Chapter 10
THE UNIT CIRCLE AND RADIAN MEASURE

EXERCISE 10A

1 **a** $180^\circ = \pi$ radians
$\therefore\ 90^\circ = \frac{\pi}{2}$ radians

b $180^\circ = \pi$ radians
$\therefore\ 60^\circ = \frac{\pi}{3}$ radians

c $180^\circ = \pi$ radians
$\therefore\ 30^\circ = \frac{\pi}{6}$ radians

d $180^\circ = \pi$ radians
$\therefore\ 18^\circ = \frac{\pi}{10}$ radians

e $180^\circ = \pi$ radians
$\therefore\ 9^\circ = \frac{\pi}{20}$ radians

f $180^\circ = \pi$ radians
$\therefore\ 45^\circ = \frac{\pi}{4}$ radians
$\therefore\ 135^\circ = \frac{3\pi}{4}$ radians

g $180^\circ = \pi$ radians
$\therefore\ 45^\circ = \frac{\pi}{4}$ radians
$\therefore\ 225^\circ = \frac{5\pi}{4}$ radians

h $180^\circ = \pi$ radians
$\therefore\ 90^\circ = \frac{\pi}{2}$ radians
$\therefore\ 270^\circ = \frac{3\pi}{2}$ radians

i $360^\circ = 2 \times 180^\circ$
$= 2\pi$ radians

j $720^\circ = 4 \times 180^\circ$
$= 4\pi$ radians

k $180^\circ = \pi$ radians
$\therefore\ 45^\circ = \frac{\pi}{4}$ radians
$\therefore\ 315^\circ = \frac{7\pi}{4}$ radians

l $180^\circ = \pi$ radians
$\therefore\ 540^\circ = 3\pi$ radians

m $180^\circ = \pi$ radians
$\therefore\ 36^\circ = \frac{\pi}{5}$ radians

n $180^\circ = \pi$ radians
$\therefore\ 10^\circ = \frac{\pi}{18}$ radians
$\therefore\ 80^\circ = \frac{8\pi}{18}$ radians
$= \frac{4\pi}{9}$ radians

o $180^\circ = \pi$ radians
$\therefore\ 10^\circ = \frac{\pi}{18}$ radians
$\therefore\ 230^\circ = \frac{23\pi}{18}$ radians

2 **a** 36.7°
$= 36.7 \times \frac{\pi}{180}$ radians
≈ 0.641 radians

b 137.2°
$= 137.2 \times \frac{\pi}{180}$ radians
≈ 2.39 radians

c 317.9°
$= 317.9 \times \frac{\pi}{180}$ radians
≈ 5.55 radians

d 219.6°
$= 219.6 \times \frac{\pi}{180}$ radians
≈ 3.83 radians

e 396.7°
$= 396.7 \times \frac{\pi}{180}$ radians
≈ 6.92 radians

3 **a** $\frac{\pi}{5}$
$= \frac{180^\circ}{5}$
$= 36^\circ$

b $\frac{3\pi}{5}$
$= \frac{3\times180^\circ}{5}$
$= 108^\circ$

c $\frac{3\pi}{4}$
$= \frac{3\times180^\circ}{4}$
$= 135^\circ$

d $\frac{\pi}{18}$
$= \frac{180^\circ}{18}$
$= 10^\circ$

e $\frac{\pi}{9}$
$= \frac{180^\circ}{9}$
$= 20^\circ$

f $\frac{7\pi}{9}$
$= \frac{7\times180^\circ}{9}$
$= 140^\circ$

g $\frac{\pi}{10}$
$= \frac{180^\circ}{10}$
$= 18^\circ$

h $\frac{3\pi}{20}$
$= \frac{3\times180^\circ}{20}$
$= 27^\circ$

i $\frac{7\pi}{6}$
$= \frac{7\times180^\circ}{6}$
$= 210^\circ$

j $\frac{\pi}{8}$
$= \frac{180^\circ}{8}$
$= 22.5^\circ$

4 **a** 2^c
$= 2 \times \frac{180}{\pi}$ degrees
$\approx 114.59^\circ$

b 1.53^c
$= 1.53 \times \frac{180}{\pi}$ degrees
$\approx 87.66^\circ$

c 0.867^c
$= 0.867 \times \frac{180}{\pi}$ degrees
$\approx 49.68^\circ$

d 3.179^c
$= 3.179 \times \frac{180}{\pi}$ degrees
$\approx 182.14^\circ$

e 5.267^c
$= 5.267 \times \frac{180}{\pi}$ degrees
$\approx 301.78^\circ$

5 **a**

Degrees	0	45	90	135	180	225	270	315	360
Radians	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π

b

Degrees	0	30	60	90	120	150	180	210	240	270	300	330	360
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π

EXERCISE 10B

1 **a** arc length $= \frac{7\pi}{4} \times 9$
≈ 49.5 cm

area $= \frac{1}{2} \times \frac{7\pi}{4} \times 9^2$
≈ 223 cm^2

b arc length $= 4.67 \times 4.93$
≈ 23.0 cm

area $= \frac{1}{2}(4.67) \times 4.93^2$
≈ 56.8 cm^2

2 **a** $\theta = 107.9°, \quad l = 5.92$

$\therefore \left(\frac{107.9}{360}\right) \times 2\pi \times r = 5.92$

$\therefore \quad r = \dfrac{5.92 \times 360}{107.9 \times 2 \times \pi}$

$\therefore \quad r \approx 3.14$ m

b area $= \left(\frac{107.9}{360}\right) \times \pi \times (3.1436)^2$
≈ 9.30 m^2

3 **a** area $= \frac{1}{2}\theta r^2$

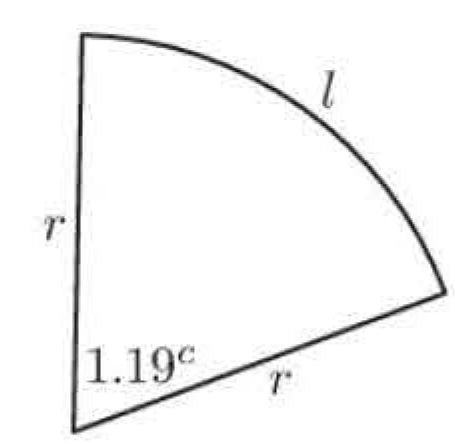
$\therefore \quad 20.8 = \frac{1}{2}(1.19) \times r^2$

$\therefore \quad \dfrac{20.8 \times 2}{1.19} = r^2$

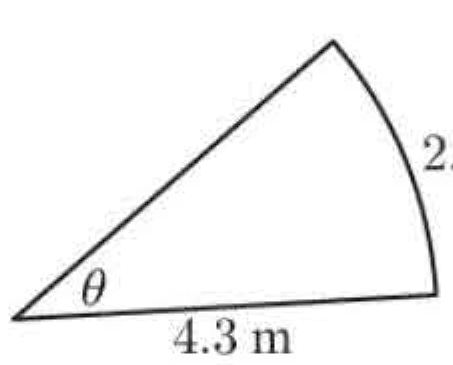
$\therefore \quad r = \sqrt{\dfrac{20.8 \times 2}{1.19}}$

$\therefore \quad r \approx 5.91$ cm

b perimeter
$= l + 2r$
$\approx 1.19 \times 5.912 + 2 \times 5.912$
≈ 18.9 cm



4 **a**



$l = \theta \times r$

$\therefore \quad 2.95 = \theta \times 4.3$

$\therefore \quad \theta \approx 0.686^c$

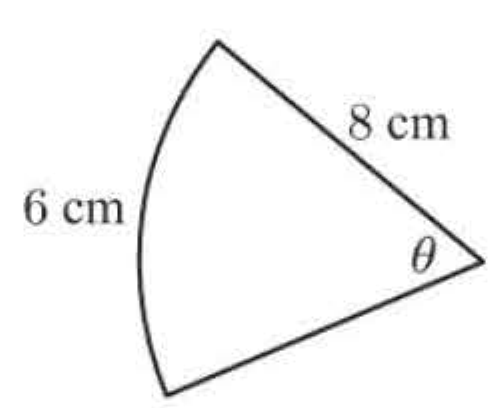
b area $= \frac{1}{2}\theta r^2$

$\therefore \quad 30 = \frac{1}{2} \times \theta \times 10^2$

$\therefore \quad \dfrac{30 \times 2}{100} = \theta$

$\therefore \quad \theta = 0.6^c$

5 **a**



$l = \theta r$

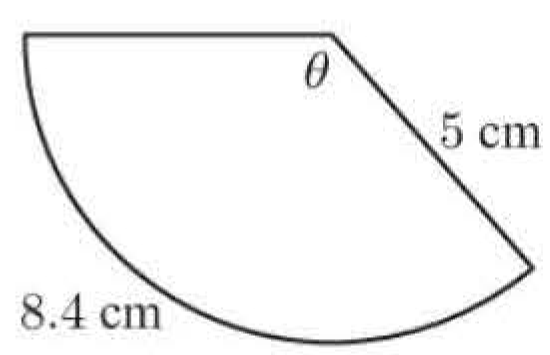
$\therefore \quad 6 = \theta \times 8$

$\therefore \quad \theta = \frac{6}{8}$

$\therefore \quad \theta = 0.75^c$

area $= \frac{1}{2}\theta r^2$
$= \frac{1}{2}(0.75) \times 8^2$
$= 24$ cm^2

b



$l = \theta r$

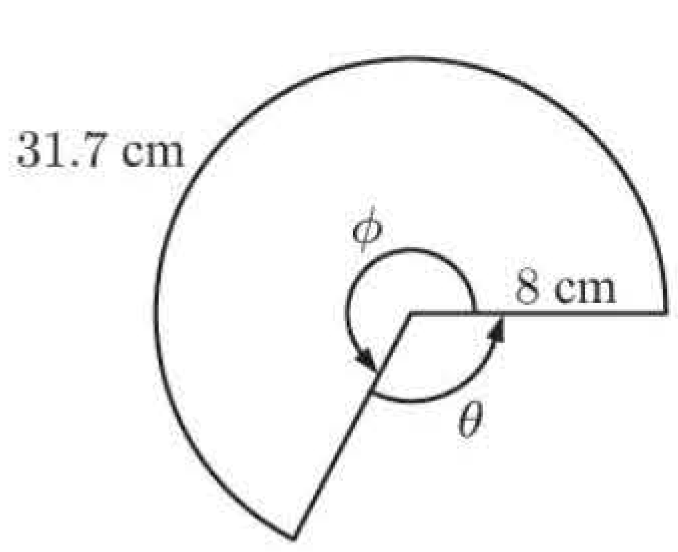
$\therefore \quad 8.4 = \theta \times 5$

$\therefore \quad \theta = \frac{8.4}{5}$

$\therefore \quad \theta = 1.68^c$

area $= \frac{1}{2}\theta r^2$
$= \frac{1}{2}(1.68) \times 5^2$
$= 21$ cm^2

c



$l = \phi r$

$\therefore \quad 31.7 = \phi \times 8$

$\therefore \quad \phi = \frac{31.7}{8}$

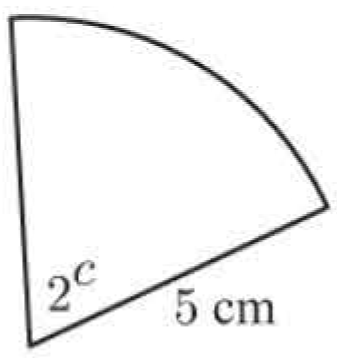
$\therefore \quad \phi \approx 3.96^c$

But $\theta = 2\pi - \phi$

$\therefore \quad \theta \approx 2.32^c$

area $= \frac{1}{2}\phi r^2$
$= \frac{1}{2} \times \frac{31.7}{8} \times 8^2$
$= 126.8$ cm^2

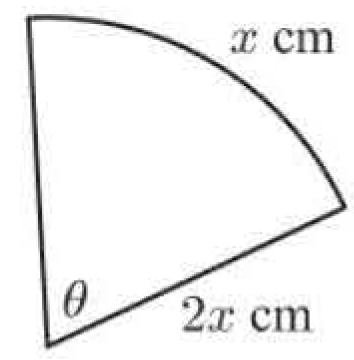
6



$$\begin{aligned}\text{arc length} &= \theta r\\ &= 2\times 5\\ &= 10 \text{ cm}\end{aligned}$$

$$\begin{aligned}\text{area} &= \tfrac{1}{2}\theta r^2\\ &= \tfrac{1}{2}\times 2\times 5^2\\ &= 25 \text{ cm}^2\end{aligned}$$

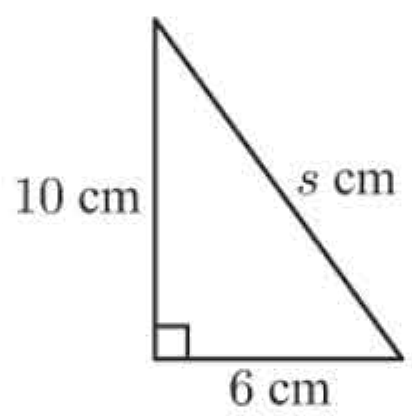
7



$$\begin{aligned}\text{arc length} &= \theta r\\ \therefore\quad x &= \theta(2x)\\ \therefore\quad \theta &= \tfrac{1}{2}\end{aligned}$$

$$\begin{aligned}\text{area} &= \tfrac{1}{2}\theta r^2\\ &= \tfrac{1}{2}\times\left(\tfrac{1}{2}\right)\times(2x)^2\\ &= x^2 \text{ cm}^2\end{aligned}$$

8 **a**



$$\begin{aligned}s^2 &= 6^2+10^2 \quad \{\text{Pythagoras}\}\\ \therefore\quad s &= \sqrt{6^2+10^2}\\ \therefore\quad s &\approx 11.6619\\ \therefore\quad s &\approx 11.7\end{aligned}$$

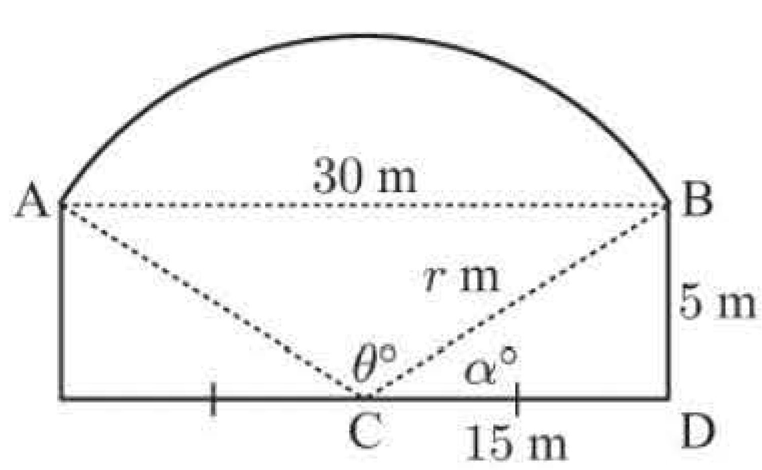
$\therefore$ slant length is 11.7 cm.

b $r = s \approx 11.7$

c
$$\begin{aligned}\text{arc length} &= \text{circumference of cone base}\\ &= 2\pi\times 6\\ &\approx 37.6991\\ &\approx 37.7 \text{ cm}\end{aligned}$$

d
$$\begin{aligned}\text{arc length} &= \theta r\\ \therefore\quad 37.6991 &\approx \theta\times 11.6619\\ \therefore\quad \theta &\approx \frac{37.6991}{11.6619}\\ \therefore\quad \theta &\approx 3.23 \text{ radians}\end{aligned}$$

9



a
$$\begin{aligned}\tan\alpha &= \tfrac{5}{15}\\ \therefore\quad \alpha &= \tan^{-1}\left(\tfrac{1}{3}\right)\\ \therefore\quad \alpha &\approx 18.43\end{aligned}$$

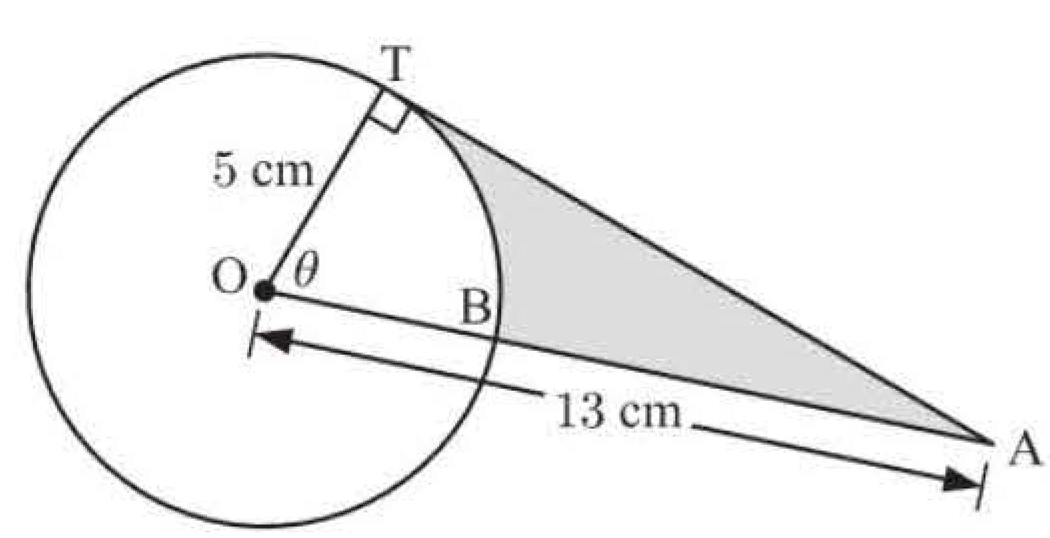
b
$$\begin{aligned}\theta + 2\alpha &= 180 \quad \{\text{angles on a line}\}\\ \therefore\quad \theta &\approx 180 - 2\times 18.43\\ \therefore\quad \theta &\approx 143.1\end{aligned}$$

c
$$\begin{aligned}\text{area} &= 2\times\text{area of }\triangle\text{CDB} + \text{area of sector}\\ &= 2\times\tfrac{1}{2}\times\text{CD}\times\text{BD} + \left(\tfrac{\theta}{360}\right)\times\pi\times r^2\end{aligned}$$

Now $r^2 = 5^2+15^2 = 250$

$$\begin{aligned}\therefore\quad \text{area} &\approx 2\times\tfrac{1}{2}\times 15\times 5 + \left(\tfrac{143.1}{360}\right)\times\pi\times 250\\ &\approx 387 \text{ m}^2\end{aligned}$$

10 Since [AT] is a tangent, $\text{O}\widehat{\text{T}}\text{A}$ is a right angle.



$$\begin{aligned}\therefore\quad \cos\theta &= \tfrac{5}{13}\\ \therefore\quad \theta &\approx 67.38°\end{aligned}$$

$$\begin{aligned}\text{arc length BT} &= \left(\tfrac{\theta}{360}\right)\times 2\pi r\\ &\approx \tfrac{67.38}{360}\times 2\times\pi\times 5\\ &\approx 5.88 \text{ cm}\end{aligned}$$

$$\begin{aligned}\text{AT}^2 + \text{OT}^2 &= \text{OA}^2 \quad \{\text{Pythagoras}\}\\ \therefore\quad \text{AT}^2 &= 13^2 - 5^2\\ \therefore\quad \text{AT} &= 12 \text{ cm}\end{aligned}$$

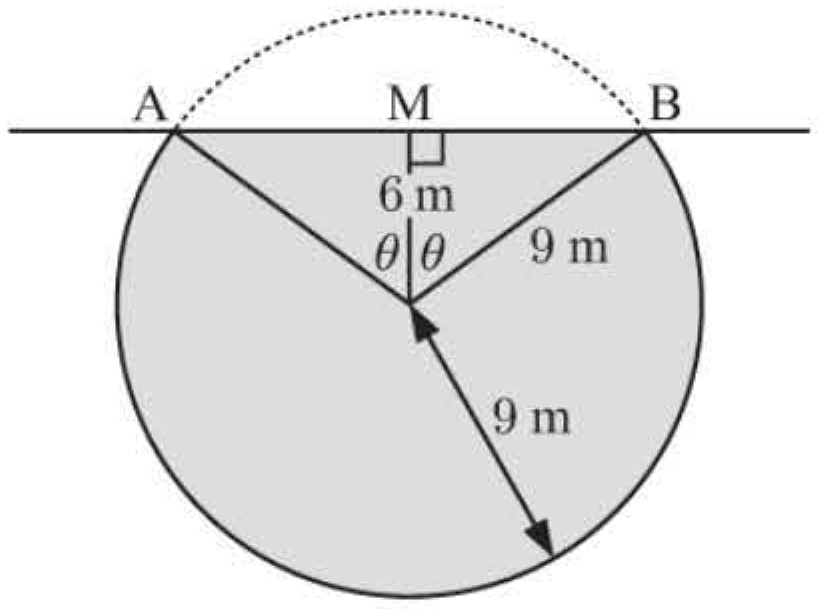
$$\begin{aligned}\therefore\quad \text{perimeter} &= \text{AT} + \text{arc length BT} + \text{AB}\\ &\approx 12 + 5.88 + (13-5)\\ &\approx 25.9 \text{ cm}\end{aligned}$$

11 **a**
$$\begin{aligned}l &= \left(\tfrac{\theta}{360}\right)\times 2\pi r\\ &= \frac{\frac{1}{60}}{360}\times 2\times\pi\times 6370 \text{ km}\\ &\approx 1.853 \text{ km}\end{aligned}$$

b $\text{speed} = \dfrac{\text{distance}}{\text{time}}$

$$\begin{aligned}\therefore\quad \text{time} &= \frac{\text{distance}}{\text{speed}}\\ &= \frac{2130 \text{ km}}{480 \text{ n miles h}^{-1}}\\ &= \frac{2130 \text{ km}}{480\times 1.853 \text{ km h}^{-1}}\\ &\approx 2.395 \text{ hours}\\ &\approx 2 \text{ hours } 24 \text{ min}\end{aligned}$$

12



$\cos\theta = \frac{6}{9} = \frac{2}{3}$

$\therefore\ \theta = \cos^{-1}\left(\frac{2}{3}\right)$

$\therefore\ \theta \approx 48.19°$

So, $360 - 2\theta \approx 263.62°$

Now $\text{MB} = \sqrt{9^2 - 6^2}$

$= \sqrt{45}$

$\therefore$ available feeding area

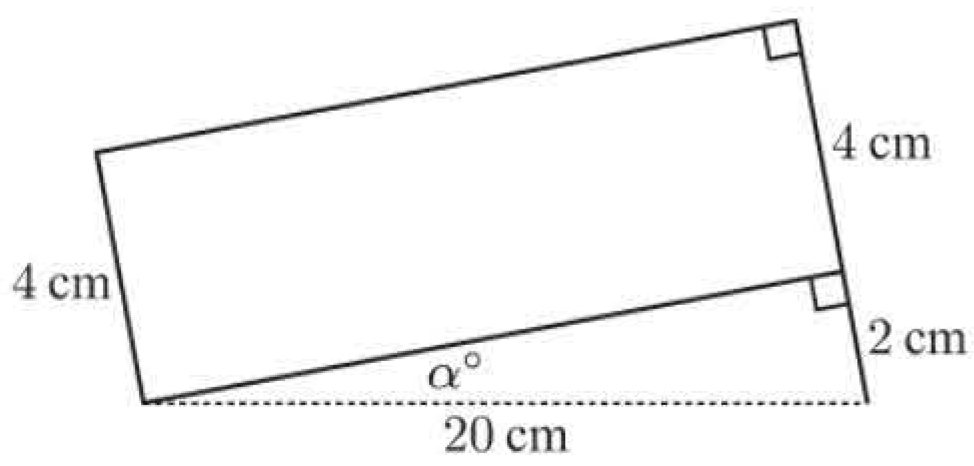
$=$ area of $\triangle$ + area of sector

$\approx \frac{1}{2} \times 2 \times \sqrt{45} \times 6$

$+ \left(\frac{263.62}{360}\right) \times \pi \times 9^2$

$\approx 227\ \text{m}^2$

13



a $\sin\alpha = \frac{2}{20} = 0.1$

$\therefore\ \alpha = \sin^{-1}(0.1)$

$\therefore\ \alpha \approx 5.739$

b $\theta + 90 + 90 + 2\alpha = 360$

$\therefore\ \theta = 180 - 2\alpha$

$\approx 180 - 2 \times 5.739$

≈ 168.5

c $\phi + \theta = 360$

$\therefore\ \phi \approx 360 - 168.5$

$\therefore\ \phi \approx 191.5$

d length of belt

$= 2 \times \sqrt{20^2 - 2^2}$

$+ \frac{\theta}{360} \times 2\pi \times 4$

$+ \frac{\phi}{360} \times 2\pi \times 6$

≈ 71.62 cm

EXERCISE 10C

1 **a** **i** A$(\cos 26°, \sin 26°)$, B$(\cos 146°, \sin 146°)$, C$(\cos 199°, \sin 199°)$

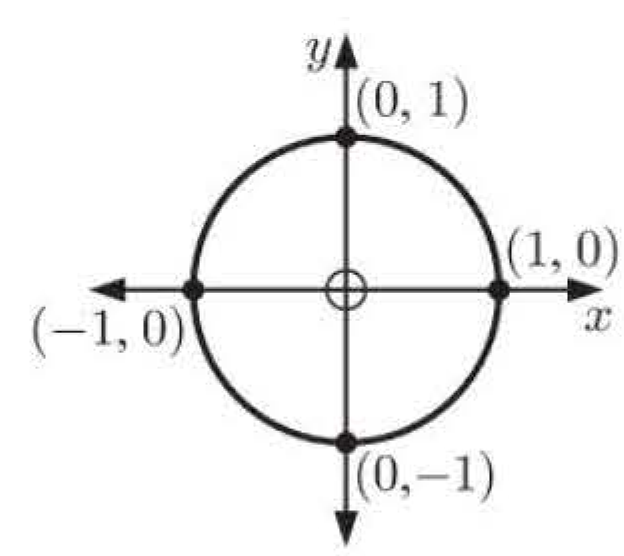
ii A$(0.899, 0.438)$, B$(-0.829, 0.559)$, C$(-0.946, -0.326)$

b **i** A$(\cos 123°, \sin 123°)$, B$(\cos 251°, \sin 251°)$, C$(\cos(-35°), \sin(-35°))$

ii A$(-0.545, 0.839)$, B$(-0.326, -0.946)$, C$(0.819, -0.574)$

2

θ (degrees)	$0°$	$90°$	$180°$	$270°$	$360°$	$450°$
θ (radians)	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$
sine	0	1	0	-1	0	1
cosine	1	0	-1	0	1	0
tangent	0	undef.	0	undef.	0	undef.



3 **a** **i** $\frac{1}{\sqrt{2}} \approx 0.707$

ii $\frac{\sqrt{3}}{2} \approx 0.866$

b

θ (degrees)	$30°$	$45°$	$60°$	$135°$	$150°$	$240°$	$315°$
θ (radians)	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{4\pi}{3}$	$\frac{7\pi}{4}$
sine	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$
cosine	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\frac{1}{\sqrt{2}}$
tangent	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	$\sqrt{3}$	-1

4 **a**

Quadrant	*Degree measure*	*Radian measure*	$\cos\theta$	$\sin\theta$	$\tan\theta$
1	$0° < \theta < 90°$	$0 < \theta < \frac{\pi}{2}$	+ve	+ve	+ve
2	$90° < \theta < 180°$	$\frac{\pi}{2} < \theta < \pi$	−ve	+ve	−ve
3	$180° < \theta < 270°$	$\pi < \theta < \frac{3\pi}{2}$	−ve	−ve	+ve
4	$270° < \theta < 360°$	$\frac{3\pi}{2} < \theta < 2\pi$	+ve	−ve	−ve

b **i** 1 and 4

ii 2 and 3

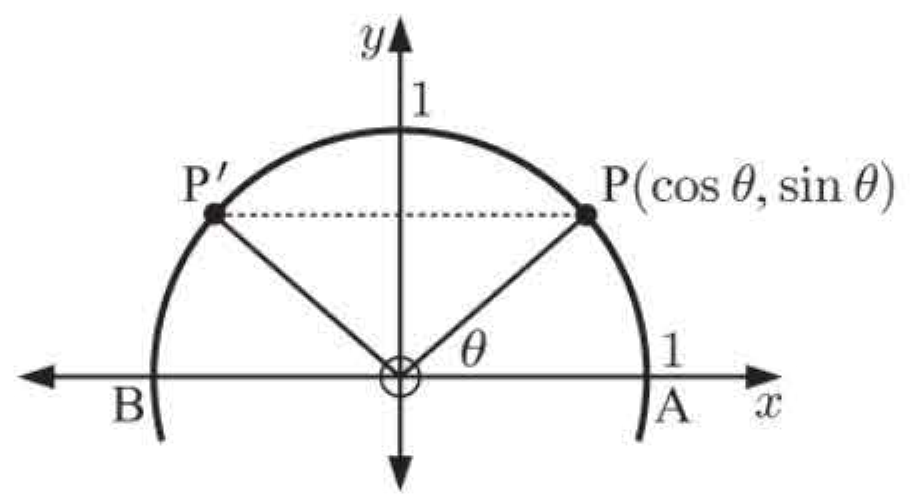
iii 3

iv 2

5 **a** **i** 0.985 **ii** 0.985 **iii** 0.866 **iv** 0.866 **v** 0.5 **vi** 0.5
vii 0.707 **viii** 0.707

b $\sin(180° - \theta) = \sin\theta$ as the points have the same y-coordinate.

c



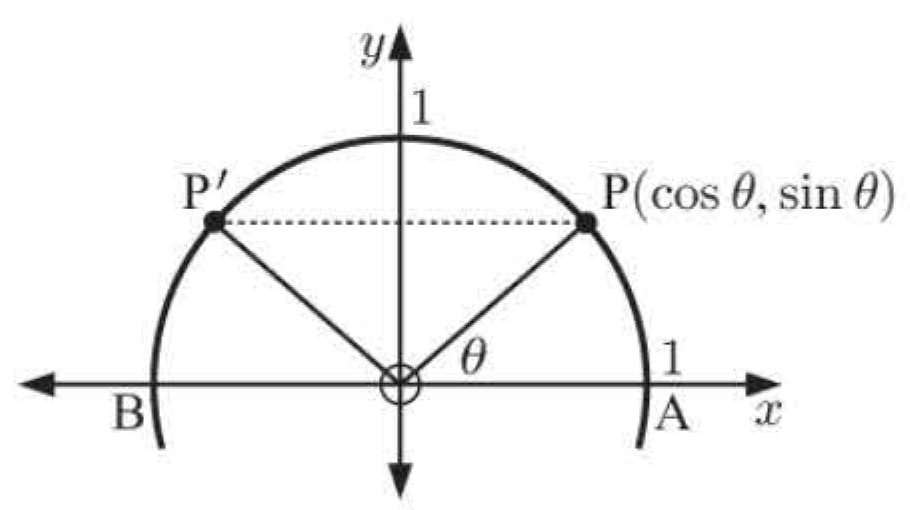
The diagram shows P reflected in the y-axis to P′, so $\mathrm{P'\widehat{O}B} = \mathrm{P\widehat{O}A} = \theta$, and P′ has coordinates $(-\cos\theta, \sin\theta)$.
But $\mathrm{A\widehat{O}P'} = 180° - \theta$ $\{\mathrm{A\widehat{O}P'} + \mathrm{P'\widehat{O}B} = 180°\}$,
so P′ has coordinates $(\cos(180° - \theta), \sin(180° - \theta))$.
$\therefore$ $\sin(180° - \theta) = \sin\theta$ {equating y-coordinates of P′}

d **i** $180° - 45° = 135°$ **ii** $180° - 51° = 129°$ **iii** $\pi - \frac{\pi}{3} = \frac{2\pi}{3}$
iv $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$ {using $\sin(180° - \theta) = \theta$}

6 **a** **i** 0.342 **ii** -0.342 **iii** 0.5 **iv** -0.5 **v** 0.906 **vi** -0.906
vii 0.174 **viii** -0.174

b $\cos(180° - \theta) = -\cos\theta$

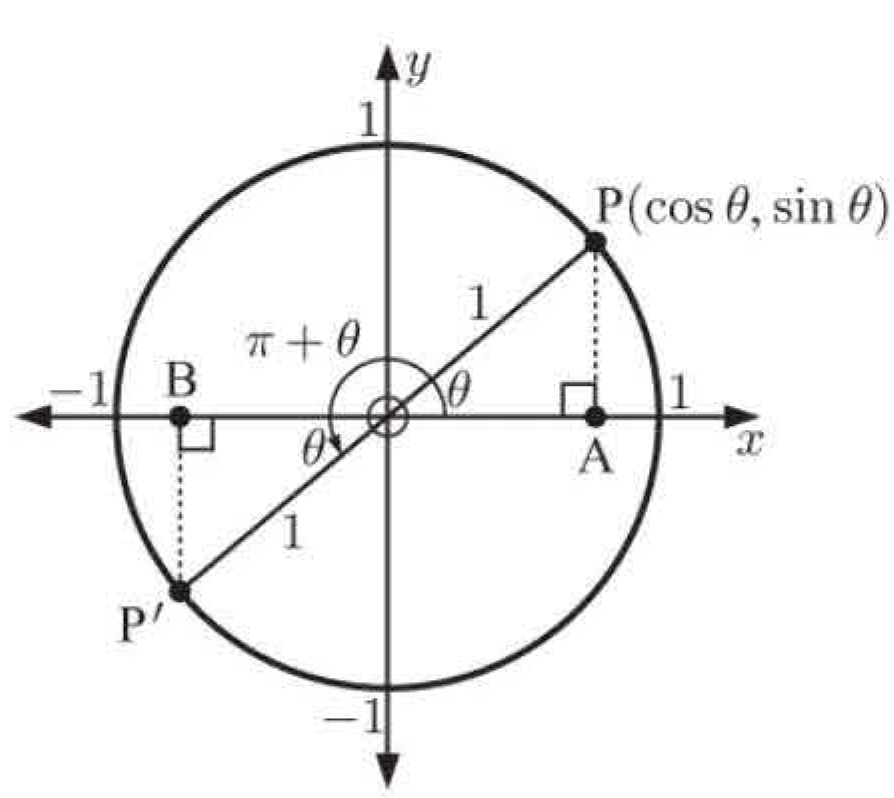
c



The diagram shows P reflected in the y-axis to P′, so $\mathrm{P'\widehat{O}B} = \mathrm{P\widehat{O}A} = \theta$, and P′ has coordinates $(-\cos\theta, \sin\theta)$.
But $\mathrm{A\widehat{O}P'} = 180° - \theta$ $\{\mathrm{A\widehat{O}P'} + \mathrm{P'\widehat{O}B} = 180°\}$,
so P′ has coordinates $(\cos(180° - \theta), \sin(180° - \theta))$.
$\therefore$ $\cos(180° - \theta) = -\cos\theta$ {equating x-coordinates of P′}

d **i** $180° - 40° = 140°$ **ii** $180° - 19° = 161°$ **iii** $\pi - \frac{\pi}{5} = \frac{4\pi}{5}$
iv $\pi - \frac{2\pi}{5} = \frac{3\pi}{5}$ {using $\cos(180° - \theta) = -\cos\theta$}

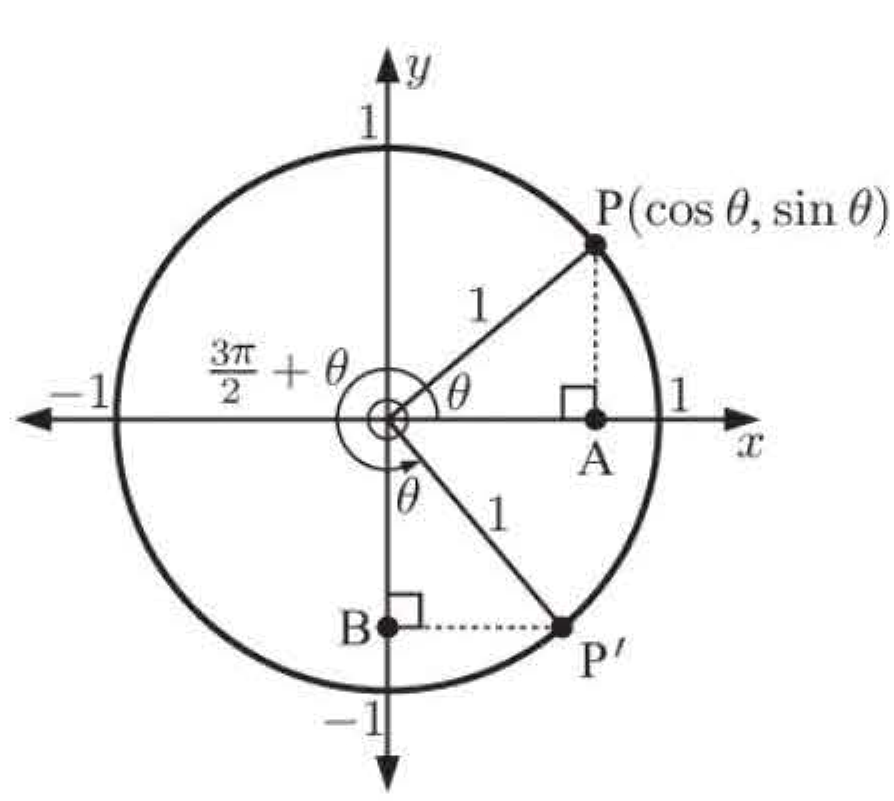
7 **a**



For $0 < \theta < \frac{\pi}{2}$:
The diagram shows P rotated through π to P′, so OP′ makes an angle of $\pi + \theta$ with the positive x-axis,
and $\mathrm{P'\widehat{O}B} = \mathrm{P\widehat{O}A} = \theta$ {vertically opposite angles}.
In $\triangle$s P′OB and POA:
- $\mathrm{OP'} = \mathrm{OP}$
- $\mathrm{P'\widehat{O}B} = \mathrm{P\widehat{O}A}$
- $\mathrm{P'\widehat{B}O} = \mathrm{P\widehat{A}O}$

$\therefore$ $\triangle$s P′OB and POA are congruent {AAcorS}
$\therefore$ $\mathrm{OB} = \mathrm{OA} = \cos\theta$
and $\mathrm{BP'} = \mathrm{AP} = \sin\theta$
$\therefore$ P′ has coordinates $(-\cos\theta, -\sin\theta)$
But P′ has coordinates $(\cos(\pi + \theta), \sin(\pi + \theta))$
$\therefore$ $\cos(\pi + \theta) = -\cos\theta$ and $\sin(\pi + \theta) = -\sin\theta$

b



For $0 < \theta < \frac{\pi}{2}$:
The diagram shows P rotated through $\frac{3\pi}{2}$ to P′,
so OP′ makes an angle of $\frac{3\pi}{2} + \theta$ with the positive x-axis.

reflex $\mathrm{A\widehat{O}B} = \frac{3\pi}{2}$ $\therefore$ $\mathrm{B\widehat{O}P'} = \text{reflex } \mathrm{A\widehat{O}P'} - \text{reflex } \mathrm{A\widehat{O}B}$

$$= \frac{3\pi}{2} + \theta - \frac{3\pi}{2}$$
$$= \theta$$

In $\triangle$s P′OB and POA:
- $\mathrm{OP'} = \mathrm{OP}$
- $\mathrm{B\widehat{O}P'} = \mathrm{A\widehat{O}P}$
- $\mathrm{P'\widehat{B}O} = \mathrm{P\widehat{A}O}$

$\therefore$ $\triangle$s P′OB and POA are congruent {AAcorS}

$\therefore$ $P'B = PA = \sin\theta$

and $OB = OA = \cos\theta$

$\therefore$ P′ has coordinates $(\sin\theta, -\cos\theta)$

But P′ has coordinates $(\cos(\frac{3\pi}{2}+\theta), \sin(\frac{3\pi}{2}+\theta))$

$\therefore$ $\cos(\frac{3\pi}{2}+\theta) = \sin\theta$ and $\sin(\frac{3\pi}{2}+\theta) = -\cos\theta$

8 **a** $\sin 137°$
$= \sin(180-137)°$
$= \sin 43°$
≈ 0.6820

b $\sin 59°$
$= \sin(180-59)°$
$= \sin 121°$
≈ 0.8572

c $\cos 143°$
$= -\cos(180-143)°$
$= -\cos 37°$
≈ -0.7986

d $\cos 24°$
$= -\cos(180-24)°$
$= -\cos 156°$
≈ 0.9135

e $\sin 115°$
$= \sin(180-115)°$
$= \sin 65°$
≈ 0.9063

f $\cos 132°$
$= -\cos(180-132)°$
$= -\cos 48°$
≈ -0.6691

9 **a** $A\widehat{O}Q = 180° - \theta$ or $\pi - \theta$ radians

b [OQ] is a reflection of [OP] in the y-axis and so Q has coordinates $(-\cos\theta, \sin\theta)$.

c $\cos(180°-\theta) = -\cos\theta$, $\sin(180°-\theta) = \sin\theta$

10 **a**

θ^c	$\sin\theta$	$\sin(-\theta)$	$\cos\theta$	$\cos(-\theta)$
0.75	0.682	−0.682	0.732	0.732
1.772	0.980	−0.980	−0.200	−0.200
3.414	−0.269	0.269	−0.963	−0.963
6.25	−0.0332	0.0332	0.999	0.999
−1.17	−0.921	0.921	0.390	0.390

b Suspect that $\sin(-\theta) = -\sin\theta$ and $\cos(-\theta) = \cos\theta$.

c **i** P is reflected in the x-axis to Q, so Q has coordinates $(\cos\theta, -\sin\theta)$.
But Q has coordinates $(\cos(-\theta), \sin(-\theta))$.
$\therefore$ $Q(\cos(-\theta), \sin(-\theta)) = Q(\cos\theta, -\sin\theta)$.
So the suspicion is correct.

ii The point Q on the unit circle corresponds to the angle $(2\pi-\theta)$ and the angle $(-\theta)$.
$\therefore$ $\cos(2\pi-\theta) = \cos(-\theta)$
But $\cos(-\theta) = \cos\theta$ {from **c i**}
$\therefore$ $\cos(2\pi-\theta) = \cos\theta$

EXERCISE 10D.1

1 **a** $\cos^2\theta + \sin^2\theta = 1$
$\therefore$ $\cos^2\theta + (\frac{1}{2})^2 = 1$
$\therefore$ $\cos^2\theta = \frac{3}{4}$
$\therefore$ $\cos\theta = \pm\frac{\sqrt{3}}{2}$

b $\cos^2\theta + \sin^2\theta = 1$
$\therefore$ $\cos^2\theta + (-\frac{1}{3})^2 = 1$
$\therefore$ $\cos^2\theta = \frac{8}{9}$
$\therefore$ $\cos\theta = \pm\frac{\sqrt{8}}{3}$
$\therefore$ $\cos\theta = \pm\frac{2\sqrt{2}}{3}$

c $\cos^2\theta + \sin^2\theta = 1$
$\therefore$ $\cos^2\theta + 0^2 = 1$
$\therefore$ $\cos\theta = \pm 1$

d $\cos^2\theta + \sin^2\theta = 1$
$\therefore$ $\cos^2\theta + (-1)^2 = 1$
$\therefore$ $\cos\theta = 0$

2 **a** $\cos^2\theta + \sin^2\theta = 1$
$\therefore\ (\frac{4}{5})^2 + \sin^2\theta = 1$
$\therefore\ \sin^2\theta = \frac{9}{25}$
$\therefore\ \sin\theta = \pm\frac{3}{5}$

b $\cos^2\theta + \sin^2\theta = 1$
$\therefore\ (-\frac{3}{4})^2 + \sin^2\theta = 1$
$\therefore\ \sin^2\theta = \frac{7}{16}$
$\therefore\ \sin\theta = \pm\frac{\sqrt{7}}{4}$

c $\cos^2\theta + \sin^2\theta = 1$
$\therefore\ 1^2 + \sin^2\theta = 1$
$\therefore\ \sin^2\theta = 0$
$\therefore\ \sin\theta = 0$

d $\cos^2\theta + \sin^2\theta = 1$
$\therefore\ 0^2 + \sin^2\theta = 1$
$\therefore\ \sin\theta = \pm 1$

3 **a** $\cos^2\theta + \sin^2\theta = 1$
$\therefore\ \frac{4}{9} + \sin^2\theta = 1$
$\therefore\ \sin^2\theta = \frac{5}{9}$
$\therefore\ \sin\theta = \pm\frac{\sqrt{5}}{3}$
But θ is in quadrant 1
where $\sin\theta > 0$
$\therefore\ \sin\theta = \frac{\sqrt{5}}{3}$

b $\cos^2\theta + \sin^2\theta = 1$
$\therefore\ \cos^2\theta + \frac{4}{25} = 1$
$\therefore\ \cos^2\theta = \frac{21}{25}$
$\therefore\ \cos\theta = \pm\frac{\sqrt{21}}{5}$
But θ is in quadrant 2
where $\cos\theta < 0$
$\therefore\ \cos\theta = -\frac{\sqrt{21}}{5}$

c $\cos^2\theta + \sin^2\theta = 1$
$\therefore\ \cos^2\theta + \frac{9}{25} = 1$
$\therefore\ \cos^2\theta = \frac{16}{25}$
$\therefore\ \cos\theta = \pm\frac{4}{5}$
But θ is in quadrant 4
where $\cos\theta > 0$
$\therefore\ \cos\theta = \frac{4}{5}$

d $\cos^2\theta + \sin^2\theta = 1$
$\therefore\ \frac{25}{169} + \sin^2\theta = 1$
$\therefore\ \sin^2\theta = \frac{144}{169}$
$\therefore\ \sin\theta = \pm\frac{12}{13}$
But θ is in quadrant 3
where $\sin\theta < 0$
$\therefore\ \sin\theta = -\frac{12}{13}$

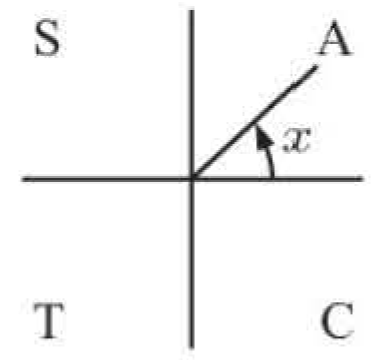
4 **a** $\cos^2 x + \sin^2 x = 1$
$\therefore\ \cos^2 x + \frac{1}{9} = 1$
$\therefore\ \cos^2 x = \frac{8}{9}$
$\therefore\ \cos x = \pm\frac{2\sqrt{2}}{3}$
But x is in quadrant 2
where $\cos x < 0$
$\therefore\ \cos x = -\frac{2\sqrt{2}}{3}$
and so $\tan x = \dfrac{\sin x}{\cos x} = \dfrac{\frac{1}{3}}{-\frac{2\sqrt{2}}{3}} = -\frac{1}{2\sqrt{2}}$

b $\cos^2 x + \sin^2 x = 1$
$\therefore\ \frac{1}{25} + \sin^2 x = 1$
$\therefore\ \sin^2 x = \frac{24}{25}$
$\therefore\ \sin x = \pm\frac{2\sqrt{6}}{5}$
But x is in quadrant 4
where $\sin x < 0$
$\therefore\ \sin x = -\frac{2\sqrt{6}}{5}$
and so $\tan x = \dfrac{\sin x}{\cos x} = \dfrac{-\frac{2\sqrt{6}}{5}}{\frac{1}{5}} = -2\sqrt{6}$

c $\cos^2 x + \sin^2 x = 1$
$\therefore\ \cos^2 x + \frac{1}{3} = 1$
$\therefore\ \cos^2 x = \frac{2}{3}$
$\therefore\ \cos x = \pm\frac{\sqrt{2}}{\sqrt{3}}$
But x is in quadrant 3
where $\cos x < 0$
$\therefore\ \cos x = -\frac{\sqrt{2}}{\sqrt{3}}$
and so $\tan x = \dfrac{\sin x}{\cos x} = \dfrac{-\frac{1}{\sqrt{3}}}{-\frac{\sqrt{2}}{\sqrt{3}}} = \frac{1}{\sqrt{2}}$

d $\cos^2 x + \sin^2 x = 1$
$\therefore\ \frac{9}{16} + \sin^2 x = 1$
$\therefore\ \sin^2 x = \frac{7}{16}$
$\therefore\ \sin x = \pm\frac{\sqrt{7}}{4}$
But x is in quadrant 2
where $\sin x > 0$
$\therefore\ \sin x = \frac{\sqrt{7}}{4}$
and so $\tan x = \dfrac{\sin x}{\cos x} = \dfrac{\frac{\sqrt{7}}{4}}{-\frac{3}{4}} = -\frac{\sqrt{7}}{3}$

5 **a** $\dfrac{\sin x}{\cos x} = \frac{2}{3}$



$\therefore\ \sin x = \frac{2}{3}\cos x$

Now $\cos^2 x + \sin^2 x = 1$

$\therefore\ \cos^2 x + \frac{4}{9}\cos^2 x = 1$

$\therefore\ \frac{13}{9}\cos^2 x = 1$

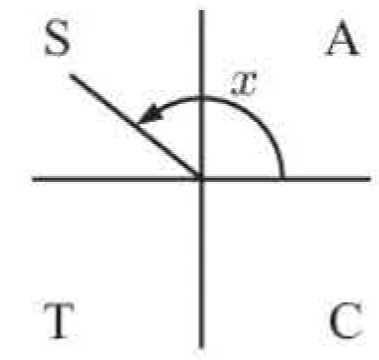
$\therefore\ \cos x = \pm\frac{3}{\sqrt{13}}$

But x is in quadrant 1

$\therefore$ $\cos x$ and $\sin x$ are positive.

$\therefore\ \cos x = \frac{3}{\sqrt{13}},\quad \sin x = \frac{2}{\sqrt{13}}$

b $\dfrac{\sin x}{\cos x} = -\frac{4}{3}$



$\therefore\ \sin x = -\frac{4}{3}\cos x$

Now $\cos^2 x + \sin^2 x = 1$

$\therefore\ \cos^2 x + \frac{16}{9}\cos^2 x = 1$

$\therefore\ \frac{25}{9}\cos^2 x = 1$

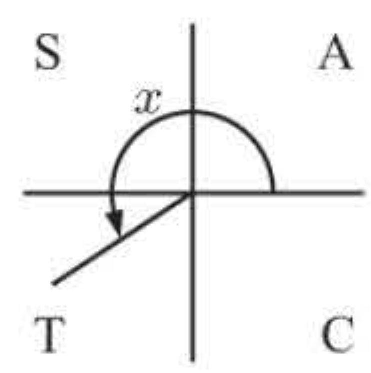
$\therefore\ \cos x = \pm\frac{3}{5}$

But x is in quadrant 2

$\therefore$ $\cos x$ is negative and $\sin x$ is positive.

$\therefore\ \cos x = -\frac{3}{5},\quad \sin x = \frac{4}{5}$

c $\dfrac{\sin x}{\cos x} = \frac{\sqrt{5}}{3}$



$\therefore\ \sin x = \frac{\sqrt{5}}{3}\cos x$

Now $\cos^2 x + \sin^2 x = 1$

$\therefore\ \cos^2 x + \frac{5}{9}\cos^2 x = 1$

$\therefore\ \frac{14}{9}\cos^2 x = 1$

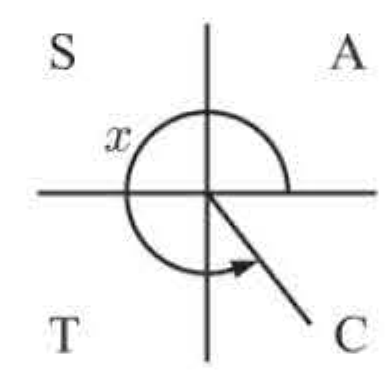
$\therefore\ \cos x = \pm\frac{3}{\sqrt{14}}$

But x is in quadrant 3

$\therefore$ $\cos x$ and $\sin x$ are both negative.

$\therefore\ \cos x = -\frac{3}{\sqrt{14}},\quad \sin x = -\frac{\sqrt{5}}{\sqrt{14}}$

d $\dfrac{\sin x}{\cos x} = -\frac{12}{5}$



$\therefore\ \sin x = -\frac{12}{5}\cos x$

Now $\cos^2 x + \sin^2 x = 1$

$\therefore\ \cos^2 x + \frac{144}{25}\cos^2 x = 1$

$\therefore\ \frac{169}{25}\cos^2 x = 1$

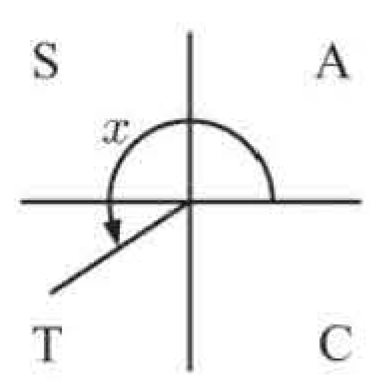
$\therefore\ \cos x = \pm\frac{5}{13}$

But x is in quadrant 4

$\therefore$ $\cos x$ is positive and $\sin x$ is negative.

$\therefore\ \cos x = \frac{5}{13},\quad \sin x = -\frac{12}{13}$

6 $\dfrac{\sin x}{\cos x} = k$



$\therefore\ \sin x = k\cos x$

Now $\cos^2 x + \sin^2 x = 1$

$\therefore\ \cos^2 x + k^2\cos^2 x = 1$

$\therefore\ (k^2+1)\cos^2 x = 1$

$\therefore\ \cos x = \dfrac{\pm 1}{\sqrt{k^2+1}}$

But x is in quadrant 3, $\therefore$ $\cos x$ and $\sin x$ are both negative.

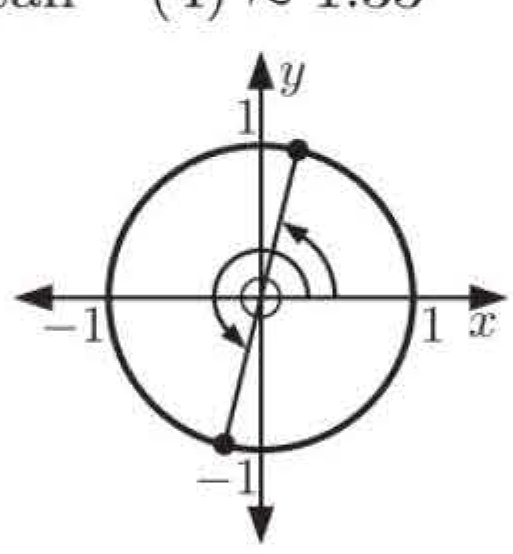
$\therefore\ \cos x = \dfrac{-1}{\sqrt{k^2+1}},\quad \sin x = \dfrac{-k}{\sqrt{k^2+1}}$

EXERCISE 10D.2

1 **a** $\tan\theta = 4$

Using technology,

$\tan^{-1}(4) \approx 1.33$



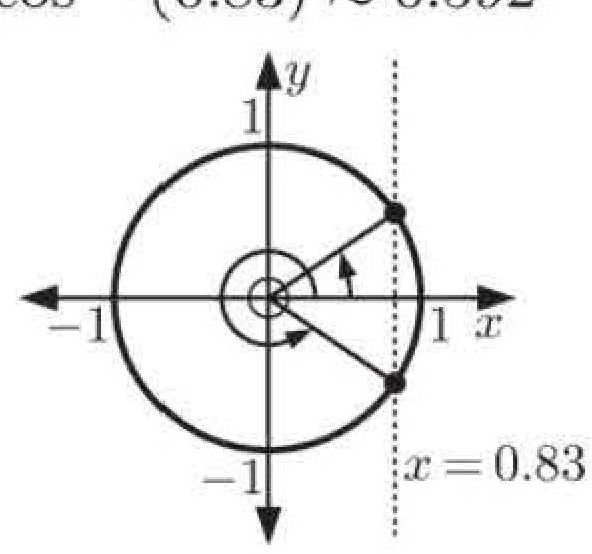
$\therefore\ \theta \approx 1.33$ or $\pi + 1.33$

$\therefore\ \theta \approx 1.33$ or 4.47

b $\cos\theta = 0.83$

Using technology,

$\cos^{-1}(0.83) \approx 0.592$



$\therefore\ \theta \approx 0.592$ or

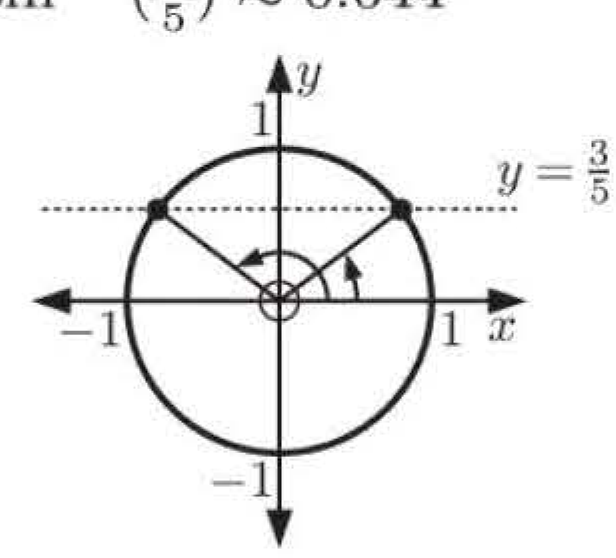
$2\pi - 0.592$

$\therefore\ \theta \approx 0.592$ or 5.69

c $\sin\theta = \frac{3}{5}$

Using technology,

$\sin^{-1}(\frac{3}{5}) \approx 0.644$



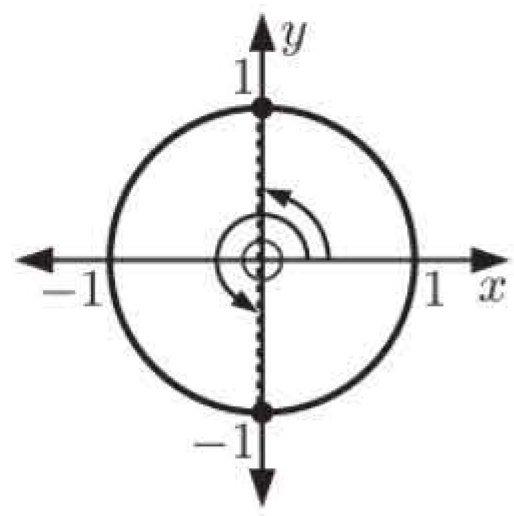
$\therefore\ \theta \approx 0.644$ or

$\pi - 0.644$

$\therefore\ \theta \approx 0.644$ or 2.50

d $\cos\theta = 0$

$\therefore \cos^{-1}(0) = \frac{\pi}{2}$



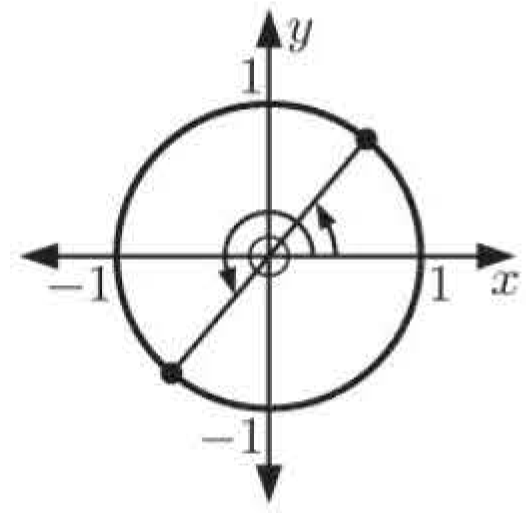
$\therefore \theta = \frac{\pi}{2}$ or $2\pi - \frac{\pi}{2}$

$\therefore \theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$

e $\tan\theta = 1.2$

Using technology,

$\tan^{-1}(1.2) \approx 0.876$



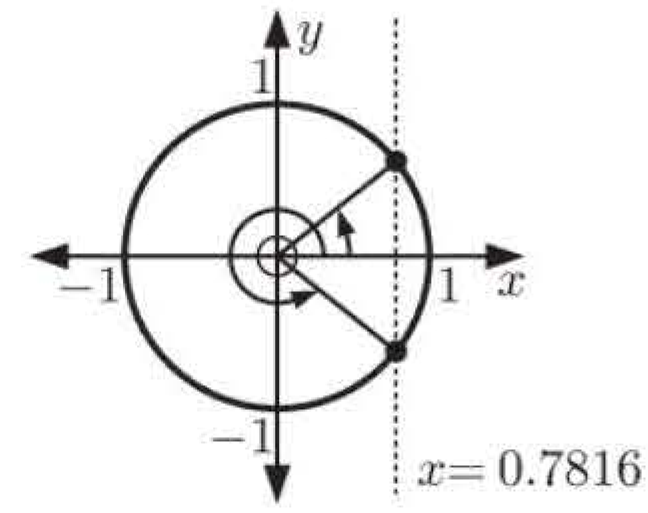
$\therefore \theta \approx 0.876$ or $\pi + 0.876$

$\therefore \theta \approx 0.876$ or 4.02

f $\cos\theta = 0.7816$

Using technology,

$\cos^{-1}(0.7816) \approx 0.674$



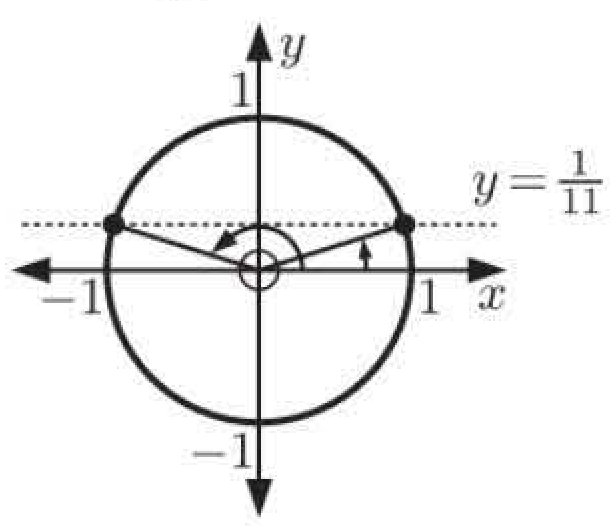
$\therefore \theta \approx 0.674$ or $2\pi - 0.674$

$\therefore \theta \approx 0.674$ or 5.61

g $\sin\theta = \frac{1}{11}$

Using technology,

$\sin^{-1}(\frac{1}{11}) \approx 0.0910$



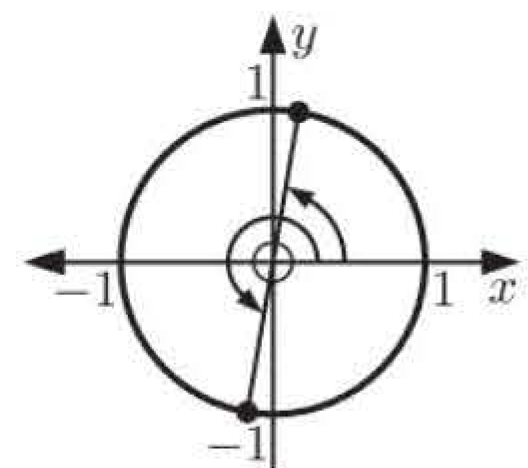
$\therefore \theta \approx 0.0910$ or $\pi - 0.0910$

$\therefore \theta \approx 0.0910$ or 3.05

h $\tan\theta = 20.2$

Using technology,

$\tan^{-1}(20.2) \approx 1.52$



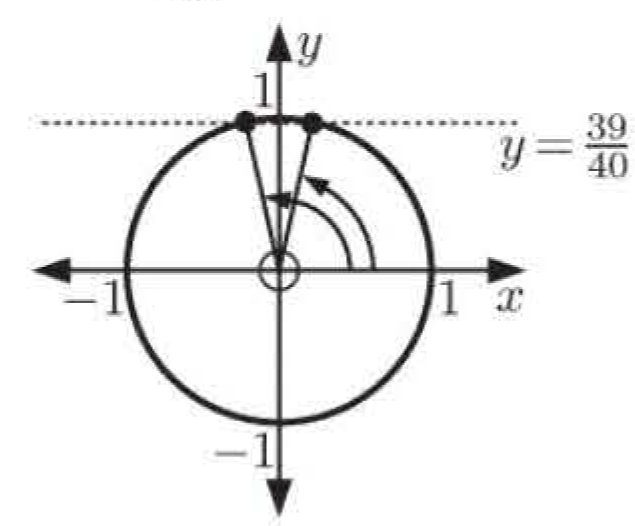
$\therefore \theta \approx 1.52$ or $\pi + 1.52$

$\therefore \theta \approx 1.52$ or 4.66

i $\sin\theta = \frac{39}{40}$

Using technology,

$\sin^{-1}(\frac{39}{40}) \approx 1.35$



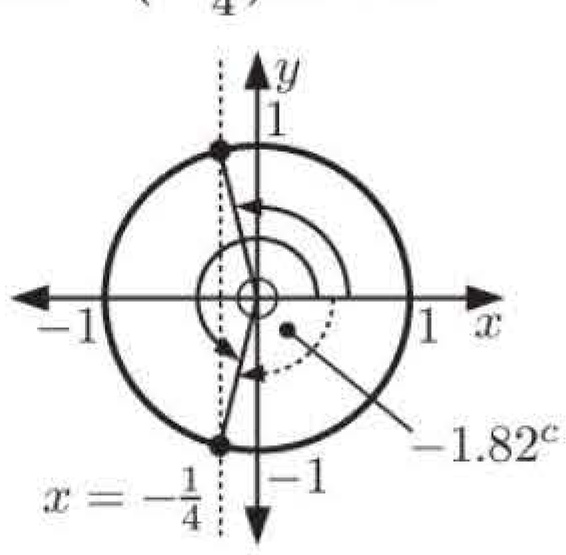
$\therefore \theta \approx 1.35$ or $\pi - 1.35$

$\therefore \theta \approx 1.35$ or 1.79

2 **a** $\cos\theta = -\frac{1}{4}$

Using technology,

$\cos^{-1}(-\frac{1}{4}) \approx 1.82$

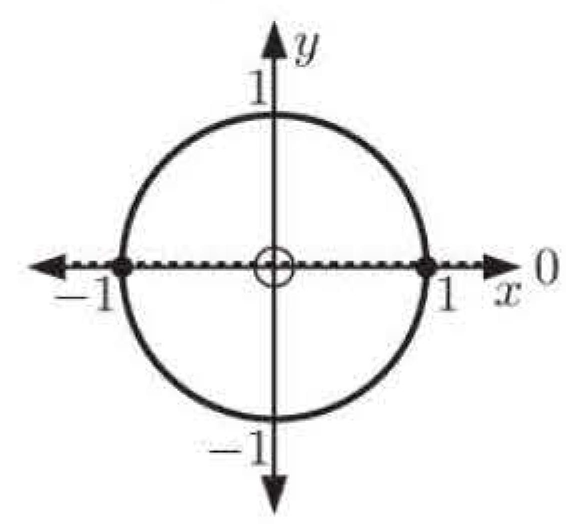


$\therefore \theta \approx 1.82$ or $2\pi - 1.82$

$\therefore \theta \approx 1.82$ or 4.46

b $\sin\theta = 0$

$\therefore \sin^{-1}(0) = 0$



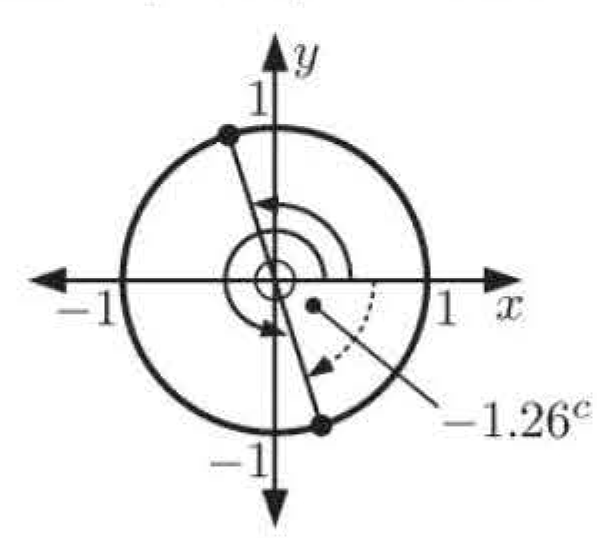
$\therefore \theta = 0$ or $\pi - 0$ or 2π

$\therefore \theta = 0, \pi$, or 2π

c $\tan\theta = -3.1$

Using technology,

$\tan^{-1}(-3.1) \approx -1.26$

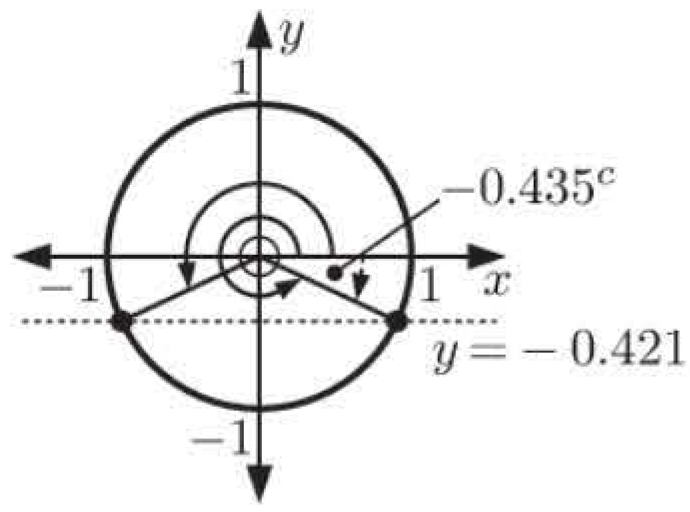


But $0 \leqslant \theta \leqslant 2\pi$

$\therefore \theta \approx \pi - 1.26$ or $2\pi - 1.26$

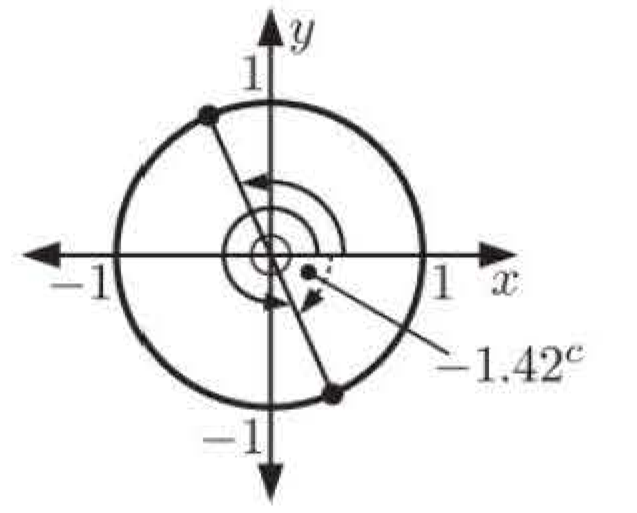
$\therefore \theta \approx 1.88$ or 5.02

d $\sin\theta = -0.421$
Using technology,
$\sin^{-1}(-0.421) \approx -0.435$



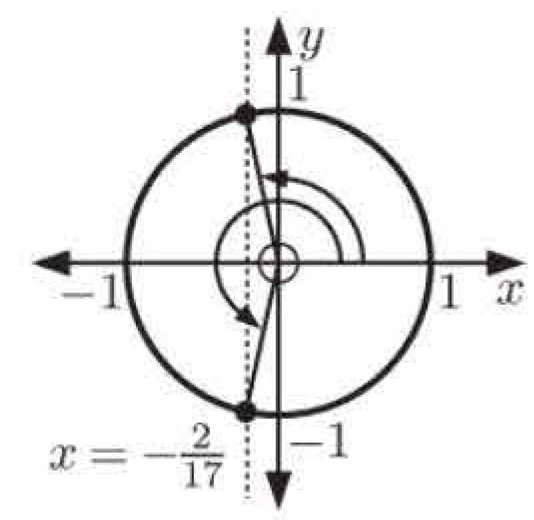
But $0 \leqslant \theta \leqslant 2\pi$
$\therefore\ \theta \approx \pi + 0.435$ or $2\pi - 0.435$
$\therefore\ \theta \approx 3.58$ or 5.85

e $\tan\theta = -6.67$
Using technology,
$\tan^{-1}(-6.67) \approx -1.42$



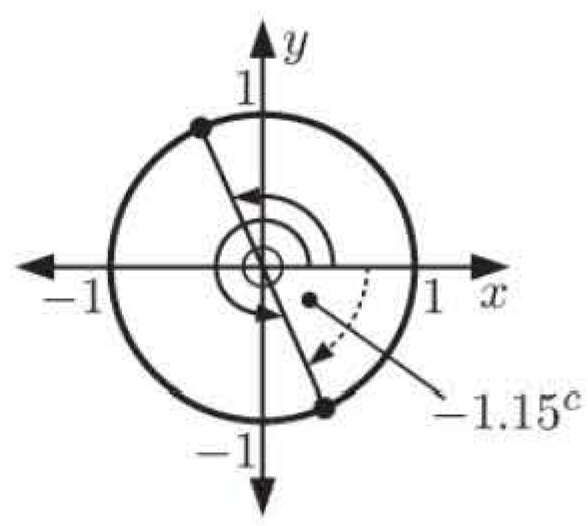
But $0 \leqslant \theta \leqslant 2\pi$
$\therefore\ \theta \approx \pi - 1.42$ or $2\pi - 1.42$
$\therefore\ \theta \approx 1.72$ or 4.86

f $\cos\theta = -\frac{2}{17}$
Using technology,
$\cos^{-1}(-\frac{2}{17}) \approx 1.69$



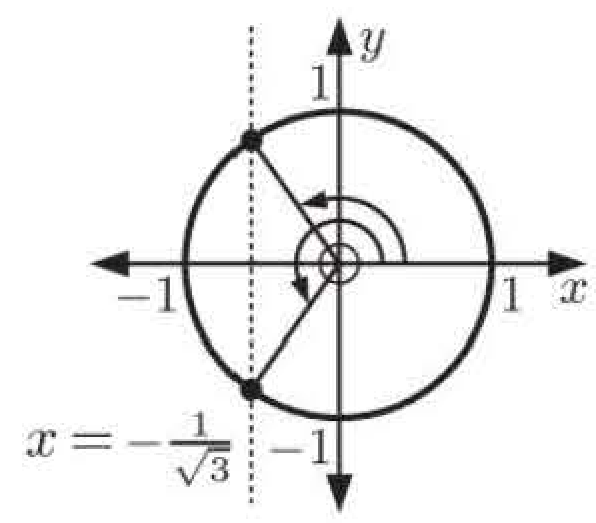
$\therefore\ \theta \approx 1.69$ or $2\pi - 1.69$
$\therefore\ \theta \approx 1.69$ or 4.59

g $\tan\theta = -\sqrt{5}$
Using technology,
$\tan^{-1}(-\sqrt{5}) \approx -1.15$



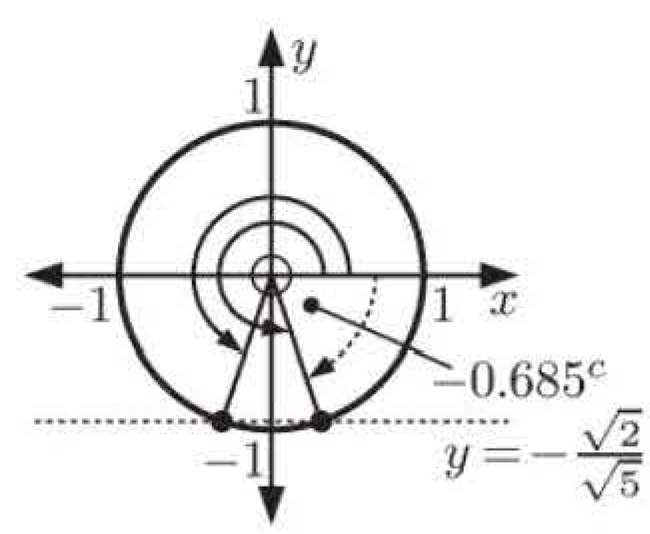
But $0 \leqslant \theta \leqslant 2\pi$
$\therefore\ \theta \approx \pi - 1.15$ or $2\pi - 1.15$
$\therefore\ \theta \approx 1.99$ or 5.13

h $\cos\theta = -\frac{1}{\sqrt{3}}$
Using technology,
$\cos^{-1}(-\frac{1}{\sqrt{3}}) \approx 2.19$



$\therefore\ \theta \approx 2.19$ or $2\pi - 2.19$
$\therefore\ \theta \approx 2.19$ or 4.10

i $\sin\theta = -\frac{\sqrt{2}}{\sqrt{5}}$
Using technology,
$\sin^{-1}(-\frac{\sqrt{2}}{\sqrt{5}}) \approx -0.685$



But $0 \leqslant \theta \leqslant 2\pi$
$\therefore\ \theta \approx \pi + 0.685$ or $2\pi - 0.685$
$\therefore\ \theta \approx 3.83$ or 5.60

EXERCISE 10E

1 **a**
$$\begin{aligned}&\sin\theta + \sin(-\theta)\\ &= \sin\theta - \sin\theta\\ &= 0\end{aligned}$$

b
$$\begin{aligned}&\tan(-\theta) - \tan\theta\\ &= -\tan\theta - \tan\theta\\ &= -2\tan\theta\end{aligned}$$

c
$$\begin{aligned}&2\cos\theta + \cos(-\theta)\\ &= 2\cos\theta + \cos\theta\\ &= 3\cos\theta\end{aligned}$$

d
$$\begin{aligned}&3\sin\theta - \sin(-\theta)\\ &= 3\sin\theta - -\sin\theta\\ &= 3\sin\theta + \sin\theta\\ &= 4\sin\theta\end{aligned}$$

e
$$\begin{aligned}&\cos^2(-\alpha)\\ &= \cos(-\alpha)\times\cos(-\alpha)\\ &= \cos\alpha\times\cos\alpha\\ &= \cos^2\alpha\end{aligned}$$

f
$$\begin{aligned}&\sin^2(-\alpha)\\ &= \sin(-\alpha)\times\sin(-\alpha)\\ &= -\sin\alpha\times -\sin\alpha\\ &= \sin^2\alpha\end{aligned}$$

g
$$\begin{aligned}&\cos(-\alpha)\cos\alpha - \sin(-\alpha)\sin\alpha\\ &= \cos\alpha\cos\alpha - -\sin\alpha\sin\alpha\\ &= \cos^2\alpha + \sin^2\alpha\\ &= 1\end{aligned}$$

2 **a**
$$\begin{aligned}&2\sin\theta - \cos(90^\circ - \theta)\\ &= 2\sin\theta - \sin\theta\\ &= \sin\theta\end{aligned}$$

b
$$\begin{aligned}&\sin(-\theta) - \cos(90^\circ - \theta)\\ &= -\sin\theta - \sin\theta\\ &= -2\sin\theta\end{aligned}$$

c
$$\begin{aligned}&\sin(90^\circ - \theta) - \cos\theta\\ &= \cos\theta - \cos\theta\\ &= 0\end{aligned}$$

d $3\cos(-\theta) - 4\sin(\frac{\pi}{2} - \theta)$
$= 3\cos\theta - 4\cos\theta$
$= -\cos\theta$

e $3\cos\theta + \sin(\frac{\pi}{2} - \theta)$
$= 3\cos\theta + \cos\theta$
$= 4\cos\theta$

f $\cos(\frac{\pi}{2} - \theta) + 4\sin\theta$
$= \sin\theta + 4\sin\theta$
$= 5\sin\theta$

3 $\sin(\theta - \phi) = \sin(-(\phi - \theta))$
$= -\sin(\phi - \theta)$
and $\cos(\theta - \phi) = \cos(-(\phi - \theta))$
$= \cos(\phi - \theta)$

4 **a** $\dfrac{\sin\theta}{\cos\theta} = \tan\theta$

b $\dfrac{\sin(-\theta)}{\cos(-\theta)} = \dfrac{-\sin\theta}{\cos\theta}$
$= -\tan\theta$

c $\dfrac{\sin(\frac{\pi}{2} - \theta)}{\cos\theta} = \dfrac{\cos\theta}{\cos\theta}$
$= 1$

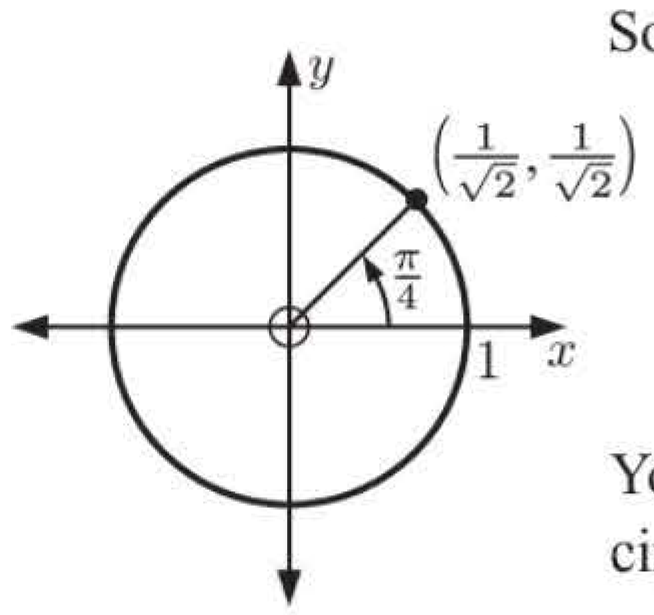
d $\dfrac{-\sin(-\theta)}{\cos\theta} = \dfrac{\sin\theta}{\cos\theta}$
$= \tan\theta$

e $\dfrac{\cos(\frac{\pi}{2} - \theta)}{\sin(\frac{\pi}{2} - \theta)} = \dfrac{\sin\theta}{\cos\theta}$
$= \tan\theta$

f $\dfrac{\cos(\frac{\pi}{2} - \theta)}{\cos\theta} = \dfrac{\sin\theta}{\cos\theta}$
$= \tan\theta$

EXERCISE 10F

1

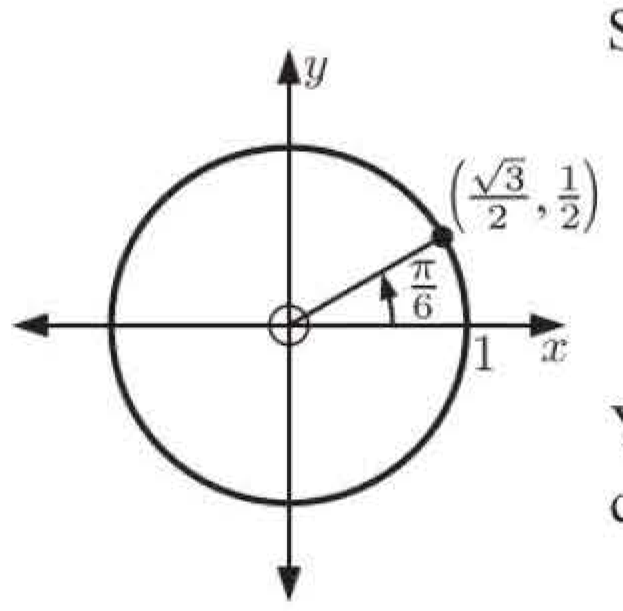


So, $\cos(\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$
$\sin(\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$
$\tan(\frac{\pi}{4}) = \dfrac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1$

You should draw separate unit circle diagrams for each case.

	a	b	c	d	e
$\sin\theta$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	0	$-\frac{1}{\sqrt{2}}$
$\cos\theta$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	-1	$-\frac{1}{\sqrt{2}}$
$\tan\theta$	1	-1	-1	0	1

2

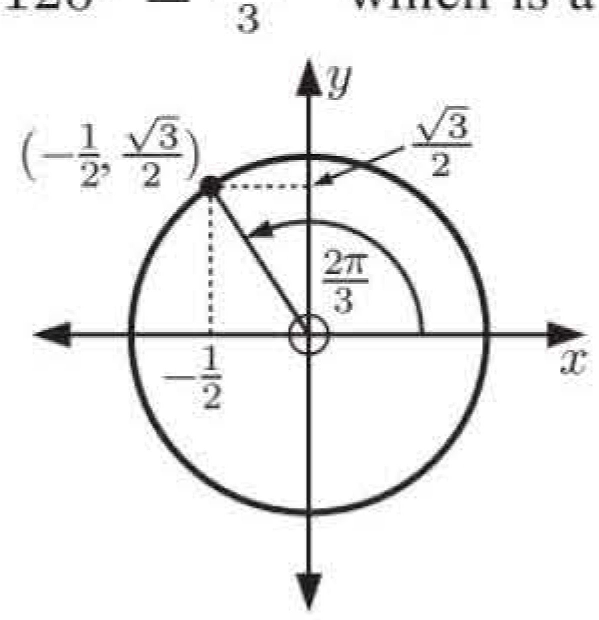


So, $\cos(\frac{\pi}{6}) = \frac{\sqrt{3}}{2}$
$\sin(\frac{\pi}{6}) = \frac{1}{2}$
$\tan(\frac{\pi}{6}) = \dfrac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$

You should draw separate unit circle diagrams for each case.

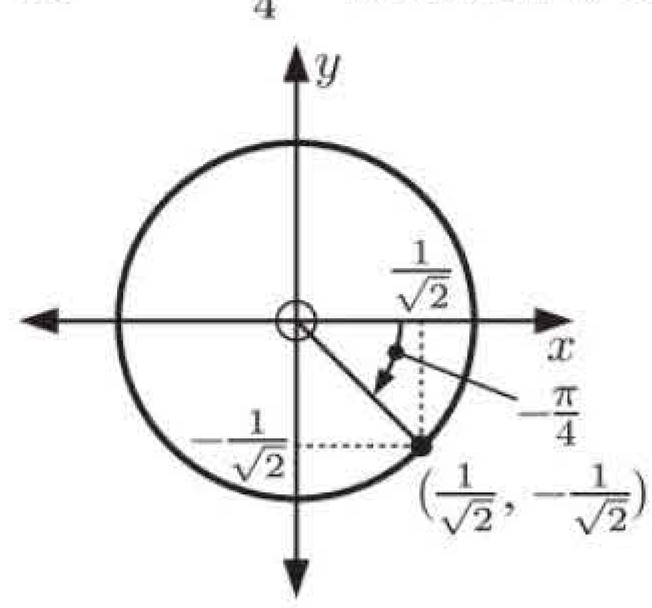
	a	b	c	d	e
$\sin\beta$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$
$\cos\beta$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\tan\beta$	$\frac{1}{\sqrt{3}}$	$-\sqrt{3}$	$\frac{1}{\sqrt{3}}$	$-\sqrt{3}$	$-\frac{1}{\sqrt{3}}$

3 **a** $120° = \frac{2\pi}{3}$ which is a multiple of $\frac{\pi}{6}$



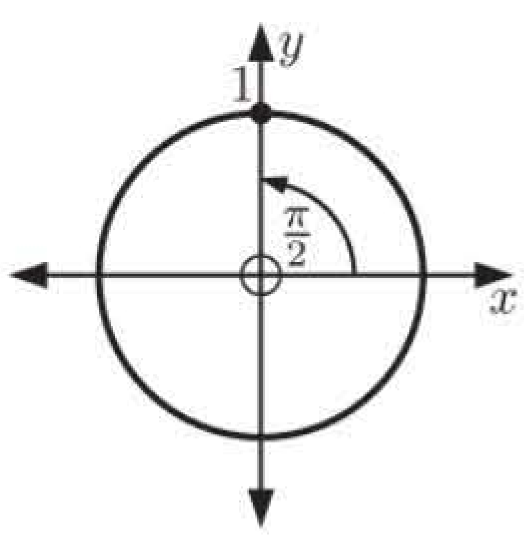
So, $\cos 120° = -\frac{1}{2}$
$\sin 120° = \frac{\sqrt{3}}{2}$
$\tan 120° = -\sqrt{3}$

b $-45° = -\frac{\pi}{4}$ which is a multiple of $\frac{\pi}{4}$



So, $\cos(-45°) = \frac{1}{\sqrt{2}}$
$\sin(-45°) = -\frac{1}{\sqrt{2}}$
$\tan(-45°) = -1$

4 **a** $90° = \frac{\pi}{2}$

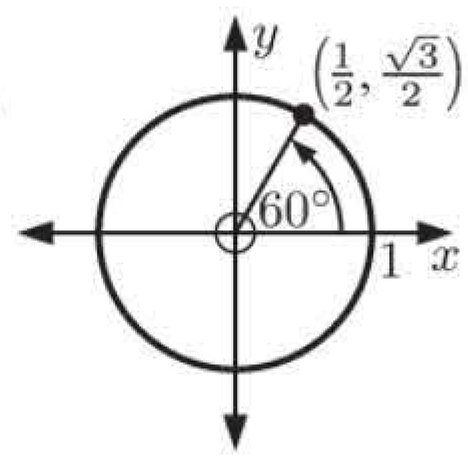


$\cos 90° = 0, \quad \sin 90° = 1$

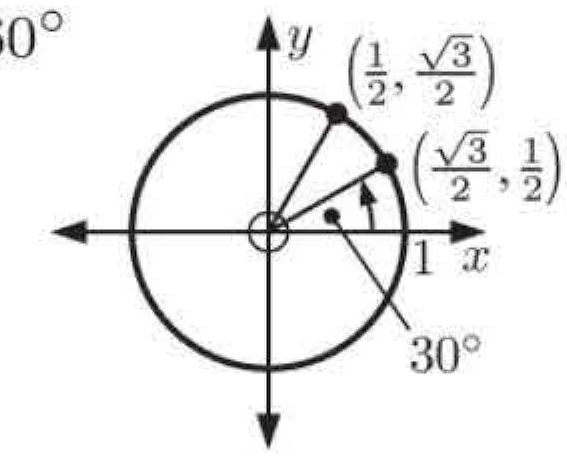
b $\tan 90° = \dfrac{\sin 90°}{\cos 90°} = \dfrac{1}{0}$

$\tan 90°$ is undefined

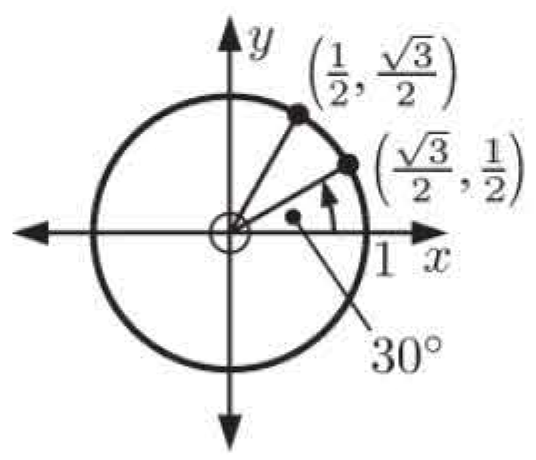
5 **a** $\sin^2 60°$
$= \sin 60° \times \sin 60°$
$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$
$= \frac{3}{4}$



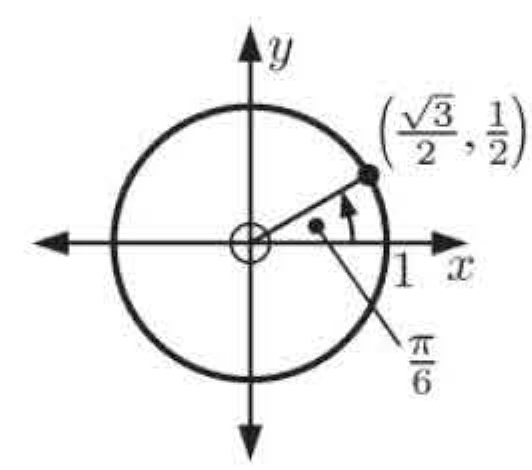
b $\sin 30° \cos 60°$
$= \frac{1}{2} \times \frac{1}{2}$
$= \frac{1}{4}$



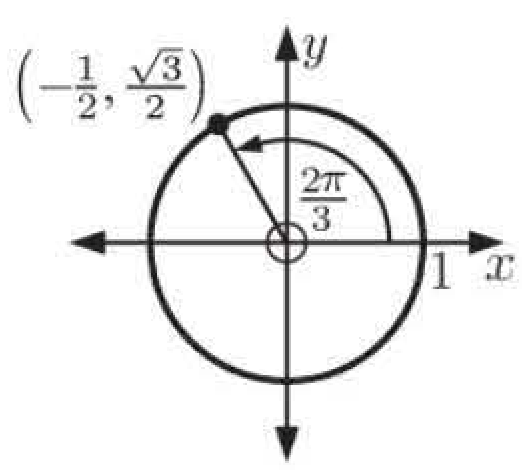
c $4 \sin 60° \cos 30°$
$= 4\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$
$= 3$



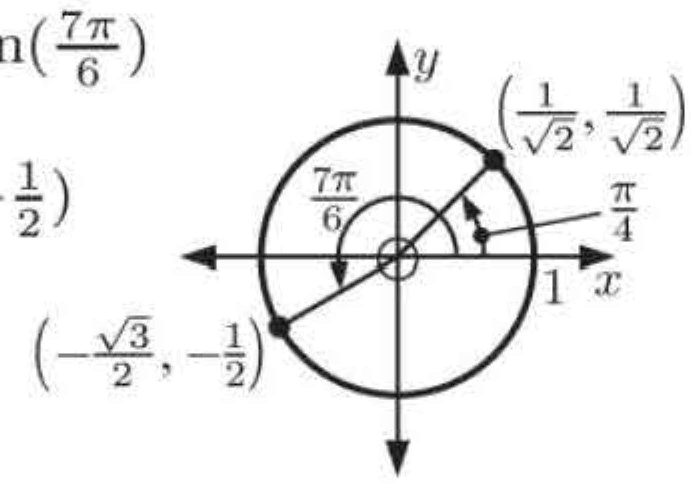
d $1 - \cos^2(\frac{\pi}{6})$
$= 1 - \left(\frac{\sqrt{3}}{2}\right)^2$
$= 1 - \frac{3}{4}$
$= \frac{1}{4}$



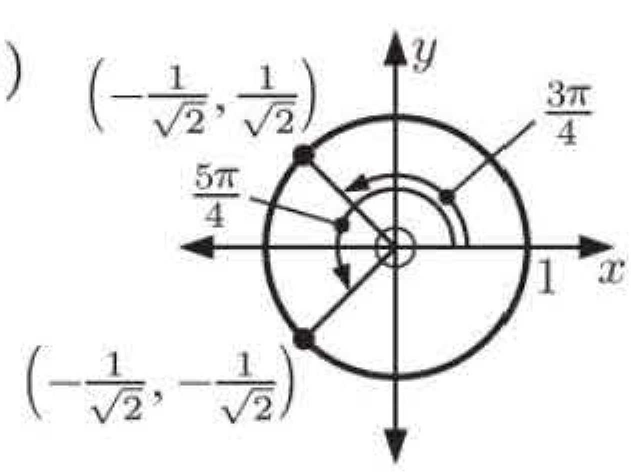
e $\sin^2(\frac{2\pi}{3}) - 1$
$= \left(\frac{\sqrt{3}}{2}\right)^2 - 1$
$= \frac{3}{4} - 1$
$= -\frac{1}{4}$



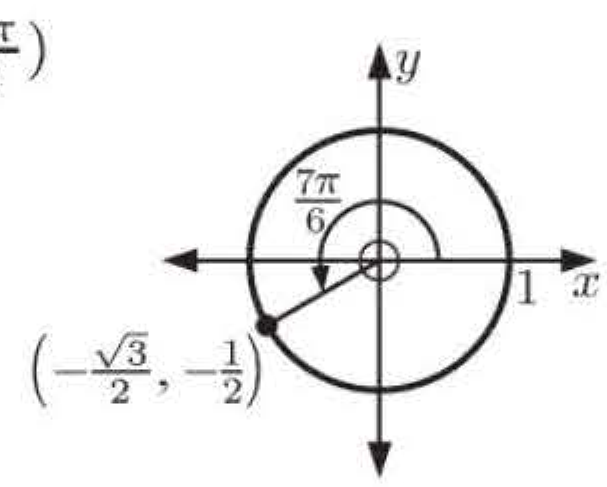
f $\cos^2(\frac{\pi}{4}) - \sin(\frac{7\pi}{6})$
$= \left(\frac{1}{\sqrt{2}}\right)^2 - (-\frac{1}{2})$
$= \frac{1}{2} + \frac{1}{2}$
$= 1$



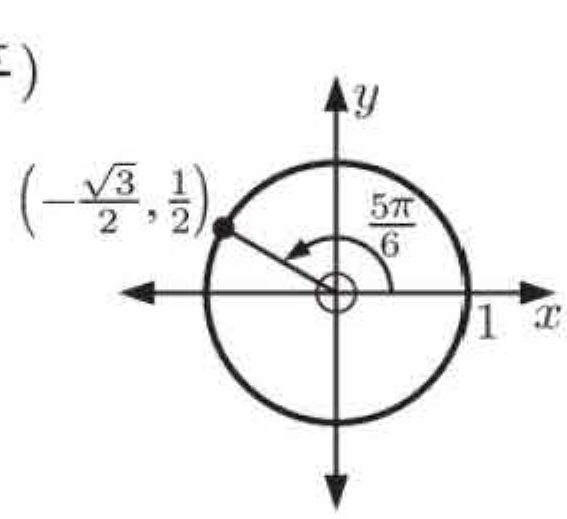
g $\sin(\frac{3\pi}{4}) - \cos(\frac{5\pi}{4})$
$= \frac{1}{\sqrt{2}} - \left(-\frac{1}{\sqrt{2}}\right)$
$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$
$= \frac{2}{\sqrt{2}}$ or $\sqrt{2}$



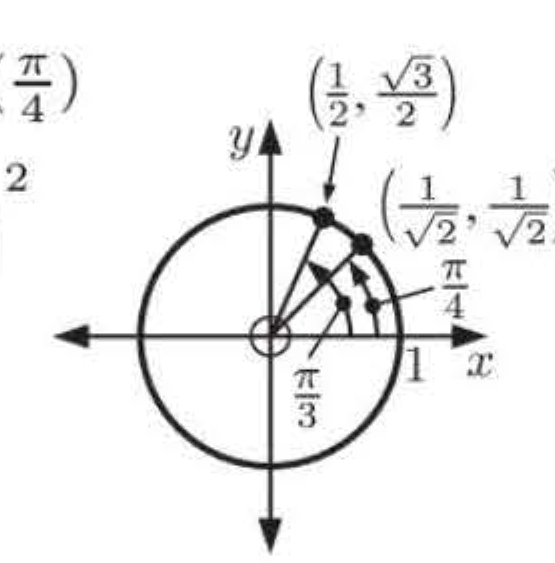
h $1 - 2\sin^2(\frac{7\pi}{6})$
$= 1 - 2(-\frac{1}{2})^2$
$= 1 - 2 \times \frac{1}{4}$
$= \frac{1}{2}$



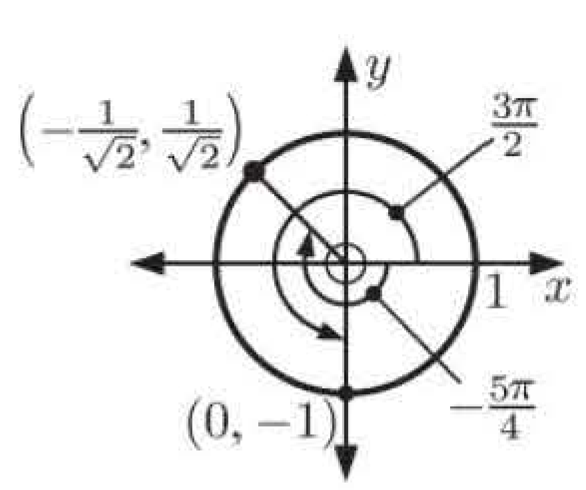
i $\cos^2(\frac{5\pi}{6}) - \sin^2(\frac{5\pi}{6})$
$= \left(-\frac{\sqrt{3}}{2}\right)^2 - (\frac{1}{2})^2$
$= \frac{3}{4} - \frac{1}{4}$
$= \frac{1}{2}$



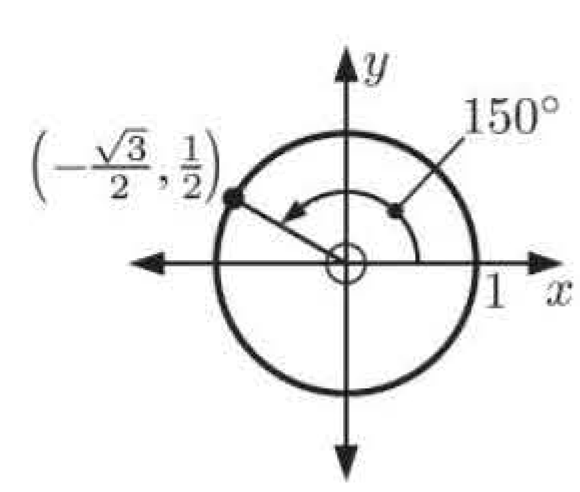
j $\tan^2(\frac{\pi}{3}) - 2\sin^2(\frac{\pi}{4})$
$= \left(\sqrt{3}\right)^2 - 2\left(\frac{1}{\sqrt{2}}\right)^2$
$= 3 - 2(\frac{1}{2})$
$= 2$



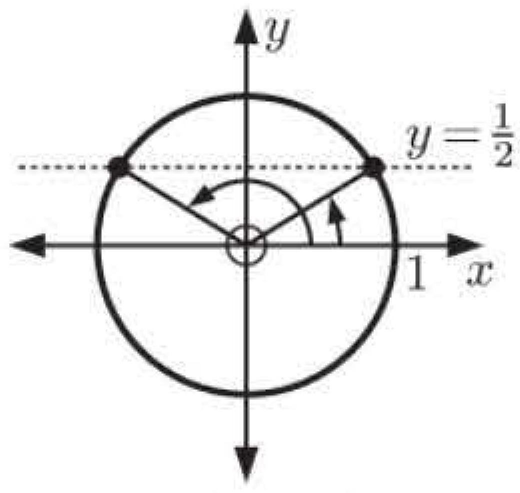
k $2\tan(-\frac{5\pi}{4}) - \sin(\frac{3\pi}{2})$
$= 2(-1) - (-1)$
$= -1$



l $\dfrac{2\tan 150°}{1 - \tan^2 150°}$
$= \dfrac{2(-\frac{1}{\sqrt{3}})}{1 - (-\frac{1}{\sqrt{3}})^2}$
$= \dfrac{-\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}}$
$= \dfrac{-\frac{2}{\sqrt{3}}}{\frac{2}{3}} = -\frac{3}{\sqrt{3}} = -\sqrt{3}$

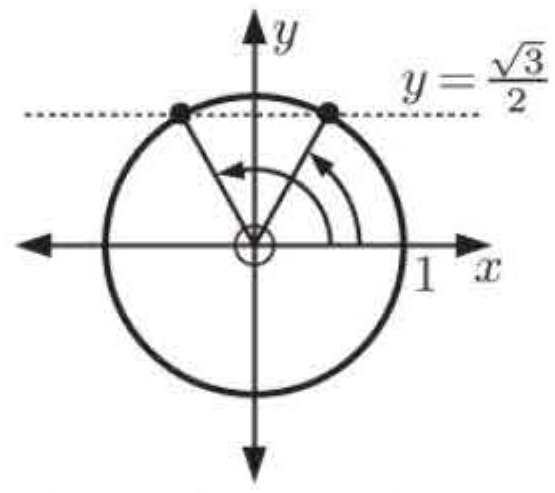


6 **a**



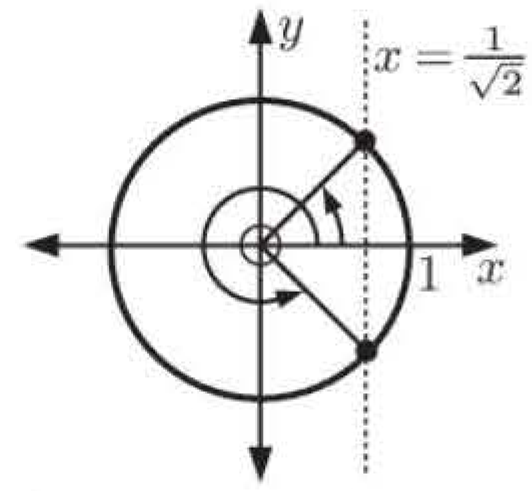
$\theta = 30°, 150°$

b



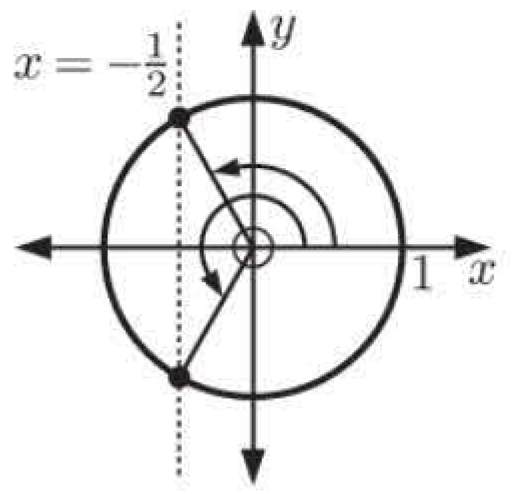
$\theta = 60°, 120°$

c



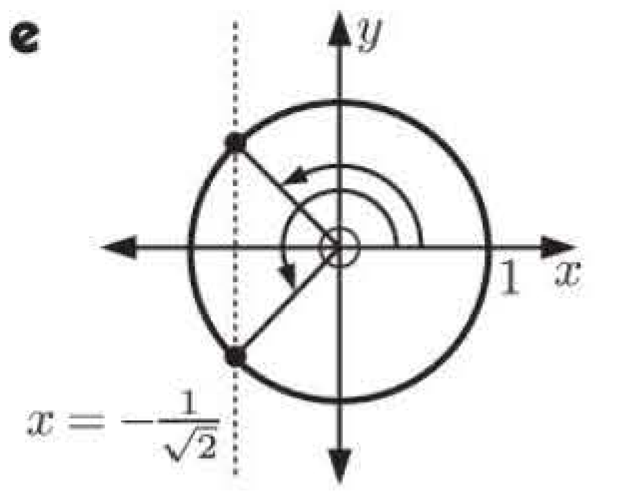
$\theta = 45°, 315°$

d



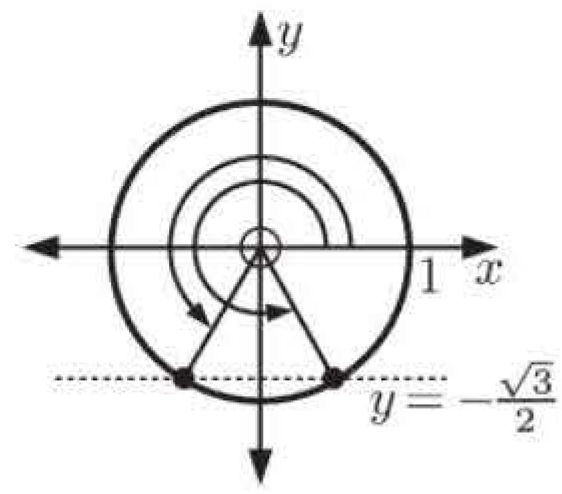
$\theta = 120°, 240°$

e



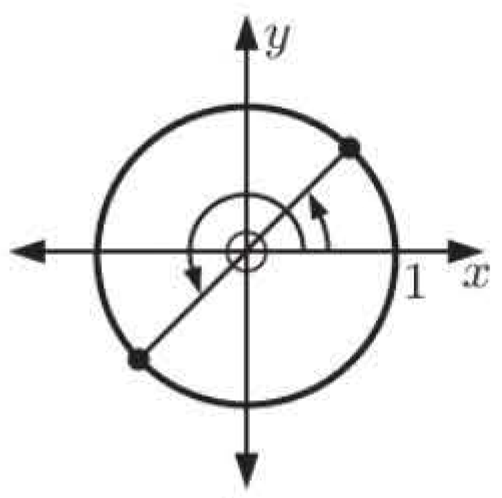
$\theta = 135°, 225°$

f



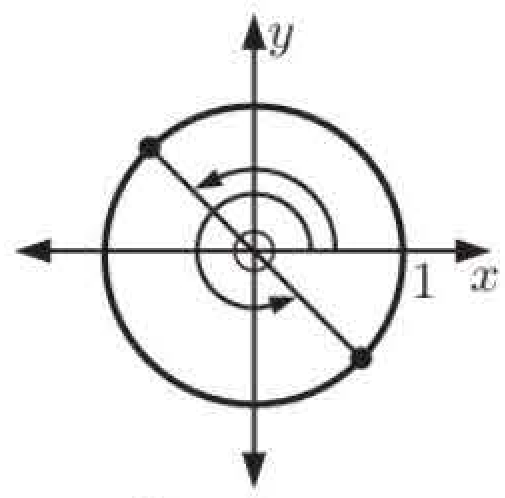
$\theta = 240°, 300°$

7 **a**



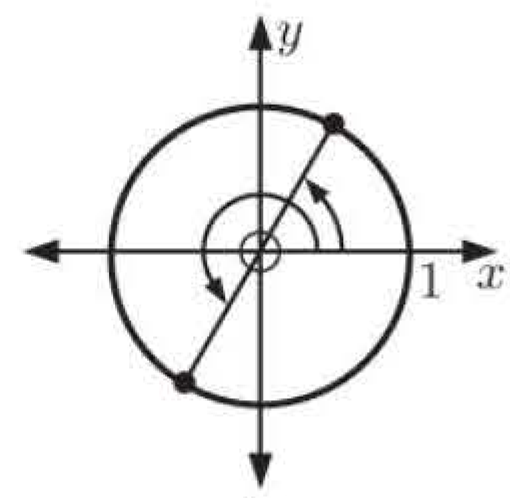
$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$

b



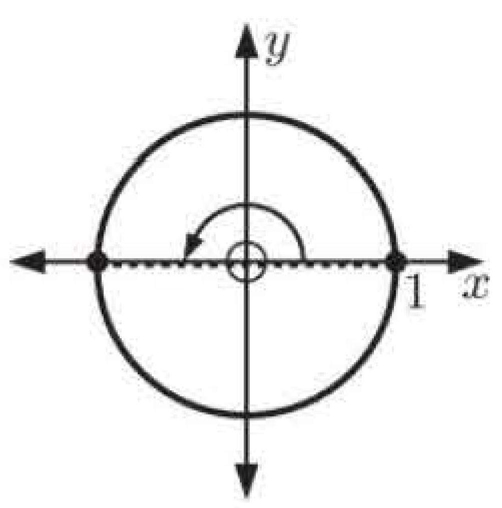
$\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$

c



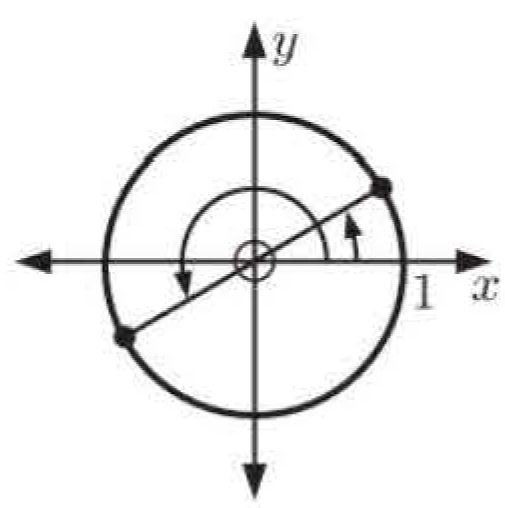
$\theta = \frac{\pi}{3}, \frac{4\pi}{3}$

d



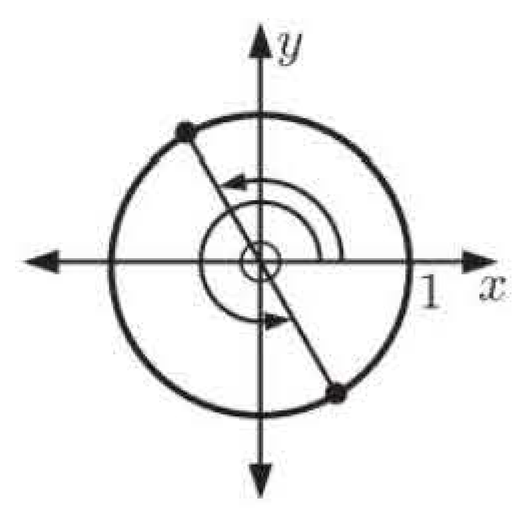
$\theta = 0, \pi, 2\pi$

e



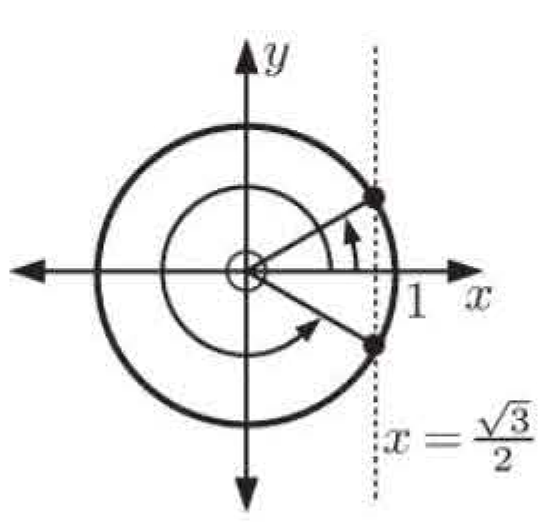
$\theta = \frac{\pi}{6}, \frac{7\pi}{6}$

f



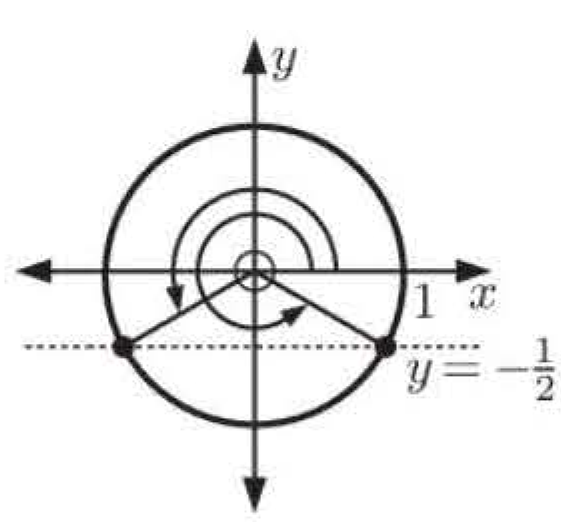
$\theta = \frac{2\pi}{3}, \frac{5\pi}{3}$

8 **a**



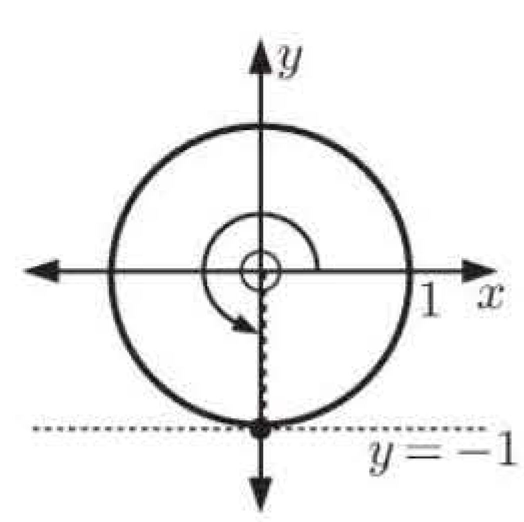
$\theta = \frac{\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}, \frac{23\pi}{6}$

b



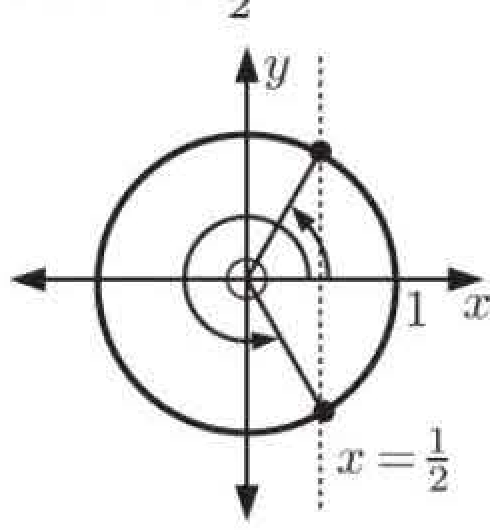
$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}$

c



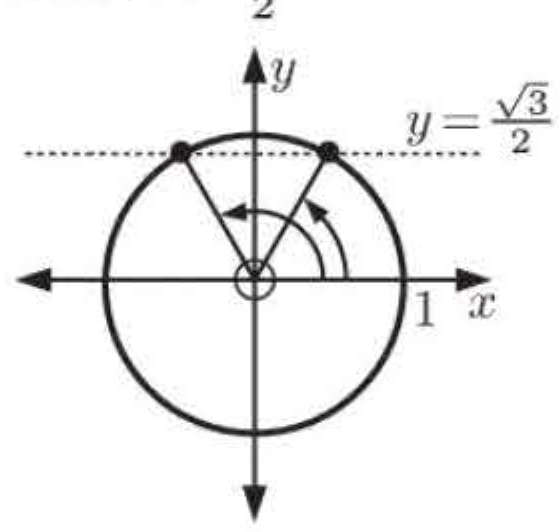
$\theta = \frac{3\pi}{2}, \frac{7\pi}{2}$

9 **a** $\cos\theta = \frac{1}{2}$



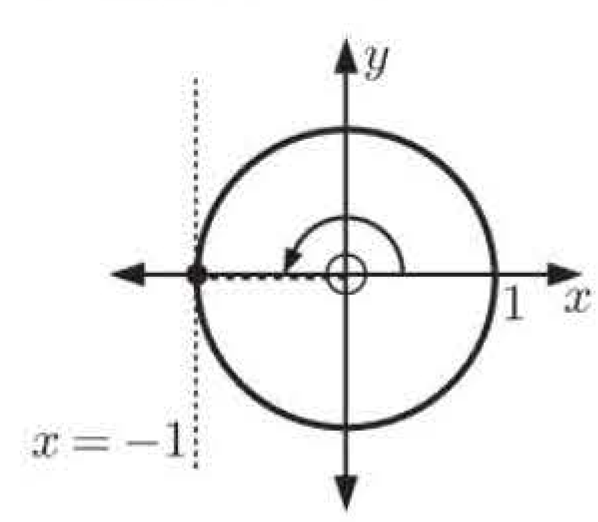
$\therefore \quad \theta = \frac{\pi}{3}, \frac{5\pi}{3}$

b $\sin\theta = \frac{\sqrt{3}}{2}$



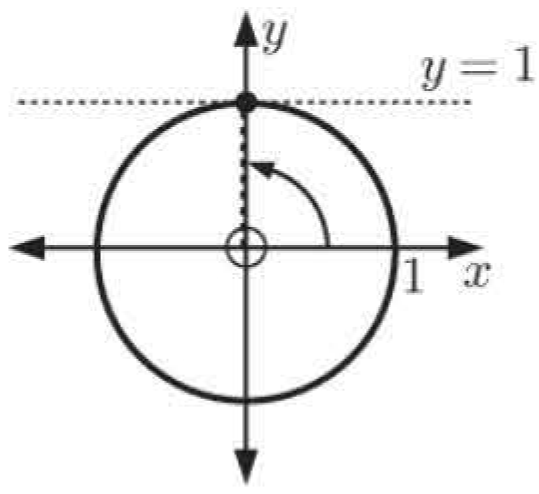
$\therefore \quad \theta = \frac{\pi}{3}, \frac{2\pi}{3}$

c $\cos\theta = -1$



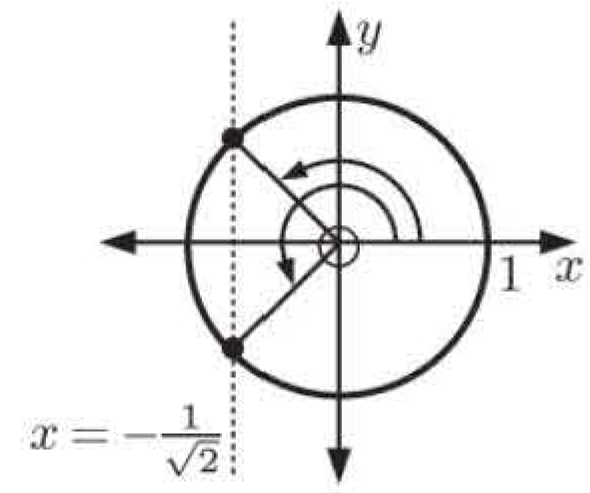
$\therefore \quad \theta = \pi$

d $\sin\theta = 1$



$\therefore\ \theta = \frac{\pi}{2}$

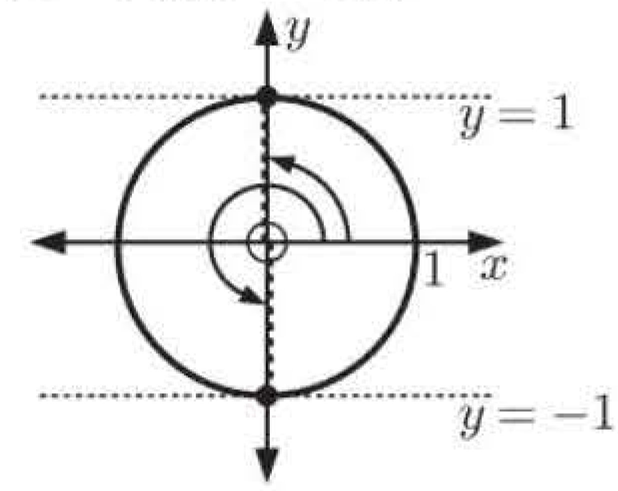
e $\cos\theta = -\frac{1}{\sqrt{2}}$



$\therefore\ \theta = \frac{3\pi}{4}, \frac{5\pi}{4}$

f $\sin^2\theta = 1$

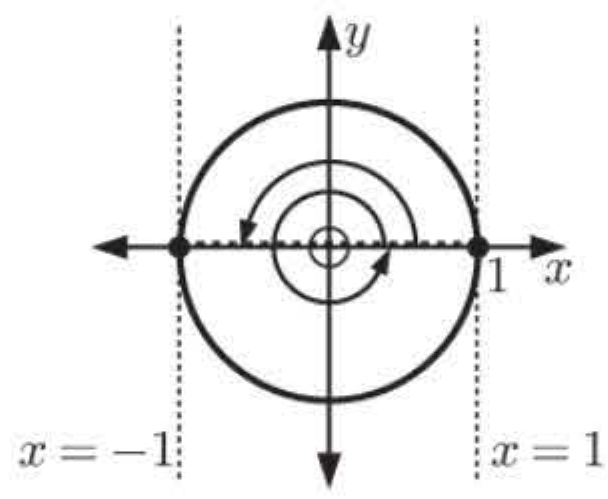
$\therefore\ \sin\theta = \pm 1$



$\therefore\ \theta = \frac{\pi}{2}, \frac{3\pi}{2}$

g $\cos^2\theta = 1$

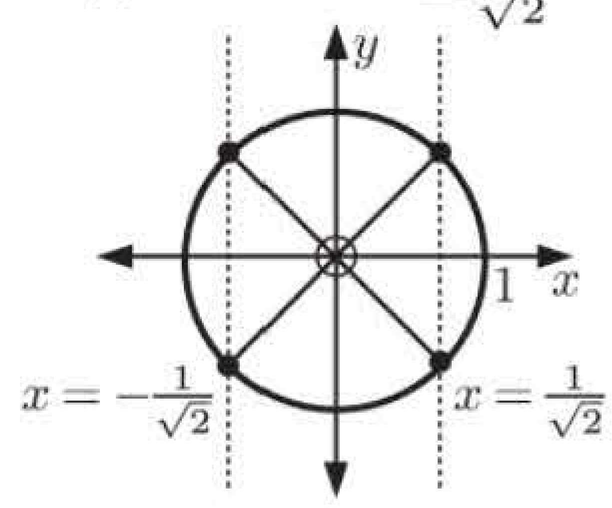
$\therefore\ \cos\theta = \pm 1$



$\therefore\ \theta = 0, \pi, 2\pi$

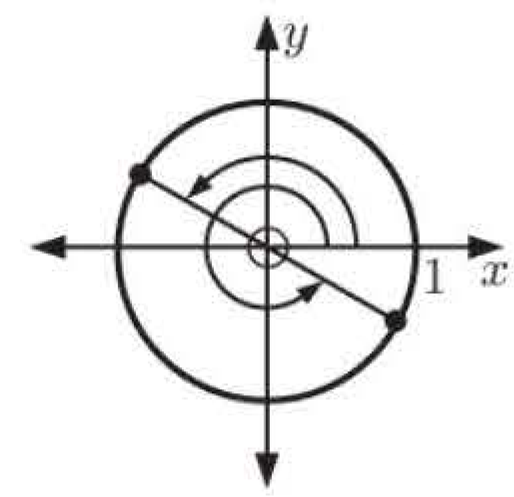
h $\cos^2\theta = \frac{1}{2}$

$\therefore\ \cos\theta = \pm\frac{1}{\sqrt{2}}$



$\therefore\ \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

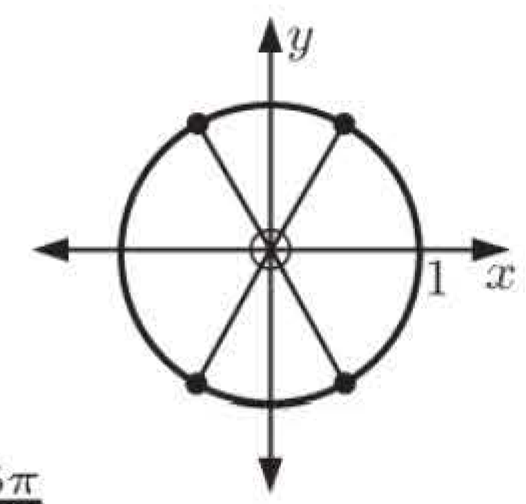
i $\tan\theta = -\frac{1}{\sqrt{3}}$



$\therefore\ \theta = \frac{5\pi}{6}, \frac{11\pi}{6}$

j $\tan^2\theta = 3$

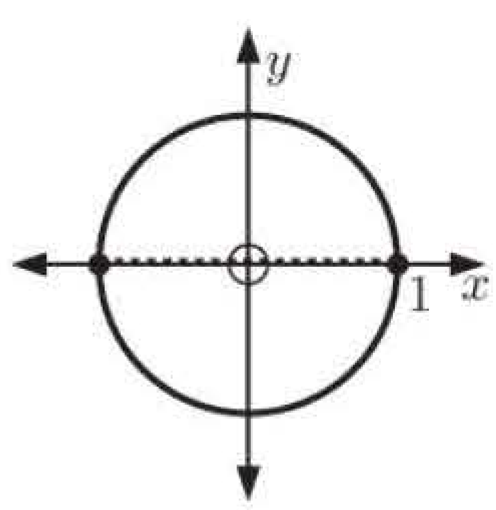
$\therefore\ \tan\theta = \pm\sqrt{3}$



$\therefore\ \theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

10 **a** $\tan\theta$ is zero when $\dfrac{\sin\theta}{\cos\theta} = \dfrac{0}{\cos\theta}$

$\therefore$ when $\sin\theta = 0$

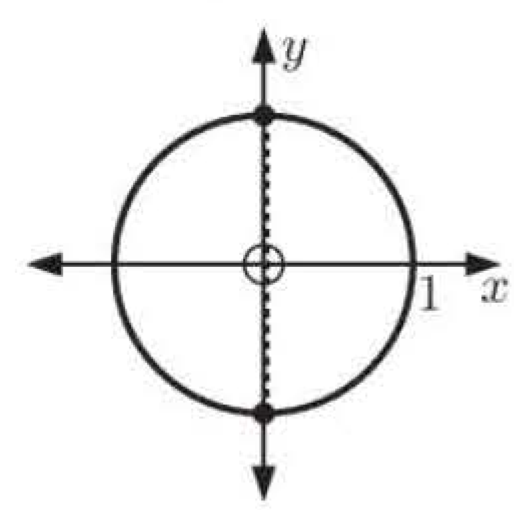


$\therefore\ \theta = ..., -\pi, 0, \pi, 2\pi, ...$

$\therefore\ \theta = k\pi,$ for $k \in \mathbb{Z}$

b $\tan\theta$ is undefined when $\dfrac{\sin\theta}{\cos\theta} = \dfrac{\sin\theta}{0}$

$\therefore$ when $\cos\theta = 0$



$\therefore\ \theta = ..., -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, ...$

$\therefore\ \theta = \frac{\pi}{2} + k\pi,$ for $k \in \mathbb{Z}$

REVIEW SET 10A

1 **a** $120°$
$= \left(120 \times \frac{\pi}{180}\right)^c$
$= \frac{2\pi}{3}^c$

b $225°$
$= 5 \times 45°$
$= 5 \times \frac{\pi}{4}^c$
$= \frac{5\pi}{4}^c$

c $150°$
$= 5 \times 30°$
$= 5 \times \frac{\pi}{6}^c$
$= \frac{5\pi}{6}^c$

d $540°$
$= 3 \times 180°$
$= 3\pi^c$

2 **a** $\sin\frac{2\pi}{3} = \sin(\pi - \frac{2\pi}{3}) = \sin\frac{\pi}{3}$

$\therefore\ \theta = \frac{\pi}{3}$

b $\sin 165° = \sin(180 - 165)° = \sin 15°$

$\therefore\ \theta = 15°$

c $\cos 276° = \cos(360 - 276)° = \cos 84°$

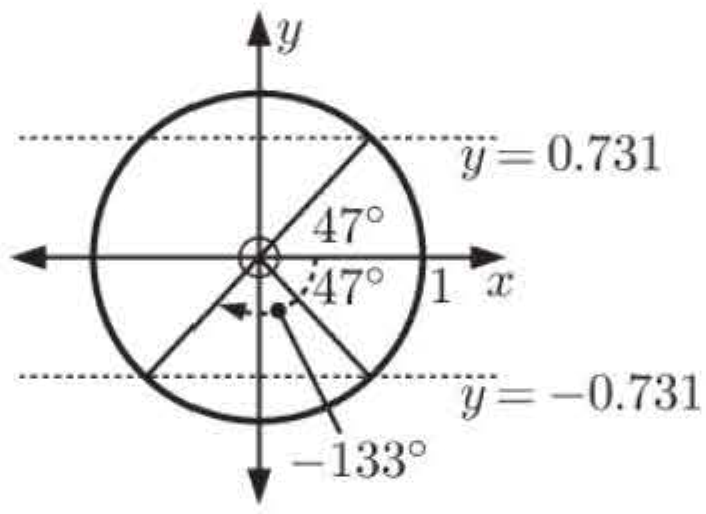
$\therefore\ \theta = 84°$

3 **a** $\sin 159^\circ$
$= \sin(180 - 159)^\circ$
$= \sin 21^\circ$
≈ 0.358

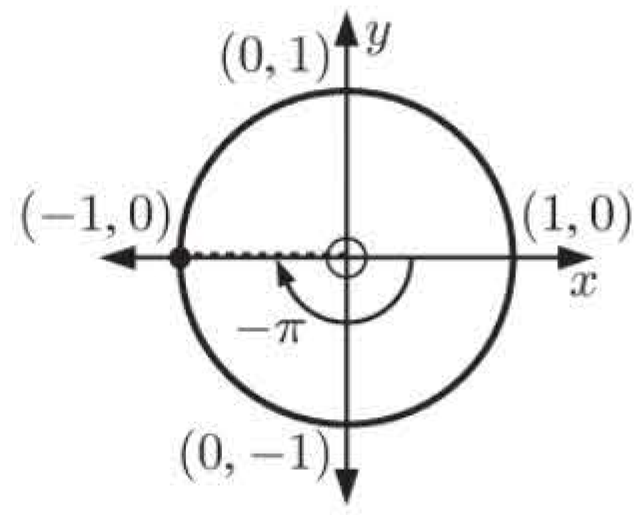
b $\cos 92^\circ$
$= -\cos(180 - 92)^\circ$
$= -\cos 88^\circ$
≈ -0.035

c $\cos 75^\circ$
$= -\cos(180 - 75)^\circ$
$= -\cos 105^\circ$
≈ 0.259

d $\sin(-133^\circ) = \sin(-47)^\circ$
$= -\sin 47^\circ$
≈ -0.731



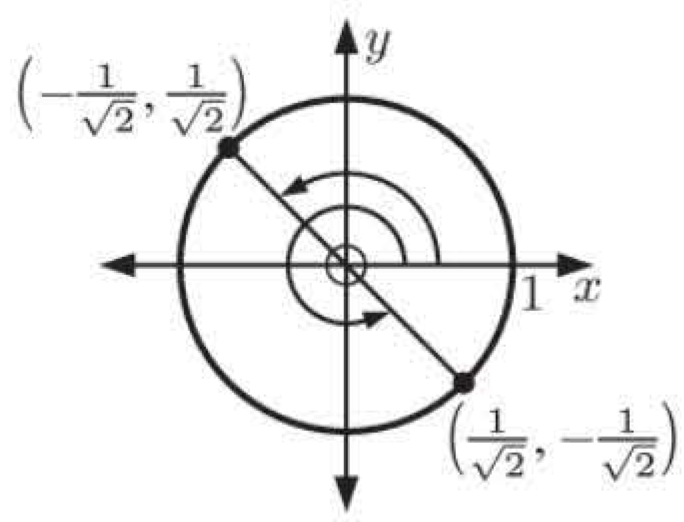
4



a $\cos 360^\circ = 1, \quad \sin 360^\circ = 0$

b $\cos(-\pi) = -1, \quad \sin(-\pi) = 0$

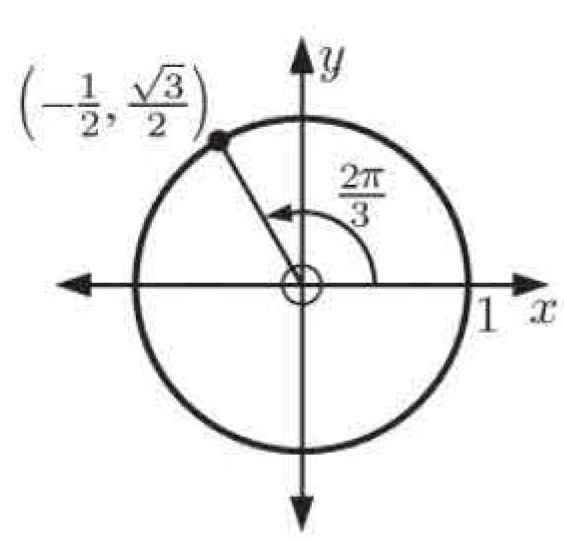
5



When $\cos\theta = -\sin\theta$,
$\dfrac{\sin\theta}{\cos\theta} = -1$
$\therefore \quad \tan\theta = -1$

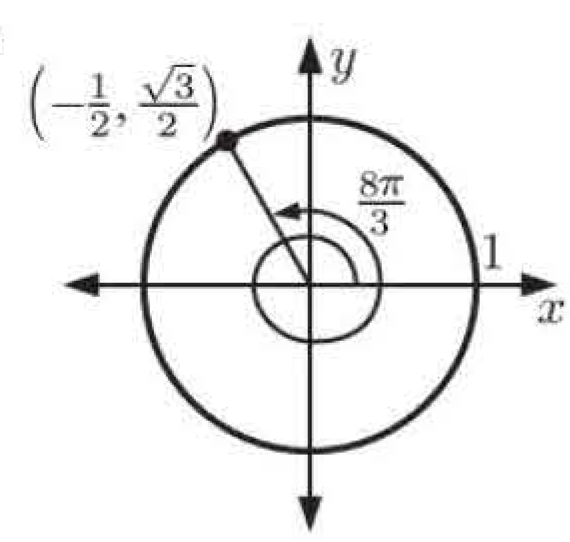
and this only occurs at the two points shown.
So, $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$

6 **a**



$\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$
$\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$
$\tan\left(\frac{2\pi}{3}\right) = \dfrac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}$
$= -\sqrt{3}$

b



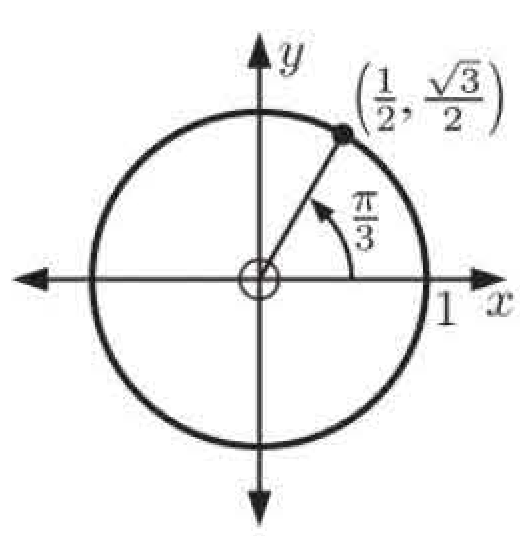
$\sin\left(\frac{8\pi}{3}\right) = \frac{\sqrt{3}}{2}$
$\cos\left(\frac{8\pi}{3}\right) = -\frac{1}{2}$
$\tan\left(\frac{8\pi}{3}\right) = \dfrac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}$
$= -\sqrt{3}$

7 $\cos^2 x + \sin^2 x = 1$
$\therefore \quad \cos^2 x + \frac{1}{16} = 1$
$\therefore \quad \cos^2 x = \frac{15}{16}$
$\therefore \quad \cos x = \pm\frac{\sqrt{15}}{4}$

But x is in quadrant 3 where $\cos x < 0$
$\therefore \quad \cos x = -\frac{\sqrt{15}}{4}$
and so $\quad \tan x = \dfrac{\sin x}{\cos x} = \dfrac{-\frac{1}{4}}{-\frac{\sqrt{15}}{4}} = \dfrac{1}{\sqrt{15}}$

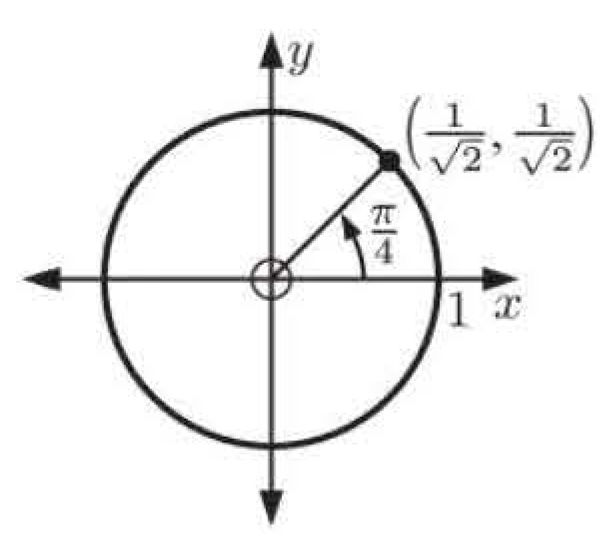
8 $\cos^2\theta + \sin^2\theta = 1$
$\therefore \quad \frac{9}{16} + \sin^2\theta = 1$
$\therefore \quad \sin^2\theta = \frac{7}{16}$
$\therefore \quad \sin\theta = \pm\frac{\sqrt{7}}{4}$

9 **a**



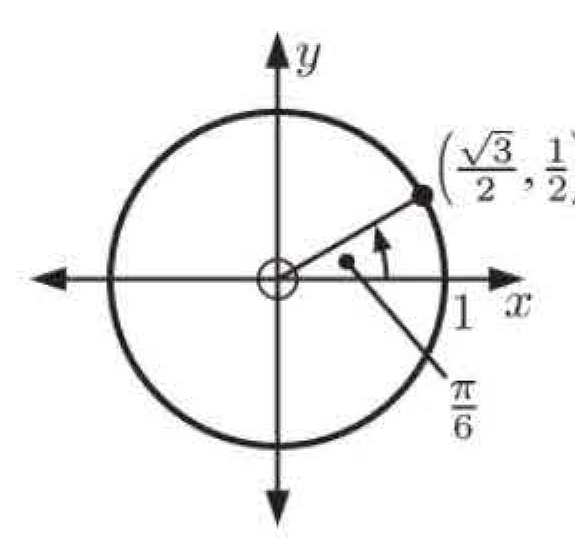
$2\sin(\frac{\pi}{3})\cos(\frac{\pi}{3})$
$= 2(\frac{\sqrt{3}}{2})(\frac{1}{2})$
$= \frac{\sqrt{3}}{2}$

b



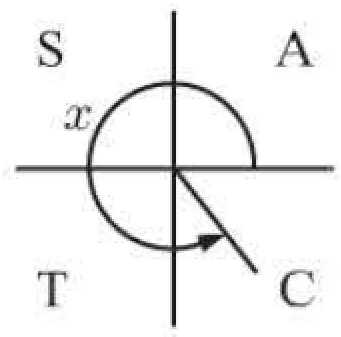
$\tan^2(\frac{\pi}{4}) - 1$
$= 1^2 - 1$
$= 0$

c



$\cos^2(\frac{\pi}{6}) - \sin^2(\frac{\pi}{6})$
$= \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2$
$= \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$

10 $\dfrac{\sin x}{\cos x} = -\frac{3}{2}$



$\therefore \sin x = -\frac{3}{2}\cos x$

Now $\cos^2 x + \sin^2 x = 1$

$\therefore \cos^2 x + \frac{9}{4}\cos^2 x = 1$

$\therefore \frac{13}{4}\cos^2 x = 1$

$\therefore \cos x = \pm\frac{2}{\sqrt{13}}$

But x is in quadrant 4, so $\cos x$ is positive and $\sin x$ is negative.

$\therefore \cos x = \frac{2}{\sqrt{13}}, \quad \sin x = -\frac{3}{\sqrt{13}}$

So, **a** $\sin x = -\frac{3}{\sqrt{13}}$ **b** $\cos x = \frac{2}{\sqrt{13}}$

11
$$\begin{aligned}\text{arc length} &= \theta r\\ &= 1\times 4\\ &= 4 \text{ units}\end{aligned}$$

$$\begin{aligned}\therefore \text{ perimeter} &= 2\times 4+4\\ &= 12 \text{ units}\end{aligned}$$

$$\begin{aligned}\text{area} &= \tfrac{1}{2}\theta r^2\\ &= \tfrac{1}{2}\times 1\times 4^2\\ &= 8 \text{ units}^2\end{aligned}$$

12
$$\cos^2\theta + \sin^2\theta = 1$$
$$\therefore \left(\frac{\sqrt{11}}{\sqrt{17}}\right)^2 + \sin^2\theta = 1$$
$$\therefore \sin^2\theta = \tfrac{6}{17}$$
$$\therefore \sin\theta = \pm\frac{\sqrt{6}}{\sqrt{17}}$$

But θ is acute, $\therefore \sin\theta = \frac{\sqrt{6}}{\sqrt{17}}$

$$\tan\theta = \frac{\frac{\sqrt{6}}{\sqrt{17}}}{\frac{\sqrt{11}}{\sqrt{17}}} = \frac{\sqrt{6}}{\sqrt{11}}$$

13 **a** $\cos(\frac{\pi}{2}-\theta) - \sin\theta$

$= \sin\theta - \sin\theta$ {complementary angle formula}

$= 0$

b $\cos(-\theta)\tan\theta$

$= \cos\theta\dfrac{\sin\theta}{\cos\theta}$ {negative angle formula}

$= \sin\theta$

c $\sin(-\alpha)\cos(\alpha - \frac{\pi}{2})$

$= -\sin\alpha\cos(-(\frac{\pi}{2}-\alpha))$ {negative angle formula}

$= -\sin\alpha\cos(\frac{\pi}{2}-\alpha)$ {negative angle formula}

$= -\sin\alpha\sin\alpha$ {complementary angle formula}

$= -\sin^2\alpha$

REVIEW SET 10B

1 **a** The point is $(\cos 320°, \sin 320°) \approx (0.766, -0.643)$.

b The point is $(\cos 163°, \sin 163°) \approx (-0.956, 0.292)$.

2 **a** $71°$ $= \left(71\times\frac{\pi}{180}\right)^c$ $\approx 1.239^c$

b $124.6°$ $= \left(124.6\times\frac{\pi}{180}\right)^c$ $\approx 2.175^c$

c $-142°$ $= \left(-142\times\frac{\pi}{180}\right)^c$ $\approx -2.478^c$

3 **a** 3^c $= \left(3\times\frac{180}{\pi}\right)^\circ$ $\approx 171.89°$

b 1.46^c $= \left(1.46\times\frac{180}{\pi}\right)^\circ$ $\approx 83.65°$

c 0.435^c $= \left(0.435\times\frac{180}{\pi}\right)^\circ$ $\approx 24.92°$

d -5.271^c $= \left(-5.271\times\frac{180}{\pi}\right)^\circ$ $\approx -302.01°$

4 area $= \frac{1}{2}\times\frac{5\pi}{12}\times 13^2$ $\approx 111 \text{ cm}^2$

5 $M(\cos 73°, \sin 73°) \approx (0.292, 0.956)$,
$N(\cos 190°, \sin 190°) \approx (-0.985, -0.174)$,
$P(\cos(-53°), \sin(-53°)) = P(\cos 307°, \sin 307°) \approx (0.602, -0.799)$

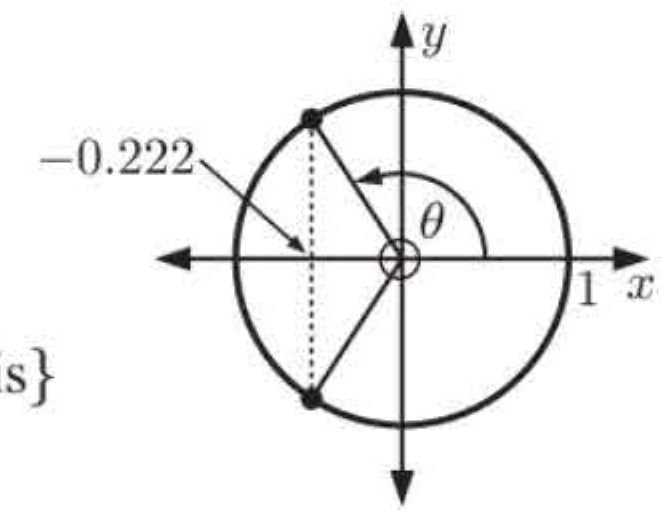
6 The x-coordinate of A $= -0.222$

$\therefore \quad \cos\theta = -0.222$

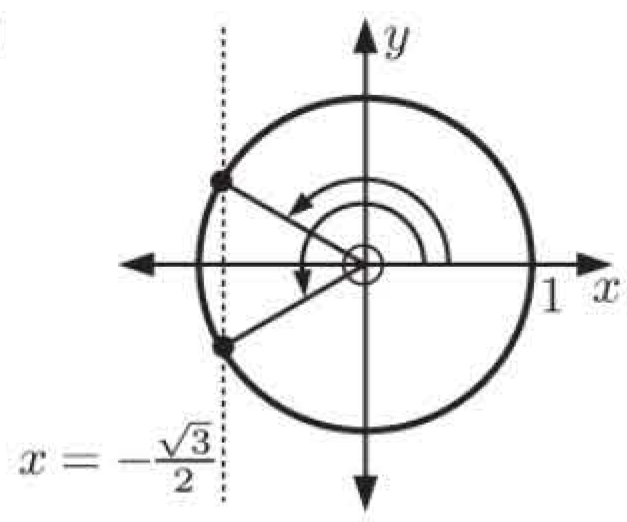
$\therefore \quad \theta = \cos^{-1}(-0.222)$

$\therefore \quad \theta \approx 102.8°,\ 257.2°$

$\therefore \quad \theta \approx 103°$ {taking angle to positive x-axis}

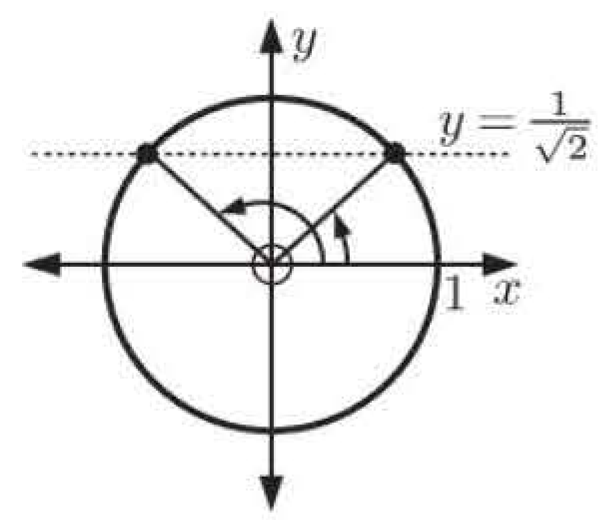


7 **a**



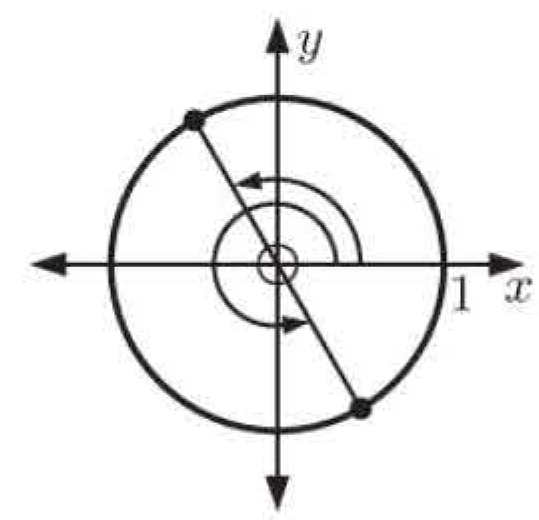
$\therefore \quad \theta = 150°$ or $210°$

b



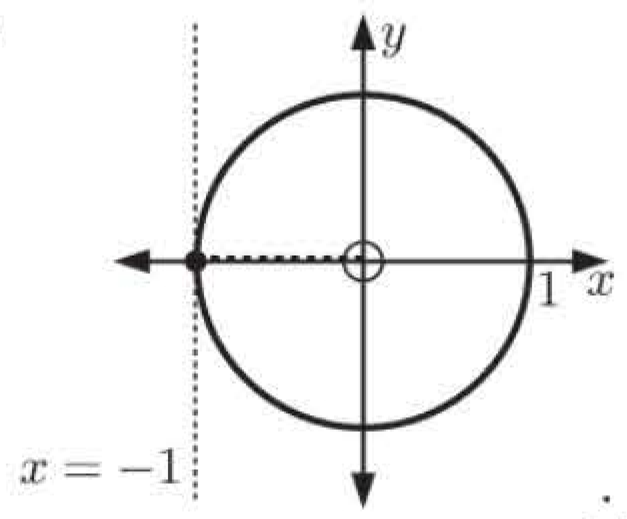
$\therefore \quad \theta = 45°$ or $135°$

c



$\therefore \quad \theta = 120°$ or $300°$

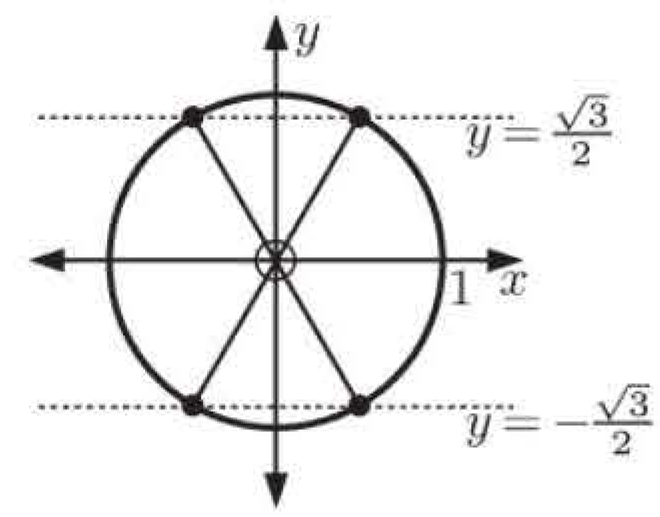
8 **a**



$\therefore \quad \theta = \pi$

b $\sin^2\theta = \frac{3}{4}$

$\therefore \quad \sin\theta = \pm\frac{\sqrt{3}}{2}$



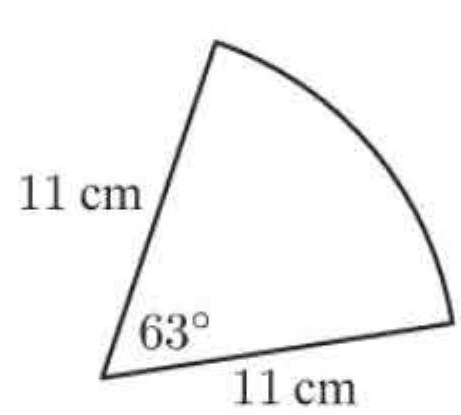
$\therefore \quad \theta = \frac{\pi}{3},\ \frac{2\pi}{3},\ \frac{4\pi}{3},\ \frac{5\pi}{3}$

9 **a** $\sin 47° = \sin(180 - 47)°$
$= \sin 133°$
$\therefore \quad \theta = 133°$

b $\sin(\frac{\pi}{15}) = \sin(\pi - \frac{\pi}{15})$
$= \sin(\frac{14\pi}{15})$
$\therefore \quad \theta = \frac{14\pi}{15}$

c $\cos 186° = \cos(360 - 186)°$
$= \cos 174°$
$\therefore \quad \theta = 174°$

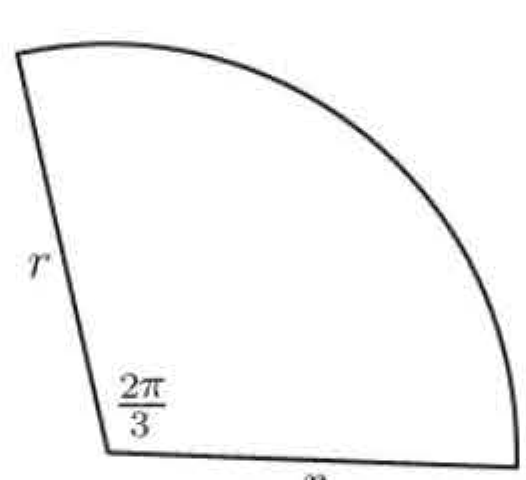
10



perimeter $= 2 \times 11 + \left(\frac{63}{360}\right) \times 2\pi \times 11$
≈ 34.1 cm

area $= \left(\frac{63}{360}\right) \times \pi \times 11^2$
$\approx 66.5\ \text{cm}^2$

11



perimeter $= 2r + (\frac{2\pi}{3})r$

$\therefore \quad 36 = r\left(2 + \frac{2\pi}{3}\right)$

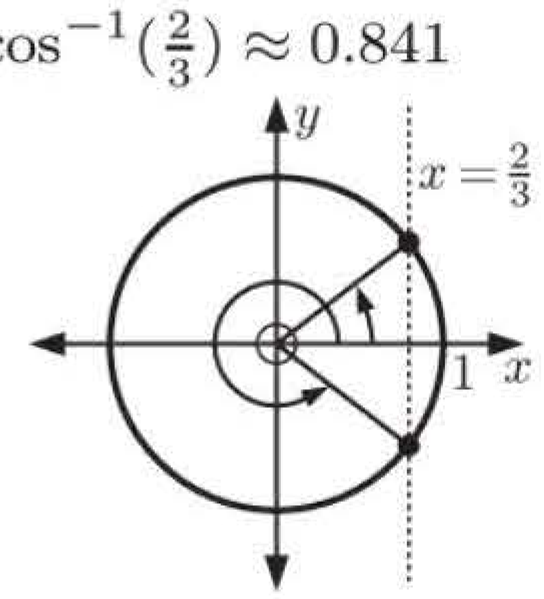
$\therefore \quad r = \dfrac{36}{2 + \frac{2\pi}{3}}$ cm

$\therefore \quad r \approx 8.79$ cm

area $\approx \frac{1}{2}(\frac{2\pi}{3}) \times (8.7925)^2$
$\approx 81.0\ \text{cm}^2$

