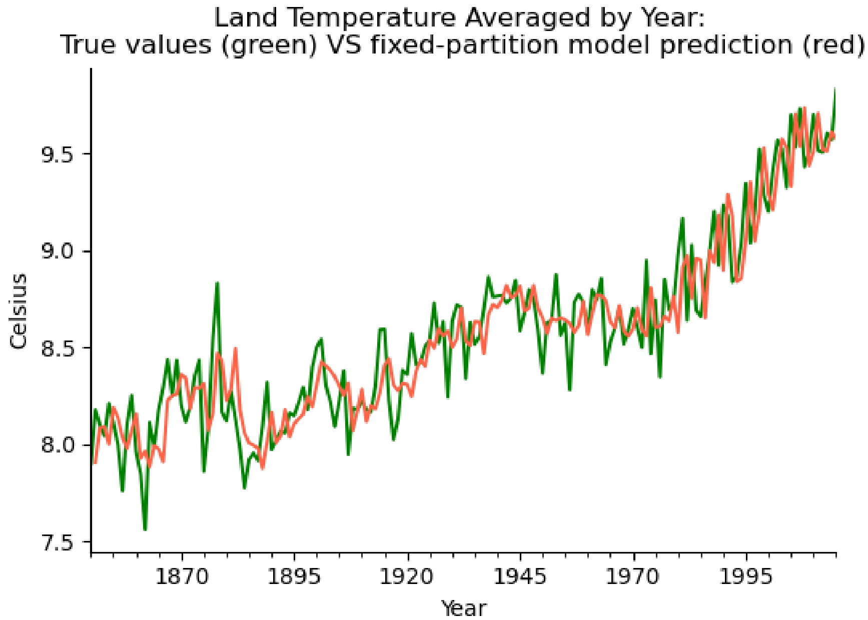
Out[64]:
```
0.14860975236678103
```

[fixed window], out-of-sample forecast of first difference

In [65]:
```
""" [fixed window], out-of-sample forecast of first difference """

forecast_diff1 = model1_fit.forecast(len(temperatures_yearly_first_diff_valid))[0]

fixed_wind_forecast1 = []
for i in range(len(temperatures_yearly_first_diff_valid)):
    if i == 0:
        fixed_wind_forecast1.append(temperatures_1850_all_yearly_train['LandAverageTemperature'][-1] + forecast_d
    else:
        fixed_wind_forecast1.append(temperatures_1850_all_yearly_valid['LandAverageTemperature'][i-1] + forecast_

fixed_wind_forecast1 = pd.Series(fixed_wind_forecast1, index = temperatures_1850_all_yearly_valid.index)
```

In [76]:
```
plt.figure(dpi=100)
temperatures_yearly_first_diff['LandAverageTemperature'].plot(ax=plt.gca())
pd.concat([pred_diff_train1, pd.Series(forecast_diff1, index = temperatures_yearly_first_diff_valid.index)], axis
ax1 = plt.gca()
ax1.get_lines()[0].set_color("green")
ax1.get_lines()[1].set_color("tomato")
plt.title('First Difference of Land Temperature Averaged by Year: \nTrue values (green) VS fixed-partition model
ax1.set_xlabel('Year')
ax1.set_ylabel('Celsius')
ax1.spines['top'].set_visible(False)
ax1.spines['right'].set_visible(False)
# plt.savefig(r'plots/LandAverageTemperature_year_AR3_fixed_partition_first_diff', dpi=150)


plt.figure(dpi=100)
temperatures_1850_all_yearly['LandAverageTemperature'].plot(ax=plt.gca())
pd.concat([predictions_train1, fixed_wind_forecast1], axis = 0).plot(ax=plt.gca())
ax1 = plt.gca()
ax1.get_lines()[0].set_color("green")
ax1.get_lines()[1].set_color("tomato")
plt.title('Land Temperature Averaged by Year: \nTrue values (green) VS fixed-partition model prediction (red)')
ax1.set_xlabel('Year')
ax1.set_ylabel('Celsius')
ax1.spines['top'].set_visible(False)
ax1.spines['right'].set_visible(False)
# ax1.get_legend().get_texts()[0].set_text('AR(3) fixed partition forecast')
# ax1.get_legend().get_texts()[1].set_text('True values')
# plt.savefig(r'plots/LandAverageTemperature_year_AR3_fixed_partition', dpi=150)
```





