Chapter 24

PROBABILITY

EXERCISE 24A

1 **a** P(inside a square) $= \dfrac{113}{145}$ ≈ 0.779

b P(on a line) $= \dfrac{32}{145}$ ≈ 0.221

2 Total frequency $= 17 + 38 + 19 + 4 = 78$

a P(20 to 39 seconds) $= \frac{38}{78} \approx 0.487$

b P($>$ 60 seconds) $= \frac{4}{78} \approx 0.051$

c P(between 20 and 59 seconds inclusive) $= \dfrac{38+19}{78} \approx 0.731$

3

Calls/day	*No. of days*
0	2
1	7
2	11
3	8
4	7
5	4
6	3
7	0
8	1

a Survey lasted $2+7+11+8+7+4+3+0+1$
$= 43$ days

b **i** P(0 calls) $\approx \dfrac{2}{43}$ ≈ 0.0465

ii P($\geqslant$ 5 calls) $\approx \dfrac{4+3+0+1}{43}$ ≈ 0.186

iii P($<$ 3 calls) $\approx \dfrac{2+7+11}{43}$ ≈ 0.465

4 Total frequency
$= 37 + 81 + 48 + 17 + 6 + 1$
$= 190$

a P(4 days gap) $\approx \dfrac{17}{190}$ ≈ 0.0895

b P(at least 4 days gap) $\approx \dfrac{17+6+1}{190}$ ≈ 0.126

EXERCISE 24B

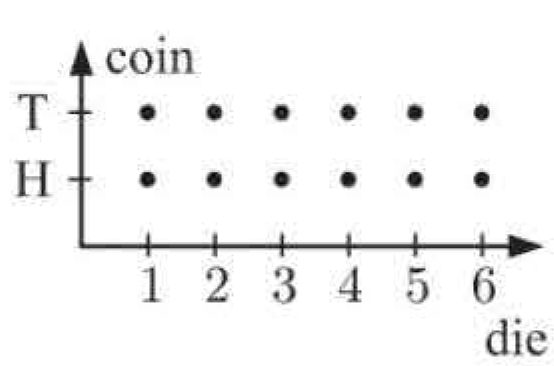
1 **a** {A, B, C, D}

b {BB, BG, GB, GG}

c {ABCD, ABDC, ACBD, ACDB, ADBC, ADCB, BACD, BADC, BCAD, BCDA, BDAC, BDCA, CABD, CADB, CBAD, CBDA, CDAB, CDBA, DABC, DACB, DBAC, DBCA, DCAB, DCBA}

d {GGG, GGB, GBG, BGG, GBB, BGB, BBG, BBB}

2 **a**



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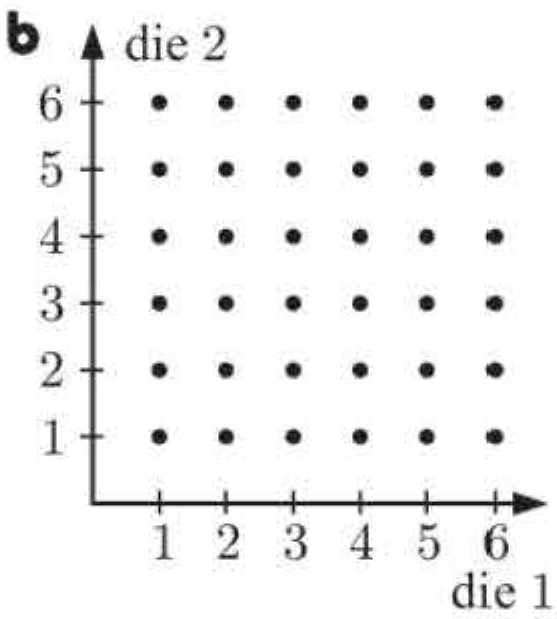

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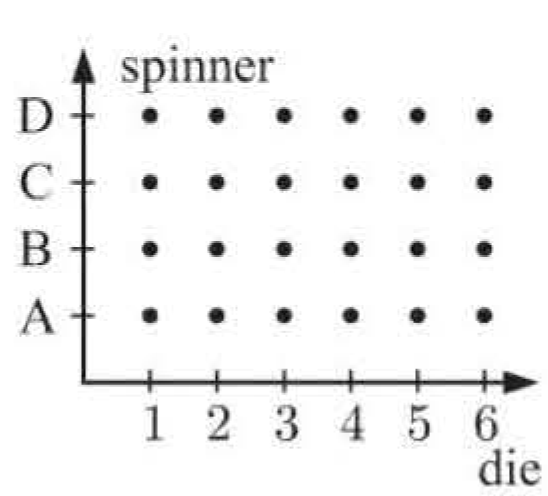

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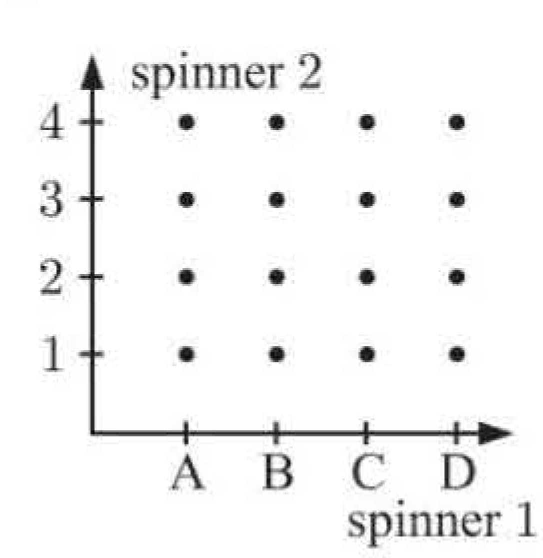
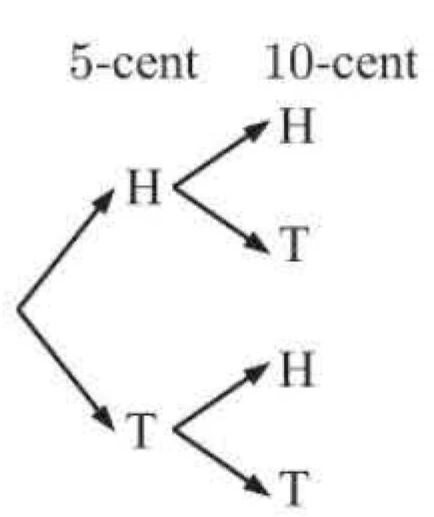


3 **a**



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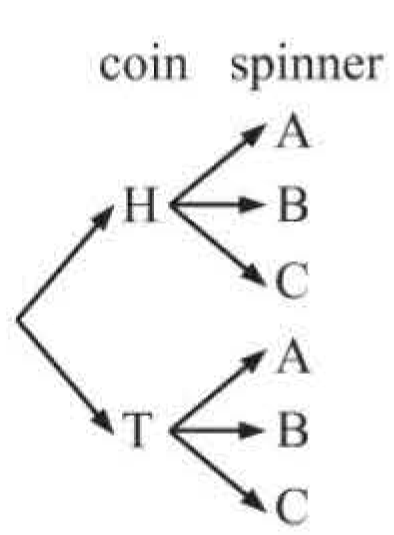

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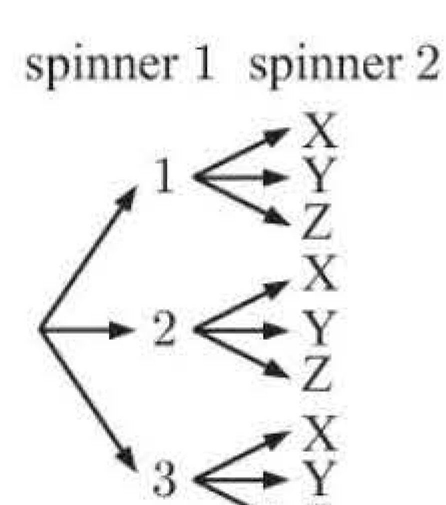

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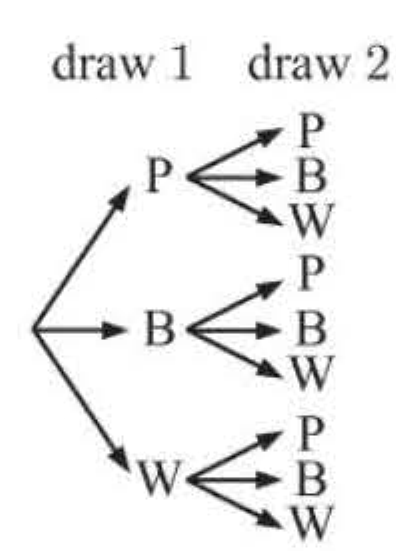


EXERCISE 24C.1

1 Total number of marbles $= 5 + 3 + 7 = 15$

a $\mathrm{P(red)} = \frac{3}{15} = \frac{1}{5}$

b $\mathrm{P(green)} = \frac{5}{15} = \frac{1}{3}$

c $\mathrm{P(blue)} = \frac{7}{15}$

d $\mathrm{P(not\ red)} = \dfrac{5+7}{15} = \frac{12}{15}$ or $\frac{4}{5}$

e $\mathrm{P(neither\ green\ nor\ blue)} = \mathrm{P(red)} = \frac{1}{5}$

f $\mathrm{P(green\ or\ red)} = \dfrac{5+3}{15} = \frac{8}{15}$

2 **a** 8 are brown and so 4 are white.

b **i** $\mathrm{P(brown)} = \frac{8}{12} = \frac{2}{3}$

ii $\mathrm{P(white)} = \frac{4}{12} = \frac{1}{3}$

3 **a** $\mathrm{P(multiple\ of\ 4)}$
$= \mathrm{P}(4, 8, 12, 16, 20, 24, 28, 32, 36)$
$= \frac{9}{36}$
$= \frac{1}{4}$

b $\mathrm{P(between\ 6\ and\ 9\ inclusive)}$
$= \mathrm{P}(6, 7, 8, \text{ or } 9)$
$= \frac{4}{36}$
$= \frac{1}{9}$

c $\mathrm{P}(> 20)$
$= \mathrm{P}(21, 22, 23, 24, ..., 35, 36)$
$= \dfrac{36-20}{36}$
$= \frac{16}{36}$
$= \frac{4}{9}$

d $\mathrm{P}(9) = \frac{1}{36}$

e $\mathrm{P(multiple\ of\ 13)}$
$= \mathrm{P}(13 \text{ or } 26)$
$= \frac{2}{36}$
$= \frac{1}{18}$

f $\mathrm{P(odd\ multiple\ of\ 3)}$
$= \mathrm{P}(3, 9, 15, 21, 27, \text{ or } 33)$
$= \frac{6}{36}$
$= \frac{1}{6}$

g $\mathrm{P(multiple\ of\ 4\ and\ 6)}$
$= \mathrm{P(multiple\ of\ 12)}$
$= \mathrm{P}(12, 24, 36)$
$= \frac{3}{36}$
$= \frac{1}{12}$

h $\mathrm{P(multiple\ of\ 4\ or\ 6)}$
$= \mathrm{P}(4, 6, 8, 12, 16, 18, 20, 24, 28, 30, 32, 36)$
$= \frac{12}{36}$
$= \frac{1}{3}$

4 **a** $\mathrm{P(on\ Tuesday)}$
$= \frac{1}{7}$

b $\mathrm{P(on\ a\ weekend)}$
$= \frac{2}{7}$

c $\mathrm{P(in\ July)}$
$= \dfrac{4 \times 31}{365 \times 3 + 366}$ {over a 4 year period}
$= \frac{124}{1461}$

d $\mathrm{P(in\ January\ or\ February)}$
$= \dfrac{4 \times 31 + 3 \times 28 + 1 \times 29}{3 \times 365 + 1 \times 366}$ {over a 4 year period}
$= \frac{237}{1461}$ $\left(= \frac{79}{487}\right)$

5 **a** Let A denote Antti, K denote Kai, and N denote Neda.
Possible orders are: {AKN, ANK, KAN, KNA, NAK, NKA}

b **i** $\mathrm{P(A\ in\ middle)} = \frac{2}{6}$
$= \frac{1}{3}$

ii $\mathrm{P(A\ at\ left\ end)} = \frac{2}{6}$
$= \frac{1}{3}$

iii P(A does not sit at right end)
$= 1 - \text{P(A at right end)}$
$= 1 - \frac{2}{6}$
$= \frac{4}{6} = \frac{2}{3}$

iv P(K and N are together) $= \frac{4}{6}$
$= \frac{2}{3}$

6 Let G denote 'a girl' and B denote 'a boy'.

a Possible orders are: {GGG, GGB, GBG, BGG, GBB, BGB, BBG, BBB}

b **i** P(all boys) = P(BBB) $= \frac{1}{8}$

ii P(all girls) = P(GGG) $= \frac{1}{8}$

iii P(boy, then girl, then girl)
= P(BGG)
$= \frac{1}{8}$

iv P(2 girls and a boy)
= P(GGB or GBG or BGG)
$= \frac{3}{8}$

v P(girl is eldest)
= P(GGG or GBG or GBB or GGB)
$= \frac{4}{8} = \frac{1}{2}$

vi P(at least one boy)
$= \frac{7}{8}$ {all except GGG}

7 **a** {ABCD, ABDC, ACBD, ACDB, ADBC, ADCB, BACD, BADC, BCAD, BCDA, BDAC, BDCA, CABD, CADB, CBAD, CBDA, CDAB, CDBA, DABC, DACB, DBAC, DBCA, DCAB, DCBA}

b **i** P(A sits on one end) $= \frac{12}{24} = \frac{1}{2}$

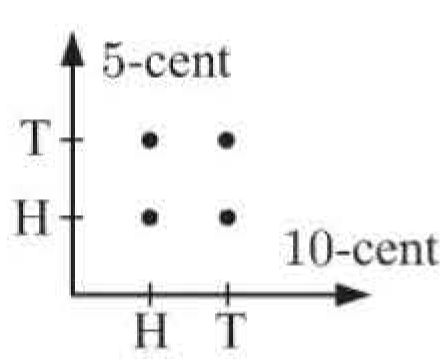
ii P(B sits on one of the two middle seats) $= \frac{12}{24} = \frac{1}{2}$

iii P(A and B are together) $= \frac{12}{24} = \frac{1}{2}$

iv P(A, B, and C are together) $= \frac{12}{24} = \frac{1}{2}$

EXERCISE 24C.2

1



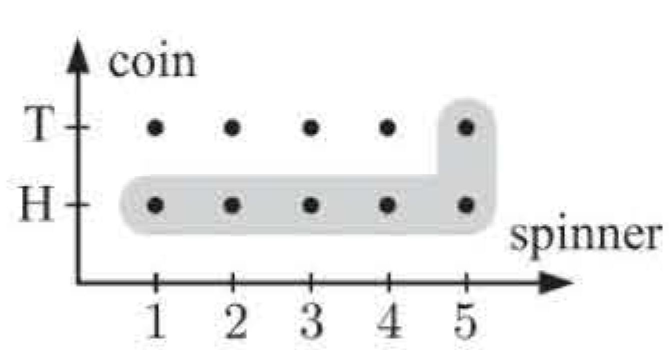
a P(2 heads) $= \frac{1}{4}$

b P(2 tails) $= \frac{1}{4}$

c P(exactly 1 head)
= P(HT or TH)
$= \frac{2}{4} = \frac{1}{2}$

d P(at least one H)
= P(HT or TH or HH)
$= \frac{3}{4}$

2 **a**



b There are $2 \times 5 = 10$ possible outcomes.

c **i** P(T and 3)
$= \frac{1}{10}$

ii P(H and even)
= P(H2 or H4)
$= \frac{2}{10} = \frac{1}{5}$

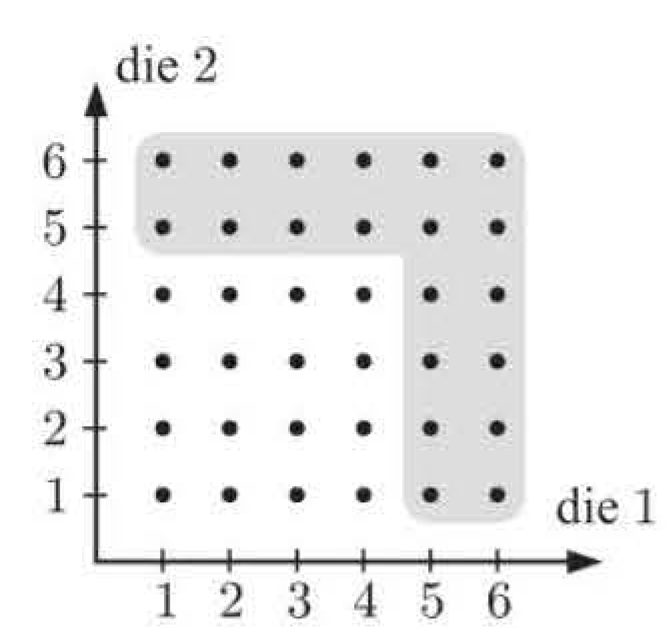
iii P(an odd)
= P(H1, T1, H3, T3, H5, T5)
$= \frac{6}{10} = \frac{3}{5}$

iv P(H or 5)
$= \frac{6}{10}$
$= \frac{3}{5}$ {shaded}

3 **a** P(two 3s)
= P((3, 3))
$= \frac{1}{36}$

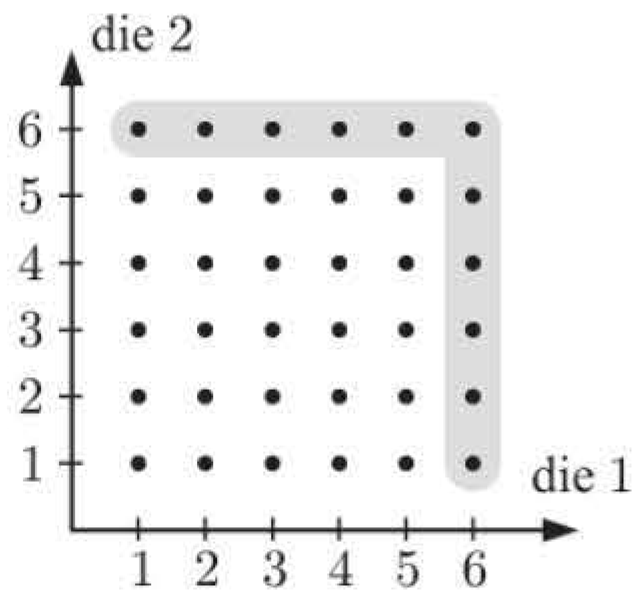
b P(5 and a 6)
= P((5, 6), (6, 5))
$= \frac{2}{36}$
$= \frac{1}{18}$

c P(5 or a 6)
$= \frac{20}{36}$
$= \frac{5}{9}$



d P(at least one 6)

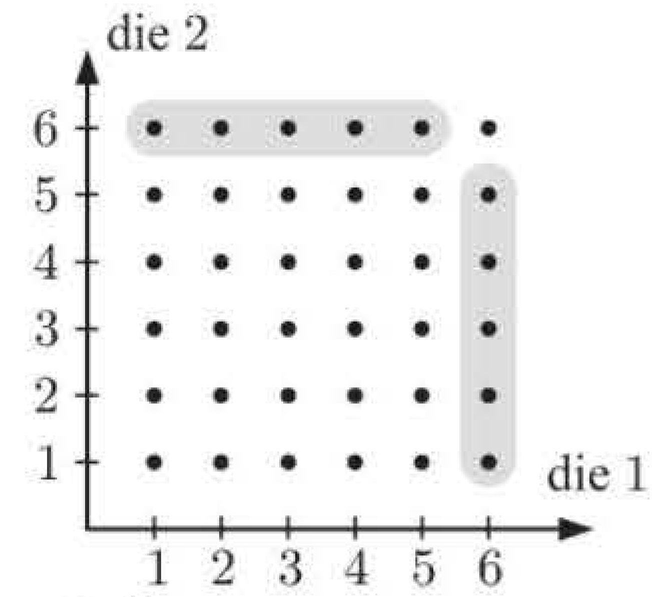
$= \frac{11}{36}$



e P(exactly one 6)

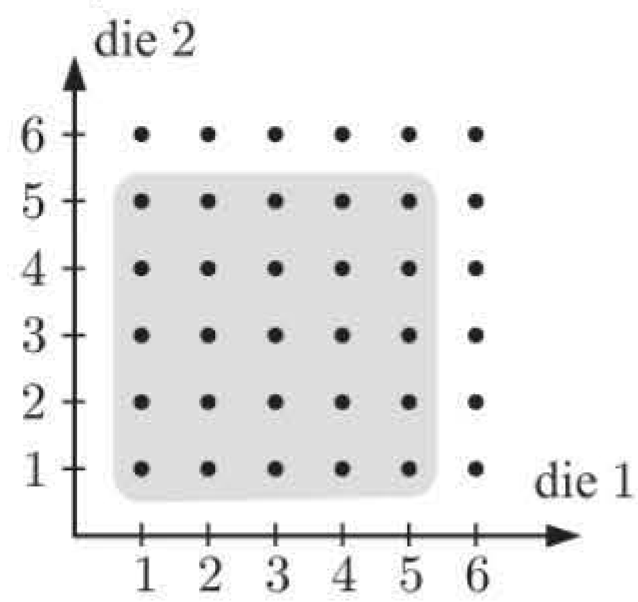
$= \frac{10}{36}$

$= \frac{5}{18}$



f P(no sixes)

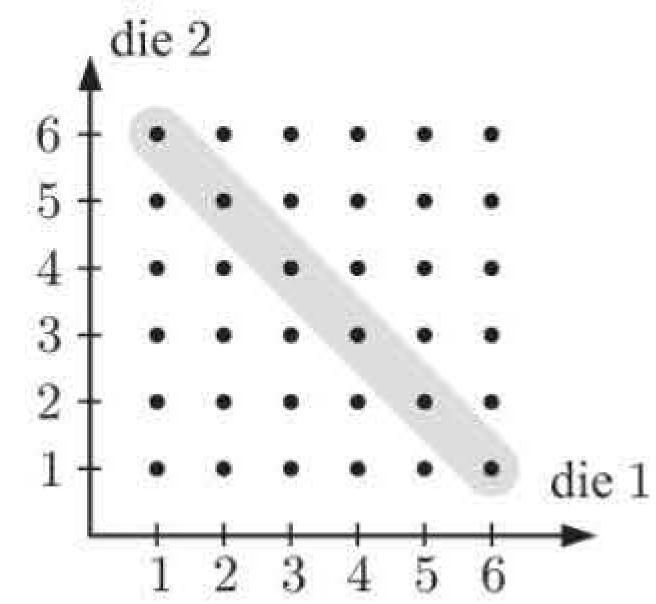
$= \frac{25}{36}$



g P(sum of 7)

$= \frac{6}{36}$

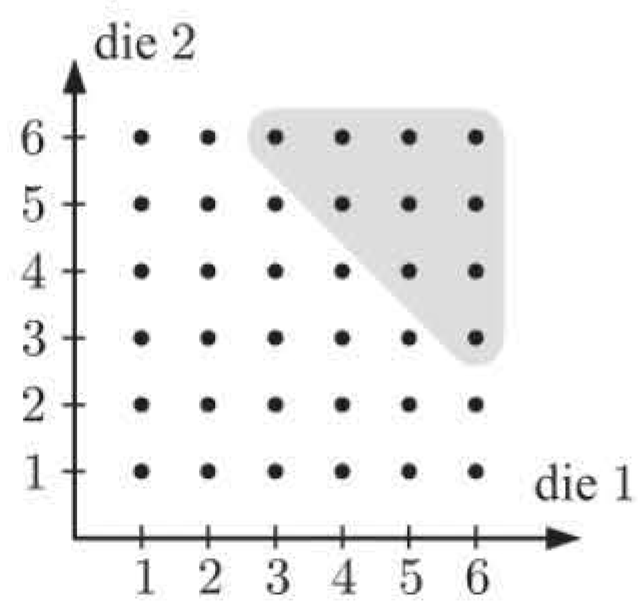
$= \frac{1}{6}$



h P(sum > 8)

$= \frac{10}{36}$

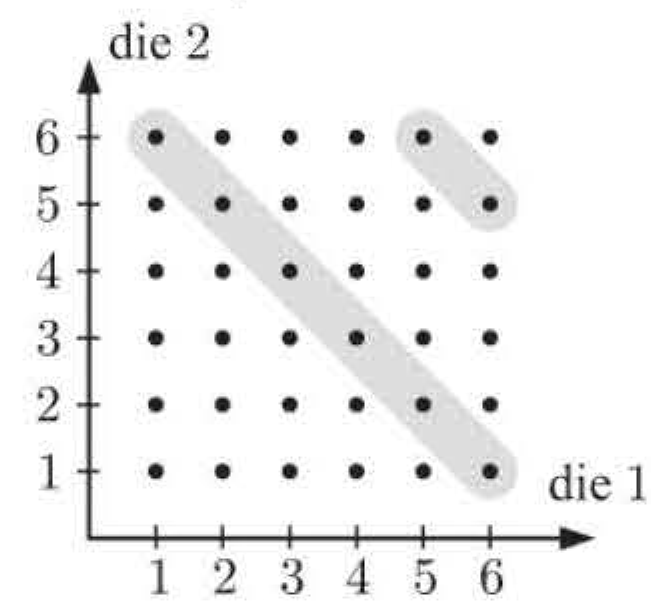
$= \frac{5}{18}$



i P(sum of 7 or 11)

$= \dfrac{6+2}{36}$

$= \frac{2}{9}$

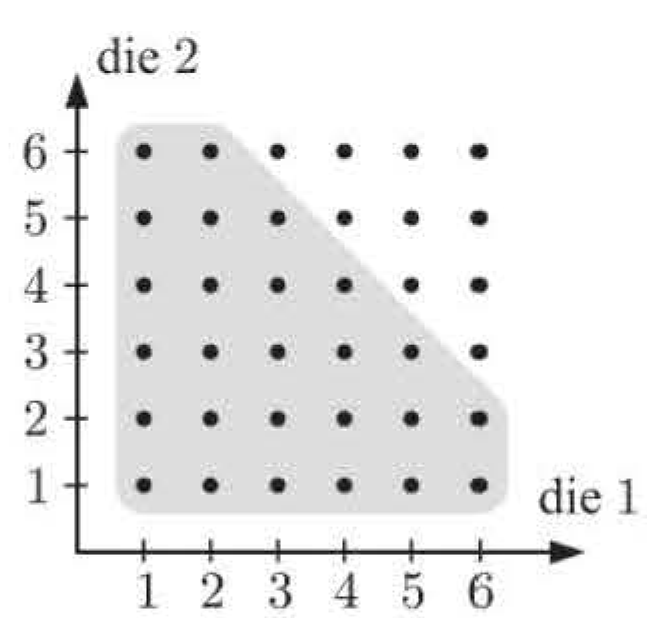


j P(sum no more than 8)

$=$ P(sum $\leqslant 8$)

$= \frac{26}{36}$

$= \frac{13}{18}$



EXERCISE 24D

1 We extend the table to include the totals:

	Employed	Unemployed	Total
Attended university	225	34	259
Did not attend university	197	81	278
Total	422	115	537

a 259 out of the 537 adults surveyed attended university.

$\therefore$ P(attended university) $\approx \frac{259}{537} \approx 0.482$

b 197 out of the 537 adults surveyed did not attend university and are currently employed.

$\therefore$ P(did not attend university and is currently employed) $\approx \frac{197}{537} \approx 0.367$

c 115 out of the 537 adults surveyed were unemployed.

$\therefore$ P(unemployed) $\approx \frac{115}{537} \approx 0.214$

d Of the 259 adults who attended university, 225 are currently employed.

$\therefore$ P(employed given that they attended university) $\approx \frac{225}{259} \approx 0.869$

e Of the 115 unemployed adults, 34 attended university.

$\therefore$ P(attended university given that they are currently unemployed) $\approx \frac{34}{115} \approx 0.296$

2 We extend the table to include the totals:

	Adult	Child	Total
Season ticket holder	1824	779	2603
Not a season ticket holder	3247	1660	4907
Total	5071	2439	7510

a Total match attendance was 7510.

b **i** P(child) $= \frac{2439}{7510} \approx 0.325$ **ii** P(not a season ticket holder) $= \frac{4907}{7510} \approx 0.653$

iii P(adult season ticket holder) $= \frac{1824}{7510} \approx 0.243$

3 We extend the table to include the totals:

	Single	Double	Family	Total
Peak season	125	220	98	443
Off-peak season	248	192	152	592
Total	373	412	250	1035

a P(peak season) $= \frac{443}{1035} \approx 0.428$

b P(single room in the off-peak season) $= \frac{248}{1035} \approx 0.240$

c P(single room or double room) $= \frac{373+412}{1035} = \frac{785}{1035} \approx 0.758$

d Of the 592 off-peak season bookings, 152 were for family rooms.

$\therefore$ P(family room given it was in the off-peak season) $= \frac{152}{592} \approx 0.257$

e $412 + 250 = 662$ bookings were not for a single room. Of these, $220 + 98 = 318$ were in the peak season.

$\therefore$ P(peak season given it was not a single room) $= \frac{318}{662} \approx 0.480$

EXERCISE 24E.1

1 **a** P(rains on any one day)
$= \frac{6}{7}$

b P(rains on 2 successive days)
$=$ P(R and R)
$= \frac{6}{7} \times \frac{6}{7}$
$= \frac{36}{49}$

c P(rains on 3 successive days)
$=$ P(R and R and R)
$= \frac{6}{7} \times \frac{6}{7} \times \frac{6}{7} = \frac{216}{343}$

2 **a** P(H, then H, then H)
$=$ P(H and H and H)
$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$
$= \frac{1}{8}$

b P(T, then H, then T)
$=$ P(T and H and T)
$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$
$= \frac{1}{8}$

3 Let A be the event of photocopier A malfunctioning and B be the event of photocopier B malfunctioning.

a P(both malfunction)
$=$ P(A and B)
$= 0.08 \times 0.12$
$= 0.0096$

b P(both work)
$=$ P(A' and B')
$= 0.92 \times 0.88$
$= 0.8096$

4 **a** P(they will be happy)
$= \text{P(B, then G, then B, then G)}$
$= \text{P(B and G and B and G)}$
$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$
$= \frac{1}{16}$

b P(they will be unhappy)
$= 1 - \text{P(they will be happy)}$
$= 1 - \frac{1}{16}$
$= \frac{15}{16}$

5 Let J be the event of Jiri hitting the target and B be the event of Benita hitting the target.

a P(both hit)
$= \text{P}(JB)$
$= 0.7 \times 0.8$
$= 0.56$

b P(both miss)
$= \text{P}(J'B')$
$= 0.3 \times 0.2$
$= 0.06$

c P(J hits and B misses)
$= \text{P}(JB')$
$= 0.7 \times 0.2$
$= 0.14$

d P(B hits and J misses)
$= \text{P}(BJ')$
$= 0.8 \times 0.3$
$= 0.24$

6 Let H be the event the archer hits the bullseye. $\therefore\ \text{P}(H) = \frac{2}{5},\ \ \text{P}(H') = \frac{3}{5}$

a P(3 hits)
$= \text{P}(HHH)$
$= \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5}$
$= \frac{8}{125}$

b P(2 hits then a miss)
$= \text{P}(HHH')$
$= \frac{2}{5} \times \frac{2}{5} \times \frac{3}{5}$
$= \frac{12}{125}$

c P(all misses)
$= \text{P}(H'H'H')$
$= \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5}$
$= \frac{27}{125}$

EXERCISE 24E.2

1 **a** P(all strawberry creams)
$= \text{P(SSS)}$
$= \frac{8}{12} \times \frac{7}{11} \times \frac{6}{10}$
$= \frac{14}{55}$

b P(none is a strawberry cream)
$= \text{P(S}'\text{S}'\text{S}')$
$= \frac{4}{12} \times \frac{3}{11} \times \frac{2}{10}$
$= \frac{1}{55}$

2 **a** P(both red)
$= \text{P(RR)}$
$= \frac{7}{10} \times \frac{6}{9}$
$= \frac{7}{15}$

b P(GR)
$= \frac{3}{10} \times \frac{7}{9}$
$= \frac{7}{30}$

c P(a green and a red)
$= \text{P(GR or RG)}$
$= \frac{3}{10} \times \frac{7}{9} + \frac{7}{10} \times \frac{3}{9}$
$= \frac{7}{15}$

3 **a** P(wins first prize) $= \frac{3}{100}$

b P(wins 1st and 2nd)
$= \text{P(WW)}$
$= \frac{3}{100} \times \frac{2}{99}$
$\approx 0.000\,606$

c P(wins all 3)
$= \text{P(WWW)}$
$= \frac{3}{100} \times \frac{2}{99} \times \frac{1}{98}$
$\approx 0.000\,006\,18$

d P(wins none of them)
$= \text{P(W}'\text{W}'\text{W}')$
$= \frac{97}{100} \times \frac{96}{99} \times \frac{95}{98}$
≈ 0.912

4 **a** P(does not contain captain)
$= \text{P(C}'\text{C}'\text{C}')$
$= \frac{6}{7} \times \frac{5}{6} \times \frac{4}{5}$
$= \frac{4}{7}$

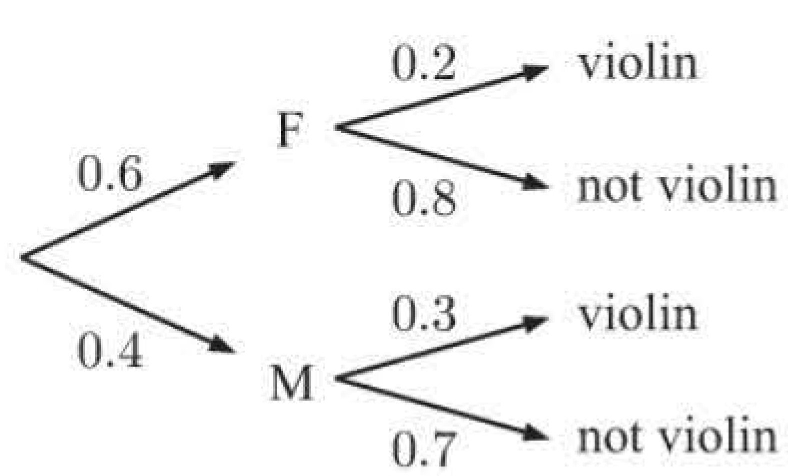
b P(does not contain captain or vice captain)
$= \text{P(OOO)} \qquad \{\text{O} \equiv \text{other}\}$
$= \frac{5}{7} \times \frac{4}{6} \times \frac{3}{5}$
$= \frac{2}{7}$

5 **a** P(two boys) = P(first selected is a boy *and* second selected is a boy)
$= \text{P(first selected is a boy)} \times \text{P(second selected is a boy)}$
$= \frac{5}{7} \times \frac{4}{6}$
$= \frac{20}{42} = \frac{10}{21}$

b P(eldest two students) $=$ P(either of the two eldest students *and* the remaining student)
$= $ P(either of the two eldest students) $\times$ P(the remaining student)
$= \frac{2}{7} \times \frac{1}{6}$
$= \frac{2}{42} = \frac{1}{21}$

EXERCISE 24F

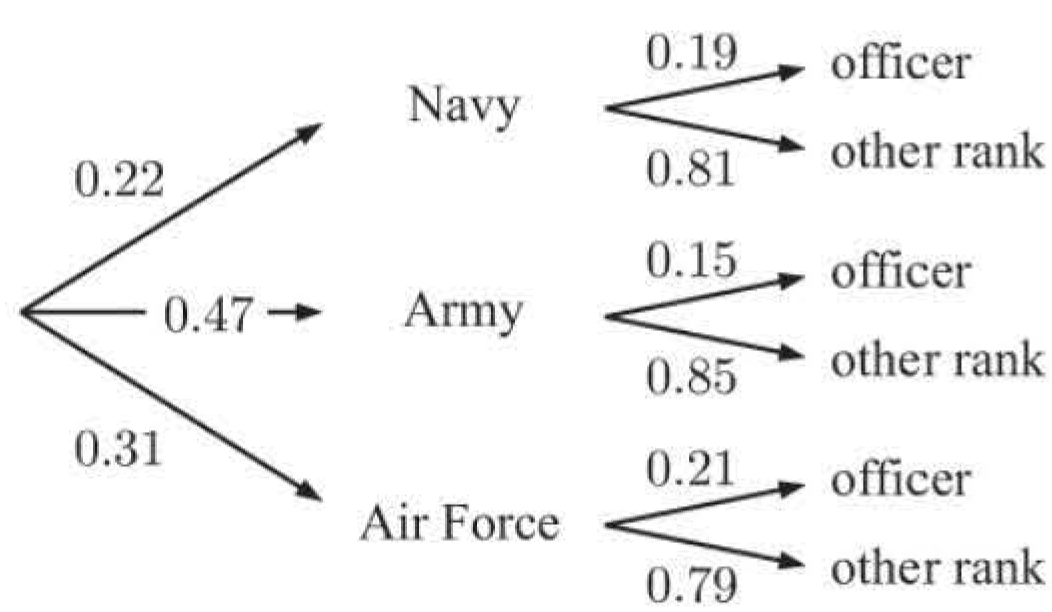
1 **a**



b **i** P(male and not violin) $= 0.4 \times 0.7$
$= 0.28$

ii P(plays the violin)
$=$ P(F and V) $+$ P(M and V)
$= 0.6 \times 0.2 + 0.4 \times 0.3$
$= 0.24$

2 **a**

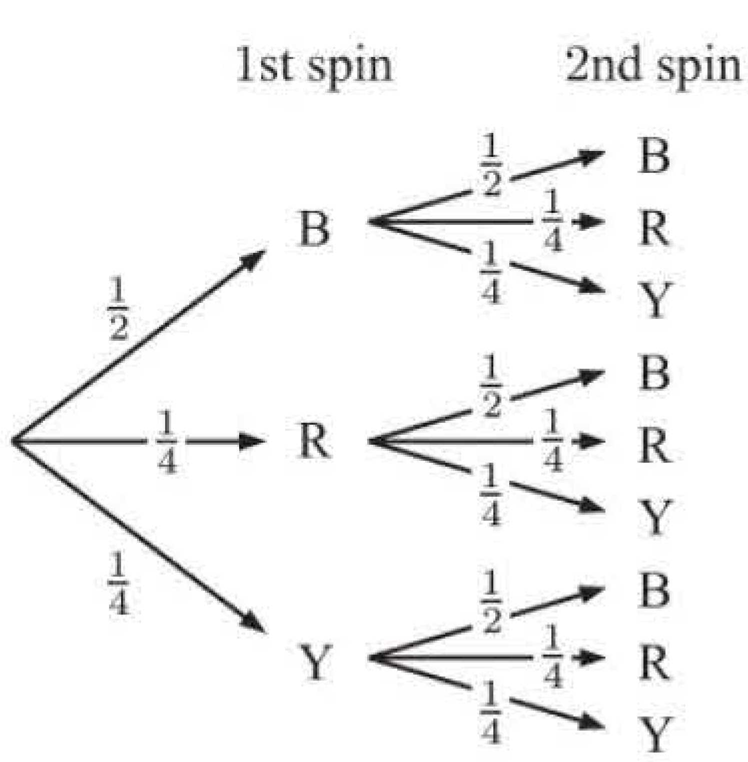


b **i** P(officer) $=$ P(N and O) $+$ P(A and O) $+$ P(AF and O) {where O represents officer}
$= 0.22 \times 0.19 + 0.47 \times 0.15 + 0.31 \times 0.21$
$= 0.1774 \approx 0.177$

ii P(not an officer in the navy)
$=$ P((N and O)$'$)
$= 1 -$ P(N and O)
$= 1 - 0.22 \times 0.19$
$= 0.9582 \approx 0.958$

iii P(not an army or air force officer)
$= 1 -$ (P(army or air force officer))
$= 1 -$ (P(A and O) $+$ P(AF and O))
$= 1 - (0.47 \times 0.15 + 0.31 \times 0.21)$
$= 0.8644 \approx 0.864$

3 **a**



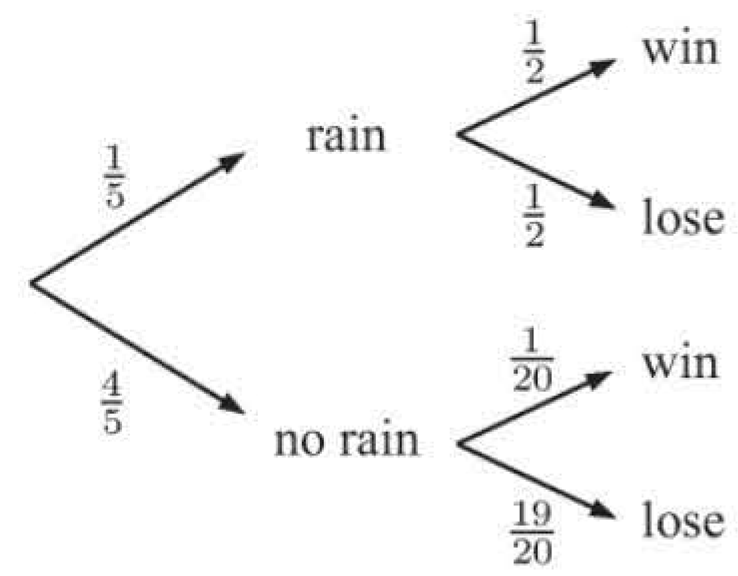
b P(both black)
$=$ P(BB)
$= \frac{1}{2} \times \frac{1}{2}$
$= \frac{1}{4}$

c P(both yellow)
$=$ P(YY)
$= \frac{1}{4} \times \frac{1}{4}$
$= \frac{1}{16}$

d P(both different)
$=$ P(BR or BY or RB or RY or YB or YR)
$= \frac{1}{2} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{4} + \frac{1}{4} \times \frac{1}{2} + \frac{1}{4} \times \frac{1}{4}$
$+ \frac{1}{4} \times \frac{1}{2} + \frac{1}{4} \times \frac{1}{4}$
$= \frac{4}{8} + \frac{2}{16}$
$= \frac{5}{8}$

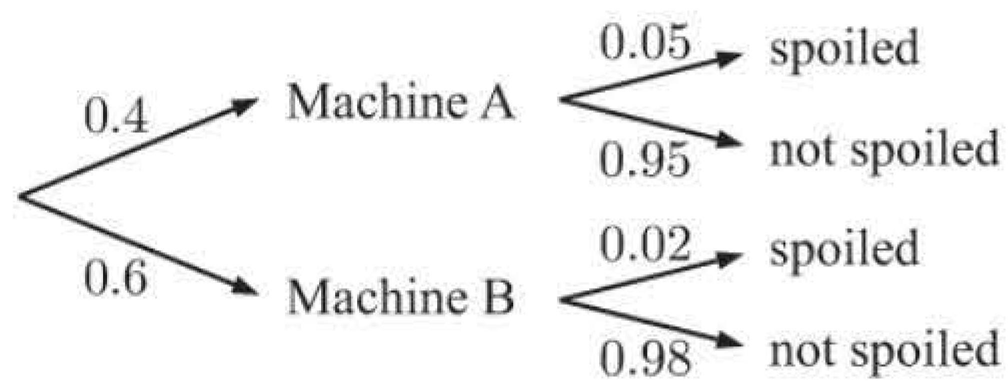
e P(B appears on either spin)
$=$ P(BB or BR or BY or RB or YB)
$= \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{4} + \frac{1}{4} \times \frac{1}{2}$
$+ \frac{1}{4} \times \frac{1}{2}$
$= 4(\frac{1}{8}) + \frac{1}{4}$
$= \frac{3}{4}$

4



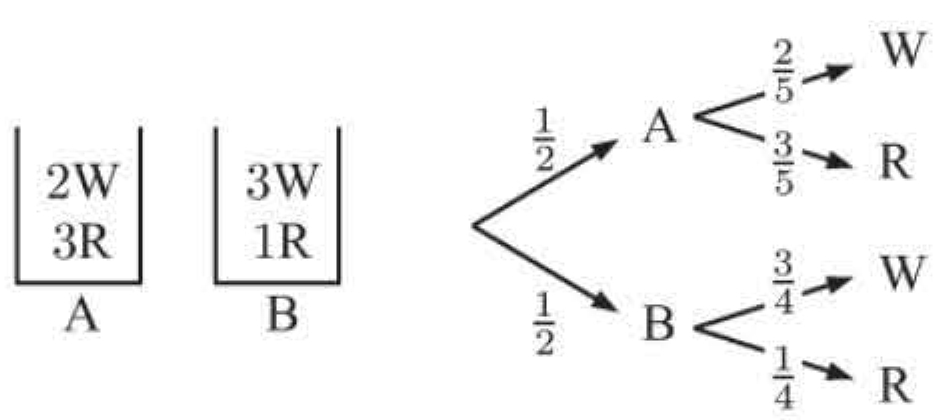
$$\begin{aligned}&\text{P(Mudlark wins)}\\&= \text{P(rain and win or no rain and win)}\\&= \tfrac{1}{5}\times\tfrac{1}{2}+\tfrac{4}{5}\times\tfrac{1}{20}\\&= \tfrac{1}{10}+\tfrac{4}{100}\\&= \tfrac{14}{100}\\&= \tfrac{7}{50}\end{aligned}$$

5



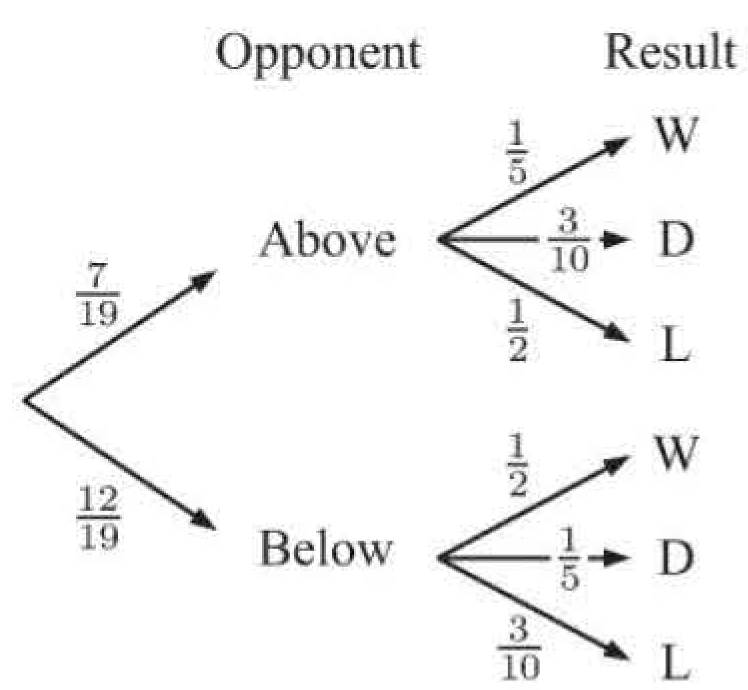
$$\begin{aligned}&\text{P(next is spoiled)}\\&= \text{P(from A and spoiled or from B and spoiled)}\\&= 0.4\times 0.05 + 0.6\times 0.02\\&= 0.020+0.012\\&= 0.032 \qquad (3.2\%)\end{aligned}$$

6



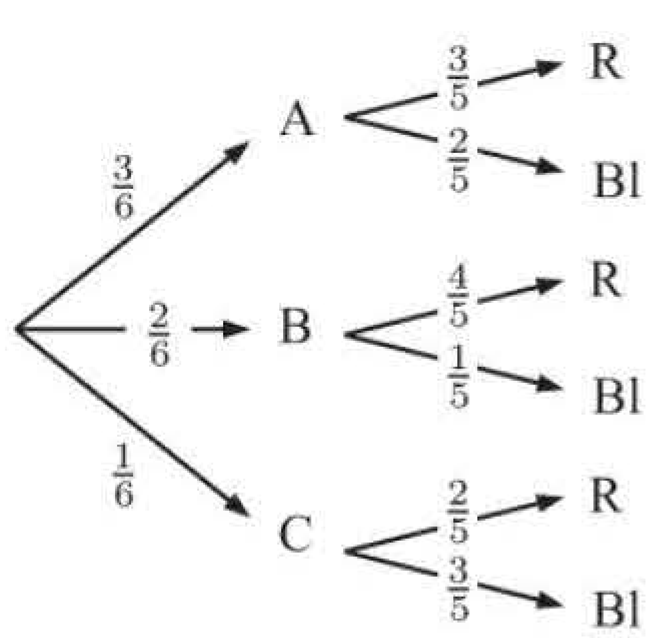
$$\begin{aligned}&\text{P(red)}\\&= \text{P(A and red or B and red)}\\&= \tfrac{1}{2}\times\tfrac{3}{5}+\tfrac{1}{2}\times\tfrac{1}{4}\\&= \tfrac{3}{10}+\tfrac{1}{8}\\&= \tfrac{17}{40}\end{aligned}$$

7 There are 7 teams above Tottenham and 12 teams below Tottenham.



$$\begin{aligned}\therefore\quad &\text{P(Draw)}\\&= \tfrac{7}{19}\times\tfrac{3}{10}+\tfrac{12}{19}\times\tfrac{1}{5}\\&= \tfrac{21}{190}+\tfrac{24}{190}\\&= \tfrac{9}{38}\end{aligned}$$

8

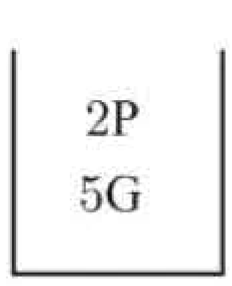


a $\text{P(blue)} = \text{P(A and Bl or B and Bl or C and Bl)}$
$$= \tfrac{3}{6}\times\tfrac{2}{5}+\tfrac{2}{6}\times\tfrac{1}{5}+\tfrac{1}{6}\times\tfrac{3}{5}$$
$$= \tfrac{11}{30}$$

b $\text{P(red)} = 1-\text{P(blue)}$
$$= 1-\tfrac{11}{30}$$
$$= \tfrac{19}{30}$$

EXERCISE 24G

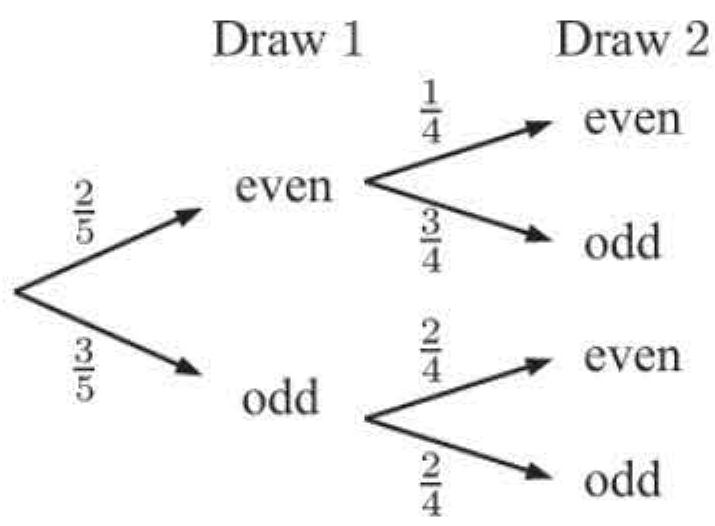
1



a
$$\begin{aligned}&\text{P(different colours)}\\&= \text{P(PG or GP)}\\&= \tfrac{2}{7}\times\tfrac{5}{7}+\tfrac{5}{7}\times\tfrac{2}{7}\\&= \tfrac{20}{49}\end{aligned}$$

b
$$\begin{aligned}&\text{P(different colours)}\\&= \text{P(PG or GP)}\\&= \tfrac{2}{7}\times\tfrac{5}{6}+\tfrac{5}{7}\times\tfrac{2}{6}\\&= \tfrac{20}{42}\\&= \tfrac{10}{21}\end{aligned}$$

2 **a**

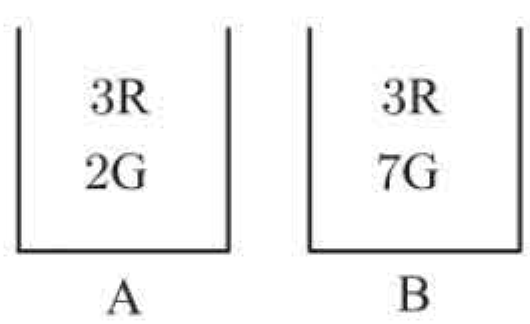


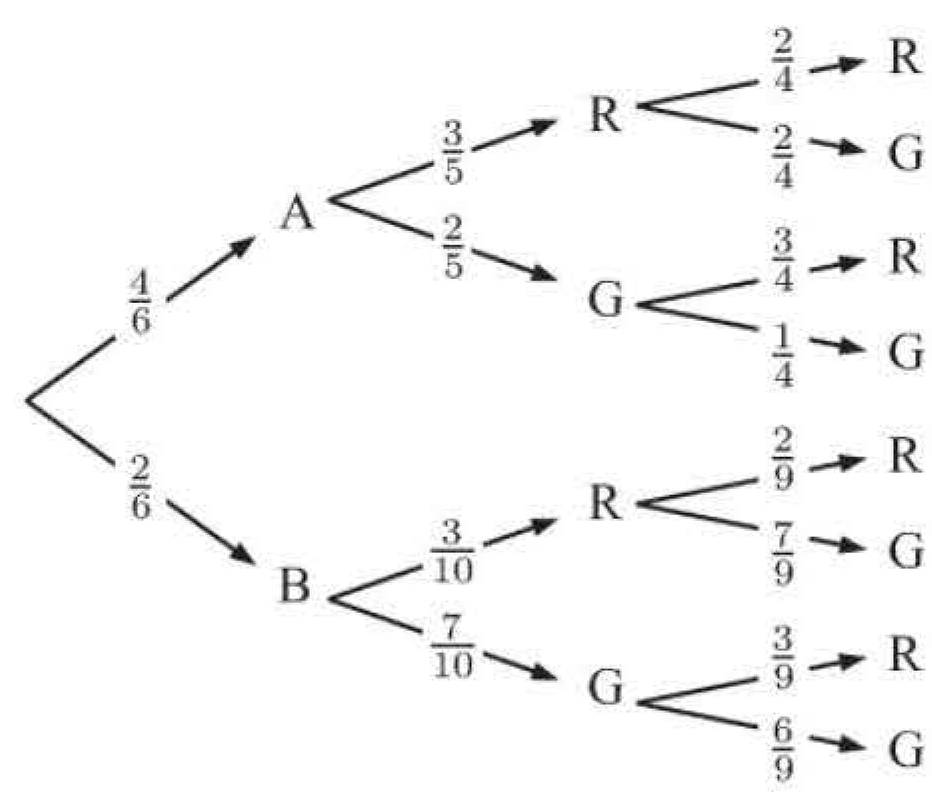
b **i** P(both odd)
$= \text{P(odd and odd)}$
$= \frac{3}{5} \times \frac{2}{4}$
$= \frac{3}{10}$

ii P(both even)
$= \text{P(even and even)}$
$= \frac{2}{5} \times \frac{1}{4}$
$= \frac{1}{10}$

iii P(one odd and other even)
$= 1 - \text{P(both odd)} - \text{P(both even)}$
$= 1 - \frac{3}{10} - \frac{1}{10}$
$= \frac{6}{10}$
$= \frac{3}{5}$

3

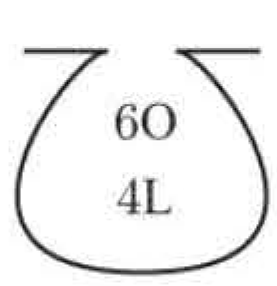


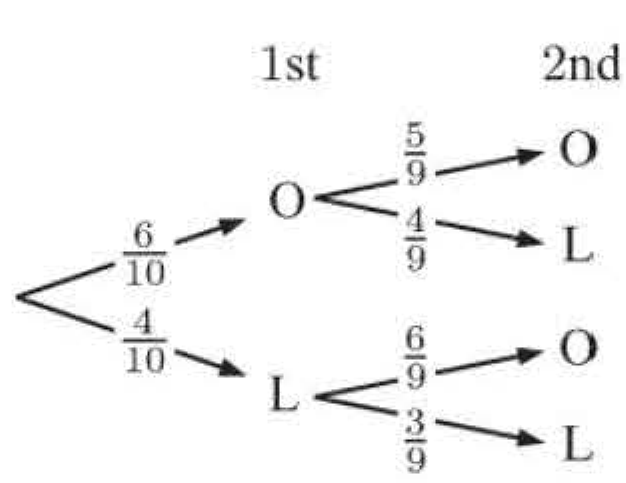


a P(both green)
$= \text{P(AGG or BGG)}$
$= \frac{4}{6} \times \frac{2}{5} \times \frac{1}{4} + \frac{2}{6} \times \frac{7}{10} \times \frac{6}{9}$
$= \frac{1}{15} + \frac{7}{45}$
$= \frac{10}{45}$
$= \frac{2}{9}$

b P(different in colour)
$= 1 - \text{P(both green)} - \text{P(both red)}$
$= 1 - \frac{2}{9} - \text{P(ARR or BRR)}$
$= \frac{7}{9} - (\frac{4}{6} \times \frac{3}{5} \times \frac{2}{4} + \frac{2}{6} \times \frac{3}{10} \times \frac{2}{9})$
$= \frac{7}{9} - (\frac{1}{5} + \frac{1}{45})$
$= \frac{5}{9}$

4





a **i** P(both O)
$= \frac{6}{10} \times \frac{5}{9}$
$= \frac{1}{3}$

ii P(both L)
$= \frac{4}{10} \times \frac{3}{9}$
$= \frac{2}{15}$

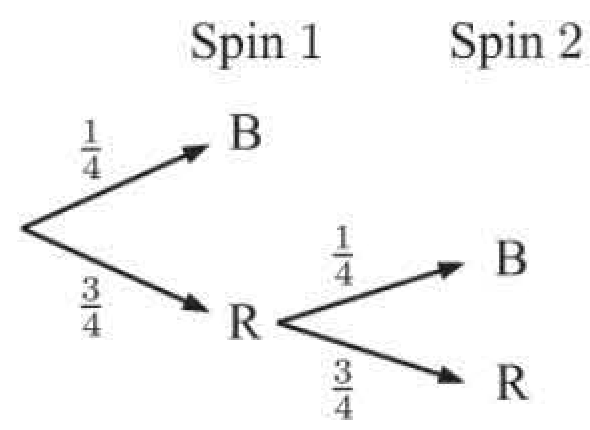
iii P(OL)
$= \frac{6}{10} \times \frac{4}{9}$
$= \frac{4}{15}$

iv P(LO)
$= \frac{4}{10} \times \frac{6}{9}$
$= \frac{4}{15}$

b $\frac{1}{3} + \frac{2}{15} + \frac{4}{15} + \frac{4}{15}$
$= \frac{5}{15} + \frac{2}{15} + \frac{4}{15} + \frac{4}{15}$
$= \frac{15}{15}$ which is 1

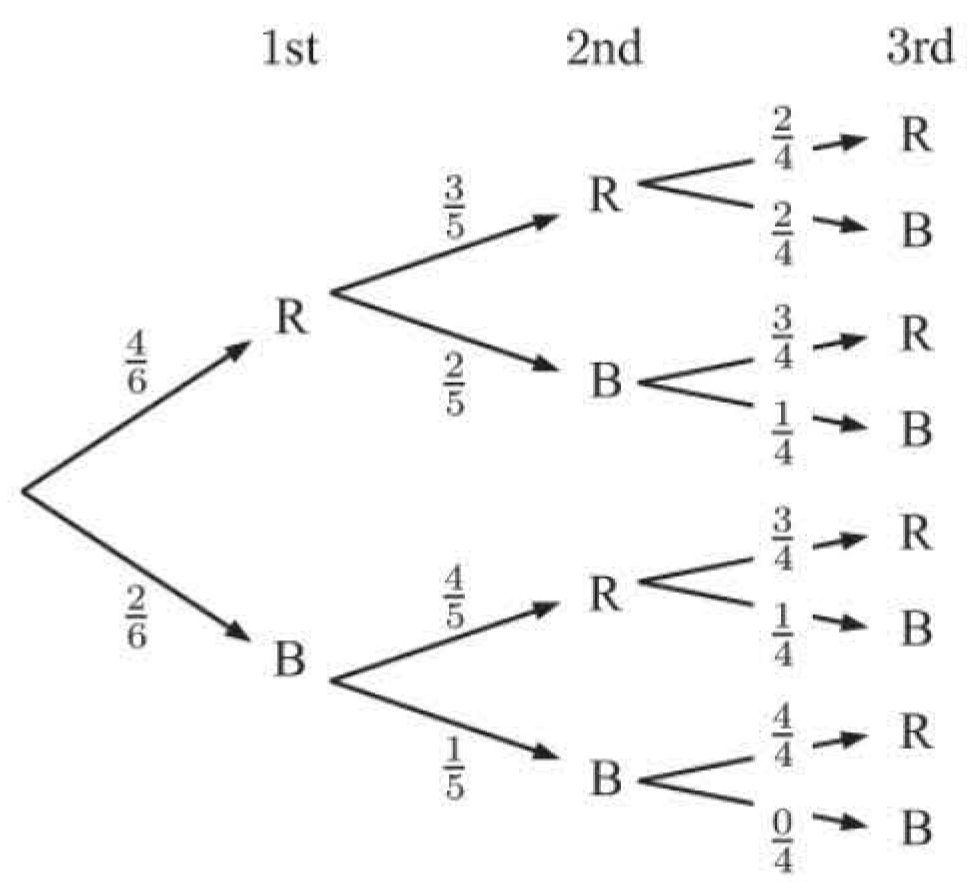
The answer must be 1 as the four categories **i**, **ii**, **iii**, **iv** are all the possibilities that could occur.

5 **a**



b P(blue)
$= \text{P}(\text{B}) + \text{P}(\text{RB})$
$= \frac{1}{4} + \frac{3}{4} \times \frac{1}{4}$
$= \frac{7}{16}$

6

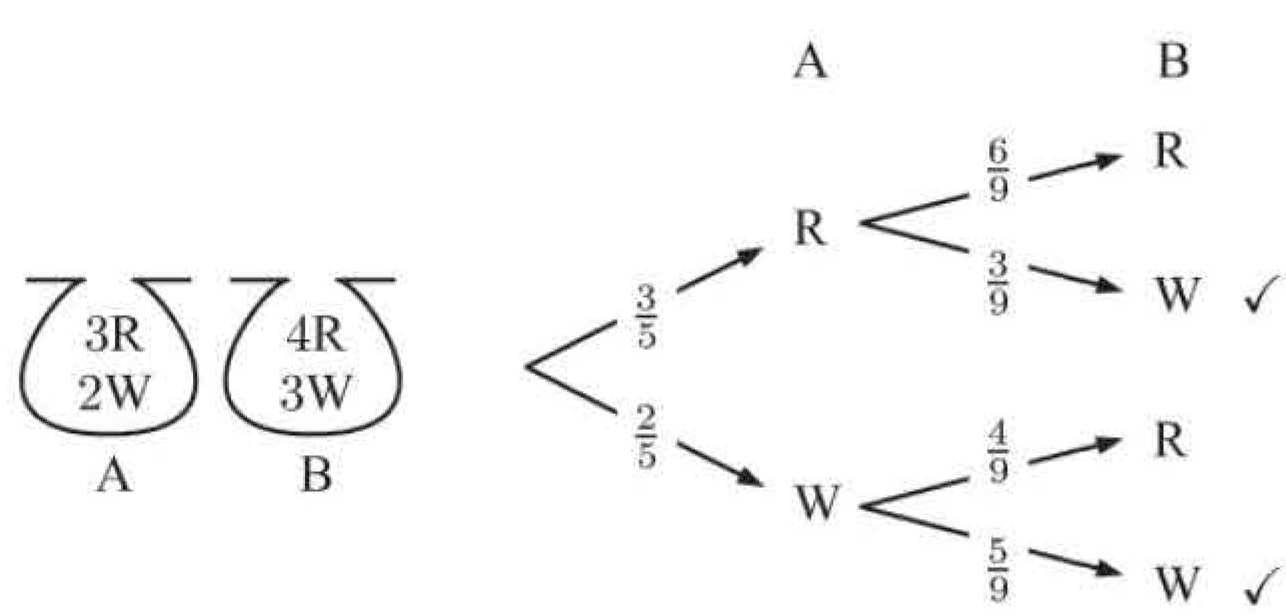


a P(all red)
$= \text{P}(\text{RRR})$
$= \frac{4}{6} \times \frac{3}{5} \times \frac{2}{4}$
$= \frac{1}{5}$

b P(only two are red)
$= \text{P}(\text{RRB or RBR or BRR})$
$= \frac{4}{6} \times \frac{3}{5} \times \frac{2}{4} + \frac{4}{6} \times \frac{2}{5} \times \frac{3}{4} + \frac{2}{6} \times \frac{4}{5} \times \frac{3}{4}$
$= 3 \times \left(\frac{24}{6 \times 5 \times 4}\right)$
$= \frac{3}{5}$

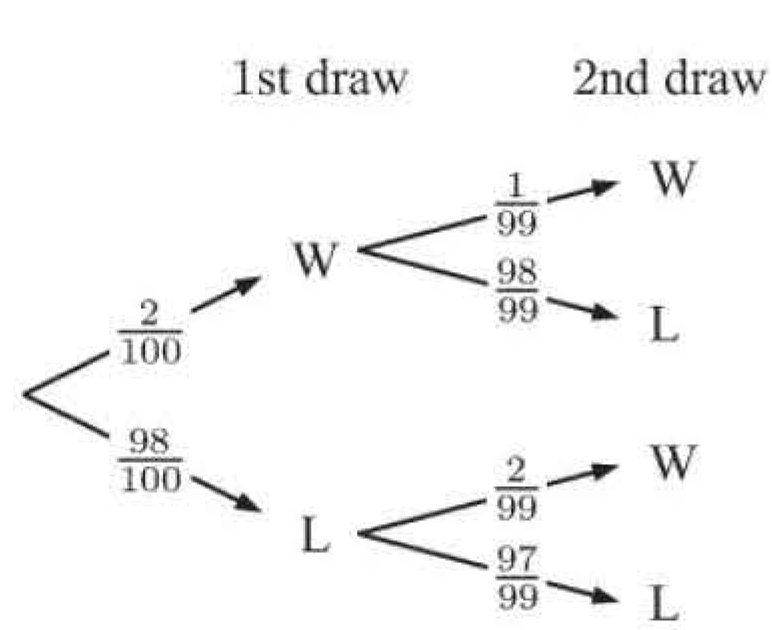
c P(at least two are red)
$= \text{P}(\text{all red or only two are red})$
$= \frac{1}{5} + \frac{3}{5}$ {from **a** and **b**}
$= \frac{4}{5}$

7



P(marble from B is W)
$= \text{P}(\text{RW or WW})$ {paths ticked}
$= \frac{3}{5} \times \frac{3}{9} + \frac{2}{5} \times \frac{5}{9}$
$= \frac{19}{45}$

8

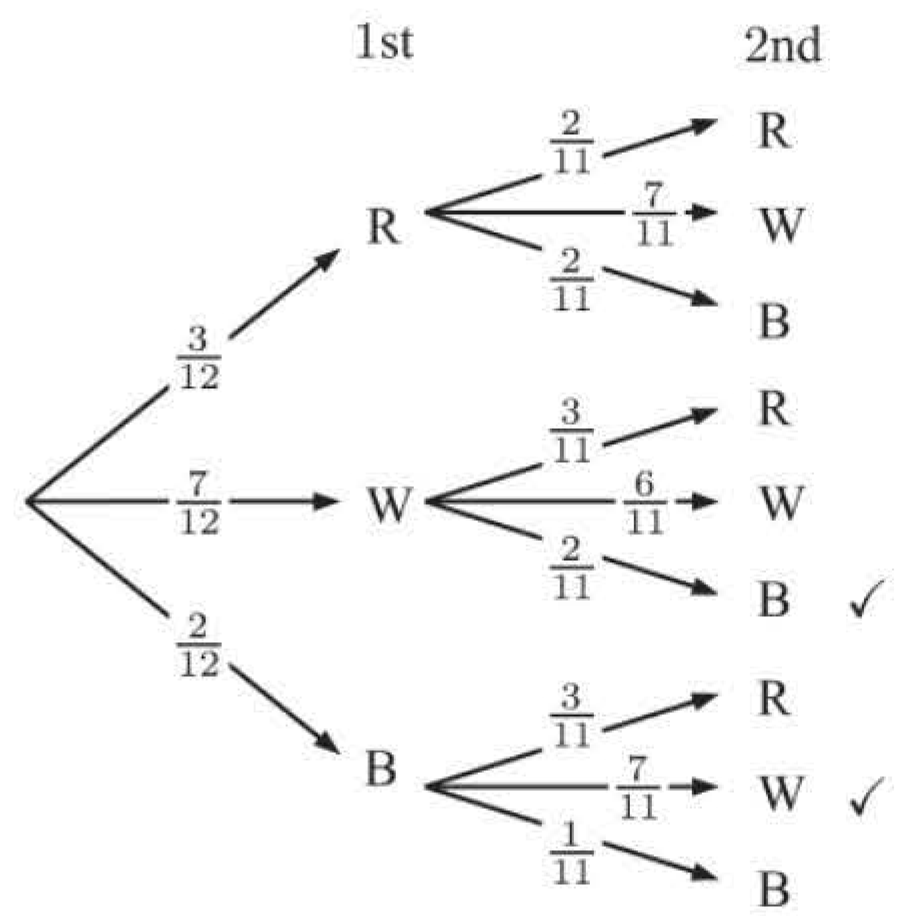


a P(wins both)
$= \text{P}(\text{WW})$
$= \frac{2}{100} \times \frac{1}{99}$
$\approx 0.000\,202$

b P(wins neither)
$= \text{P}(\text{LL})$
$= \frac{98}{100} \times \frac{97}{99}$
≈ 0.960

c P(wins at least one prize)
$= 1 - \text{P}(\text{wins neither})$
$= 1 - \frac{98}{100} \times \frac{97}{99}$
≈ 0.0398

9



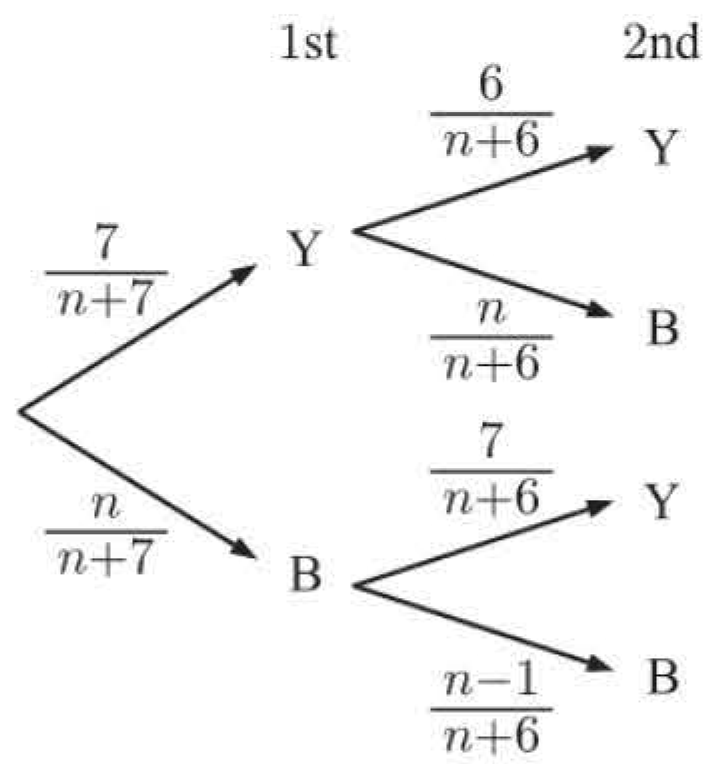
P(one white and one black)

$= \text{P(WB or BW)}$ {paths ticked}

$= \frac{7}{12} \times \frac{2}{11} + \frac{2}{12} \times \frac{7}{11}$

$= \frac{7}{33}$

10 There are $(n+7)$ markers in total.



$$\text{P(YY)} = \frac{3}{13}$$

$$\therefore \quad \frac{7}{n+7} \times \frac{6}{n+6} = \frac{3}{13}$$

$$\therefore \quad \frac{42}{n^2+13n+42} = \frac{3}{13}$$

$$\therefore \quad 546 = 3n^2 + 39n + 126$$

$$\therefore \quad 3(n^2 + 13n - 140) = 0$$

$$\therefore \quad 3(n-7)(n+20) = 0$$

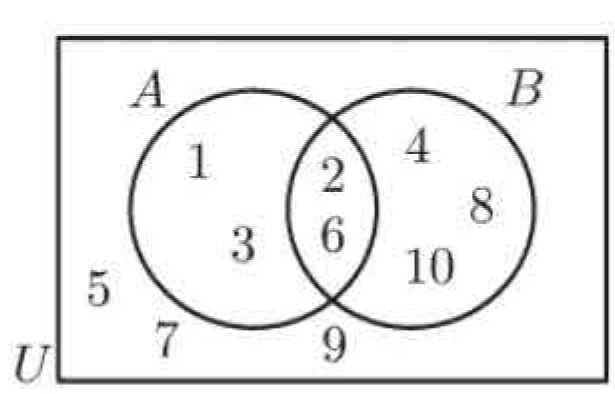
$$\therefore \quad n = 7 \qquad \{n \geqslant 0\}$$

$\therefore$ there are 7 blue markers in the bag to start with.

EXERCISE 24H.1

1 **a** $A = \{1, 2, 3, 6\}$, $B = \{2, 4, 6, 8, 10\}$

b

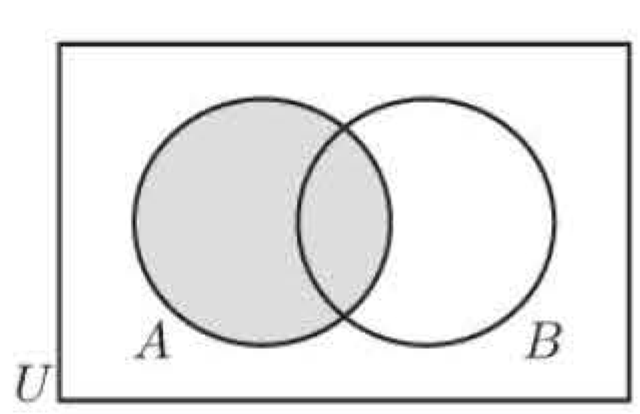


c **i** $n(A) = 4$

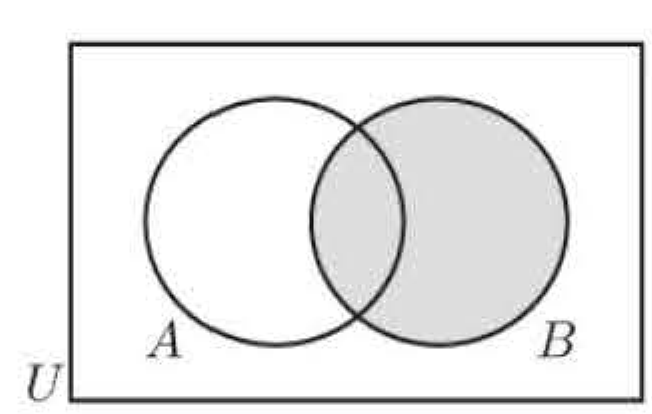
ii $A \cup B = \{1, 2, 3, 4, 6, 8, 10\}$

iii $A \cap B = \{2, 6\}$

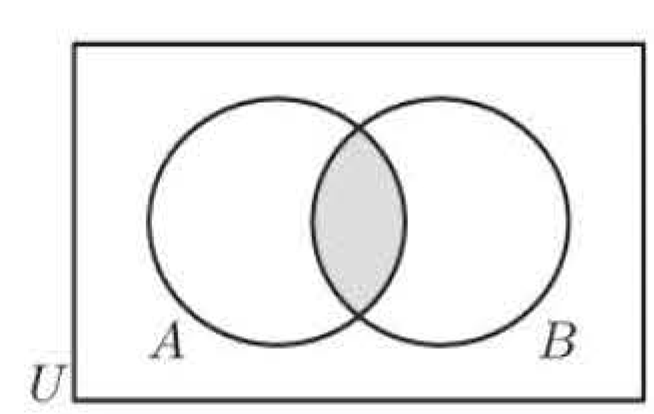
2 **a**



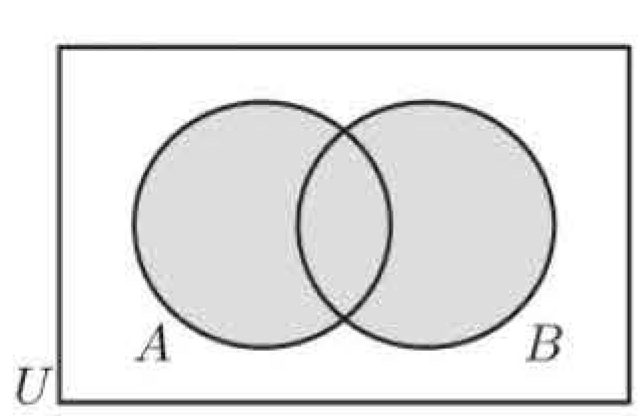
b



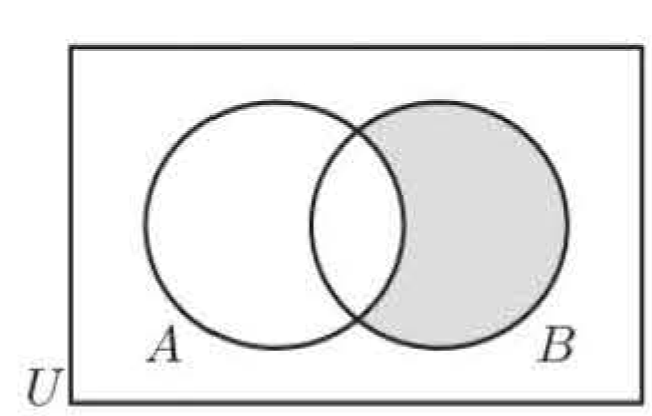
c



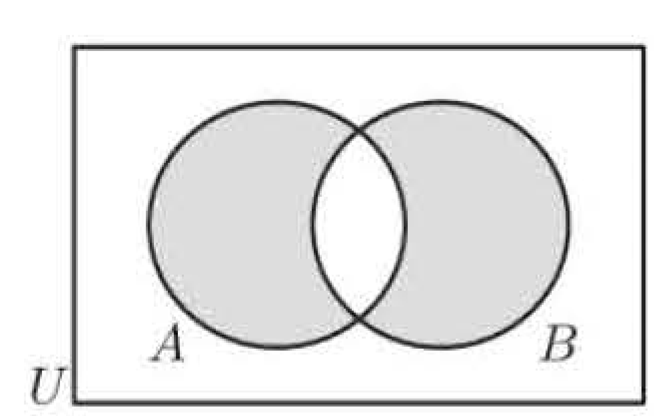
d



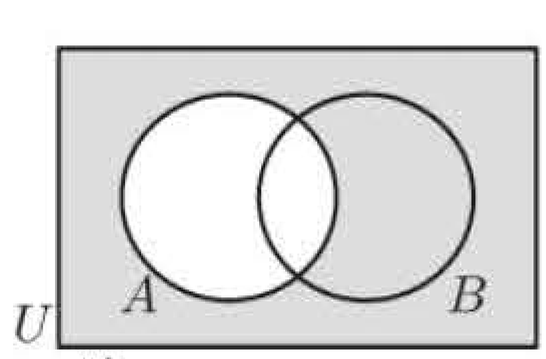
e



f

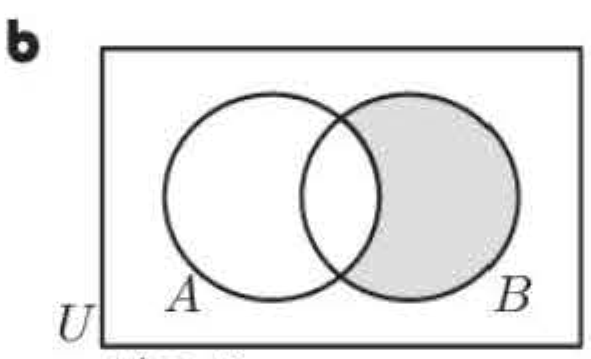


3 **a**



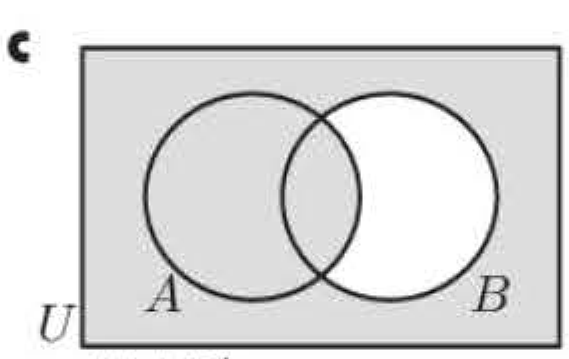
A'

b



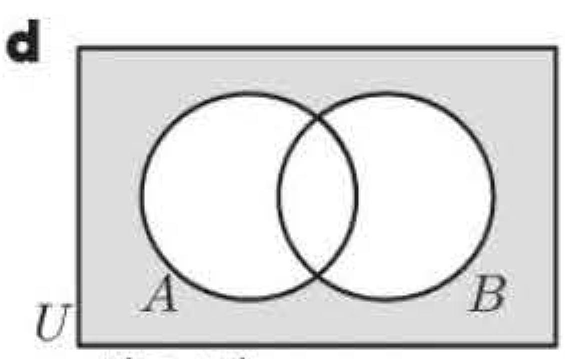
$A' \cap B$

c

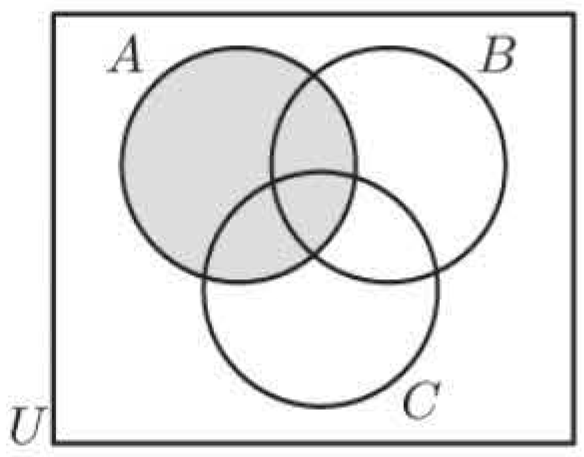


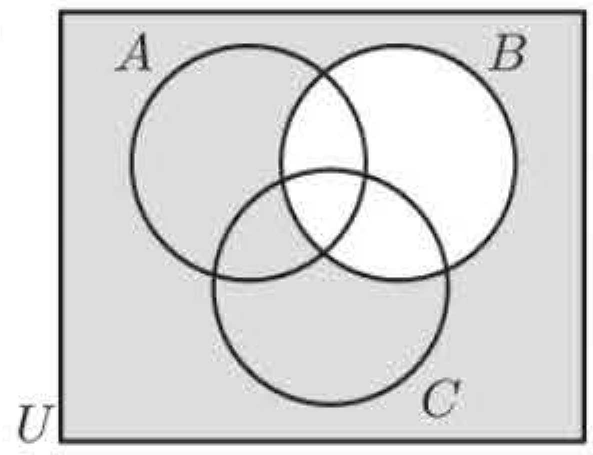
$A \cup B'$

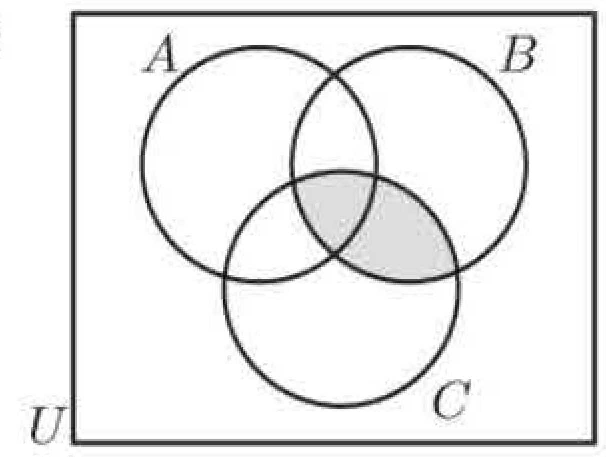
d

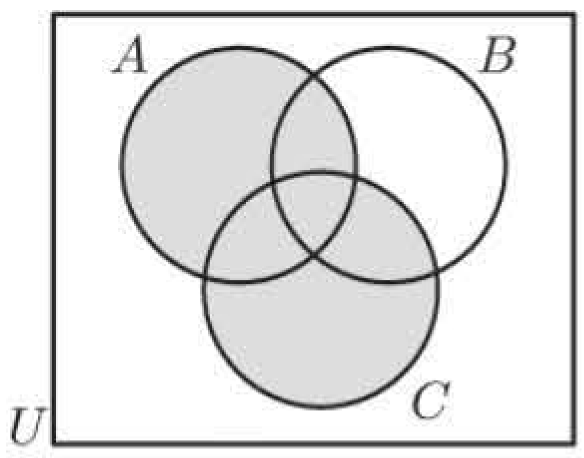


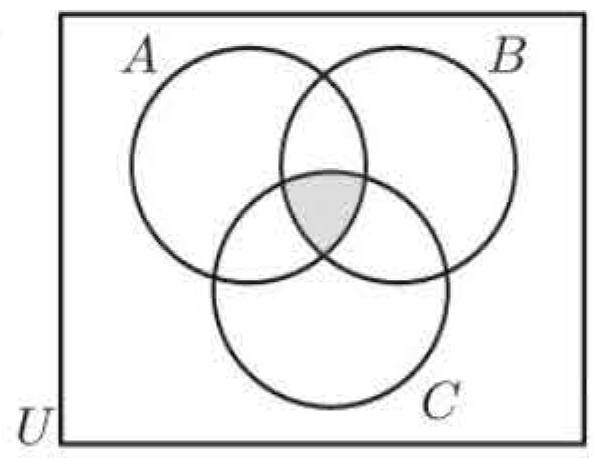
$A' \cap B'$

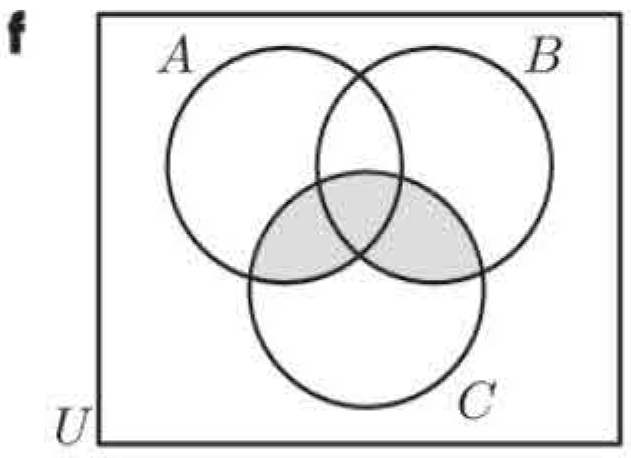
4 **a** 

b 

c 

d 

e 

f 

5 **a** Total number in the class $= 3 + 5 + 17 + 4 = 29$

b Number who study both $= 17$ {the intersection}

c Number who study at least one $= 5 + 17 + 4 = 26$ {the union}

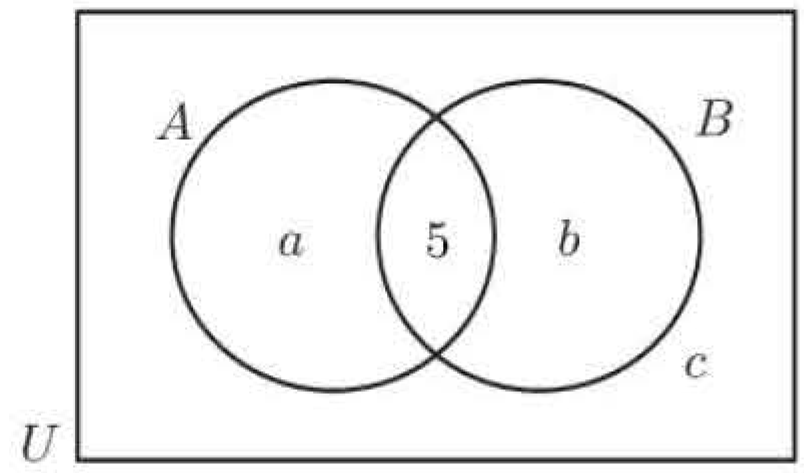
d Number who study only Chemistry $= 5$

6 **a** Total number in the survey $= 37 + 9 + 15 + 4 = 65$

b Number who liked both $= 9$ {the intersection}

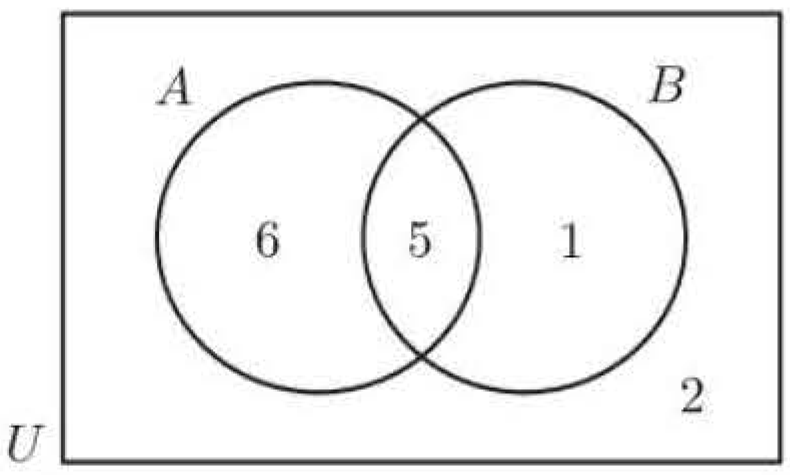
c Number who liked neither $= 4$

d Number who liked exactly one $= 37 + 15 = 52$

7 **a** 

$$\begin{aligned} a + 5 &= 11 \quad \{\text{since } n(A) = 11\} \\ \therefore\ a &= 6 \\ 6 + 5 + b &= 12 \quad \{\text{since } n(A \cup B) = 12\} \\ \therefore\ b &= 1 \\ 6 + c &= 8 \quad \{\text{since } n(B') = 8\} \\ \therefore\ c &= 2 \end{aligned}$$

$\therefore$ the completed Venn diagram is:

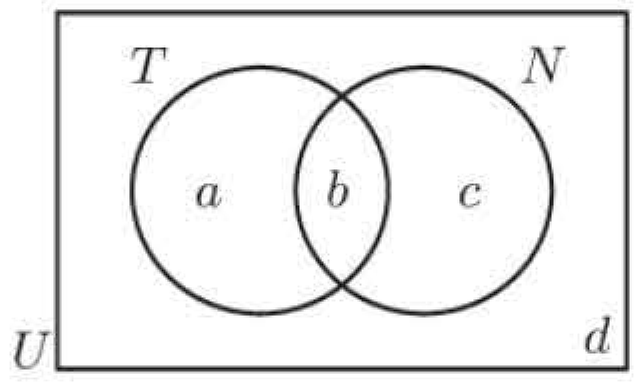


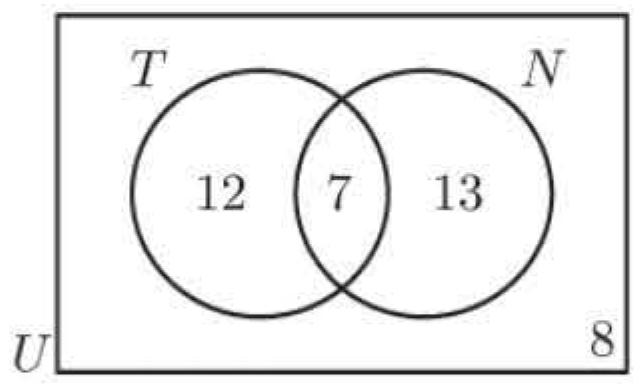
b **i** $n(U) = 6 + 5 + 1 + 2 = 14$

$$\therefore\ P(A \cup B) = \frac{n(A \cup B)}{n(U)} = \frac{6+5+1}{14} = \frac{12}{14} = \frac{6}{7}$$

ii $$P(A') = \frac{n(A')}{n(U)} = \frac{1+2}{14} = \frac{3}{14}$$

8





T represents those playing tennis
N represents those playing netball

$$\therefore \quad \begin{cases} a+b+c+d=40 \\ a+b=19 \\ b+c=20 \\ d=8 \end{cases}$$

So, $a+b+c=32$

$\therefore\ 19+c=32$ and $a+20=32$

$\therefore\ c=13$ and $a=12$

Hence, $12+b=19$

$\therefore\ b=7$

a P(plays tennis) $= \dfrac{12+7}{40} = \frac{19}{40}$

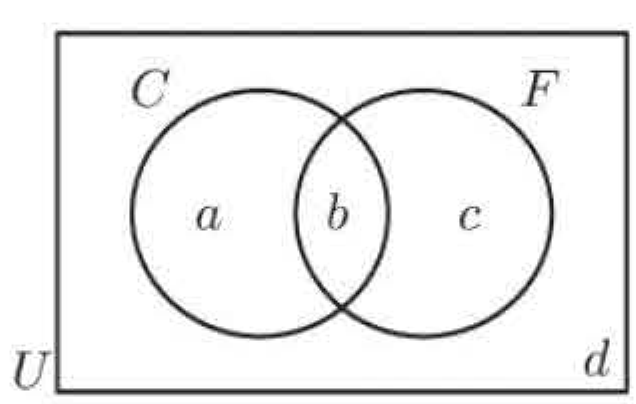
b P(does not play netball) $= \dfrac{12+8}{40} = \frac{1}{2}$

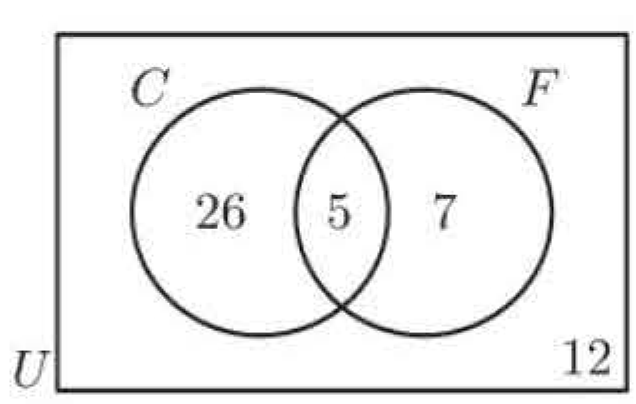
c P(plays at least one) $= \dfrac{12+7+13}{40} = \frac{32}{40} = \frac{4}{5}$

d P(plays one and only one) $= \dfrac{12+13}{40} = \frac{25}{40} = \frac{5}{8}$

e P(plays netball, but not tennis) $= \frac{13}{40}$

9





C represents men who gave chocolates.
F represents men who gave flowers.

$$\therefore \quad \begin{cases} a+b+c+d=50 \\ a+b=31 \\ b+c=12 \\ b=5 \end{cases}$$

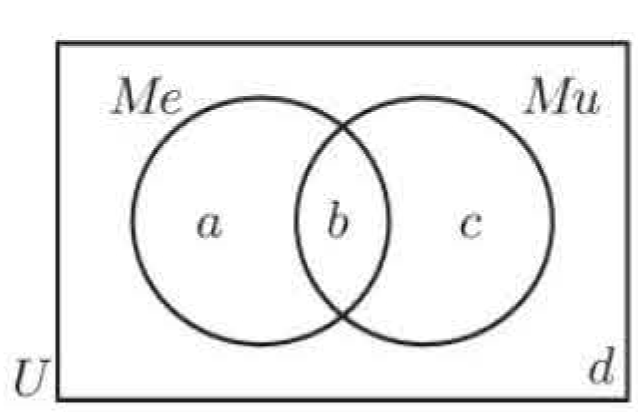
Thus $c=7$, $a=26$ and $26+5+7+d=50 \quad \therefore\ d=12$

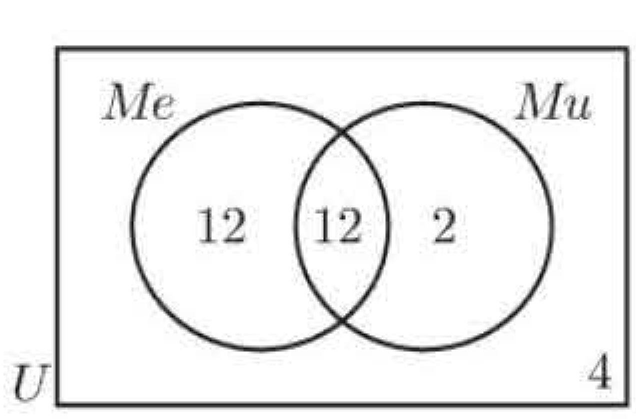
a P(C or F) $= \dfrac{26+5+7}{50} = \frac{38}{50} = \frac{19}{25}$

b P(C but not F) $= \frac{26}{50} = \frac{13}{25}$

c P(neither C nor F) $= \frac{12}{50} = \frac{6}{25}$

10





Me represents children who had measles.
Mu represents children who had mumps.

$$\therefore \quad \begin{cases} a+b+c+d=30 \\ a+b=24 \\ b=12 \\ a+b+c=26 \end{cases}$$

$\therefore\ 26+d=30 \quad \therefore\ d=4$

$24+c=26 \quad \therefore\ c=2$

and $a+12=24 \quad \therefore\ a=12$

a $\text{P}(Mu)$
$= \frac{14}{30}$
$= \frac{7}{15}$

b $\text{P}(Mu, \text{ but not } Me)$
$= \frac{2}{30}$
$= \frac{1}{15}$

c $\text{P}(\text{neither } Mu \text{ nor } Me)$
$= \frac{4}{30}$
$= \frac{2}{15}$

11 **a** $4+2+1+a = 10$ {10 watched a movie}
$\therefore\ a = 3$

$4+2+1+3+6+12+9+b = 40$ {40 individuals in total}
$\therefore\ 37+b = 40$
$\therefore\ b = 3$

b **i** $\text{P(sport)} = \dfrac{6+2+1+3}{40} = \frac{12}{40} = \frac{3}{10}$

ii $\text{P(drama and sport)} = \dfrac{3+1}{40} = \frac{4}{40} = \frac{1}{10}$

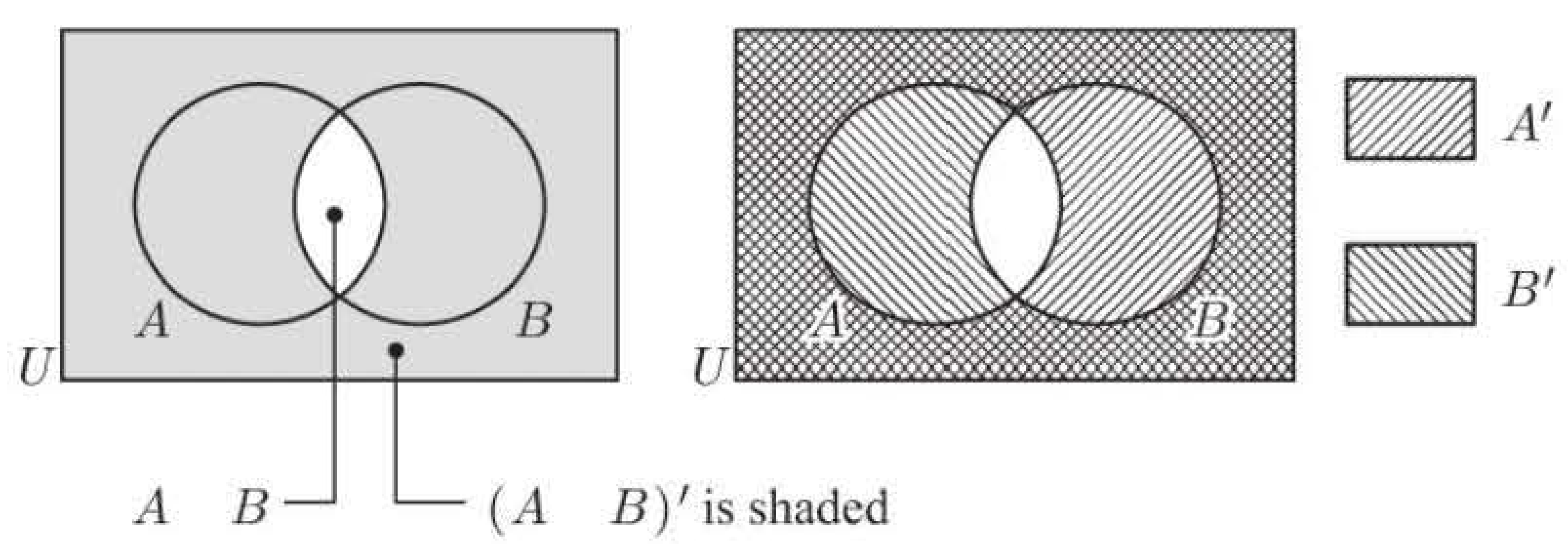
iii $\text{P(movie but not sport)} = \dfrac{4+3}{40} = \frac{7}{40}$

iv $\text{P(drama but not movie)} = \dfrac{12+3}{40} = \frac{15}{40} = \frac{3}{8}$

v $\text{P(drama or a movie)} = \dfrac{12+3+3+1+4+2}{40} = \frac{25}{40} = \frac{5}{8}$

EXERCISE 24H.2

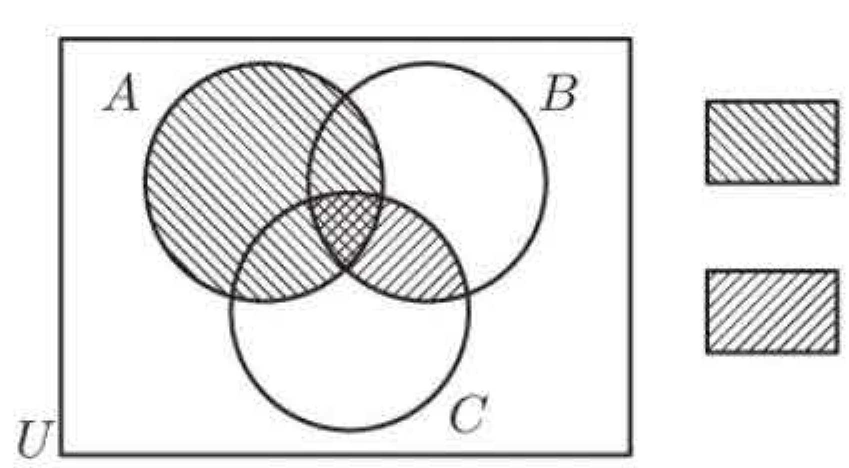
1 **a**

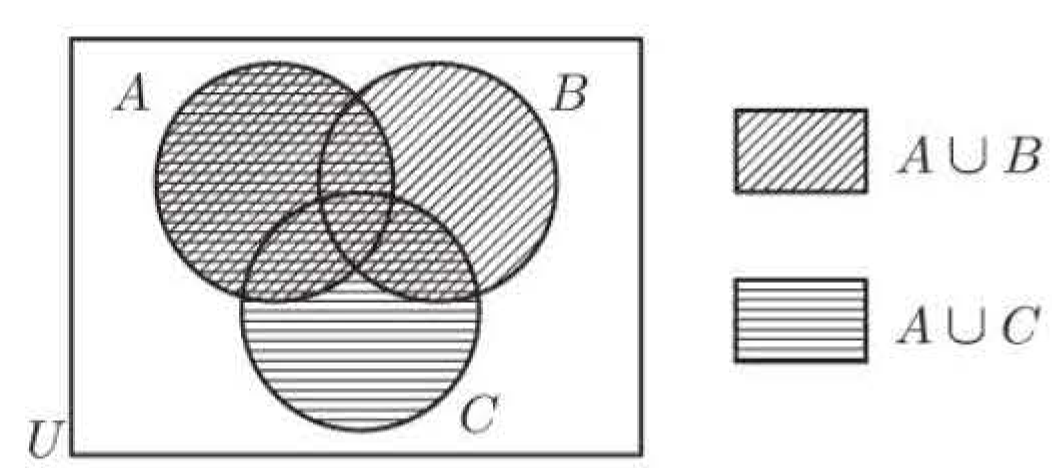


So $A' \cup B'$ is the region containing either type of shading.

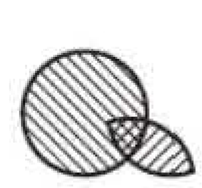
Thus, as the regions are the same, $(A \cap B)' = A' \cup B'$ is verified.

b





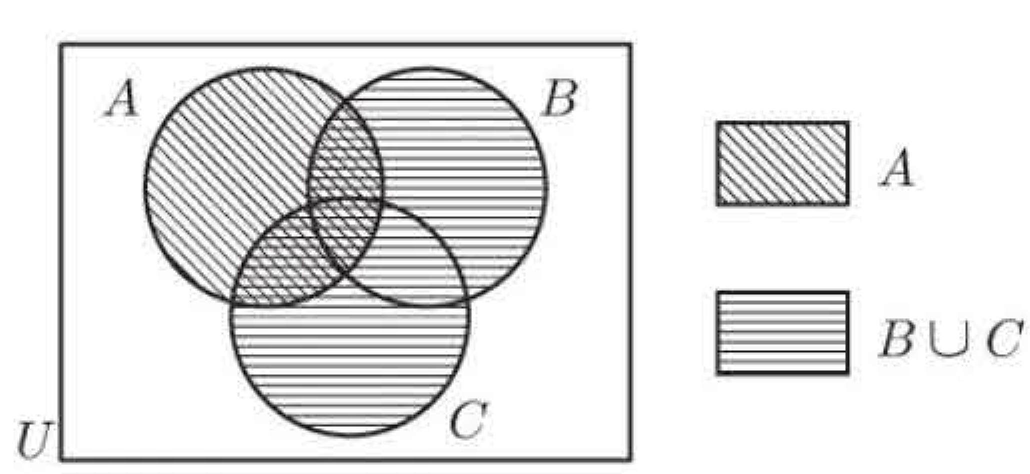
$A \cup (B \cap C)$ consists of the shaded region

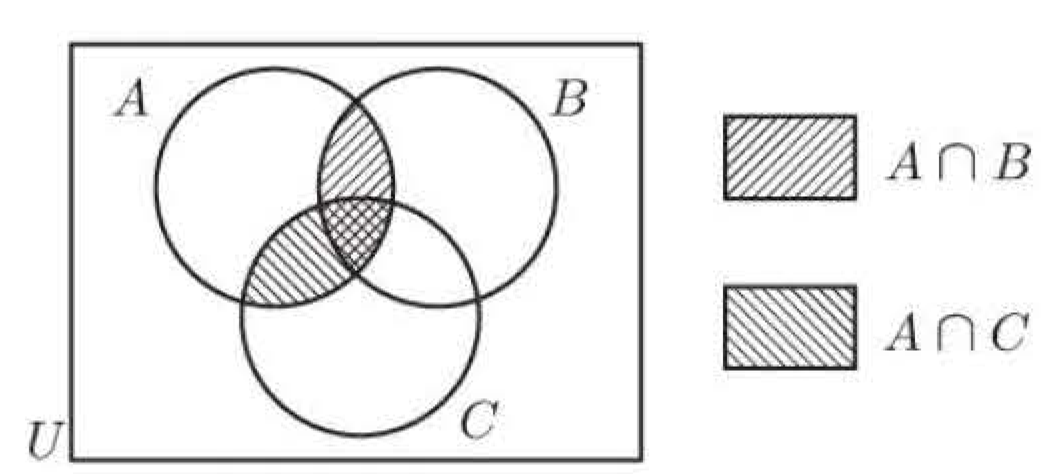


$(A \cup B) \cap (A \cup C)$ consists of the 'double shaded' region.

As the two regions are identical
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ is verified.

c





$A \cap (B \cup C)$ consists of the double shaded region

$(A \cap B) \cup (A \cap C)$ consists of the region shaded. (all forms and)

As the regions are identical, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ is verified.

2 **a** $A = \{7, 14, 21, 28, 35, ..., 98\}$
$B = \{5, 10, 15, 20, 25, ..., 95\}$

i as $98 = 7 \times 14$, $n(A) = 14$

ii as $95 = 5 \times 19$, $n(B) = 19$

iii $A \cap B = \{35, 70\}$ $\therefore$ $n(A \cap B) = 2$

iv $A \cup B = \{5, 7, 10, 14, 15, 20, 21, 25, 28, 30, 35, 40, 42, 45, 49, 50, 55, 56, 60, 63, 65, 70, 75, 77, 80, 84, 85, 90, 91, 95, 98\}$ $\therefore$ $n(A \cup B) = 31$

b $n(A) + n(B) - n(A \cap B)$
$= 14 + 19 - 2$
$= 31$
$= n(A \cup B)$ ✓

c From the diagram, $n(A) + n(B) - n(A \cap B)$
$= (a + b) + (b + c) - b$
$= a + b + c$
$= n(A \cup B)$

3 **a** **i** $\text{P}(B)$
$= \dfrac{n(B)}{n(U)}$
$= \dfrac{b + c}{a + b + c + d}$

ii $\text{P}(A \cap B)$
$= \dfrac{n(A \cap B)}{n(U)}$
$= \dfrac{b}{a + b + c + d}$

iii $\text{P}(A \cup B)$
$= \dfrac{n(A \cup B)}{n(U)}$
$= \dfrac{a + b + c}{a + b + c + d}$

iv $\text{P}(A) + \text{P}(B) - \text{P}(A \cap B) = \dfrac{a + b + b + c - b}{a + b + c + d}$
$= \dfrac{a + b + c}{a + b + c + d}$

b $\text{P}(A \cup B) = \text{P}(A) + \text{P}(B) - \text{P}(A \cap B)$ {using **iii** and **iv**}

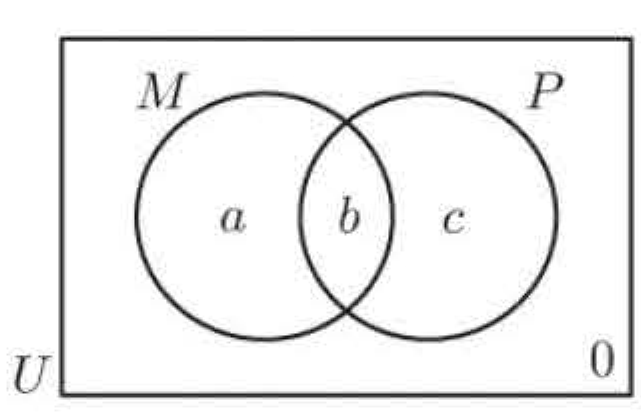
EXERCISE 24I

1 $\text{P}(A \cup B) = \text{P}(A) + \text{P}(B) - \text{P}(A \cap B)$
$\therefore$ $0.9 = 0.4 + \text{P}(B) - 0.1$
$\therefore$ $\text{P}(B) = 0.6$

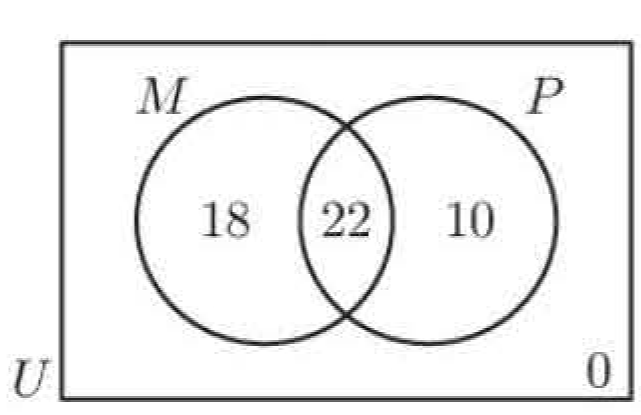
2 $\text{P}(X \cup Y) = \text{P}(X) + \text{P}(Y) - \text{P}(X \cap Y)$
$\therefore$ $0.9 = 0.6 + 0.5 - \text{P}(X \cap Y)$
$\therefore$ $\text{P}(X \cap Y) = 0.2$

3 $\text{P}(A \cup B) = \text{P}(A) + \text{P}(B)$ {A and B are mutually exclusive}
$\therefore$ $0.8 = \text{P}(A) + 0.45$
$\therefore$ $\text{P}(A) = 0.35$

4 **a**



$a + b + c = 50$ $\therefore$ $a + 32 = 50$, $\therefore$ $a = 18$
$a + b = 40$ $\therefore$ $18 + b = 40$, $\therefore$ $b = 22$
$b + c = 32$ $\therefore$ $22 + c = 32$, $\therefore$ $c = 18$

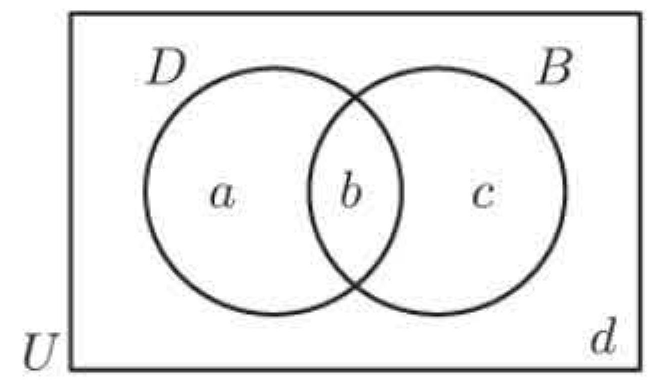


So 22 study both.

b **i** $\text{P}(M \text{ but not } P)$
$= \frac{18}{50}$
$= \frac{9}{25}$

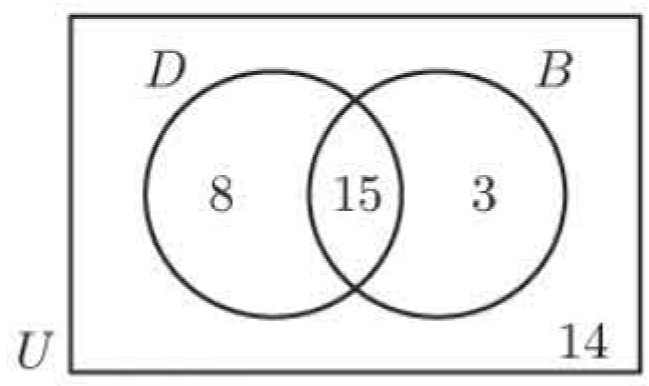
ii $\text{P}(P \text{ given } M)$
$= \dfrac{22}{18 + 22}$
$= \frac{22}{40}$
$= \frac{11}{20}$

5



$$\begin{aligned} a+b+c+d &= 40 \quad .... (1) \\ a+b &= 23 \quad .... (2) \\ b+c &= 18 \quad .... (3) \\ a+b+c &= 26 \quad .... (4) \end{aligned}$$

$\therefore\ d = 14$ {using (1) and (4)}
$23 + c = 26$ and $a + 18 = 26$
$\therefore\ c = 3$ and $a = 8$
Thus $b = 18 - c = 15$



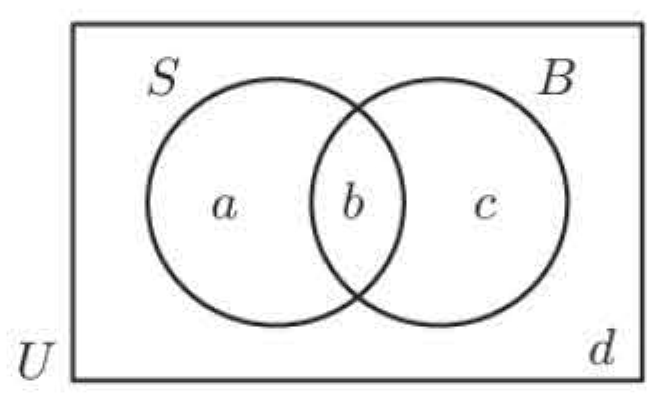
a P(D and B)
$= \frac{15}{40}$
$= \frac{3}{8}$

b P(neither D nor B)
$= \frac{14}{40}$
$= \frac{7}{20}$

c P(D, but not B)
$= \frac{8}{40}$
$= \frac{1}{5}$

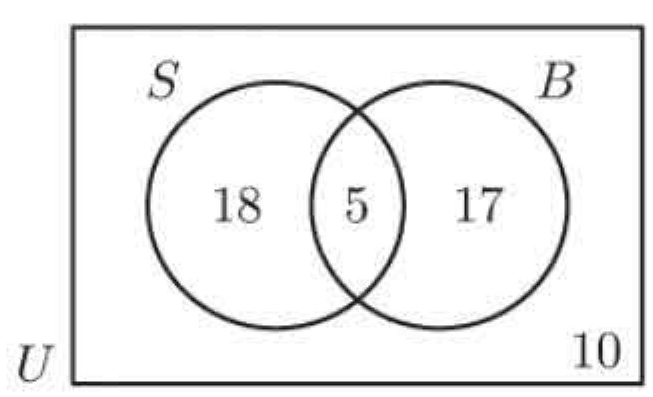
d P(B given D)
$= \frac{15}{23}$

6



$$\begin{aligned} a+b+c+d &= 50 \\ a+b &= 23 \\ b+c &= 22 \\ b &= 5 \end{aligned}$$

$\therefore\ c = 17,\ a = 18$
and $18 + 5 + 17 + d = 50$
$\therefore\ d = 10$



a P(not B)
$= \text{P}(B')$
$= \frac{28}{50}$
$= \frac{14}{25}$

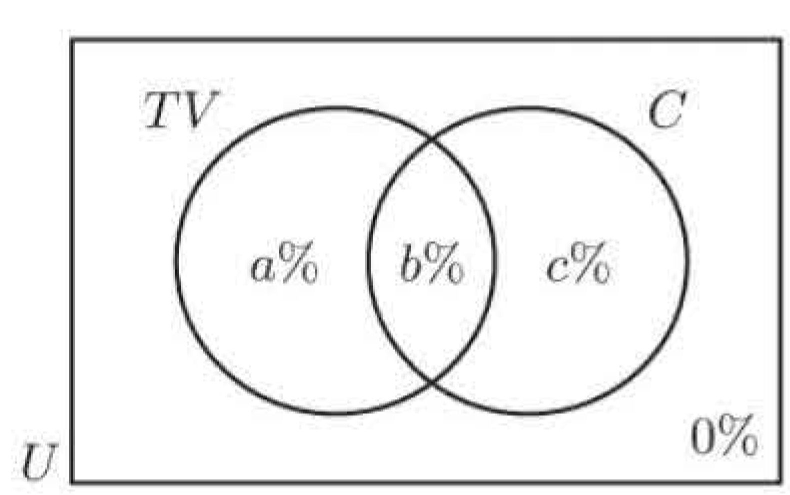
b P(B or S)
$= \dfrac{18+5+17}{50}$
$= \frac{40}{50}$
$= \frac{4}{5}$

c P(neither B nor S)
$= \frac{10}{50}$
$= \frac{1}{5}$

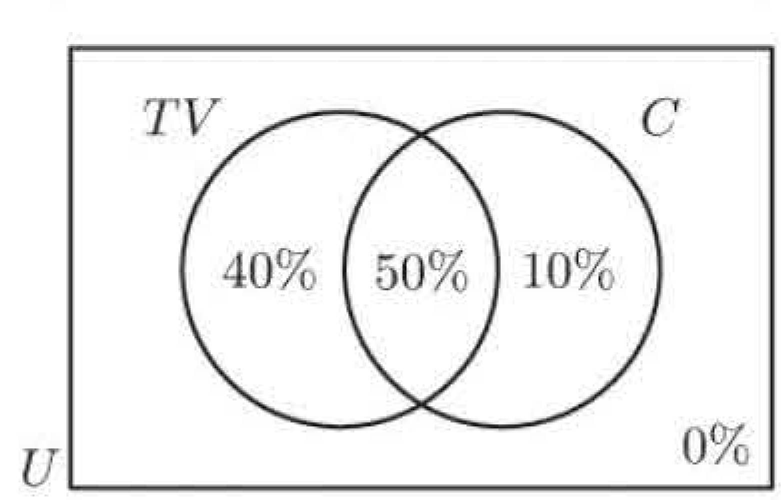
d P(B, given S)
$= \dfrac{5}{18+5}$
$= \frac{5}{23}$

e P(S, given B')
$= \dfrac{18}{18+10}$
$= \frac{18}{28}$
$= \frac{9}{14}$

7

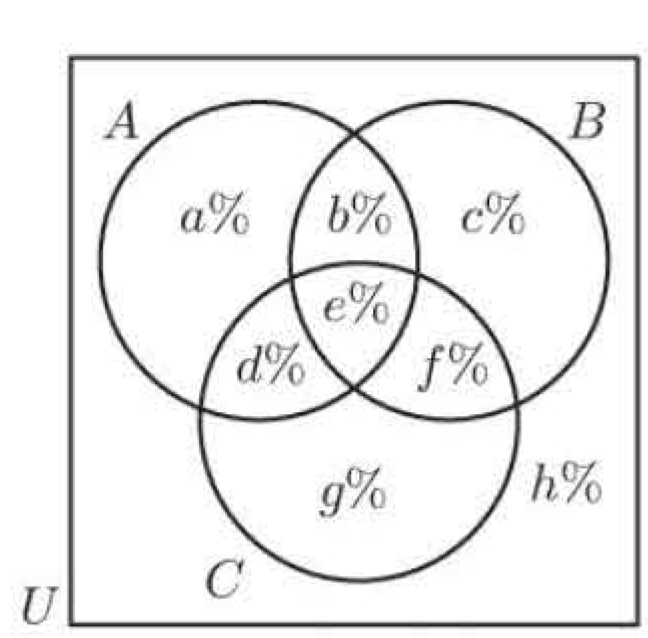


$$\begin{cases} a+b+c = 100 \\ a+b = 90 \quad \therefore\ c = 10 \text{ and } a = 40 \\ b+c = 60 \quad \therefore\ b = 50 \end{cases}$$

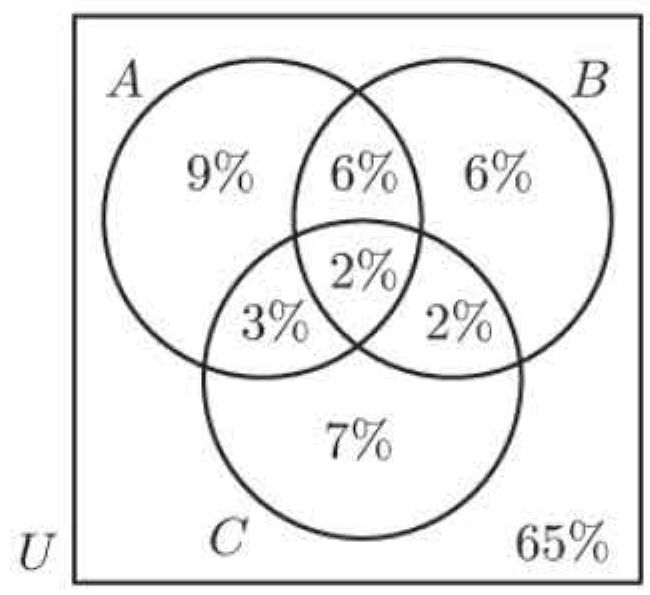


$$\text{P}(TV\text{, given } C) = \frac{50}{50+10} = \frac{5}{6}$$

8



$$\begin{aligned} a+b+c+d+e+f+g+h &= 100 \\ a+b+d+e &= 20 \\ b+c+e+f &= 16 \\ d+e+f+g &= 14 \\ b+e &= 8 \\ d+e &= 5 \\ e+f &= 4 \\ e &= 2 \end{aligned}$$



$\therefore\ e=2,\ f=2,\ d=3,\ b=6,\quad \begin{cases} a+6+3+2=20 \\ 6+c+2+2=16 \\ 3+2+2+g=14 \end{cases} \quad \therefore\ \begin{cases} a=9 \\ c=6 \\ g=7 \end{cases}$

a P(none) $= \frac{65}{100}$ $= \frac{13}{20}$

b P(at least one) $= 1 - \text{P(none)}$ $= 1 - \frac{13}{20}$ $= \frac{7}{20}$

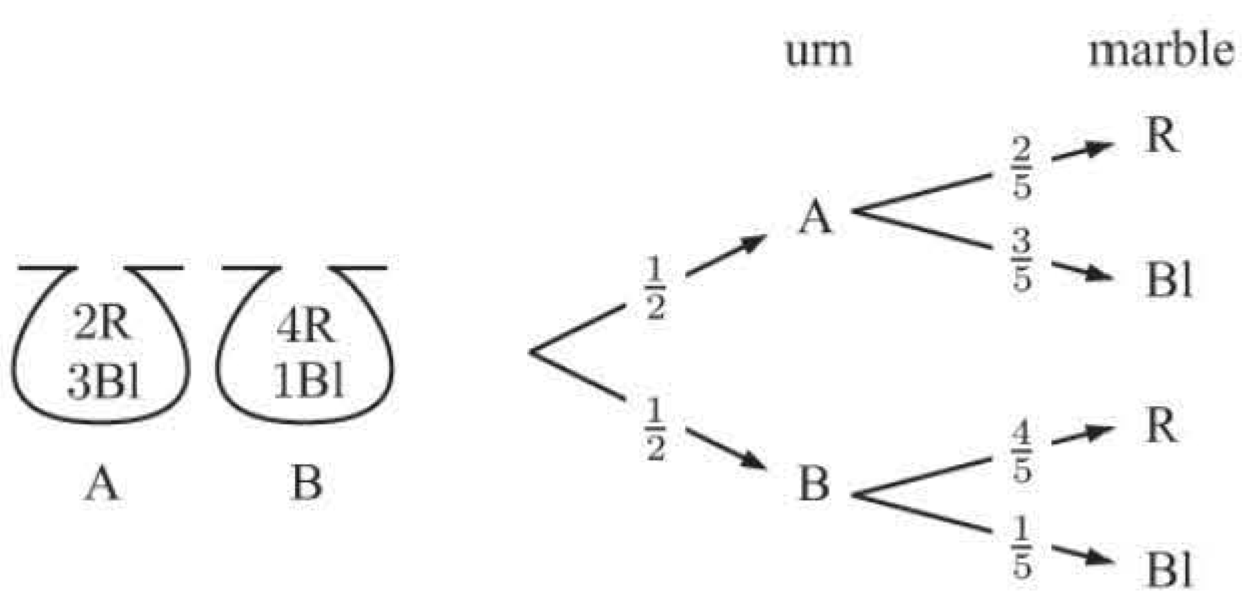
c P(exactly one) $= \dfrac{9+6+7}{100}$ $= \frac{22}{100}$ $= \frac{11}{50}$

d P(A or B) $= \frac{9+6+6+3+2+2}{100}$ $= \frac{28}{100}$ $= \frac{7}{25}$

e P(A, given at least one) $= \frac{9+6+2+3}{35}$ $= \frac{20}{35}$ $= \frac{4}{7}$

f P(C, given A or B or both) $= \frac{3+2+2}{9+6+6+3+2+2}$ $= \frac{7}{28}$ $= \frac{1}{4}$

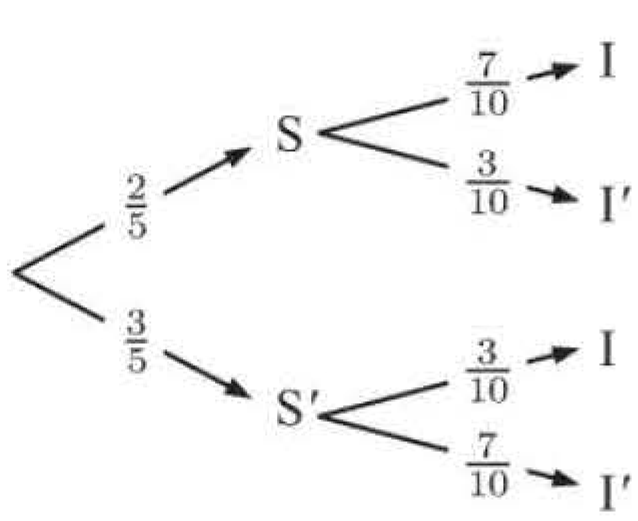
9



a $\text{P(R)} = \frac{1}{2} \times \frac{2}{5} + \frac{1}{2} \times \frac{4}{5}$
$= \frac{3}{5}$

b $\text{P(B} \mid \text{R)} = \dfrac{\text{P(B} \cap \text{R)}}{\text{P(R)}}$
$= \dfrac{\frac{1}{2} \times \frac{4}{5}}{\frac{3}{5}}$
$= \frac{2}{3}$

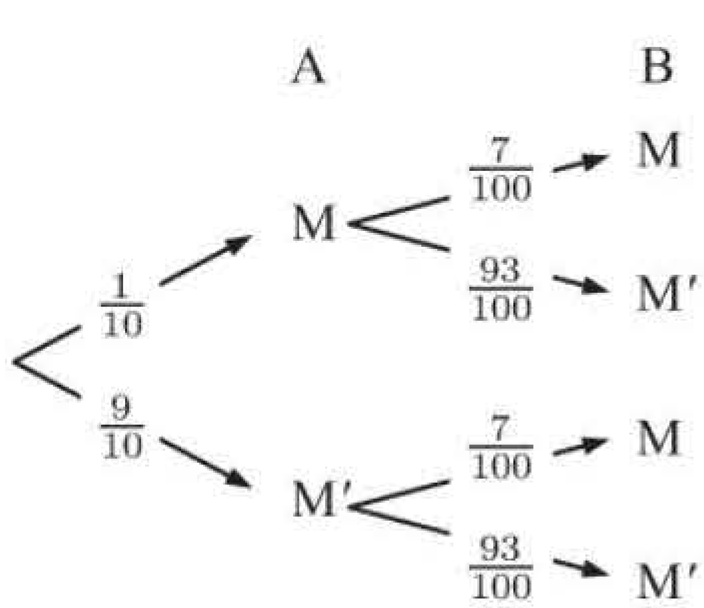
10



a $\text{P(I)} = \frac{2}{5} \times \frac{7}{10} + \frac{3}{5} \times \frac{3}{10}$
$= \frac{23}{50}$ (or 0.46)

b $\text{P(S} \mid \text{I)} = \dfrac{\text{P(S} \cap \text{I)}}{\text{P(I)}}$
$= \dfrac{\frac{2}{5} \times \frac{7}{10}}{\frac{23}{50}}$
$= \frac{14}{23}$

11



P(B | at least one malfunctions)

$= \dfrac{\text{P(B} \cap \text{at least one malfunctions)}}{\text{P(at least one malfunctions)}}$

$= \dfrac{\frac{1}{10} \times \frac{7}{100} + \frac{9}{10} \times \frac{7}{100}}{\frac{1}{10} \times \frac{7}{100} + \frac{1}{10} \times \frac{93}{100} + \frac{9}{10} \times \frac{7}{100}}$

$= \frac{7+63}{7+93+63}$

$= \frac{70}{163}$

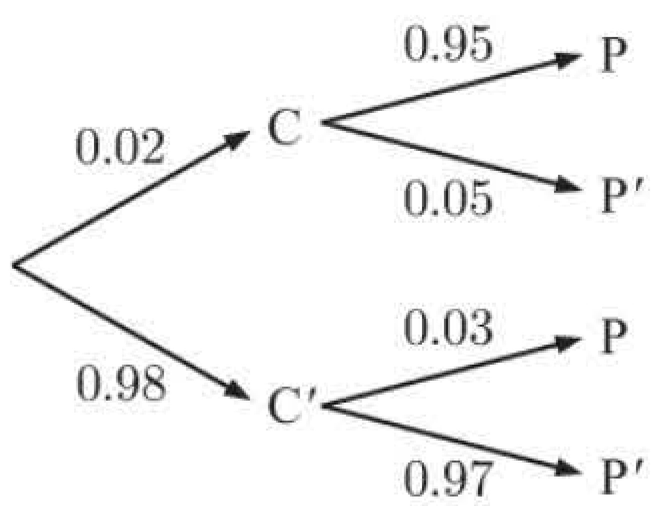
12 $\text{P}(B) = 0.5$, $\text{P}(G) = 0.6$, $\text{P}(G \mid B) = 0.9$, where B is "the boy eats his lunch" and G is "the girl eats her lunch"

a P(both eat lunch)
$= \text{P}(B \cap G)$
$= \text{P}(G \mid B) \times \text{P}(B) \qquad \left\{\text{as } \text{P}(G \mid B) = \dfrac{\text{P}(G \cap B)}{\text{P}(B)}\right\}$
$= 0.9 \times 0.5$
$= 0.45$

b $\text{P}(B \mid G)$
$= \dfrac{\text{P}(B \cap G)}{\text{P}(G)}$
$= \dfrac{0.45}{0.6}$
$= 0.75$

c $\quad \text{P(at least one eats lunch)}$
$= \mathrm{P}(B \cup G)$
$= \mathrm{P}(B) + \mathrm{P}(G) - \mathrm{P}(B \cap G)$
$= 0.5 + 0.6 - 0.45$
$= 0.65$

13



a $\quad \mathrm{P(P)}$
$= 0.02 \times 0.95 + 0.98 \times 0.03$
$= 0.0484$

b $\quad \mathrm{P(C \mid P)}$
$= \dfrac{\mathrm{P(C \cap P)}}{\mathrm{P(P)}}$
$= \dfrac{0.02 \times 0.95}{0.0484}$
≈ 0.393

14 The coins are H, H T, T and H, T.

Any one of these 6 faces could be seen uppermost, $\therefore$ P(falls H) $= \frac{3}{6} = \frac{1}{2}$

$$\text{Now} \quad \mathrm{P(HH\ coin \mid falls\ H)} = \frac{\mathrm{P(HH\ coin \cap falls\ H)}}{\mathrm{P(falls\ H)}} = \frac{\mathrm{P(HH)}}{\mathrm{P(falls\ H)}} = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$

EXERCISE 24J

1 $\quad \mathrm{P}(R \cap S)$
$= \mathrm{P}(R) + \mathrm{P}(S) - \mathrm{P}(R \cup S)$
$= 0.4 + 0.5 - 0.7$
$= 0.2$

Also, $\quad \mathrm{P}(R) \times \mathrm{P}(S)$
$= 0.4 \times 0.5$
$= 0.2$

So, $\mathrm{P}(R \cap S) = \mathrm{P}(R) \times \mathrm{P}(S)$ and hence R and S are independent events.

2 **a** $\quad \mathrm{P}(A \cap B)$
$= \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cup B)$
$= \frac{2}{5} + \frac{1}{3} - \frac{1}{2}$
$= \frac{7}{30}$

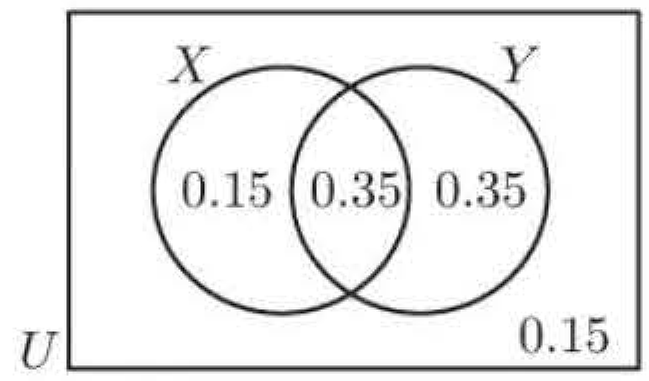
b $\quad \mathrm{P}(B \mid A)$
$= \dfrac{\mathrm{P}(B \cap A)}{\mathrm{P}(A)}$
$= \dfrac{\frac{7}{30}}{\frac{2}{5}}$
$= \frac{7}{12}$

c $\quad \mathrm{P}(A \mid B)$
$= \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$
$= \dfrac{\frac{7}{30}}{\frac{1}{3}}$
$= \frac{7}{10}$

A and B are not independent as $\mathrm{P}(A \mid B) \neq \mathrm{P}(A)$.

3 **a** As X and Y are independent
$\mathrm{P}(X \cap Y) = \mathrm{P}(X) \times \mathrm{P}(Y)$
$= 0.5 \times 0.7$
$= 0.35$
$\therefore$ P(both X and Y) $= 0.35$

b $\quad \mathrm{P}(X \text{ or } Y)$
$= \mathrm{P}(X \cup Y)$
$= \mathrm{P}(X) + \mathrm{P}(Y) - \mathrm{P}(X \cap Y)$
$= 0.5 + 0.7 - 0.35$
$= 0.85$



c $\quad$ P(neither X nor Y)
$= 0.15$

d $\quad$ P(X but not Y)
$= 0.15$

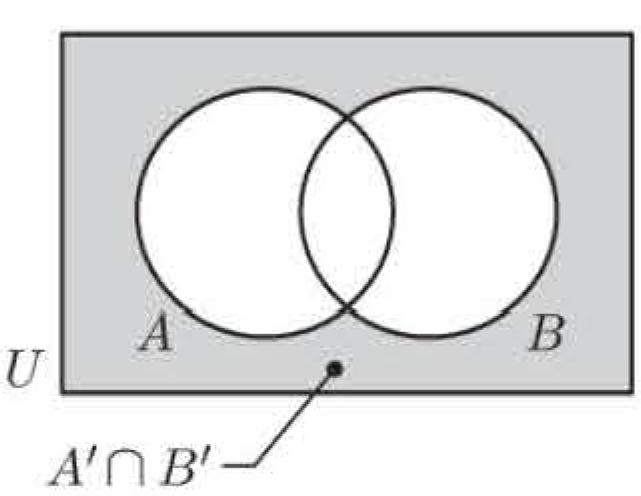
e $\mathrm{P}(X \mid Y) = \dfrac{\mathrm{P}(X \cap Y)}{\mathrm{P}(Y)} = \dfrac{0.35}{0.70} = 0.5$

4 $\quad$ P(at least one solves it)
$= 1 - \text{P(no-one solves it)}$
$= 1 - \text{P(A}' \text{ and B}' \text{ and C}')$
$= 1 - \frac{2}{5} \times \frac{1}{3} \times \frac{1}{2}$ {each student's ability to solve the problem is independent}
$= 1 - \frac{1}{15}$
$= \frac{14}{15}$

5 **a** $\quad$ P(at least one 6)
$= 1 - \text{P(no 6s)}$
$= 1 - \text{P}(6' \text{ and } 6' \text{ and } 6')$
$= 1 - \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6}$
$= 1 - \frac{125}{216}$
$= \frac{91}{216}$

b $\quad$ P(at least one 6 in n throws)
$= 1 - (\frac{5}{6})^n$
So we want $1 - (\frac{5}{6})^n > 0.99$
$\therefore\ -(\frac{5}{6})^n > -0.01$
$\therefore\ (\frac{5}{6})^n < 0.01$
$\therefore\ n \log(\frac{5}{6}) < \log(0.01)$
$\therefore\ n > \dfrac{\log(0.01)}{\log(\frac{5}{6})}$ {as $\log(\frac{5}{6}) < 0$}
$\therefore\ n > 25.2585$
$\therefore\ n = 26$

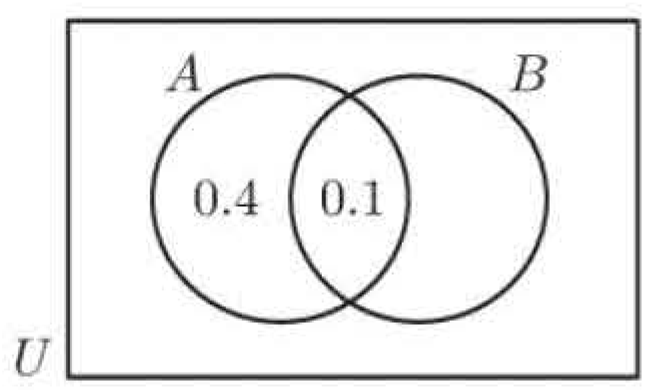
6 A and B are independent, so $\text{P}(A \cap B) = \text{P}(A)\,\text{P}(B)$ .... (1)



Now $\quad \text{P}(A' \cap B')$
$= 1 - \text{P}(A \cup B)$
$= 1 - [\text{P}(A) + \text{P}(B) - \text{P}(A \cap B)]$
$= 1 - \text{P}(A) - \text{P}(B) + \text{P}(A \cap B)$
$= 1 - \text{P}(A) - \text{P}(B) + \text{P}(A)\,\text{P}(B)$ {using (1)}
$= [1 - \text{P}(A)]\,[1 - \text{P}(B)]$
$= \text{P}(A')\,\text{P}(B')$

$\therefore$ A' and B' are also independent.

7



$\therefore\ \text{P}(A) = 0.5$
and $\quad \text{P}(A \cap B) = \text{P}(A) \times \text{P}(B)$ {A and B are independent}
$\therefore\ 0.1 = 0.5 \times \text{P}(B)$
$\therefore\ \text{P}(B) = 0.2$
Now $\quad \text{P}(A \cup B') = \text{P}(A) + \text{P}(B') - \text{P}(A \cap B')$
$= 0.5 + 0.8 - 0.4$
$= 0.9$

8 **a** **i** $\text{P}(C \mid D) = \dfrac{\text{P}(C \cap D)}{\text{P}(D)}$, so $\text{P}(C \cap D) = \text{P}(C \mid D)\,\text{P}(D)$
Similarly, $\text{P}(C \cap D') = \text{P}(C \mid D')\,\text{P}(D')$
Now $\text{P}(C \cap D) + \text{P}(C \cap D') = \text{P}(C)$
$\therefore\ \text{P}(C \mid D)\,\text{P}(D) + \text{P}(C \mid D')\,\text{P}(D') = \text{P}(C)$
$\therefore\ \frac{6}{13}\,\text{P}(D) + \frac{3}{7}\,[1 - \text{P}(D)] = \frac{9}{20}$
$\therefore\ \frac{6}{13}\,\text{P}(D) + \frac{3}{7} - \frac{3}{7}\,\text{P}(D) = \frac{9}{20}$
$\therefore\ \frac{3}{91}\,\text{P}(D) = \frac{3}{140}$
$\therefore\ \text{P}(D) = \frac{91}{140} = \frac{13}{20}$

ii $P(C \cap D) = P(C \mid D)\, P(D) = \frac{6}{13} \times \frac{13}{20} = \frac{3}{10}$

Now $P(C' \cup D') = 1 - P(C \cap D) = 1 - \frac{3}{10} = \frac{7}{10}$

b $P(C \mid D) = \frac{6}{13}$ and $P(C) = \frac{9}{20}$

$\therefore$ C and D are not independent as $P(C \mid D) \neq P(C)$

or

$P(C \cap D) = \frac{3}{10}$ and $P(C)\, P(D) = \frac{9}{20} \times \frac{13}{20} = \frac{117}{400}$

$\therefore$ C and D are not independent as $P(C \cap D) \neq P(C)\, P(D)$

9 **a** **i** P(Ruba wins on her third turn) = P(4 non-aces, then an ace)

$$= \left(\tfrac{12}{13}\right)^4 \times \left(\tfrac{1}{13}\right) \approx 0.0558$$

ii P(Ruba wins on her nth turn) = P($2(n-1)$ non-aces, then an ace)

$$= \left(\tfrac{12}{13}\right)^{2(n-1)} \times \tfrac{1}{13}$$

$\therefore$ P(Ruba wins prior to her $(n+1)$th turn)

= P(Ruba wins on her 1st or 2nd or 3rd or .... or nth turn)

$$= \left(\tfrac{12}{13}\right)^0 \times \tfrac{1}{13} + \left(\tfrac{12}{13}\right)^2 \times \tfrac{1}{13} + \left(\tfrac{12}{13}\right)^4 \times \tfrac{1}{13} + \dots + \left(\tfrac{12}{13}\right)^{2(n-1)} \times \tfrac{1}{13}$$

$$= \tfrac{1}{13}\Bigg(\underbrace{1 + \left(\tfrac{12}{13}\right)^2 + \left(\tfrac{12}{13}\right)^4 + \dots + \left(\tfrac{12}{13}\right)^{2(n-1)}}_{\text{geometric series with } u_1 = 1,\ r = \left(\frac{12}{13}\right)^2,\ \text{“}n\text{”} = n}\Bigg)$$

$$= \tfrac{1}{13}\left(\frac{1 - \left(\frac{12}{13}\right)^{2n}}{1 - \left(\frac{12}{13}\right)^2}\right) = \tfrac{1}{13}\left(\frac{1 - \left(\frac{12}{13}\right)^{2n}}{\frac{25}{169}}\right)$$

$$= \frac{169}{13 \times 25}\left(1 - \left(\tfrac{12}{13}\right)^{2n}\right)$$

$$= \tfrac{13}{25}\left(1 - \left(\tfrac{12}{13}\right)^{2n}\right)$$

iii As $n \to \infty$, $1 - \left(\frac{12}{13}\right)^{2n} \to 1$

$\therefore$ $\frac{13}{25}\left(1 - \left(\frac{12}{13}\right)^{2n}\right) \to \frac{13}{25}$

$\therefore$ P(Ruba wins the game) $= \frac{13}{25}$

b Let X be the number of times Ruba wins the game.

Then X is binomial with $n = 7$ trials of probability $p = \frac{13}{25}$.

$\therefore$ P(Ruba will win more games than Hania) $= P(X \geqslant 4)$

$$= 1 - P(X \leqslant 3)$$
$$\approx 1 - 0.456$$
$$\approx 0.544$$

10 The man will step over the cliff in his first four steps if either:

(1) he steps towards the cliff on his first step

(2) he steps away from the cliff on his first step, but towards the cliff on his next two steps.

P(case (1)) $= \frac{2}{5}$

P(case (2)) $= \frac{3}{5} \times \left(\frac{2}{5}\right)^2 = \frac{12}{125}$

$\therefore$ P(the man steps over the cliff in his first four steps) $= \frac{2}{5} + \frac{12}{125} = \frac{62}{125}$

$\therefore$ P(the man does **not** step over the cliff in his first four steps) $= 1 - \frac{62}{125}$

$$= \frac{63}{125}$$

EXERCISE 24K

1 The total number of different committees is $\binom{11}{4}$.

The number of ways of both sisters being on a committee with any 2 others is $\binom{2}{2} \times \binom{9}{2}$.

$\therefore$ P(both sisters are on the committee) $= \dfrac{\binom{2}{2} \times \binom{9}{2}}{\binom{11}{4}} = \frac{6}{55}$

2 AIDS and SAID are 2 of the 4! different orderings. $\therefore$ P(AIDS or SAID) $= \dfrac{2}{4!} = \frac{1}{12}$

3 There are $\binom{12}{7}$ different teams that can be selected.

$\therefore$ P(captain and vice captain are chosen) $= \dfrac{\binom{2}{2} \times \binom{10}{5}}{\binom{12}{7}} \approx 0.318$

4 P(none of the golfers was killed) $= \dfrac{\binom{3}{0} \times \binom{19}{4}}{\binom{22}{4}} \approx 0.530$

5

5	4	3	2	1

$\therefore$ there are 5! different possible seating arrangements.

a

2	3	2	1	1

There are $2 \times 3!$ seating arrangements if K and J sit at the ends

$\therefore$ P(K and J sit at the ends) $= \dfrac{2 \times 3!}{5!} = \frac{1}{10}$

b K and J can sit together in 2! ways. They as a pair plus the other three people can then be ordered in 4! ways.

$\therefore$ P(sit together) $= \dfrac{2! \times 4!}{5!} = \frac{2}{5}$

6 There are $\binom{16}{5}$ different committees possible.

a P(all men)

$= \dfrac{\binom{9}{5} \times \binom{7}{0}}{\binom{16}{5}}$

$= 0.0288$

b P(at least 3 men)

$=$ P(3 men or 4 men or 5 men)

$= \dfrac{\binom{9}{3}\binom{7}{2} + \binom{9}{4}\binom{7}{1} + \binom{9}{5}\binom{7}{0}}{\binom{16}{5}}$

≈ 0.635

c P(at least one of each sex)

$= 1 -$ P(no men or no women)

$= 1 - \dfrac{\binom{9}{0}\binom{7}{5} + \binom{9}{5}\binom{7}{0}}{\binom{16}{5}}$

≈ 0.966

7 If there are no restrictions there are 6! different orderings possible. A, B, and C can be ordered in 3! ways. This triple together with the 3 others can be ordered in 4! ways.

$\therefore$ P(A, B, C together) $= \dfrac{3! \times 4!}{6!} = \frac{1}{5}$

8 There are $\binom{14}{7}$ different committees possible.

a P(only senior students) $= \dfrac{\binom{11}{7}\binom{3}{0}}{\binom{14}{7}}$

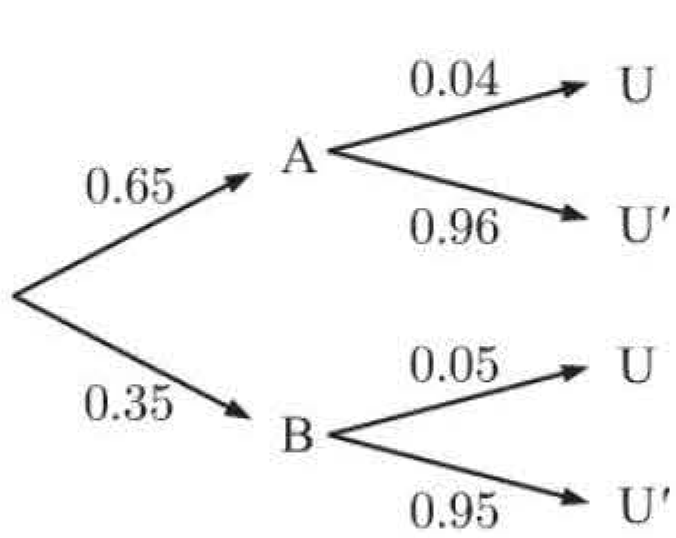
≈ 0.0962

b P(all three junior students chosen) $= \dfrac{\binom{11}{4}\binom{3}{3}}{\binom{14}{7}}$

≈ 0.0962

EXERCISE 24L

1



a P(underfilled)

$=$ P(A and U or B and U)

$= 0.65 \times 0.04 + 0.35 \times 0.05$

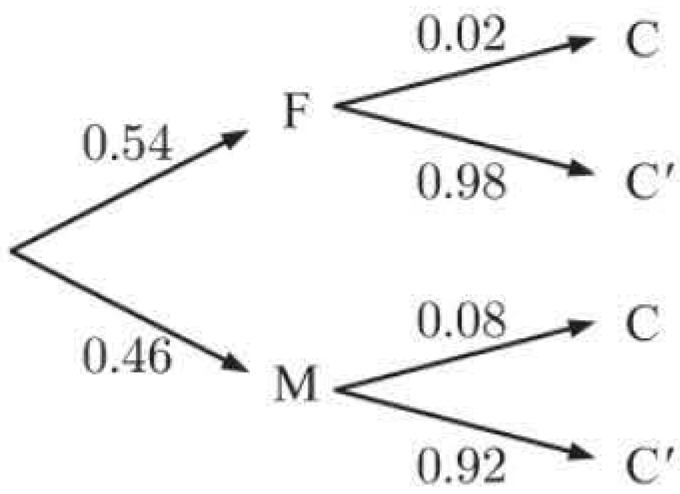
$= 0.0435$

b $\text{P}(A \mid U) = \dfrac{\text{P}(U \mid A)\,\text{P}(A)}{\text{P}(U)}$

$= \dfrac{0.04 \times 0.65}{0.0435}$

≈ 0.598

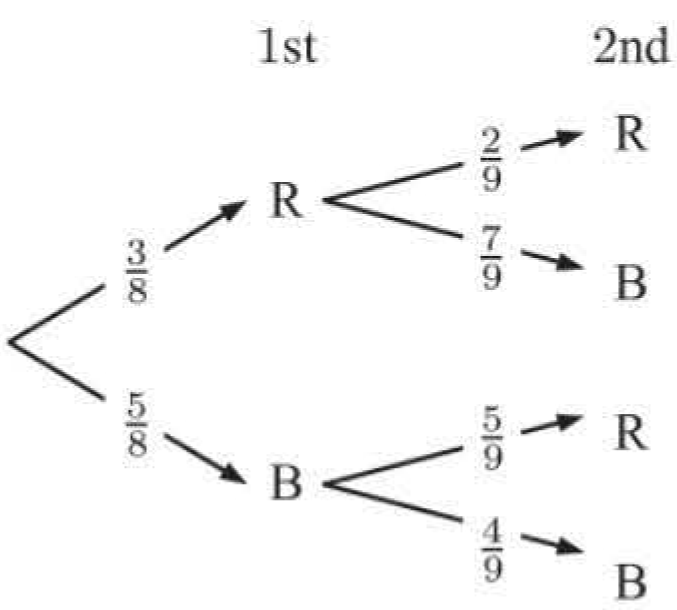
2



a $$\mathrm{P}(\mathrm{M} \mid \mathrm{C}) = \frac{\mathrm{P}(\mathrm{C} \mid \mathrm{M}) \times \mathrm{P}(\mathrm{M})}{\mathrm{P}(\mathrm{C} \mid \mathrm{M}) \times \mathrm{P}(\mathrm{M}) + \mathrm{P}(\mathrm{C} \mid \mathrm{F}) \times \mathrm{P}(\mathrm{F})} = \frac{0.08 \times 0.46}{0.08 \times 0.46 + 0.02 \times 0.54} \approx 0.773$$

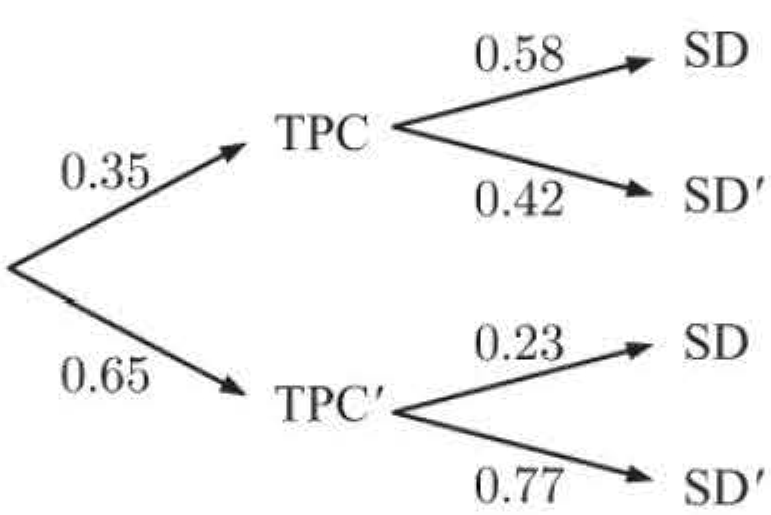
b $$\mathrm{P}(\mathrm{F} \mid \mathrm{C}') = \frac{\mathrm{P}(\mathrm{C}' \mid \mathrm{F}) \times \mathrm{P}(\mathrm{F})}{\mathrm{P}(\mathrm{C}' \mid \mathrm{F}) \times \mathrm{P}(\mathrm{F}) + \mathrm{P}(\mathrm{C}' \mid \mathrm{M}) \times \mathrm{P}(\mathrm{M})} = \frac{0.98 \times 0.54}{0.98 \times 0.54 + 0.92 \times 0.46} \approx 0.556$$

3



$$\mathrm{P}(\mathrm{BB} \mid \mathrm{RR \text{ or } BB}) = \frac{\mathrm{P}(\mathrm{BB} \cap (\mathrm{RR \text{ or } BB}))}{\mathrm{P}(\mathrm{RR \text{ or } BB})} = \frac{\mathrm{P}(\mathrm{BB})}{\mathrm{P}(\mathrm{RR \text{ or } BB})} = \frac{\frac{5}{8} \times \frac{4}{9}}{\frac{3}{8} \times \frac{2}{9} + \frac{5}{8} \times \frac{4}{9}} = \frac{20}{26} = \frac{10}{13}$$

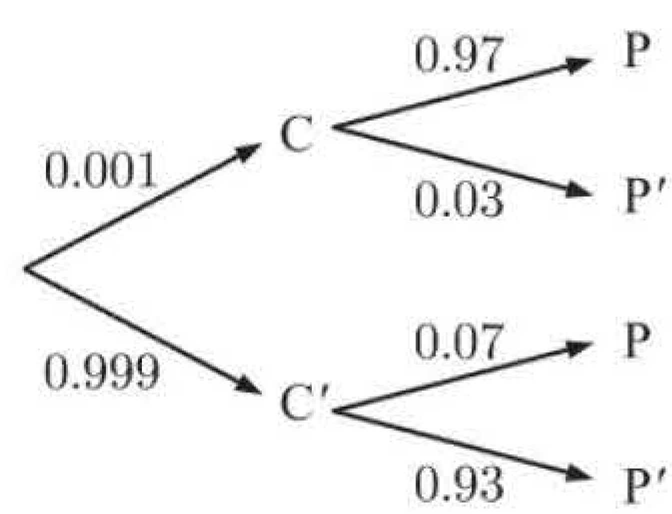
4



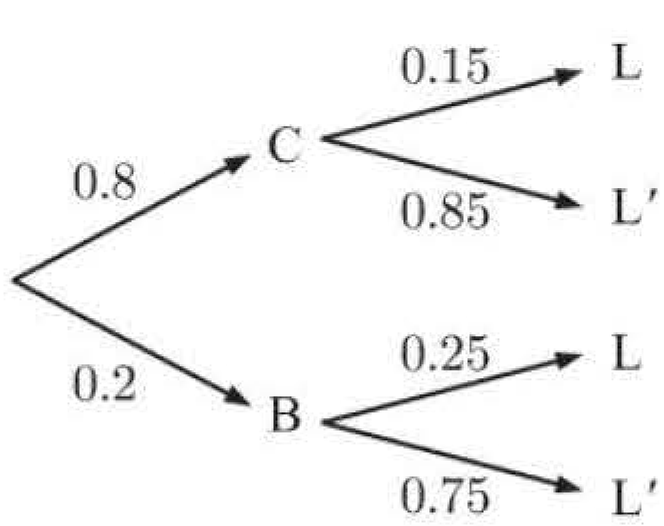
$$\mathrm{P}(\mathrm{TPC}' \mid \mathrm{SD}) = \frac{\mathrm{P}(\mathrm{TPC}' \cap \mathrm{SD})}{\mathrm{P}(\mathrm{SD})} = \frac{0.65 \times 0.23}{0.35 \times 0.58 + 0.65 \times 0.23} \approx 0.424$$

5

$$\mathrm{P}(\mathrm{C} \mid \mathrm{P}) = \frac{\mathrm{P}(\mathrm{P} \mid \mathrm{C}) \times \mathrm{P}(\mathrm{C})}{\mathrm{P}(\mathrm{P} \mid \mathrm{C}) \times \mathrm{P}(\mathrm{C}) + \mathrm{P}(\mathrm{P} \mid \mathrm{C}') \times \mathrm{P}(\mathrm{C}')} = \frac{0.97 \times 0.001}{0.97 \times 0.001 + 0.07 \times 0.999} \approx 0.0137$$

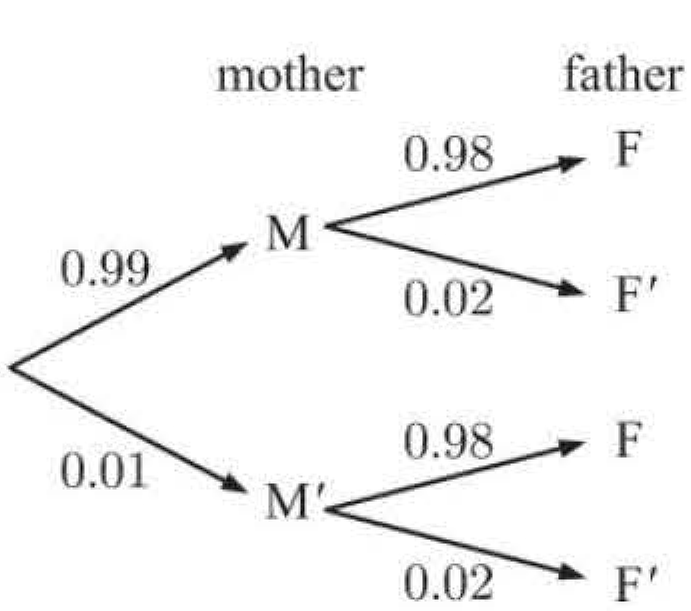


6



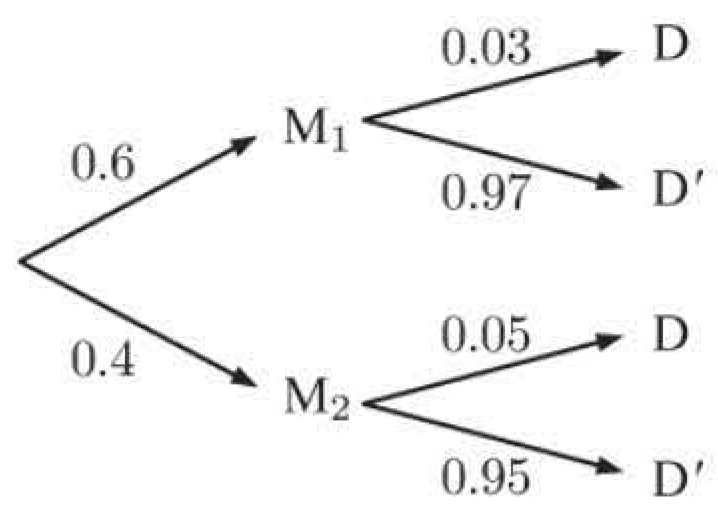
$$\mathrm{P}(\mathrm{B} \mid \mathrm{L}') = \frac{\mathrm{P}(\mathrm{L}' \mid \mathrm{B}) \times \mathrm{P}(\mathrm{B})}{\mathrm{P}(\mathrm{L}' \mid \mathrm{B}) \times \mathrm{P}(\mathrm{B}) + \mathrm{P}(\mathrm{L}' \mid \mathrm{C}) \times \mathrm{P}(\mathrm{C})} = \frac{0.75 \times 0.2}{0.75 \times 0.2 + 0.85 \times 0.8} = \frac{15}{83} \approx 0.181$$

7



$$\mathrm{P}(\mathrm{M} \mid \text{only 1 alive}) = \frac{\mathrm{P}(\mathrm{M} \cap (\text{only 1 alive}))}{\mathrm{P}(\text{only 1 alive})} = \frac{\mathrm{P}(\mathrm{MF}')}{\mathrm{P}(\mathrm{MF}' \text{ or } \mathrm{M}'\mathrm{F})} = \frac{0.99 \times 0.02}{0.99 \times 0.02 + 0.01 \times 0.98} = \frac{99}{148} \approx 0.669$$

8



a $\text{P}(\text{M}_1 \mid \text{D}) = \dfrac{\text{P}(\text{D} \mid \text{M}_1) \times \text{P}(\text{M}_1)}{\text{P}(\text{D} \mid \text{M}_1) \times \text{P}(\text{M}_1) + \text{P}(\text{D} \mid \text{M}_2) \times \text{P}(\text{M}_2)}$

$= \dfrac{0.03 \times 0.6}{0.03 \times 0.6 + 0.05 \times 0.4}$

$= \frac{9}{19} \approx 0.474$

b $\text{P}(\text{M}_2 \mid \text{D}) = 1 - \frac{9}{19}$

$= \frac{10}{19}$

9 **a** $\text{P}(B) = \text{P}(B \text{ and in } A_1 \text{ or } B \text{ and in } A_2 \text{ or } B \text{ and in } A_3)$

$= \text{P}((B \cap A_1) \cup (B \cap A_2) \cup (B \cap A_3))$

$= \text{P}(B \cap A_1) + \text{P}(B \cap A_2) + \text{P}(B \cap A_3)$

as $B \cap A_1$, $B \cap A_2$, $B \cap A_3$ are disjoint

$= \text{P}(B \mid A_1)\ \text{P}(A_1) + \text{P}(B \mid A_2)\ \text{P}(A_2) + \text{P}(B \mid A_3)\ \text{P}(A_3)$

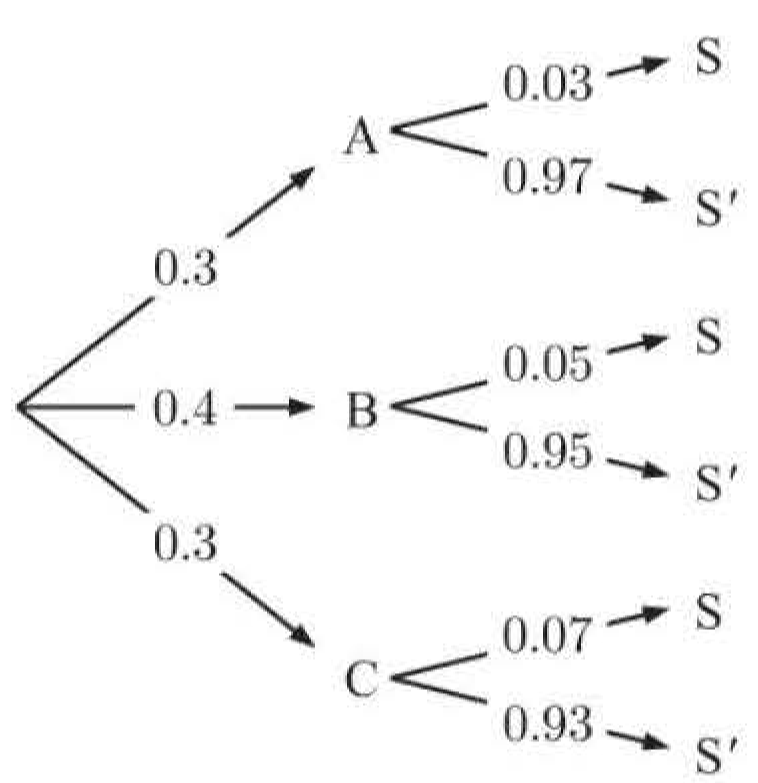
$\left\{\text{since } \text{P}(X \mid Y) = \dfrac{\text{P}(X \cap Y)}{\text{P}(Y)} \quad \Rightarrow \quad \text{P}(X \cap Y) = \text{P}(X \mid Y)\ \text{P}(Y)\right\}$

b $\text{P}(A_i \mid B) = \dfrac{\text{P}(A_i \cap B)}{\text{P}(B)} = \dfrac{\text{P}(B \mid A_i)\ \text{P}(A_i)}{\text{P}(B)}$

where $\text{P}(B) = \text{P}(B \mid A_1)\ \text{P}(A_1) + \text{P}(B \mid A_2)\ \text{P}(A_2) + \text{P}(B \mid A_3)\ \text{P}(A_3)$

$= \sum_{j=1}^{3} \text{P}(B \mid A_j)\ \text{P}(A_j)$

10



a $\text{P}(\text{S}') = \text{P}(\text{A} \cap \text{S}' \text{ or } \text{B} \cap \text{S}' \text{ or } \text{C} \cap \text{S}')$

$= 0.3 \times 0.97 + 0.4 \times 0.95 + 0.3 \times 0.93$

$= 0.95$

b $\text{P}(\text{A} \mid \text{S}') = \dfrac{\text{P}(\text{A} \cap \text{S}')}{\text{P}(\text{S}')} = \dfrac{0.3 \times 0.97}{0.95} \approx 0.306$

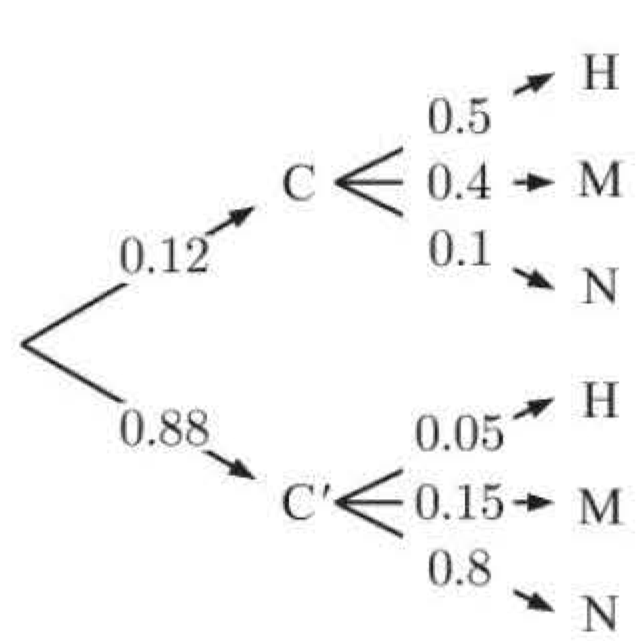
c $\text{P}(\text{A or C} \mid \text{S}) = \dfrac{\text{P}((\text{A} \cup \text{C}) \cap \text{S})}{\text{P}(\text{S})}$

$= \dfrac{\text{P}((\text{A} \cap \text{S}) \cup (\text{C} \cap \text{S}))}{1 - \text{P}(\text{S}')}$

$= \dfrac{0.3 \times 0.03 + 0.3 \times 0.07}{0.05}$

$= 0.6$

11



a $\text{P}(\text{H})$

$= \text{P}(\text{C} \cap \text{H} \text{ or } \text{C}' \cap \text{H})$

$= 0.12 \times 0.5 + 0.88 \times 0.05$

$= 0.104$

b $\text{P}(\text{C} \mid \text{M}) = \dfrac{\text{P}(\text{C} \cap \text{M})}{\text{P}(\text{M})}$

$= \dfrac{0.12 \times 0.4}{0.12 \times 0.4 + 0.88 \times 0.15}$

≈ 0.267

c $\text{P}(\text{C} \mid \text{N}) = \dfrac{\text{P}(\text{C} \cap \text{N})}{\text{P}(\text{N})}$

$= \dfrac{0.12 \times 0.1}{0.12 \times 0.1 + 0.88 \times 0.8}$

≈ 0.0168

REVIEW SET 24A

1 ABCD, ABDC, ACBD, ACDB, ADBC, ADCB, BACD, BADC, BCAD, BCDA, BDAC, BDCA, CABD, CADB, CBAD, CBDA, CDAB, CDBA, DABC, DACB, DBAC, DBCA, DCAB, DCBA

a There are 24 possible orderings.

