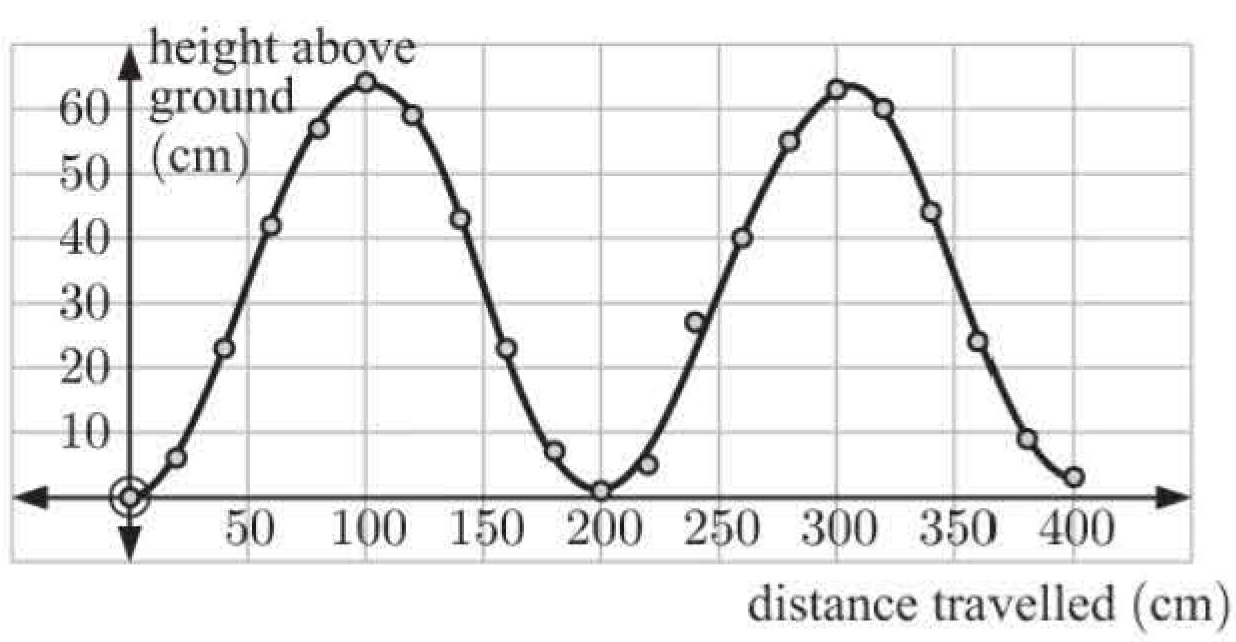
Chapter 12
TRIGONOMETRIC FUNCTIONS

EXERCISE 12A

1 **a** periodic **b** periodic **c** periodic **d** not periodic **e** periodic
f periodic **g** not periodic **h** not periodic

2 **a**



c The data is periodic.

i The minimum value from the table is 0 and the maximum value is 64.

So, the principal axis is $y \approx \frac{0+64}{2}$

$\therefore\ y \approx 32$

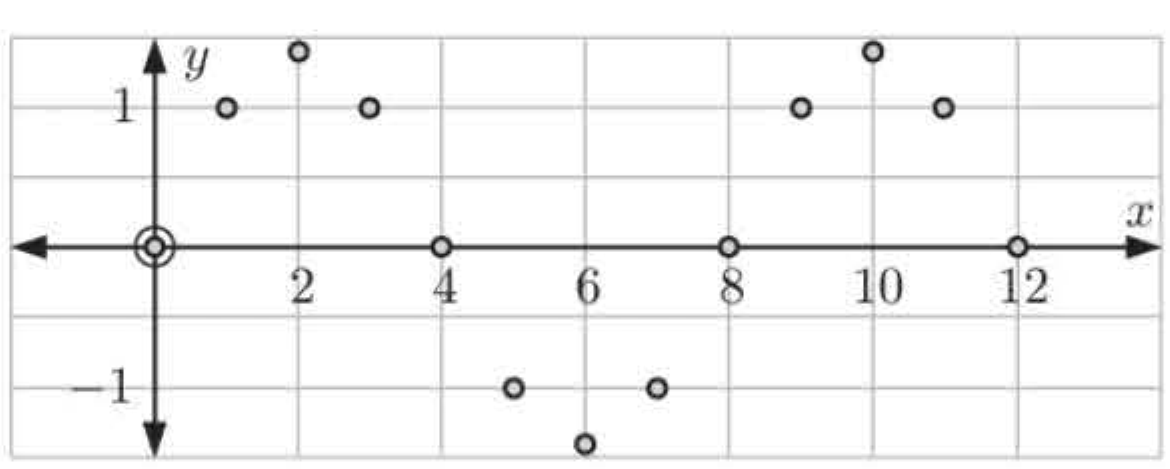
ii The maximum value is ≈ 64 cm.

iii The period is ≈ 200 cm.

iv The amplitude is ≈ 32 cm.

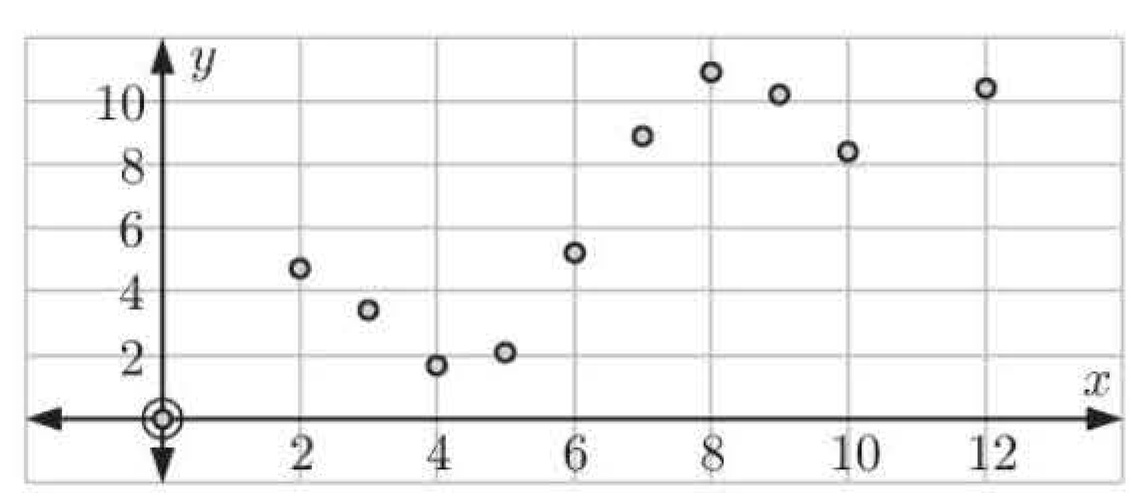
b A curve can be fitted to the data as the distance travelled is continuous.

3 **a**



Data exhibits periodic behaviour.

b



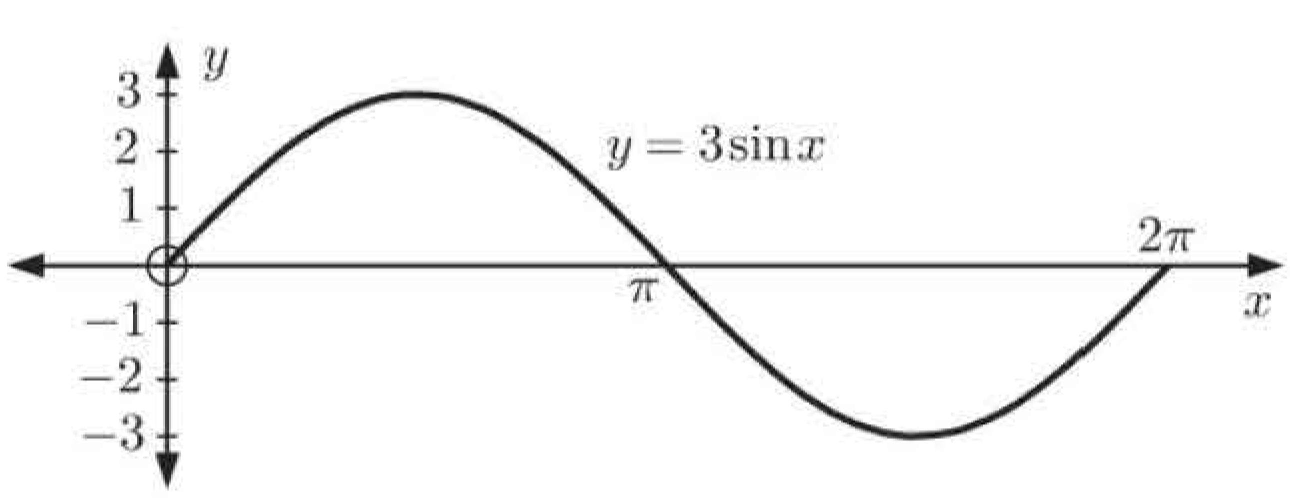
Not enough information to say data is periodic.

EXERCISE 12B.1

1 **a** $y = 3\sin x$

has amplitude 3 and period $\frac{2\pi}{1} = 2\pi$

When $x = 0,\ y = 0.$

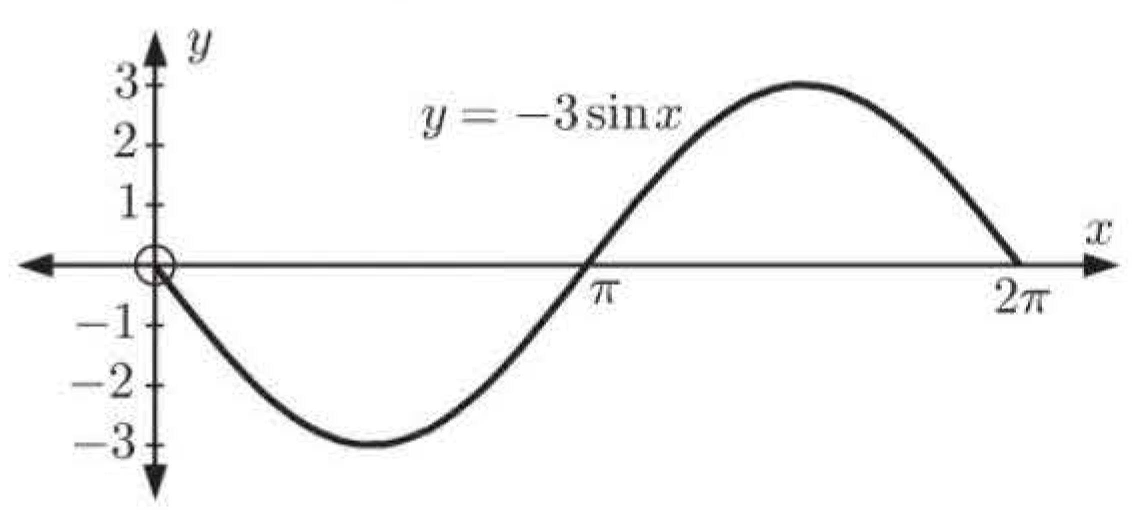


b $y = -3\sin x$

has amplitude $|-3| = 3$

and period $\frac{2\pi}{1} = 2\pi.$

When $x = 0,\ y = 0.$

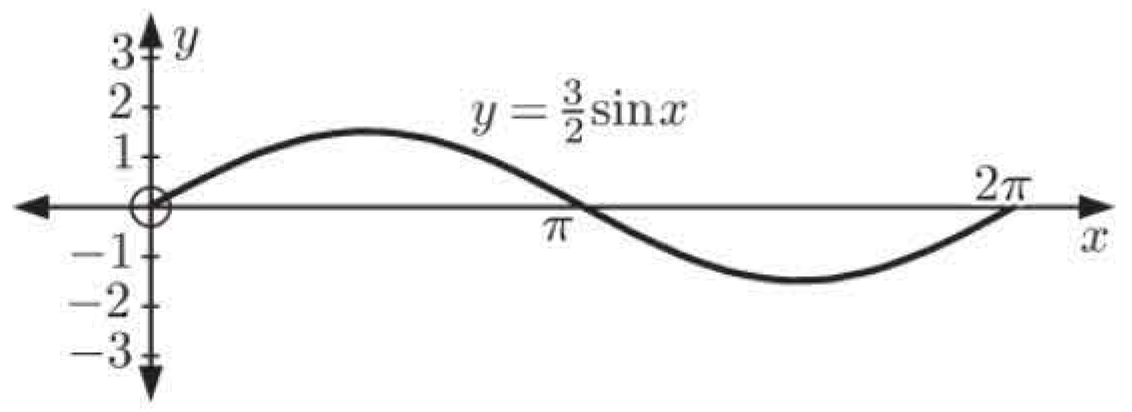


It is the reflection of $y = 3\sin x$ in the x-axis.

c $y = \frac{3}{2}\sin x$

has amplitude $\frac{3}{2}$ and period $\frac{2\pi}{1} = 2\pi$.

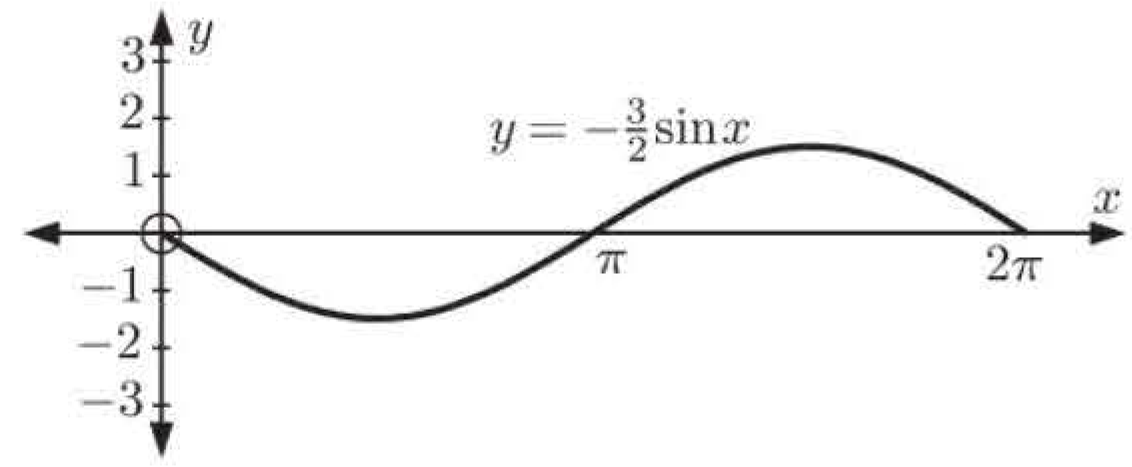
When $x = 0$, $y = 0$.



d $y = -\frac{3}{2}\sin x$

has amplitude $\left|-\frac{3}{2}\right| = \frac{3}{2}$

and period $\frac{2\pi}{1} = 2\pi$.

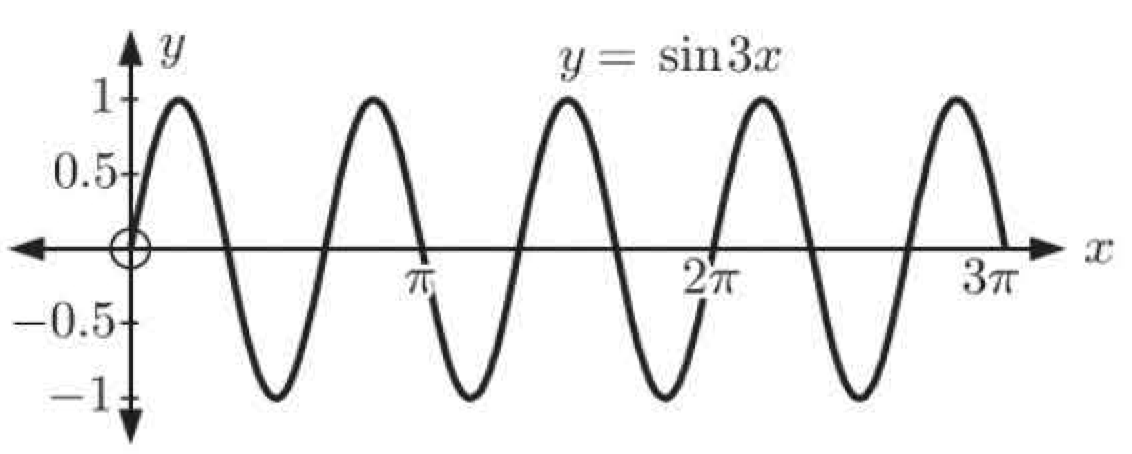


It is the reflection of $y = \frac{3}{2}\sin x$ in the x-axis.

2 **a** $y = \sin 3x$

has amplitude 1 and period $\frac{2\pi}{3}$.

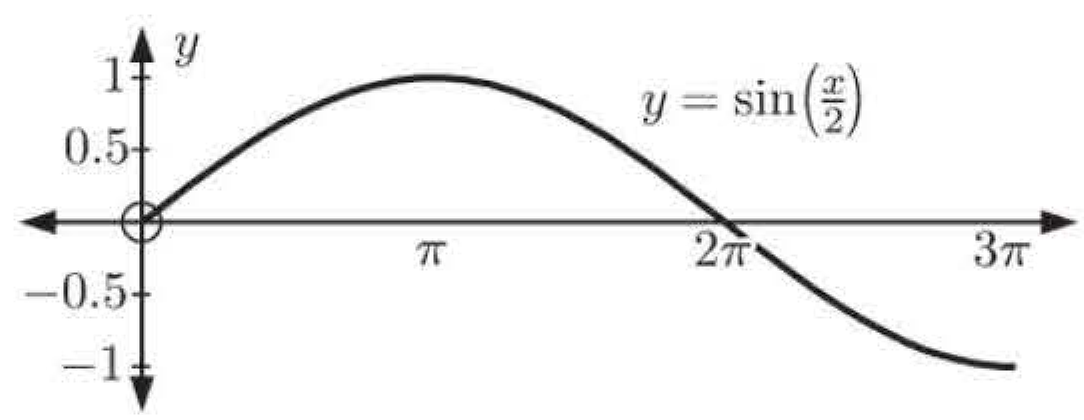
When $x = 0$, $y = 0$.



b $y = \sin\left(\frac{x}{2}\right)$

has amplitude 1 and period $\dfrac{2\pi}{\frac{1}{2}} = 4\pi$.

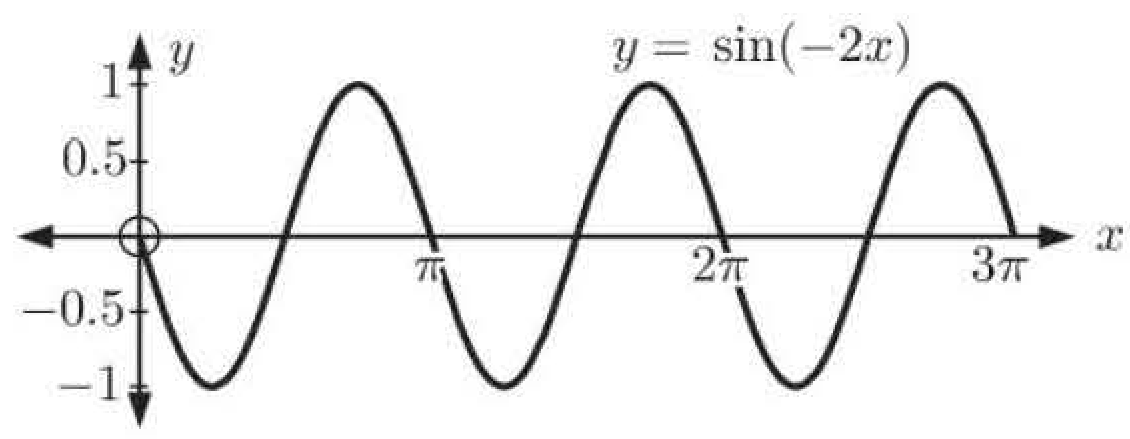
When $x = 0$, $y = 0$.



c $y = \sin(-2x)$

has amplitude 1 and period $\dfrac{2\pi}{|-2|} = \pi$.

When $x = 0$, $y = 0$.



It is the reflection of $y = \sin 2x$ in the y-axis.

3 **a** $\text{period} = \dfrac{2\pi}{4} = \frac{\pi}{2}$

b $\text{period} = \dfrac{2\pi}{|-4|} = \frac{\pi}{2}$

c $\text{period} = \dfrac{2\pi}{\left(\frac{1}{3}\right)} = 6\pi$

d $\text{period} = \dfrac{2\pi}{0.6} = \frac{20\pi}{6} = \frac{10\pi}{3}$

4 **a** $\dfrac{2\pi}{b} = 5\pi$ $\therefore\ b = \frac{2}{5}$

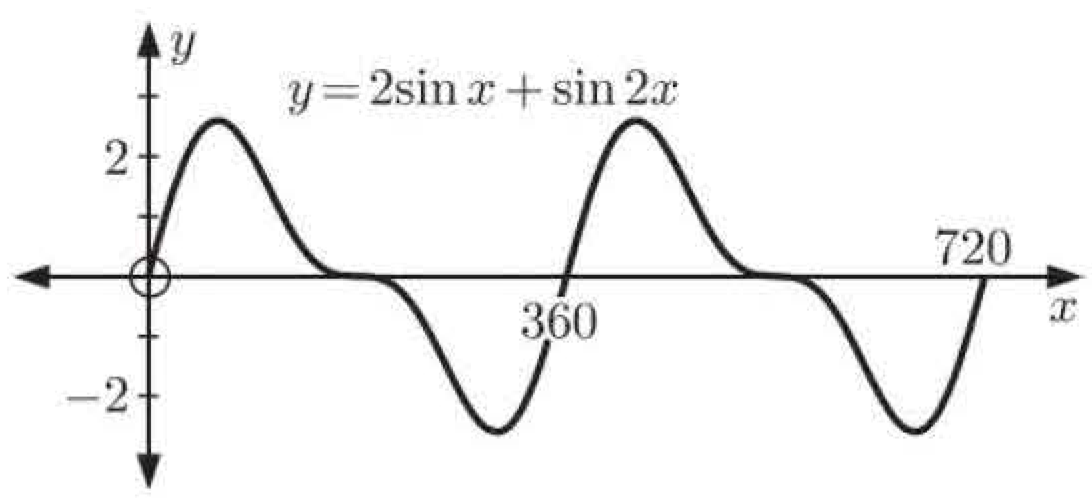
b $\dfrac{2\pi}{b} = \dfrac{2\pi}{3}$ $\therefore\ b = 3$

c $\dfrac{2\pi}{b} = 12\pi$ $\therefore\ b = \frac{1}{6}$

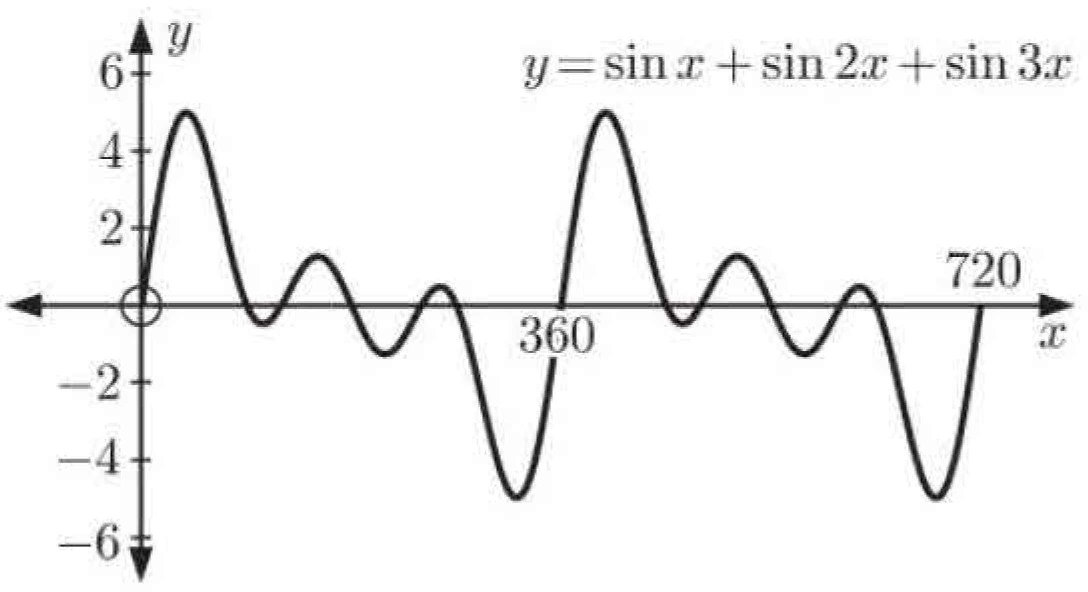
d $\dfrac{2\pi}{b} = 4$ $\therefore\ b = \frac{\pi}{2}$

e $\dfrac{2\pi}{b} = 100$ $\therefore\ b = \frac{2\pi}{100} = \frac{\pi}{50}$

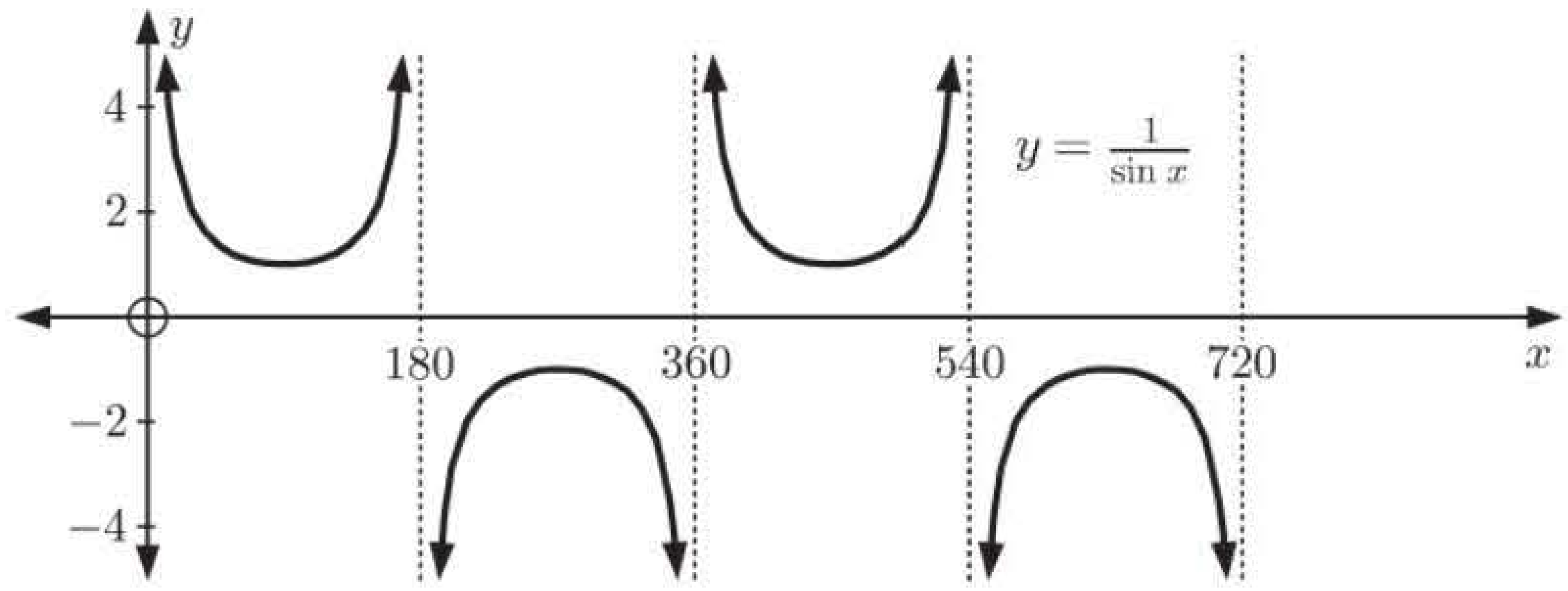
5 **a**



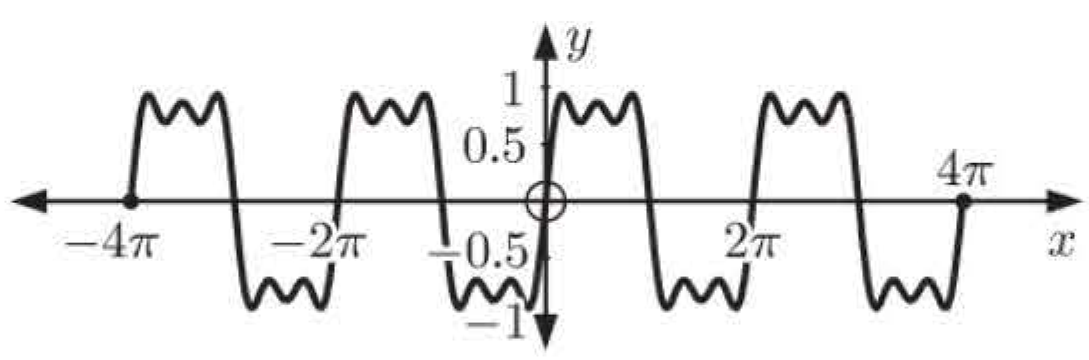
b



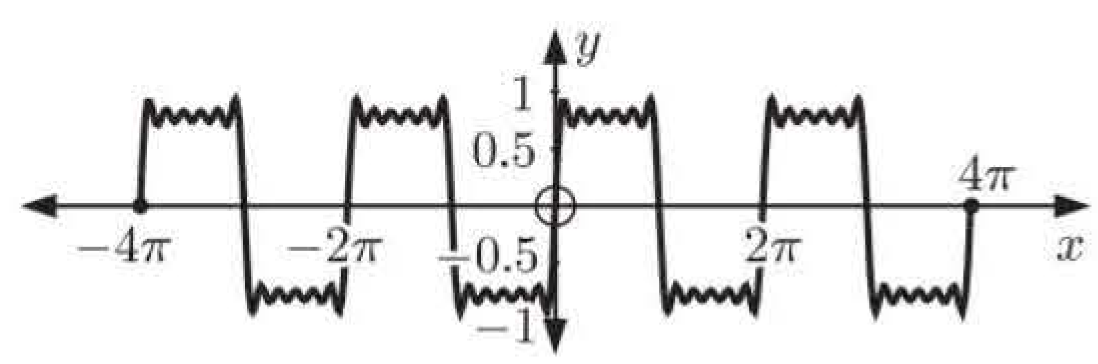
c



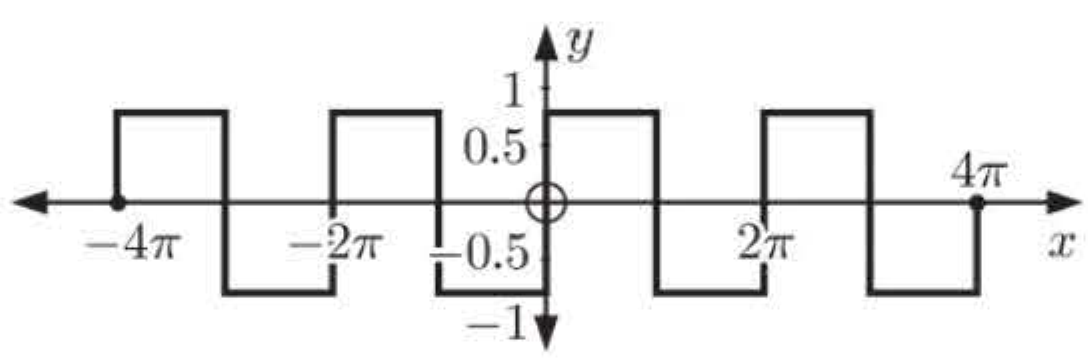
6 **a** **i**



ii

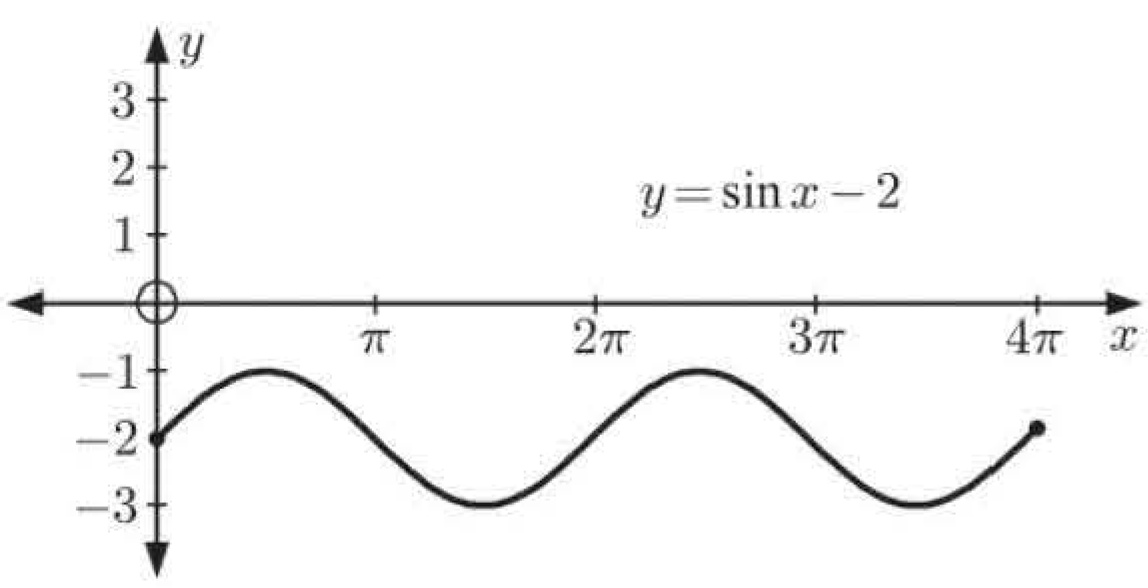


b Prediction:



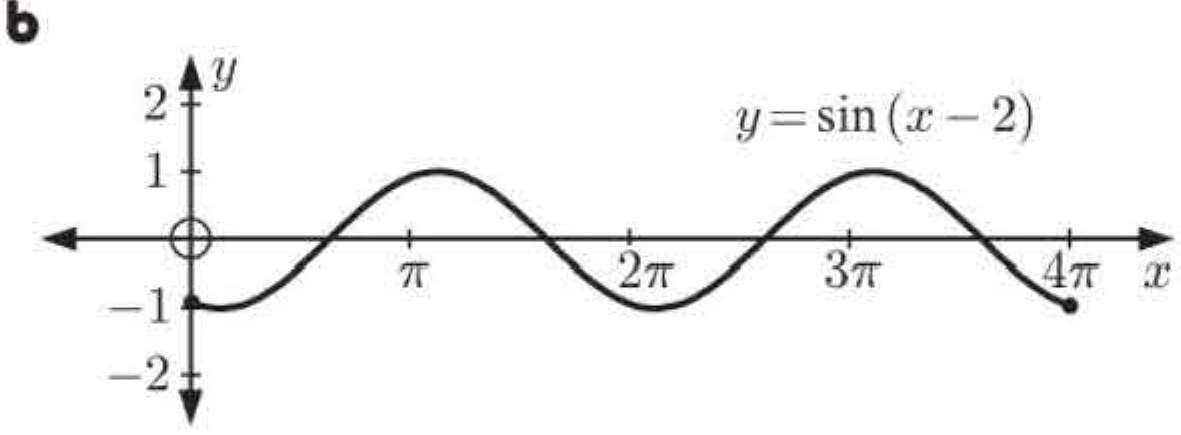
EXERCISE 12B.2

1 **a**



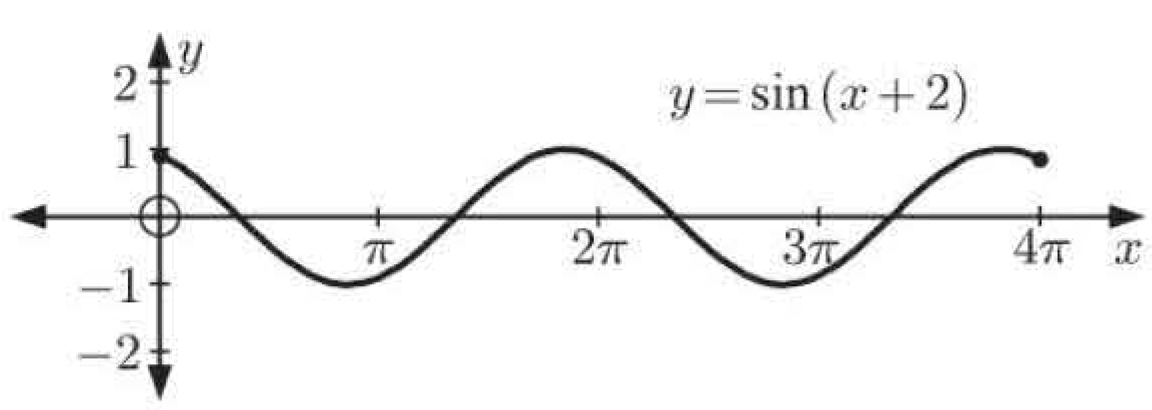
This is the graph of $y = \sin x$ translated by $\binom{0}{-2}$.

b



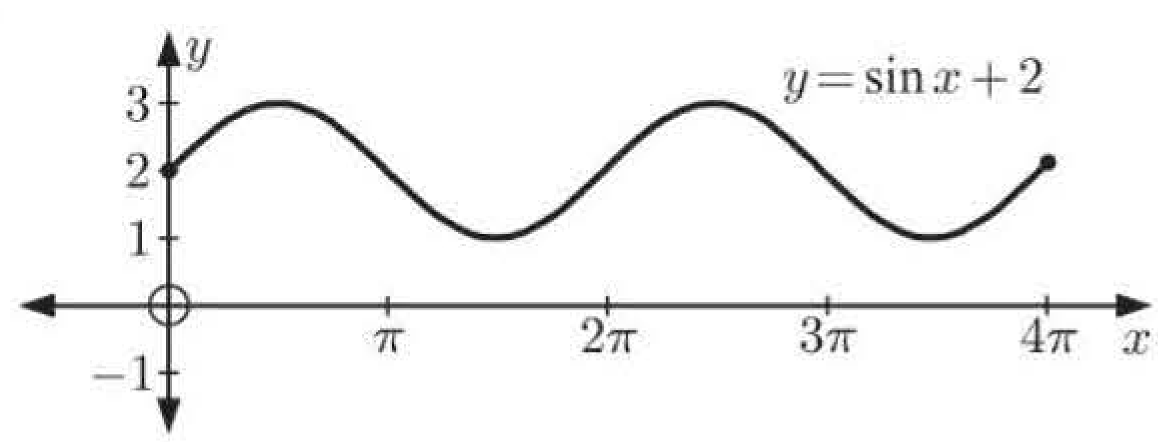
This is the graph of $y = \sin x$ translated by $\binom{2}{0}$.

c



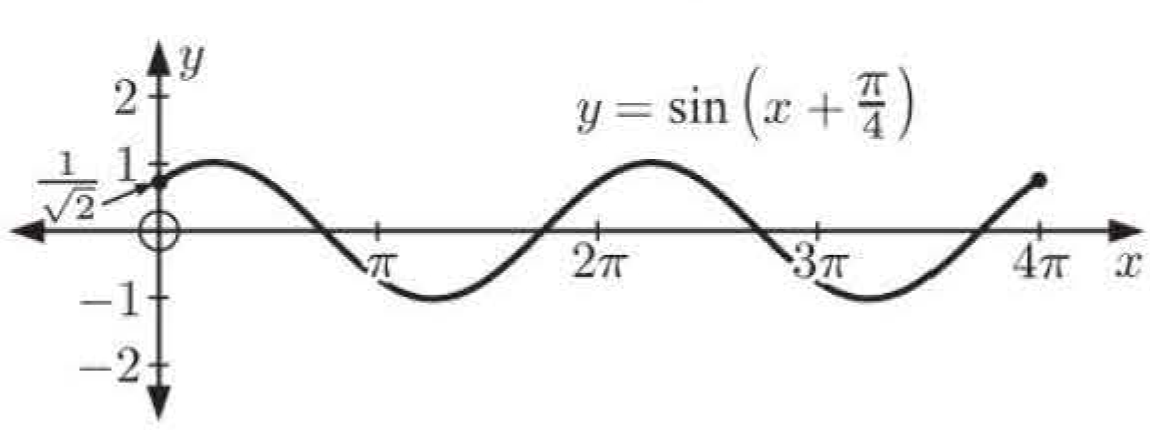
This is the graph of $y = \sin x$ translated by $\binom{-2}{0}$.

d



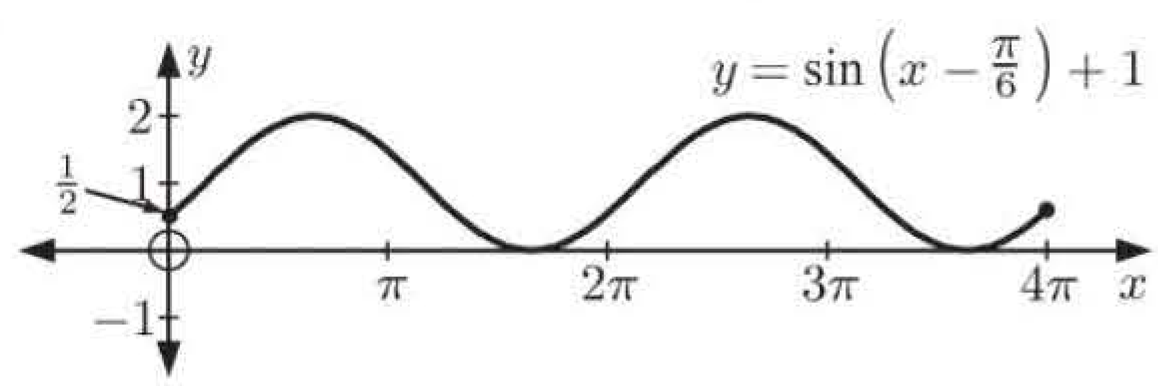
This is the graph of $y = \sin x$ translated by $\binom{0}{2}$.

e



This is the graph of $y = \sin x$ translated by $\begin{pmatrix} -\frac{\pi}{4} \\ 0 \end{pmatrix}$.

f



This is the graph of $y = \sin x$ translated by $\begin{pmatrix} \frac{\pi}{6} \\ 1 \end{pmatrix}$.

2 **a** period $= \dfrac{2\pi}{5} = \frac{2\pi}{5}$ **b** period $= \dfrac{2\pi}{(\frac{1}{4})} = 8\pi$ **c** period $= \dfrac{2\pi}{|-2|} = \pi$

3 **a** $\dfrac{2\pi}{b} = 3\pi$ $\therefore\ b = \frac{2}{3}$

b $\dfrac{2\pi}{b} = \dfrac{\pi}{10}$ $\therefore\ b = 20$

c $\dfrac{2\pi}{b} = 100\pi$ $\therefore\ b = \frac{2}{100} = \frac{1}{50}$

d $\dfrac{2\pi}{b} = 50$ $\therefore\ b = \frac{2\pi}{50} = \frac{\pi}{25}$

4 **a** A translation of $\binom{0}{-1}$, or vertically down 1 unit.

b A translation of $\begin{pmatrix} \frac{\pi}{4} \\ 0 \end{pmatrix}$, or horizontally $\frac{\pi}{4}$ units right.

c A vertical stretch of factor 2.

d A horizontal stretch of factor $\frac{1}{4}$.

e A vertical stretch of factor $\frac{1}{2}$.

f A horizontal stretch of factor 4.

g A reflection in the x-axis.

h A translation of $\binom{-2}{-3}$.

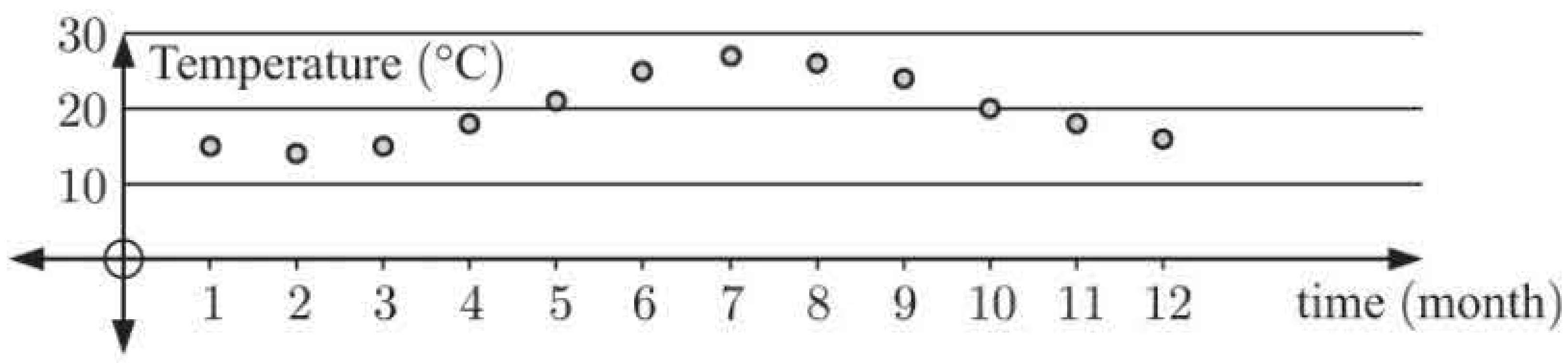
i A vertical stretch of factor 2 followed by a horizontal stretch of factor $\frac{1}{3}$.

j A translation of $\begin{pmatrix} \frac{\pi}{3} \\ 2 \end{pmatrix}$.

EXERCISE 12C

1 **a**

Month, t	1	2	3	4	5	6	7	8	9	10	11	12
Temp, T	15	14	15	18	21	25	27	26	24	20	18	16



The period is 12 months so $\dfrac{2\pi}{b} = 12$ $\therefore\ b = \frac{\pi}{6}$ {assuming $b > 0$}.

Amplitude, $a \approx \dfrac{\text{max.} - \text{min.}}{2} \approx \dfrac{27-14}{2} \approx 6.5$

As the principal axis is midway between min. and max., then $d \approx \dfrac{27+14}{2} \approx 20.5$

When T is 20.5 (midway between min. and max.), $c \approx \dfrac{2+7}{2} \approx 4.5$ {average of t values}

$\therefore\ T \approx 6.5\sin(\frac{\pi}{6}(t-4.5)) + 20.5$ where $\frac{\pi}{6} \approx 0.524$

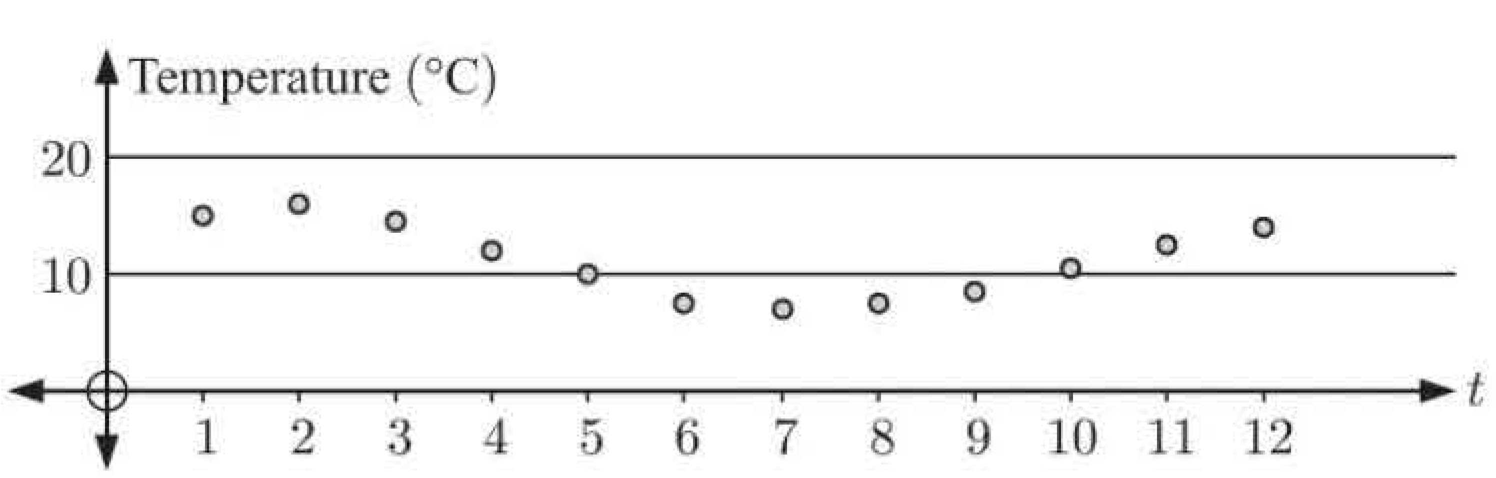
b Using technology, $T \approx 6.14\sin(0.575t - 2.70) + 20.4$

$\therefore\ T \approx 6.14\sin(0.575(t - 4.70)) + 20.4$

The model fits reasonably well.

2 **a**

Month, t	1	2	3	4	5	6	7	8	9	10	11	12
Temp, T	15	16	$14\frac{1}{2}$	12	10	$7\frac{1}{2}$	7	$7\frac{1}{2}$	$8\frac{1}{2}$	$10\frac{1}{2}$	$12\frac{1}{2}$	14



The period is $\dfrac{2\pi}{b} = 12$ $\therefore\ b = \frac{\pi}{6}$ $\{b > 0\}$

Amplitude, $a \approx \dfrac{\text{max.} - \text{min.}}{2} \approx \dfrac{16-7}{2} \approx 4.5$

As the principal axis is midway between min. and max. then $d \approx \dfrac{16+7}{2} \approx 11.5$

At min., $t = 7$ and at max., $t = 2 + 12 = 14$ $\therefore$ $c \approx \dfrac{7+14}{2} \approx 10.5$

So, $T \approx 4.5\sin(\frac{\pi}{6}(t - 10.5)) + 11.5$

b Using technology, $T \approx 4.29\sin(0.533t + 0.769) + 11.2$

$\therefore$ $T \approx 4.29\sin(0.533(t + 1.44)) + 11.2$

Note: (1) $\frac{\pi}{6} \approx 0.524$ ✓

(2) $1.44 - (-10.5)$
$= 11.94 \approx 12$

3 **a** For the model $H = a\sin(b(t - c)) + d$

$$\text{period} = \frac{2\pi}{b} = 12.4 \text{ hours} \quad \therefore \quad b = \frac{2\pi}{12.4} \approx 0.507$$

We let the principal axis be 0, so $d = 0$

$\therefore$ the amplitude $a = 7$, so the min. is -7, and the max. is $+7$

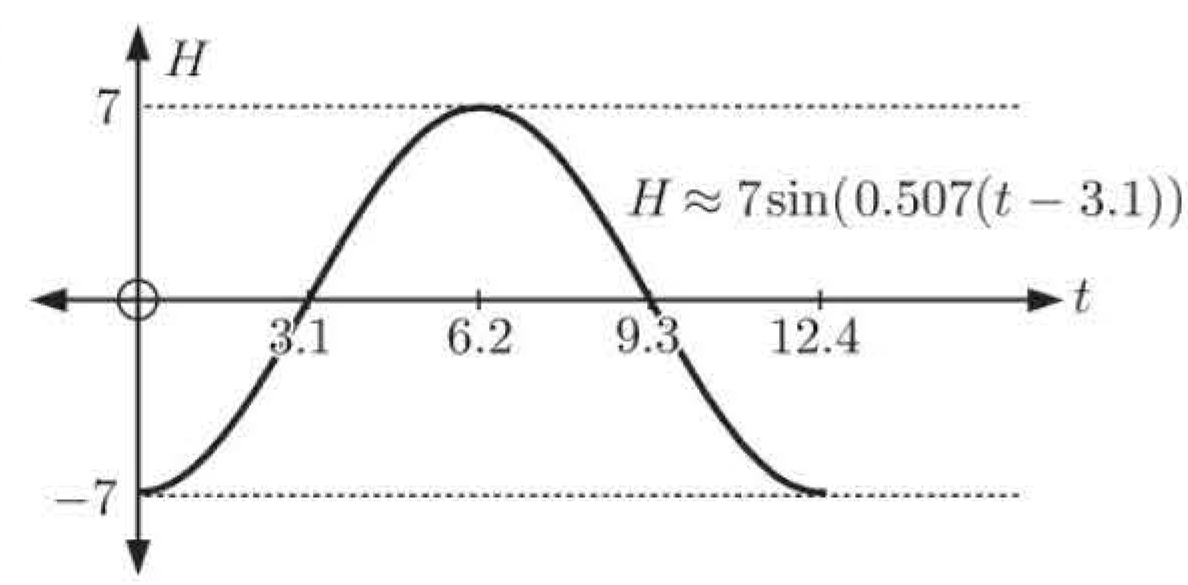
Let $t = 0$ correspond to 'low tide' $\therefore$ $t = 6.2$ corresponds to 'high tide'

$\therefore$ $c = \dfrac{0 + 6.2}{2} = 3.1$

So, $H \approx 7\sin(0.507(t - 3.1)) + 0$

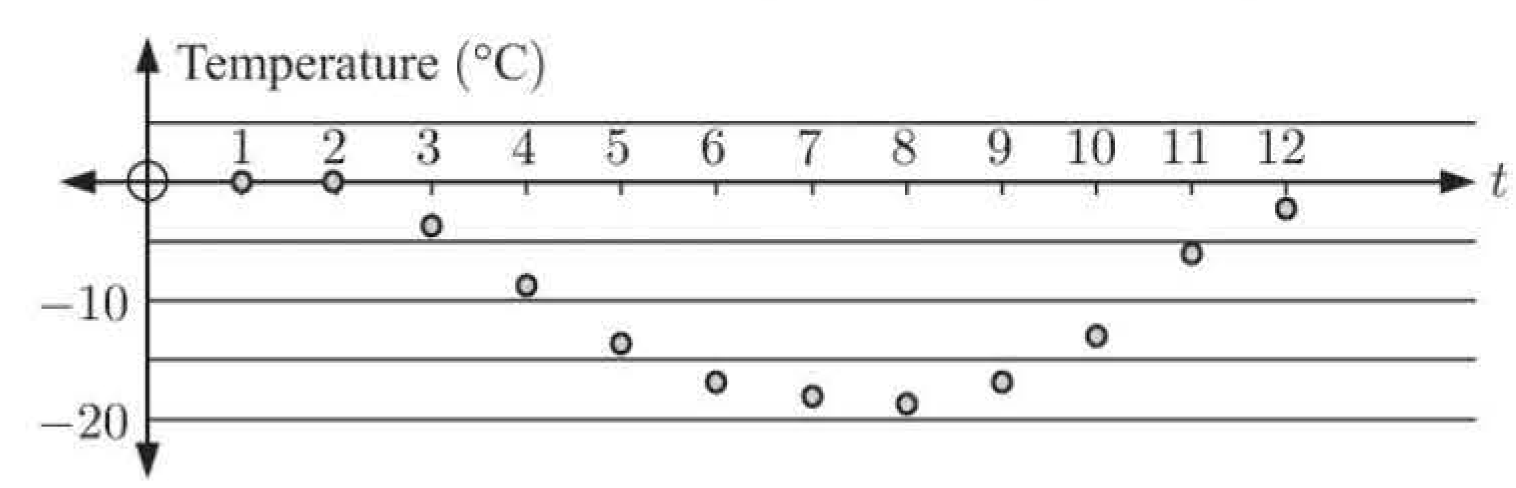
$\therefore$ $H \approx 7\sin(0.507(t - 3.1))$

b



4 **a**

Month, t	1	2	3	4	5	6	7	8	9	10	11	12
Temp, T	0	0	-4	-9	-14	-17	-18	-19	-17	-13	-6	-2



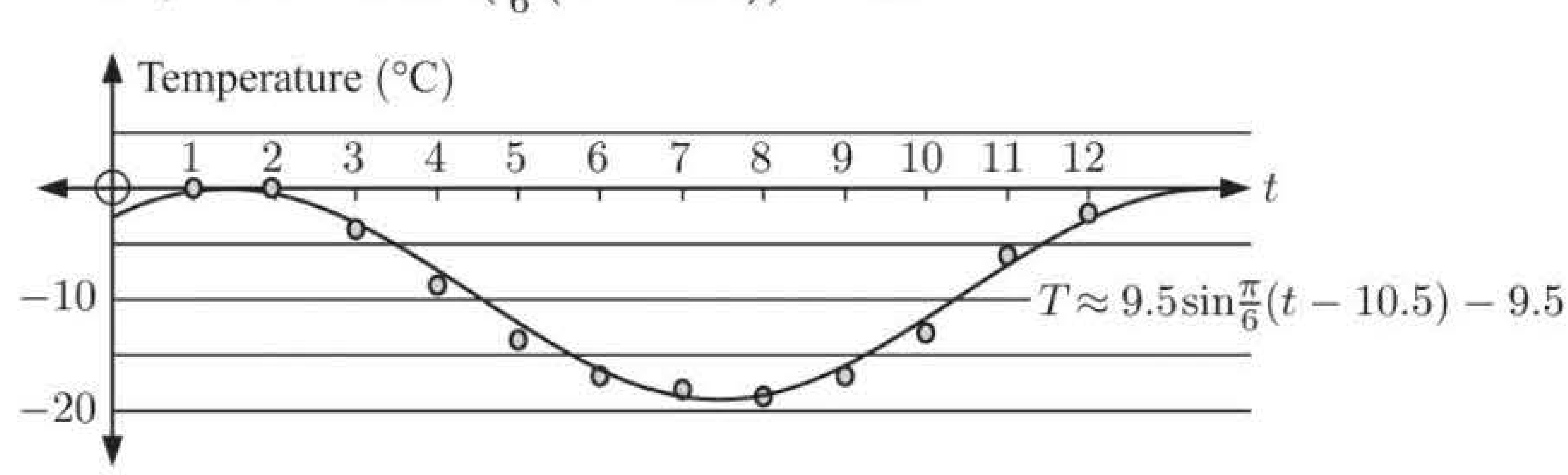
The period is $\dfrac{2\pi}{b} = 12$ $\therefore$ $b = \frac{\pi}{6}$ $\{b > 0\}$

$$\text{Amplitude,} \quad a \approx \frac{\text{max.} - \text{min.}}{2} \approx \frac{0 - (-19)}{2} \approx 9.5$$

$$d \approx \frac{\text{max.} + \text{min.}}{2} \approx \frac{0 + (-19)}{2} \approx -9.5$$

At min., $t = 8$ and at max., $t = 1 + 12 = 13$ $\therefore$ $c \approx \dfrac{8 + 13}{2} \approx 10.5$

So, $T \approx 9.5\sin(\frac{\pi}{6}(t - 10.5)) - 9.5$



b The model is reasonably appropriate.

5 Let the model be $H = a\sin(b(t-c)) + d$ metres

When $t = 0$, $\underset{\text{min.}}{\underset{\uparrow}{H = 2}}$ and when $t = 50$, $\underset{\text{max.}}{\underset{\uparrow}{H = 22}}$

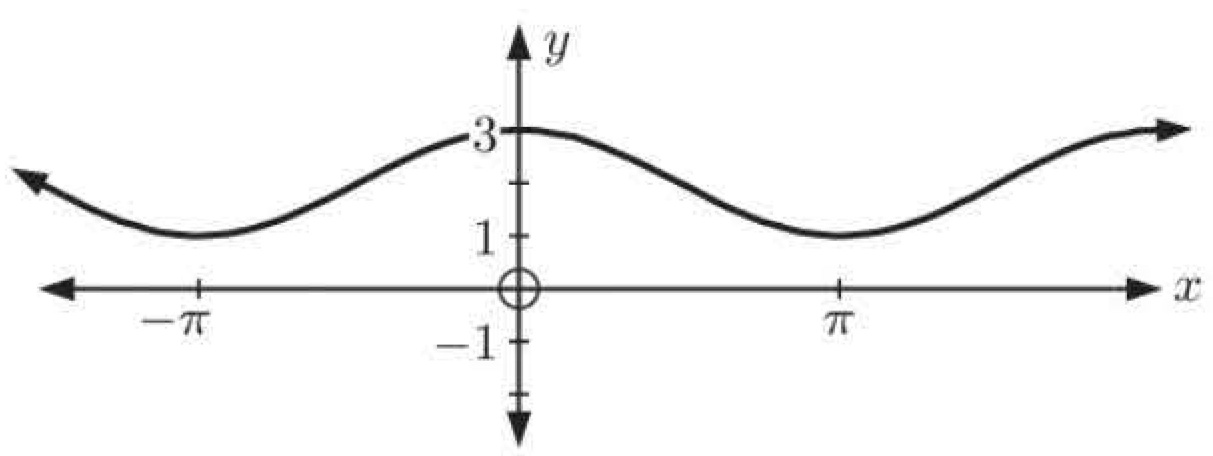
period $= \dfrac{2\pi}{b} = 100 \quad \therefore \quad b = \dfrac{2\pi}{100} = \dfrac{\pi}{50}$

$a = 10$ {from the diagram}, $\quad d = \dfrac{\text{max.} + \text{min.}}{2} = \dfrac{22+2}{2} = 12$

$c = \dfrac{0+50}{2} = 25$ {values of t at min. and max.} $\quad \therefore \quad H = 10\sin(\frac{\pi}{50}(t-25)) + 12$

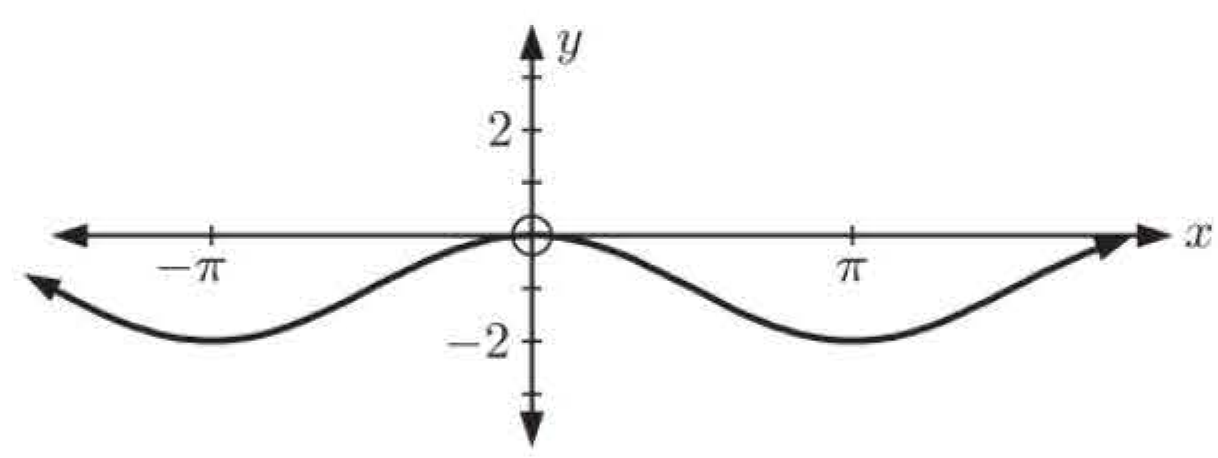
EXERCISE 12D

1 **a** $y = \cos x + 2$



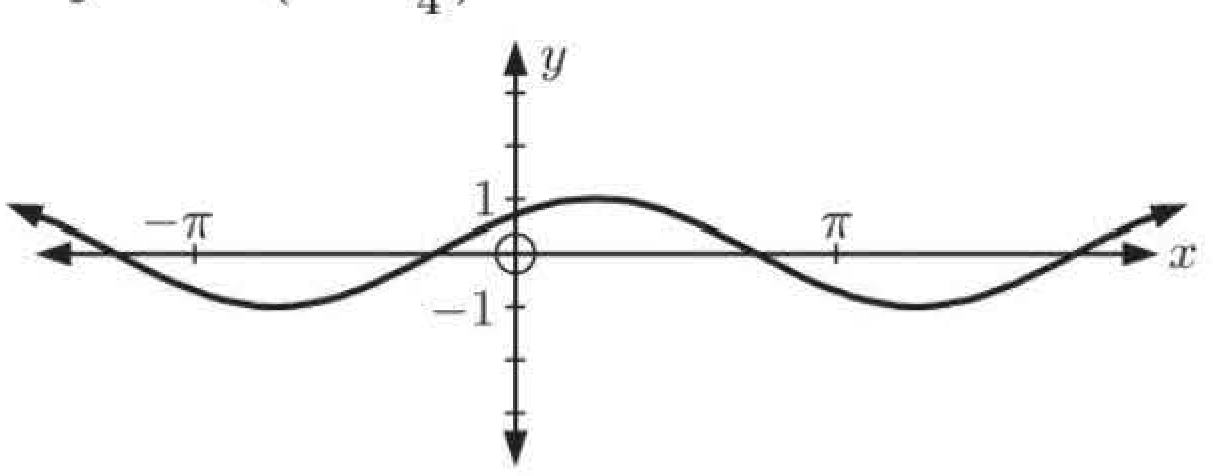
This is a vertical translation of $y = \cos x$ through $\binom{0}{2}$.

b $y = \cos x - 1$



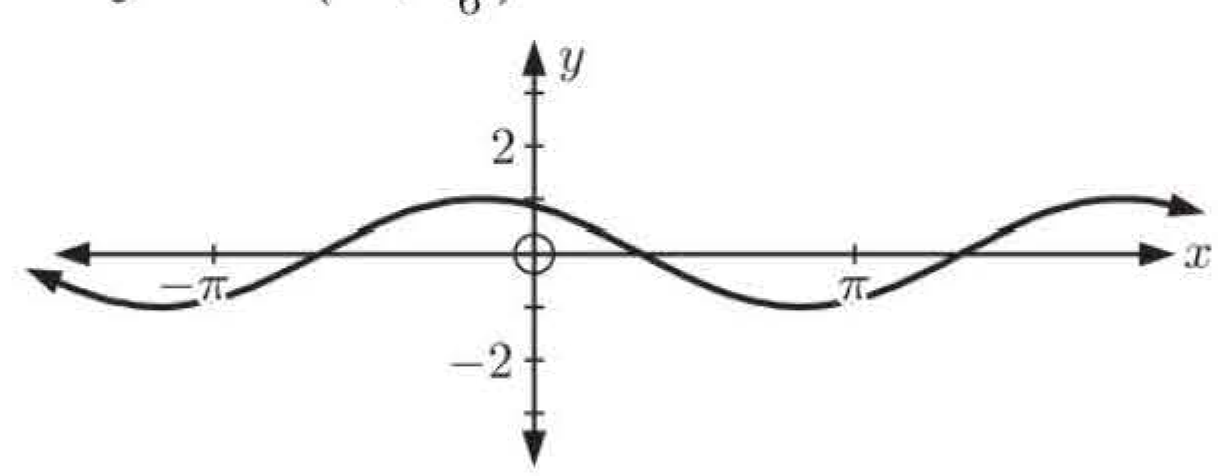
This is a vertical translation of $y = \cos x$ through $\binom{0}{-1}$.

c $y = \cos(x - \frac{\pi}{4})$



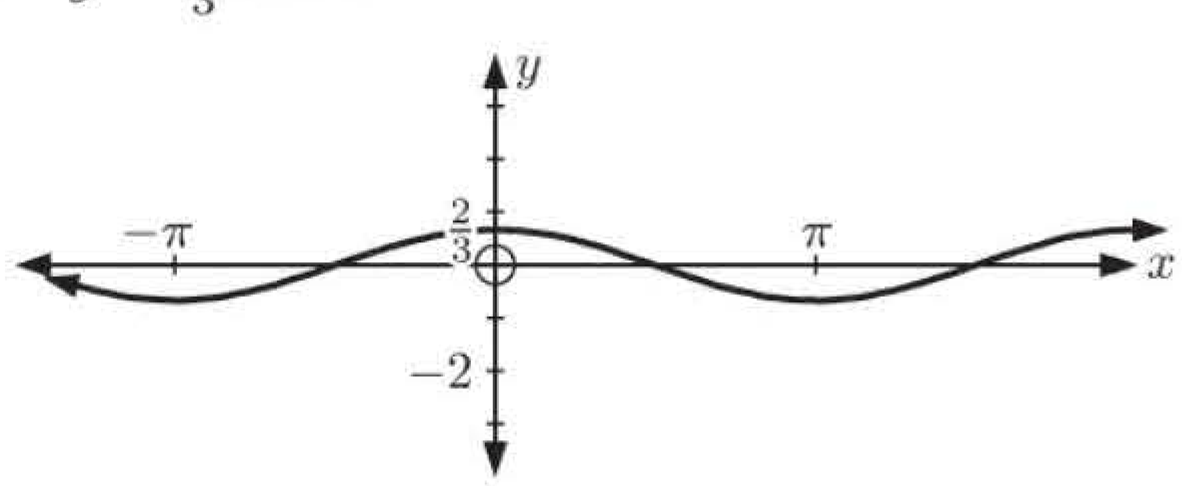
This is a horizontal translation of $y = \cos x$ through $\begin{pmatrix} \frac{\pi}{4} \\ 0 \end{pmatrix}$.

d $y = \cos(x + \frac{\pi}{6})$



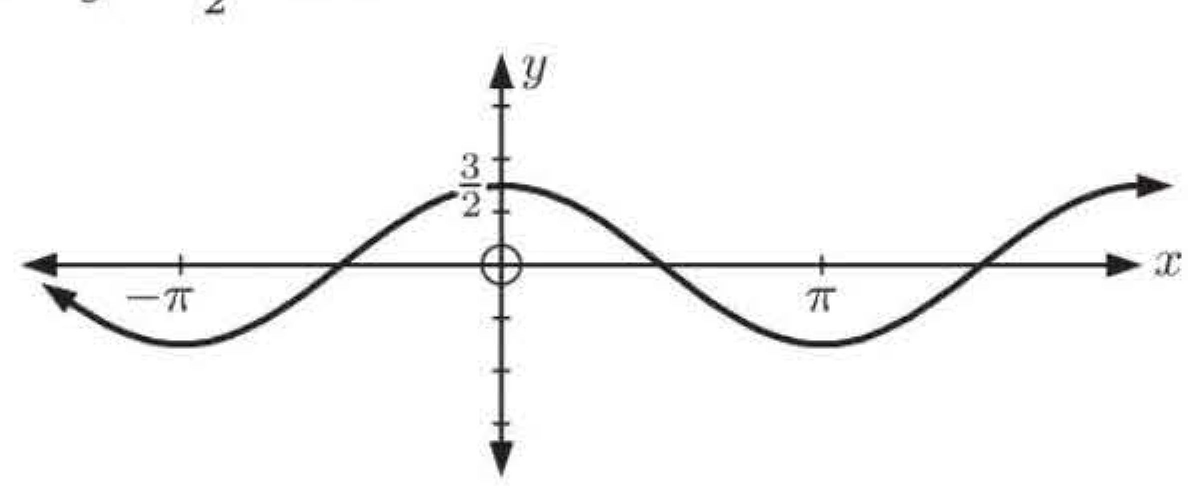
This is a horizontal translation of $y = \cos x$ through $\begin{pmatrix} -\frac{\pi}{6} \\ 0 \end{pmatrix}$.

e $y = \frac{2}{3}\cos x$



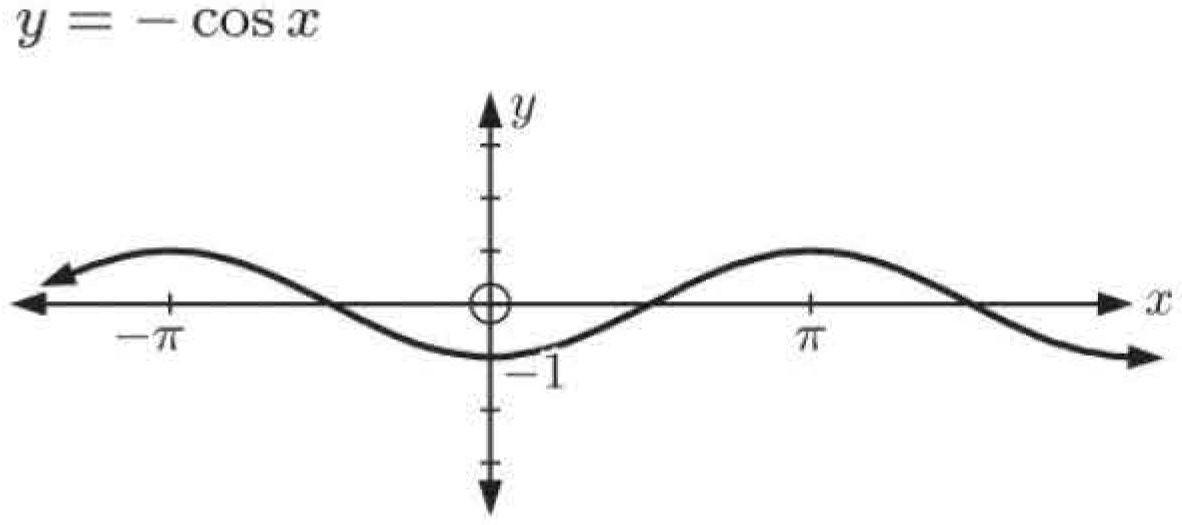
This is a vertical stretch of $y = \cos x$ with factor $\frac{2}{3}$.

f $y = \frac{3}{2}\cos x$



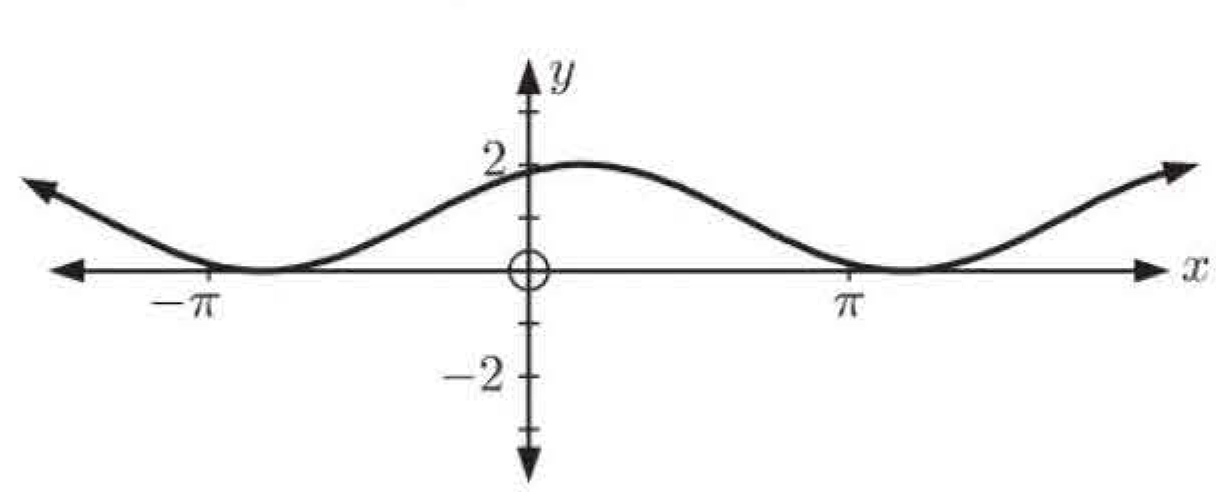
This is a vertical stretch of $y = \cos x$ with factor $\frac{3}{2}$.

g $y = -\cos x$



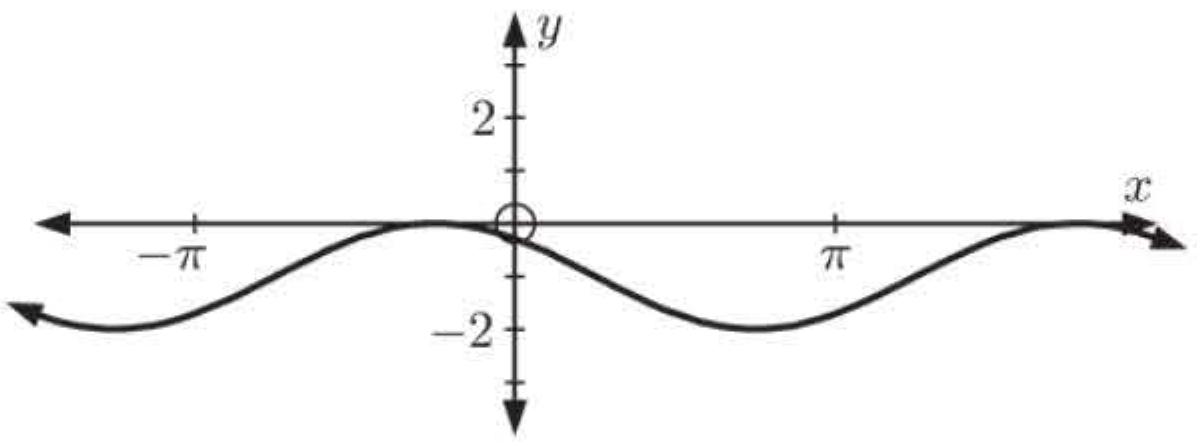
This is a reflection of $y = \cos x$ in the x-axis.

h $y = \cos(x - \frac{\pi}{6}) + 1$



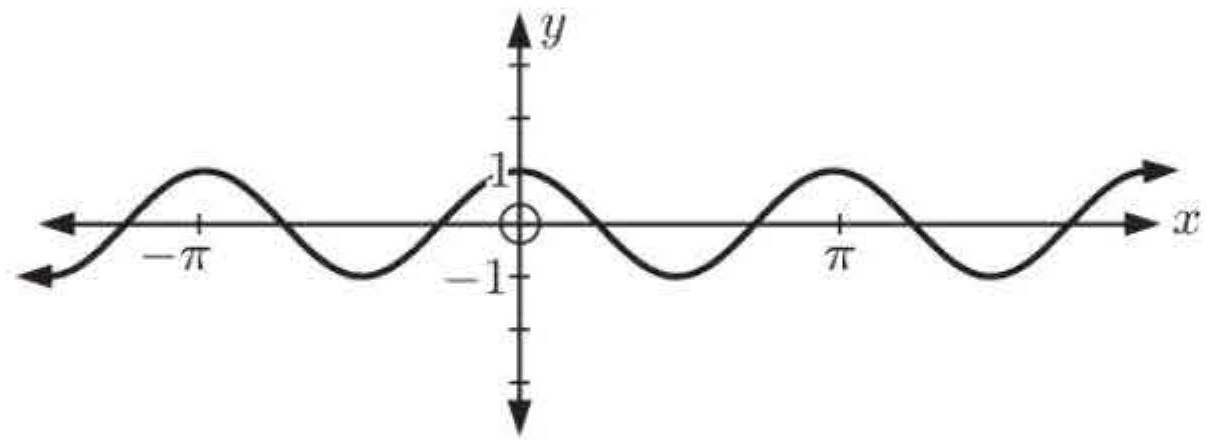
This is a translation of $\begin{pmatrix} \frac{\pi}{6} \\ 1 \end{pmatrix}$.

i $y = \cos(x + \frac{\pi}{4}) - 1$



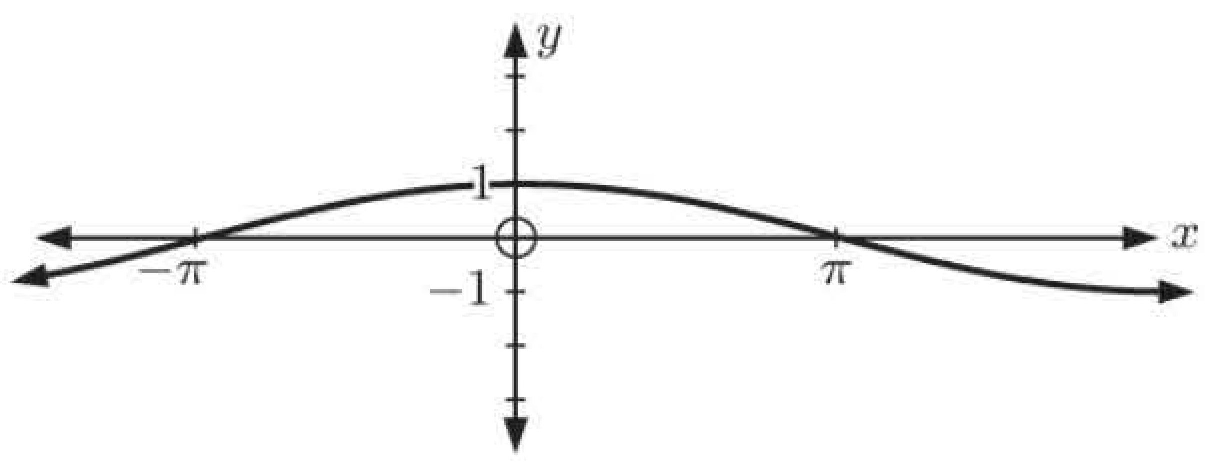
This is a translation of $\begin{pmatrix} -\frac{\pi}{4} \\ -1 \end{pmatrix}$.

j $y = \cos 2x$



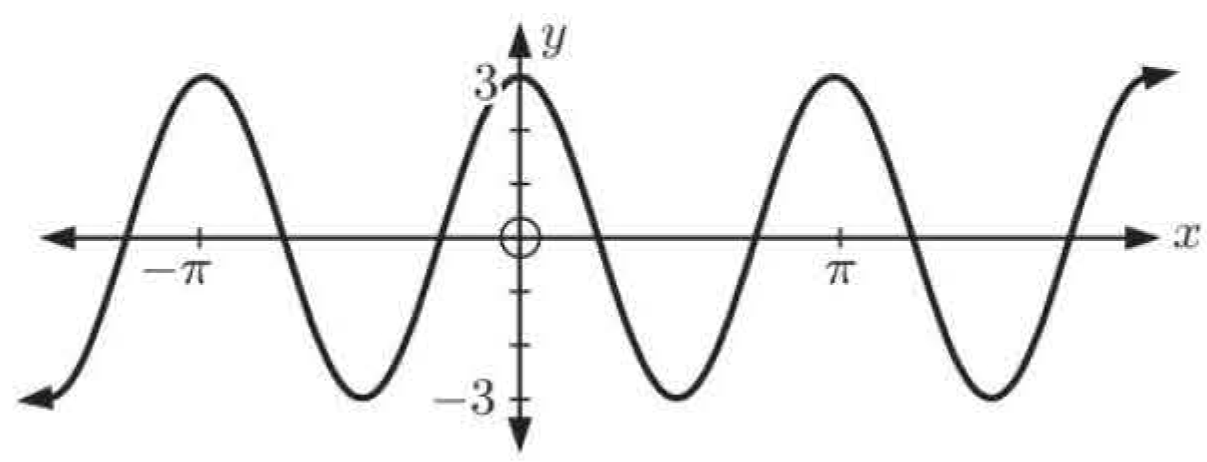
This is a horizontal stretch of factor $\frac{1}{2}$.

k $y = \cos\left(\frac{x}{2}\right)$



This is a horizontal stretch of factor 2.

l $y = 3\cos 2x$



This is a horizontal stretch of factor $\frac{1}{2}$ followed by a vertical stretch of factor 3.

2 **a** period $= \frac{2\pi}{3}$ **b** period $= \dfrac{2\pi}{\frac{1}{3}} = 6\pi$ **c** period $= \dfrac{2\pi}{\frac{\pi}{50}} = 100$

3 a controls the amplitude {amplitude $= |a|$}. b controls the period {period $= \dfrac{2\pi}{|b|}$}.

c controls the horizontal translation. d controls the vertical translation.

4 **a** If $y = a\cos(b(x-c)) + d$, then $a = 2$, $\pi = \dfrac{2\pi}{b}$ $\therefore$ $b = 2$

c and d are 0 as there is no horizontal or vertical shift. $\therefore$ $y = 2\cos(2x)$

b If $y = a\cos(b(x-c)) + d$, then $a = 1$, $4\pi = \dfrac{2\pi}{b}$ $\therefore$ $b = \frac{1}{2}$

A vertical shift of 2 units, no horizontal shift $\therefore$ $d = 2$, $c = 0$.

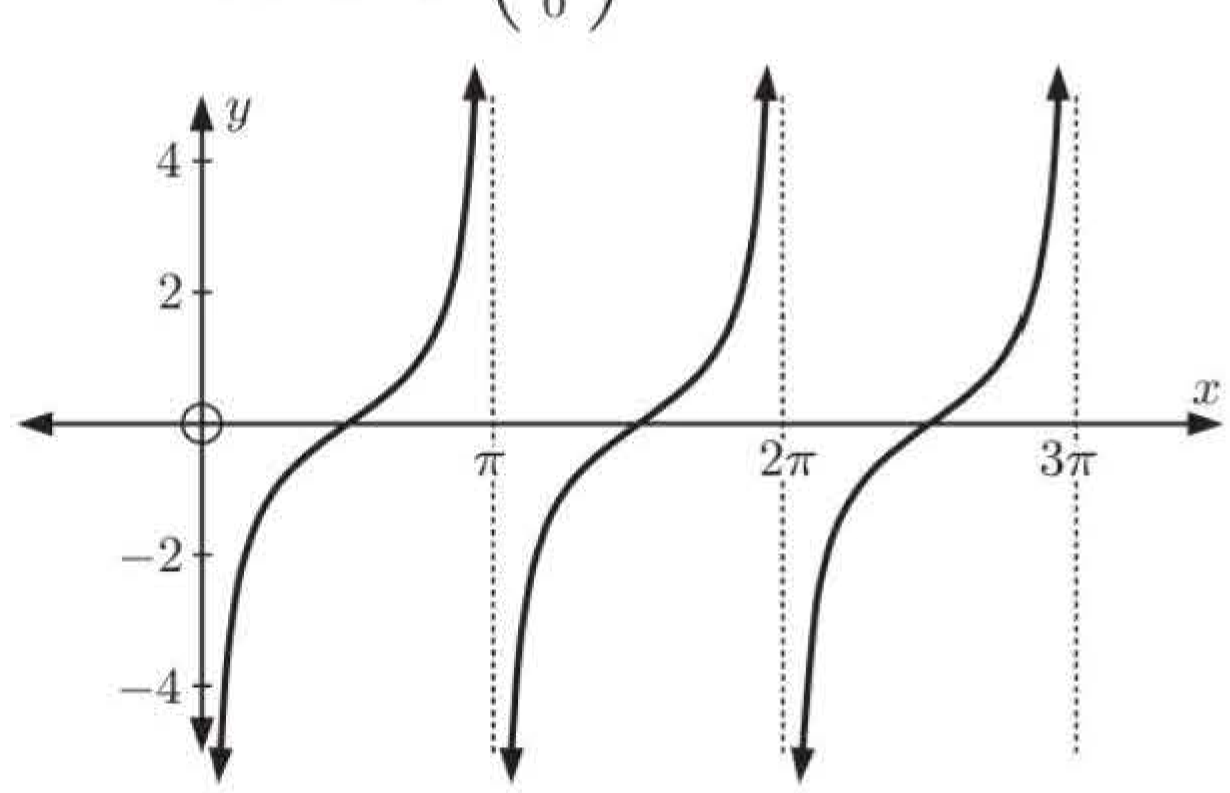
So, $y = \cos(\frac{1}{2}x) + 2$ or $y = \cos\left(\frac{x}{2}\right) + 2$.

c If $y = a\cos(b(x-c)) + d$, then $a = -5$, $6 = \dfrac{2\pi}{b}$ $\therefore$ $b = \frac{\pi}{3}$

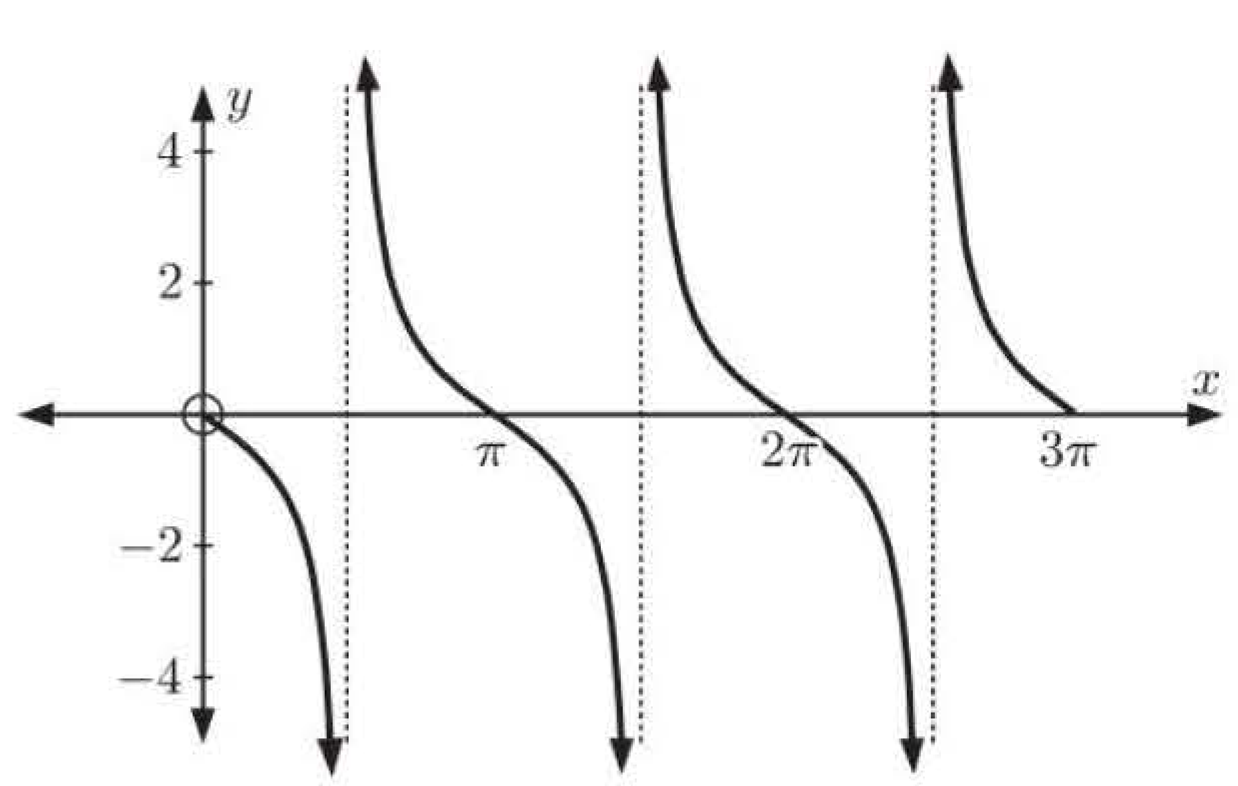
$c = d = 0$ {as there is no translation} $\therefore$ $y = -5\cos\left(\frac{\pi}{3}x\right)$

EXERCISE 12E

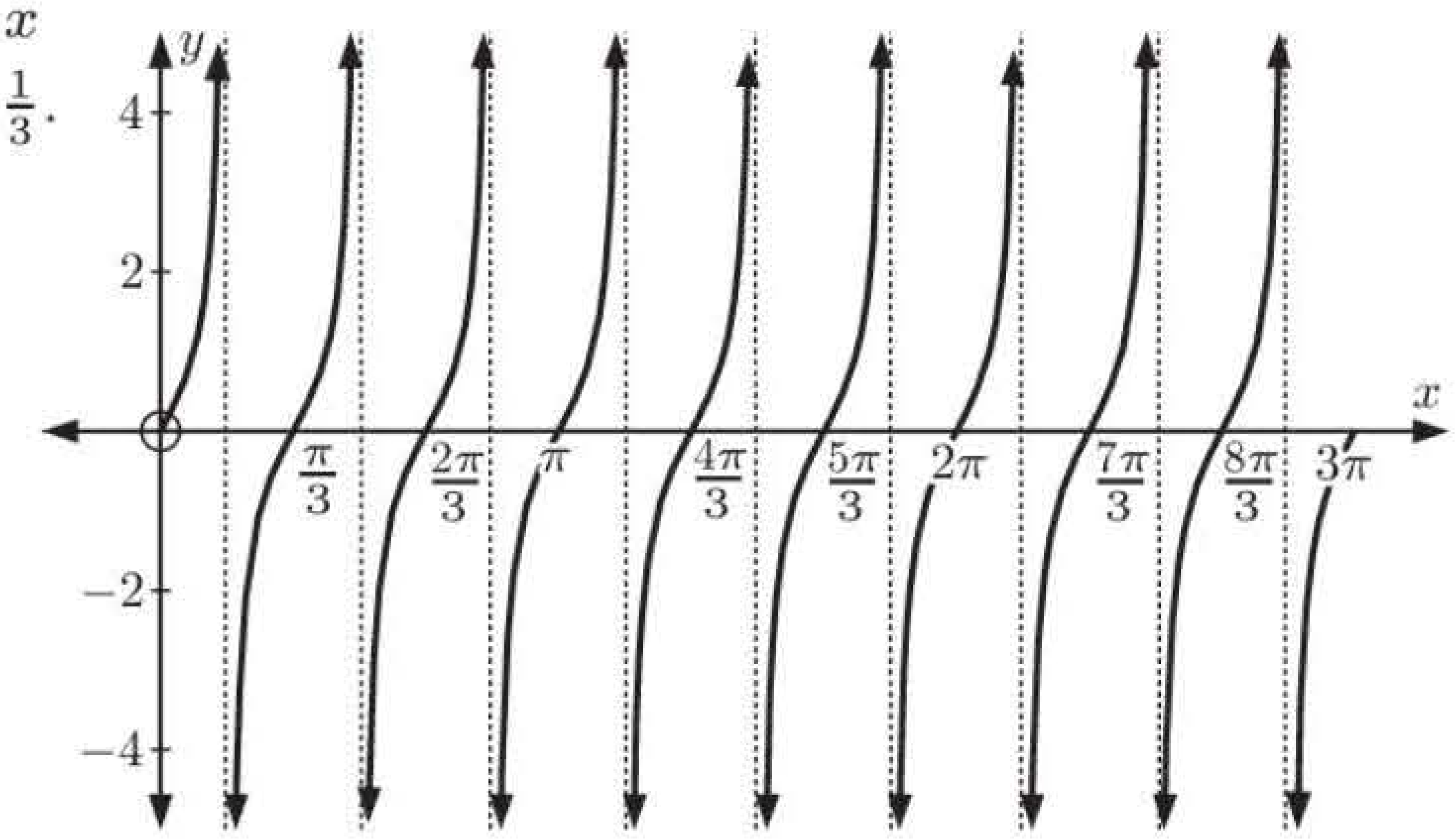
1 **a** **i** $y = \tan(x - \frac{\pi}{2})$ is $y = \tan x$ translated $\begin{pmatrix} \frac{\pi}{2} \\ 0 \end{pmatrix}$.



ii $y = -\tan x$ is $y = \tan x$ reflected in the x-axis.



iii $y = \tan 3x$ comes from $y = \tan x$ under a horizontal stretch of factor $\frac{1}{3}$.



2 **a** translation through $\binom{1}{2}$ **b** reflection in x-axis

c horizontal stretch, factor 2; vertical stretch, factor 2

3 **a** period $= \frac{\pi}{1} = \pi$ **b** period $= \frac{\pi}{3}$ **c** period $= \frac{\pi}{n}$

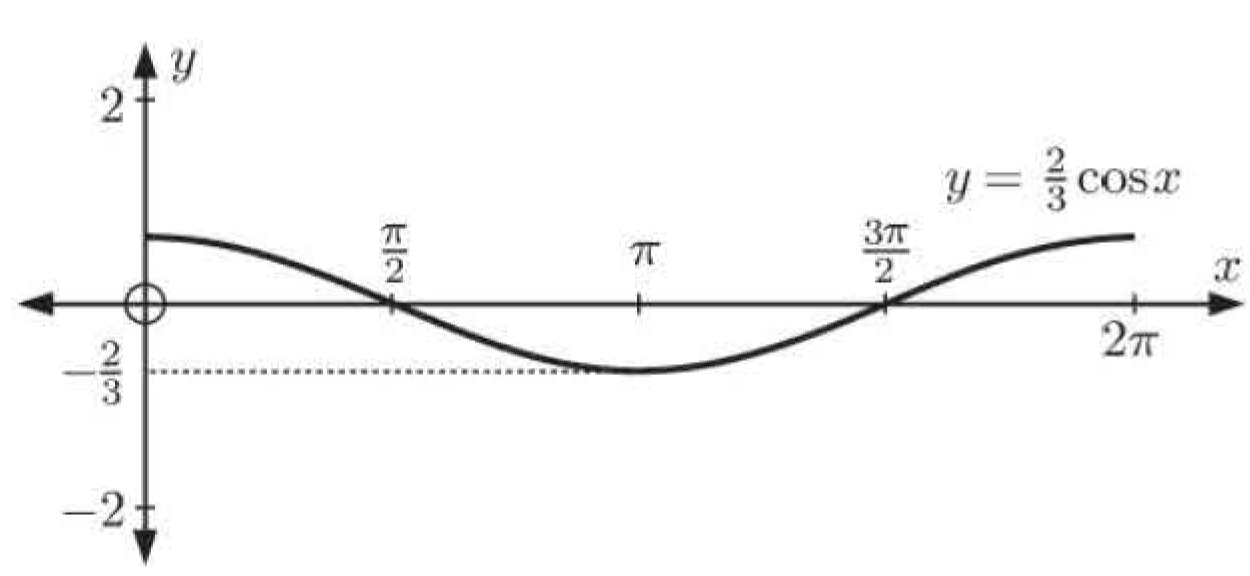
EXERCISE 12F

1 **a** amplitude $= |1| = 1$ **b** amplitude undefined **c** amplitude $= |-1| = 1$

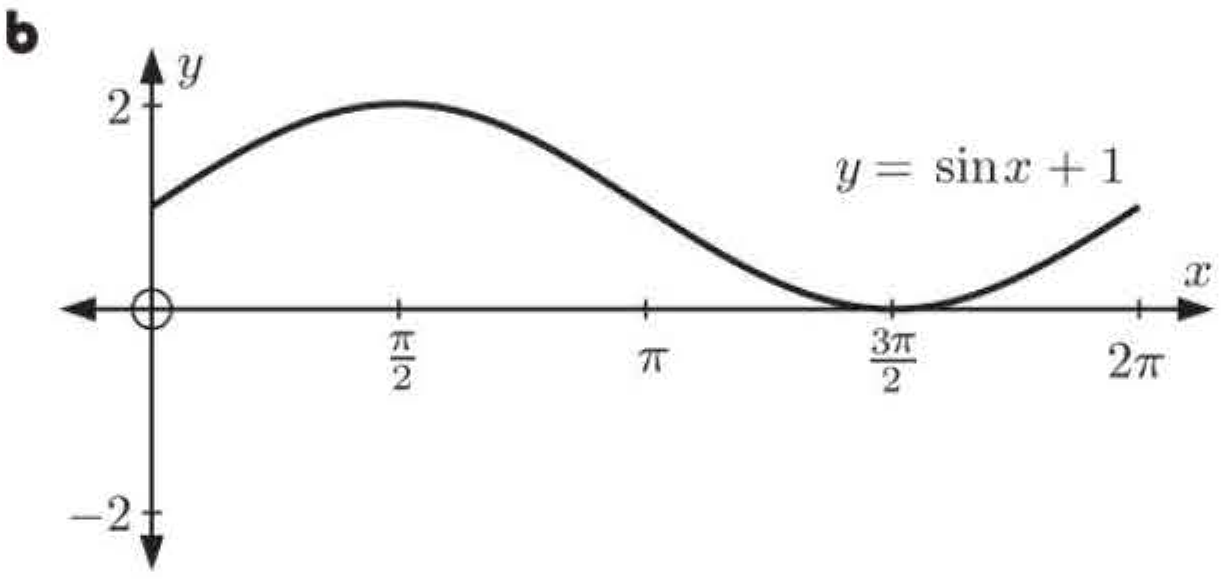
2 **a** period $= \dfrac{\pi}{1} = \pi$ **b** period $= \dfrac{2\pi}{\frac{1}{3}} = 6\pi$ **c** period $= \dfrac{2\pi}{2} = \pi$

3 **a** $\dfrac{2\pi}{b} = 2\pi$ $\therefore\ b = 1$ **b** $\dfrac{2\pi}{b} = \dfrac{2\pi}{3}$ $\therefore\ b = 3$ **c** $\dfrac{\pi}{b} = \dfrac{\pi}{2}$ $\therefore\ b = 2$ **d** $\dfrac{2\pi}{b} = 4$ $\therefore\ b = \frac{\pi}{2}$

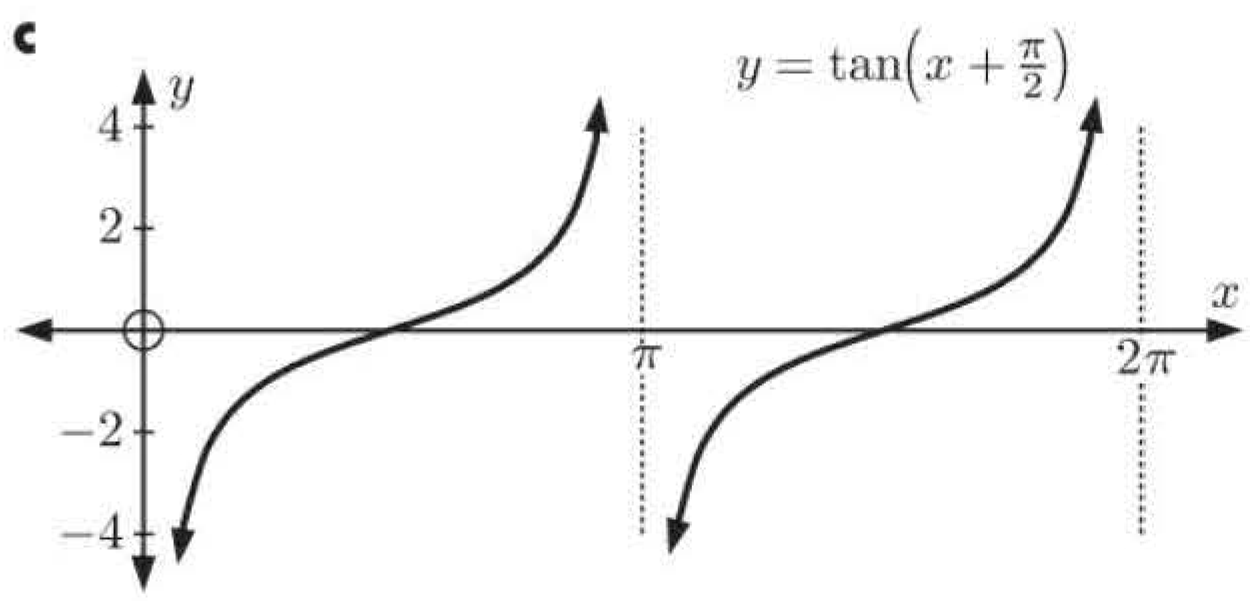
4 **a**



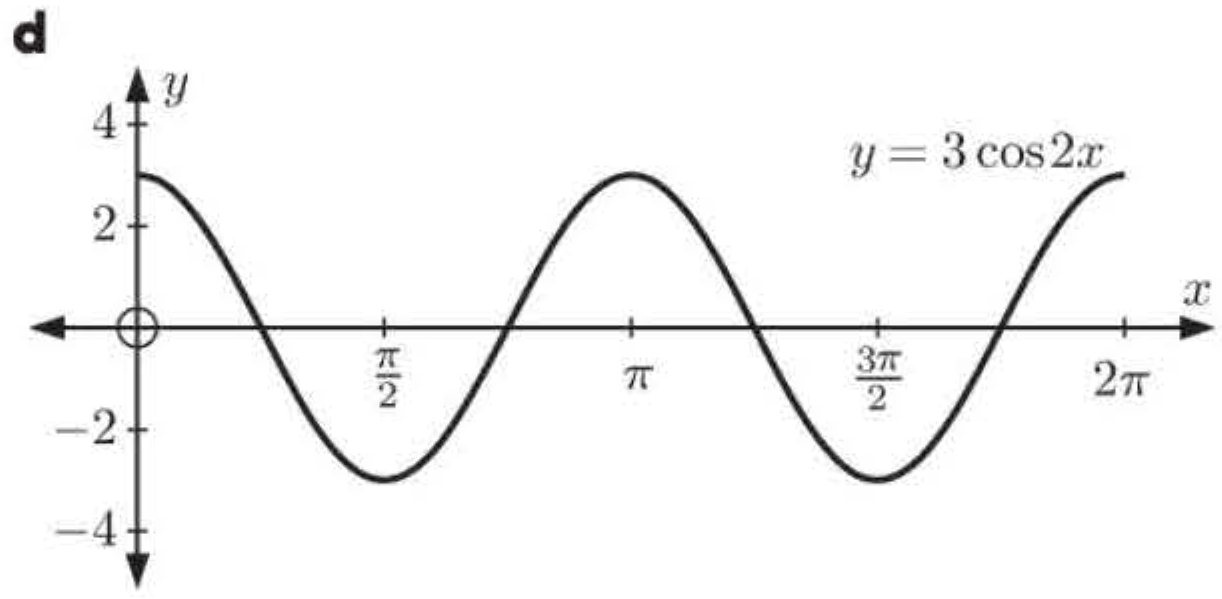
b



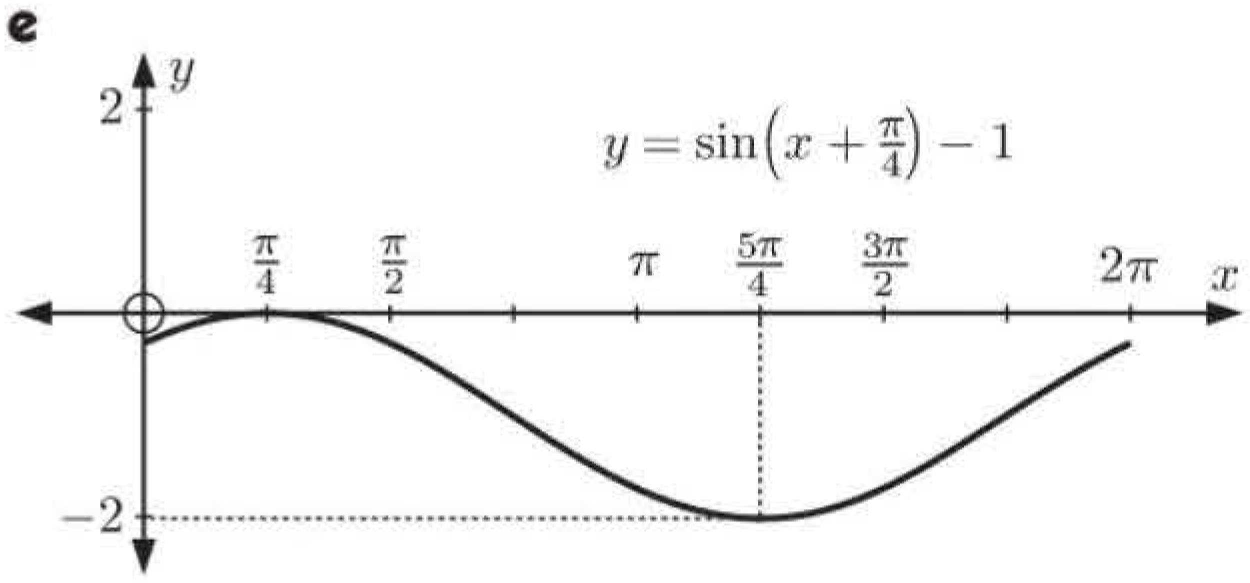
c



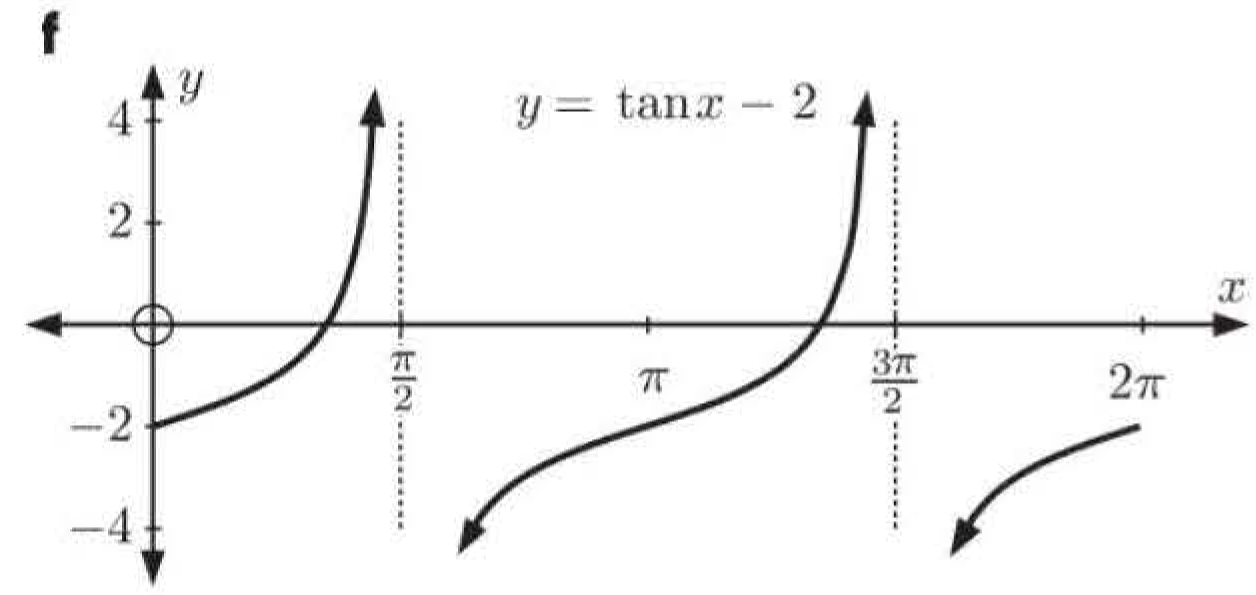
d



e



f



5 **a** $y = -\sin 5x$ has maximum value $-(-1) = 1$ $\{$when $\sin 5x = -1\}$
and minimum value $-(1) = -1$ $\{$when $\sin 5x = 1\}$

b $y = 3\cos x$ has maximum value $3(1) = 3$ $\{$when $\cos x = 1\}$
and minimum value $3(-1) = -3$ $\{$when $\cos x = -1\}$

c $y = 2\tan x$ has no maximum or minimum values.

d $y = -\cos 2x + 3$ has maximum value $-(-1) + 3 = 4$ $\{$when $\cos 2x = -1\}$
and minimum value $-(1) + 3 = 2$ $\{$when $\cos 2x = 1\}$

e $y = 1 + 2\sin x$ has maximum value $1 + 2(1) = 3$ $\{$when $\sin x = 1\}$
and minimum value $1 + 2(-1) = -1$ $\{$when $\sin x = -1\}$

f $y = \sin\left(x - \frac{\pi}{2}\right) - 3$ has maximum value $1 - 3 = -2$ $\{$when $\sin\left(x - \frac{\pi}{2}\right) = 1\}$
and minimum value $-1 - 3 = -4$ $\{$when $\sin\left(x - \frac{\pi}{2}\right) = -1\}$

6 **a** vertical stretch, factor $\frac{1}{2}$
b horizontal stretch, factor 4
c reflection in the x-axis
d vertical translation down 2 units
e horizontal translation $\frac{\pi}{4}$ units to the left
f reflection in the y-axis

7 The amplitude is 2, so $m = 2$.
The principal axis is $y = -3$, so $n = -3$.

8 The period is 2π, so $\dfrac{\pi}{p} = 2\pi$

$\therefore\ p = \frac{1}{2}$

The graph has undergone a vertical translation of 1 unit, so $q = 1$.

EXERCISE 12G

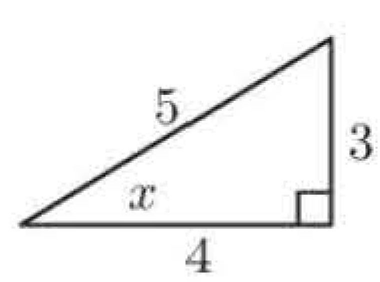
1 **a** $\sin(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}$
$\therefore\ \csc(\frac{\pi}{3}) = \frac{2}{\sqrt{3}}$

b $\tan(\frac{2\pi}{3}) = -\sqrt{3}$
$\therefore\ \cot(\frac{2\pi}{3}) = -\frac{1}{\sqrt{3}}$

c $\cos(\frac{5\pi}{6}) = -\frac{\sqrt{3}}{2}$
$\therefore\ \sec(\frac{5\pi}{6}) = -\frac{2}{\sqrt{3}}$

d $\tan(\pi) = 0$
$\therefore\ \cot(\pi)$ is undefined.

2 **a** $\sin x = \frac{3}{5},\ \ 0 \leqslant x \leqslant \frac{\pi}{2}$

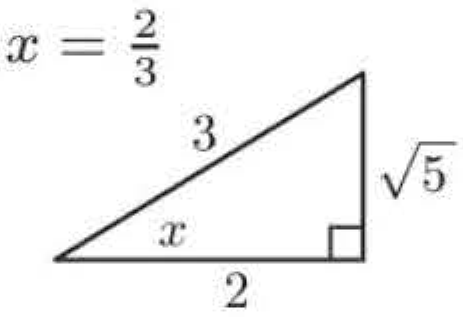


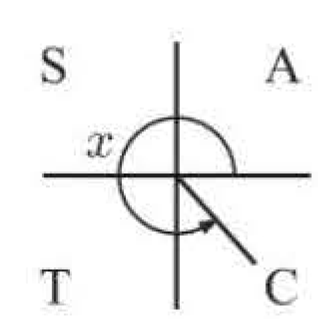
$\therefore\ \csc x = \dfrac{1}{\sin x} = \frac{5}{3}$

$\sec x = \dfrac{1}{\cos x} = \frac{5}{4}$

$\cot x = \dfrac{1}{\tan x} = \frac{4}{3}$

b $\cos x = \frac{2}{3}$





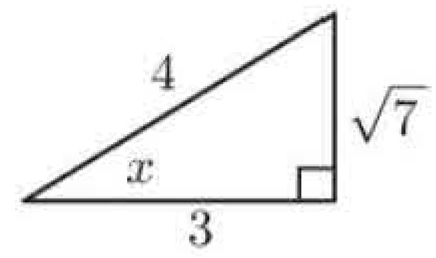
$\therefore\ \sin x = -\frac{\sqrt{5}}{3}$ and $\tan x = -\frac{\sqrt{5}}{2}$

$\therefore\ \csc x = -\frac{3}{\sqrt{5}}$

$\sec x = \frac{3}{2}$

$\cot x = -\frac{2}{\sqrt{5}}$

3 **a** $\cos x = \frac{3}{4}$



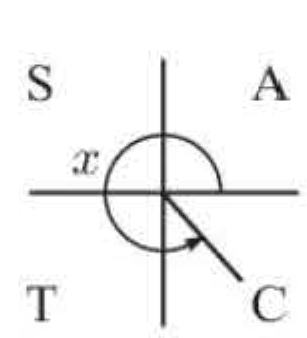
$\therefore\ \sin x = -\frac{\sqrt{7}}{4}$

$\tan x = -\frac{\sqrt{7}}{3}$

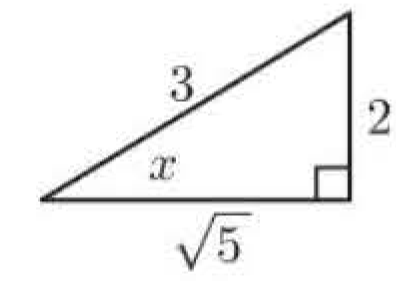
$\csc x = -\frac{4}{\sqrt{7}}$

$\sec x = \frac{4}{3}$

$\cot x = -\frac{3}{\sqrt{7}}$



b $\sin x = -\frac{2}{3}$



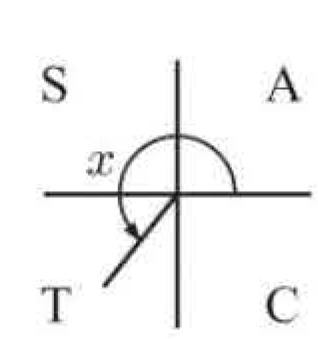
$\therefore\ \cos x = -\frac{\sqrt{5}}{3}$

$\tan x = \frac{2}{\sqrt{5}}$

$\csc x = -\frac{3}{2}$

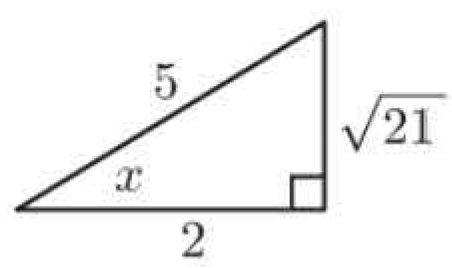
$\sec x = -\frac{3}{\sqrt{5}}$

$\cot x = \frac{\sqrt{5}}{2}$



c $\sec x = \frac{5}{2}$

$\therefore \quad \cos x = \frac{2}{5}$

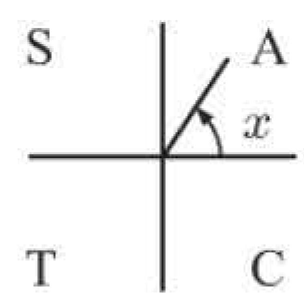


$\therefore \quad \sin x = \frac{\sqrt{21}}{5}$

$\tan x = \frac{\sqrt{21}}{2}$

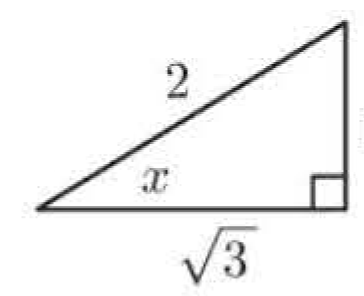
$\csc x = \frac{5}{\sqrt{21}}$

$\cot x = \frac{2}{\sqrt{21}}$



d $\csc x = 2$

$\therefore \quad \sin x = \frac{1}{2}$

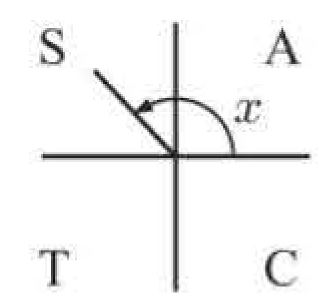


$\therefore \quad \cos x = -\frac{\sqrt{3}}{2}$

$\tan x = -\frac{1}{\sqrt{3}}$

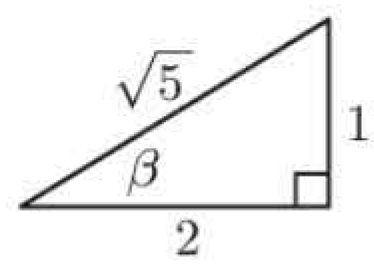
$\sec x = -\frac{2}{\sqrt{3}}$

$\cot x = -\sqrt{3}$



e $\tan \beta = \frac{1}{2}$

$\therefore \quad \cot \beta = 2$

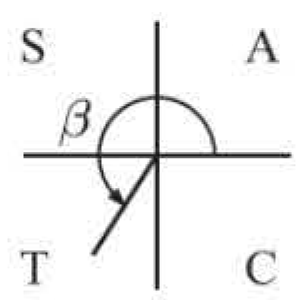


$\therefore \quad \sin \beta = -\frac{1}{\sqrt{5}}$

$\cos \beta = -\frac{2}{\sqrt{5}}$

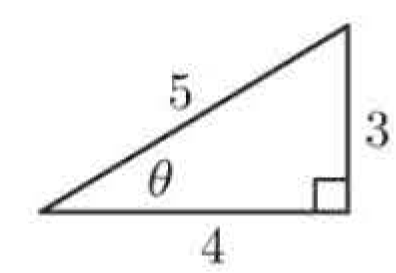
$\csc \beta = -\sqrt{5}$

$\sec \beta = -\frac{\sqrt{5}}{2}$



f $\cot \theta = \frac{4}{3}$

$\therefore \quad \tan \theta = \frac{3}{4}$

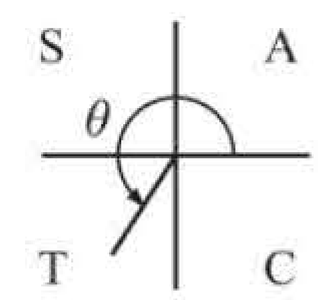


$\therefore \quad \sin \theta = -\frac{3}{5}$

$\cos \theta = -\frac{4}{5}$

$\csc \theta = -\frac{5}{3}$

$\sec \theta = -\frac{5}{4}$



4 **a** $\tan x \cot x$

$= \dfrac{\sin x}{\cos x} \times \dfrac{\cos x}{\sin x}$

$= 1$

b $\sin x \csc x$

$= \sin x \times \dfrac{1}{\sin x}$

$= 1$

c $\csc x \cot x$

$= \dfrac{1}{\sin x} \times \dfrac{\cos x}{\sin x}$

$= \dfrac{\cos x}{\sin^2 x}$

d $\sin x \cot x$

$= \sin x \times \dfrac{\cos x}{\sin x}$

$= \cos x$

e $\dfrac{\cot x}{\csc x}$

$= \dfrac{\cos x}{\sin x} \div \dfrac{1}{\sin x}$

$= \dfrac{\cos x}{\sin x} \times \dfrac{\sin x}{1}$

$= \cos x$

f $\dfrac{2\sin x \cot x + 3\cos x}{\cot x}$

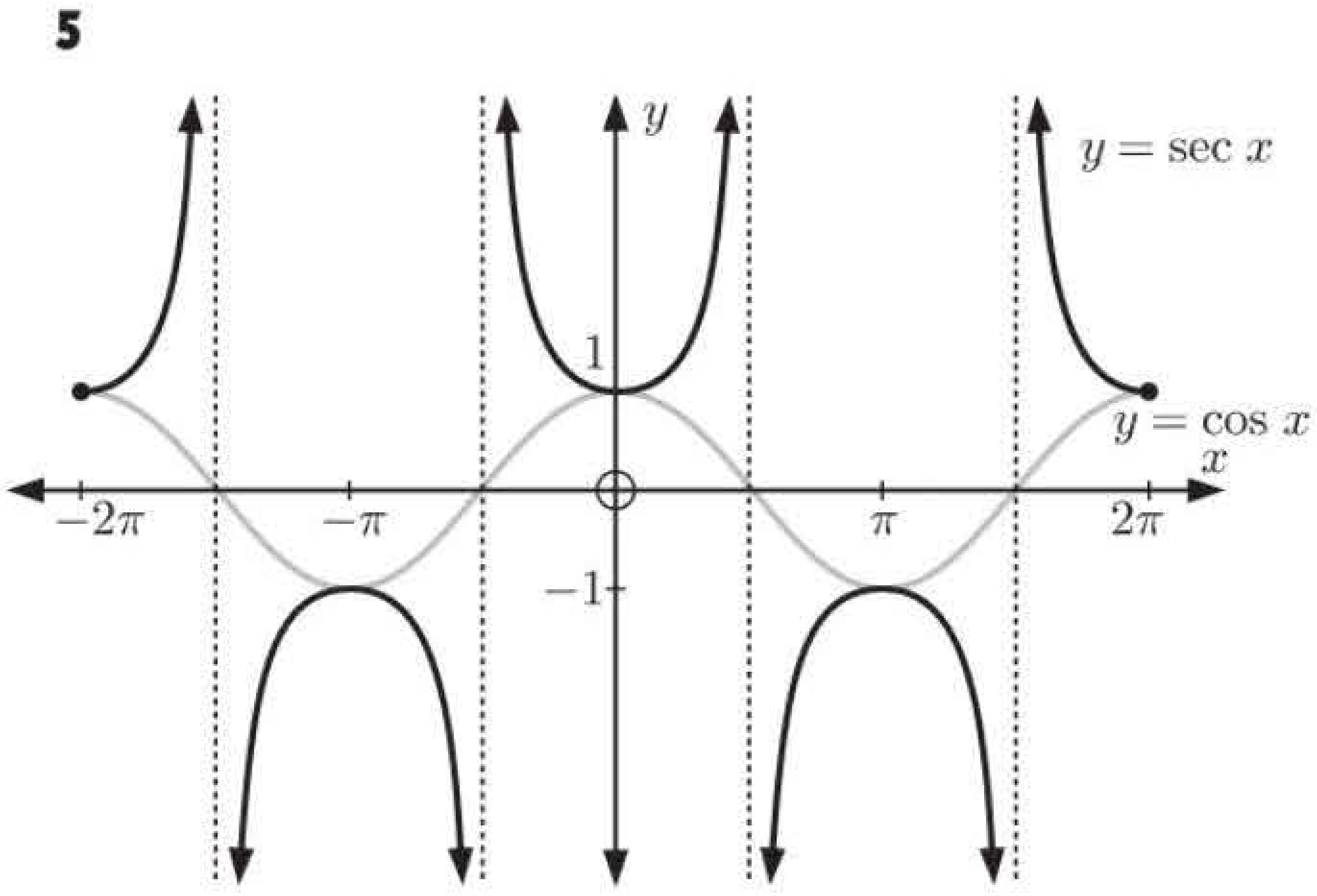
$= \dfrac{2\sin x \times \dfrac{\cos x}{\sin x} + 3\cos x}{\dfrac{\cos x}{\sin x}}$

$= (2\cos x + 3\cos x) \times \dfrac{\sin x}{\cos x}$

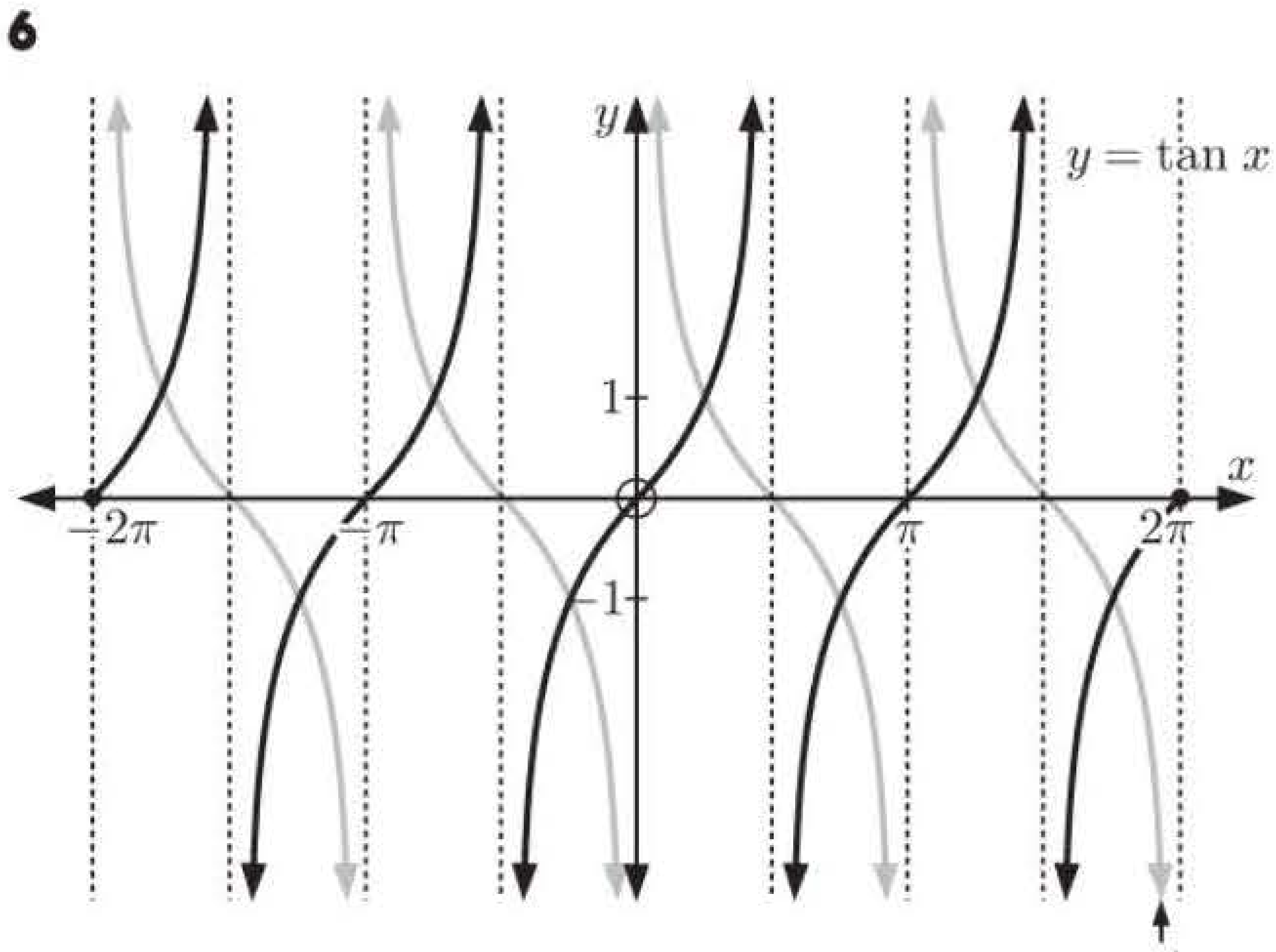
$= 5\cos x \times \dfrac{\sin x}{\cos x}$

$= 5\sin x$

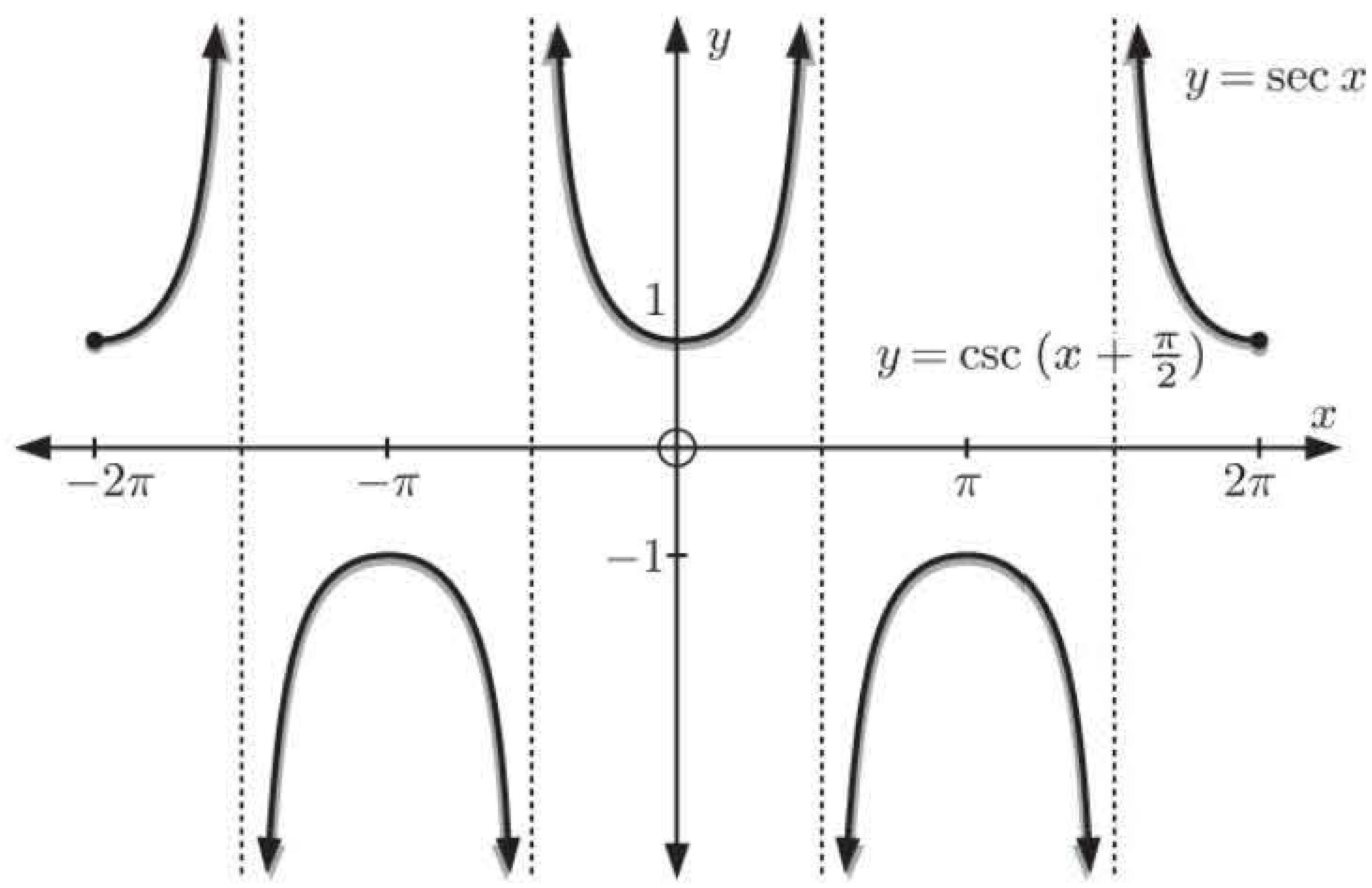
5



6



7



$\sin(x+\frac{\pi}{2}) = \cos x \quad \therefore \quad \csc(x+\frac{\pi}{2}) = \dfrac{1}{\sin(x+\frac{\pi}{2})} = \dfrac{1}{\cos x} = \sec x$

EXERCISE 12H

1

Function	*Restricted domain*	*Restricted range*	*Inverse function*	*Domain*	*Range*
$y = \sin x$	$-\frac{\pi}{2} \leqslant x \leqslant \frac{\pi}{2}$	$-1 \leqslant y \leqslant 1$	$y = \arcsin x$	$-1 \leqslant x \leqslant 1$	$-\frac{\pi}{2} \leqslant y \leqslant \frac{\pi}{2}$
$y = \cos x$	$0 \leqslant x \leqslant \pi$	$-1 \leqslant y \leqslant 1$	$y = \arccos x$	$-1 \leqslant x \leqslant 1$	$0 \leqslant y \leqslant \pi$
$y = \tan x$	$-\frac{\pi}{2} < x < \frac{\pi}{2}$	$y \in \mathbb{R}$	$y = \arctan x$	$x \in \mathbb{R}$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

2 **a** $\arccos(1) = 0$ **b** $\arcsin(-1) = -\frac{\pi}{2}$ **c** $\arctan(1) = \frac{\pi}{4}$

d $\arctan(-1) = -\frac{\pi}{4}$ **e** $\arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$ **f** $\arccos\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$

g $\arctan(\sqrt{3}) = \frac{\pi}{3}$ **h** $\arccos\left(-\frac{1}{\sqrt{2}}\right) = \frac{3\pi}{4}$ **i** $\arctan\left(-\frac{1}{\sqrt{3}}\right) = -\frac{\pi}{6}$

j $\sin^{-1}(-0.767) \approx -0.874$ **k** $\cos^{-1}(0.327) \approx 1.24$ **l** $\tan^{-1}(-50) \approx -1.55$

3 **a** The inverse transformation from $y = \sin x$ to $y = \arcsin x$ has an invariant point where $\sin x = \arcsin x$ $\therefore$ at $(0, 0)$.

b The inverse transformation from $y = \tan x$ to $y = \arctan x$ has an invariant point where $\tan x = \arctan x$ $\therefore$ at $(0, 0)$.

c The inverse transformation from $y = \cos x$ to $y = \arccos x$ has an invariant point where $\cos x = \arccos x$ $\therefore$ at $(0.739, 0.739)$.

4 **a** $y = \arctan x$ has horizontal asymptotes $y = -\frac{\pi}{2}$ and $y = \frac{\pi}{2}$.

b The functions $y = \arcsin x$ and $y = \arccos x$ each have points on the lines $x = -1$ and $x = 1$. So, these functions do not have vertical asymptotes.

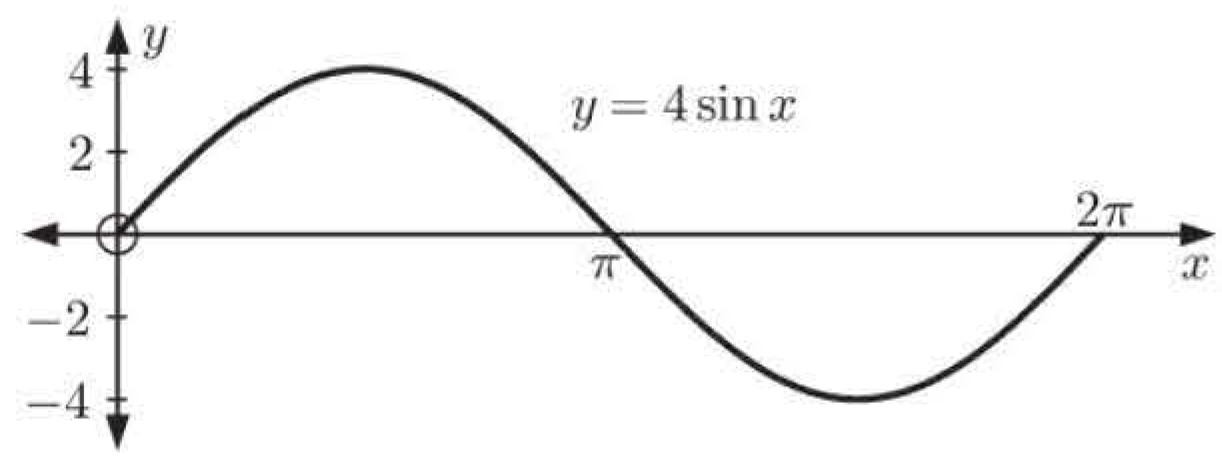
5 **a** $\arcsin(\sin\frac{\pi}{3}) = \frac{\pi}{3}$ **b** $\arccos(\cos(-\frac{\pi}{6})) = \frac{\pi}{6}$ **c** $\tan(\arctan(0.3)) = 0.3$

d $\cos(\arccos(-\frac{1}{2})) = -\frac{1}{2}$ **e** $\arctan(\tan\pi) = 0$ **f** $\arcsin(\sin\frac{4\pi}{3}) = -\frac{\pi}{3}$

REVIEW SET 12A

1 **a** not periodic **b** periodic

2 $y = 4\sin x$ has amplitude 4.



3 **a** $-1 \leqslant \sin x \leqslant 1$

$\therefore$ $1 + \sin x$ has minimum $1 + (-1) = 0$ and maximum $1 + 1 = 2$.

b $-1 \leqslant \cos 3x \leqslant 1$

$\therefore$ $-2\cos 3x$ has minimum $-2(1) = -2$ and maximum $-2(-1) = 2$.

4 **a** period $= \dfrac{2\pi}{\frac{1}{5}} = 10\pi$

b period $= \dfrac{2\pi}{4} = \frac{\pi}{2}$

c period $= \dfrac{2\pi}{\frac{1}{2}} = 4\pi$

d period $= \frac{\pi}{3}$

5

Function	*Period*	*Amplitude*	*Domain*	*Range*
$y = -3\sin(\frac{x}{4}) + 1$	8π	3	$x \in \mathbb{R}$	$-2 \leqslant y \leqslant 4$
$y = \tan 2x$	$\frac{\pi}{2}$	undefined	$x \neq \pm\frac{\pi}{4}, \pm\frac{3\pi}{4}, ...$	$y \in \mathbb{R}$
$y = 3\cos \pi x$	2	3	$x \in \mathbb{R}$	$-3 \leqslant y \leqslant 3$

6 **a** If $y = a\cos(b(x - c)) + d$

then $a = -4$, $\dfrac{2\pi}{b} = \pi$

$\therefore$ $b = 2$

$c = d = 0$

$\therefore$ $y = -4\cos 2x$

b If $y = a\cos(b(x - c)) + d$

then $a = 1$, $\dfrac{2\pi}{b} = 8$ $\therefore$ $b = \frac{\pi}{4}$

$d = \dfrac{\text{max.} + \text{min.}}{2} = \dfrac{3 + 1}{2} = 2$

$c = 0$

So, $y = \cos\left(\frac{\pi}{4}x\right) + 2$

7 **a** A vertical stretch with scale factor 3 followed by a horizontal stretch with scale factor $\frac{1}{2}$.

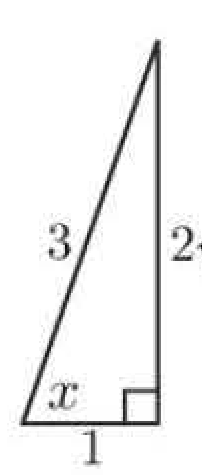
b A translation of $\begin{pmatrix} \frac{\pi}{3} \\ -1 \end{pmatrix}$.