Validation MAE

In [67]:
```
print(mean_absolute_error(temperatures_yearly_first_diff_valid['LandAverageTemperature'], pd.Series(forecast_diff
print(mean_absolute_error(temperatures_1850_all_yearly_valid['LandAverageTemperature'], fixed_wind_forecast1))
```

```
0.18836564357881894
0.1883656435788189
```

The forecast first difference of the fixed-partitioning model is basically zero; this reduces to a naive forecast by the value of the previous time-step.

Rolling forecast

Walk-forward validation

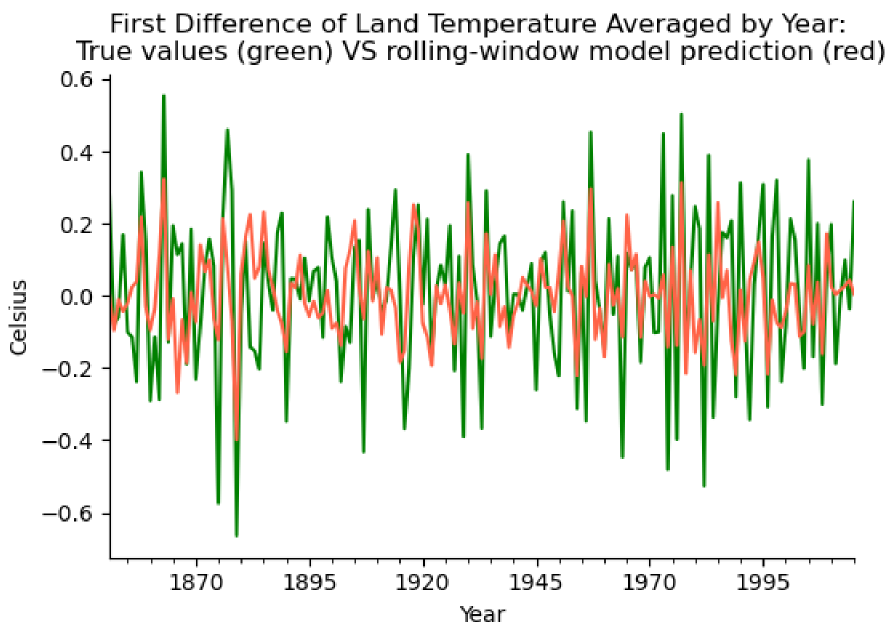
In [ ]:
```
size = 136
# temperatures_1850_all_yearly.index[136]
# Out[159]: Timestamp('1986-12-31 00:00:00', freq='A-DEC')
history1 = temperatures_yearly_first_diff_train['LandAverageTemperature'].to_list()
rolling_forecast_first_diff1 = []


for t in range(len(temperatures_1850_all_yearly_valid)):
        model = ARIMA(history1, order=(3,0,0))
        model_fit = model.fit()
        output = model_fit.forecast() # outputs an array of 1 element
        yhat = output[0][0] # the forecast one time step ahead
        rolling_forecast_first_diff1.append(yhat)
        obs = temperatures_yearly_first_diff_valid['LandAverageTemperature'].iloc[t]
        history1.append(obs) # append value of current time step for next model
```

