Chapter 11
NON-RIGHT ANGLED TRIANGLE TRIGONOMETRY

EXERCISE 11A

1 **a** area
$= \frac{1}{2} \times 9 \times 10 \times \sin 40°$
$\approx 28.9\ \text{cm}^2$

b area
$= \frac{1}{2} \times 25 \times 31 \times \sin 82°$
$\approx 384\ \text{km}^2$

c area
$= \frac{1}{2} \times 10.2 \times 6.4 \times \sin(\frac{2\pi}{3})$
$\approx 28.3\ \text{cm}^2$

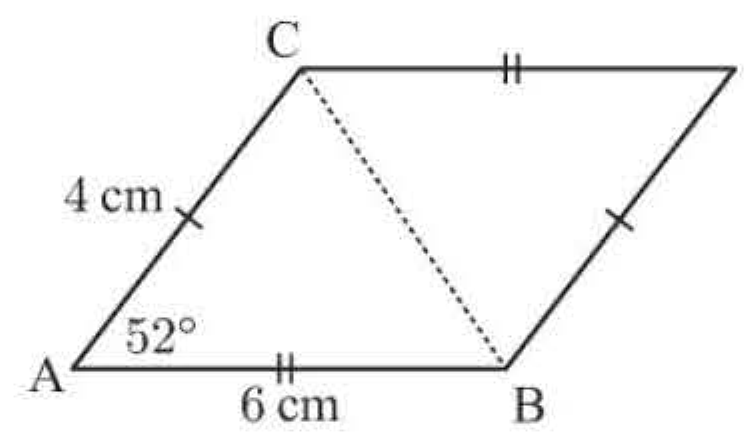
2 area $= 150\ \text{cm}^2$

$\therefore\ \frac{1}{2} \times 17 \times x \times \sin 68° = 150$

$\therefore\ x = \dfrac{2 \times 150}{17 \times \sin 68°}$

$\therefore\ x \approx 19.0$

3



area $= 2 \times$ area $\triangle$ABC
$= 2 \times \frac{1}{2} \times 4 \times 6 \times \sin 52°$
$\approx 18.9\ \text{cm}^2$

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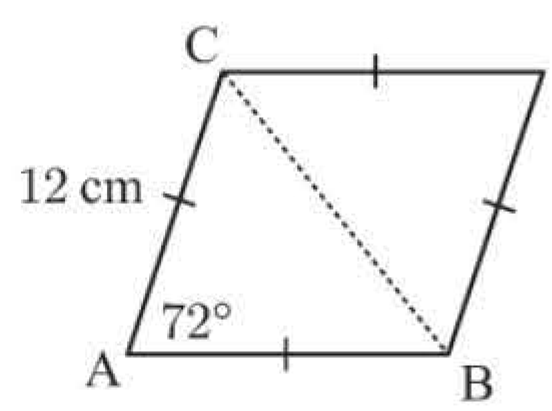

area $= 2 \times$ area $\triangle$ABC
$= 2 \times \frac{1}{2} \times 12^2 \times \sin 72°$
$\approx 137\ \text{cm}^2$

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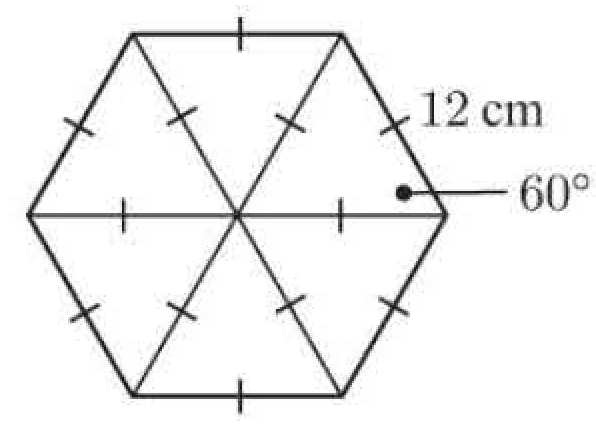

area $= 6 \times$ area of $\triangle$
$= 6 \times \frac{1}{2} \times 12^2 \times \sin 60°$
$\approx 374\ \text{cm}^2$

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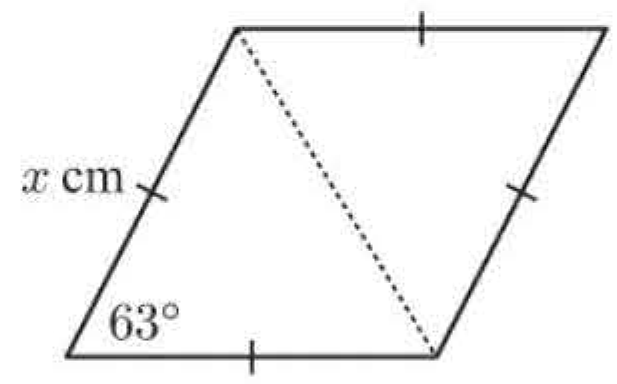


area $= 2 \times \frac{1}{2}x^2 \sin 63°$

$\therefore\ x^2 \sin 63° = 50$

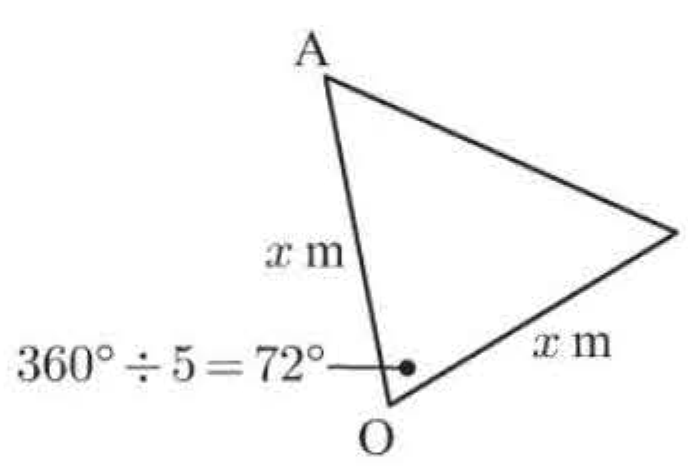
$\therefore\ x^2 = \dfrac{50}{\sin 63°}$

$\therefore\ x = \sqrt{\dfrac{50}{\sin 63°}}\quad \{x > 0\}$

$\therefore\ x \approx 7.49$

So, sides are 7.49 cm long.

7



area of $\triangle = \frac{338}{5}$

$\therefore\ \frac{1}{2}x^2 \sin 72° = \frac{338}{5}$

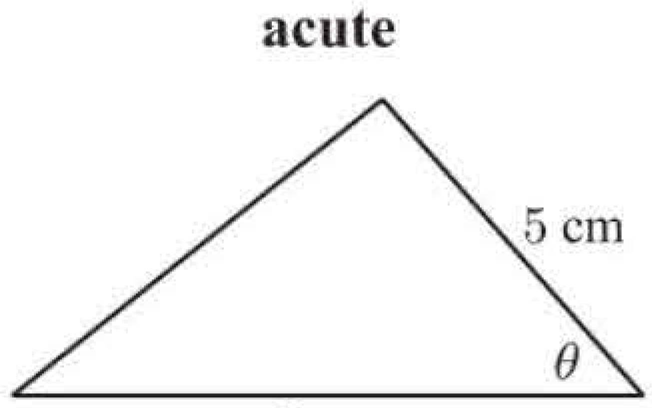
$\therefore\ x^2 = \dfrac{2 \times 338}{5 \times \sin 72°}$

$\therefore\ x = \sqrt{\dfrac{2 \times 338}{5 \times \sin 72°}}\quad \{x > 0\}$

$\therefore\ x \approx 11.9$

So, OA ≈ 11.9 m

8 **a** **acute**



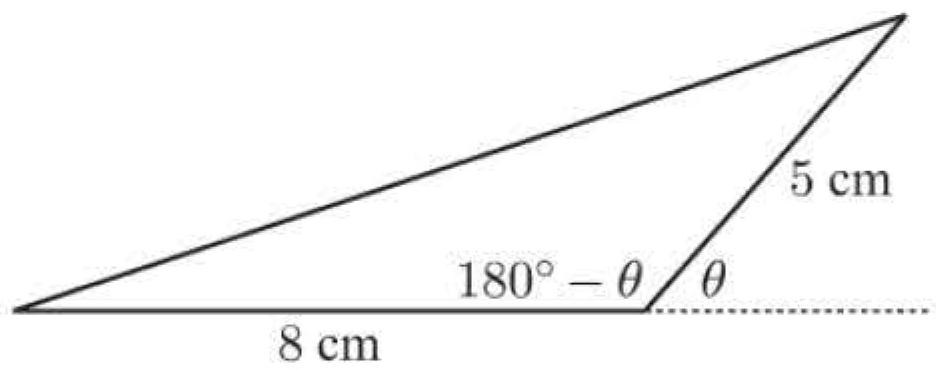
area $= \frac{1}{2} \times 5 \times 8 \times \sin\theta$

$\therefore\ 15 = 20 \sin\theta$

$\therefore\ \sin\theta = \frac{3}{4}$

$\therefore\ \theta = \sin^{-1}(\frac{3}{4})$
$\approx 48.6°$

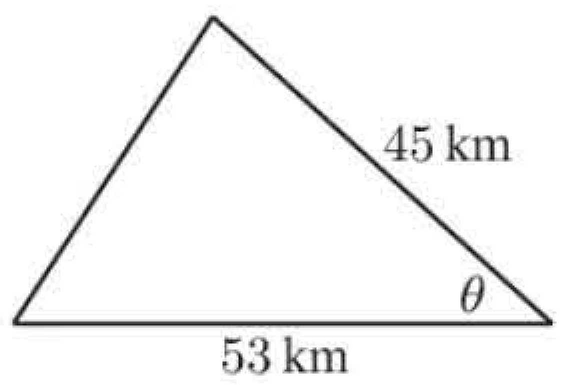
or **obtuse**



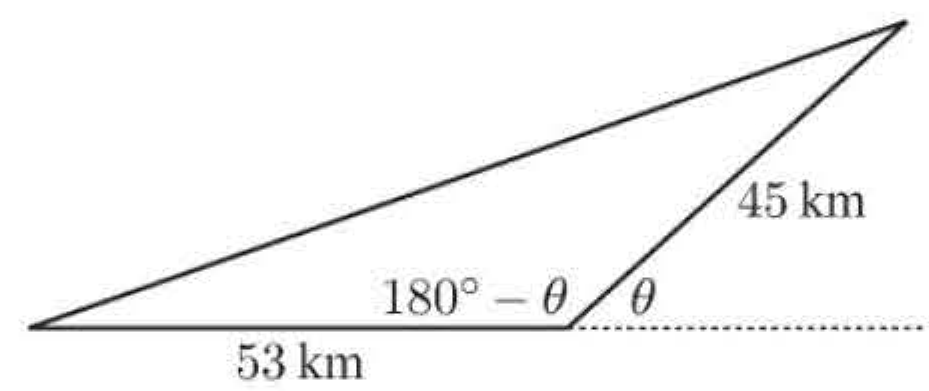
also $180° - \theta \approx 180° - 48.6° \approx 131.4°$

So, if the included angle is acute then its value is $\approx 48.6°$, otherwise if the included angle is obtuse then its value is $\approx 131.4°$.

b **acute**



or **obtuse**

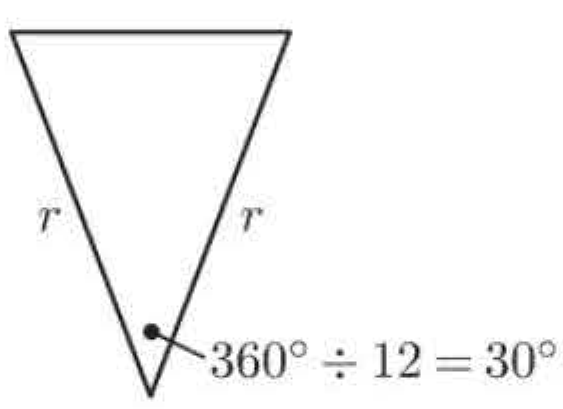


$$\text{area} = \tfrac{1}{2} \times 45 \times 53 \times \sin\theta$$
$$\therefore\ 800 = \frac{45 \times 53}{2} \times \sin\theta$$
$$\therefore\ \sin\theta = \frac{2 \times 800}{45 \times 53}$$
$$\therefore\ \theta = \sin^{-1}\left(\frac{2 \times 800}{45 \times 53}\right) \approx 42.1°$$

also $180° - \theta \approx 180° - 42.1° \approx 137.9°$

So, if the included angle is acute then its value is $\approx 42.1°$, otherwise if the included angle is obtuse then its value is $\approx 137.9°$.

9



total area of 8 coins
$= 8 \times 12 \times \frac{1}{2}r^2 \sin 30°$
$= 48r^2(\frac{1}{2})$
$= 24r^2$

area of \$10 note
$= 8r \times 4r$
$= 32r^2$

fraction covered
$= \dfrac{24r^2}{32r^2}$
$= \frac{3}{4}$ $\therefore$ $\frac{1}{4}$ is not covered

10 **a** shaded area
$= \text{area of sector} - \text{area of triangle}$
$= \frac{1}{2} \times 1.5 \times 12^2 - \frac{1}{2} \times 12 \times 12 \times \sin(1.5^c)$
$\approx 36.2\ \text{cm}^2$

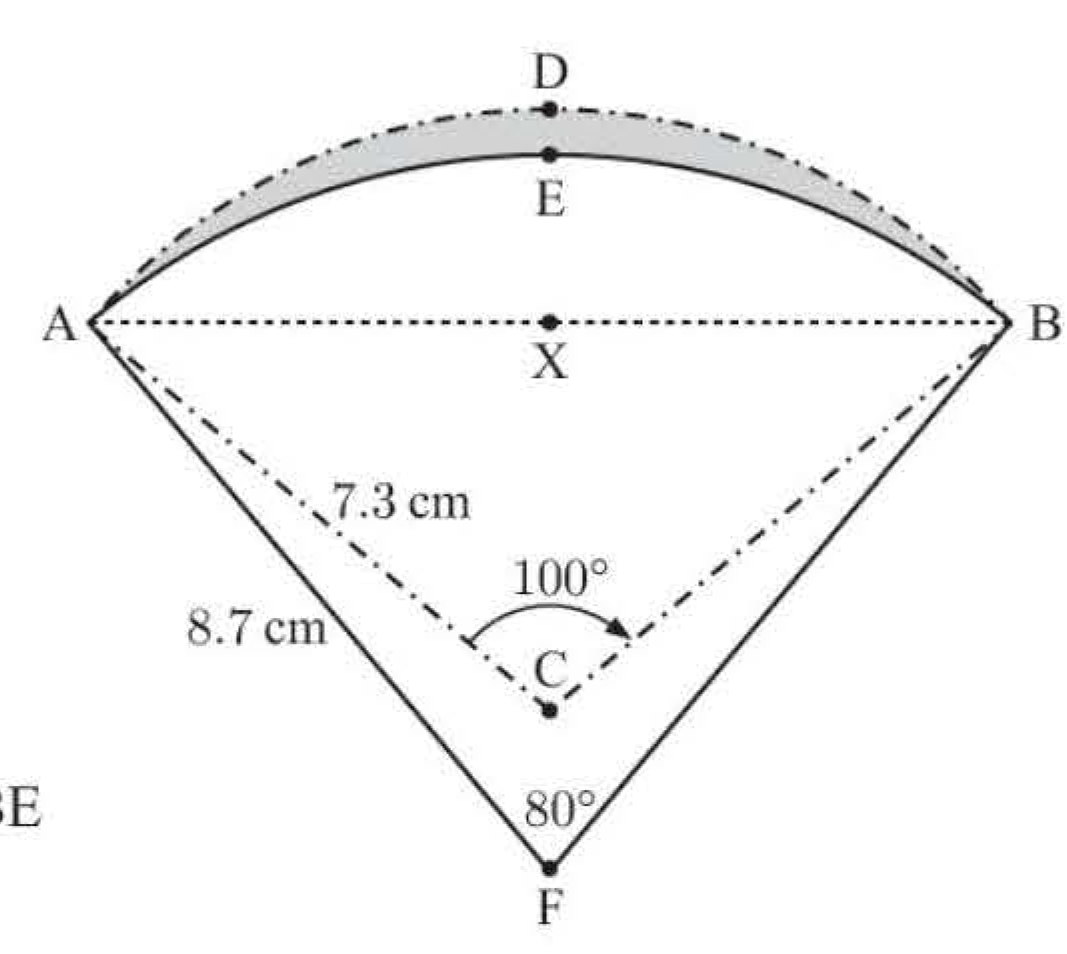
b shaded area
$= \text{area of triangle} - \text{area of sector}$
$= \frac{1}{2} \times 12 \times 30 \times \sin(0.66^c) - \frac{1}{2} \times 0.66 \times 12^2$
$\approx 62.8\ \text{cm}^2$

c shaded area
$= \text{area of sector} - \text{area of triangle}$
$= \left(\frac{135}{360}\right) \times \pi \times 7^2 - \frac{1}{2} \times 7 \times 7 \times \sin 135°$
$\approx 40.4\ \text{mm}^2$

11 area segment AXBD
$= \text{area sector ACBD} - \text{area } \triangle\text{ACB}$
$= \left(\frac{100}{360}\right) \times \pi \times 7.3^2 - \frac{1}{2} \times 7.3 \times 7.3 \times \sin 100°$
$\approx 20.264\ \text{cm}^2$

area segment AXBE
$= \text{area sector AFBE} - \text{area } \triangle\text{AFB}$
$= \left(\frac{80}{360}\right) \times \pi \times 8.7^2 - \frac{1}{2} \times 8.7 \times 8.7 \times \sin 80°$
$\approx 15.572\ \text{cm}^2$

$\therefore$ shaded area $=$ area segment AXBD $-$ area segment AXBE
$\approx 20.264 - 15.572 \approx 4.69\ \text{cm}^2$



EXERCISE 11B

1 **a** $\text{BC}^2 = 21^2 + 15^2 - 2 \times 21 \times 15 \times \cos 105°$

$\therefore \quad \text{BC} = \sqrt{21^2 + 15^2 - 2 \times 21 \times 15 \times \cos 105°} \approx 28.8 \text{ cm}$

b $\text{PQ}^2 = 6.3^2 + 4.8^2 - 2 \times 6.3 \times 4.8 \times \cos 32°$

$\therefore \quad \text{PQ} = \sqrt{6.3^2 + 4.8^2 - 2 \times 6.3 \times 4.8 \times \cos 32°} \approx 3.38 \text{ km}$

c $\text{KM}^2 = 6.2^2 + 14.8^2 - 2 \times 6.2 \times 14.8 \times \cos 72°$

$\therefore \quad \text{KM} = \sqrt{6.2^2 + 14.8^2 - 2 \times 6.2 \times 14.8 \times \cos 72°} \approx 14.2 \text{ m}$

2 $\cos \text{B}\widehat{\text{A}}\text{C} = \dfrac{12^2 + 13^2 - 11^2}{2 \times 12 \times 13}$

$\therefore \quad \text{B}\widehat{\text{A}}\text{C} = \cos^{-1}\left(\frac{192}{312}\right)$

$\therefore \quad \text{B}\widehat{\text{A}}\text{C} \approx 52.0°$

$\cos \text{A}\widehat{\text{B}}\text{C} = \dfrac{13^2 + 11^2 - 12^2}{2 \times 13 \times 11}$

$\therefore \quad \text{A}\widehat{\text{B}}\text{C} = \cos^{-1}\left(\frac{146}{286}\right)$

$\therefore \quad \text{A}\widehat{\text{B}}\text{C} \approx 59.3°$

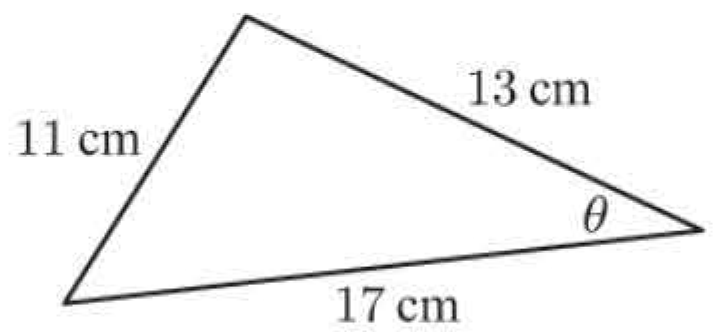
$\text{A}\widehat{\text{C}}\text{B} = 180° - \text{B}\widehat{\text{A}}\text{C} - \text{A}\widehat{\text{B}}\text{C}$
$\approx 180° - 52.0° - 59.3°$
$\approx 68.7°$

3 **a** $\cos \text{P}\widehat{\text{Q}}\text{R} = \dfrac{5^2 + 7^2 - 10^2}{2 \times 5 \times 7}$

$\therefore \quad \text{P}\widehat{\text{Q}}\text{R} = \cos^{-1}\left(\frac{-26}{70}\right) \approx 112°$

b $\text{area} \approx \frac{1}{2} \times 5 \times 7 \times \sin 112°$
$\approx 16.2 \text{ cm}^2$

4 **a**



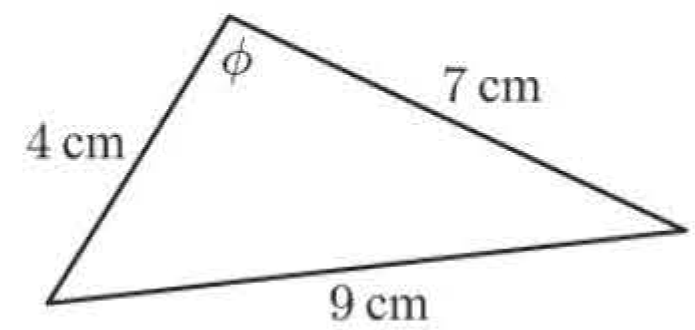
The smallest angle is opposite the shortest side.

$\cos \theta = \dfrac{13^2 + 17^2 - 11^2}{2 \times 13 \times 17}$

$\therefore \quad \theta = \cos^{-1}\left(\frac{337}{442}\right) \approx 40.3°$

So, the smallest angle measures $40.3°$.

b



The largest angle is opposite the longest side.

$\cos \phi = \dfrac{4^2 + 7^2 - 9^2}{2 \times 4 \times 7}$

$\therefore \quad \phi = \cos^{-1}\left(-\frac{16}{56}\right) \approx 106.60°$

So, the largest angle measures about $107°$.

5 **a** $\cos \theta = \dfrac{2^2 + 5^2 - 4^2}{2 \times 2 \times 5}$
$= \frac{13}{20}$
$= 0.65$

b $x^2 = 5^2 + 3^2 - 2 \times 5 \times 3 \times \cos \theta$

$\therefore \quad x = \sqrt{5^2 + 3^2 - 2 \times 5 \times 3 \times 0.65}$

$\therefore \quad x \approx 3.81$

6 **a** $7^2 = x^2 + 6^2 - 2 \times x \times 6 \times \cos 60°$

$\therefore \quad 49 = x^2 + 36 - 12x \times \left(\frac{1}{2}\right)$

$\therefore \quad x^2 - 6x - 13 = 0$

$\therefore \quad x = \dfrac{6 \pm \sqrt{36 - 4(1)(-13)}}{2}$
$= \dfrac{6 \pm \sqrt{88}}{2}$
$= 3 \pm \sqrt{22}$

But $x > 0$, so $x = 3 + \sqrt{22}$

b $5^2 = x^2 + 3^2 - 2 \times x \times 3 \times \cos 120°$

$\therefore \quad 25 = x^2 + 9 - 6x \times \left(-\frac{1}{2}\right)$

$\therefore \quad x^2 + 3x - 16 = 0$

$\therefore \quad x = \dfrac{-3 \pm \sqrt{9 - 4(1)(-16)}}{2}$
$= \dfrac{-3 \pm \sqrt{73}}{2}$

But $x > 0$, so $x = \dfrac{-3 + \sqrt{73}}{2}$

c $5^2 = (2x)^2 + x^2 - 2 \times (2x) \times x \times \cos 60°$

$\therefore \quad 25 = 4x^2 + x^2 - 4x^2\left(\frac{1}{2}\right)$

$\therefore \quad 3x^2 = 25$

$\therefore \quad x^2 = \frac{25}{3}$

$\therefore \quad x = \pm\frac{5}{\sqrt{3}}$

But $x > 0$, so $x = \frac{5}{\sqrt{3}}$

7 **a** Area $= 11.6\text{ m}^2$

$\therefore\ 11.6 = \frac{1}{2} \times 6 \times 4 \times \sin\theta$

$\therefore\ \sin\theta = \frac{29}{30}$

$\therefore\ \theta = \sin^{-1}\left(\frac{29}{30}\right)$

$\therefore\ \theta \approx 75.2°$

b Let the third side have length x m.

By the cosine rule,

$x^2 = 6^2 + 4^2 - 2 \times 6 \times 4 \times \cos 75.2°$

$\therefore\ x = \sqrt{6^2 + 4^2 - 2 \times 6 \times 4 \times \cos 75.2°}$

$\therefore\ x \approx 6.30$

The third side has length 6.30 m.

8 **a** $11^2 = x^2 + 8^2 - 2 \times x \times 8 \times \cos 70°$

$\therefore\ 121 = x^2 + 64 - 16x\cos 70°$

$\therefore\ x^2 - (16\cos 70°)x - 57 = 0$

Using the quadratic formula or technology,

$x \approx -5.29$ or 10.8.

But $x > 0$, so $x \approx 10.8$.

b $13^2 = x^2 + 5^2 - 2 \times x \times 5 \times \cos 130°$

$\therefore\ 169 = x^2 + 25 - 10x\cos 130°$

$\therefore\ x^2 - (10\cos 130°)x - 144 = 0$

Using the quadratic formula or technology,

$x \approx -15.6$ or 9.21.

But $x > 0$, so $x \approx 9.21$.

9 $5^2 = x^2 + 6^2 - 2 \times x \times 6 \times \cos 40°$

$\therefore\ 25 = x^2 + 36 - 12x\cos 40°$

$\therefore\ x^2 - (12\cos 40°)x + 11 = 0$

Using the quadratic formula or technology, $x \approx 1.41$ or 7.78.

10 **a** $(3x+1)^2 = (x+2)^2 + (x+3)^2 - 2(x+2)(x+3)\cos\theta$

$\therefore\ 9x^2 + 6x + 1 = x^2 + 4x + 4 + x^2 + 6x + 9 - 2(x^2 + 5x + 6)(-\frac{1}{5})$

$\therefore\ 9x^2 + 6x + 1 = 2x^2 + 10x + 13 + \frac{2}{5}x^2 + 2x + \frac{12}{5}$

$\therefore\ \frac{33}{5}x^2 - 6x - \frac{72}{5} = 0$

$\therefore\ 33x^2 - 30x - 72 = 0$

$\therefore\ 3(11x + 12)(x - 2) = 0$

$\therefore\ x = -\frac{12}{11}$ or 2

But $3x + 1 > 0$, so $x = 2$

b $\cos^2\theta + \sin^2\theta = 1$

$\therefore\ \frac{1}{25} + \sin^2\theta = 1$

$\therefore\ \sin^2\theta = \frac{24}{25}$

$\therefore\ \sin\theta = \pm\frac{\sqrt{24}}{5}$

But $0° < \theta < 180°$, so $\sin\theta > 0$

$\therefore\ \sin\theta = \frac{\sqrt{24}}{5}$

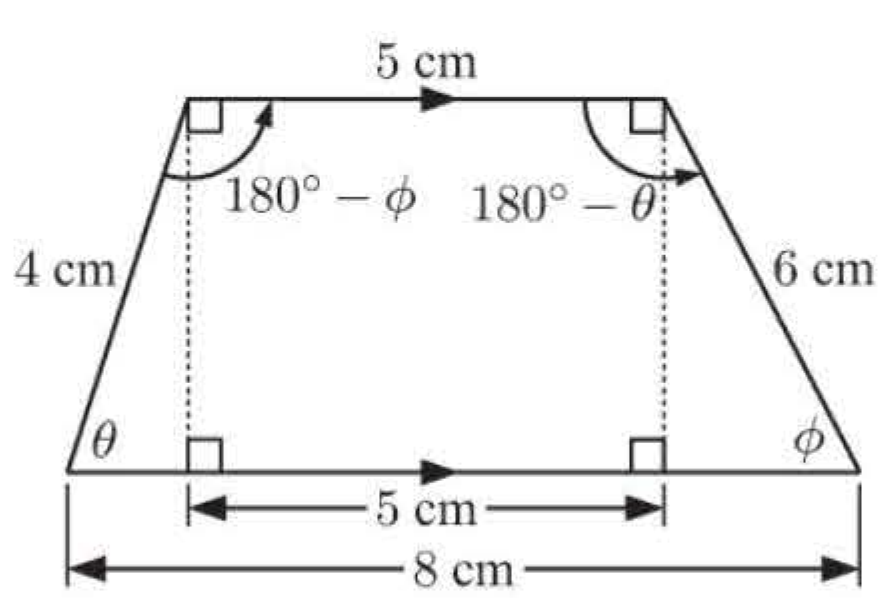
$\therefore$ area $= \frac{1}{2} \times (x+2) \times (x+3) \times \sin\theta$

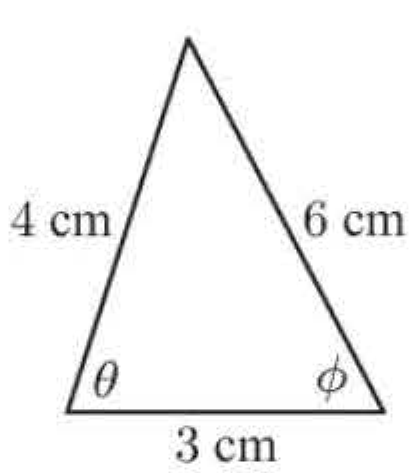
$= \frac{1}{2} \times 4 \times 5 \times \frac{\sqrt{24}}{5}$

$= 2\sqrt{24}$

$= 4\sqrt{6}\text{ cm}^2$

11





$\cos\theta = \dfrac{4^2 + 3^2 - 6^2}{2 \times 3 \times 4}$

$\therefore\ \theta = \cos^{-1}\left(\dfrac{16 + 9 - 36}{24}\right)$

$= \cos^{-1}(-\frac{11}{24})$

$\approx 117.3°$

$\cos\phi = \dfrac{3^2 + 6^2 - 4^2}{2 \times 3 \times 6}$

$\therefore\ \phi = \cos^{-1}\left(\dfrac{9 + 36 - 16}{36}\right)$

$= \cos^{-1}(\frac{29}{36})$

$\approx 36.3°$

$\therefore\ 180° - \theta \approx 180° - 117.3° \approx 62.7°$ and $180° - \phi \approx 180° - 36.3° \approx 143.7°$

So, the angles are $36.3°$, $62.7°$, $117.3°$, and $143.7°$.

EXERCISE 11C.1

1 **a** By the sine rule,

$$\frac{x}{\sin 48^\circ} = \frac{23}{\sin 37^\circ}$$
$$\therefore \quad x = \frac{23 \times \sin 48^\circ}{\sin 37^\circ}$$
$$\therefore \quad x \approx 28.4$$

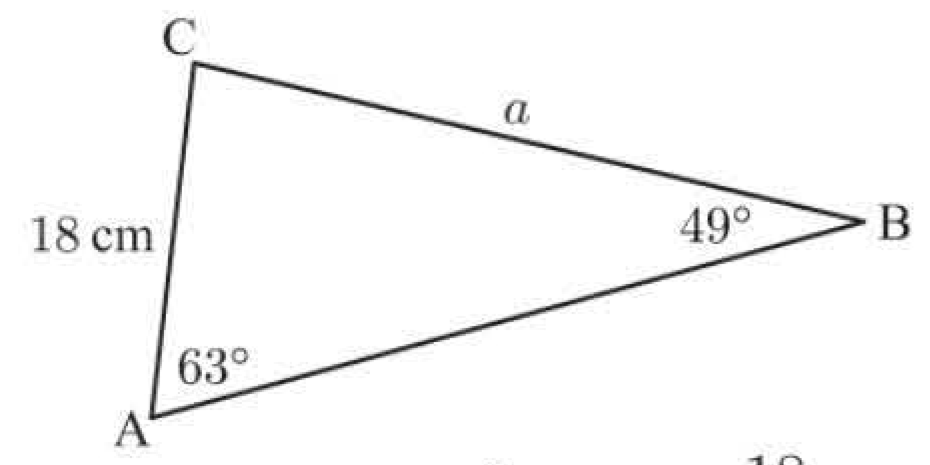
b By the sine rule,

$$\frac{x}{\sin 115^\circ} = \frac{11}{\sin 48^\circ}$$
$$\therefore \quad x = \frac{11 \times \sin 115^\circ}{\sin 48^\circ}$$
$$\therefore \quad x \approx 13.4$$

c By the sine rule,

$$\frac{x}{\sin 51^\circ} = \frac{4.8}{\sin 80^\circ}$$
$$\therefore \quad x = \frac{4.8 \times \sin 51^\circ}{\sin 80^\circ}$$
$$\therefore \quad x \approx 3.79$$

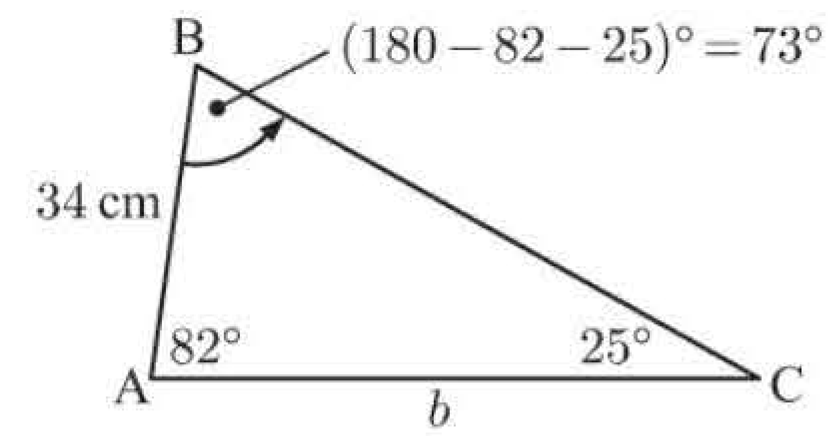
2 **a**



By the sine rule, $\dfrac{a}{\sin 63^\circ} = \dfrac{18}{\sin 49^\circ}$

$$\therefore \quad a = \frac{18 \times \sin 63^\circ}{\sin 49^\circ}$$
$$\therefore \quad a \approx 21.3 \text{ cm}$$

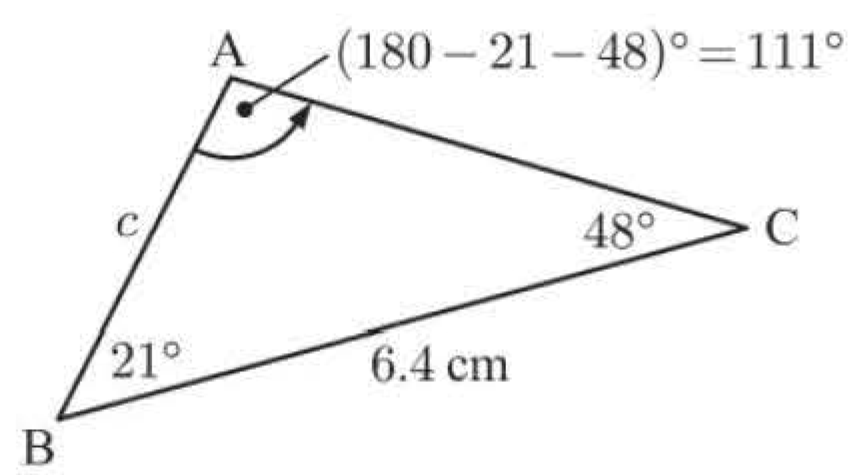
b



By the sine rule, $\dfrac{b}{\sin 73^\circ} = \dfrac{34}{\sin 25^\circ}$

$$\therefore \quad b = \frac{34 \times \sin 73^\circ}{\sin 25^\circ}$$
$$\therefore \quad b \approx 76.9 \text{ cm}$$

c



By the sine rule, $\dfrac{c}{\sin 48^\circ} = \dfrac{6.4}{\sin 111^\circ}$

$$\therefore \quad c = \frac{6.4 \times \sin 48^\circ}{\sin 111^\circ}$$
$$\therefore \quad c \approx 5.09 \text{ cm}$$

EXERCISE 11C.2

1 By the sine rule, $\dfrac{\sin C}{11} = \dfrac{\sin 40^\circ}{8}$

$$\therefore \quad \sin C = \frac{11 \times \sin 40^\circ}{8}$$
$$\therefore \quad C = \sin^{-1}\left(\frac{11 \times \sin 40^\circ}{8}\right) \quad \text{or its supplement}$$
$$\therefore \quad C \approx 62.1^\circ \quad \text{or} \quad (180 - 62.1)^\circ$$
$$\therefore \quad C \approx 62.1^\circ \quad \text{or} \quad 117.9^\circ$$

2 **a**

$$\frac{\sin \widehat{BAC}}{a} = \frac{\sin \widehat{ABC}}{b}$$
$$\therefore \quad \sin \widehat{BAC} = \frac{14.6 \times \sin 65^\circ}{17.4}$$
$$\therefore \quad \widehat{BAC} = \sin^{-1}\left(\frac{14.6 \times \sin 65^\circ}{17.4}\right)$$

or its supplement

$$\therefore \quad \widehat{BAC} \approx 49.5^\circ \quad \text{or} \quad 180^\circ - 49.5^\circ$$
$$\therefore \quad \widehat{BAC} \approx 49.5^\circ \quad \text{or} \quad 130.5^\circ$$

Check: $\widehat{BAC} = 130.5^\circ$ is impossible as $\widehat{BAC} + \widehat{ABC} = 130.5^\circ + 65^\circ$ is already over 180°. $\therefore \widehat{BAC} \approx 49.5^\circ$

b

$$\frac{\sin \widehat{ABC}}{43.8} = \frac{\sin 43^\circ}{31.4}$$
$$\therefore \quad \sin \widehat{ABC} = \frac{43.8 \times \sin 43^\circ}{31.4}$$
$$\therefore \quad \widehat{ABC} = \sin^{-1}\left(\frac{43.8 \times \sin 43^\circ}{31.4}\right)$$

or its supplement

$$\therefore \quad \widehat{ABC} \approx 72.0^\circ \quad \text{or} \quad 108^\circ$$

both of which are possible as $108 + 43 = 151$ which is < 180.

c $\dfrac{\sin \widehat{ACB}}{4.8} = \dfrac{\sin 71°}{6.5}$

$\therefore \sin \widehat{ACB} = \dfrac{4.8 \times \sin 71°}{6.5}$

$\therefore \widehat{ACB} = \sin^{-1}\left(\dfrac{4.8 \times \sin 71°}{6.5}\right)$ or its supplement

$\therefore \widehat{ACB} \approx 44.3°$ or $135.7°$

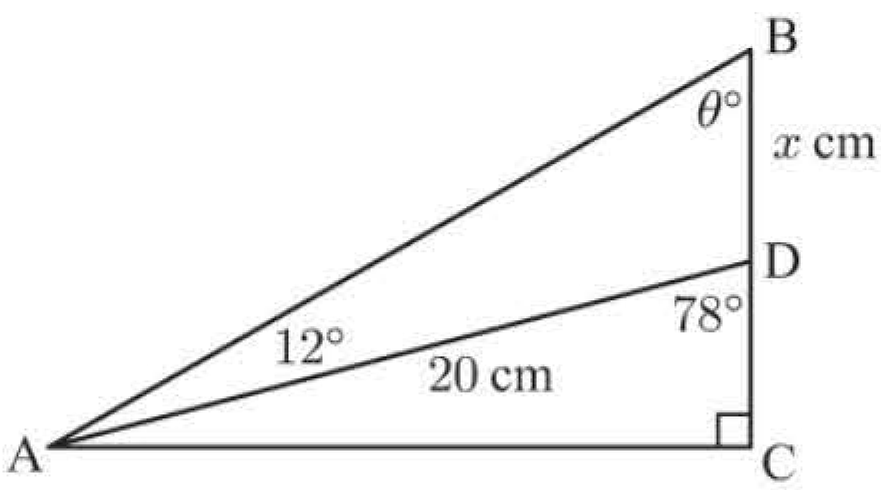
But $135.7 + 71 > 180$, so this case is impossible. $\therefore \widehat{ACB} \approx 44.3°$

3 The third angle is $180° - 85° - 68° = 27°$

Now $\dfrac{\sin 85°}{11.4} \approx 0.087\,38$ and $\dfrac{\sin 27°}{9.8} \approx 0.046\,32$

This is not possible since $\dfrac{\sin 85°}{11.4} \neq \dfrac{\sin 27°}{9.8}$ violates the sine rule.

4



In $\triangle ABD$,

$\theta = 78 - 12$

$\therefore \widehat{ABC} = 66°$

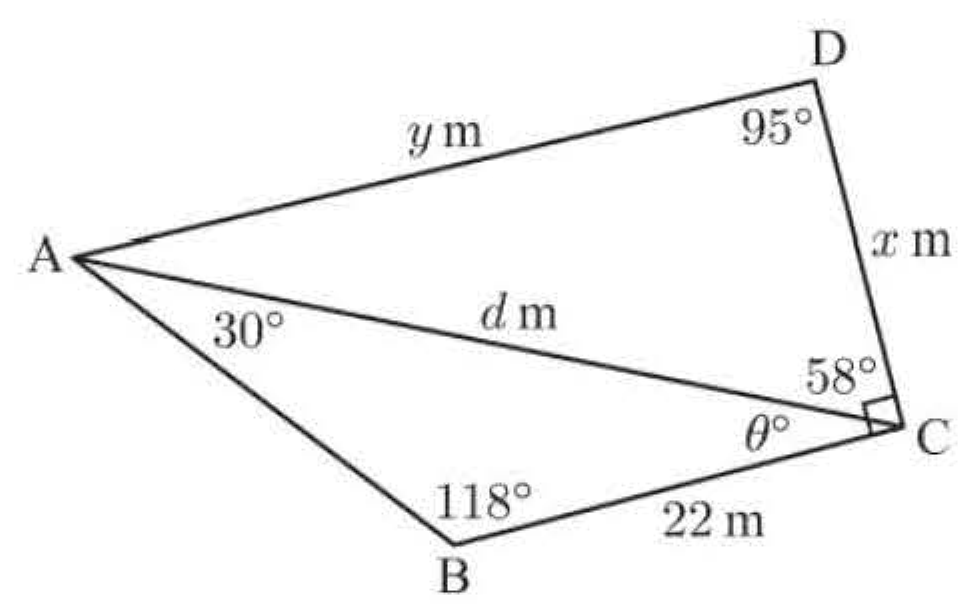
Now $\dfrac{x}{\sin 12°} = \dfrac{20}{\sin 66°}$

$\therefore x = \dfrac{20 \times \sin 12°}{\sin 66°}$

$\therefore x \approx 4.55$

$\therefore BD \approx 4.55$ cm

5 First we find the length of the diagonal, d m.



$\dfrac{d}{\sin 118°} = \dfrac{22}{\sin 30°}$

$\therefore d = \dfrac{22 \times \sin 118°}{\sin 30°}$

$\therefore d \approx 38.85$

Now $\theta = 180 - 30 - 118 = 32$

$\therefore \widehat{ACD} = 90° - 32° = 58°$

Using the sine rule, $\dfrac{y}{\sin 58°} = \dfrac{38.85}{\sin 95°}$ and $\dfrac{x}{\sin(180 - 95 - 58)°} \approx \dfrac{38.85}{\sin 95°}$

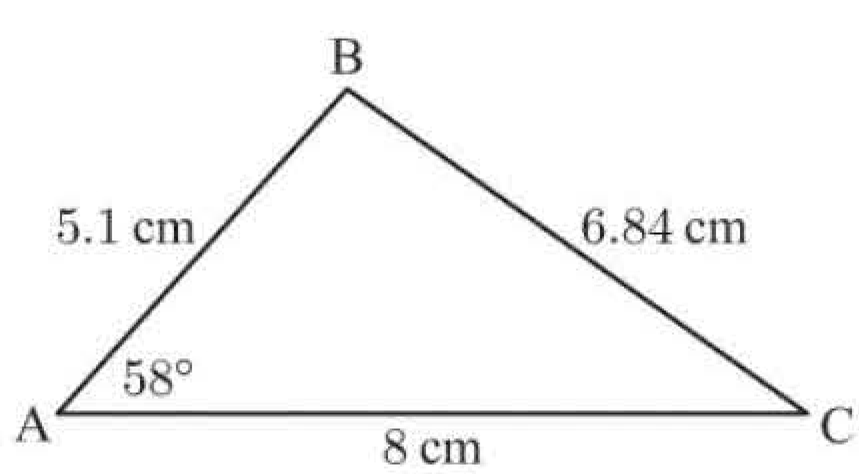
$\therefore y \approx \dfrac{38.85 \times \sin 58°}{\sin 95°}$

$\therefore y \approx 33.1$

$\therefore x \approx \dfrac{38.85 \times \sin 27°}{\sin 95°}$

$\therefore x \approx 17.7$

6



a $\dfrac{\sin \widehat{B}}{8} = \dfrac{\sin 58°}{6.84}$

$\therefore \sin \widehat{B} = \dfrac{8 \sin 58°}{6.84}$

$\therefore \widehat{B} = \sin^{-1}\left(\dfrac{8 \sin 58°}{6.84}\right)$ or its supplement

$\therefore \widehat{B} \approx 83°$ or $(180 - 83)°$

$\therefore \widehat{B} \approx 83°$ or $97°$

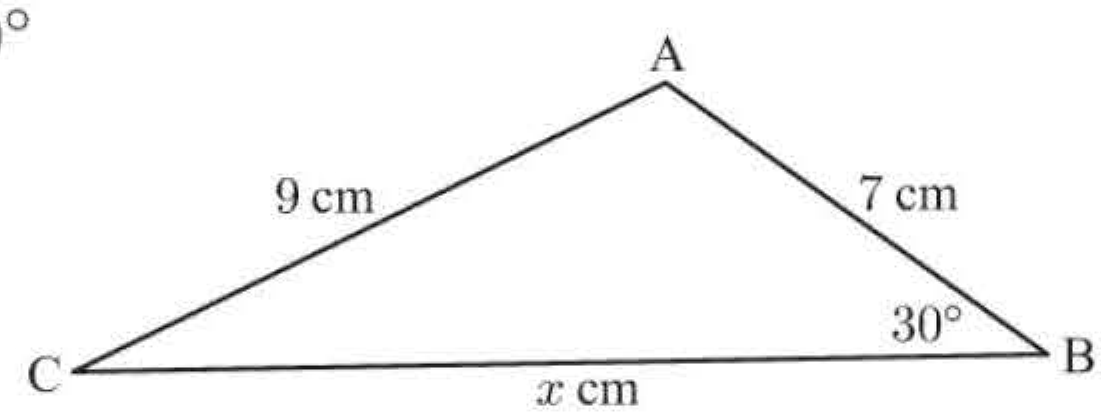
b $\cos \widehat{B} = \dfrac{5.1^2 + 6.84^2 - 8^2}{2 \times 5.1 \times 6.84}$

$\therefore \widehat{B} = \cos^{-1}\left(\dfrac{5.1^2 + 6.84^2 - 8^2}{2 \times 5.1 \times 6.84}\right)$

$\therefore \widehat{B} \approx 83°$

c When faced with using either the sine rule or the cosine rule, it is better to use the cosine rule as it avoids the ambiguous case.

7



$$9^2 = x^2 + 7^2 - 2 \times x \times 7 \times \cos 30°$$
$$\therefore\ 81 = x^2 + 49 - 14x(\tfrac{\sqrt{3}}{2})$$
$$\therefore\ x^2 - \tfrac{14\sqrt{3}}{2}x - 32 = 0$$

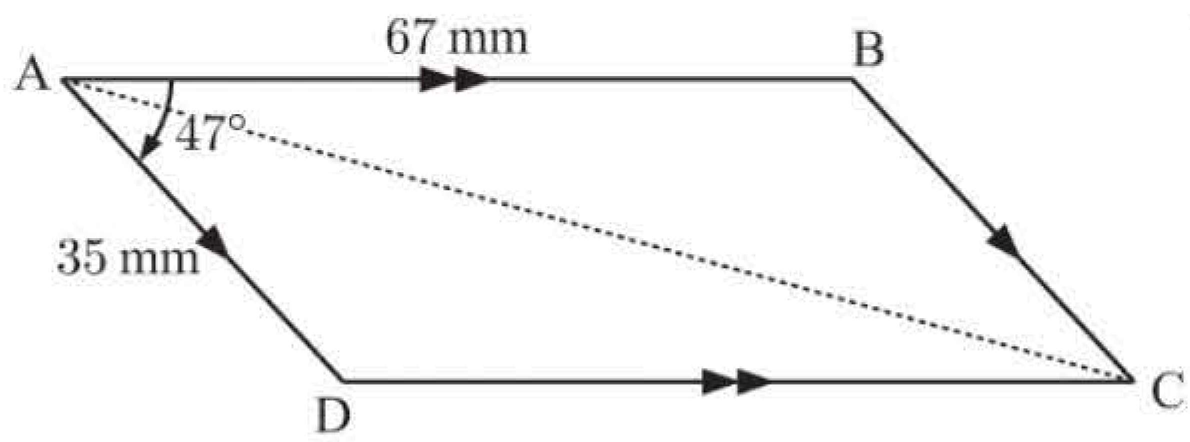
Using the quadratic formula or technology,

$$x \approx -2.23 \ \text{ or } \ 14.35$$

but $x > 0$, so $x \approx 14.35$

$$\therefore\ \text{area of triangle} \approx \tfrac{1}{2} \times 7 \times 14.35 \times \sin 30° \approx 25.1\ \text{cm}^2$$

8



$$\text{A}\widehat{\text{B}}\text{C} = 180° - 47° = 133°$$

$$\cos 133° = \frac{67^2 + 35^2 - \text{AC}^2}{2 \times 67 \times 35} \quad \{\text{cosine rule}\}$$
$$\text{AC}^2 = 5714 - 4690\cos 133°$$
$$\text{AC} = 94.41\ \text{mm} \quad \{\text{as } \text{AC} > 0\}$$

$$\frac{\sin 133°}{94.41} = \frac{\sin \text{B}\widehat{\text{A}}\text{C}}{35}$$
$$\text{B}\widehat{\text{A}}\text{C} = \sin^{-1}\left(\frac{35 \sin 133°}{94.41}\right) \approx 15.7°$$

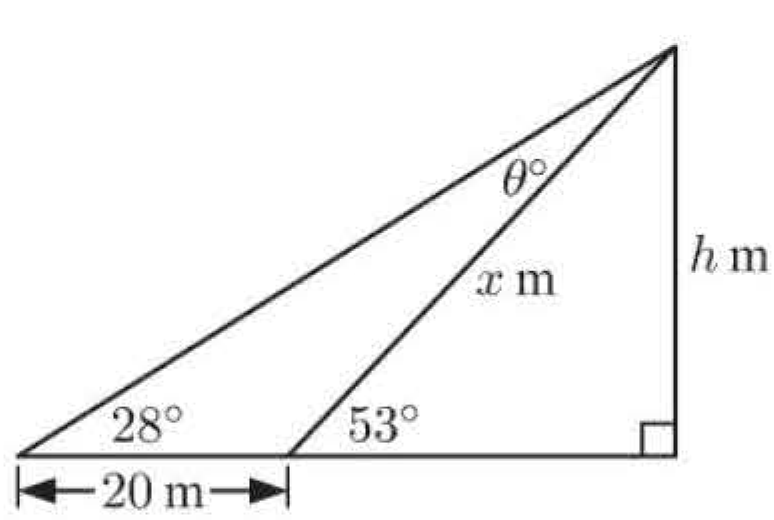
9

$$\frac{2x-5}{\sin 45°} = \frac{x+3}{\sin 30°}$$
$$\therefore\ (2x-5)\sin 30° = (x+3)\sin 45°$$
$$\therefore\ \frac{2x-5}{2} = \frac{x+3}{\sqrt{2}}$$
$$\therefore\ 2\sqrt{2}x - 5\sqrt{2} = 2x + 6$$
$$\therefore\ -6 - 5\sqrt{2} = x(2 - 2\sqrt{2})$$

$$\therefore\ x = \left(\frac{-6-5\sqrt{2}}{2-2\sqrt{2}}\right)\left(\frac{2+2\sqrt{2}}{2+2\sqrt{2}}\right)$$
$$= \frac{-12 - 12\sqrt{2} - 10\sqrt{2} - 10(2)}{4 - 4(2)}$$
$$= \frac{-32 - 22\sqrt{2}}{-4}$$
$$= 8 + \tfrac{11}{2}\sqrt{2}$$

EXERCISE 11D

1



$$\theta° + 28° = 53°$$
{exterior angle of a △ theorem}
$$\therefore\ \theta = 25$$

By the sine rule,

$$\frac{x}{\sin 28°} = \frac{20}{\sin 25°}$$
$$\therefore\ x \approx \frac{20 \times \sin 28°}{\sin 25°}$$
$$\therefore\ x \approx 22.22$$

and $\sin 53° = \dfrac{h}{x}$

$$\therefore\ h = x \sin 53° \approx 22.22 \times \sin 53° \approx 17.7\ \text{m}$$

$\therefore$ the pole is 17.7 m high.

2 $\text{PR}^2 = 63^2 + 175^2 - 2 \times 63 \times 175 \times \cos 112°$

$\therefore\ \text{PR} = \sqrt{63^2 + 175^2 - 2 \times 63 \times 175 \times \cos 112°}$

$\therefore\ \text{PR} \approx 207$ m

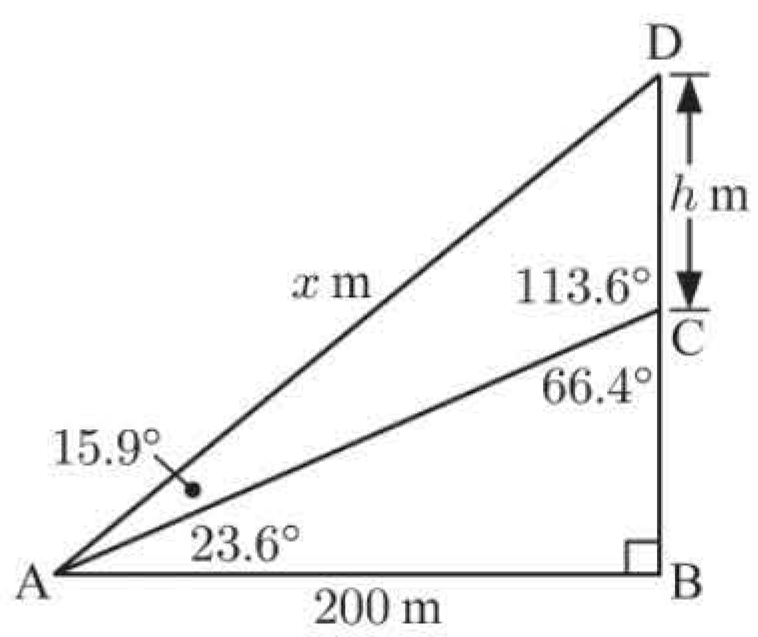
3 $\cos T = \dfrac{220^2 + 340^2 - 165^2}{2 \times 220 \times 340}$

$\therefore\ T = \cos^{-1}\left(\dfrac{136\,775}{149\,600}\right)$

$\therefore\ T \approx 23.9$

$\therefore$ the tee shot was 23.9° off line.

4



In $\triangle$ABD,

$$\cos(23.6+15.9)^\circ = \frac{200}{x}$$

$$\therefore\ x = \frac{200}{\cos 39.5^\circ}$$

$$\therefore\ x \approx 259.2$$

In $\triangle$ACD,

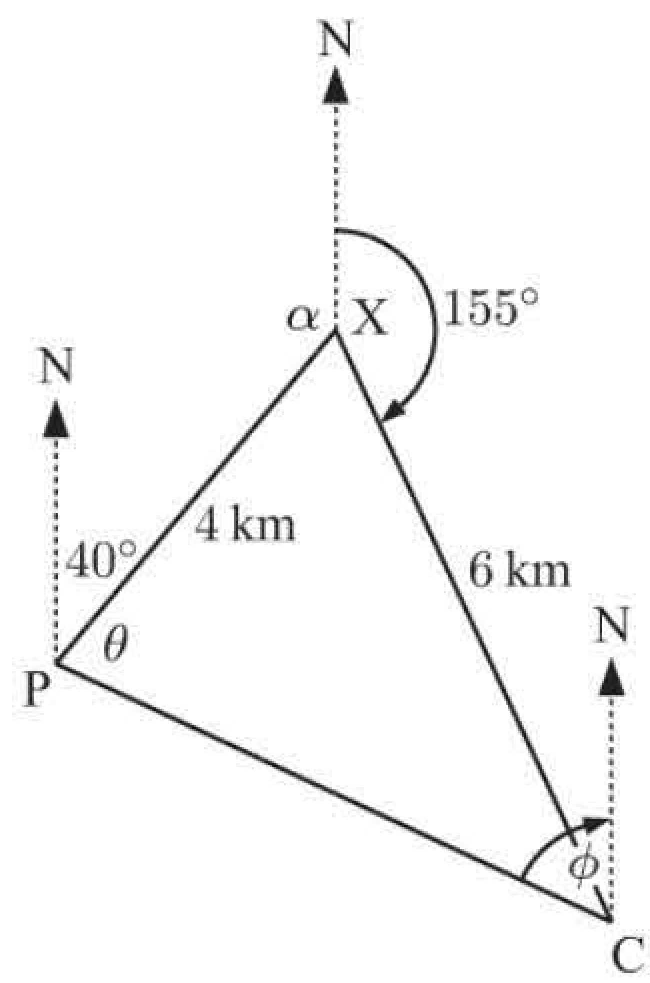
$$\frac{h}{\sin 15.9^\circ} = \frac{x}{\sin 113.6^\circ}$$

$$\therefore\ h \approx \frac{259.2\times\sin 15.9^\circ}{\sin 113.6^\circ}$$

$$\therefore\ h \approx 77.5$$

$\therefore$ the tower is 77.5 m high.

5



a $\alpha = 140^\circ$ {co-interior angles}

$$\therefore\ \widehat{\text{PXC}} = 360^\circ - 140^\circ - 155^\circ \quad \text{\{angles at a point\}}$$
$$= 65^\circ$$

So, $\text{PC}^2 = 4^2 + 6^2 - 2\times 4\times 6\cos 65^\circ$

$$\therefore\ \text{PC} = \sqrt{16+36-48\cos 65^\circ}$$
$$\approx 5.6315 \text{ km}$$

$\therefore$ Esko hikes 5.63 km.

b
$$\cos\theta \approx \frac{4^2+5.6315^2-6^2}{2\times 4\times 5.6315}$$

$$\therefore\ \theta \approx 74.9^\circ$$

$\therefore$ bearing $= 40^\circ + \theta$
$\approx 114.9^\circ$

$\therefore$ Esko hikes on a bearing of 115°.

c **i**
$$\text{speed} = \frac{\text{distance}}{\text{time}} \quad\Rightarrow\quad \text{time} = \frac{\text{distance}}{\text{speed}}$$

$$\therefore\ \text{time}_{\text{Ritva}} = \frac{4+6}{10} = 1 \text{ hour} \quad \text{and} \quad \text{time}_{\text{Esko}} \approx \frac{5.6315}{6} \approx 0.9386 \text{ hours}$$
$$\approx 56.32 \text{ min}$$

So Esko arrives at the campsite first.

ii $60 - 56.32 = 3.68$

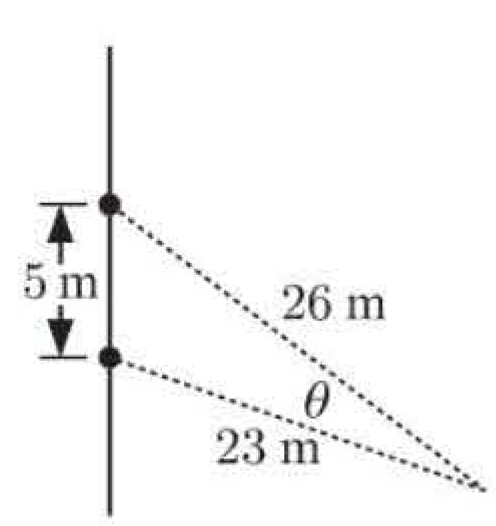
Esko needs to wait about 3.68 minutes before Ritva arrives.

d $\phi \approx 180^\circ - 114.9^\circ \approx 65.1^\circ$ {co-interior angles}

$\therefore\ 360^\circ - \phi \approx 295^\circ$

The return bearing is 295°.

6



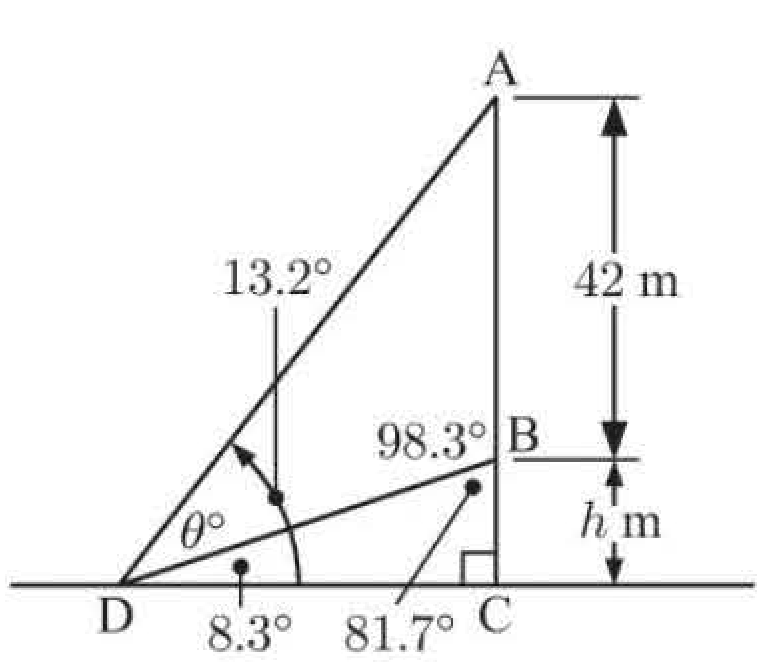
$$\cos\theta = \frac{23^2+26^2-5^2}{2\times 23\times 26}$$

$$\therefore\ \theta = \cos^{-1}\left(\tfrac{1180}{1196}\right)$$

$$\therefore\ \theta \approx 9.38^\circ$$

$\therefore$ the angle of view is 9.38°.

7



$\theta = 13.2^\circ - 8.3^\circ = 4.9^\circ$

In $\triangle$ABD,

$$\frac{\text{AD}}{\sin 98.3^\circ} = \frac{42}{\sin 4.9^\circ}$$

$$\therefore\ \text{AD} = \frac{42\times\sin 98.3^\circ}{\sin 4.9^\circ}$$

$$\therefore\ \text{AD} \approx 486.56 \text{ m}$$

In $\triangle$ADC,

$$\sin 13.2^\circ = \frac{h+42}{\text{AD}}$$

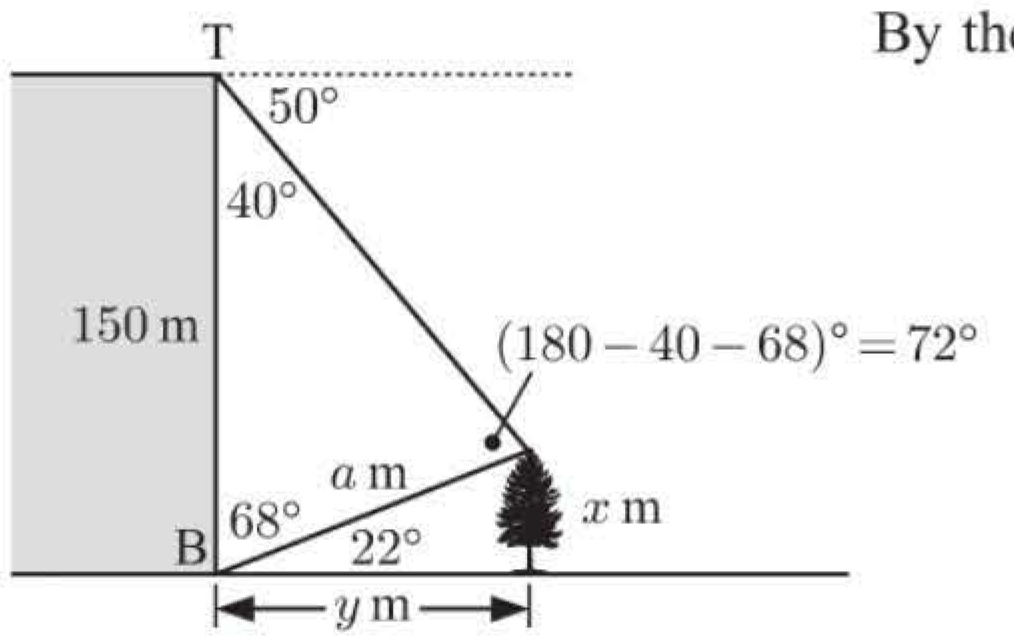
$$\therefore\ h+42 \approx 486.56\times\sin 13.2^\circ$$

$$\therefore\ h+42 \approx 111.1$$

$$\therefore\ h \approx 69.1$$

$\therefore$ the hill is 69.1 m high.

8



By the sine rule, $\dfrac{a}{\sin 40°} = \dfrac{150}{\sin 72°}$

$\therefore\ a = \dfrac{150 \times \sin 40°}{\sin 72°}$

$\therefore\ a \approx 101.38$

a $\sin 22° \approx \dfrac{x}{101.38}$

$\therefore\ x \approx 101.38 \times \sin 22°$

$\therefore\ x \approx 38.0$

$\therefore$ the tree is 38.0 m high.

b $\cos 22° \approx \dfrac{y}{101.38}$

$\therefore\ y \approx 101.38 \times \cos 22°$

$\therefore\ y \approx 94.0$

$\therefore$ the tree is 94.0 m from the building.

9 Using Pythagoras' theorem

$\text{RQ} = \sqrt{4^2 + 7^2} = \sqrt{65}$ cm

$\text{PQ} = \sqrt{8^2 + 7^2} = \sqrt{113}$ cm

$\text{PR} = \sqrt{8^2 + 4^2} = \sqrt{80}$ cm

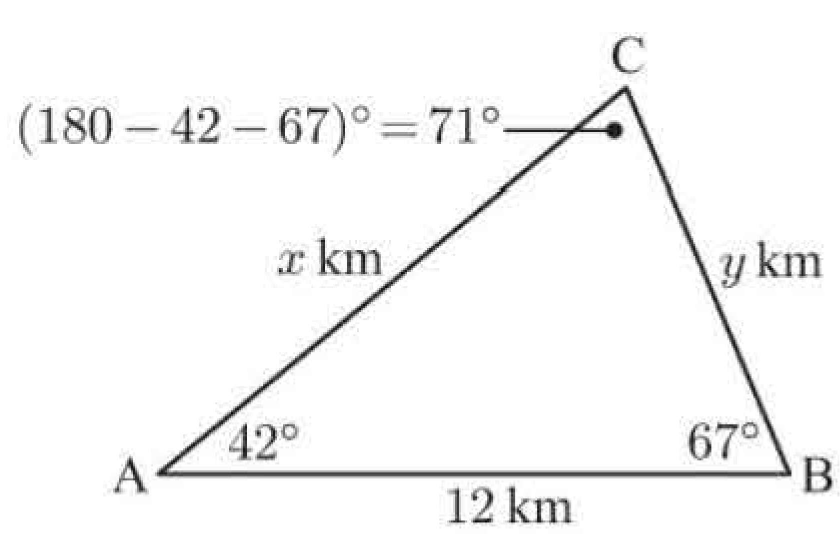
Now $\cos \text{P}\widehat{\text{Q}}\text{R} = \dfrac{(\sqrt{113})^2 + (\sqrt{65})^2 - (\sqrt{80})^2}{2 \times \sqrt{113} \times \sqrt{65}}$

$\therefore\ \cos \text{P}\widehat{\text{Q}}\text{R} \approx \left(\frac{98}{171.4}\right)$

$\therefore\ \text{P}\widehat{\text{Q}}\text{R} \approx \cos^{-1}\left(\frac{98}{171.4}\right)$

$\therefore\ \text{P}\widehat{\text{Q}}\text{R} \approx 55.1$ So, $\text{P}\widehat{\text{Q}}\text{R}$ measures $55.1°$.

10



$\dfrac{x}{\sin 67°} = \dfrac{12}{\sin 71°} = \dfrac{y}{\sin 42°}$

$\therefore\ x = \dfrac{12 \times \sin 67°}{\sin 71°}$ and $y = \dfrac{12 \times \sin 42°}{\sin 71°}$

$\therefore\ x \approx 11.7$ $\therefore\ y \approx 8.49$

So, C is 11.7 km from A and 8.49 km from B.

11 **a** $\text{QS} = \sqrt{8^2 + 12^2 - 2 \times 8 \times 12 \times \cos 70°}$

≈ 11.93

$\therefore$ area $\approx \frac{1}{2} \times 8 \times 12 \times \sin 70° + \frac{1}{2} \times 10 \times 11.93 \times \sin 30°$

$\approx 74.9\ \text{km}^2$

b 1 ha is 100 m $\times$ 100 m

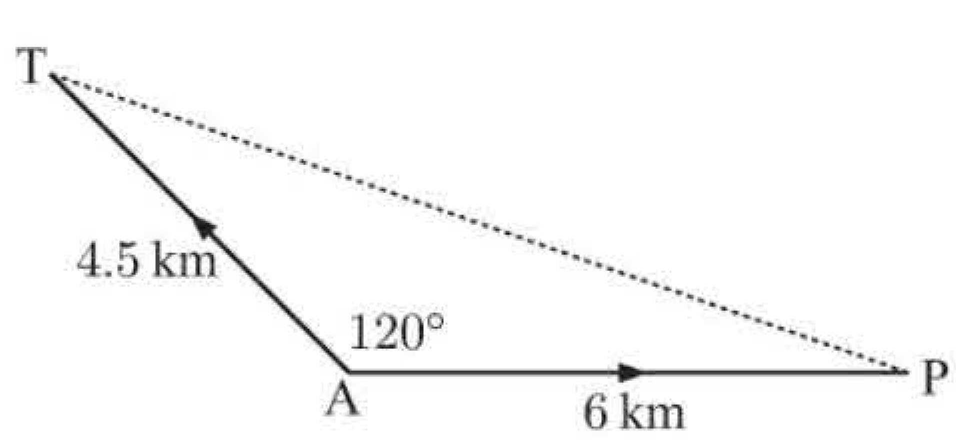
$= 0.1$ km $\times$ 0.1 km

$= 0.01\ \text{km}^2$

$\therefore\ 1\ \text{km}^2 = 100$ ha

$\therefore$ area ≈ 7490 ha

12



Distance = speed $\times$ time

So, after 45 min = 0.75 h, AT $= 6 \times 0.75 = 4.5$ km

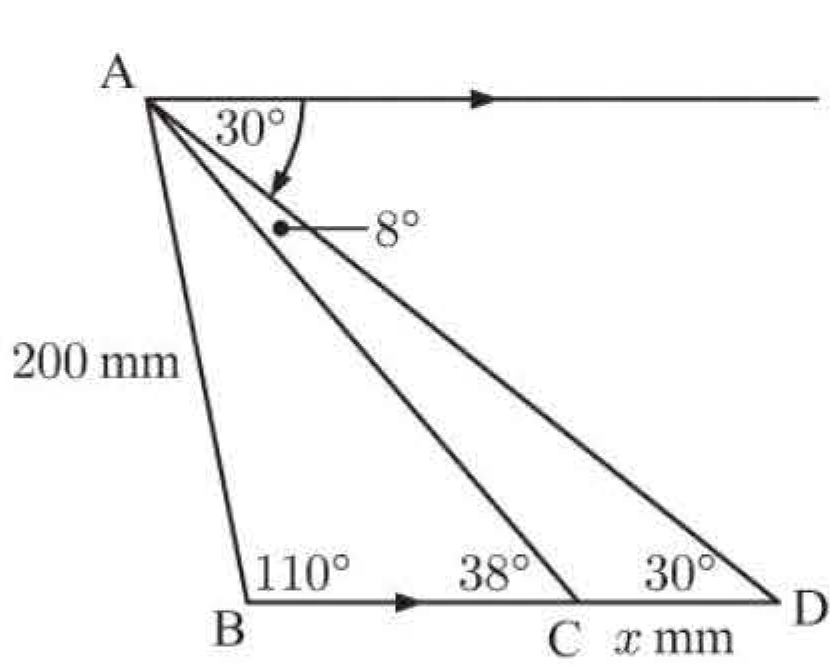
AP $= 8 \times 0.75 = 6$ km

Now $\text{PT} = \sqrt{4.5^2 + 6^2 - 2 \times 4.5 \times 6 \times \cos 120°}$

$\therefore\ \text{PT} \approx 9.12$

So, they are 9.12 km apart.

13



In $\triangle$ABC, $\dfrac{\text{AC}}{\sin 110°} = \dfrac{200}{\sin 38°}$

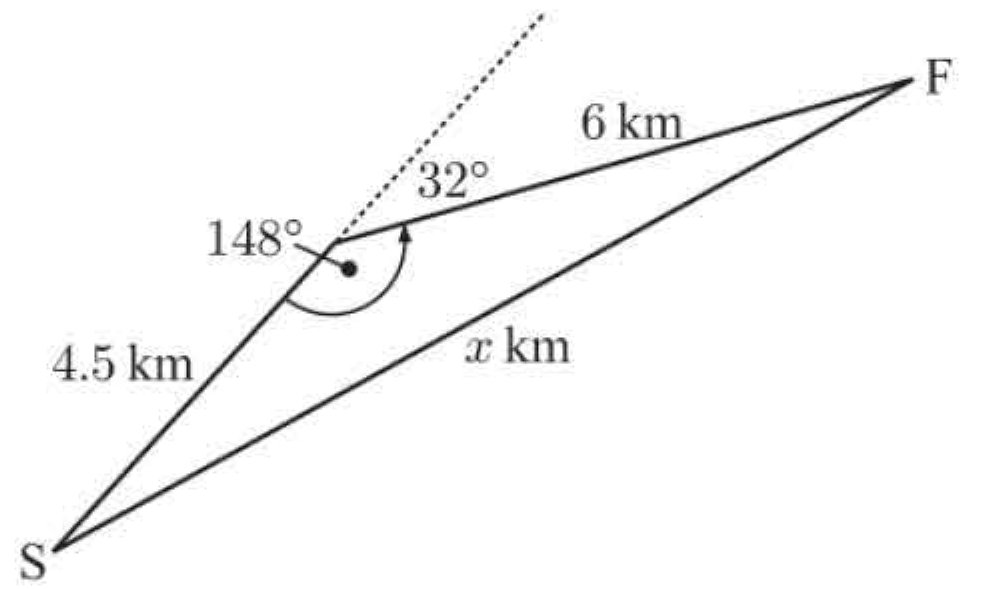
$\therefore\ \text{AC} = \dfrac{200 \times \sin 110°}{\sin 38°} \approx 305.26$

and in $\triangle$ACD, $\dfrac{x}{\sin 8°} \approx \dfrac{305.26}{\sin 30°}$

$\therefore\ x \approx \dfrac{305.26 \times \sin 8°}{\sin 30°} \approx 84.968$

$\therefore$ the metal strip is 85.0 mm wide.

14

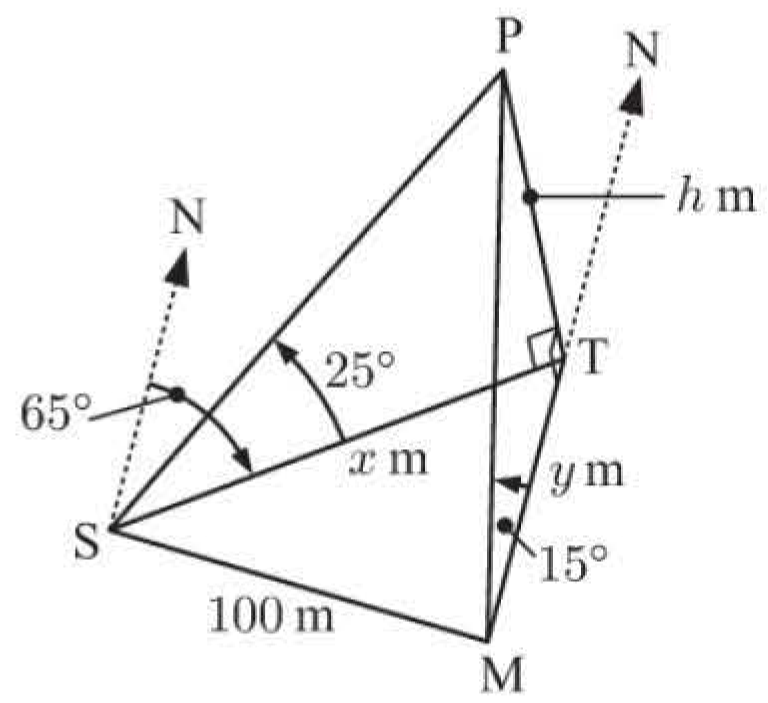


$x = \sqrt{6^2 + (4.5)^2 - 2 \times 6 \times 4.5 \times \cos 148°}$

$\therefore \quad x \approx 10.1$

$\therefore$ the orienteer is 10.1 km from the start.

15



In $\triangle$PST, $\tan 25° = \dfrac{h}{x}$

$\therefore \quad x = \dfrac{h}{\tan 25°}$

$\approx 2.145h$

In $\triangle$PMT, $\tan 15° = \dfrac{h}{y}$

$\therefore \quad y = \dfrac{h}{\tan 15°}$

$\approx 3.732h$

But $\widehat{STM} = 65°$ {equal alternate angles}

and $100^2 = x^2 + y^2 - 2xy\cos 65°$

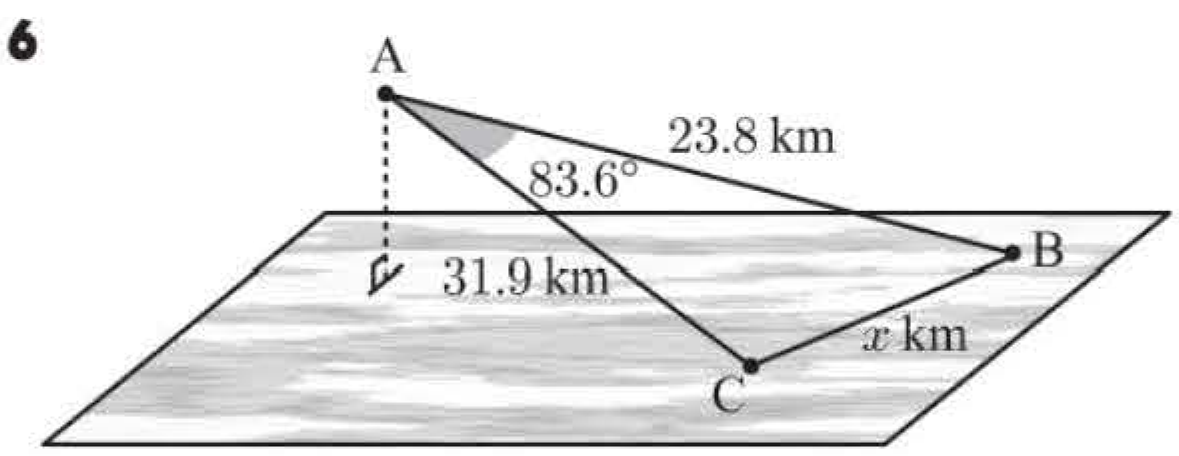
$\therefore \quad 10\,000 \approx (2.145h)^2 + (3.732h)^2 - 2 \times (2.145)(3.732)h^2 \cos 65°$

$\therefore \quad 10\,000 \approx 11.762\,h^2$

$\therefore \quad h^2 \approx 850.17$

$\therefore \quad h \approx 29.2$ So, the tree is 29.2 m high.

16



By the cosine rule

$x^2 = 23.8^2 + 31.9^2 - 2 \times 23.8 \times 31.9 \times \cos 83.6°$

$\therefore \quad x = \sqrt{23.8^2 + 31.9^2 - 2 \times 23.8 \times 31.9 \times \cos 83.6°}$

$\therefore \quad x \approx 37.6$

$\therefore$ B and C are 37.6 km apart.

REVIEW SET 11A

1 area $= \frac{1}{2} \times 7 \times 8 \times \sin 30°$

$= 28 \times \frac{1}{2}$

$= 14 \text{ km}^2$

2 If the unknown is an angle, use the cosine rule to avoid the ambiguous case.

3 **a** By the cosine rule, $7^2 = 8^2 + x^2 - 2 \times 8 \times x \times \cos 60°$

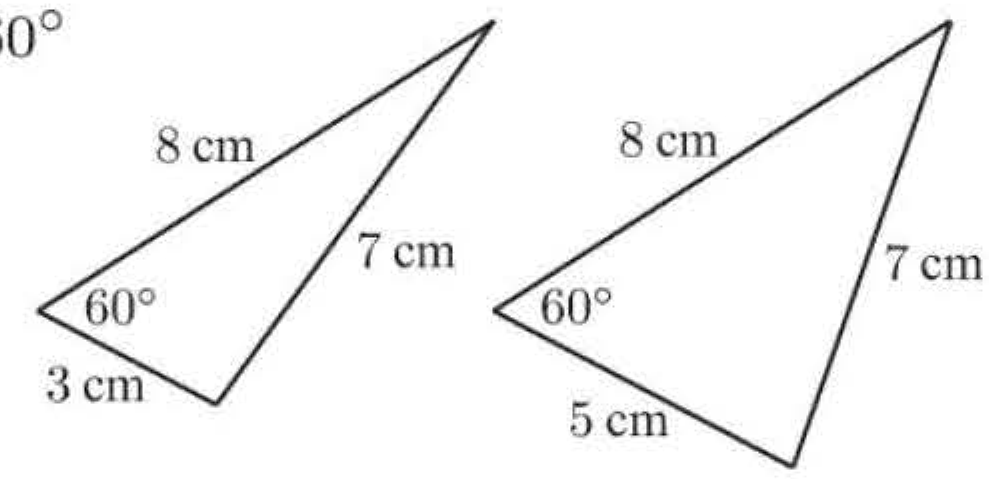
$\therefore \quad 49 = 64 + x^2 - 16x\left(\frac{1}{2}\right)$

$\therefore \quad 49 = 64 + x^2 - 8x$

$\therefore \quad x^2 - 8x + 15 = 0$

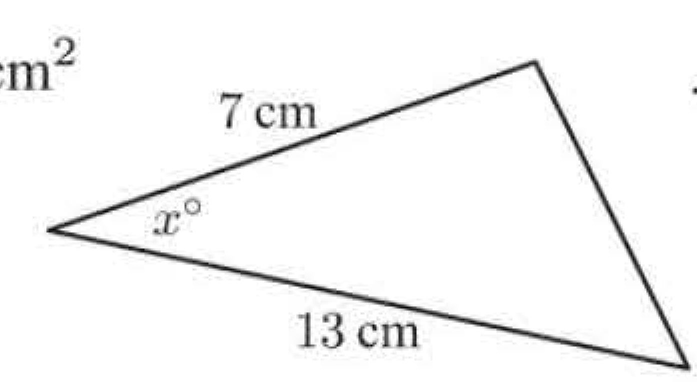
$\therefore \quad (x-3)(x-5) = 0$

$\therefore \quad x = 3$ or 5



b Kady's response should be "Please supply me with additional information as there are two possibilities. Which one do you want?"

4 area $= 42 \text{ cm}^2$

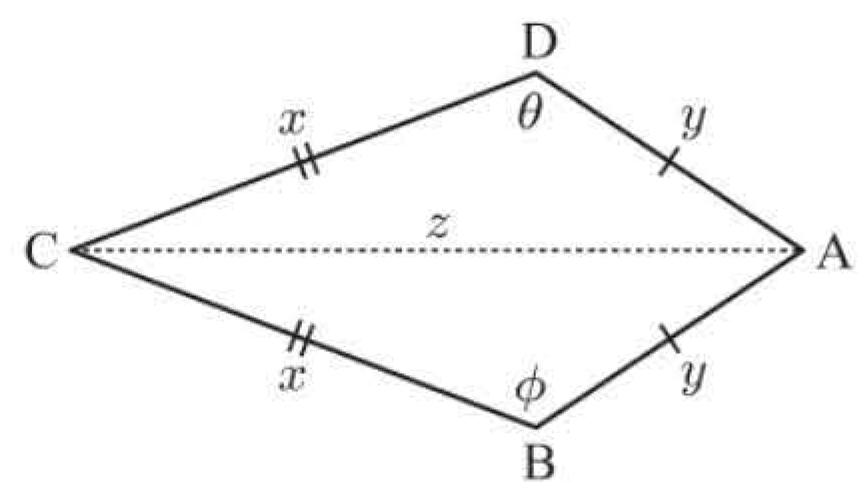


$\therefore \quad \frac{1}{2} \times 7 \times 13 \times \sin x° = 42$

$\therefore \quad \sin x° = \dfrac{42 \times 2}{7 \times 13}$

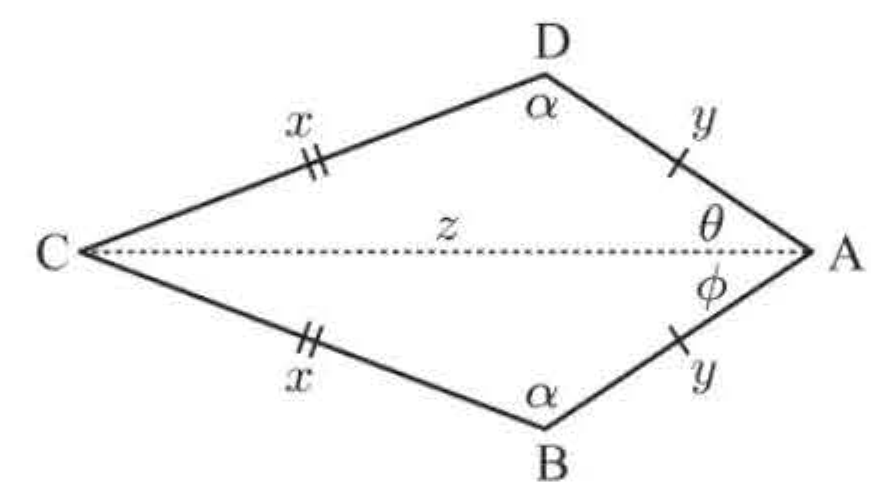
$= \frac{12}{13}$

5 **a**



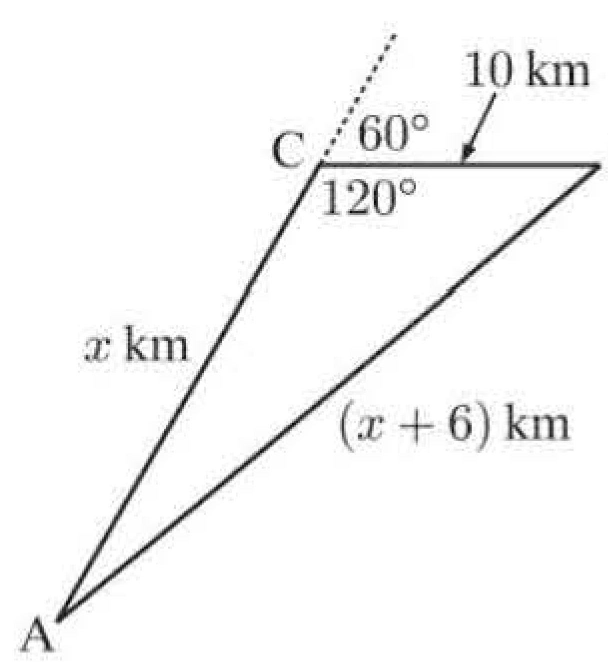
Using $\triangle$ADC,
$z^2 = x^2 + y^2 - 2xy\cos\theta$ .... (1)
Using $\triangle$ABC,
$z^2 = x^2 + y^2 - 2xy\cos\phi$ .... (2)
Equating (1) and (2),
$\cos\theta = \cos\phi$
and since $0 < \theta,\ \phi < 180$,
$\theta = \phi$
$\therefore\ \widehat{\text{ADC}} = \widehat{\text{ABC}}$

b



Using $\triangle$DAC,
$\dfrac{\sin\theta}{x} = \dfrac{\sin\alpha}{z}$ .... (1)
Using $\triangle$BAC,
$\dfrac{\sin\phi}{x} = \dfrac{\sin\alpha}{z}$ .... (2)
$\{\widehat{\text{ADC}} = \widehat{\text{ABC}}$ from **a**$\}$
Equating (1) and (2),
$\sin\theta = \sin\phi$
$\therefore\ \theta = \phi$ or $\theta = 180 - \phi$
but $\widehat{\text{DAB}} = \theta + \phi < 180°$
$\therefore\ \theta = \phi$
$\therefore\ \widehat{\text{DAC}} = \widehat{\text{BAC}}$

6 Total distance travelled $= x + 10$ km
$\therefore$ AB $= (x + 10) - 4 = x + 6$ km
Now $(x+6)^2 = x^2 + 10^2 - 2 \times x \times 10 \times \cos 120°$
$\therefore\ x^2 + 12x + 36 = x^2 + 100 - 20x(-\frac{1}{2})$
$\therefore\ 12x + 36 = 100 + 10x$
$\therefore\ 2x = 64$
$\therefore\ x = 32$
$\therefore$ the boat travelled $x + 10 = 42$ km.

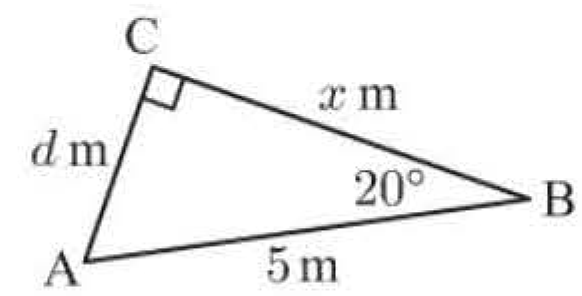


7 shaded area $=$ area of sector $-$ area of $\triangle$
$= \frac{1}{2} \times \frac{13\pi}{18} \times 7^2 - \frac{1}{2} \times 7 \times 7 \times \sin\left(\frac{13\pi}{18}\right)$
$= \frac{49}{2}\left(\frac{13\pi}{18} - \sin(\frac{13\pi}{18})\right)$

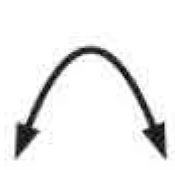
8 **a** $d^2 = x^2 + 5^2 - 2 \times x \times 5 \times \cos 20°$
$\therefore\ d^2 = x^2 - (10\cos 20°)x + 25$

b d^2 is minimised when $x = \dfrac{-b}{2a}$
$\therefore\ x = \dfrac{10\cos 20°}{2}$
$\therefore\ x = 5\cos 20°$
$\therefore$ d is minimised when $x = 5\cos 20°$

c If $\widehat{\text{BCA}}$ is a right angle, then we have



Now $\cos 20° = \dfrac{x}{5}$
$\therefore\ x = 5\cos 20°$
and from **b**, d is minimised when $x = 5\cos 20°$
$\therefore$ d is minimised when $\widehat{\text{BCA}}$ is a right angle.

9 **a** $y = -x^2 + 12x - 20$ has $a = -1 < 0$
$\therefore$ its shape is , and y is maximised when $x = \dfrac{-b}{2a} = \dfrac{-12}{-2} = 6$.
When $x = 6$, $y = -6^2 + 12(6) - 20 = -36 + 72 - 20 = 16$
$\therefore$ the maximum value of $y = -x^2 + 12x - 20$ is 16, which occurs when $x = 6$.

b **i** The perimeter is 20

$\therefore\ x+y+8=20$

$\therefore\ y=12-x$

ii $y^2=x^2+8^2-2\times x\times 8\times\cos\theta$

$\therefore\ y^2=x^2+64-16x\cos\theta$

iii Since $y=12-x$, $(12-x)^2=x^2+64-16x\cos\theta$

$\therefore\ 144-24x+\cancel{x^2}=\cancel{x^2}+64-16x\cos\theta$

$\therefore\ 16x\cos\theta=24x-80$

$\therefore\ \cos\theta=\dfrac{24x-80}{16x}$

$=\dfrac{3x-10}{2x}$

c Area $A=\frac{1}{2}\times x\times 8\times\sin\theta$

$=4x\sin\theta$

$\therefore\ A^2=16x^2\sin^2\theta$

$=16x^2(1-\cos^2\theta)$

$=16x^2\left[1-\left(\dfrac{3x-10}{2x}\right)^2\right]$

$=16x^2\left[1-\dfrac{9x^2-60x+100}{4x^2}\right]$

$=16x^2-4(9x^2-60x+100)$

$=16x^2-36x^2+240x-400$

$=-20x^2+240x-400$

$=20(-x^2+12x-20)$

d A is maximised when A^2 is maximised since $A>0$.

From **a**, $-x^2+12x-20$ has a maximum value of 16 when $x=6$.

When $x=6$, $A^2=20(16)$

$=320$

$\therefore\ A=\sqrt{320}\ \ \{A>0\}$

$=8\sqrt{5}$

Also, when $x=6$, $y=12-6$

$=6$

$\therefore$ the maximum area of the triangle is $8\sqrt{5}$ units^2, and the triangle is isosceles when this occurs.

REVIEW SET 11B

1 **a** $\cos x^\circ=\dfrac{13^2+19^2-11^2}{2\times 13\times 19}$

$\therefore\ \cos x^\circ=\frac{409}{494}$

$\therefore\ x^\circ=\cos^{-1}\left(\frac{409}{494}\right)$

$\therefore\ x\approx 34.1$

b $x^2=15^2+17^2-2\times 15\times 17\times\cos 72^\circ$

$\therefore\ x=\sqrt{15^2+17^2-2\times 15\times 17\times\cos 72^\circ}$

$\therefore\ x\approx 18.9$

2 $AC^2=11^2+9.8^2-2\times 11\times 9.8\times\cos 74^\circ$

$\therefore\ AC=\sqrt{11^2+9.8^2-2\times 11\times 9.8\times\cos 74^\circ}$

$\therefore\ AC\approx 12.554$ cm

$\therefore\ AC\approx 12.6$ cm

Now $\dfrac{\sin C}{11}=\dfrac{\sin 74^\circ}{AC}$

$\therefore\ \sin C\approx\dfrac{11\times\sin 74^\circ}{12.554}$

$\therefore\ C\approx\sin^{-1}\left(\dfrac{11\times\sin 74^\circ}{12.554}\right)$ or its supplement

$\therefore\ C\approx 57.4^\circ$ or 122.6°

impossible as $122.6+74>180$

$\therefore\ C$ measures 57.4°

$\therefore\ A$ measures $180^\circ-74^\circ-57.4^\circ=48.6^\circ$.

3 $DB^2=7^2+11^2-2\times 7\times 11\times\cos 110^\circ$

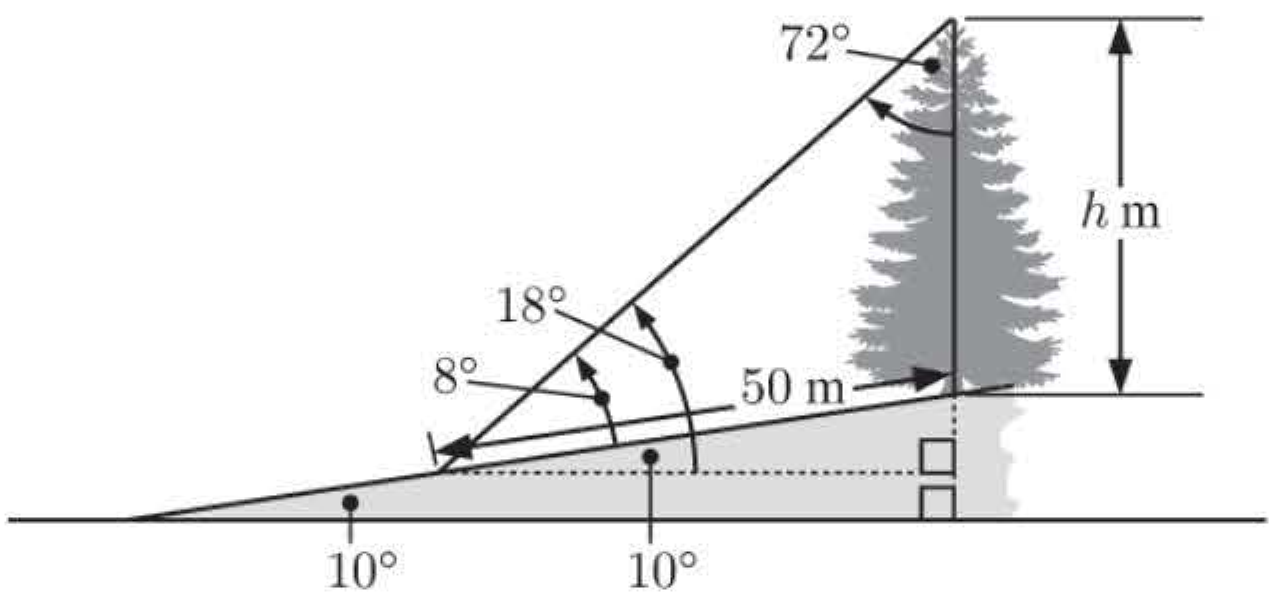
$\therefore\ DB=\sqrt{7^2+11^2-2\times 7\times 11\times\cos 110^\circ}\approx 14.922$ cm

$\therefore$ total area = area $\triangle ABD$ + area $\triangle BCD$

$\approx\frac{1}{2}\times 7\times 11\times\sin 110^\circ+\frac{1}{2}\times 16\times 14.922\times\sin 40^\circ$

≈ 113 cm^2

4



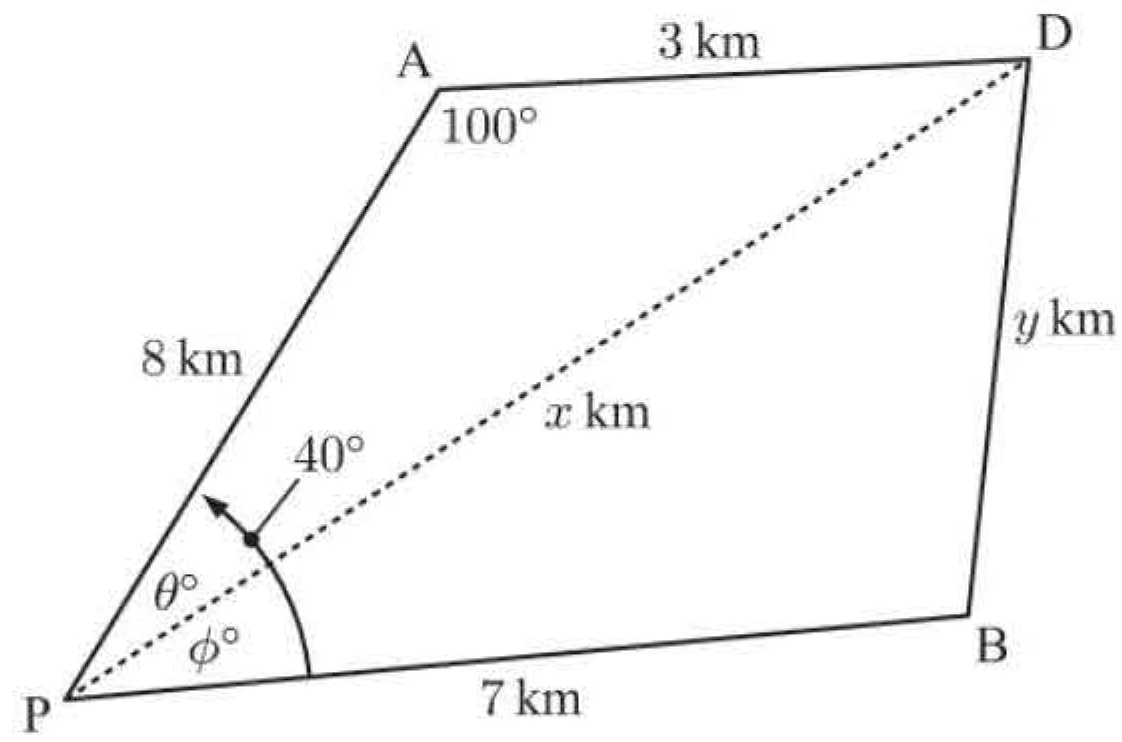
$$\frac{h}{\sin 8°} = \frac{50}{\sin 72°}$$

$$\therefore\ h = \frac{50 \times \sin 8°}{\sin 72°}$$

$$\therefore\ h \approx 7.32$$

So, the tree is 7.32 m high.

5 $x^2 = 8^2 + 3^2 - 2 \times 8 \times 3 \times \cos 100°$



$\therefore\ x = \sqrt{8^2 + 3^2 - 48\cos 100°}$

$\therefore\ x \approx 9.0186$

Now $\dfrac{\sin\theta°}{3} \approx \dfrac{\sin 100°}{9.0186}$

$\therefore\ \sin\theta° \approx \dfrac{3 \times \sin 100°}{9.0186}$

$\therefore\ \theta \approx \sin^{-1}\left(\dfrac{3 \times \sin 100°}{9.0186}\right)$ or its supplement

$\therefore\ \theta \approx 19.1$ or $\underset{\text{impossible}}{\underset{\uparrow}{160.9}}$

$\therefore\ \theta \approx 19.1$

$\therefore\ \phi \approx 40 - 19.1 \approx 20.9$

$\therefore\ y^2 = x^2 + 7^2 - 2 \times x \times 7 \times \cos\phi°$

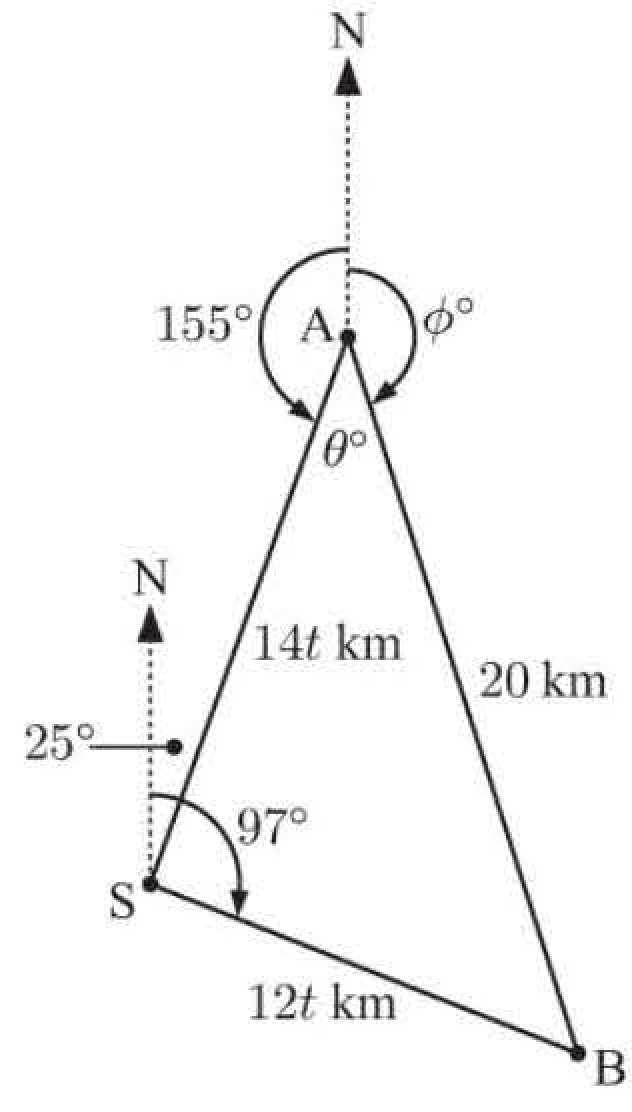
$\therefore\ y \approx \sqrt{(9.0186)^2 + 7^2 - 2 \times (9.0186) \times 7 \times \cos 20.9°}$

$\therefore\ y \approx 3.52$

So, Brett still has to walk 3.52 km.

6 **a** speed $= \dfrac{\text{distance}}{\text{time}}$ $\quad\therefore$ distance = speed × time

$\therefore$ in t hours, runner A travels $14 \times t = 14t$ km
and runner B travels $12 \times t = 12t$ km



Now $\widehat{ASB} = 97° - 25° = 72°$

$\therefore\ 20^2 = (14t)^2 + (12t)^2 - 2(14t)(12t)\cos 72°$

$\therefore\ 400 = 196t^2 + 144t^2 - 336t^2\cos 72°$

$\therefore\ 400 \approx 236.2t^2$

$\therefore\ t^2 \approx 1.69$

$\therefore\ t \approx 1.30 \quad \{t > 0\}$

$\therefore$ A and B are 20 km apart after 1 hour 18 minutes, at 2:18 pm.

b When $t \approx 1.30$, $SA \approx 14 \times 1.30 \approx 18.22$ km
and $SB \approx 12 \times 1.30 \approx 15.62$ km

$\therefore\ \cos\theta° \approx \dfrac{18.22^2 + 20^2 - 15.62^2}{2 \times 18.22 \times 20}$

$\therefore\ \theta° \approx \cos^{-1}\left(\frac{488.1}{728.8}\right)$

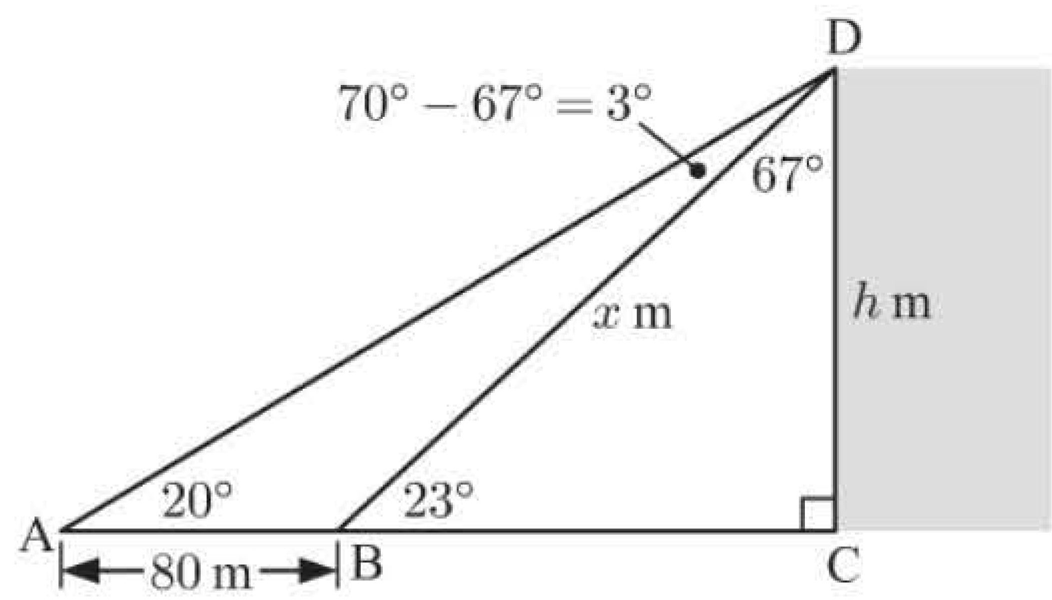
$\therefore\ \theta \approx 48.0$

$\therefore\ \phi \approx 360 - 155 - 48 \quad \{180° - 25° = 155°,\ \text{co-interior angles}\}$

≈ 157

$\therefore$ B is on a bearing of 157° from A.

7



In $\triangle$ABD, $\dfrac{x}{\sin 20°} = \dfrac{80}{\sin 3°}$

$\therefore\ x = \dfrac{80 \times \sin 20°}{\sin 3°} \approx 522.8$

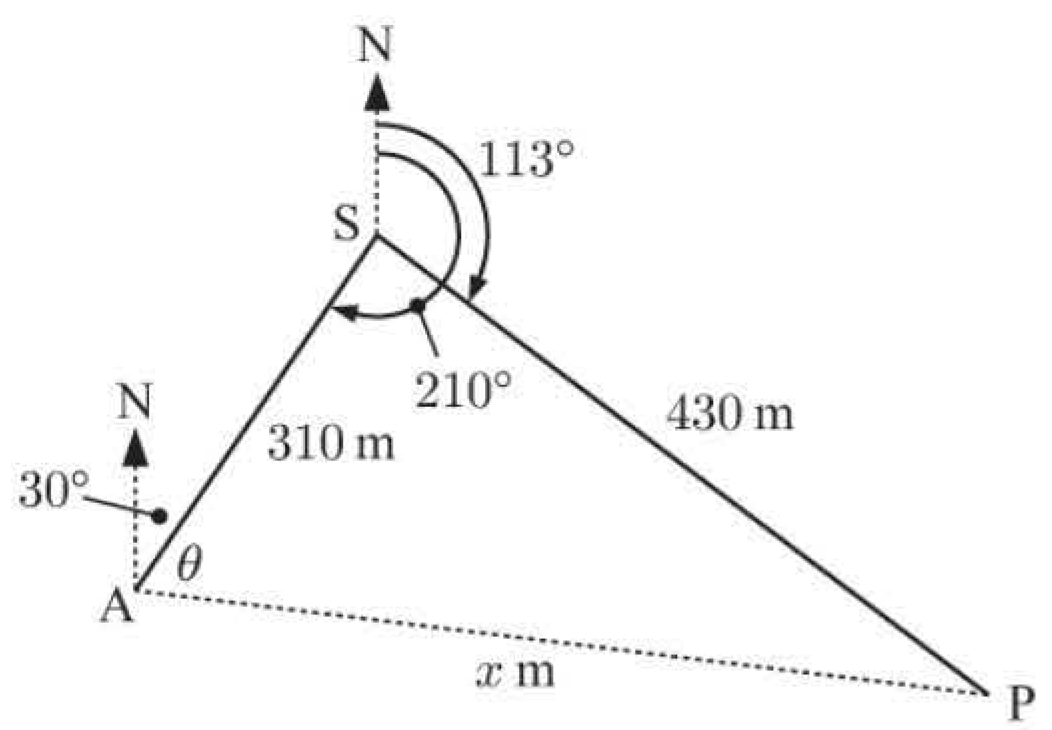
Now $\sin 23° = \dfrac{h}{x}$

$\therefore\ h \approx 522.8 \times \sin 23°$

$\therefore\ h \approx 204$

So the building is 204 m tall.

8



$\widehat{ASP} = 210° - 113° = 97°$

$\therefore\ x^2 = 310^2 + 430^2 - 2 \times 310 \times 430 \times \cos 97°$

$\therefore\ x = \sqrt{310^2 + 430^2 - 2 \times 310 \times 430 \times \cos 97°}$

$\therefore\ x \approx 559.9$

$\therefore$ Peter and Alix are 560 m apart.

and $\cos\theta \approx \dfrac{310^2 + 559.9^2 - 430^2}{2 \times 310 \times 559.9}$

$\therefore\ \theta \approx 49.7$

and $30 + \theta \approx 79.7$

$\therefore$ the bearing of Peter from Alix is $079.7°$.

REVIEW SET 11C

1 **a** $\cos x° = \dfrac{11^2 + 19^2 - 13^2}{2 \times 11 \times 19}$

$\therefore\ \cos x° = \frac{313}{418}$

$\therefore\ x° = \cos^{-1}\left(\frac{313}{418}\right)$

$\therefore\ x \approx 41.5$

b $x^2 = 14^2 + 21^2 - 2 \times 14 \times 21 \times \cos 47°$

$\therefore\ x = \sqrt{14^2 + 21^2 - 2 \times 14 \times 21 \times \cos 47°}$

$\therefore\ x \approx 15.4$

2 **a** area $= 80$ cm^2

$\therefore\ \frac{1}{2} \times 11.3 \times 19.2 \times \sin x° = 80$

$\therefore\ \sin x° = \dfrac{2 \times 80}{11.3 \times 19.2}$

$\therefore\ x° = \sin^{-1}\left(\dfrac{160}{216.96}\right)$

$\approx 47.5°$

$\therefore\ x \approx 47.5$ or $180 - 47.5$

$\therefore\ x \approx 47.5$ or 132.5

b $\text{AC}^2 = 19.2^2 + 11.3^2 - 2 \times 19.2 \times 11.3 \times \cos x°$

$\text{AC} = \sqrt{368.64 + 127.69 - 433.92 \cos x°}$

But $x \approx 47.5$ or 132.5

$\therefore\ \text{AC} = \sqrt{496.33 - 433.92 \cos 47.5°}$ or $\text{AC} = \sqrt{496.33 - 433.92 \cos 132.5°}$

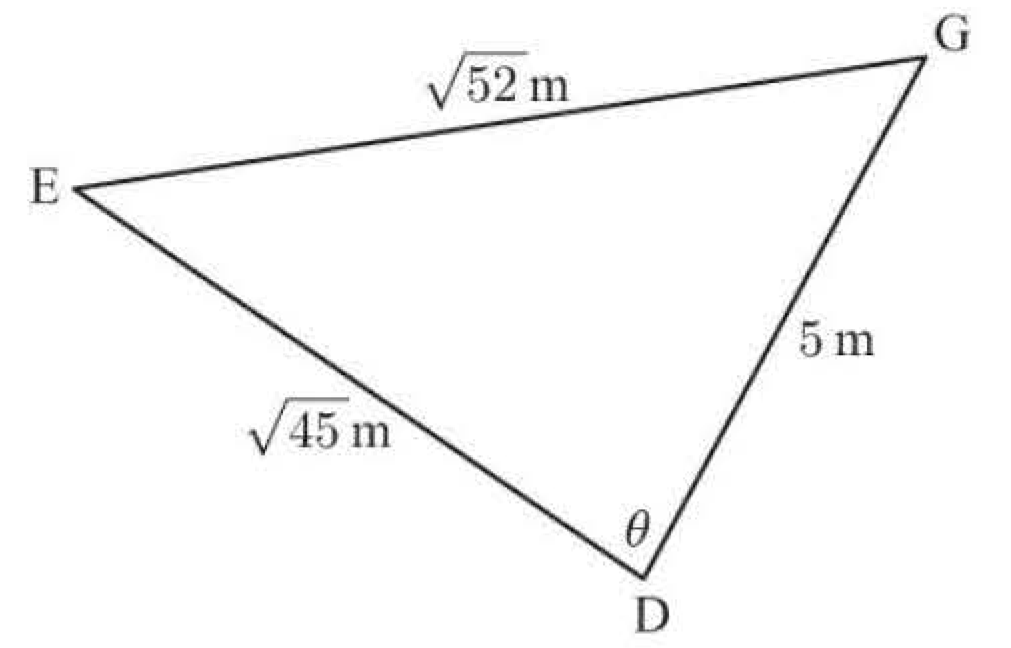
≈ 14.3 cm or ≈ 28.1 cm

3 Using Pythagoras,

$\text{ED} = \sqrt{6^2 + 3^2} = \sqrt{45}$ m

$\text{DG} = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$ m

$\text{EG} = \sqrt{6^2 + 4^2} = \sqrt{52}$ m



Using the cosine rule, $\cos\theta = \dfrac{(\sqrt{45})^2 + 5^2 - (\sqrt{52})^2}{2 \times \sqrt{45} \times 5}$

$\therefore\ \theta = \cos^{-1}\left(\frac{18}{10\sqrt{45}}\right)$

$\therefore\ \theta \approx 74.4°$ Thus $\widehat{EDG}$ measures $74.4°$.

4 **a** $\text{BD}^2 = 120^2 + 125^2 - 2 \times 120 \times 125 \cos 75°$

$\therefore \ \text{BD} = \sqrt{120^2 + 125^2 - 2 \times 120 \times 125 \cos 75°}$

≈ 149.2 m

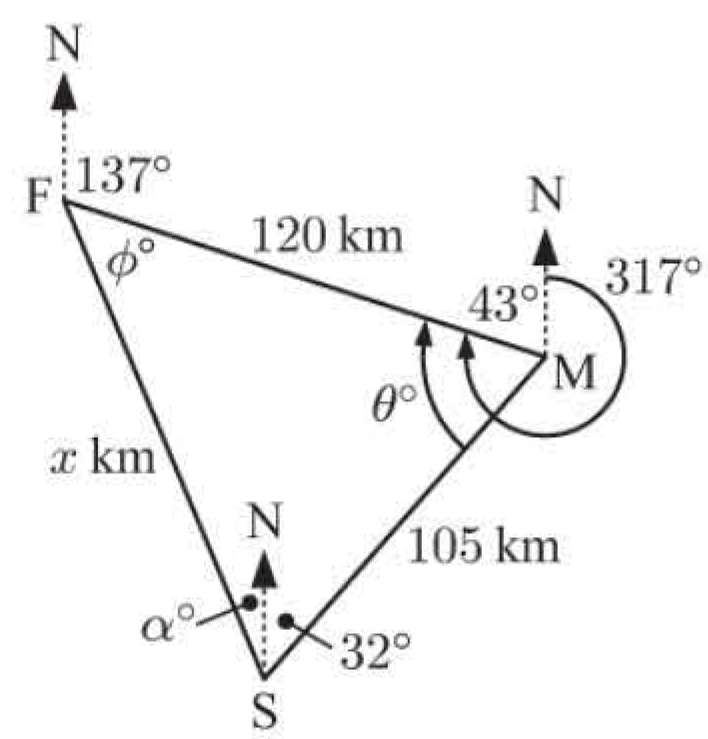
The area of the block $=$ area of $\triangle$ABD $+$ area of $\triangle$BCD

$\approx \frac{1}{2} \times 120 \times 125 \times \sin 75° + \frac{1}{2} \times 149.2 \times 90 \times \sin 30°$

$\approx 10\,600\ \text{m}^2$

b ≈ 1.06 ha $\{10\,000\ \text{m}^2 = 1\ \text{ha}\}$

5



distance $=$ speed $\times$ time

So, in 45 minutes, $140 \times \frac{3}{4} = 105$ km is travelled. $\{45 \text{ minutes} = \frac{3}{4} \text{ hour}\}$

In 40 minutes, $180 \times \frac{2}{3} = 120$ km is travelled. $\{40 \text{ minutes} = \frac{2}{3} \text{ hour}\}$

We notice that $\theta + 43 + 32 = 180$ {co-interior angles add to $180°$}

$\therefore \ \theta = 105$

Using the cosine rule, $x^2 = 120^2 + 105^2 - 2 \times 120 \times 105 \times \cos 105°$

$\therefore \ x = \sqrt{120^2 + 105^2 - 2 \times 120 \times 105 \times \cos 105°}$

$\therefore \ x \approx 178.74$

So, the car is 179 km from the start.

Now $\dfrac{\sin \phi°}{105} \approx \dfrac{\sin 105°}{178.74}$

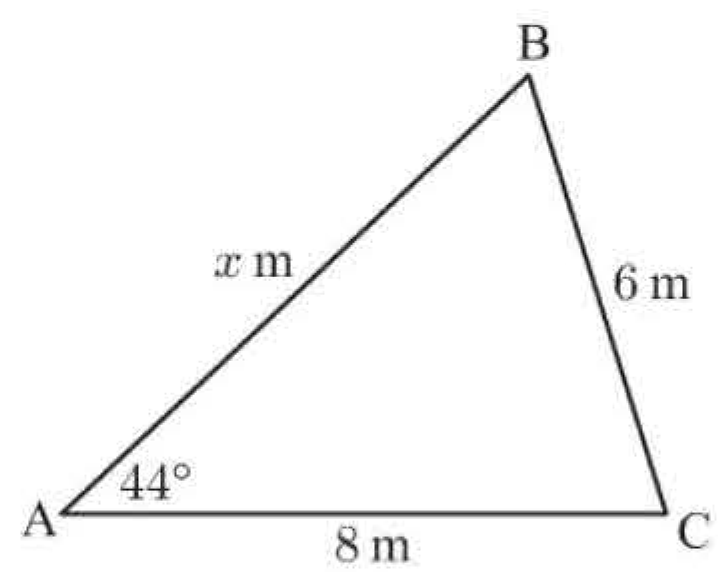
$\therefore \ \sin \phi° \approx \dfrac{105 \times \sin 105°}{178.74}$

$\therefore \ \phi \approx 34.6$

$\therefore \ \alpha \approx 180 - 105 - 34.6 - 32 \approx 8.4 \approx 8$

So, the bearing from its starting point is $360° - 8° = 352°$.

6 **a**



By the cosine rule, $6^2 = x^2 + 8^2 - 2 \times x \times 8 \times \cos 44°$

$\therefore \ 36 = x^2 + 64 - 16x \times \cos 44°$

$\therefore \ x^2 - 11.51x + 28 \approx 0$

$\therefore \ x \approx \dfrac{11.51 \pm \sqrt{11.51^2 - 4(1)(28)}}{2}$

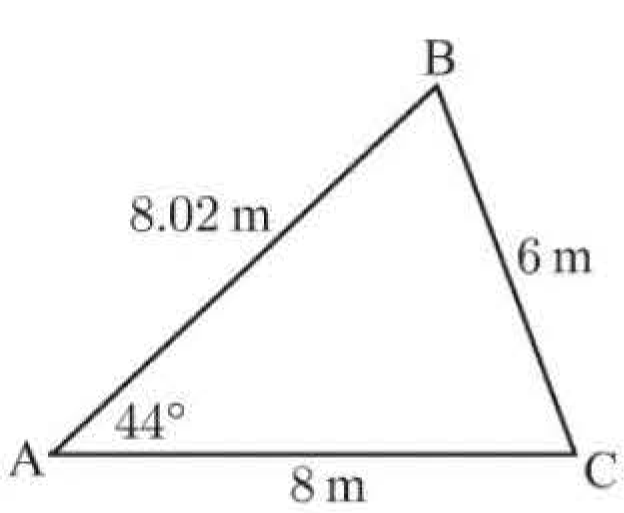
$\therefore \ x \approx \dfrac{11.51 \pm 4.524}{2}$

$\therefore \ x \approx 8.02$ or 3.49

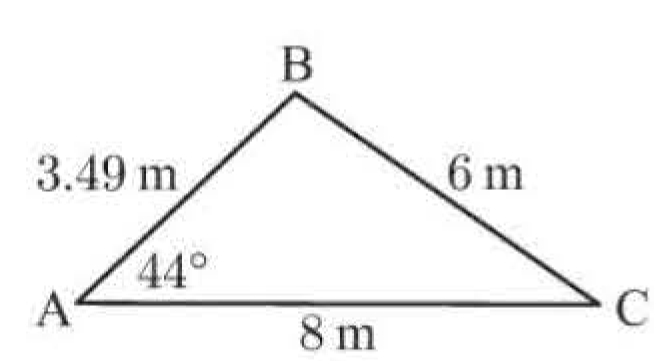
Frank needs additional information as there are two possible cases:

(1) when AB ≈ 8.02 m and

(2) when AB ≈ 3.49 m



Case (1)



Case (2)

b Volume $=$ area $\times$ depth

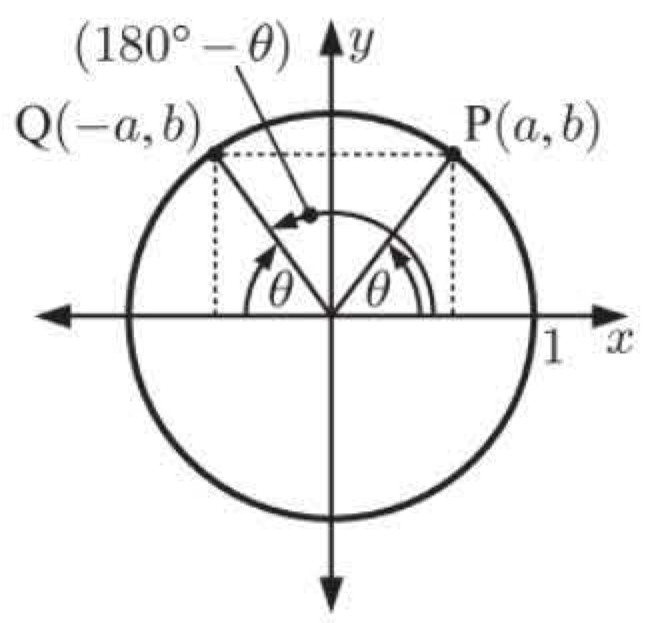
$= \frac{1}{2} \times 8 \times x \times \sin 44° \times 0.1$ and is a maximum when $x \approx 8.02$ m

$\approx 4 \times 8.02 \times \sin 44° \times 0.1$

$\approx 2.23\ \text{m}^3$

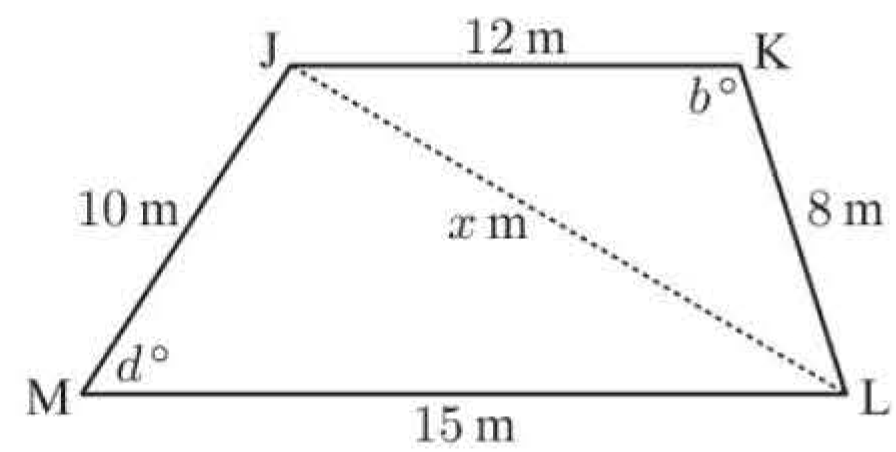
So, the maximum volume of soil needed is $2.23\ \text{m}^3$.

7 **a**



$\therefore \quad \cos(180° - \theta) = -a$
$= -\cos\theta \quad \{\cos\theta = a\}$

b **i** Using $\triangle$JLM,



$x^2 = 10^2 + 15^2 - 2 \times 10 \times 15 \cos d°$
$\therefore \quad x^2 = 325 - 300 \cos d° \quad .... (1)$

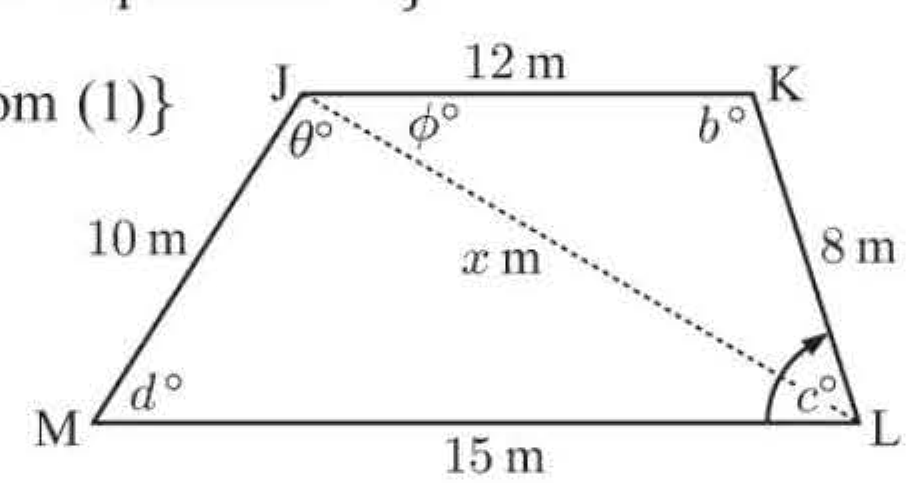
Using $\triangle$JLK,
$x^2 = 12^2 + 8^2 - 2 \times 12 \times 8 \cos b°$
$\therefore \quad x^2 = 208 - 192 \cos b° \quad .... (2)$

Equating (1) and (2), $\quad 325 - 300 \cos d° = 208 - 192 \cos b°$
$\therefore \quad 300 \cos d° - 192 \cos b° = 117$

ii If $b + d = 180$, then $b = 180 - d$
$\therefore \quad 300 \cos d° - 192 \cos(180 - d)° = 117$
$\therefore \quad 300 \cos d° + 192 \cos d° = 117 \quad$ {from **a**}
$\therefore \quad 492 \cos d° = 117$
$\therefore \quad d = \cos^{-1}\left(\frac{117}{492}\right)$
$\therefore \quad d \approx 76.2$
and $\quad b = 180 - d \approx 103.8$

iii If $b + d = 180$, then $a + c = 180$ also {angles in a quadrilateral}

If $d \approx 76.2$, then $x \approx \sqrt{325 - 300\cos(76.2)°}$ {from (1)}
$\therefore \quad x \approx 15.93$

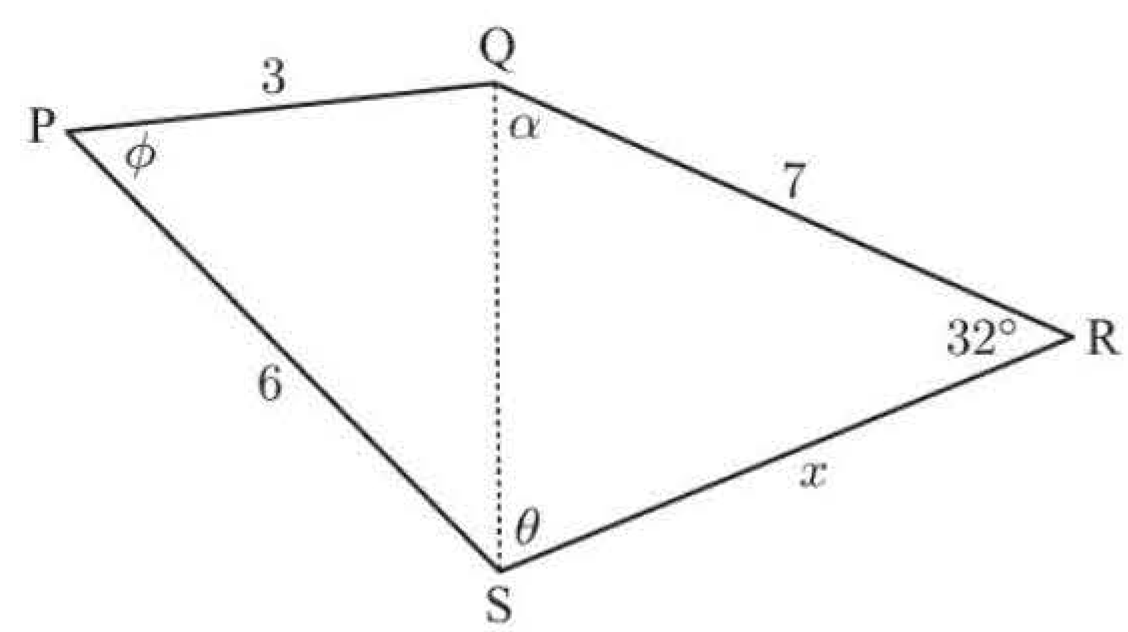


In $\triangle$JLM, $\cos\theta° \approx \dfrac{10^2 + 15.93^2 - 15^2}{2 \times 10 \times 15.93}$
$\therefore \quad \theta° \approx \cos^{-1}\left(\frac{128.7}{318.5}\right)$
$\therefore \quad \theta \approx 66.2$

In $\triangle$JLK, $\cos\phi° \approx \dfrac{12^2 + 15.93^2 - 8^2}{2 \times 12 \times 15.93}$
$\therefore \quad \phi° \approx \cos^{-1}\left(\frac{333.7}{382.2}\right)$
$\therefore \quad \phi \approx 29.2$

$\therefore \quad a = \theta + \phi \approx 95.4$ and $c = 180 - a \approx 84.6$

8 **a** $QS^2 = 6^2 + 3^2 - 2 \times 6 \times 3 \times \cos\phi$
$\therefore \quad QS = \sqrt{45 - 36\cos\phi}$



b **i** If $\phi = 50°$, $QS = \sqrt{45 - 36\cos 50°}$
≈ 4.675

$\therefore \quad \dfrac{\sin\theta}{7} \approx \dfrac{\sin 32°}{4.675}$

$\therefore \quad \sin\theta \approx \dfrac{7 \times \sin 32°}{4.675}$

$\therefore \quad \theta \approx \sin^{-1}\left(\dfrac{7 \times \sin 32°}{4.675}\right)$ or its supplement

$\therefore \quad \theta \approx 52.5°$ or $(180 - 52.5)°$

$\therefore \quad \widehat{RSQ} \approx 52.5°$ or $127.5°$

but since $\widehat{RSQ}$ is acute it must be $\approx 52.5°$.

ii $\alpha = 180° - 32° - \theta \quad \{\theta \approx 52.5°\}$

$\approx 95.5°$

$$\therefore \quad \frac{x}{\sin 95.5°} \approx \frac{7}{\sin 52.5°}$$

$$\therefore \quad x \approx \frac{7 \times \sin 95.5°}{\sin 52.5°}$$

$$\therefore \quad x \approx 8.78$$

$$\therefore \quad \text{perimeter} \approx 6 + 3 + 7 + 8.78$$

$$\approx 24.8 \text{ units}$$

iii area of PQRS = area of $\triangle$PQS + area of $\triangle$QRS

$$= \tfrac{1}{2} \times 3 \times 6 \times \sin 50° + \tfrac{1}{2} \times 7 \times 8.78 \times \sin 32°$$

$$\approx 23.2 \text{ units}^2$$

or if θ is obtuse we can similarly calculate that the area of PQRS ≈ 12.6 units^2.