12 **a** $\cos\theta = \frac{2}{3}$

Using technology,
$\cos^{-1}(\frac{2}{3}) \approx 0.841$

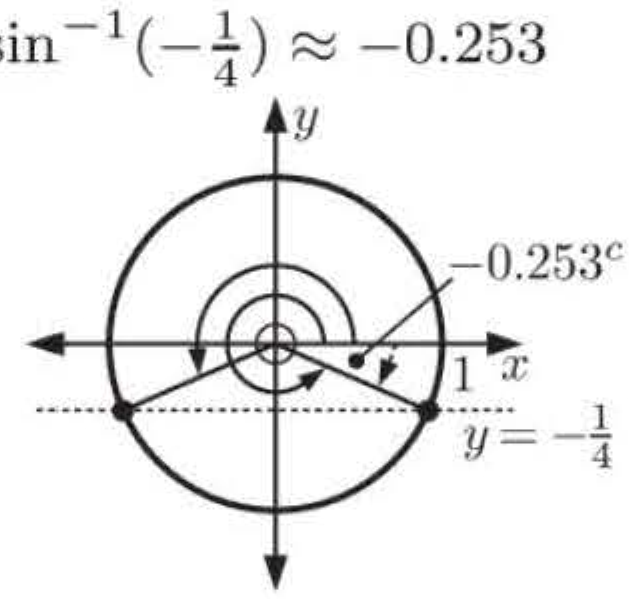


$\therefore \quad \theta \approx 0.841$ or $2\pi - 0.841$

$\therefore \quad \theta \approx 0.841$ or 5.44

b $\sin\theta = -\frac{1}{4}$

Using technology,
$\sin^{-1}(-\frac{1}{4}) \approx -0.253$



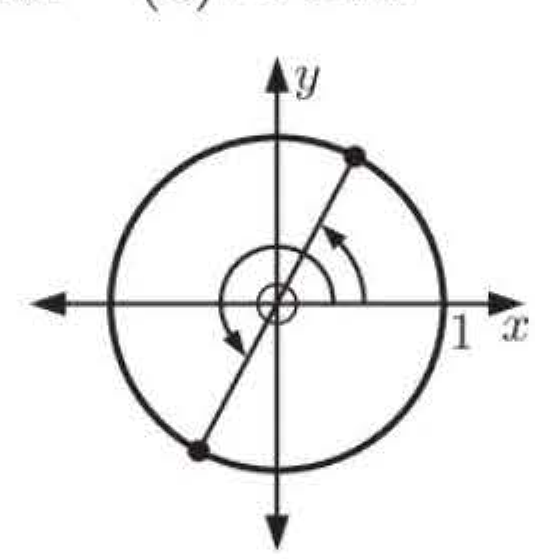
But $0 \leqslant \theta \leqslant 2\pi$

$\therefore \quad \theta \approx \pi + 0.253$ or $2\pi + (-0.253)$

$\therefore \quad \theta \approx 3.39$ or 6.03

c $\tan\theta = 3$

Using technology,
$\tan^{-1}(3) \approx 1.25$



$\therefore \quad \theta \approx 1.25$ or $\pi + 1.25$

$\therefore \quad \theta \approx 1.25$ or 4.39

REVIEW SET 10C

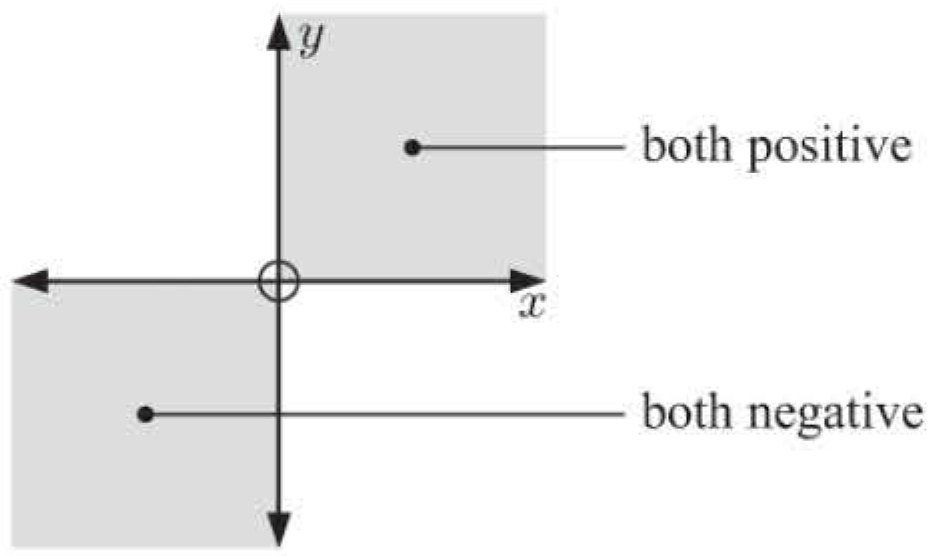
1 **a** $\frac{2\pi}{5} = \frac{2\times 180^\circ}{5}$
$= 72^\circ$

b $\frac{5\pi}{4} = \frac{5\times 180^\circ}{4}$
$= 225^\circ$

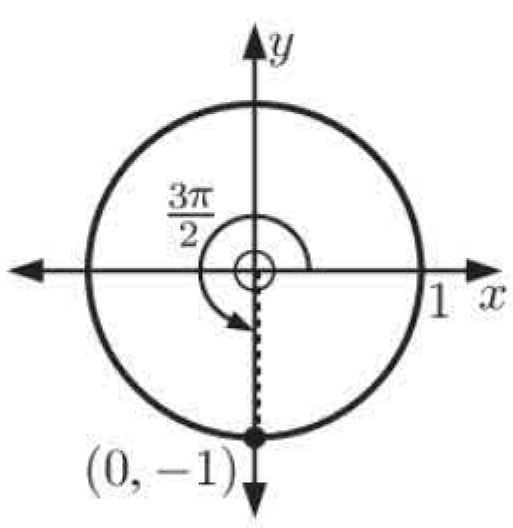
c $\frac{7\pi}{9} = \frac{7\times 180^\circ}{9}$
$= 140^\circ$

d $\frac{11\pi}{6} = \frac{11\times 180^\circ}{6}$
$= 330^\circ$

2

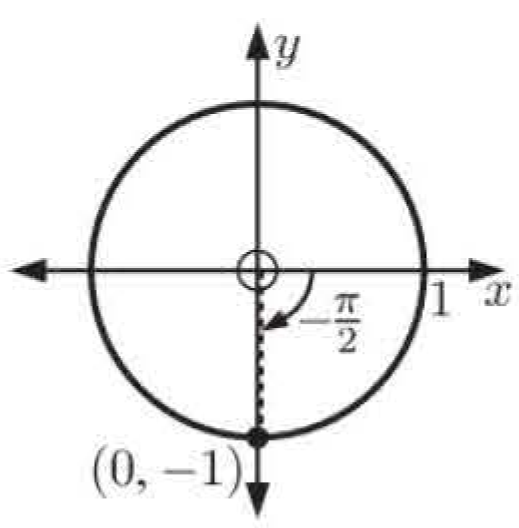


3 **a**



$\therefore \quad \cos\left(\frac{3\pi}{2}\right) = 0$
$\sin\left(\frac{3\pi}{2}\right) = -1$

b



$\therefore \quad \cos\left(-\frac{\pi}{2}\right) = 0$
$\sin\left(-\frac{\pi}{2}\right) = -1$

4 **a** $\sin(\pi - \theta) = \sin\theta$
$\therefore \sin(\pi - p) = \sin p$
$= m$

b $\sin(\theta + 2\pi) = \sin\theta$
$\therefore \sin(p + 2\pi) = \sin p$
$= m$

c $\cos^2 p + \sin^2 p = 1$
$\therefore \cos^2 p + m^2 = 1$
$\therefore \cos^2 p = 1 - m^2$
$\therefore \cos p = \pm\sqrt{1 - m^2}$
But p is acute, $\therefore \cos p = \sqrt{1 - m^2}$

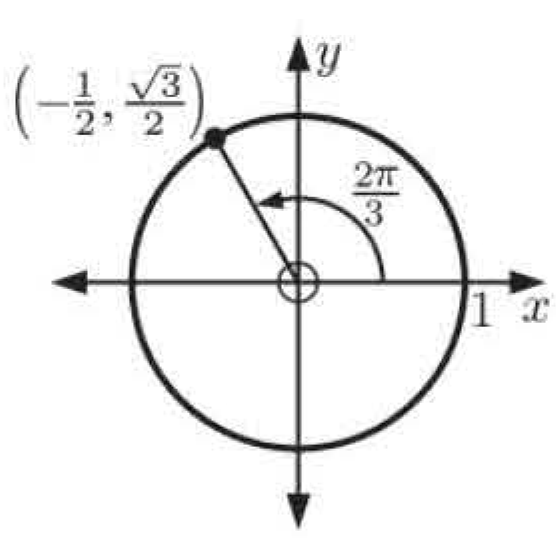
d $\tan p = \dfrac{\sin p}{\cos p}$
$= \dfrac{m}{\sqrt{1 - m^2}}$

5 **a** **i** $\theta = 60^\circ$ {equilateral triangle} **ii** $\theta = \frac{\pi}{3}$ radians

b arc length $= \theta r = \frac{\pi}{3}$ units

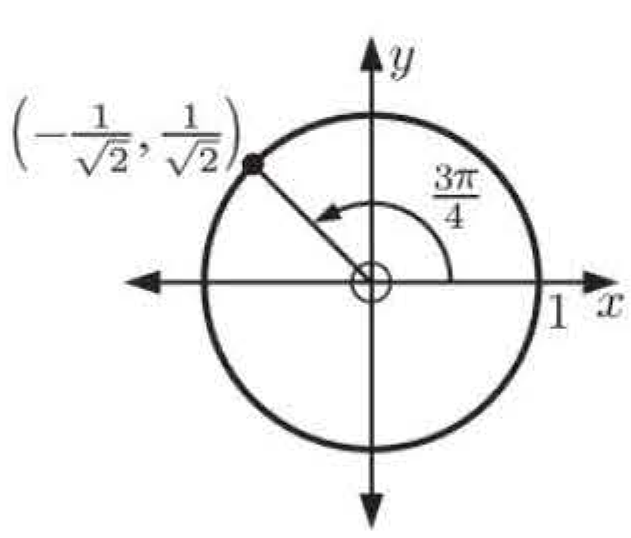
c sector area $= \frac{1}{2}\theta r^2 = \frac{\pi}{6}$ units^2

6



$\tan^2\left(\frac{2\pi}{3}\right)$
$= (-\sqrt{3})^2$
$= 3$

7

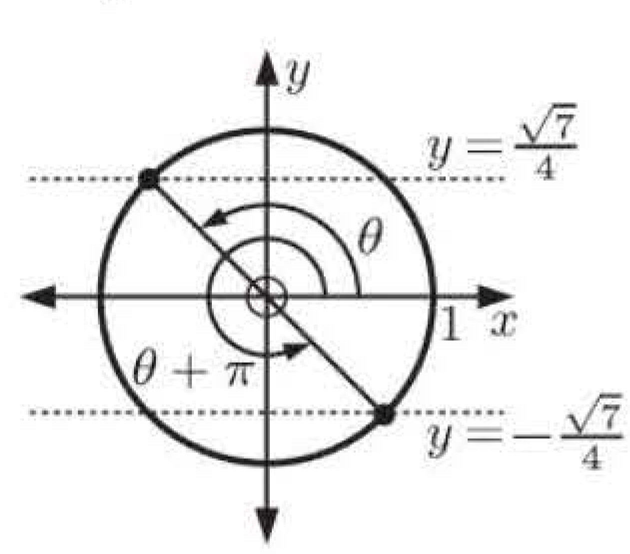


$\cos\left(\frac{3\pi}{4}\right) - \sin\left(\frac{3\pi}{4}\right)$
$= -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}$
$= -\frac{2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$
$= -\sqrt{2}$

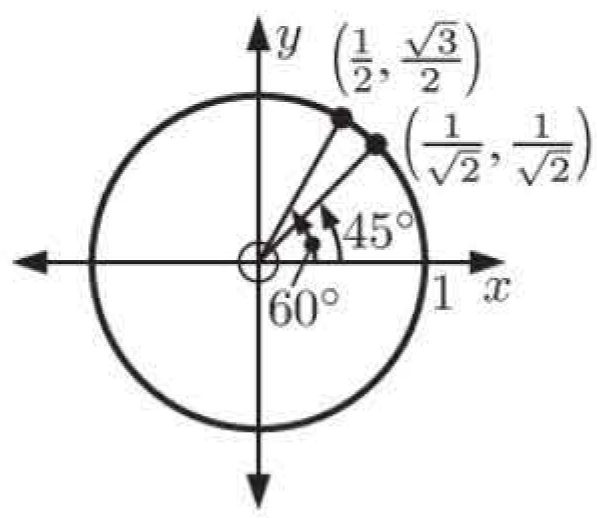
8 **a** $\cos^2\theta + \sin^2\theta = 1$
$\therefore \frac{9}{16} + \sin^2\theta = 1$
$\therefore \sin^2\theta = \frac{7}{16}$
$\therefore \sin\theta = \pm\frac{\sqrt{7}}{4}$
But θ is in quadrant 2 where $\sin\theta > 0$
$\therefore \sin\theta = \frac{\sqrt{7}}{4}$

b $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\frac{\sqrt{7}}{4}}{-\frac{3}{4}} = -\frac{\sqrt{7}}{3}$

c $\sin(\theta + \pi)$
$= -\sin\theta$
$= -\frac{\sqrt{7}}{4}$

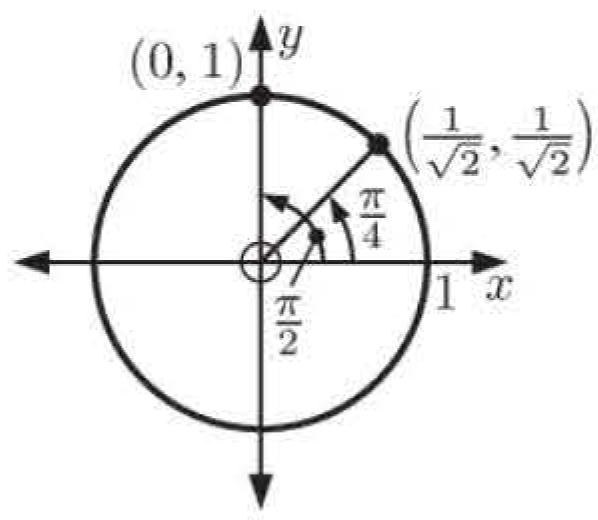


9 **a**



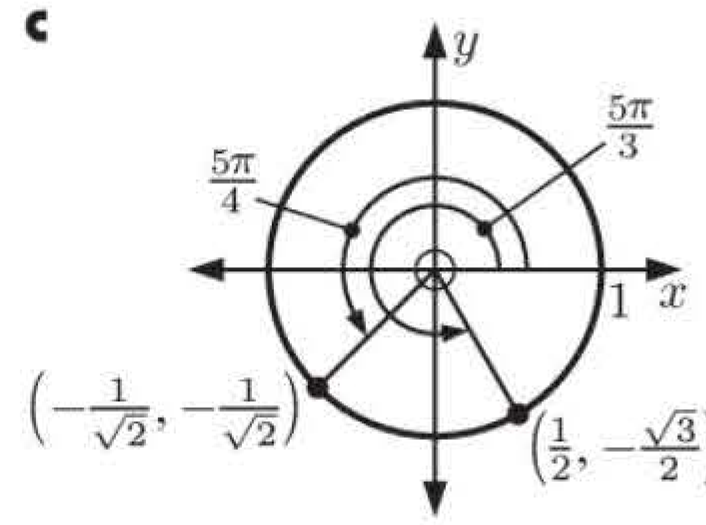
$\tan^2 60° - \sin^2 45°$
$= (\sqrt{3})^2 - \left(\frac{1}{\sqrt{2}}\right)^2$
$= 3 - \frac{1}{2}$
$= 2\frac{1}{2}$

b



$\cos^2(\frac{\pi}{4}) + \sin(\frac{\pi}{2})$
$= \left(\frac{1}{\sqrt{2}}\right)^2 + 1$
$= \frac{1}{2} + 1$
$= 1\frac{1}{2}$

c

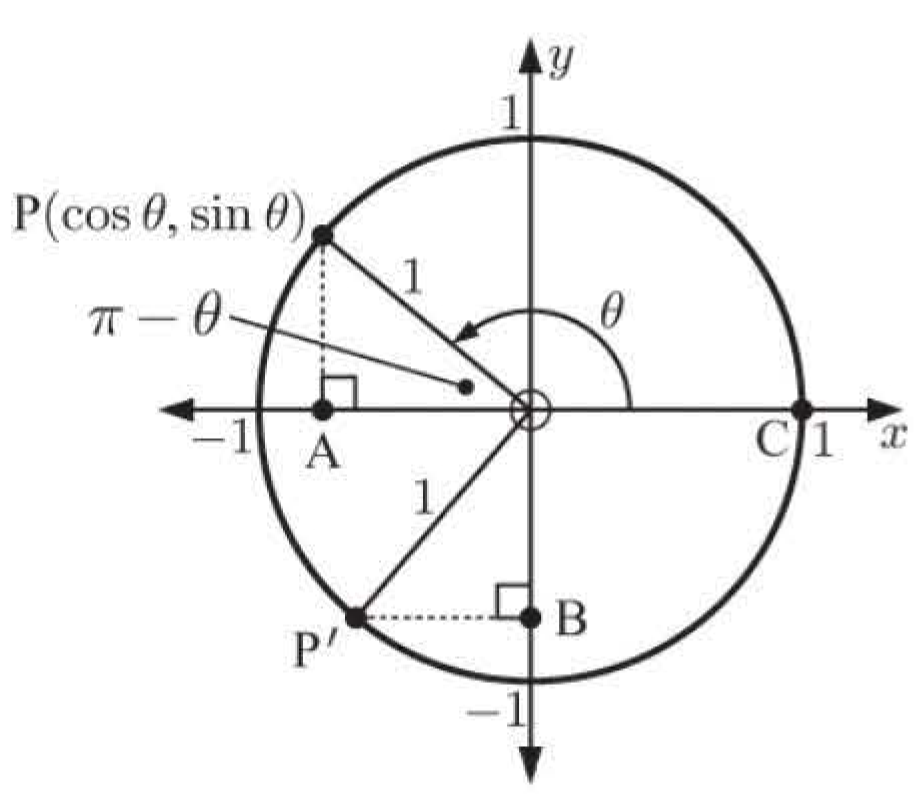


$\cos(\frac{5\pi}{3}) - \tan(\frac{5\pi}{4})$
$= \frac{1}{2} - 1$
$= -\frac{1}{2}$

10 **a** $\sin(\pi - \theta) - \sin\theta = \sin\theta - \sin\theta$
$= 0$

b $\sin(\frac{\pi}{2} - \theta) - 2\cos\theta = \cos\theta - 2\cos\theta$
$= -\cos\theta$

11



For $\frac{\pi}{2} < \theta < \pi$:

The diagram shows P rotated through $\frac{\pi}{2}$ to P′, so OP′ makes an angle of $\frac{\pi}{2} + \theta$ with the positive x-axis.

$\mathrm{P\widehat{O}A} = \pi - \theta$ and $\mathrm{P'\widehat{O}B} = \text{reflex } \mathrm{C\widehat{O}B} - \text{reflex } \mathrm{C\widehat{O}P'}$
$= \frac{3\pi}{2} - (\frac{\pi}{2} + \theta)$
$= \pi - \theta$

In $\triangle$s P′OB and POA:
- $\mathrm{OP'} = \mathrm{OP}$
- $\mathrm{P'\widehat{O}B} = \mathrm{P\widehat{O}A}$
- $\mathrm{P'\widehat{B}O} = \mathrm{P\widehat{A}O}$

$\therefore$ $\triangle$s P′OB and POA are congruent {AAcorS}

$\therefore$ $\mathrm{P'B} = \mathrm{PA} = \sin\theta$

So P′ has x-coordinate $-\sin\theta$

But P′ has x-coordinate $\cos(\frac{\pi}{2} + \theta)$

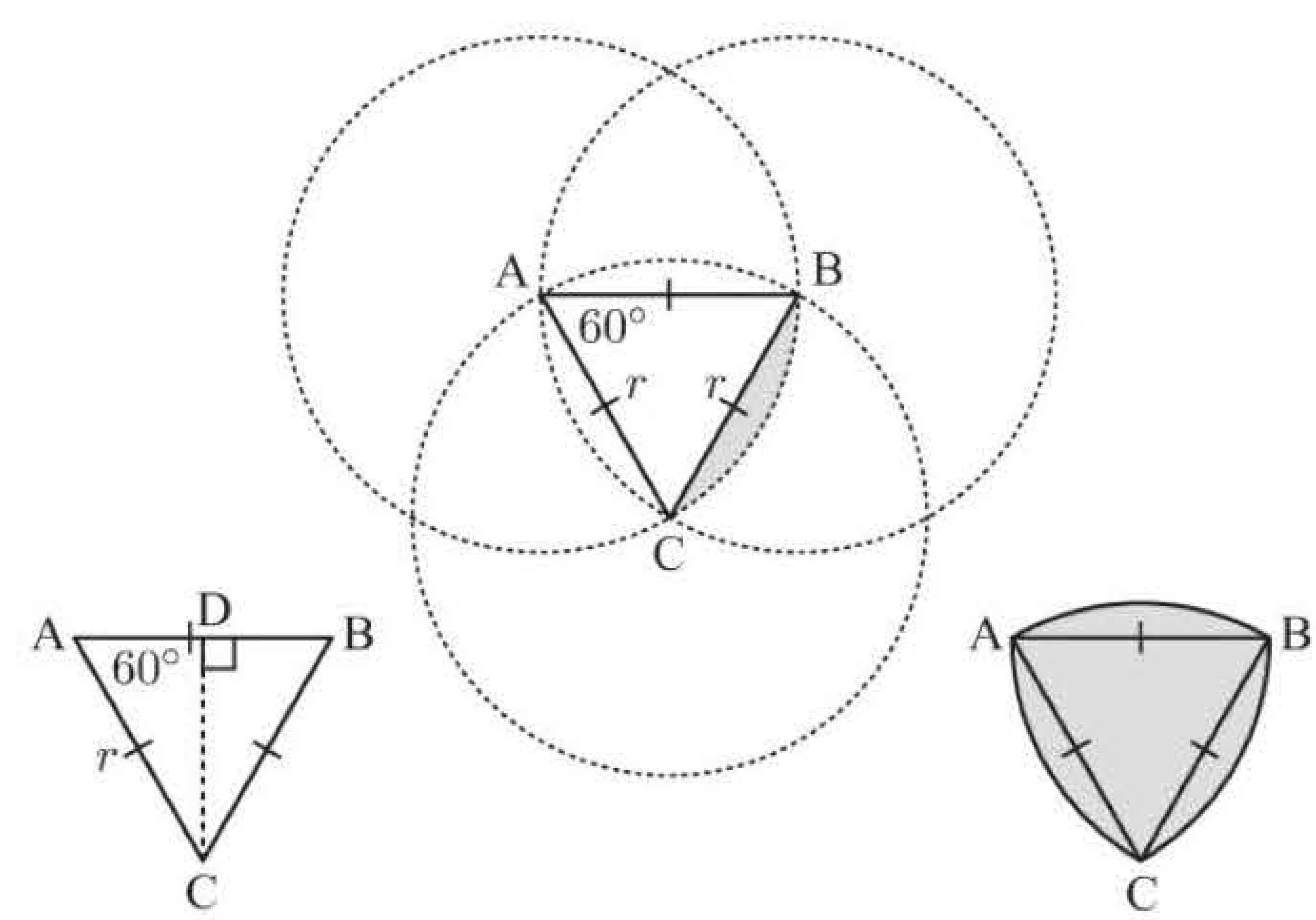
$\therefore$ $\cos(\frac{\pi}{2} + \theta) = -\sin\theta$

12 [AB], [AC], and [BC] are all radii,
so $\mathrm{AB} = \mathrm{AC} = \mathrm{BC} = r$.
Hence $\triangle$ABC is equilateral
and so $\mathrm{C\widehat{A}B} = 60°$.

$\therefore$ $\sin 60° = \dfrac{\mathrm{CD}}{\mathrm{AC}}$

$\therefore$ $\mathrm{CD} = \sin 60° \times \mathrm{AC} = \frac{\sqrt{3}}{2}r$

$\therefore$ area of $\triangle = \frac{1}{2}(r)(\frac{\sqrt{3}}{2}r)$
$= \frac{\sqrt{3}}{4}r^2$



shaded area of sector
$=$ area of sector $-$ area of $\triangle$
$= \frac{60}{360}\pi r^2 - \frac{\sqrt{3}}{4}r^2$
$= \frac{\pi}{6}r^2 - \frac{\sqrt{3}}{4}r^2$

$\therefore$ shaded area of figure $= 3\left[\frac{\pi}{6}r^2 - \frac{\sqrt{3}}{4}r^2\right] + \frac{\sqrt{3}}{4}r^2$
$= \frac{\pi}{2}r^2 - \frac{3\sqrt{3}}{4}r^2 + \frac{\sqrt{3}}{4}r^2$
$= \frac{\pi}{2}r^2 - \frac{2}{4}\sqrt{3}r^2$
$= \dfrac{r^2}{2}\left(\pi - \sqrt{3}\right)$