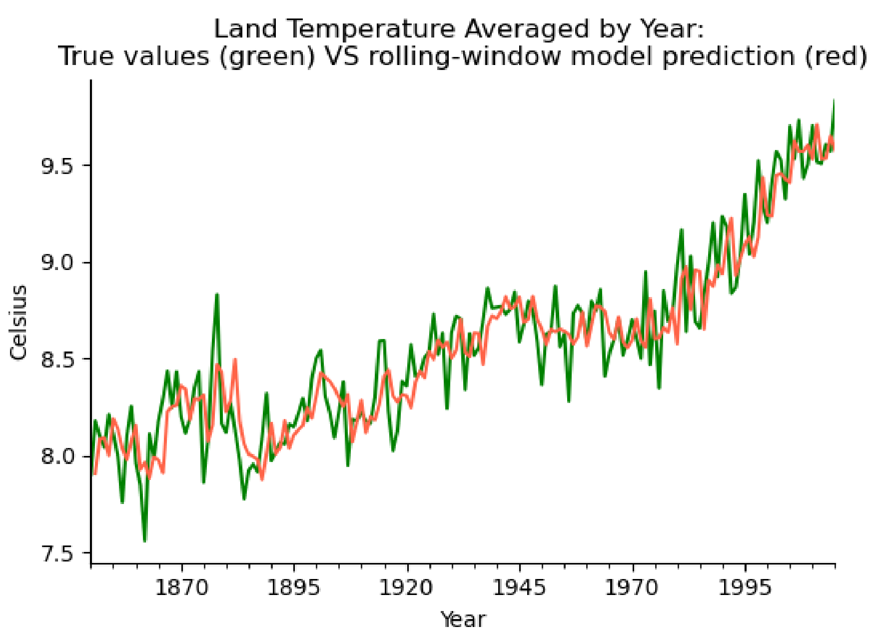
[rolling window], out-of-sample forecast from first difference

In [78]:
```
""" [rolling window], out-of-sample forecast of first difference """

rolling_forecast1 = []
for i in range(len(temperatures_yearly_first_diff_valid)):
    if i == 0:
        rolling_forecast1.append(temperatures_1850_all_yearly_train['LandAverageTemperature'][-1] + rolling_forec
    else:
        rolling_forecast1.append(temperatures_1850_all_yearly_valid['LandAverageTemperature'][i-1] + rolling_fore

rolling_forecast1 = pd.Series(rolling_forecast1, index = temperatures_1850_all_yearly_valid.index)
```

In [80]:
```
plt.figure(dpi=100)
```

```
temperatures_yearly_first_diff['LandAverageTemperature'].plot(ax=plt.gca())
pd.concat([pred_diff_train1, pd.Series(rolling_forecast_first_diff1, index = temperatures_yearly_first_diff_valid
ax1 = plt.gca()
ax1.get_lines()[0].set_color("green")
ax1.get_lines()[1].set_color("tomato")
plt.title('First Difference of Land Temperature Averaged by Year: \nTrue values (green) VS rolling-window model p
ax1.set_xlabel('Year')
ax1.set_ylabel('Celsius')
ax1.spines['top'].set_visible(False)
ax1.spines['right'].set_visible(False)
# plt.savefig(r'plots/LandAverageTemperature_year_AR3_rolling_forecast_first_diff', dpi=150)


plt.figure(dpi=100)
temperatures_1850_all_yearly['LandAverageTemperature'].plot(ax=plt.gca())
pd.concat([predictions_train1, rolling_forecast1], axis = 0).plot(ax=plt.gca())
ax1 = plt.gca()
ax1.get_lines()[0].set_color("green")
ax1.get_lines()[1].set_color("tomato")
plt.title('Land Temperature Averaged by Year: \nTrue values (green) VS rolling-window model prediction (red)')
ax1.set_xlabel('Year')
ax1.set_ylabel('Celsius')
ax1.spines['top'].set_visible(False)
ax1.spines['right'].set_visible(False)
```





Validation MAE

In [79]:
```
print(mean_absolute_error(temperatures_yearly_first_diff_valid['LandAverageTemperature'], pd.Series(rolling_forec
)
```

```
0.15548113397624228
```

Smaller error indeed, compared to fixed-partition forecast.

Summary of Results

Validation (data from 1986 onward) MAE:

Naive forecast (lag 1): 0.18930555555555575

AR(3) fixed partitioning: 0.1883656435788189

AR(3) rolling forecast: 0.15548113397624228

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]: