Chapter 3
EXPONENTIALS

EXERCISE 3A

1 **a** $2^1 = 2,\ 2^2 = 4,\ 2^3 = 8,\ 2^4 = 16,\ 2^5 = 32,\ 2^6 = 64$

b $3^1 = 3,\ 3^2 = 9,\ 3^3 = 27,\ 3^4 = 81,\ 3^5 = 243,\ 3^6 = 729$

c $4^1 = 4,\ 4^2 = 16,\ 4^3 = 64,\ 4^4 = 256,\ 4^5 = 1024,\ 4^6 = 4096$

2 **a** $5^1 = 5,\ 5^2 = 25,\ 5^3 = 125,\ 5^4 = 625$

b $6^1 = 6,\ 6^2 = 36,\ 6^3 = 216,\ 6^4 = 1296$

c $7^1 = 7,\ 7^2 = 49,\ 7^3 = 343,\ 7^4 = 2401$

3 **a** $$\begin{aligned}&(-1)^5\\ &= (-1)\times(-1)\times(-1)\times(-1)\times(-1)\\ &= 1\times 1\times(-1)\\ &= -1\end{aligned}$$

b $$\begin{aligned}&(-1)^6\\ &= (-1)^5\times(-1)\\ &= (-1)\times(-1)\\ &= 1\end{aligned}$$

c $$\begin{aligned}&(-1)^{14}\\ &= 1\end{aligned}$$

d $$\begin{aligned}&(-1)^{19}\\ &= -1\end{aligned}$$

e $$\begin{aligned}&(-1)^8\\ &= 1\end{aligned}$$

f $$\begin{aligned}&-1^8\\ &= -(1^8)\\ &= -1\end{aligned}$$

g $$\begin{aligned}&-(-1)^8\\ &= -(1)\\ &= -1\end{aligned}$$

h $$\begin{aligned}&(-2)^5\\ &= (-2)\times(-2)\times(-2)\times(-2)\times(-2)\\ &= 4\times 4\times(-2)\\ &= -32\end{aligned}$$

i $$\begin{aligned}&-2^5\\ &= -(2^5)\\ &= -32\end{aligned}$$

j $$\begin{aligned}&-(-2)^6\\ &= -(-2)^5\times(-2)\\ &= 32\times(-2)\\ &= -64\end{aligned}$$

k $$\begin{aligned}&(-5)^4\\ &= (-5)\times(-5)\times(-5)\times(-5)\\ &= 25\times 25\\ &= 625\end{aligned}$$

l $$\begin{aligned}&-(-5)^4\\ &= -(-5)\times(-5)\times(-5)\times(-5)\\ &= -25\times 25\\ &= -625\end{aligned}$$

4 **a** $4^7 = 16\,384$ **b** $7^4 = 2401$ **c** $-5^5 = -3125$ **d** $(-5)^5 = -3125$

e $8^6 = 262\,144$ **f** $(-8)^6 = 262\,144$ **g** $-8^6 = -262\,144$

h $2.13^9 \approx 902.436\,039\,6$ **i** $-2.13^9 \approx -902.436\,039\,6$ **j** $(-2.13)^9 \approx -902.436\,039\,6$

5 **a** $9^{-1} = 0.\overline{1}$ **b** $\dfrac{1}{9^1} = 0.\overline{1}$ **c** $6^{-2} = 0.02\overline{7}$ **d** $\dfrac{1}{6^2} = 0.02\overline{7}$

e $3^{-4} \approx 0.012\,345\,679$ **f** $\dfrac{1}{3^4} \approx 0.012\,345\,679$ **g** $17^0 = 1$ **h** $(0.366)^0 = 1$

We notice that $a^{-n} = \dfrac{1}{a^n}$ and $a^0 = 1$ for $a \neq 0$.

6 $3^{101} = \underbrace{3^4\times 3^4\times 3^4\times \ldots\times 3^4}_{\text{25 of these}}\times 3^1$

But $3^4 = 81$ which ends in a 1

$\therefore\ \underbrace{3^4\times 3^4\times 3^4\times \ldots\times 3^4}_{\text{25 of these}}$ ends in a 1

$\therefore\ 3^{101}$ ends in a 3

7 $7^1 = 7,\quad 7^2 = 49,\quad 7^3 = 343,\quad 7^4 = 2401,\quad 7^5 = 16\,807$

Now $7^{217} = \underbrace{7^4\times 7^4\times 7^4\times \ldots\times 7^4}_{\text{54 of these, so this part ends in a 1}}\times 7^1$

$\therefore\ 7^{217}$ ends in $1\times 7 = 7$.

EXERCISE 3B

1

a $5^4 \times 5^7 = 5^{4+7}$
$= 5^{11}$

b $d^2 \times d^6 = d^{2+6}$
$= d^8$

c $\dfrac{k^8}{k^3} = k^{8-3}$
$= k^5$

d $\dfrac{7^5}{7^6} = 7^{5-6}$
$= 7^{-1}$
$= \frac{1}{7}$

e $(x^2)^5 = x^{2\times 5}$
$= x^{10}$

f $(3^4)^4 = 3^{4\times 4}$
$= 3^{16}$

g $\dfrac{p^3}{p^7} = p^{3-7}$
$= p^{-4}$ or $\dfrac{1}{p^4}$

h $n^3 \times n^9 = n^{3+9}$
$= n^{12}$

i $(5^t)^3 = 5^{t\times 3}$
$= 5^{3t}$

j $7^x \times 7^2 = 7^{x+2}$

k $\dfrac{10^3}{10^q} = 10^{3-q}$

l $(c^4)^m = c^{4\times m}$
$= c^{4m}$

2

a $4 = 2 \times 2$
$= 2^2$

b $\dfrac{1}{4} = \dfrac{1}{2^2}$
$= 2^{-2}$

c $8 = 2 \times 2 \times 2$
$= 2^3$

d $\dfrac{1}{8} = \dfrac{1}{2^3}$
$= 2^{-3}$

e 32
$= 2 \times 2 \times 2 \times 2 \times 2$
$= 2^5$

f $\dfrac{1}{32} = \dfrac{1}{2^5}$
$= 2^{-5}$

g $2 = 2^1$

h $\dfrac{1}{2} = \dfrac{1}{2^1}$
$= 2^{-1}$

i $64 = 32 \times 2$
$= 2^5 \times 2^1$
$= 2^6$

j $\dfrac{1}{64} = \dfrac{1}{2^6}$
$= 2^{-6}$

k $128 = 64 \times 2$
$= 2^6 \times 2^1$
$= 2^7$

l $\dfrac{1}{128} = \dfrac{1}{2^7}$
$= 2^{-7}$

3

a $9 = 3 \times 3$
$= 3^2$

b $\dfrac{1}{9} = \dfrac{1}{3^2}$
$= 3^{-2}$

c $27 = 3 \times 3 \times 3$
$= 3^3$

d $\dfrac{1}{27} = \dfrac{1}{3^3}$
$= 3^{-3}$

e $3 = 3^1$

f $\dfrac{1}{3} = \dfrac{1}{3^1}$
$= 3^{-1}$

g $81 = 3 \times 3 \times 3 \times 3$
$= 3^4$

h $\dfrac{1}{81} = \dfrac{1}{3^4}$
$= 3^{-4}$

i $1 = 3^0$

j $243 = 81 \times 3$
$= 3^4 \times 3^1$
$= 3^5$

k $\dfrac{1}{243} = \dfrac{1}{3^5}$
$= 3^{-5}$

4

a $2 \times 2^a = 2^1 \times 2^a$
$= 2^{a+1}$

b $4 \times 2^b = 2^2 \times 2^b$
$= 2^{b+2}$

c $8 \times 2^t = 2^3 \times 2^t$
$= 2^{t+3}$

d $(2^{x+1})^2 = 2^{2(x+1)}$
$= 2^{2x+2}$

e $(2^{1-n})^{-1} = 2^{-(1-n)}$
$= 2^{n-1}$

f $\dfrac{2^c}{4} = \dfrac{2^c}{2^2} = 2^{c-2}$

g $\dfrac{2^m}{2^{-m}} = 2^{m-(-m)}$
$= 2^{2m}$

h $\dfrac{4}{2^{1-n}} = \dfrac{2^2}{2^{1-n}}$
$= 2^{2-(1-n)}$
$= 2^{n+1}$

i $\dfrac{2^{x+1}}{2^x} = 2^{x+1-x}$
$= 2^1$

j $\dfrac{4^x}{2^{1-x}} = \dfrac{(2^2)^x}{2^{1-x}}$
$= 2^{2x-(1-x)}$
$= 2^{3x-1}$

5 **a** $9 \times 3^p = 3^2 \times 3^p$
$= 3^{p+2}$

b $27^a = (3^3)^a$
$= 3^{3a}$

c $3 \times 9^n = 3^1 \times (3^2)^n$
$= 3^{2n+1}$

d $27 \times 3^d = 3^3 \times 3^d$
$= 3^{d+3}$

e $9 \times 27^t = 3^2 \times (3^3)^t$
$= 3^{3t+2}$

f $\dfrac{3^y}{3} = \dfrac{3^y}{3^1} = 3^{y-1}$

g $\dfrac{3}{3^y} = \dfrac{3^1}{3^y}$
$= 3^{1-y}$

h $\dfrac{9}{27^t} = \dfrac{3^2}{(3^3)^t}$
$= 3^{2-3t}$

i $\dfrac{9^a}{3^{1-a}} = \dfrac{(3^2)^a}{3^{1-a}}$
$= 3^{2a-(1-a)}$
$= 3^{3a-1}$

j $\dfrac{9^{n+1}}{3^{2n-1}} = \dfrac{(3^2)^{n+1}}{3^{2n-1}}$
$= 3^{2n+2-(2n-1)}$
$= 3^3$

6 **a** $(2a)^2 = 2^2 \times a^2$
$= 4a^2$

b $(3b)^3 = 3^3 \times b^3$
$= 27b^3$

c $(ab)^4 = a^4 \times b^4$
$= a^4b^4$

d $(pq)^3 = p^3 \times q^3$
$= p^3q^3$

e $\left(\dfrac{m}{n}\right)^2 = \dfrac{m^2}{n^2}$

f $\left(\dfrac{a}{3}\right)^3 = \dfrac{a^3}{3^3} = \dfrac{a^3}{27}$

g $\left(\dfrac{b}{c}\right)^4 = \dfrac{b^4}{c^4}$

h $\left(\dfrac{2a}{b}\right)^0 = 1, \ b \neq 0$

i $\left(\dfrac{m}{3n}\right)^4 = \dfrac{m^4}{3^4 \times n^4} = \dfrac{m^4}{81n^4}$

j $\left(\dfrac{xy}{2}\right)^3 = \dfrac{x^3y^3}{2^3} = \dfrac{x^3y^3}{8}$

7 **a** $(-2a)^2$
$= (-2)^2a^2$
$= 4a^2$

b $(-6b^2)^2$
$= (-6)^2b^4$
$= 36b^4$

c $(-2a)^3$
$= (-2)^3a^3$
$= -8a^3$

d $(-3m^2n^2)^3$
$= (-3)^3m^6n^6$
$= -27m^6n^6$

e $(-2ab^4)^4$
$= (-2)^4a^4b^{16}$
$= 16a^4b^{16}$

f $\left(\dfrac{-2a^2}{b^2}\right)^3$
$= \dfrac{(-2)^3a^6}{b^6}$
$= -\dfrac{8a^6}{b^6}$

g $\left(\dfrac{-4a^3}{b}\right)^2$
$= \dfrac{(-4)^2a^6}{b^2}$
$= \dfrac{16a^6}{b^2}$

h $\left(\dfrac{-3p^2}{q^3}\right)^2$
$= \dfrac{(-3)^2p^4}{q^6}$
$= \dfrac{9p^4}{q^6}$

8 **a** $ab^{-2} = \dfrac{a}{b^2}$

b $(ab)^{-2} = \dfrac{1}{(ab)^2}$
$= \dfrac{1}{a^2b^2}$

c $(2ab^{-1})^2 = 2^2a^2b^{-2}$
$= \dfrac{4a^2}{b^2}$

d $(3a^{-2}b)^2 = 3^2a^{-4}b^2$
$= \dfrac{9b^2}{a^4}$

e $\dfrac{a^2b^{-1}}{c^2} = \dfrac{a^2}{bc^2}$

f $\dfrac{a^2b^{-1}}{c^{-2}} = \dfrac{a^2c^2}{b}$

g $\dfrac{1}{a^{-3}} = a^3$

h $\dfrac{a^{-2}}{b^{-3}} = \dfrac{b^3}{a^2}$

i $\dfrac{2a^{-1}}{d^2} = \dfrac{2}{ad^2}$

j $\dfrac{12a}{m^{-3}} = 12am^3$

9 **a** $\dfrac{1}{a^n} = a^{-n}$

b $\dfrac{1}{b^{-n}} = b^n$

c $\dfrac{1}{3^{2-n}} = 3^{n-2}$

d $\dfrac{a^n}{b^{-m}} = a^nb^m$

e $\dfrac{a^{-n}}{a^{2+n}} = a^{-n-(2+n)}$
$= a^{-2n-2}$

10 **a** $(\frac{5}{3})^0 = 1$ **b** $(\frac{7}{4})^{-1} = \frac{4}{7}$ **c** $(\frac{1}{6})^{-1} = \frac{6}{1} = 6$ **d** $\dfrac{3^3}{3^0} = \frac{27}{1} = 27$

e $(\frac{4}{3})^{-2} = \dfrac{3^2}{4^2} = \frac{9}{16}$ **f** $2^1 + 2^{-1} = 2 + \frac{1}{2} = \frac{5}{2}$ **g** $(1\frac{2}{3})^{-3} = (\frac{5}{3})^{-3} = \dfrac{3^3}{5^3} = \frac{27}{125}$ **h** $5^2 + 5^1 + 5^{-1} = 25 + 5 + \frac{1}{5} = \frac{151}{5}$

11 **a** $5^3 = 21 + 23 + 25 + 27 + 29$ **b** $7^3 = 43 + 45 + 47 + 49 + 51 + 53 + 55$

c $12^3 = 133 + 135 + 137 + 139 + 141 + 143 + 145 + 147 + 149 + 151 + 153 + 155$

EXERCISE 3C

1 **a** $\sqrt[5]{2} = 2^{\frac{1}{5}}$ **b** $\dfrac{1}{\sqrt[5]{2}} = \dfrac{1}{2^{\frac{1}{5}}} = 2^{-\frac{1}{5}}$ **c** $2\sqrt{2} = 2^1 \times 2^{\frac{1}{2}} = 2^{\frac{3}{2}}$ **d** $4\sqrt{2} = 2^2 \times 2^{\frac{1}{2}} = 2^{\frac{5}{2}}$

e $\dfrac{1}{\sqrt[3]{2}} = \dfrac{1}{2^{\frac{1}{3}}} = 2^{-\frac{1}{3}}$ **f** $2 \times \sqrt[3]{2} = 2^1 \times 2^{\frac{1}{3}} = 2^{\frac{4}{3}}$ **g** $\dfrac{4}{\sqrt{2}} = \dfrac{2^2}{2^{\frac{1}{2}}} = 2^{\frac{3}{2}}$

h $(\sqrt{2})^3 = (2^{\frac{1}{2}})^3 = 2^{\frac{3}{2}}$ **i** $\dfrac{1}{\sqrt[3]{16}} = \dfrac{1}{\sqrt[3]{2^4}} = \dfrac{1}{2^{\frac{4}{3}}} = 2^{-\frac{4}{3}}$ **j** $\dfrac{1}{\sqrt{8}} = \dfrac{1}{\sqrt{2^3}} = \dfrac{1}{2^{\frac{3}{2}}} = 2^{-\frac{3}{2}}$

2 **a** $\sqrt[3]{3} = 3^{\frac{1}{3}}$ **b** $\dfrac{1}{\sqrt[3]{3}} = \dfrac{1}{3^{\frac{1}{3}}} = 3^{-\frac{1}{3}}$ **c** $\sqrt[4]{3} = 3^{\frac{1}{4}}$ **d** $3\sqrt{3} = 3^1 \times 3^{\frac{1}{2}} = 3^{\frac{3}{2}}$

e $\dfrac{1}{9\sqrt{3}} = \dfrac{1}{3^2 \times 3^{\frac{1}{2}}} = \dfrac{1}{3^{\frac{5}{2}}} = 3^{-\frac{5}{2}}$

3 **a** $\sqrt[3]{7} = 7^{\frac{1}{3}}$ **b** $\sqrt[4]{27} = \sqrt[4]{3^3} = 3^{\frac{3}{4}}$ **c** $\sqrt[5]{16} = \sqrt[5]{2^4} = 2^{\frac{4}{5}}$ **d** $\sqrt[3]{32} = \sqrt[3]{2^5} = 2^{\frac{5}{3}}$

e $\sqrt[7]{49} = \sqrt[7]{7^2} = 7^{\frac{2}{7}}$ **f** $\dfrac{1}{\sqrt[3]{7}} = \dfrac{1}{7^{\frac{1}{3}}} = 7^{-\frac{1}{3}}$ **g** $\dfrac{1}{\sqrt[4]{27}} = \dfrac{1}{3^{\frac{3}{4}}} = 3^{-\frac{3}{4}}$ **h** $\dfrac{1}{\sqrt[5]{16}} = \dfrac{1}{2^{\frac{4}{5}}} = 2^{-\frac{4}{5}}$

i $\dfrac{1}{\sqrt[3]{32}} = \dfrac{1}{2^{\frac{5}{3}}} = 2^{-\frac{5}{3}}$ **j** $\dfrac{1}{\sqrt[7]{49}} = \dfrac{1}{7^{\frac{2}{7}}} = 7^{-\frac{2}{7}}$

4 **a** $3^{\frac{3}{4}} \approx 2.28$ **b** $2^{\frac{7}{8}} \approx 1.83$ **c** $2^{-\frac{1}{3}} \approx 0.794$ **d** $4^{-\frac{3}{5}} \approx 0.435$

e $\sqrt[4]{8} \approx 1.68$ **f** $\sqrt[5]{27} \approx 1.93$ **g** $\dfrac{1}{\sqrt[3]{7}} \approx 0.523$

5 **a** $4^{\frac{3}{2}} = (2^2)^{\frac{3}{2}} = 2^3 = 8$ **b** $8^{\frac{5}{3}} = (2^3)^{\frac{5}{3}} = 2^5 = 32$ **c** $16^{\frac{3}{4}} = (2^4)^{\frac{3}{4}} = 2^3 = 8$ **d** $25^{\frac{3}{2}} = (5^2)^{\frac{3}{2}} = 5^3 = 125$

e $32^{\frac{2}{5}} = (2^5)^{\frac{2}{5}}$
$= 2^2$
$= 4$

f $4^{-\frac{1}{2}} = (2^2)^{-\frac{1}{2}}$
$= 2^{-1}$
$= \frac{1}{2}$

g $9^{-\frac{3}{2}} = (3^2)^{-\frac{3}{2}}$
$= 3^{-3}$
$= \frac{1}{27}$

h $8^{-\frac{4}{3}} = (2^3)^{-\frac{4}{3}}$
$= 2^{-4}$
$= \frac{1}{16}$

i $27^{-\frac{4}{3}} = (3^3)^{-\frac{4}{3}}$
$= 3^{-4}$
$= \frac{1}{81}$

j $125^{-\frac{2}{3}} = (5^3)^{-\frac{2}{3}}$
$= 5^{-2}$
$= \frac{1}{25}$

EXERCISE 3D.1

1 **a** $x^2(x^3 + 2x^2 + 1)$
$= x^2 \times x^3 + x^2 \times 2x^2 + x^2 \times 1$
$= x^5 + 2x^4 + x^2$

b $2^x(2^x + 1)$
$= 2^x \times 2^x + 2^x \times 1$
$= 2^{2x} + 2^x$
$= 4^x + 2^x$

c $x^{\frac{1}{2}}(x^{\frac{1}{2}} + x^{-\frac{1}{2}})$
$= x^{\frac{1}{2}} \times x^{\frac{1}{2}} + x^{\frac{1}{2}} \times x^{-\frac{1}{2}}$
$= x^1 + x^0$
$= x + 1$

d $7^x(7^x + 2)$
$= 7^x \times 7^x + 7^x \times 2$
$= 7^{2x} + 2(7^x)$
$= 49^x + 2(7^x)$

e $3^x(2 - 3^{-x})$
$= 3^x \times 2 - 3^x \times 3^{-x}$
$= 2(3^x) - 3^0$
$= 2(3^x) - 1$

f $x^{\frac{1}{2}}(x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + 3x^{-\frac{1}{2}})$
$= x^{\frac{1}{2}} \times x^{\frac{3}{2}} + x^{\frac{1}{2}} \times 2x^{\frac{1}{2}} + x^{\frac{1}{2}} \times 3x^{-\frac{1}{2}}$
$= x^2 + 2x^1 + 3x^0$
$= x^2 + 2x + 3$

g $2^{-x}(2^x + 5)$
$= 2^{-x} \times 2^x + 2^{-x} \times 5$
$= 2^0 + 5(2^{-x})$
$= 1 + 5(2^{-x})$

h $5^{-x}(5^{2x} + 5^x)$
$= 5^{-x} \times 5^{2x} + 5^{-x} \times 5^x$
$= 5^x + 5^0$
$= 5^x + 1$

i $x^{-\frac{1}{2}}(x^2 + x + x^{\frac{1}{2}})$
$= x^{-\frac{1}{2}} \times x^2 + x^{-\frac{1}{2}} \times x^1 + x^{-\frac{1}{2}} \times x^{\frac{1}{2}}$
$= x^{\frac{3}{2}} + x^{\frac{1}{2}} + x^0$
$= x^{\frac{3}{2}} + x^{\frac{1}{2}} + 1$

2 **a** $(2^x - 1)(2^x + 3)$
$= 2^x \times 2^x + 2^x \times 3 - 1 \times 2^x - 3$
$= 2^{2x} + 2(2^x) - 3$
$= 4^x + 2^{x+1} - 3$

b $(3^x + 2)(3^x + 5)$
$= 3^x \times 3^x + 3^x \times 5 + 2 \times 3^x + 10$
$= 3^{2x} + 7(3^x) + 10$
$= 9^x + 7(3^x) + 10$

c $(5^x - 2)(5^x - 4)$
$= 5^x \times 5^x - 5^x \times 4 - 2 \times 5^x + 8$
$= 5^{2x} - 6(5^x) + 8$
$= 25^x - 6(5^x) + 8$

d $(2^x + 3)^2$
$= (2^x)^2 + 2 \times 2^x \times 3 + 3^2$
$= 2^{2x} + 6(2^x) + 9$
$= 4^x + 6(2^x) + 9$

e $(3^x - 1)^2$
$= (3^x)^2 - 2 \times 3^x \times 1 + 1^2$
$= 3^{2x} - 2(3^x) + 1$
$= 9^x - 2(3^x) + 1$

f $(4^x + 7)^2$
$= (4^x)^2 + 2 \times 4^x \times 7 + 7^2$
$= 4^{2x} + 14(4^x) + 49$
$= 16^x + 14(4^x) + 49$

g $(x^{\frac{1}{2}} + 2)(x^{\frac{1}{2}} - 2)$
$= (x^{\frac{1}{2}})^2 - 2^2$
$= x - 4$

h $(2^x + 3)(2^x - 3)$
$= (2^x)^2 - 3^2$
$= 2^{2x} - 9$
$= 4^x - 9$

i $(x^{\frac{1}{2}}+x^{-\frac{1}{2}})(x^{\frac{1}{2}}-x^{-\frac{1}{2}})$
$=(x^{\frac{1}{2}})^2-(x^{-\frac{1}{2}})^2$
$=x^1-x^{-1}$
$=x-x^{-1}$

j $\left(x+\frac{2}{x}\right)^2$
$=x^2+2\times x\times\left(\frac{2}{x}\right)+\left(\frac{2}{x}\right)^2$
$=x^2+4+\frac{4}{x^2}$

k $(7^x-7^{-x})^2$
$=(7^x)^2-2\times 7^x\times 7^{-x}+(7^{-x})^2$
$=7^{2x}-2\times 7^0+7^{-2x}$
$=7^{2x}-2+7^{-2x}$

l $(5-2^{-x})^2$
$=5^2-2\times 5\times 2^{-x}+(2^{-x})^2$
$=25-10(2^{-x})+2^{-2x}$
$=25-10(2^{-x})+4^{-x}$

EXERCISE 3D.2

1 **a** $5^{2x}+5^x$
$=5^x\times 5^x+5^x$
$=5^x(5^x+1)$

b $3^{n+2}+3^n$
$=3^n\times 3^2+3^n$
$=3^n(3^2+1)$
$=10(3^n)$

c 7^n+7^{3n}
$=7^n+7^n\times 7^{2n}$
$=7^n(1+7^{2n})$

d $5^{n+1}-5$
$=5\times 5^n-5$
$=5(5^n-1)$

e $6^{n+2}-6$
$=6\times 6^{n+1}-6$
$=6(6^{n+1}-1)$

f $4^{n+2}-16$
$=4^2\times 4^n-16$
$=16\times 4^n-16$
$=16(4^n-1)$

2 **a** 9^x-4
$=(3^x)^2-2^2$
$=(3^x+2)(3^x-2)$

b 4^x-25
$=(2^x)^2-5^2$
$=(2^x+5)(2^x-5)$

c $16-9^x$
$=4^2-(3^x)^2$
$=(4+3^x)(4-3^x)$

d $25-4^x$
$=5^2-(2^x)^2$
$=(5+2^x)(5-2^x)$

e 9^x-4^x
$=(3^x)^2-(2^x)^2$
$=(3^x+2^x)(3^x-2^x)$

f $4^x+6(2^x)+9$
$=(2^x)^2+6(2^x)+9$
$=(2^x+3)^2$
$\{a^2+6a+9=(a+3)^2\}$

g $9^x+10(3^x)+25$
$=(3^x)^2+10(3^x)+25$
$=(3^x+5)^2$
$\{a^2+10a+25=(a+5)^2\}$

h $4^x-14(2^x)+49$
$=(2^x)^2-14(2^x)+49$
$=(2^x-7)^2$
$\{a^2-14a+49=(a-7)^2\}$

i $25^x-4(5^x)+4$
$=(5^x)^2-4(5^x)+4$
$=(5^x-2)^2$
$\{a^2-4a+4=(a-2)^2\}$

3 **a** $4^x+9(2^x)+18$
$=(2^x)^2+9(2^x)+18$
$=(2^x+3)(2^x+6)$
$\{a^2+9a+18=(a+3)(a+6)\}$

b 4^x-2^x-20
$=(2^x)^2-2^x-20$
$=(2^x+4)(2^x-5)$
$\{a^2-a-20=(a+4)(a-5)\}$

c $9^x+9(3^x)+14$
$=(3^x)^2+9(3^x)+14$
$=(3^x+2)(3^x+7)$
$\{a^2+9a+14=(a+2)(a+7)\}$

d $9^x+4(3^x)-5$
$=(3^x)^2+4(3^x)-5$
$=(3^x+5)(3^x-1)$
$\{a^2+4a-5=(a+5)(a-1)\}$

e 25^x+5^x-2
$=(5^x)^2+5^x-2$
$=(5^x+2)(5^x-1)$
$\{a^2+a-2=(a+2)(a-1)\}$

f $49^x-7^{x+1}+12$
$=(7^x)^2-7(7^x)+12$
$=(7^x-4)(7^x-3)$
$\{a^2-7a+12=(a-4)(a-3)\}$

4 **a** $\frac{12^n}{6^n} = \left(\frac{12}{6}\right)^n$
$= 2^n$

b $\frac{20^a}{2^a} = \left(\frac{20}{2}\right)^a$
$= 10^a$

c $\frac{6^b}{2^b} = \left(\frac{6}{2}\right)^b$
$= 3^b$

d $\frac{4^n}{20^n} = \left(\frac{4}{20}\right)^n$
$= \left(\frac{1}{5}\right)^n$
$= \frac{1}{5^n}$

e $\frac{35^x}{7^x} = \left(\frac{35}{7}\right)^x$
$= 5^x$

f $\frac{6^a}{8^a} = \left(\frac{6}{8}\right)^a$
$= \left(\frac{3}{4}\right)^a$

g $\frac{5^{n+1}}{5^n} = \frac{5 \times \cancel{5^n}}{\cancel{5^n}_1}$
$= 5$

h $\frac{5^{n+1}}{5} = \frac{\cancel{5} \times 5^n}{\cancel{5}_1}$
$= 5^n$

5 **a** $\frac{6^m + 2^m}{2^m}$
$= \frac{2^m 3^m + 2^m}{2^m}$
$= \frac{\cancel{2^m}(3^m + 1)}{\cancel{2^m}\,1}$
$= 3^m + 1$

b $\frac{2^n + 12^n}{2^n}$
$= \frac{2^n + 2^n 6^n}{2^n}$
$= \frac{\cancel{2^n}(1 + 6^n)}{\cancel{2^n}\,1}$
$= 1 + 6^n$

c $\frac{8^n + 4^n}{2^n}$
$= \frac{2^n 4^n + 2^n 2^n}{2^n}$
$= \frac{\cancel{2^n}(4^n + 2^n)}{\cancel{2^n}\,1}$
$= 4^n + 2^n$

d $\frac{12^x - 3^x}{3^x}$
$= \frac{3^x 4^x - 3^x}{3^x}$
$= \frac{\cancel{3^x}(4^x - 1)}{\cancel{3^x}\,1}$
$= 4^x - 1$

e $\frac{6^n + 12^n}{1 + 2^n}$
$= \frac{6^n + 6^n 2^n}{1 + 2^n}$
$= \frac{6^n \cancel{(1 + 2^n)}}{\cancel{1 + 2^n}\,1}$
$= 6^n$

f $\frac{5^{n+1} - 5^n}{4}$
$= \frac{5^n \times 5 - 5^n}{4}$
$= \frac{5^n \cancel{(5 - 1)}}{\cancel{4}\,1}$
$= 5^n$

g $\frac{5^{n+1} - 5^n}{5^n}$
$= \frac{5^n \times 5 - 5^n}{5^n}$
$= \frac{\cancel{5^n}(5 - 1)}{\cancel{5^n}\,1}$
$= 4$

h $\frac{4^n - 2^n}{2^n}$
$= \frac{2^n 2^n - 2^n}{2^n}$
$= \frac{\cancel{2^n}(2^n - 1)}{\cancel{2^n}\,1}$
$= 2^n - 1$

i $\frac{2^n - 2^{n-1}}{2^n}$
$= \frac{2^{n-1} \times 2 - 2^{n-1}}{2^{n-1} \times 2}$
$= \frac{\cancel{2^{n-1}}(2 - 1)}{\cancel{2^{n-1}} \times 2}$
$= \frac{1}{2}$

6 **a** $2^n(n+1) + 2^n(n-1)$
$= 2^n(n + 1 + n - 1)$
$= 2^n(2n)$
$= n2^{n+1}$

b $3^n\left(\frac{n-1}{6}\right) - 3^n\left(\frac{n+1}{6}\right)$
$= 3^n\left(\frac{n-1}{6} - \frac{n+1}{6}\right)$
$= 3^n(-\frac{1}{3})$
$= 3^n \times -3^{-1}$
$= -3^{n-1}$

EXERCISE 3E

1 **a** $2^x = 8$
$\therefore\ 2^x = 2^3$
$\therefore\ x = 3$

b $5^x = 25$
$\therefore\ 5^x = 5^2$
$\therefore\ x = 2$

c $3^x = 81$
$\therefore\ 3^x = 3^4$
$\therefore\ x = 4$

d $7^x = 1$
$\therefore\ 7^x = 7^0$
$\therefore\ x = 0$

e $3^x = \frac{1}{3}$
$\therefore\ 3^x = 3^{-1}$
$\therefore\ x = -1$

f $2^x = \sqrt{2}$
$\therefore\ 2^x = 2^{\frac{1}{2}}$
$\therefore\ x = \frac{1}{2}$

g $5^x = \frac{1}{125}$
$\therefore\ 5^x = 5^{-3}$
$\therefore\ x = -3$

h $4^{x+1} = 64$
$\therefore\ 4^{x+1} = 4^3$
$\therefore\ x+1 = 3$
$\therefore\ x = 2$

i $2^{x-2} = \frac{1}{32}$
$\therefore\ 2^{x-2} = 2^{-5}$
$\therefore\ x-2 = -5$
$\therefore\ x = -3$

j $3^{x+1} = \frac{1}{27}$
$\therefore\ 3^{x+1} = 3^{-3}$
$\therefore\ x+1 = -3$
$\therefore\ x = -4$

k $7^{x+1} = 343$
$\therefore\ 7^{x+1} = 7^3$
$\therefore\ x+1 = 3$
$\therefore\ x = 2$

l $5^{1-2x} = \frac{1}{5}$
$\therefore\ 5^{1-2x} = 5^{-1}$
$\therefore\ 1-2x = -1$
$\therefore\ -2x = -2$
$\therefore\ x = 1$

2 **a** $8^x = 32$
$\therefore\ 2^{3x} = 2^5$
$\therefore\ 3x = 5$
$\therefore\ x = \frac{5}{3}$

b $4^x = \frac{1}{8}$
$\therefore\ 2^{2x} = 2^{-3}$
$\therefore\ 2x = -3$
$\therefore\ x = -\frac{3}{2}$

c $9^x = \frac{1}{27}$
$\therefore\ 3^{2x} = 3^{-3}$
$\therefore\ 2x = -3$
$\therefore\ x = -\frac{3}{2}$

d $25^x = \frac{1}{5}$
$\therefore\ 5^{2x} = 5^{-1}$
$\therefore\ 2x = -1$
$\therefore\ x = -\frac{1}{2}$

e $27^x = \frac{1}{9}$
$\therefore\ 3^{3x} = 3^{-2}$
$\therefore\ 3x = -2$
$\therefore\ x = -\frac{2}{3}$

f $16^x = \frac{1}{32}$
$\therefore\ 2^{4x} = 2^{-5}$
$\therefore\ 4x = -5$
$\therefore\ x = -\frac{5}{4}$

g $4^{x+2} = 128$
$\therefore\ 2^{2(x+2)} = 2^7$
$\therefore\ 2x+4 = 7$
$\therefore\ 2x = 3$
$\therefore\ x = \frac{3}{2}$

h $25^{1-x} = \frac{1}{125}$
$\therefore\ 5^{2(1-x)} = 5^{-3}$
$\therefore\ 2-2x = -3$
$\therefore\ -2x = -5$
$\therefore\ x = \frac{5}{2}$

i $4^{4x-1} = \frac{1}{2}$
$\therefore\ 2^{2(4x-1)} = 2^{-1}$
$\therefore\ 8x-2 = -1$
$\therefore\ 8x = 1$
$\therefore\ x = \frac{1}{8}$

j $9^{x-3} = 27$
$\therefore\ 3^{2(x-3)} = 3^3$
$\therefore\ 2x-6 = 3$
$\therefore\ 2x = 9$
$\therefore\ x = \frac{9}{2}$

k $\left(\frac{1}{2}\right)^{x+1} = 8$
$\therefore\ \left(2^{-1}\right)^{x+1} = 2^3$
$\therefore\ -x-1 = 3$
$\therefore\ -x = 4$
$\therefore\ x = -4$

l $\left(\frac{1}{3}\right)^{x+2} = 9$
$\therefore\ \left(3^{-1}\right)^{x+2} = 3^2$
$\therefore\ -x-2 = 2$
$\therefore\ -x = 4$
$\therefore\ x = -4$

m $81^x = 27^{-x}$
$\therefore\ 3^{4x} = 3^{-3x}$
$\therefore\ 4x = -3x$
$\therefore\ 7x = 0$
$\therefore\ x = 0$

n $\left(\frac{1}{4}\right)^{1-x} = 32$
$\therefore\ \left(2^{-2}\right)^{1-x} = 2^5$
$\therefore\ -2+2x = 5$
$\therefore\ 2x = 7$
$\therefore\ x = \frac{7}{2}$

o $\left(\frac{1}{7}\right)^x = 49$
$\therefore\ 7^{-x} = 7^2$
$\therefore\ -x = 2$
$\therefore\ x = -2$

p $\left(\frac{1}{3}\right)^{x+1} = 243$
$\therefore\ \left(3^{-1}\right)^{x+1} = 3^5$
$\therefore\ -x-1 = 5$
$\therefore\ -x = 6$
$\therefore\ x = -6$

3 **a** $4^{2x+1} = 8^{1-x}$
$\therefore\ \left(2^2\right)^{2x+1} = \left(2^3\right)^{1-x}$
$\therefore\ 4x+2 = 3-3x$
$\therefore\ 7x = 1$
$\therefore\ x = \frac{1}{7}$

b $9^{2-x} = \left(\frac{1}{3}\right)^{2x+1}$
$\therefore\ \left(3^2\right)^{2-x} = \left(3^{-1}\right)^{2x+1}$
$\therefore\ 4-2x = -2x-1$
$\therefore\ 4 = -1$
This is clearly false, so no solutions exist.

c $2^x \times 8^{1-x} = \frac{1}{4}$
$\therefore\ 2^x \times \left(2^3\right)^{1-x} = 2^{-2}$
$\therefore\ x+3-3x = -2$
$\therefore\ -2x = -5$
$\therefore\ x = \frac{5}{2}$
(or $2\frac{1}{2}$)

4 **a** $3 \times 2^x = 24$
$\therefore\ 2^x = 8$
$\therefore\ 2^x = 2^3$
$\therefore\ x = 3$

b $7 \times 2^x = 56$
$\therefore\ 2^x = 8$
$\therefore\ 2^x = 2^3$
$\therefore\ x = 3$

c $3 \times 2^{x+1} = 24$
$\therefore\ 2^{x+1} = 8$
$\therefore\ 2^{x+1} = 2^3$
$\therefore\ x + 1 = 3$
$\therefore\ x = 2$

d $12 \times 3^{-x} = \frac{4}{3}$
$\therefore\ 3^{-x} = \frac{4}{3} \div 12$
$\therefore\ 3^{-x} = \frac{4}{3} \times \frac{1}{12}$
$\therefore\ 3^{-x} = \frac{1}{9}$
$\therefore\ 3^{-x} = 3^{-2}$
$\therefore\ x = 2$

e $4 \times \left(\frac{1}{3}\right)^x = 36$
$\therefore\ \left(\frac{1}{3}\right)^x = 9$
$\therefore\ \left(3^{-1}\right)^x = 3^2$
$\therefore\ 3^{-x} = 3^2$
$\therefore\ -x = 2$
$\therefore\ x = -2$

f $5 \times \left(\frac{1}{2}\right)^x = 20$
$\therefore\ \left(\frac{1}{2}\right)^x = 4$
$\therefore\ \left(2^{-1}\right)^x = 2^2$
$\therefore\ -x = 2$
$\therefore\ x = -2$

5 **a** $4^x - 6(2^x) + 8 = 0$
$\therefore\ (2^x)^2 - 6(2^x) + 8 = 0$
$\therefore\ (2^x - 2)(2^x - 4) = 0 \quad \{a^2 - 6a + 8 = (a-2)(a-4)\}$
$\therefore\ 2^x = 2 \text{ or } 4$
$\therefore\ 2^x = 2^1 \text{ or } 2^2$
$\therefore\ x = 1 \text{ or } 2$

b $4^x - 2^x - 2 = 0$
$\therefore\ (2^x)^2 - 2^x - 2 = 0$
$\therefore\ (2^x - 2)(2^x + 1) = 0 \quad \{a^2 - a - 2 = (a-2)(a+1)\}$
$\therefore\ 2^x = 2 \text{ or } -1$
$\therefore\ 2^x = 2^1 \quad \{\text{since } 2^x \text{ cannot be negative}\}$
$\therefore\ x = 1$

c $9^x - 12(3^x) + 27 = 0$
$\therefore\ (3^x)^2 - 12(3^x) + 27 = 0$
$\therefore\ (3^x - 3)(3^x - 9) = 0 \quad \{a^2 - 12a + 27 = (a-3)(a-9)\}$
$\therefore\ 3^x = 3 \text{ or } 9$
$\therefore\ 3^x = 3^1 \text{ or } 3^2$
$\therefore\ x = 1 \text{ or } 2$

d $9^x = 3^x + 6$
$\therefore\ (3^x)^2 - 3^x - 6 = 0$
$\therefore\ (3^x - 3)(3^x + 2) = 0 \quad \{a^2 - a - 6 = (a-3)(a+2)\}$
$\therefore\ 3^x = 3 \text{ or } -2$
$\therefore\ 3^x = 3^1 \quad \{\text{since } 3^x \text{ cannot be negative}\}$
$\therefore\ x = 1$

e $25^x - 23(5^x) - 50 = 0$
$\therefore\ (5^x)^2 - 23(5^x) - 50 = 0$
$\therefore\ (5^x - 25)(5^x + 2) = 0 \quad \{a^2 - 23a - 50 = (a-25)(a+2)\}$
$\therefore\ 5^x = 25 \text{ or } -2$
$\therefore\ 5^x = 5^2 \quad \{\text{since } 5^x \text{ cannot be negative}\}$
$\therefore\ x = 2$

f $49^x + 1 = 2(7^x)$
$\therefore\ (7^x)^2 - 2(7^x) + 1 = 0$
$\therefore\ (7^x - 1)^2 = 0 \quad \{a^2 - 2a + 1 = (a-1)^2\}$
$\therefore\ 7^x = 1$
$\therefore\ 7^x = 7^0$
$\therefore\ x = 0$

EXERCISE 3F

1 **a** When $x = \frac{1}{2}$, $y = 2^{\frac{1}{2}}$

From point A, $y \approx 1.4$

$\therefore \quad 2^{\frac{1}{2}} \approx 1.4$

b When $x = 0.8$, $y = 2^{0.8}$

From point B, $y \approx 1.7$

$\therefore \quad 2^{0.8} \approx 1.7$

c When $x = 1.5$, $y = 2^{1.5}$

From point C, $y \approx 2.8$

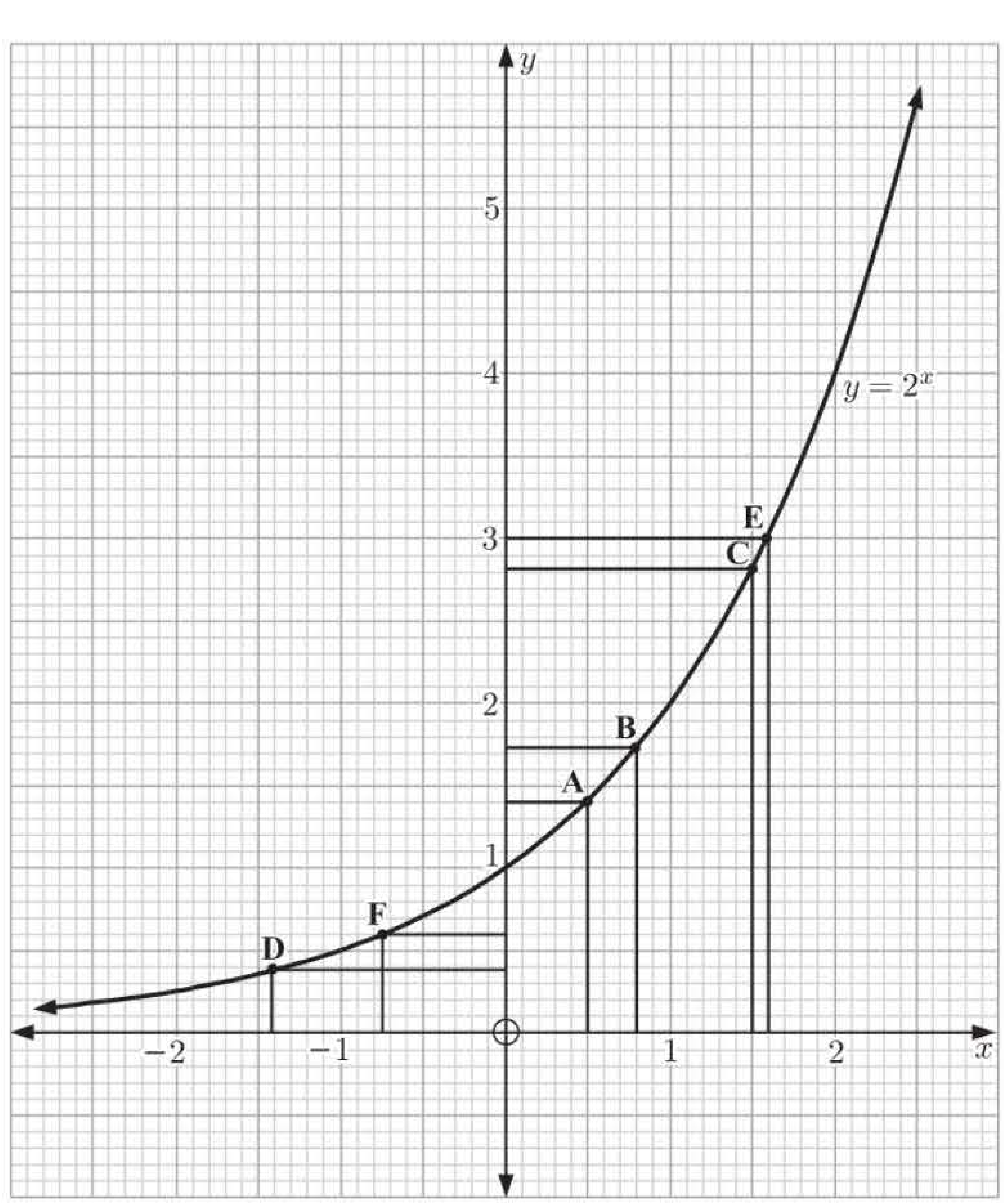
$\therefore \quad 2^{1.5} \approx 2.8$

d When $x = -\sqrt{2}$, $y = 2^{-\sqrt{2}}$

Using **a** we know $x \approx -1.4$

From point D, $y \approx 0.4$

$\therefore \quad 2^{-\sqrt{2}} \approx 0.4$



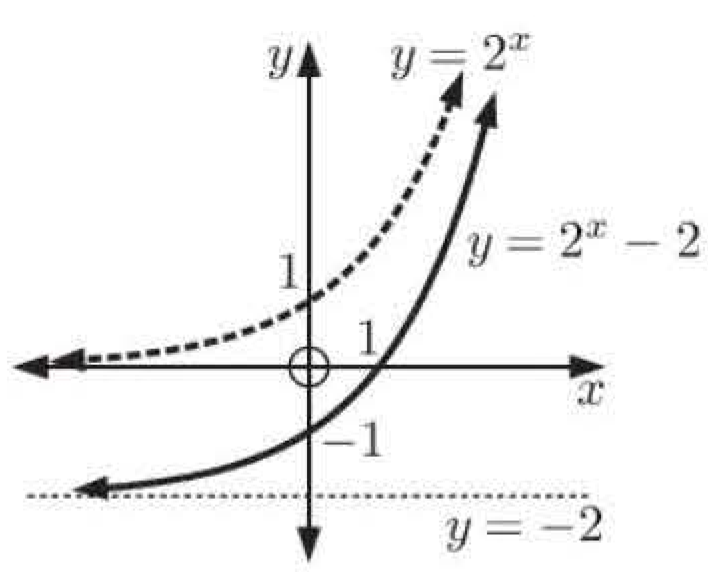
2 **a** When $2^x = 3$, $x \approx 1.6$ from point E. **b** When $2^x = 0.6$, $x \approx -0.7$ from point F.

3 The graph of $y = 2^x$ has a horizontal asymptote of $y = 0$.

$\therefore$ there is no value of x such that $2^x = 0$.

$\therefore$ $2^x = 0$ has no solutions.

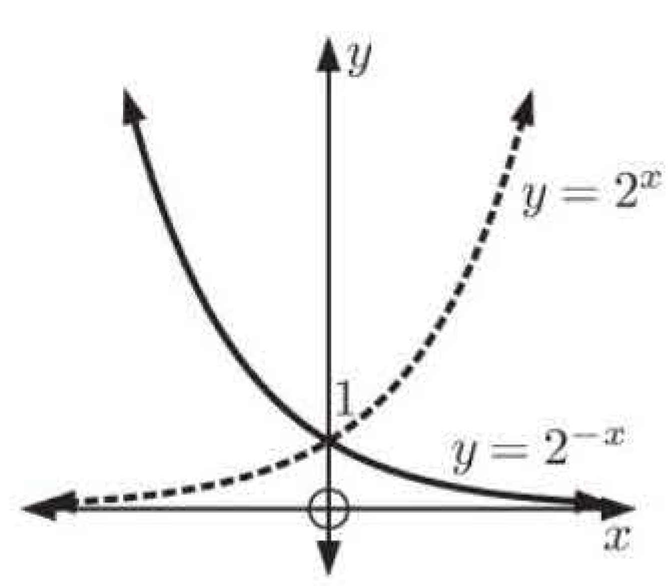
4 **a**



a vertical translation of 2 units downwards

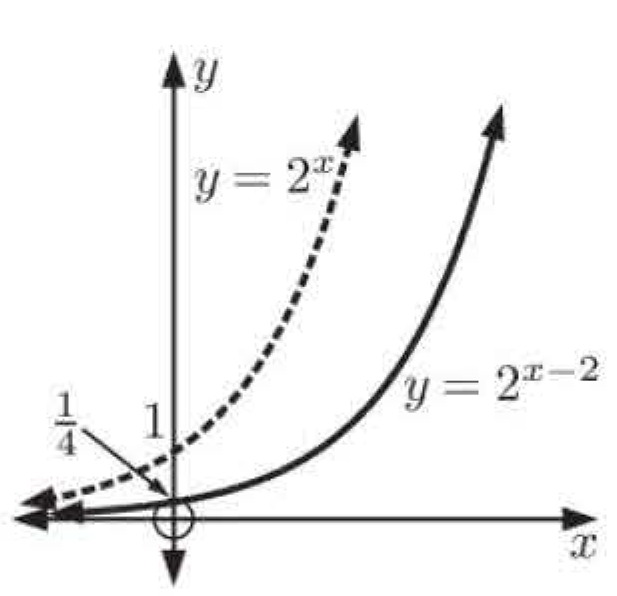
$y = -2$ is the H.A.

b



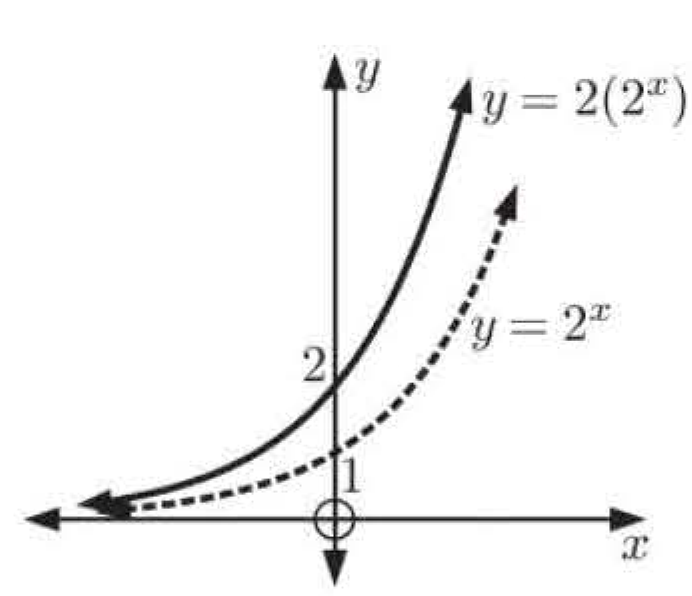
a reflection in the y-axis

c



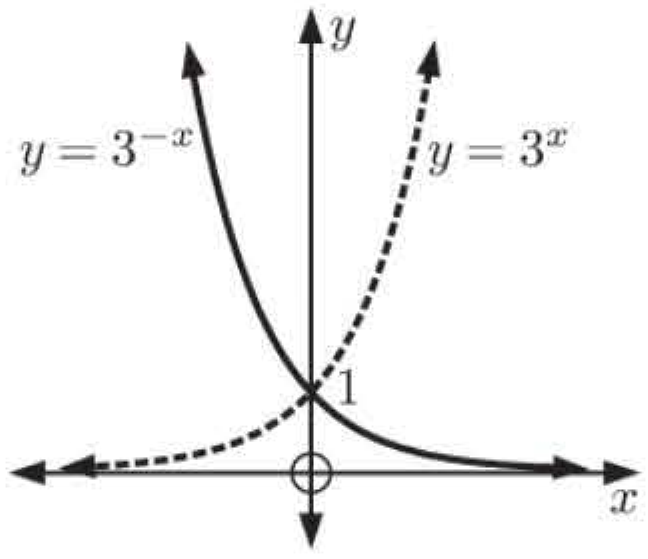
a horizontal translation of 2 units right

d



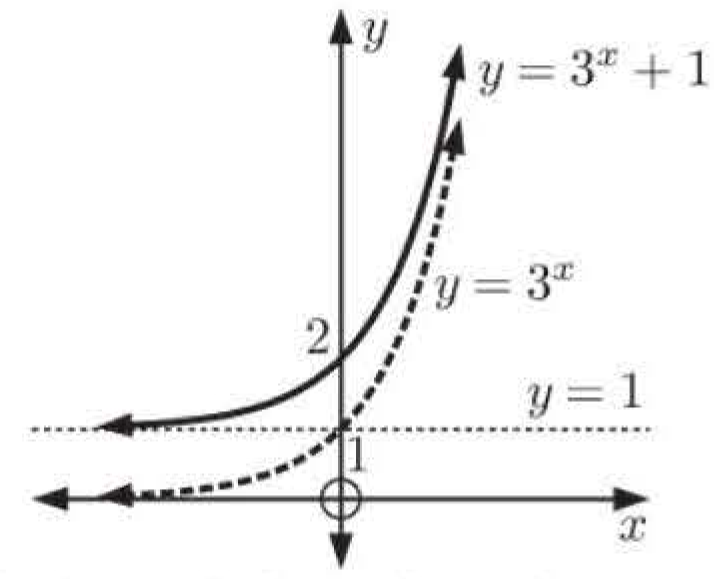
a vertical stretch of factor 2

5 **a**



a reflection in the y-axis

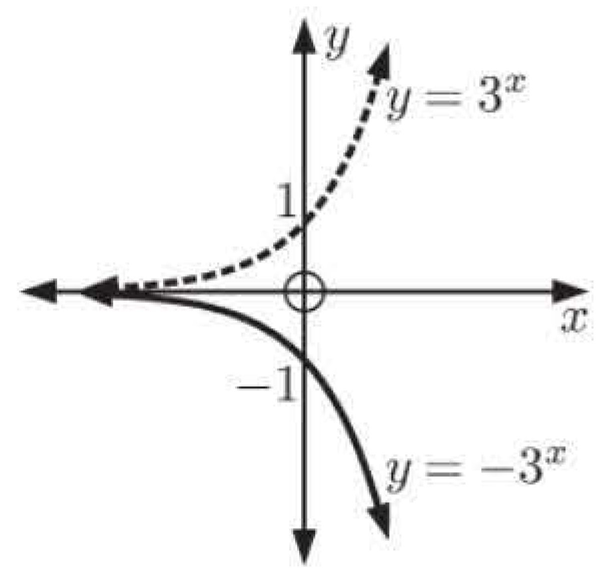
b



a vertical translation of 1 unit upwards

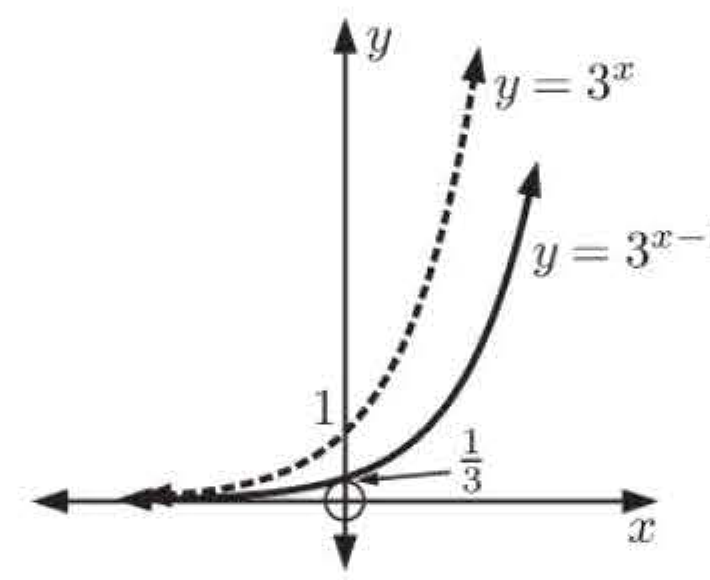
$y = 1$ is the H.A.

c



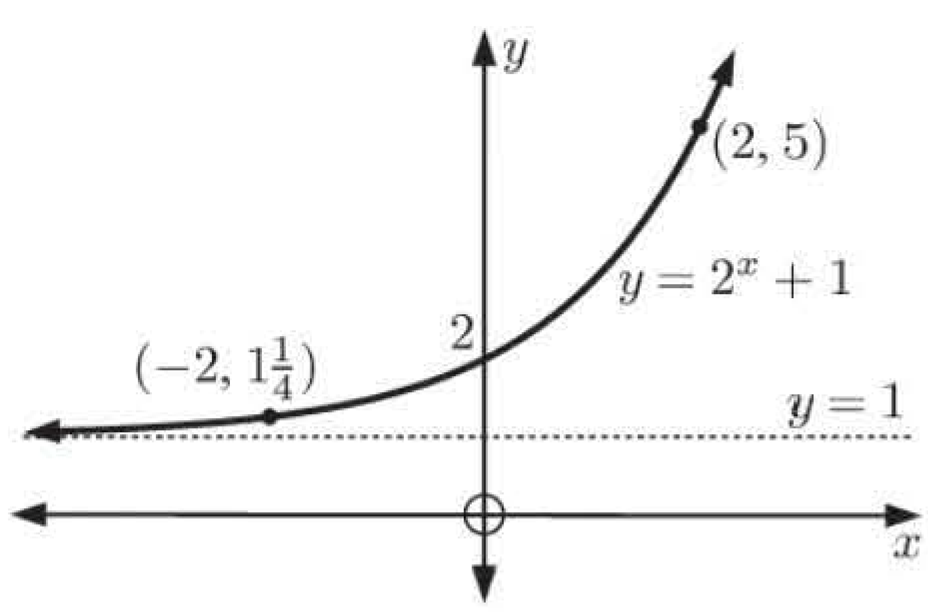
a reflection in the x-axis

d



a horizontal translation of 1 unit right

6 **a** **i**



a vertical translation of 1 unit upwards

When $x = 2$, $y = 4 + 1 = 5$

When $x = -2$, $y = \frac{1}{4} + 1 = 1\frac{1}{4}$

ii Domain $= \{x \mid x \in \mathbb{R}\}$,

Range $= \{y \mid y > 1\}$

iii Using technology, when

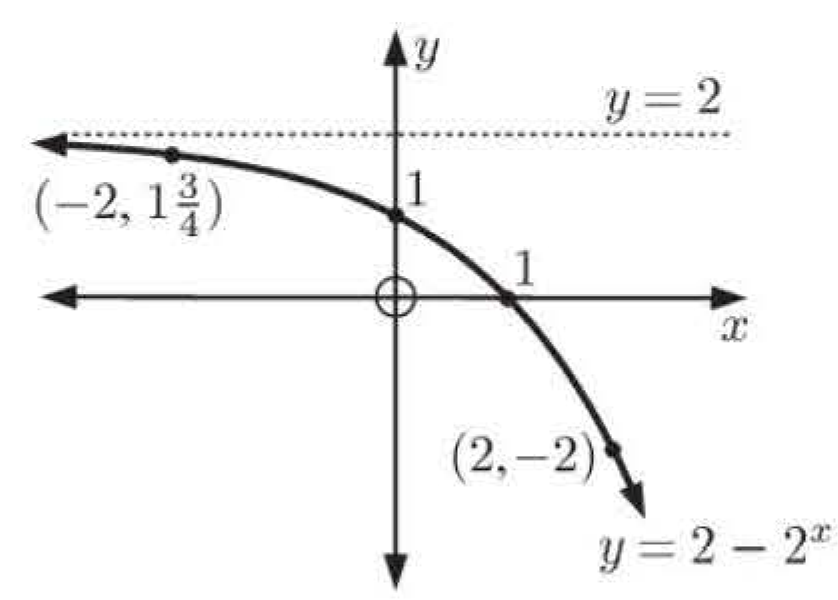
$x = \sqrt{2}$, $y \approx 3.67$

iv As $x \to \infty$, $y \to \infty$

As $x \to -\infty$, $y \to 1^+$

v The horizontal asymptote is $y = 1$.

b **i**



When $x = 0$, $y = 2 - 2^0 = 2 - 1 = 1$

$\therefore$ the y-intercept is 1

When $x = 1$, $y = 2 - 2 = 0$

When $x = 2$, $y = 2 - 4 = -2$

When $x = -2$, $y = 2 - \frac{1}{4} = 1\frac{3}{4}$

ii Domain $= \{x \mid x \in \mathbb{R}\}$,

Range $= \{y \mid y < 2\}$

iii Using technology, when

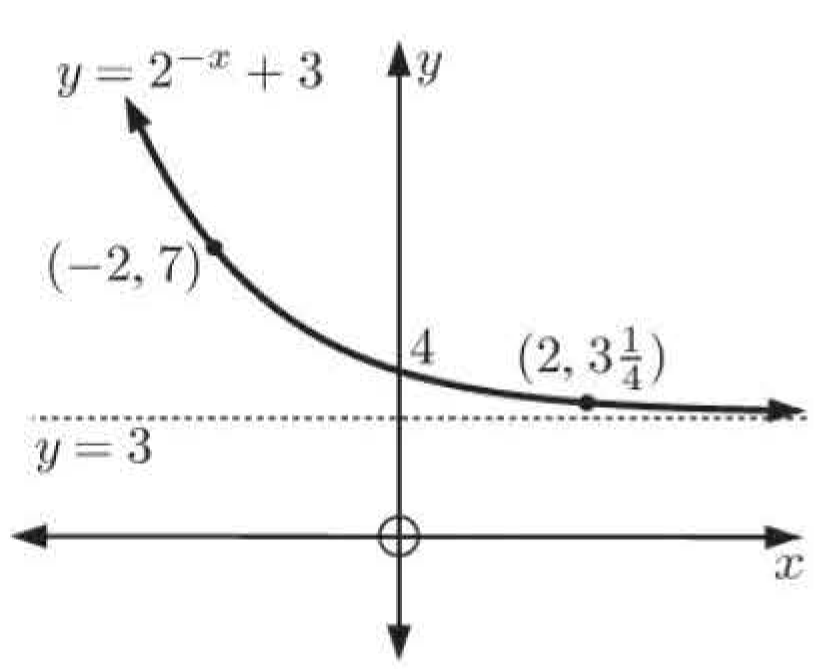
$x = \sqrt{2}$, $y \approx -0.665$

iv As $x \to \infty$, $y \to -\infty$

As $x \to -\infty$, $y \to 2^-$

v The horizontal asymptote is $y = 2$.

c **i**



When $x = 0$, $y = 1 + 3 = 4$

When $x = 2$, $y = \frac{1}{4} + 3 = 3\frac{1}{4}$

When $x = -2$, $y = 2^2 + 3 = 7$

ii Domain $= \{x \mid x \in \mathbb{R}\}$,

Range $= \{y \mid y > 3\}$

iii Using technology, when

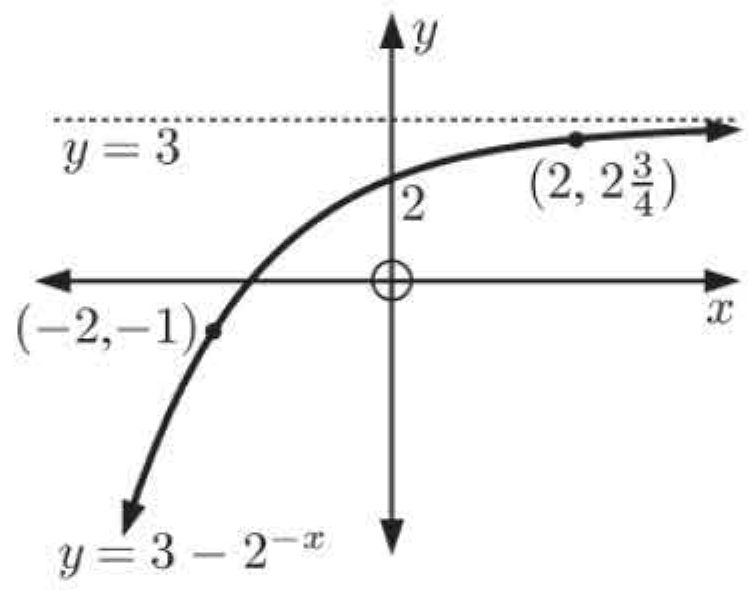
$x = \sqrt{2}$, $y \approx 3.38$

iv As $x \to \infty$, $y \to 3^+$

As $x \to -\infty$, $y \to \infty$

v The horizontal asymptote is $y = 3$.

d **i**



When $x = 0$, $\quad y = 3 - 1 = 2$

When $x = 2$, $\quad y = 3 - \frac{1}{4} = 2\frac{3}{4}$

When $x = -2$, $\quad y = 3 - 4 = -1$

ii Domain $= \{x \mid x \in \mathbb{R}\}$,
Range $= \{y \mid y < 3\}$

iii Using technology, when
$x = \sqrt{2}$, $\quad y \approx 2.62$

iv As $x \to \infty$, $\quad y \to 3^-$
As $x \to -\infty$, $\quad y \to -\infty$

v The horizontal asymptote is $y = 3$.