$\therefore$ P(A is next to C)

$= \frac{12}{24}$ {12 have A next to C}

$= \frac{1}{2}$

b P(exactly one person between A and C)

$= \frac{8}{24}$ {8 have one person between A and C}

$= \frac{1}{3}$

2 **a** $\text{P}(A') = 1 - \text{P}(A)$

$= 1 - m$

b m can be any value between 0 and 1 inclusive.

$\therefore \; 0 \leqslant m \leqslant 1$

c **i** P(A exactly once)

$= \text{P}(AA') + \text{P}(A'A)$

$= m(1-m) + (1-m)m$

$= 2m(1-m)$

ii P(A at least once)

$= 1 - \text{P}(A'A')$

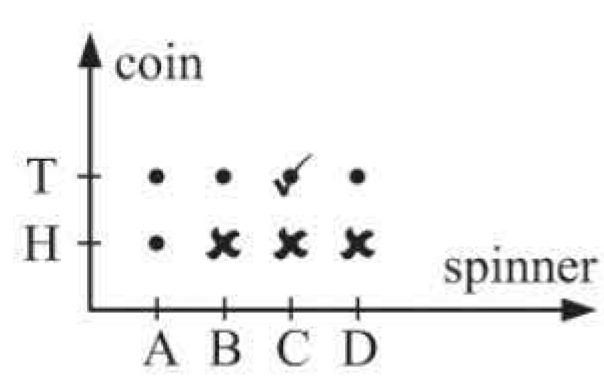
$= 1 - (1-m)(1-m)$

$= 1 - (1 - 2m + m^2)$

$= 1 - 1 + 2m - m^2$

$= 2m - m^2$

3



a Consonants are B, C and D

$\therefore$ P(H and a consonant)

$= \frac{3}{8}$ {those with a ✘}

b P(T and C)

$= \frac{1}{8}$ {those with a ✓}

c P(T or vowel)

$=$ P(T or A)

$=$ P(T) + P(A) − P(T and A)

$= \frac{4}{8} + \frac{2}{8} - \frac{1}{8}$

$= \frac{5}{8}$

4 $\text{P}(M) = \frac{3}{5}$, $\text{P}(W) = \frac{2}{3}$, where M is the event "the man is alive in 25 years", and W is the event "the woman is alive in 25 years".

a P(M and W)

$= \frac{3}{5} \times \frac{2}{3}$

{assuming independence}

$= \frac{2}{5}$

b P(at least one)

$=$ P(M or W)

$= \text{P}(M) + \text{P}(W) - \text{P}(M \text{ and } W)$

$= \frac{3}{5} + \frac{2}{3} - \frac{2}{5}$

$= \frac{13}{15}$

c P(M' and W)

$= (1 - \frac{3}{5}) \times \frac{2}{3}$

$= \frac{2}{5} \times \frac{2}{3}$

$= \frac{4}{15}$

5 **a** $\text{P}(X \cap Y) = 0$ {X and Y are mutually exclusive events}

b $\text{P}(X \cup Y) = \text{P}(X) + \text{P}(Y) - \text{P}(X \cap Y)$

$\therefore \; 0.8 = \text{P}(X) + 0.35 - 0$

$\therefore \; \text{P}(X) = 0.45$

c P(X or Y but not both) $=$ P(X or Y) {X and Y mutually exclusive}

$= \text{P}(X \cup Y)$

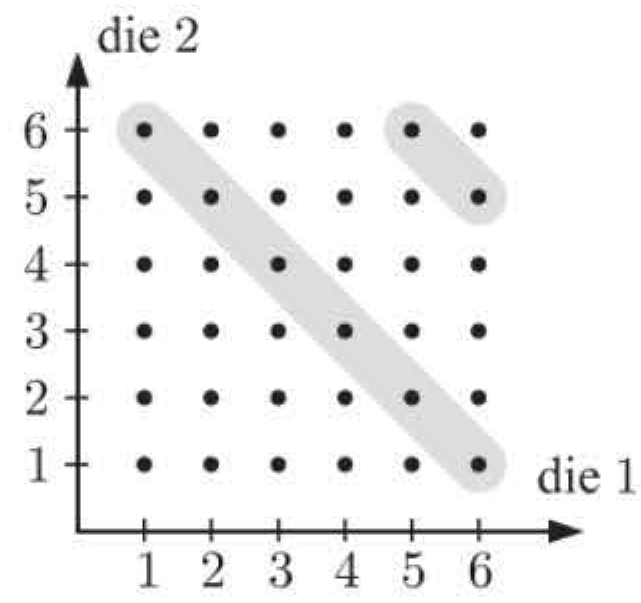
$= 0.8$

6 **a** Two events are independent if the occurrence of either event does not affect the probability that the other occurs. For A and B independent, $\text{P}(A) \times \text{P}(B) = \text{P}(A \text{ and } B)$.

b Two events A and B are mutually exclusive if they have no common outcomes.

$\therefore$ P(A and B) $= 0$ and so P(A or B) $=$ P(A) + P(B).

7 **a**

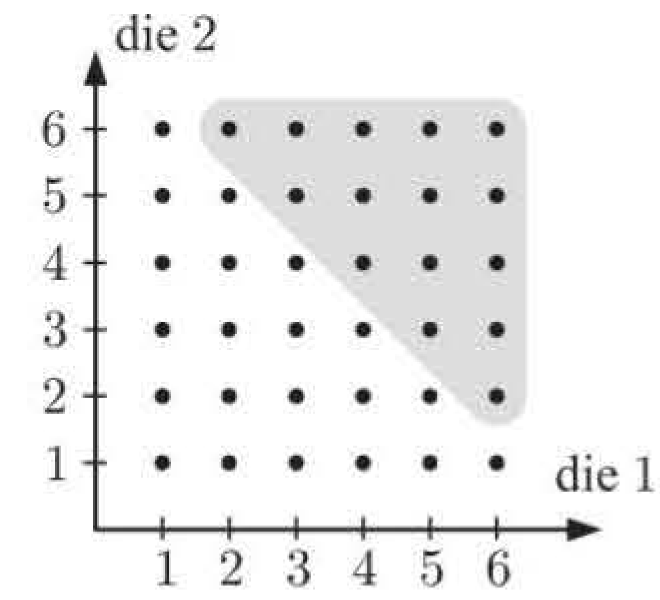


P(sum of 7 or 11)

$= \frac{8}{36}$

$= \frac{2}{9}$

b

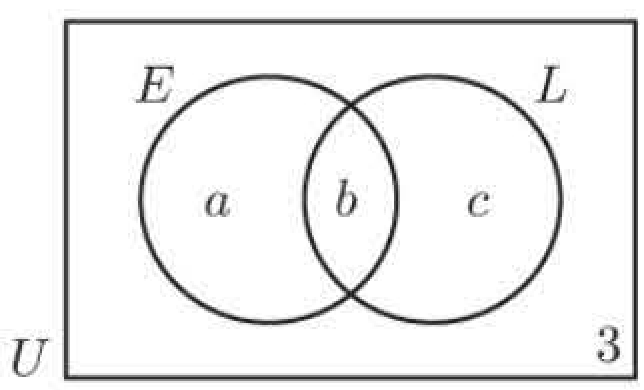


P(sum of at least 8)

$= \frac{15}{36}$

$= \frac{5}{12}$

8



$a+b+c=37$ $\quad \therefore\ 22+c=37$ and $a+25=37$

$a+b=22$ $\quad \therefore\ c=15$ and $a=12$

$b+c=25$ $\quad$ Hence, $b=22-a=10$

a $\quad$ P(E and L)

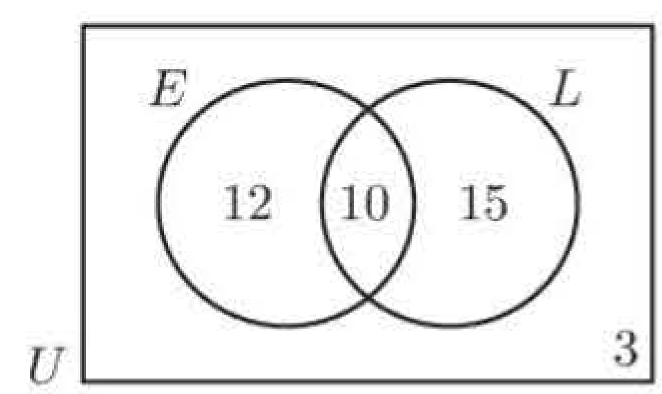
$= \frac{10}{40}$

$= \frac{1}{4}$

b $\quad$ P(at least one)

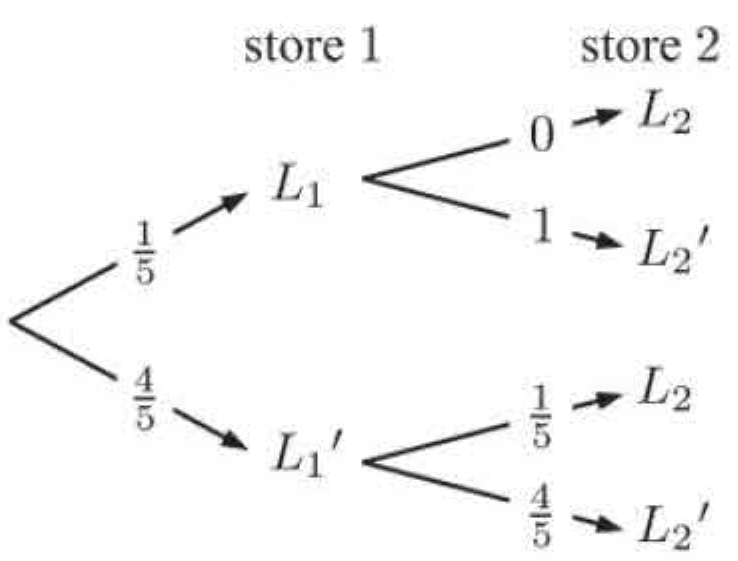
$= \frac{12+10+15}{40}$

$= \frac{37}{40}$



c $\mathrm{P}(E \mid L) = \frac{10}{15+10} = \frac{10}{25} = \frac{2}{5}$

9 Let L_i be the event that the salesman leaves his sunglasses in store i.



$$\mathrm{P}(L_1 \mid L_1 \text{ or } L_2) = \frac{\mathrm{P}(L_1 \cap (L_1 \text{ or } L_2))}{\mathrm{P}(L_1 \text{ or } L_2)}$$

$$= \frac{\mathrm{P}(L_1)}{\mathrm{P}(L_1 L_2' \text{ or } L_1' L_2)}$$

$$= \frac{\frac{1}{5}}{\frac{1}{5} + \frac{4}{5} \times \frac{1}{5}}$$

$$= \frac{5}{9}$$

10 $\mathrm{P(M)} = \frac{4}{5}$, $\mathrm{P(M')} = \frac{1}{5}$

a There are $\binom{5}{3} = 10$ ways in which Mae wins 3 games and Ravi wins 2 games.

P(M wins 3 games)

$= 10(\frac{4}{5})^3(\frac{1}{5})^2$

≈ 0.205

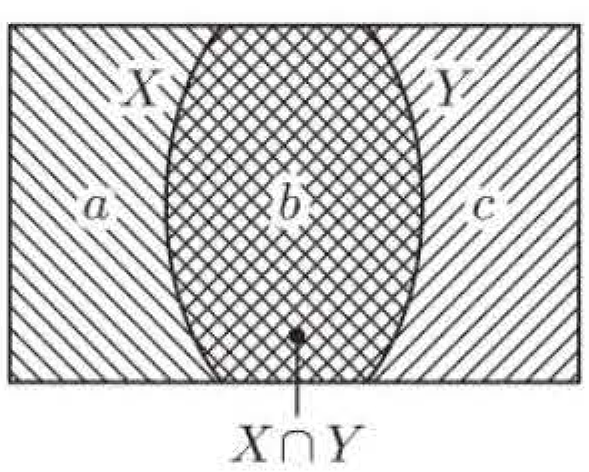
b $\quad$ P(M wins 4 or 5 games)

$= \binom{5}{4}(\frac{4}{5})^4(\frac{1}{5})^1 + \binom{5}{5}(\frac{4}{5})^5$

≈ 0.737

11 Since $X' \cap Y' = \varnothing$, every element in the universal set is in either X or Y or both.

$\therefore$ we can construct the Venn diagram alongside:



Now $\quad \mathrm{P}(X' \mid Y) = \frac{2}{3}$

$\therefore \quad \dfrac{\mathrm{P}(X' \cap Y)}{\mathrm{P}(Y)} = \frac{2}{3}$

$\therefore \quad \mathrm{P}(X' \cap Y) = \frac{2}{3} \times \mathrm{P}(Y)$

$= \frac{2}{3} \times \frac{5}{6}$

$= \frac{5}{9}$

So, $\quad c = \frac{5}{9}$

and $\quad \mathrm{P}(Y) = b + c$

$\therefore \quad b = \frac{5}{6} - \frac{5}{9} = \frac{5}{18}$

$\therefore \quad a = 1 - b - c$

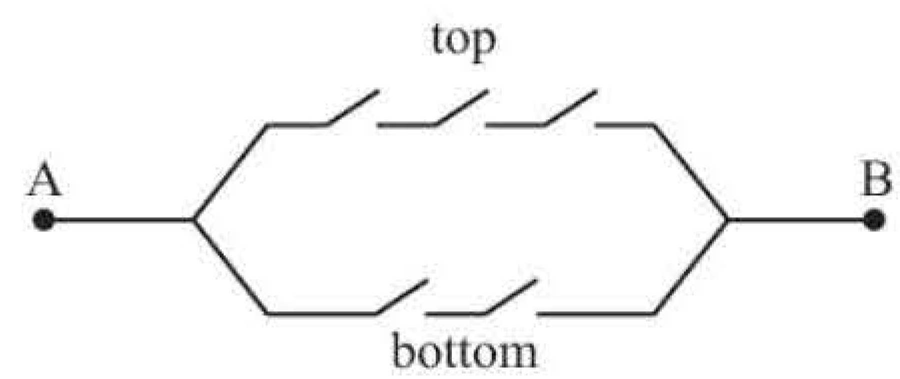
$= 1 - \frac{5}{18} - \frac{5}{9}$

$= \frac{1}{6}$

$\therefore \quad \mathrm{P}(X) = a + b$

$= \frac{1}{6} + \frac{5}{18} = \frac{4}{9}$

12 $\mathrm{P}(\text{any switch is closed}) = \frac{2}{3}$



Current will flow through a wire if all switches along that wire are closed.

$\therefore \quad \mathrm{P}(\text{current flows through top wire}) = \left(\frac{2}{3}\right)^3$

$\mathrm{P}(\text{current flows through bottom wire}) = \left(\frac{2}{3}\right)^2$

$\mathrm{P}(\text{current flows through both wires}) = \mathrm{P}(\text{all five switches closed})$

$= \left(\frac{2}{3}\right)^5$

$\therefore \quad \mathrm{P}(\text{current flows from A to B}) = \mathrm{P}(\text{top}) + \mathrm{P}(\text{bottom}) - \mathrm{P}(\text{top and bottom})$

$= \left(\frac{2}{3}\right)^3 + \left(\frac{2}{3}\right)^2 - \left(\frac{2}{3}\right)^5$

$= \frac{148}{243}$

13 **a** E and S are the only letters common to all three names.

$\therefore \quad \mathrm{P}(\text{all letters are the same}) = \mathrm{P}(\text{all Es}) + \mathrm{P}(\text{all Ss})$

$= \frac{1}{5} \times \frac{2}{6} \times \frac{1}{5} + \frac{1}{5} \times \frac{1}{6} \times \frac{1}{5}$

$= \frac{1}{50}$

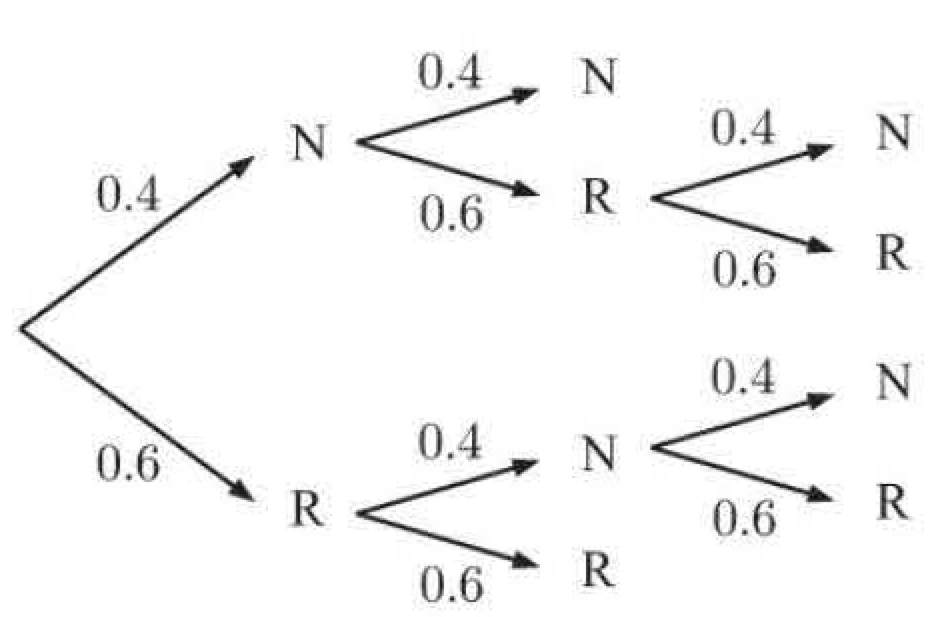
b Exactly two letters will be the same if:

N is selected from JONES and EVANS	$\frac{1}{5} \times \frac{1}{5} = \frac{1}{25}$
E is selected from JONES and PETERS (but not EVANS)	$\frac{1}{5} \times \frac{2}{6} \times \frac{4}{5} = \frac{8}{150}$
E is selected from JONES and EVANS (but not PETERS)	$\frac{1}{5} \times \frac{1}{5} \times \frac{4}{6} = \frac{4}{150}$
E is selected from PETERS and EVANS (but not JONES)	$\frac{2}{6} \times \frac{1}{5} \times \frac{4}{5} = \frac{8}{150}$
S is selected from JONES and PETERS (but not EVANS)	$\frac{1}{5} \times \frac{1}{6} \times \frac{4}{5} = \frac{4}{150}$
S is selected from JONES and EVANS (but not PETERS)	$\frac{1}{5} \times \frac{1}{5} \times \frac{5}{6} = \frac{5}{150}$
S is selected from PETERS and EVANS (but not JONES)	$\frac{1}{6} \times \frac{1}{5} \times \frac{4}{5} = \frac{4}{150}$
	Total $= \frac{13}{50}$

$\therefore \quad \mathrm{P}(\text{only two of the letters are the same}) = \frac{13}{50}$

REVIEW SET 24B

1



$\mathrm{P}(\text{Niklas wins})$

$= (0.4)(0.4) + (0.4)(0.6)(0.4) + (0.6)(0.4)(0.4)$

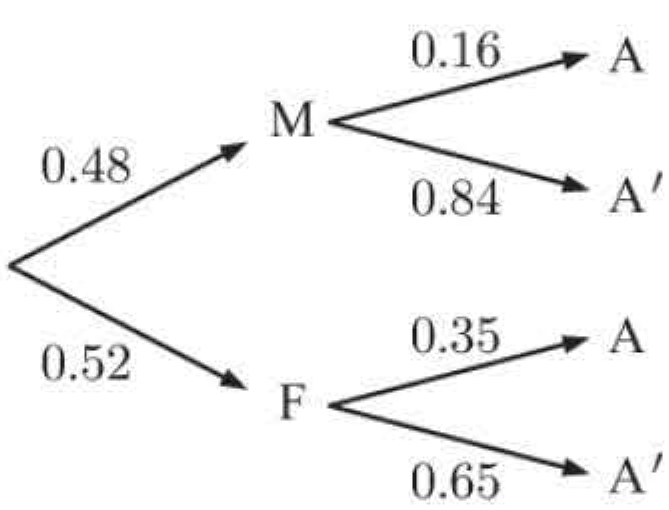
$= \frac{44}{125}$

$= 0.352$

2 **a** P(win first 3 prizes)
$= \text{P(WWW)}$
$= \frac{4}{500} \times \frac{3}{499} \times \frac{2}{498}$
$\approx 0.000\,000\,193$

b P(win at least one of the 3 prizes)
$= 1 - \text{P(wins none of them)}$
$= 1 - \text{P(W}'\text{W}'\text{W}')$
$= 1 - \frac{496}{500} \times \frac{495}{499} \times \frac{494}{498}$
≈ 0.0239

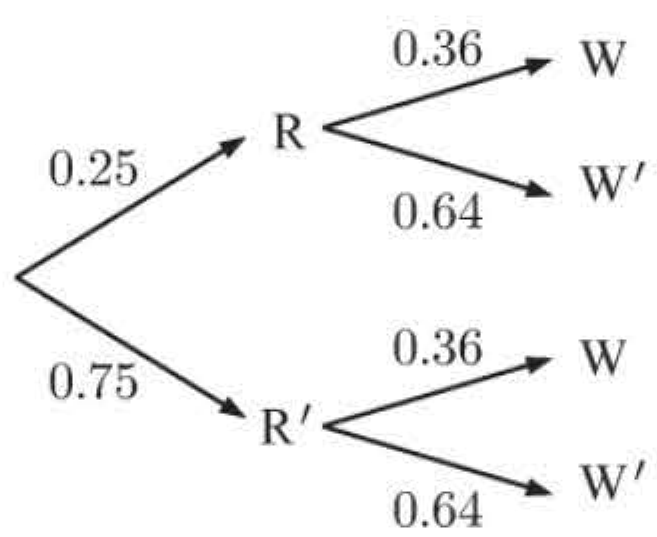
3



a $\text{P(A)} = \text{P(M} \cap \text{A or F} \cap \text{A)}$
$= 0.48 \times 0.16 + 0.52 \times 0.35$
$= 0.2588 \approx 0.259$

b $\text{P(F} \mid \text{A)} = \dfrac{\text{P(F} \cap \text{A)}}{\text{P(A)}}$
$= \dfrac{0.52 \times 0.35}{0.2588}$
≈ 0.703

4 **a**



b **i** P(R and W)
$= 0.25 \times 0.36$
$= 0.09$

ii P(R or W)
$= \text{P(R)} + \text{P(W)} - \text{P(R and W)}$
$= 0.25 + 0.36 - 0.09$
$= 0.52$

or $\text{P(R or W)} = 1 - \text{P(R}'\text{W}')$
$= 1 - 0.75 \times 0.64$
$= 0.52$

c We have assumed that the two events (rain and wind) are independent.