EXERCISE 3G.1

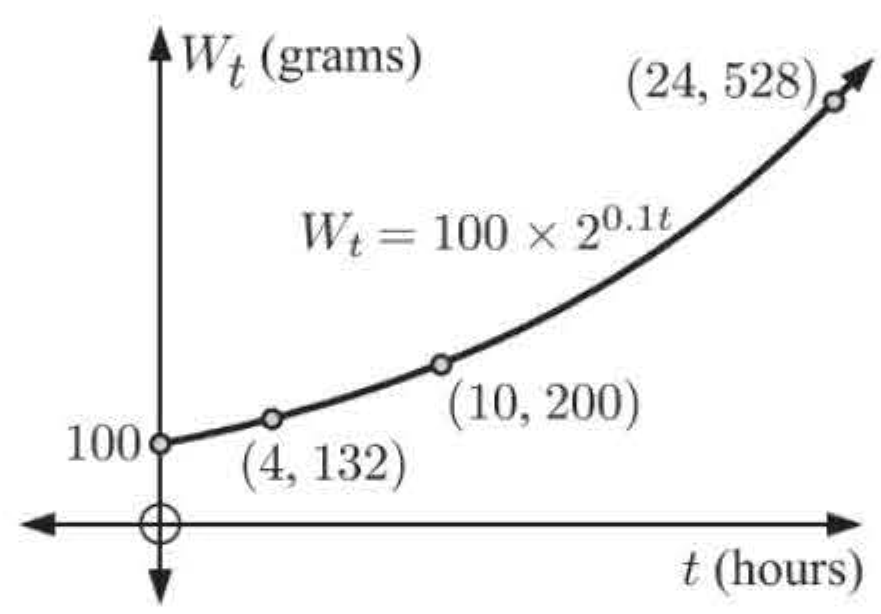
1 **a** When $t = 0$, $W_0 = 100$ grams $=$ the initial weight

b **i** When $t = 4$,
$$\begin{aligned} W_4 &= 100 \times 2^{0.1 \times 4} \\ &= 100 \times 2^{0.4} \\ &\approx 132 \text{ grams} \end{aligned}$$

ii When $t = 10$,
$$\begin{aligned} W_{10} &= 100 \times 2^1 \\ &= 200 \text{ grams} \end{aligned}$$

iii When $t = 24$,
$$\begin{aligned} W_{24} &= 100 \times 2^{0.1 \times 24} \\ &= 100 \times 2^{2.4} \\ &\approx 528 \text{ grams} \end{aligned}$$

c



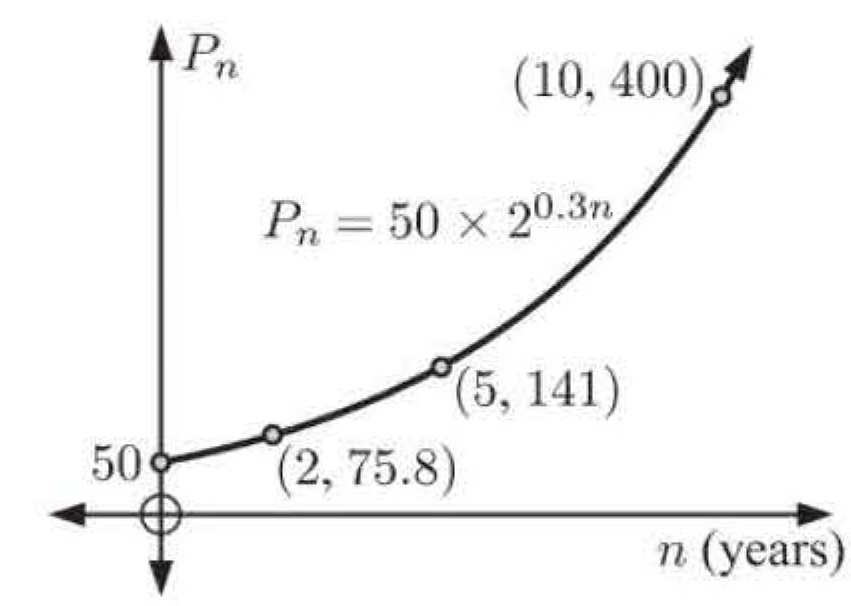
2 **a** $P_0 = 50$ (the initial population)

b **i** When $n = 2$,
$$\begin{aligned} P_2 &= 50 \times 2^{0.3 \times 2} \\ &= 50 \times 2^{0.6} \\ &\approx 75.785 \end{aligned}$$
So, the expected population is 76 possums.

ii When $n = 5$,
$$\begin{aligned} P_5 &= 50 \times 2^{0.3 \times 5} \\ &= 50 \times 2^{1.5} \\ &\approx 141.421 \end{aligned}$$
So, the expected population is 141 possums.

iii When $n = 10$,
$$\begin{aligned} P_{10} &= 50 \times 2^{0.3 \times 10} \\ &= 50 \times 2^3 = 400 \end{aligned}$$
So, the expected population is 400 possums.

c



3 **a** $B_0 = 6$ pairs $= 12$ bears

b In 2018, $t = 20$
$$\begin{aligned} \therefore\ B_{20} &= 12 \times 2^{0.18 \times 20} \\ &= 12 \times 2^{3.6} \\ &\approx 145.509 \\ &\approx 146 \text{ bears} \end{aligned}$$

c In 2008, $t = 10$
$$\begin{aligned} \therefore\ \%\text{ increase} &= \left(\frac{B_{20} - B_{10}}{B_{10}}\right) \times 100\% \\ &= \left(\frac{12 \times 2^{3.6} - 12 \times 2^{1.8}}{12 \times 2^{1.8}}\right) \times 100\% \\ &= \left(\frac{2^{3.6} - 2^{1.8}}{2^{1.8}}\right) \times 100\% \\ &\approx 248\% \end{aligned}$$

4 **a** **i** When $t = 0$,
$$\begin{aligned} V_0 &= V_0 \times 2^0 \\ &= V_0 \end{aligned}$$
So, the speed is V_0.

ii When $t = 20$,
$$\begin{aligned} V_{20} &= V_0 \times 2^{0.05 \times 20} \\ &= V_0 \times 2^1 \\ &= 2V_0 \end{aligned}$$
So, the speed is $2V_0$.

b V_0 becomes $2V_0$. So, there was a 100% increase in speed.

c $\left(\dfrac{V_{50}-V_{20}}{V_{20}}\right)\times 100\% = \left(\dfrac{V_0\times 2^{2.5}-V_0\times 2^1}{V_0\times 2^1}\right)\times 100\%$

$= \left(\dfrac{2^{2.5}-2^1}{2^1}\right)\times 100\%$

$\approx 183\%$

This expression is the percentage increase in speed from the speed at 20°C to the speed at 50°C. ($V_{50}-V_{20}$ is the increase in speed.)

EXERCISE 3G.2

1 $W(t) = 250\times(0.998)^t$ grams

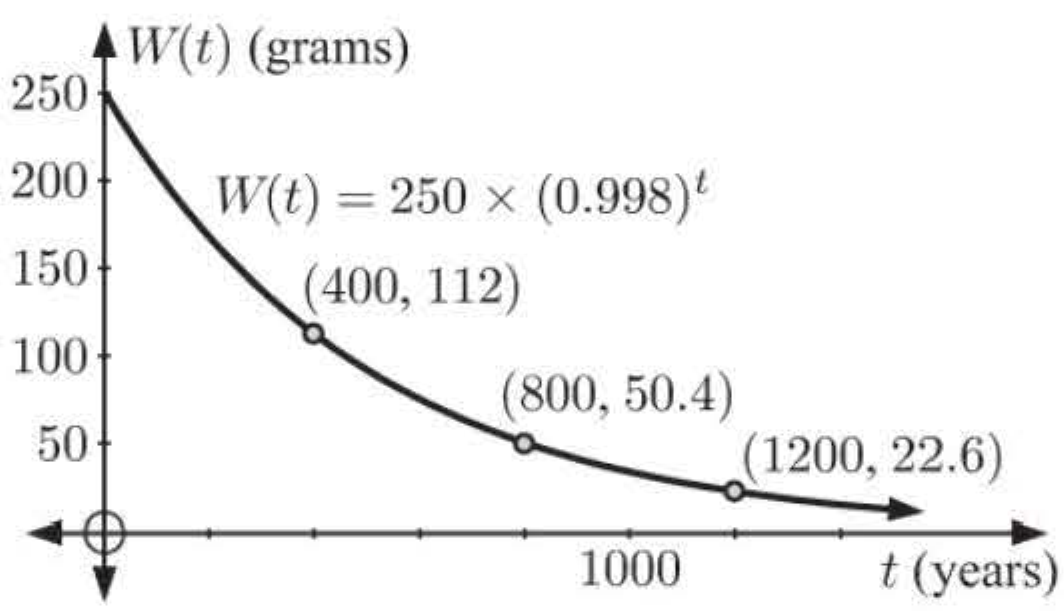
a $W(0) = 250\times(0.998)^0$
$= 250\times 1 = 250$ grams $\quad\therefore$ 250 g of radioactive substance was put aside.

b **i** When $t = 400$,
$W(400)$
$= 250\times(0.998)^{400}$
≈ 112 grams

ii When $t = 800$,
$W(800)$
$= 250\times(0.998)^{800}$
≈ 50.4 grams

iii When $t = 1200$,
$W(1200)$
$= 250\times(0.998)^{1200}$
≈ 22.6 grams

c



d When $W(t) = 125$
$250\times(0.998)^t = 125$
$\therefore\ (0.998)^t = 0.5$
$\therefore\ t\approx 346.2$ {using technology}
It takes approximately 346 years.

2 $T(t) = 100\times 2^{-0.02t}$

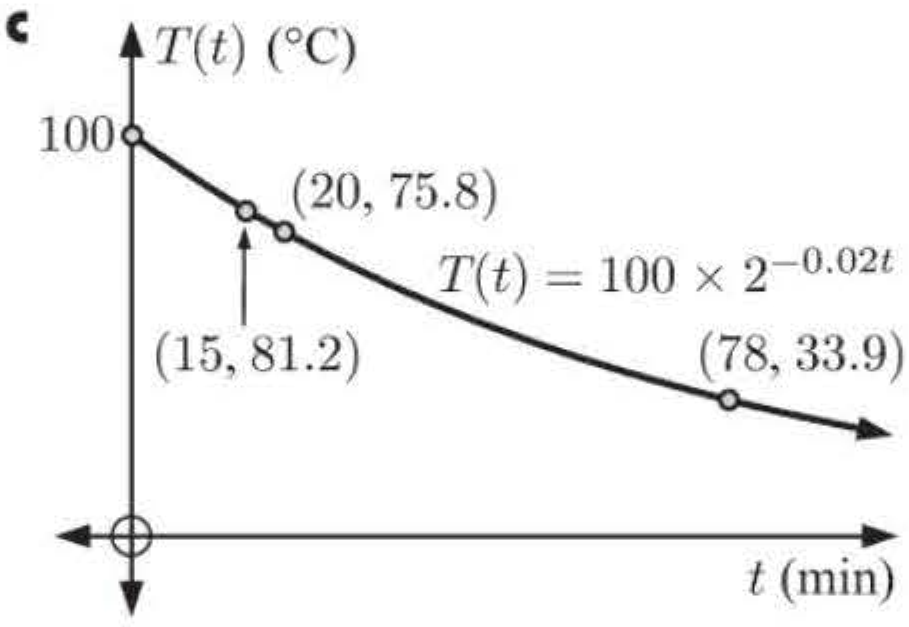
a $T(0) = 100\times 2^0$
$= 100\times 1$
$= 100$°C

b **i** $T(15) = 100\times 2^{-0.02\times 15}$
$= 100\times 2^{-0.3}$
≈ 81.2°C

ii $T(20) = 100\times 2^{-0.02\times 20}$
$= 100\times 2^{-0.4}$
≈ 75.8°C

iii $T(78) = 100\times 2^{-0.02\times 78}$
$= 100\times 2^{-1.56}$
≈ 33.9°C

c



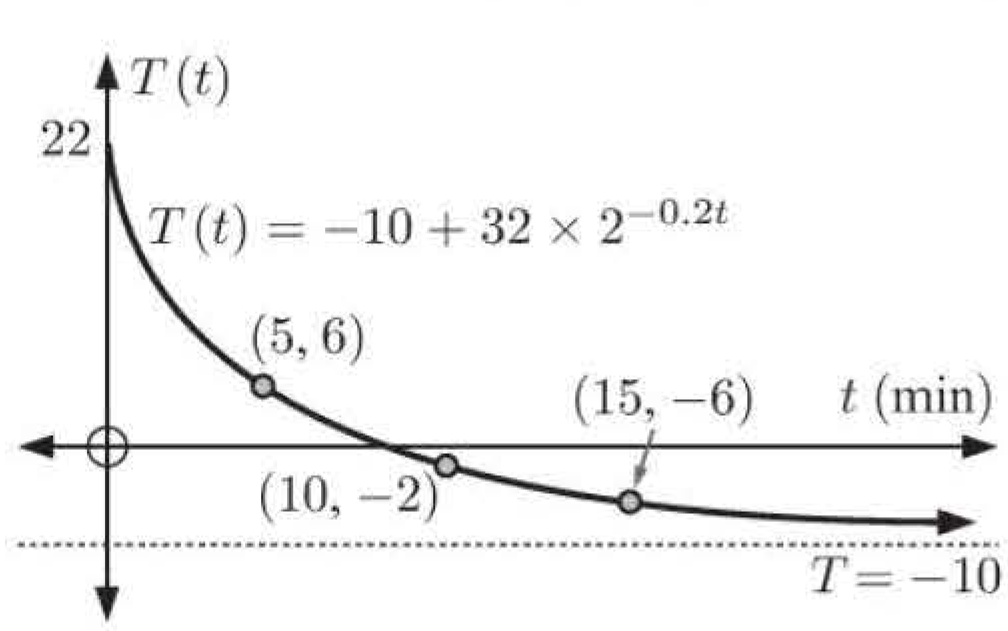
3 **a** **i** $T(0) = -10+32\times 2^0$
$= -10+32\times 1 = 22$°C

ii $T(5) = -10+32\times 2^{-0.2\times 5}$
$= -10+32\times 2^{-1} = 6$°C

iii $T(10) = -10+32\times 2^{-0.2\times 10}$
$= -10+32\times 2^{-2} = -2$°C

iv $T(15) = -10+32\times 2^{-0.2\times 15}$
$= -10+32\times 2^{-3} = -6$°C

b



c $32\times 2^{-0.2t}$ is always > 0 since 2^t is always > 0
$\therefore\ -10+32\times 2^{-0.2t}$ is always > -10
$\therefore$ the temperature of the packet of peas will never reach -10°C.

4 $W_t = 1000 \times 2^{-0.03t}$

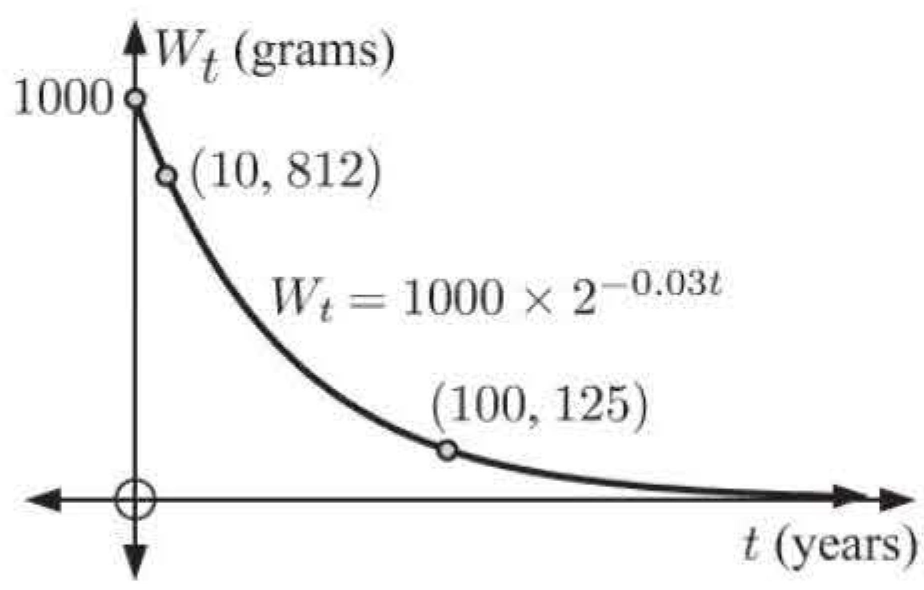
a $W_0 = 1000 \times 2^0$
$= 1000 \times 1$
$= 1000$ g

b **i** W_{10}
$= 1000 \times 2^{-0.3}$
≈ 812 g

ii W_{100}
$= 1000 \times 2^{-3}$
$= 125$ g

iii W_{1000}
$= 1000 \times 2^{-30}$
$\approx 9.31 \times 10^{-7}$ g

c



d When $W_t = 10$, $1000 \times 2^{-0.03t} = 10$
$\therefore \quad (2^{-0.03})^t = 0.01$
$\therefore \quad t \approx 221.46$ {using technology}

There is 10 g of the substance remaining after approximately 221 years.

e Initial weight $= W_0 = 1000$ g

Amount remaining after t years $= W_t = 1000 \times 2^{-0.03t}$

Amount that has decayed after t years $= W_0 - W_t$
$= 1000 - 1000 \times 2^{-0.03t}$
$= 1000(1 - 2^{-0.03t})$ g

5 **a** When $t = 0$, $W_0 = W_0 2^0$
$= W_0$ grams

$\therefore$ the original weight was W_0 grams.

b $\%$ change $= \left(\dfrac{W_{1000} - W_0}{W_0}\right) \times 100\%$
$= \left(\dfrac{W_0 \times 2^{-0.2} - W_0}{W_0}\right) \times 100\%$
$= (2^{-0.2} - 1) \times 100\%$
$\approx -12.9\%$

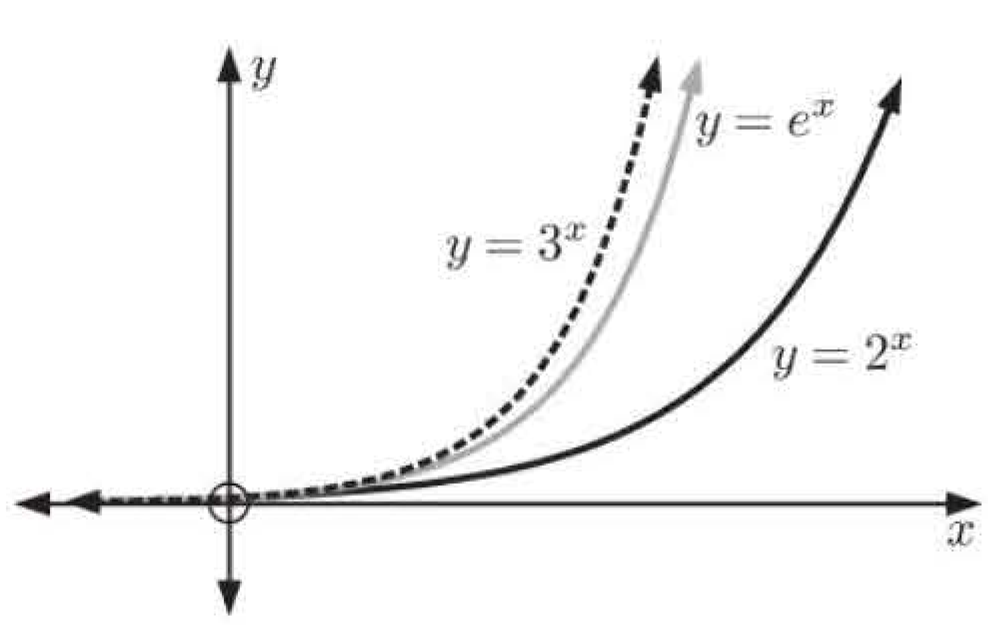
The weight loss was about 12.9%.

c $W_0 \times 2^{-0.0002t} = \frac{1}{512} W_0$
$\therefore \quad (2^{-0.0002})^t = \frac{1}{512}$
$\therefore \quad t = 45\,000$ {using technology}

It would take 45 000 years.

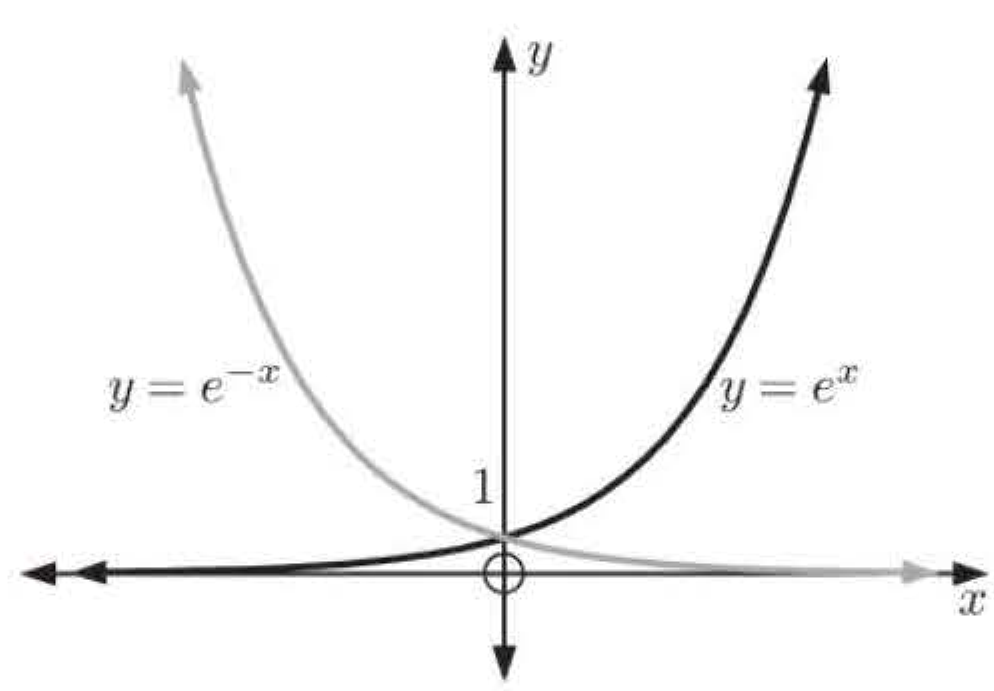
EXERCISE 3H

1



The graph of $y = e^x$ lies between $y = 2^x$ and $y = 3^x$.

2



One is the other reflected in the y-axis.

3 When $x = 0$, $y = ae^0 = a \times 1 = a$ $\quad \therefore$ the y-intercept is a.

4 **a** The graph of $y = e^x$ is entirely above the x-axis.
$y > 0$ for all x
$\therefore \quad e^x > 0$ for all x
$\therefore \quad 2e^x > 0$ for all x
$\therefore \quad y = 2e^x$ cannot be negative.

b **i** When $x = -20$, $y = 2e^{-20} \approx 4.12 \times 10^{-9}$
$\approx 0.000\,000\,004\,12$

ii When $x = 20$, $y = 2e^{20} \approx 9.70 \times 10^8$
$\approx 970\,000\,000$

5 **a** $e^2 \approx 7.39$ **b** $e^3 \approx 20.1$ **c** $e^{0.7} \approx 2.01$ **d** $\sqrt{e} \approx 1.65$ **e** $e^{-1} \approx 0.368$

6 **a** $\sqrt{e} = e^{\frac{1}{2}}$

b $\dfrac{1}{\sqrt{e}}$
$= \dfrac{1}{e^{\frac{1}{2}}}$
$= e^{-\frac{1}{2}}$

c $\dfrac{1}{e^2} = e^{-2}$

d $e\sqrt{e}$
$= e^1 e^{\frac{1}{2}}$
$= e^{\frac{3}{2}}$

7 **a** $\left(e^{0.36}\right)^{\frac{t}{2}}$
$= e^{0.36 \times \frac{t}{2}}$
$= e^{0.18t}$

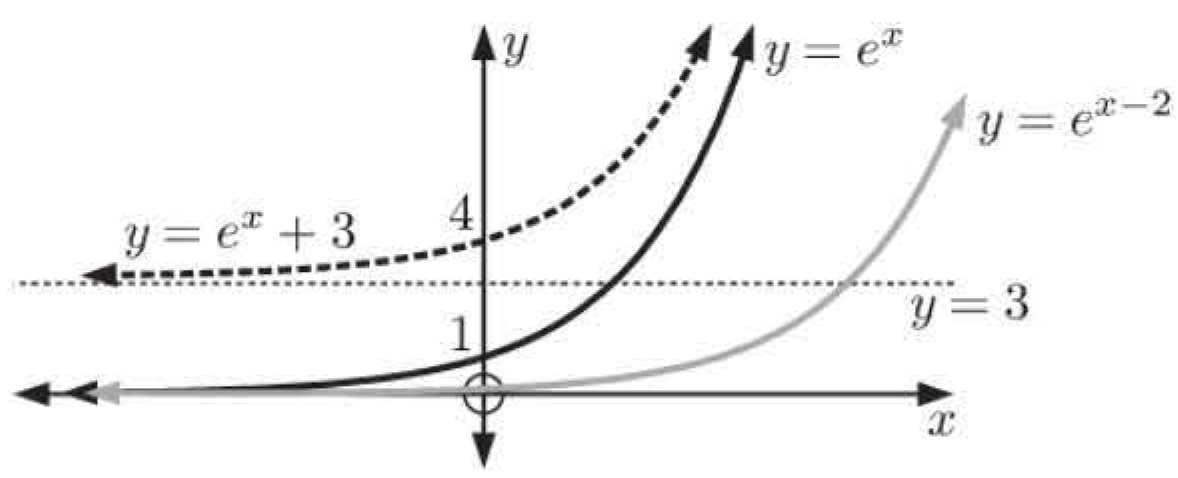
b $\left(e^{0.064}\right)^{\frac{t}{16}}$
$= e^{0.064 \times \frac{t}{16}}$
$= e^{0.004t}$

c $\left(e^{-0.04}\right)^{\frac{t}{8}}$
$= e^{-0.04 \times \frac{t}{8}}$
$= e^{-0.005t}$

d $\left(e^{-0.836}\right)^{\frac{t}{5}}$
$= e^{-0.836 \times \frac{t}{5}}$
$\approx e^{-0.167t}$

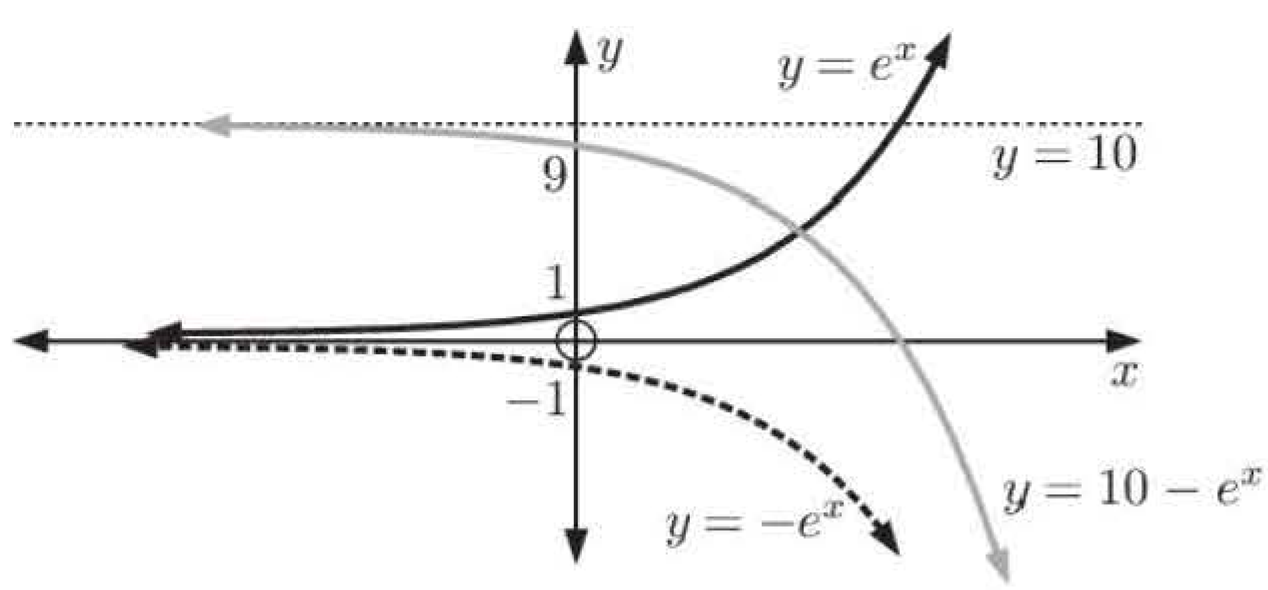
8 **a** ≈ 10.074 **b** $\approx 0.099\,261$ **c** ≈ 125.09 **d** $\approx 0.007\,994\,5$
e ≈ 41.914 **f** ≈ 42.429 **g** ≈ 3540.3 **h** $\approx 0.006\,342\,4$

9



Domain of f, g, and h is $\{x \mid x \in \mathbb{R}\}$
Range of f is $\{y \mid y > 0\}$
Range of g is $\{y \mid y > 0\}$
Range of h is $\{y \mid y > 3\}$

10



Domain of f, g, and h is $\{x \mid x \in \mathbb{R}\}$
Range of f is $\{y \mid y > 0\}$
Range of g is $\{y \mid y < 0\}$
Range of h is $\{y \mid y < 10\}$

11 **a** $(e^x + 1)^2$
$= (e^x)^2 + 2 \times e^x \times 1 + 1^2$
$= e^{2x} + 2e^x + 1$

b $(1 + e^x)(1 - e^x)$
$= 1^2 - (e^x)^2$
$= 1 - e^{2x}$

c $e^x(e^{-x} - 3)$
$= e^x \times e^{-x} - e^x \times 3$
$= e^0 - 3e^x$
$= 1 - 3e^x$

12 $W(t) = 2e^{\frac{t}{2}}$ grams

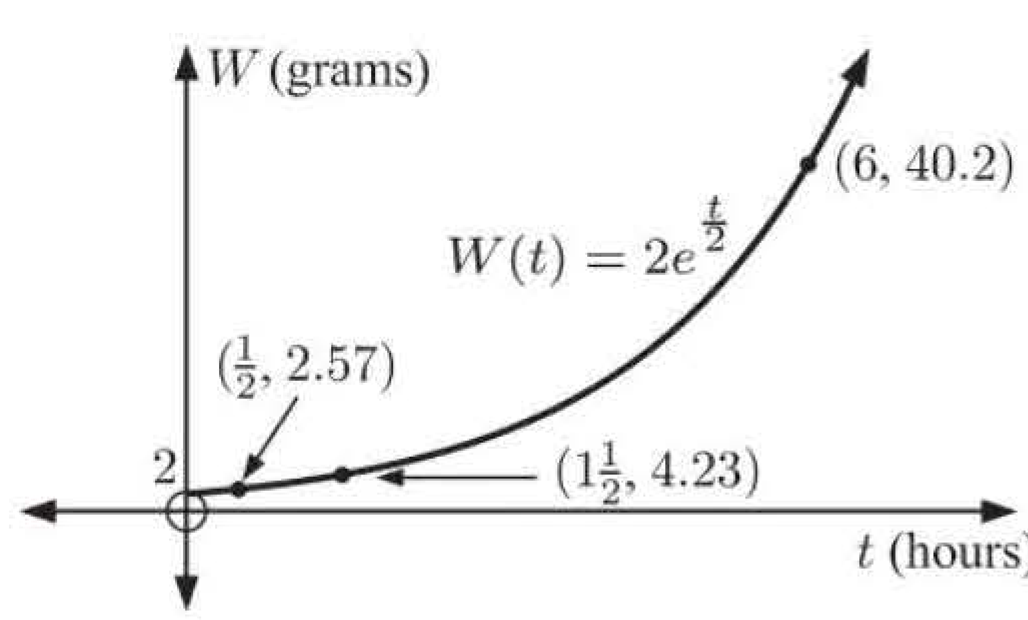
a **i** $W(0) = 2e^0$
$= 2 \times 1$
$= 2$ g

ii $W(\frac{1}{2}) = 2e^{\frac{1}{4}}$
≈ 2.57 g

iii $W(1\frac{1}{2}) = 2e^{\frac{3}{4}}$
≈ 4.23 g

iv $W(6) = 2e^3$
≈ 40.2 g

b



13 **a** $e^x = \sqrt{e}$
$\therefore\ e^x = e^{\frac{1}{2}}$
$\therefore\ x = \frac{1}{2}$

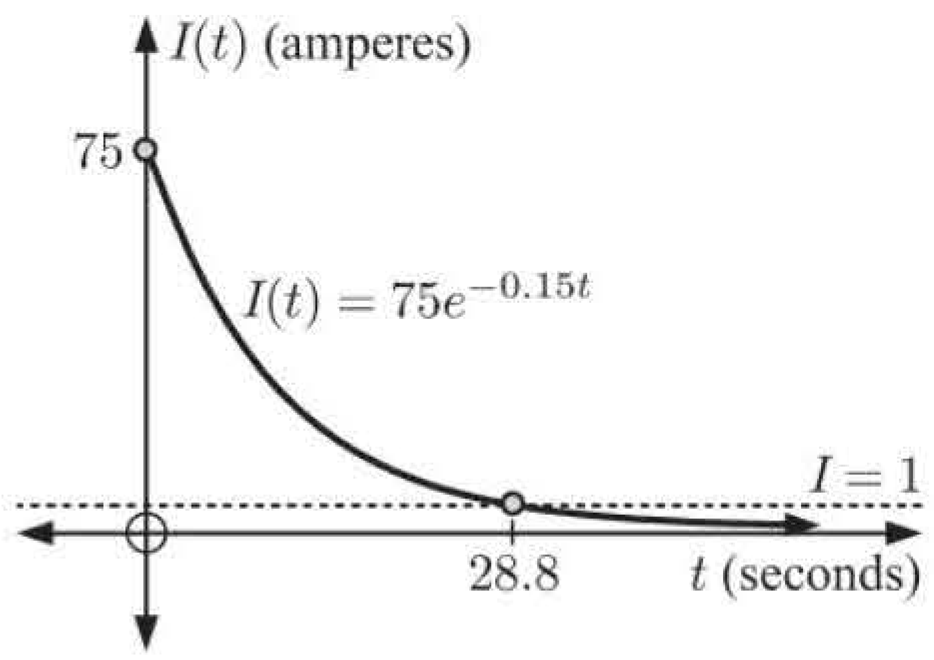
b $e^{\frac{1}{2}x} = \dfrac{1}{e^2}$
$\therefore\ e^{\frac{1}{2}x} = e^{-2}$
$\therefore\ \frac{1}{2}x = -2$
$\therefore\ x = -4$

14 $I(t) = 75e^{-0.15t}$

a **i** $I(1) = 75e^{-0.15}$
≈ 64.6 amps

ii $I(10) = 75e^{-1.5}$
≈ 16.7 amps

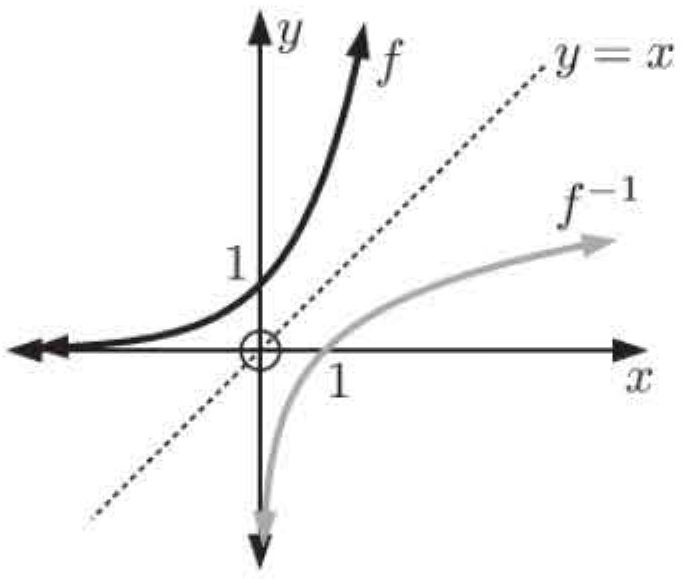
b



c We need to solve $75e^{-0.15t} = 1$.
Using technology, $t \approx 28.8$ s

15 $f(x) = e^x$

a



b Domain of f^{-1} is $\{x \mid x > 0\}$
Range of f^{-1} is $\{y \mid y \in \mathbb{R}\}$

REVIEW SET 3A

1 **a** $-(-1)^{10}$
$= -1$

b $-(-3)^3$
$= -(-27)$
$= 27$

c $3^0 - 3^{-1}$
$= 1 - \frac{1}{3}$
$= \frac{2}{3}$

2 **a** $a^4b^5 \times a^2b^2$
$= a^{4+2} \times b^{5+2}$
$= a^6b^7$

b $6xy^5 \div 9x^2y^5$
$= \frac{6}{9}x^{1-2}y^{5-5}$
$= \frac{2}{3}x^{-1}y^0$
$= \dfrac{2}{3x}$

c $\dfrac{5(x^2y)^2}{(5x^2)^2}$
$= \dfrac{5 \times x^4y^2}{25x^4}$
$= \frac{1}{5}x^0y^2$
$= \dfrac{y^2}{5}$

3 **a** **i** $f(4) = 3^4$
$= 81$

ii $f(-1) = 3^{-1}$
$= \frac{1}{3}$

b $f(x+2) = kf(x)$
$\therefore\ 3^{x+2} = k \times 3^x$
$\therefore\ 3^2 \times 3^x = k \times 3^x$
$\therefore\ 3^2 = k$
$\therefore\ k = 9$

4 **a** $x^{-2} \times x^{-3}$
$= x^{-2+(-3)}$
$= x^{-5}$
$= \dfrac{1}{x^5}$

b $2(ab)^{-2}$
$= 2 \times \dfrac{1}{(ab)^2}$
$= \dfrac{2}{a^2b^2}$

c $2ab^{-2}$
$= 2a \times \left(\dfrac{1}{b^2}\right)$
$= \dfrac{2a}{b^2}$

5 **a** $\dfrac{27}{9^a} = \dfrac{3^3}{(3^2)^a}$
$= 3^{3-2a}$

b $\left(\sqrt{3}\right)^{1-x} \times 9^{1-2x} = (3^{\frac{1}{2}})^{1-x} \times (3^2)^{1-2x}$
$= 3^{\frac{1}{2} - \frac{1}{2}x + 2 - 4x}$
$= 3^{\frac{5}{2} - \frac{9}{2}x}$

6 **a** $8^{\frac{2}{3}} = \left(2^3\right)^{\frac{2}{3}} = 2^2 = 4$

b $27^{-\frac{2}{3}} = \left(3^3\right)^{-\frac{2}{3}} = 3^{-2} = \dfrac{1}{3^2} = \frac{1}{9}$

7 **a**
$$\begin{aligned} &mn^{-2} \\ &= m \times \frac{1}{n^2} \\ &= \frac{m}{n^2} \end{aligned}$$

b
$$\begin{aligned} &(mn)^{-3} \\ &= \frac{1}{(mn)^3} \\ &= \frac{1}{m^3n^3} \end{aligned}$$

c
$$\begin{aligned} &\frac{m^2n^{-1}}{p^{-2}} \\ &= m^2\left(\frac{1}{n}\right)p^2 \\ &= \frac{m^2p^2}{n} \end{aligned}$$

d
$$\begin{aligned} &(4m^{-1}n)^2 \\ &= 4^2m^{-2}n^2 \\ &= \frac{16n^2}{m^2} \end{aligned}$$

8 **a**
$$\begin{aligned} &(3-e^x)^2 \\ &= 3^2 - 2\times 3\times e^x + (e^x)^2 \\ &= 9 - 6e^x + e^{2x} \end{aligned}$$

b
$$\begin{aligned} &(\sqrt{x}+2)(\sqrt{x}-2) \\ &= (\sqrt{x})^2 - 2^2 \\ &= x - 4 \end{aligned}$$

c
$$\begin{aligned} &2^{-x}(2^{2x}+2^x) \\ &= 2^{-x+2x} + 2^{-x+x} \\ &= 2^x + 2^0 \\ &= 2^x + 1 \end{aligned}$$

9 **a**
$$\begin{aligned} 2^{x-3} &= \tfrac{1}{32} \\ \therefore\ 2^{x-3} &= 2^{-5} \\ \therefore\ x-3 &= -5 \\ \therefore\ x &= -2 \end{aligned}$$

b
$$\begin{aligned} 9^x &= 27^{2-2x} \\ \therefore\ (3^2)^x &= (3^3)^{2-2x} \\ \therefore\ 2x &= 6-6x \\ \therefore\ 8x &= 6 \\ \therefore\ x &= \tfrac{6}{8} = \tfrac{3}{4} \end{aligned}$$

c
$$\begin{aligned} e^{2x} &= \frac{1}{\sqrt{e}} \\ \therefore\ e^{2x} &= e^{-\frac{1}{2}} \\ \therefore\ 2x &= -\tfrac{1}{2} \\ \therefore\ x &= -\tfrac{1}{4} \end{aligned}$$

10 Use the general exponential function $y = a\times b^{x-c} + d$.

a $y = -e^x$

$\left.\begin{aligned} a=-1 \quad &\therefore\ a<0 \\ b=e \quad &\therefore\ b>1 \end{aligned}\right\}$ function is decreasing

When $x=0$, $y=-e^0=-1$

$\therefore$ y-intercept is $y=-1$.

$\therefore$ the graph is **C**.

b $y = 3\times 2^x$

$\left.\begin{aligned} a=3 \quad &\therefore\ a>0 \\ b=2 \quad &\therefore\ b>1 \end{aligned}\right\}$ function is increasing

When $x=0$, $y=3\times 2^0=3$

$\therefore$ y-intercept is $y=3$.

$\therefore$ the graph is **E**.

c $y = e^x + 1$

$\left.\begin{aligned} a=1 \quad &\therefore\ a>0 \\ b=e \quad &\therefore\ b>1 \end{aligned}\right\}$ function is increasing

When $x=0$, $y=e^0+1=2$

$\therefore$ y-intercept is $y=2$.

$d=1$ $\therefore$ $y=1$ is a horizontal asymptote.

$\therefore$ the graph is **A**.

d $y = 3^{-x} = \dfrac{1}{3^x} = \left(\frac{1}{3}\right)^x$

$\left.\begin{aligned} a=1 \quad &\therefore\ a>0 \\ b=\tfrac{1}{3} \quad &\therefore\ 0<b<1 \end{aligned}\right\}$ function is decreasing

When $x=0$, $y=3^0=1$

$\therefore$ y-intercept is $y=1$.

$\therefore$ the graph is **B**.

e $y = -e^{-x} = -\dfrac{1}{e^x} = -\left(\dfrac{1}{e}\right)^x$

$\left.\begin{aligned} a=-1 \quad &\therefore\ a<0 \\ b=\frac{1}{e} \quad &\therefore\ 0<b<1 \end{aligned}\right\}$ function is increasing

When $x=0$, $y=-e^0=-1$

$\therefore$ y-intercept is $y=-1$.

$\therefore$ the graph is **D**.

11 $y = a^x$

a $a^{2x} = (a^x)^2 = y^2$

b $a^{-x} = (a^x)^{-1} = y^{-1}$

c $\dfrac{1}{\sqrt{a^x}} = \dfrac{1}{\sqrt{y}} = y^{-\frac{1}{2}}$

REVIEW SET 3B

1 **a** 4×2^n
$= 2^2 \times 2^n$
$= 2^{n+2}$

b $7^{-1} - 7^0$
$= \frac{1}{7} - 1$
$= -\frac{6}{7}$

c $\left(\frac{2}{3}\right)^{-3}$
$= \left(\frac{3}{2}\right)^3$
$= \frac{27}{8}$
$= 3\frac{3}{8}$

d $\left(\dfrac{2a^{-1}}{b^2}\right)^2$
$= \dfrac{2^2 a^{-2}}{b^4}$
$= \dfrac{4}{a^2 b^4}$

2 **a** $3^{\frac{3}{4}} \approx 2.28$ **b** $27^{-\frac{1}{5}} \approx 0.517$ **c** $\sqrt[4]{100} \approx 3.16$

3 $f(x) = 3 \times 2^x$

a $f(0) = 3 \times 2^0$
$= 3 \times 1$
$= 3$

b $f(3) = 3 \times 2^3$
$= 3 \times 8$
$= 24$

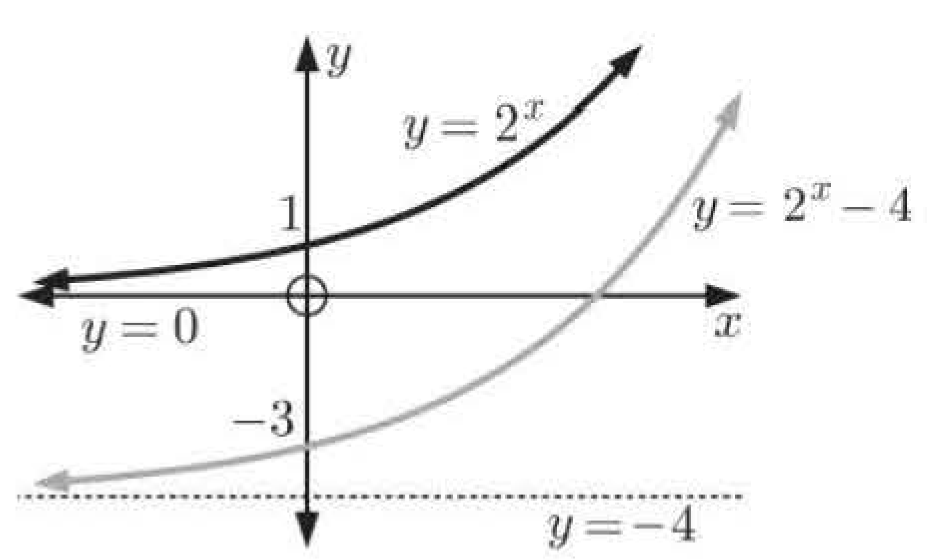
c $f(-2) = 3 \times 2^{-2}$
$= 3 \times \dfrac{1}{2^2} = \frac{3}{4}$

4 $f(x) = 2^{-x} + 1$

a $f(\frac{1}{2}) = 2^{-\frac{1}{2}} + 1$
$= \dfrac{1}{\sqrt{2}} + 1$
≈ 1.71

b $f(a) = 3$
$\therefore \; 2^{-a} + 1 = 3$
$\therefore \; 2^{-a} = 2$
$\therefore \; 2^{-a} = 2^1$
$\therefore \; -a = 1$
$\therefore \; a = -1$

5



$y = 2^x$ has y-intercept 1 and horizontal asymptote $y = 0$
$y = 2^x - 4$ has y-intercept -3 and horizontal asymptote $y = -4$

6 $T = 80 \times (0.913)^t$ °C

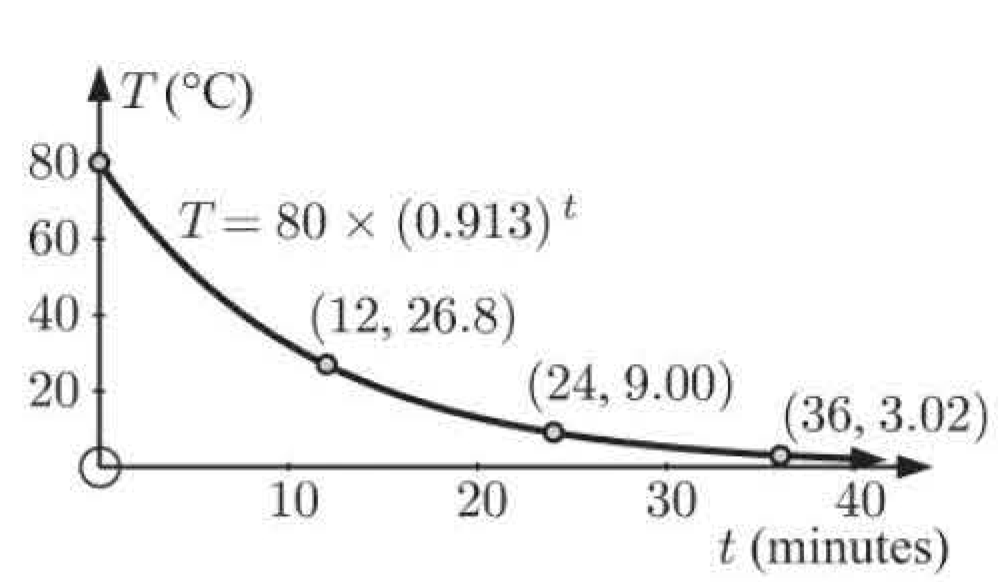
a When $t = 0$, $T = 80 \times (0.913)^0$
$= 80 \times 1$
$= 80$ $\therefore$ the initial temperature was 80°C.

b **i** When $t = 12$,
$T = 80 \times (0.913)^{12}$
≈ 26.8°C

ii When $t = 24$,
$T = 80 \times (0.913)^{24}$
≈ 9.00°C

iii When $t = 36$,
$T = 80 \times (0.913)^{36}$
≈ 3.02°C

c

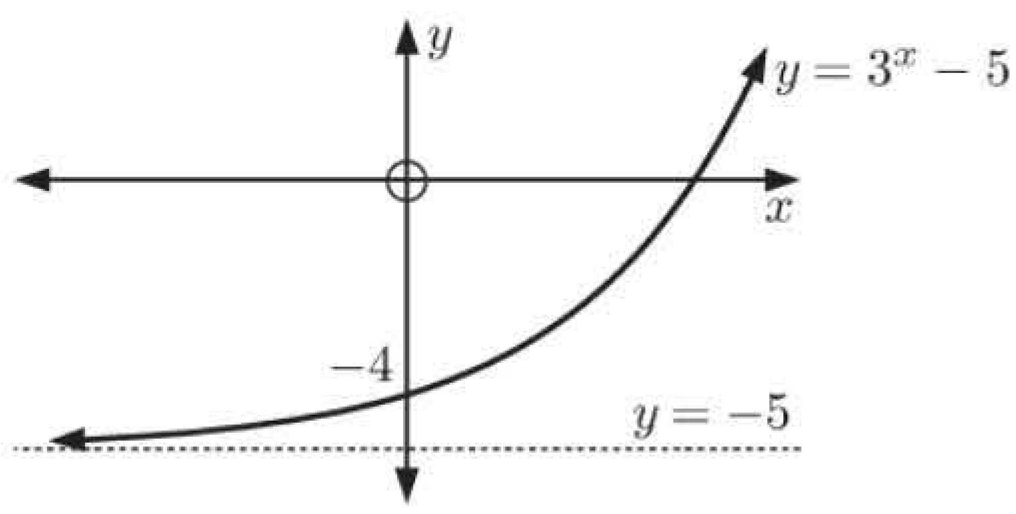


d When $T = 25$
$80 \times (0.913)^t = 25$
$\therefore \; 0.913^t = 0.3125$
$\therefore \; t \approx 12.8$ min {using technology}

7 **a** When $x = 0$, $y = 3^0 - 5 = 1 - 5 = -4$
When $x = 1$, $y = 3^1 - 5 = 3 - 5 = -2$
When $x = 2$, $y = 3^2 - 5 = 9 - 5 = 4$
When $x = -1$, $y = 3^{-1} - 5 = \frac{1}{3} - 5 = -4\frac{2}{3}$
When $x = -2$, $y = 3^{-2} - 5 = \frac{1}{9} - 5 = -4\frac{8}{9}$

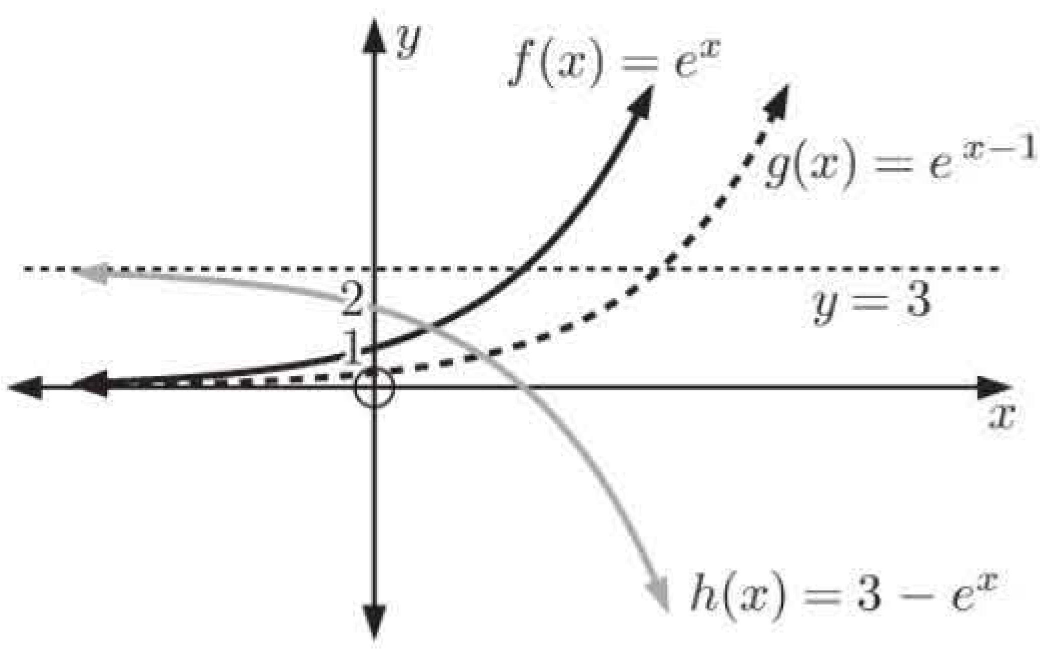
b As $x \to \infty$, $3^x \to \infty$ and so $y \to \infty$
As $x \to -\infty$, $3^x \to 0$ and so $y \to -5^+$

c



d $y = -5$ is the horizontal asymptote.

8 **a**

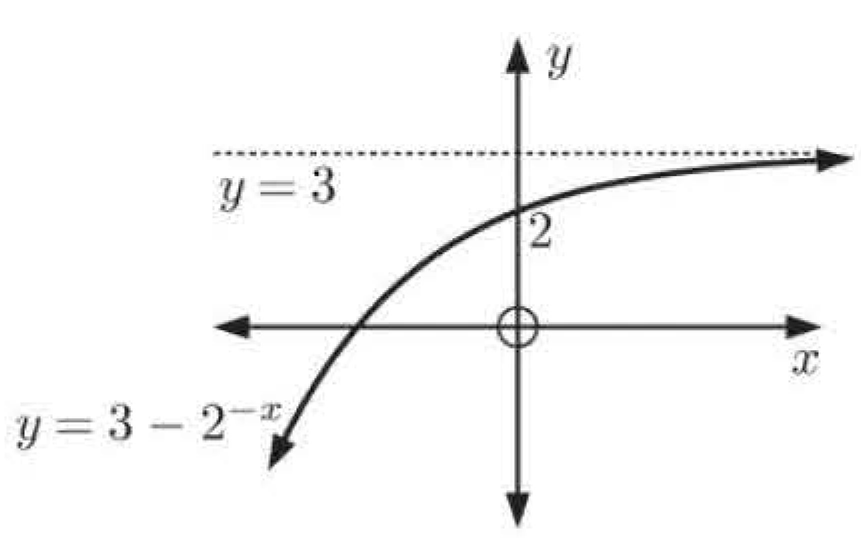


b Domain of f, g, and h is $\{x \mid x \in \mathbb{R}\}$
Range of f is $\{y \mid y > 0\}$
Range of g is $\{y \mid y > 0\}$
Range of h is $\{y \mid y < 3\}$

9 **a** When $x = 0$, $y = 3 - 2^0 = 3 - 1 = 2$
When $x = 1$, $y = 3 - 2^{-1} = 3 - \frac{1}{2} = 2\frac{1}{2}$
When $x = 2$, $y = 3 - 2^{-2} = 3 - \frac{1}{4} = 2\frac{3}{4}$
When $x = -1$, $y = 3 - 2^1 = 3 - 2 = 1$
When $x = -2$, $y = 3 - 2^2 = 3 - 4 = -1$

b As $x \to \infty$, $2^{-x} \to 0$, $\therefore\ y \to 3^-$
As $x \to -\infty$, $2^{-x} \to \infty$, $\therefore\ y \to -\infty$

c



d horizontal asymptote is $y = 3$

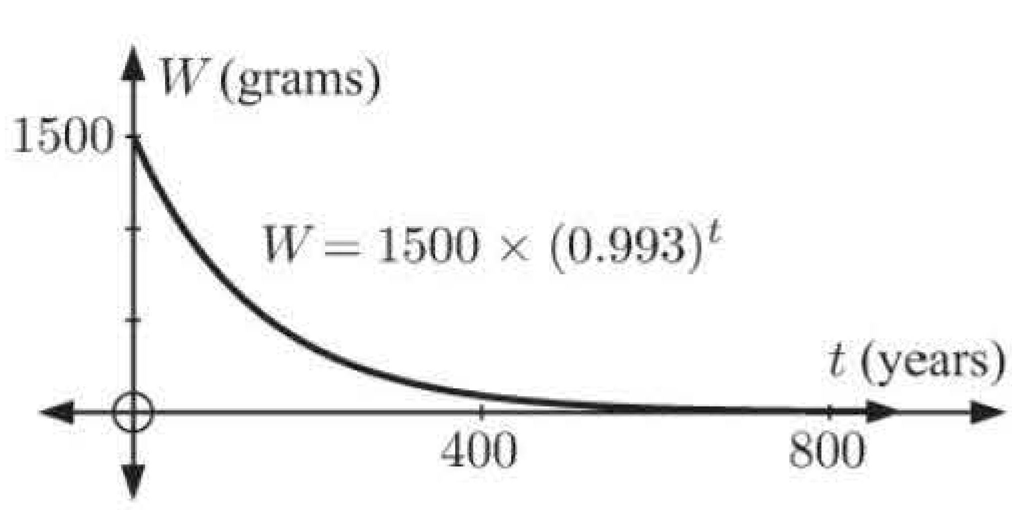
10 $W = 1500 \times (0.993)^t$ grams

a When $t = 0$,
$W = 1500 \times (0.993)^0$
$= 1500 \times 1$
$= 1500$ grams

b **i** When $t = 400$,
$W = 1500 \times (0.993)^{400}$
≈ 90.3 grams

ii When $t = 800$,
$W = 1500 \times (0.993)^{800}$
≈ 5.44 grams

c



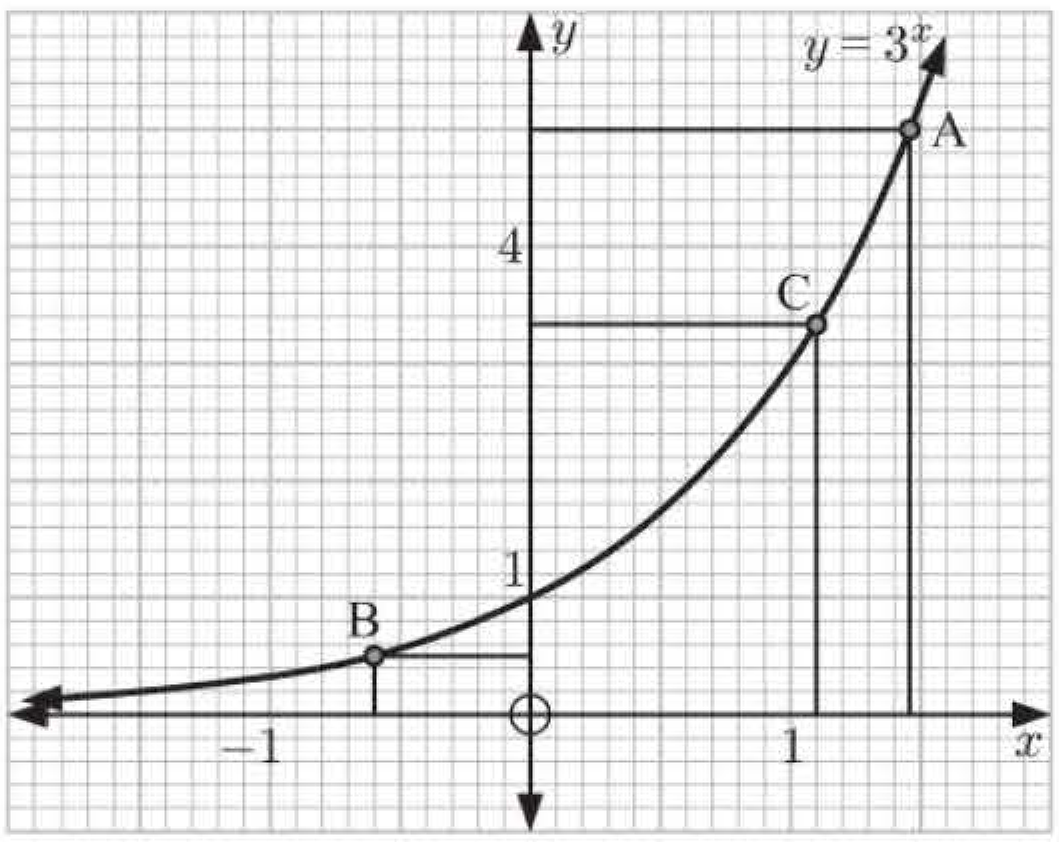
d When $W = 100$,
$1500 \times (0.993)^t = 100$
$\therefore\ (0.993)^t \approx 0.0667$
$\therefore\ t \approx 385.5$ {using technology}
So, it will take about 386 years.

REVIEW SET 3C

1 **a** When $y = 3^x = 5$,
$x \approx 1.5$ from point A.

b When $y = 3^x = \frac{1}{2}$,
$x \approx -0.6$ from point B.

c $6 \times 3^x = 20$
$\therefore\ 3^x = \frac{20}{6} = 3\frac{1}{3}$
When $y = 3^x = 3\frac{1}{3}$,
$x \approx 1.1$ from point C.



2 **a** $(a^7)^3$
$= a^{7\times 3}$
$= a^{21}$

b $pq^2 \times p^3q^4$
$= p^{1+3}q^{2+4}$
$= p^4q^6$

c $\dfrac{8ab^5}{2a^4b^4}$
$= \frac{8}{2}a^{1-4}b^{5-4}$
$= 4a^{-3}b^1$
$= \dfrac{4b}{a^3}$

3 **a** 2×2^{-4}
$= 2^1 \times 2^{-4}$
$= 2^{1+(-4)}$
$= 2^{-3}$

b $16 \div 2^{-3}$
$= 2^4 \div 2^{-3}$
$= 2^{4-(-3)}$
$= 2^7$

c 8^4
$= (2^3)^4$
$= 2^{12}$

4 **a** $b^{-3} = \dfrac{1}{b^3}$

b $(ab)^{-1}$
$= a^{-1}b^{-1}$
$= \dfrac{1}{ab}$

c ab^{-1}
$= a \times \dfrac{1}{b}$
$= \dfrac{a}{b}$

5 $\dfrac{2^{x+1}}{2^{1-x}} = 2^{x+1-(1-x)}$
$= 2^{x+1-1+x}$
$= 2^{2x}$

6 **a** $1 = 5^0$

b $5\sqrt{5}$
$= 5^1 \times 5^{\frac{1}{2}}$
$= 5^{\frac{3}{2}}$

c $\dfrac{1}{\sqrt[4]{5}}$
$= \dfrac{1}{5^{\frac{1}{4}}}$
$= 5^{-\frac{1}{4}}$

d 25^{a+3}
$= (5^2)^{a+3}$
$= 5^{2a+6}$

7 **a** $e^x(e^{-x} + e^x)$
$= e^0 + e^{2x}$
$= 1 + e^{2x}$

b $(2^x + 5)^2$
$= (2^x)^2 + 2 \times 2^x \times 5 + 5^2$
$= 2^{2x} + 5 \times 2^{x+1} + 25$
$= 4^x + 5 \times 2^{x+1} + 25$
$\{or\ \ 2^{2x} + 10(2^x) + 25\}$

c $(x^{\frac{1}{2}} - 7)(x^{\frac{1}{2}} + 7)$
$= (x^{\frac{1}{2}})^2 - 7^2$
$= x^1 - 49$
$= x - 49$

8 **a** $6 \times 2^x = 192$
$\therefore\ 2^x = 32$
$\therefore\ 2^x = 2^5$
$\therefore\ x = 5$

b $4 \times (\frac{1}{3})^x = 324$
$\therefore\ (\frac{1}{3})^x = 81$
$\therefore\ (3^{-1})^x = 3^4$
$\therefore\ 3^{-x} = 3^4$
$\therefore\ x = -4$

9 The point $(1, \sqrt{8})$ lies on the graph of $y = 2^{kx}$.
$\therefore\ 2^{k \times 1} = \sqrt{8}$
$\therefore\ 2^k = \sqrt{2^3}$
$\therefore\ 2^k = 2^{\frac{3}{2}}$
$\therefore\ k = \frac{3}{2}$

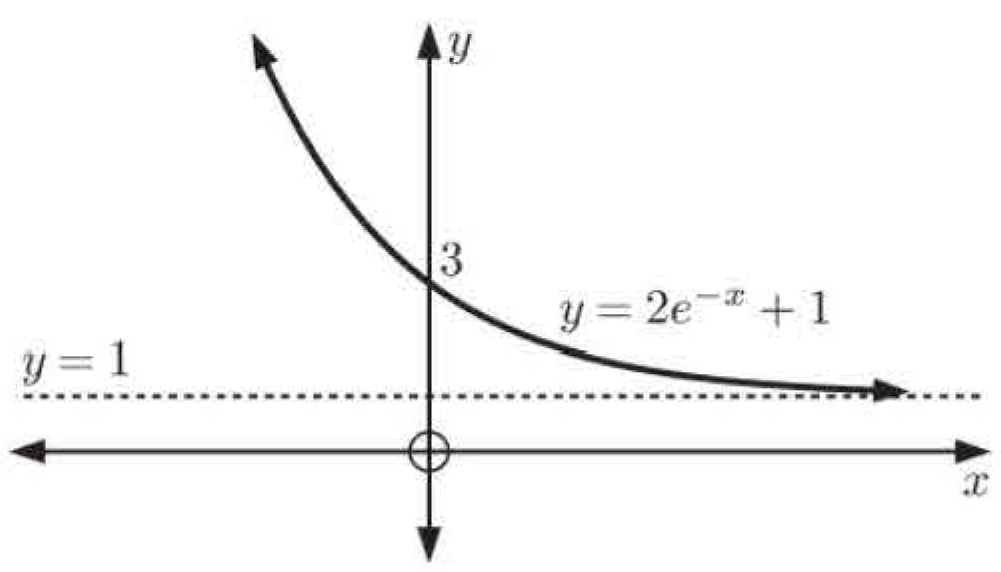
10 **a** $2^{x+1} = 32$
$\therefore\ 2^{x+1} = 2^5$
$\therefore\ x + 1 = 5$
$\therefore\ x = 4$

b $4^{x+1} = \left(\frac{1}{8}\right)^x$
$\therefore\ (2^2)^{x+1} = (2^{-3})^x$
$\therefore\ 2x + 2 = -3x$
$\therefore\ 5x = -2$
$\therefore\ x = -\frac{2}{5}$

11 **a** When $x = 0$, $y = 2e^{-0} + 1 = 3$
When $x = 1$, $y = 2e^{-1} + 1 \approx 1.74$
When $x = 2$, $y = 2e^{-2} + 1 \approx 1.27$
When $x = -1$, $y = 2e^{1} + 1 \approx 6.44$
When $x = -2$, $y = 2e^{2} + 1 \approx 15.8$

b As $x \to \infty$, $y \to 1^+$
As $x \to -\infty$, $y \to \infty$

c



d $y = 1$ is a horizontal asymptote.