5 $\text{P(A)} = 0.1, \quad \text{P(B)} = 0.2, \quad \text{P(C)} = 0.3$

$\therefore$ P(group solves it) = P(at least one solves it)
$= 1 - \text{P(no-one solves it)}$
$= 1 - \text{P(A}' \text{ and B}' \text{ and C}')$
$= 1 - (0.9 \times 0.8 \times 0.7)$
$= 0.496$

6 **a** **i** $\text{P}(A') = 1 - \text{P}(A)$
$= 1 - 0.11$
$= 0.89$

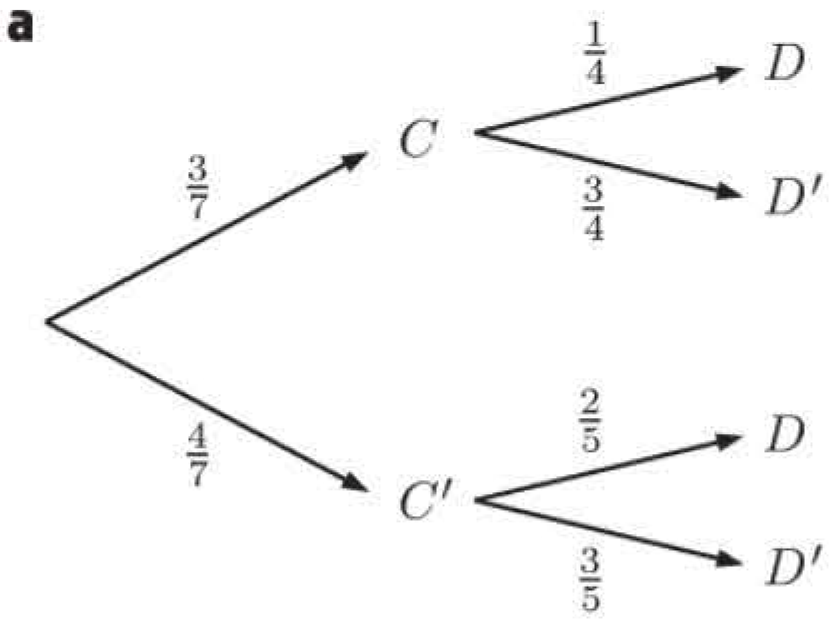
ii $\text{P}(B) = 0.7$
$\therefore \dfrac{14}{n(U)} = 0.7$
$\therefore n(U) = \dfrac{14}{0.7}$
$= 20$

b **i** $\text{P}(A \cap B) = \text{P}(A) \times \text{P}(B)$
$= 0.11 \times 0.7$
$= 0.077$

ii $\text{P}(A \mid B) = \text{P}(A)$
$= 0.11$

c $\text{P}(A \cup B) = \text{P}(A) + \text{P}(B)$
$= 0.11 + 0.7$
$= 0.81$

7 **a**



b **i** $\text{P}(CD) = \frac{3}{7} \times \frac{1}{4}$
$= \frac{3}{28}$

ii $\text{P}(\text{at least one pet}) = 1 - \text{P}(C'D')$
$= 1 - \frac{4}{7} \times \frac{3}{5}$
$= \frac{23}{35}$

8 **a**

	Female	*Male*	*Total*
Smoker	$60 - 40 = 20$	40	60
Non-smoker	$90 - 20 = 70$	$110 - 40 = 70$	$200 - 60 = 140$
Total	90	$200 - 90 = 110$	200

b **i** $\text{P}(\text{female non-smoker}) = \frac{70}{200} = \frac{7}{20}$

ii $\text{P}(\text{male given non-smoker}) = \frac{70}{140} = \frac{1}{2}$

c **i** P(two non-smoking females)
$= \frac{70}{200} \times \frac{69}{199}$
≈ 0.121

ii P(one is a smoker and the other is not)
$= \text{P}(\text{SS}') + \text{P}(\text{S}'\text{S})$
$= \frac{60}{200} \times \frac{140}{199} + \frac{140}{200} \times \frac{60}{199}$
$= \frac{42}{199} + \frac{42}{199}$
$= \frac{84}{199}$
≈ 0.422

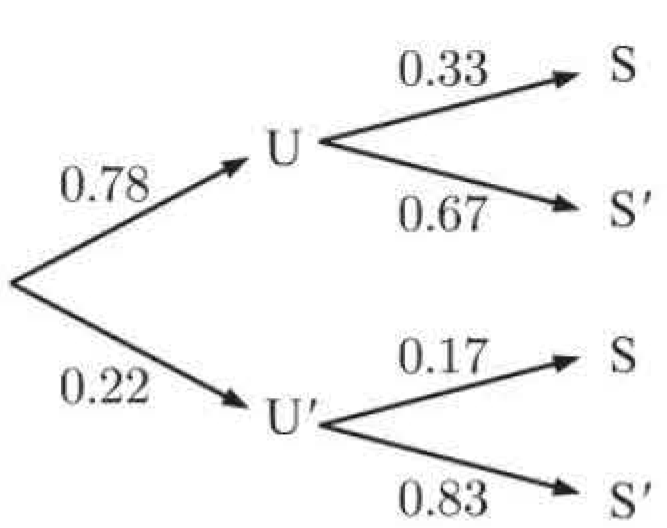
9 **a** $\text{P}(M) \times \text{P}(C) = 0.91 \times 0.88$
≈ 0.801
and $\text{P}(M \cap C) = 0.85$
$\therefore \text{P}(M \cap C) \neq \text{P}(M)\ \text{P}(C)$,
so M and C are not independent.

b $\text{P}(M' \mid C) = \dfrac{\text{P}(M' \cap C)}{\text{P}(C)}$
Now $\text{P}(M' \cap C) + \text{P}(M \cap C) = \text{P}(C)$
$\therefore \text{P}(M' \cap C) = \text{P}(C) - \text{P}(M \cap C)$
$= 0.88 - 0.85$
$= 0.03$
$\therefore \text{P}(M' \mid C) = \frac{0.03}{0.88} = \frac{3}{88}$

10 There are $\binom{10}{5}$ different ways to choose the group.

$\therefore \text{P}(2 \text{ Year } 12, 2 \text{ Year } 11) = \dfrac{\binom{3}{2}\binom{4}{2}\binom{3}{1}}{\binom{10}{5}}$
$= \frac{3}{14}$

11

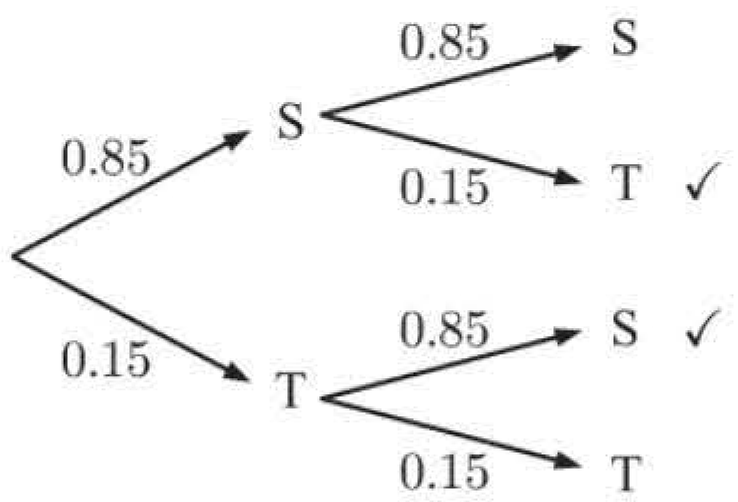


$\text{P}(\text{U}' \mid \text{S}) = \dfrac{\text{P}(\text{U}' \cap \text{S})}{\text{P}(\text{S})}$
$= \dfrac{0.22 \times 0.17}{0.22 \times 0.17 + 0.78 \times 0.33}$
≈ 0.127

12 **a** $\text{P}(\text{all doctors}) = \dfrac{\binom{6}{5} \times \binom{4}{0}}{\binom{10}{5}} \approx 0.0238$

b $\text{P}(\text{at least 2 doctors}) = 1 - \text{P}(1 \text{ doctor}) = 1 - \dfrac{\binom{6}{1} \times \binom{4}{4}}{\binom{10}{5}} \approx 0.976$

13



$$\begin{aligned}\text{P(twins first} \mid \text{3 children)} &= \frac{\text{P(twins first} \cap \text{3 children)}}{\text{P(3 children)}}\\ &= \frac{0.15 \times 0.85}{0.85 \times 0.15 + 0.15 \times 0.85}\\ &= \tfrac{1}{2}\end{aligned}$$

14 There are $\binom{10}{4} = 210$ ways to select the four numbers.

a 2 cannot be the second largest number, as there is only 1 number smaller than 2.
$\therefore\ \text{P}(X = 2) = 0$

b There are $\binom{6}{2}\binom{1}{1}\binom{3}{1} = 45$ ways to choose the numbers so that $X = 7$.

2 numbers below 7   7   1 number above 7

$\therefore\ \text{P}(X = 7) = \frac{45}{210} = \frac{3}{14}$

c There are $\binom{8}{2}\binom{1}{1}\binom{1}{1} = 28$ ways to choose the numbers so that $X = 9$.

2 numbers below 9   9   10

$\therefore\ \text{P}(X = 9) = \frac{28}{210} = \frac{2}{15}$

REVIEW SET 24C

1 BBBB, BBBG, BBGB, BGBB, GBBB, BBGG, BGBG, BGGB, GBBG, GBGB, GGBB, BGGG, GBGG, GGBG, GGGB, GGGG

P(2B and 2G)
$= \frac{6}{16}$ $\leftarrow$ 6 have 2B and 2G
$= \frac{3}{8}$

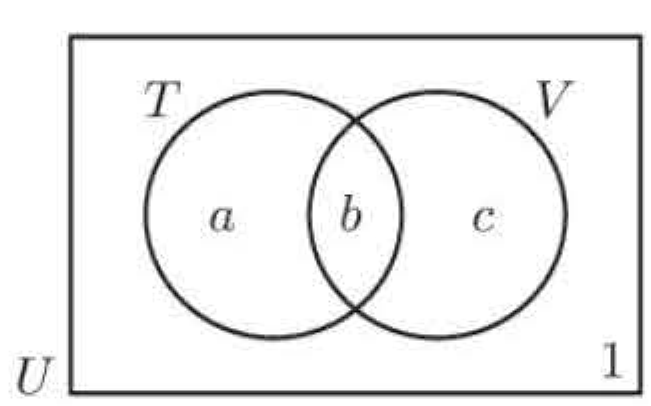
2 **a** P(both blue)
$= \text{P(BB)}$
$= \frac{5}{12} \times \frac{4}{11}$
$= \frac{5}{33}$

b P(both same colour)
$= \text{P(BB or RR or YY)}$
$= \frac{5}{12} \times \frac{4}{11} + \frac{3}{12} \times \frac{2}{11} + \frac{4}{12} \times \frac{3}{11}$
$= \frac{19}{66}$

c P(at least one R)
$= 1 - \text{P(no reds)}$
$= 1 - \text{P(R}'\text{R}')$
$= 1 - \frac{9}{12} \times \frac{8}{11}$
$= 1 - \frac{6}{11}$
$= \frac{5}{11}$

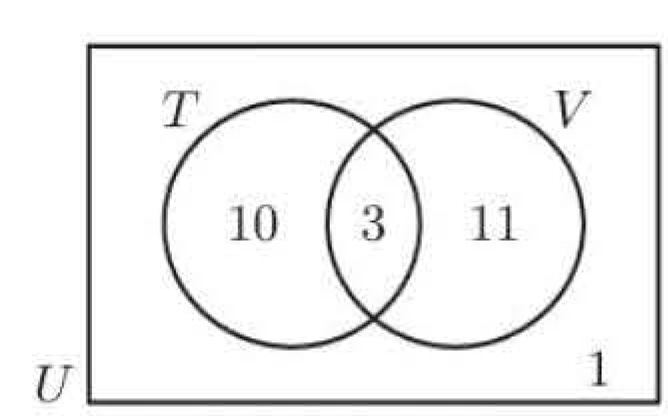
d P(exactly one Y)
$= \text{P(YY}' \text{ or Y}'\text{Y)}$
$= \frac{4}{12} \times \frac{8}{11} + \frac{8}{12} \times \frac{4}{11}$
$= \frac{16}{33}$

3 **a**



$a + b + c = 24$
$a + b = 13$
$b + c = 14$
Also $b = 13 - a$
$= 3$

$\therefore\ 13 + c = 24$ and $a + 14 = 24$
$\therefore\ c = 11$ and $a = 10$



i $\text{P}(T \text{ and } V)$
$= \frac{3}{25}$

ii P(at least one)
$= 1 - \text{P(neither)}$
$= 1 - \frac{1}{25}$
$= \frac{24}{25}$

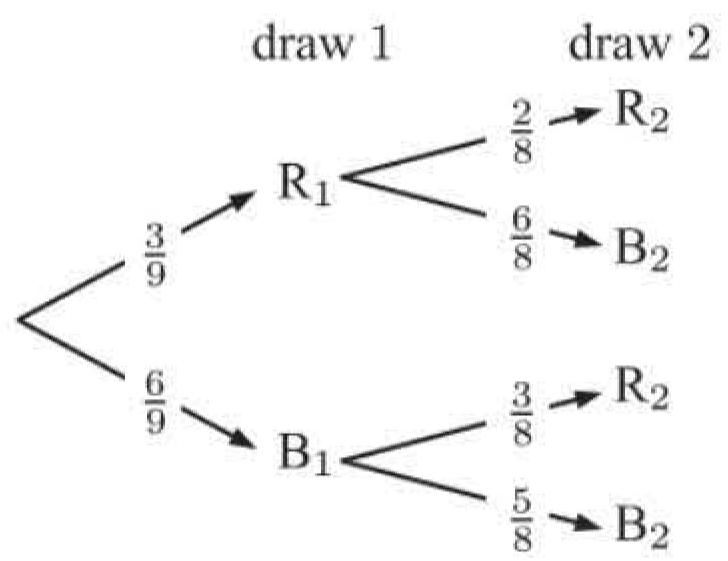
iii $\text{P}(V \mid T')$
$= \dfrac{11}{11 + 1}$
$= \frac{11}{12}$

b **i** $P(T'T'T')$
$= \frac{12}{25} \times \frac{11}{24} \times \frac{10}{23}$
$= \frac{11}{115}$

ii P(at least one plays tennis)
$= 1 - \text{P(none play tennis)}$
$= 1 - \frac{11}{115}$
$= \frac{104}{115}$

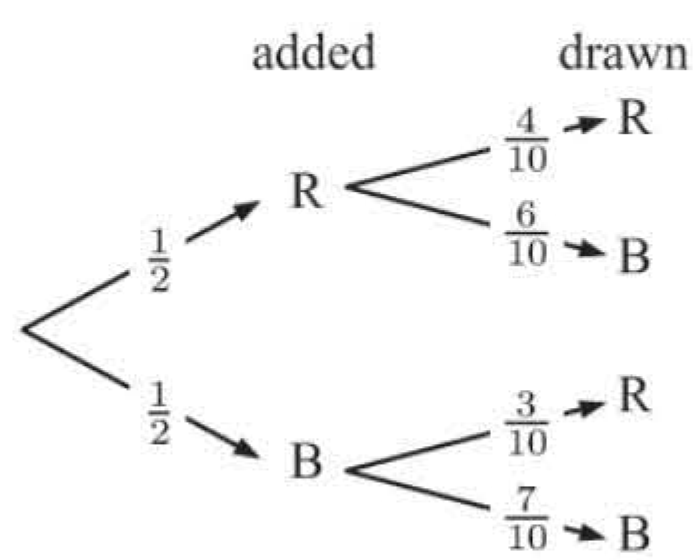
4 **a** There are now 3 red and 5 blue balls remaining.
$\therefore\ \text{P(blue)} = \frac{5}{8}$

b



$$P(R_1 \mid R_2) = \frac{P(R_1 \cap R_2)}{P(R_2)} = \frac{\frac{3}{9} \times \frac{2}{8}}{\frac{3}{9} \times \frac{2}{8} + \frac{6}{9} \times \frac{3}{8}} = \frac{1}{4}$$

c



$$\text{P(red added} \mid \text{blue drawn)} = \frac{\text{P(red added} \cap \text{ blue drawn)}}{\text{P(blue drawn)}} = \frac{\frac{1}{2} \times \frac{6}{10}}{\frac{1}{2} \times \frac{6}{10} + \frac{1}{2} \times \frac{7}{10}} = \frac{6}{13}$$

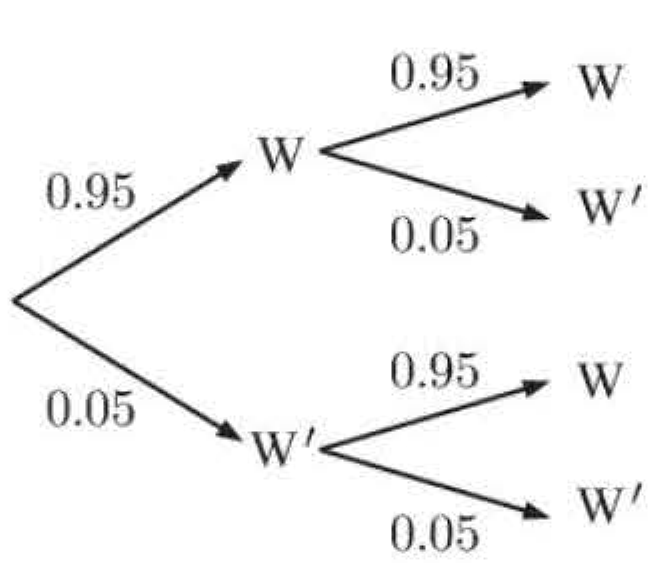
5 **a** $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= P(A) + P(B) - P(A)\,P(B)$ $\quad$ {A and B are independent}
$= 0.8 + 0.65 - 0.8 \times 0.65$
$= 0.93$

b $P(A \mid B) = P(A)$ $\quad$ {A and B are independent}
$= 0.8$

c $$P(A' \mid B') = \frac{P(A' \cap B')}{P(B')} = \frac{1 - P(A \cup B)}{1 - P(B)} = \frac{1 - 0.93}{1 - 0.65} = 0.2$$

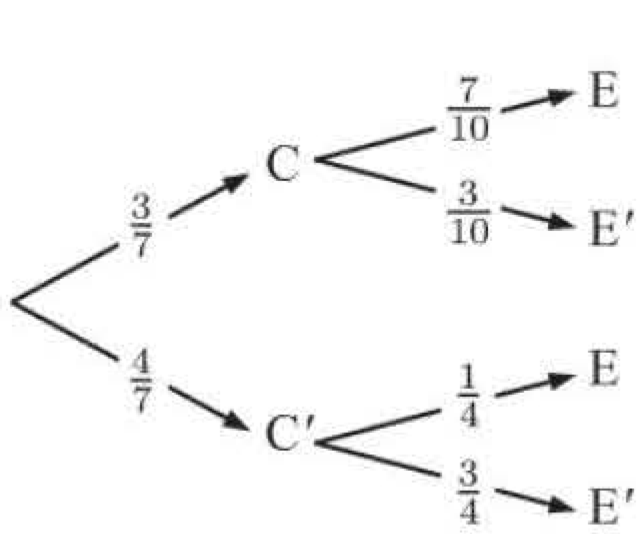
d $P(B \mid A) = P(B)$
$= 0.65$

6



P(works on at least one day)
$= 0.95 \times 0.95 + 0.95 \times 0.05 + 0.05 \times 0.95$
$= 0.9975$

7



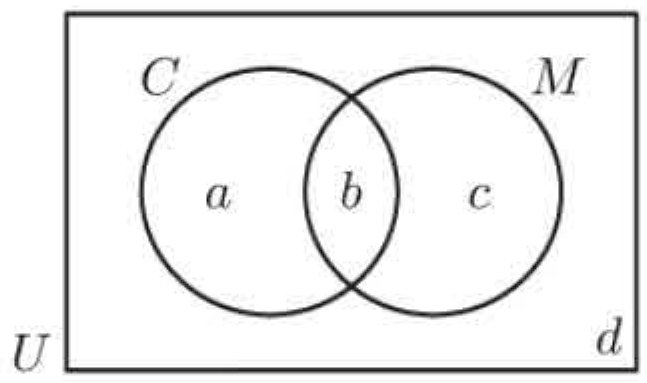
a $P(E) = \frac{3}{7} \times \frac{7}{10} + \frac{4}{7} \times \frac{1}{4}$
$= \frac{3}{10} + \frac{1}{7}$
$= \frac{31}{70}$

b $$P(C \mid E) = \frac{\text{P(C and E)}}{P(E)} = \frac{\frac{3}{7} \times \frac{7}{10}}{\frac{31}{70}} = \frac{21}{31}$$

8 **a**

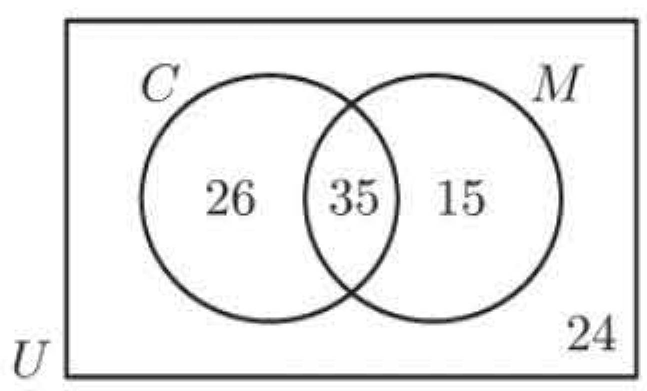
	Men	*Women*	*Total*
Tea	15	24	$15+24=39$
Coffee	$50-15=35$	$50-24=26$	$35+26=61$
Total	50	50	100

b



$$\begin{aligned} b &= 35 && \{35 \text{ men prefer coffee}\} \\ a+35 &= 61 && \{61 \text{ people prefer coffee}\} \\ \therefore\ a &= 26 \\ 35+c &= 50 && \{50 \text{ men were surveyed}\} \\ \therefore\ c &= 15 \\ 26+35+15+d &= 100 && \{100 \text{ people were surveyed}\} \\ \therefore\ d &= 24 \end{aligned}$$

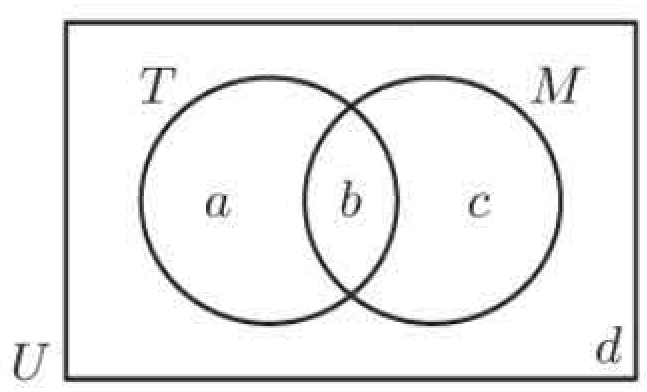
$\therefore$ the Venn diagram is:



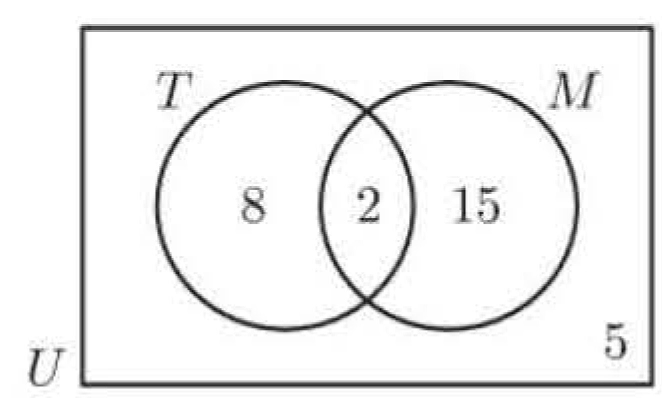
c **i** $\mathrm{P}(C') = \dfrac{15+24}{100} = \frac{39}{100} = 0.39$

ii $\mathrm{P}(M \mid C) = \dfrac{35}{26+35} = \frac{35}{61} \approx 0.574$

9 **a**



$$\begin{aligned} a+b &= 10 && \{n(T)=10\} \\ b+c &= 17 && \{n(M)=17\} \\ d &= 5 && \{n((T\cup M)')=5\} \\ \therefore\ a+b+c &= 30-5 && \{n(U)=30\} \\ \therefore\ a+17 &= 25 \\ \therefore\ a &= 8 \\ \text{and}\quad 8+b &= 10 \\ \therefore\ b &= 2 \\ \text{and}\quad 2+c &= 17 \\ \therefore\ c &= 15 \end{aligned}$$



b **i** $\mathrm{P}(T\cap M) = \frac{2}{30} = \frac{1}{15}$

ii $\mathrm{P}((T\cap M) \mid M) = \frac{2}{17}$

10 **b** Of 100 000 females born, 98 956 are still alive at age 15.
Of 100 000 males born, 98 555 are still alive at age 15.

$$\therefore\ \mathrm{P}(\text{reaching the age of 15}) = \frac{98\,956+98\,555}{200\,000} = \frac{197\,511}{200\,000} = 0.987\,555 \approx 0.9876$$

c **i** There are 98 555 boys alive at age 15, and 53 942 still alive at 75.

$$\therefore\ \text{probability} = \frac{53\,942}{98\,555} \approx 0.547$$

ii There are 98 956 females alive at age 15, and 72 656 alive at age 75.

$\therefore$ P(15 year old girl does *not* reach 75)

$$= 1 - \frac{72\,656}{98\,956} = \frac{26\,300}{98\,956} \approx 0.266$$

d A 20 year old of either gender is expected to live for longer than 30 years, so it is unlikely the insurance company will have to pay out the policy.

11 **a** P(at least one component needs replacing) $= 1 - \text{P(no components need replacing)}$

$$= 1 - \tfrac{19}{20} \times \tfrac{49}{50} \times \tfrac{99}{100}$$
$$= 0.078\,31$$

b P(exactly one component needs replacing) $= \tfrac{1}{20} \times \tfrac{49}{50} \times \tfrac{99}{100} + \tfrac{19}{20} \times \tfrac{1}{50} \times \tfrac{99}{100} + \tfrac{19}{20} \times \tfrac{49}{50} \times \tfrac{1}{100}$

$$= 0.076\,63$$

12 **A** Peter will win at least two consecutive games out of 3 serving first if

(1) he wins the second game (served by John), **and**

(2) he wins at least one of the other two games (served by Peter).

P(event 1) $= 1 - q$ {John loses his serve}

$$\begin{aligned}\text{P(event 2)} &= 1 - \text{P(Peter loses both)}\\ &= 1 - (1-p)^2\\ &= 1 - (1 - 2p + p^2)\\ &= p(2-p)\end{aligned}$$

$\therefore$ P(**A**) $= p(1-q)(2-p)$

B Peter will win at least two consecutive games out of 3 when John serves first if

(1) he wins the second game (served by Peter), **and**

(2) he wins at least one of the other two games (served by John).

P(event 1) $= p$ {Peter wins his serve}

$$\begin{aligned}\text{P(event 2)} &= 1 - \text{P(Peter loses both)}\\ &= 1 - q^2\\ &= (1-q)(1+q)\end{aligned}$$

$\therefore$ P(**B**) $= p(1-q)(1+q)$

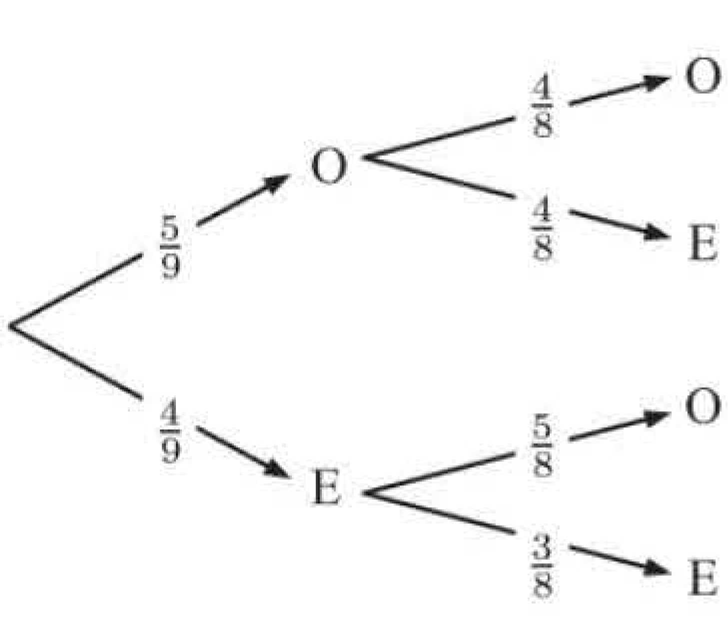
Now $p + q > 1$

$\therefore\ q > 1 - p$

$\therefore\ 1 + q > 2 - p$

$\therefore$ P(**B**) $>$ P(**A**), and so **B** is more likely than **A**.

13



If the sum of the numbers is even, then the numbers are either both even or both odd.

$$\therefore\ \text{P(both odd} \mid \text{sum even)} = \frac{\text{P(both odd} \cap \text{sum even)}}{\text{P(sum even)}}$$
$$= \frac{\text{P(OO)}}{\text{P(OO or EE)}}$$
$$= \frac{\frac{5}{9} \times \frac{4}{8}}{\frac{5}{9} \times \frac{4}{8} + \frac{4}{9} \times \frac{3}{8}}$$
$$= \tfrac{5}{8}$$

14 **a** There are $\binom{52}{5}$ possible poker hands.

4 of these are a royal flush {10, J, Q, K, A of hearts, clubs, diamonds, or spades}

$\therefore$ P(royal flush in any order) $= \dfrac{4}{\binom{52}{5}} \approx 1.54 \times 10^{-6}$

b When order is important, there are $52 \times 51 \times 50 \times 49 \times 48$ possible poker hands.

4 of these are a royal flush in the order 10, J, Q, K, A.

$\therefore$ P(royal flush in the order 10, J, Q, K, A) $= \dfrac{4}{52 \times 51 \times 50 \times 49 \times 48} \approx 1.28 \times 10^{-8}$