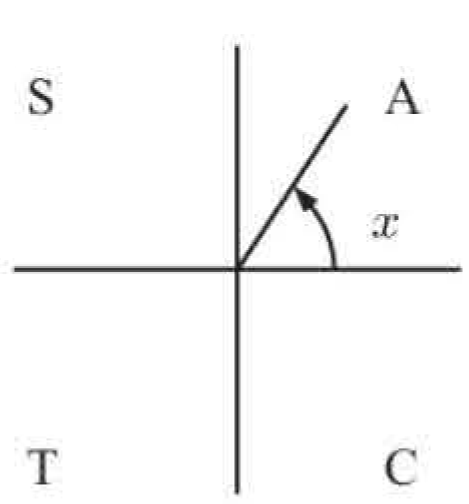
8 **a** $\cos x = \frac{1}{3}$

$0 < x < \pi$

but $\cos x > 0$

$\therefore$ $0 < x < \frac{\pi}{2}$





$\therefore$ $\sin x = \frac{2\sqrt{2}}{3}$

$\tan x = 2\sqrt{2}$

$\csc x = \frac{3}{2\sqrt{2}}$

$\sec x = 3$

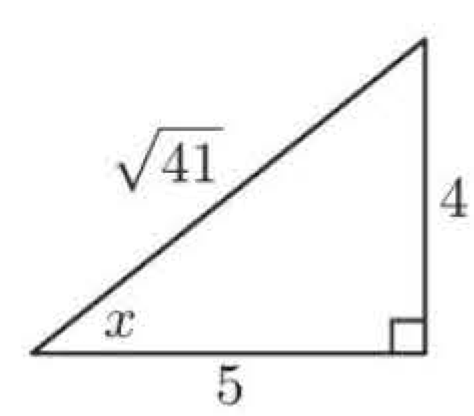
$\cot x = \frac{1}{2\sqrt{2}}$

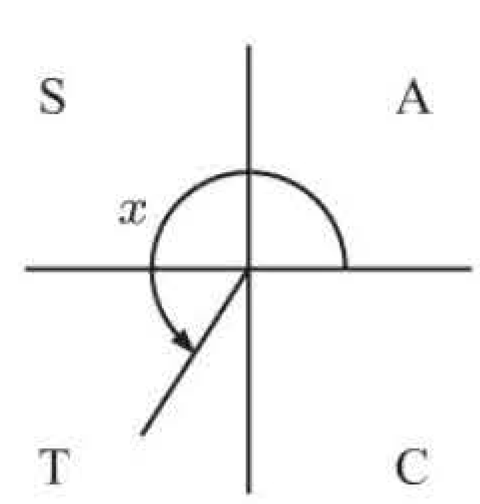
b $\tan x = \frac{4}{5}$

$\pi < x < 2\pi$

but $\tan x > 0$

$\therefore$ $\pi < x < \frac{3\pi}{2}$





$\therefore$ $\sin x = -\frac{4}{\sqrt{41}}$

$\cos x = -\frac{5}{\sqrt{41}}$

$\csc x = -\frac{\sqrt{41}}{4}$

$\sec x = -\frac{\sqrt{41}}{5}$

$\cot x = \frac{5}{4}$

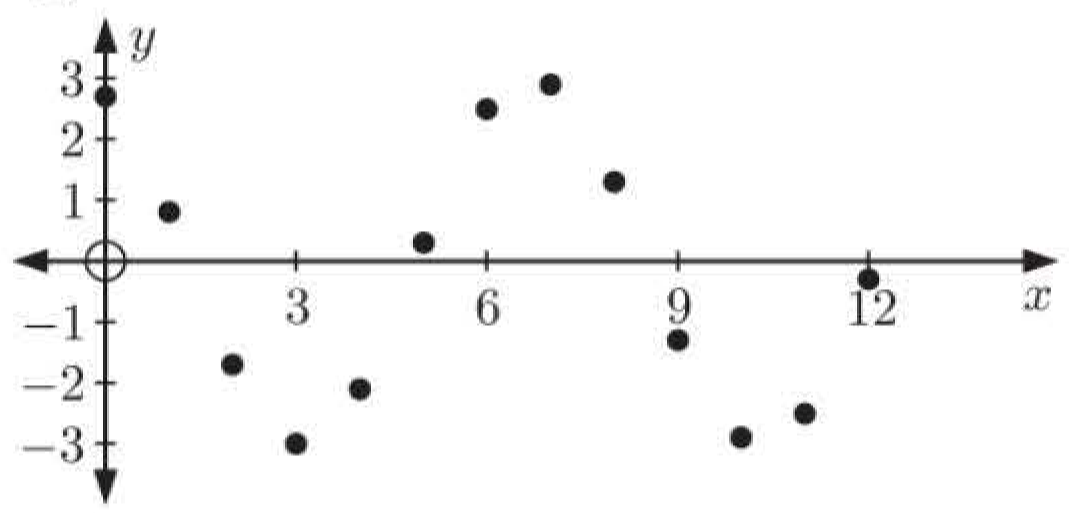
9 **a** $\arctan(\tan(-0.5)) = -0.5$

b $\arcsin(\sin(-\frac{\pi}{6})) = -\frac{\pi}{6}$

c $\arccos(\cos 2\pi) = 0$

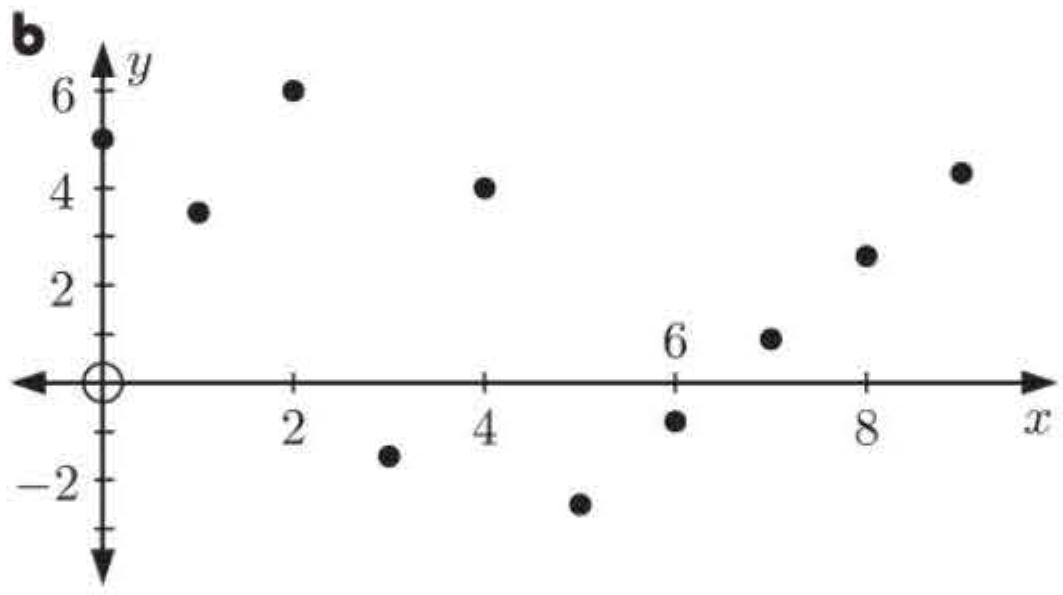
REVIEW SET 12B

1 **a**



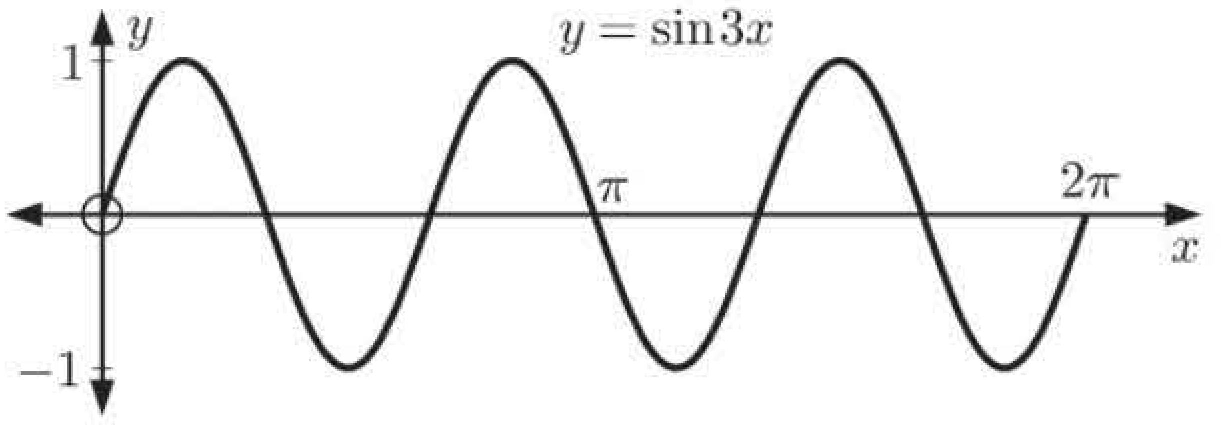
approximately periodic

b



not periodic

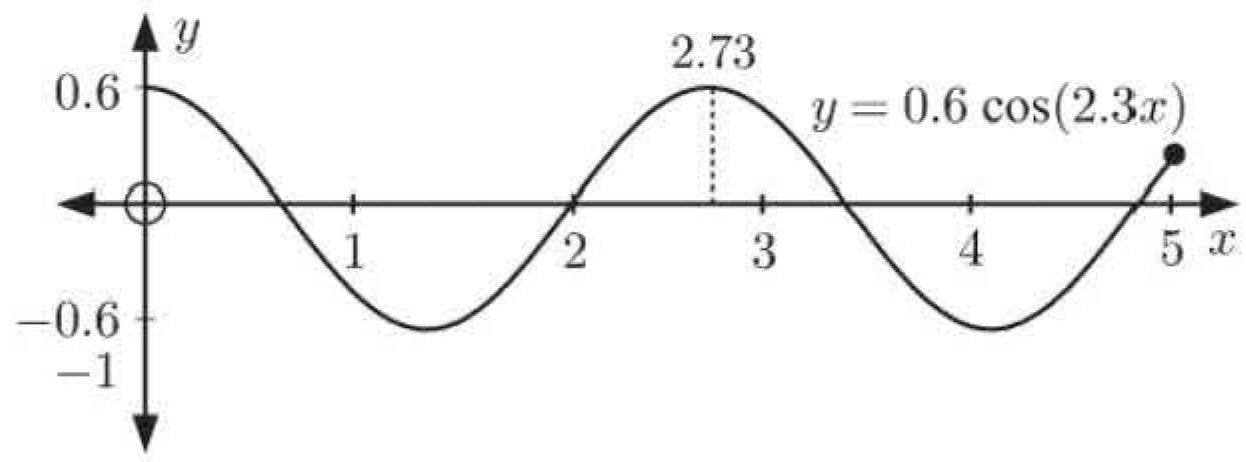
2 $y = \sin 3x$ has period $\frac{2\pi}{3}$.



3 **a** period $= \dfrac{2\pi}{\frac{1}{3}} = 6\pi$

b period $= \frac{\pi}{4}$

4 $y = 0.6\cos(2.3x)$ has period $\frac{2\pi}{2.3} \approx 2.73$



5 **a** maximum $= -5°$C, minimum $= -79°$C

b amplitude $= \dfrac{-5 - -79}{2} = 37°$C, so $a = 37$

principal axis is $y = \dfrac{-5 + -79}{2} = -42$, so $c = -42$

Now, we see that the temperature is $-68°$C and rising on days 600 and 1300, so we estimate the period to be 700 days.

$\therefore\ b \approx \frac{2\pi}{700} \approx 0.008\,98$

So, $T \approx 37\sin(0.008\,98n) - 42$ °C

c A Mars year is equivalent to one period of the temperature pattern, so 1 Mars year $\approx$ 700 Mars days.

6 Minimum = mean value − amplitude $= c - |a|$, maximum = mean value + amplitude $= c + |a|$.

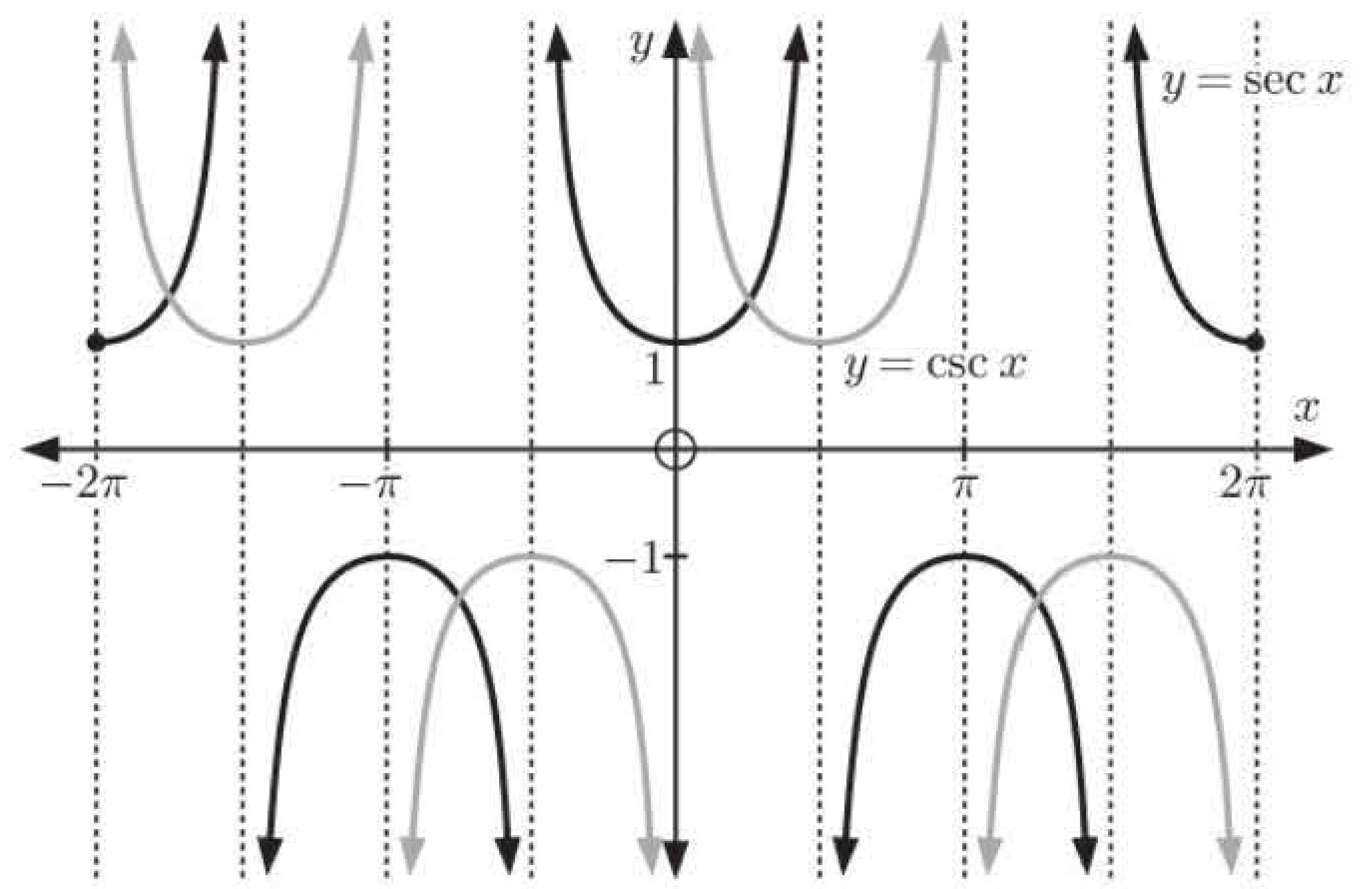
a $y = 5\sin x - 3$ has $a = 5,\ c = -3$
so $\min = -3 - 5 = -8$
and $\max = -3 + 5 = 2$

b $y = \frac{1}{3}\cos x + 1$ has $a = \frac{1}{3},\ c = 1$
so $\min = 1 - \frac{1}{3} = \frac{2}{3}$
and $\max = 1 + \frac{1}{3} = 1\frac{1}{3}$

7 **a** A reflection in the x-axis followed by a horizontal stretch with scale factor $\frac{1}{2}$.

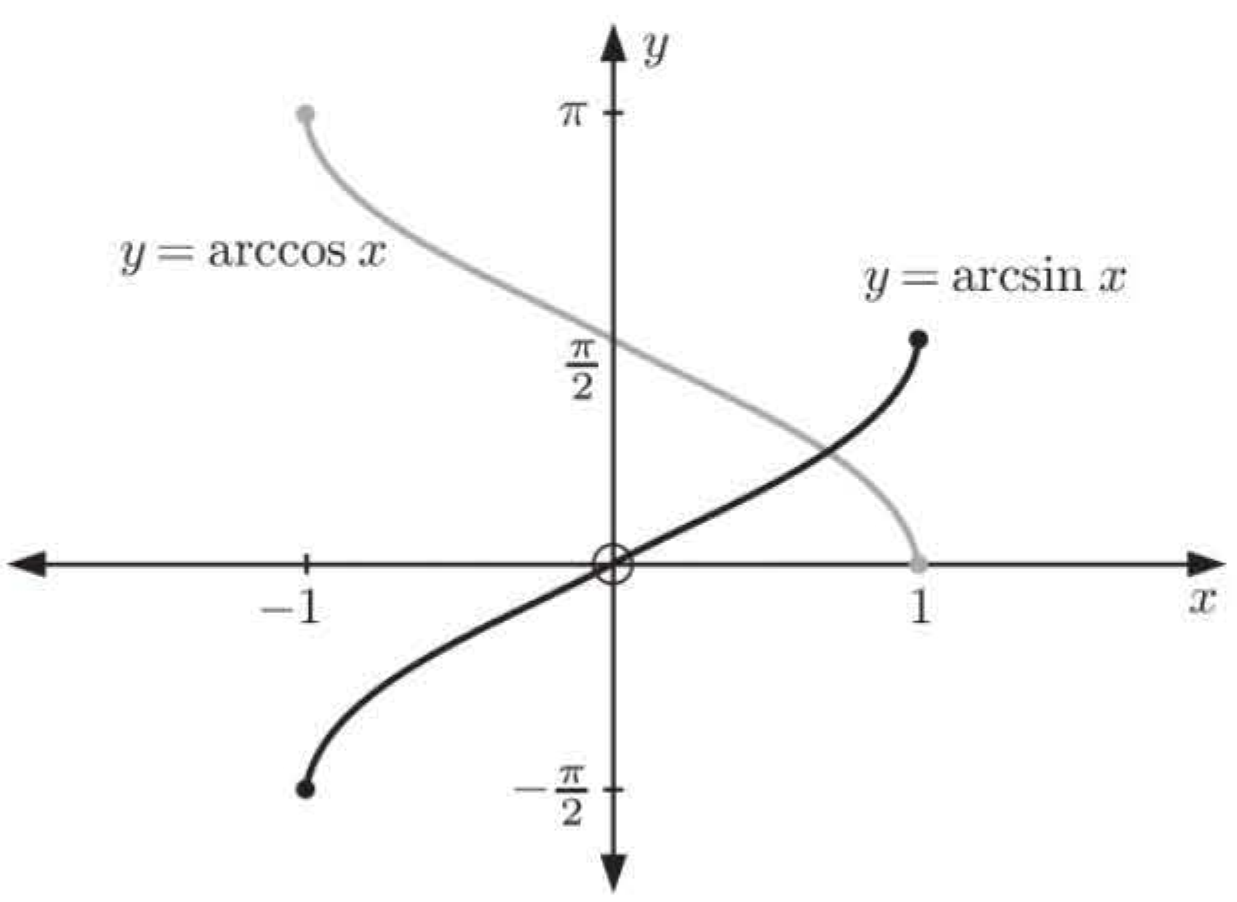
b A vertical stretch with scale factor 2, followed by a translation of $\begin{pmatrix} \frac{\pi}{4} \\ \frac{1}{2} \end{pmatrix}$, followed by a horizontal stretch with scale factor 2.

8 **a**



b a translation of $\begin{pmatrix} \frac{\pi}{2} \\ 0 \end{pmatrix}$

9 **a**



b $y = \arcsin x$:

Domain $= \{x \mid -1 \leqslant x \leqslant 1\}$
Range $= \{y \mid -\frac{\pi}{2} \leqslant y \leqslant \frac{\pi}{2}\}$

$y = \arccos x$:

Domain $= \{x \mid -1 \leqslant x \leqslant 1\}$
Range $= \{y \mid 0 \leqslant y \leqslant \pi\}$

c A reflection in the y-axis (*or* a reflection in the x-axis), followed by a translation of $\begin{pmatrix} 0 \\ \frac{\pi}{2} \end{pmatrix}$.

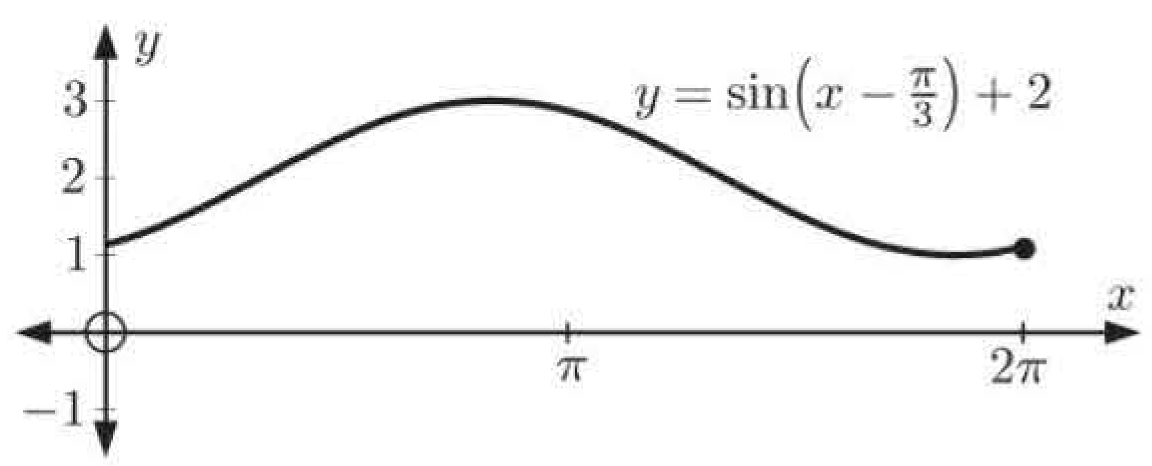
REVIEW SET 12C

1 **a** period $= \dfrac{2\pi}{b} = 6\pi$
$\therefore\ b = \frac{1}{3}$

b period $= \dfrac{2\pi}{b} = \frac{\pi}{12}$
$\therefore\ b = 24$

c period $= \dfrac{2\pi}{b} = 9$
$\therefore\ b = \frac{2\pi}{9}$

2 **a**

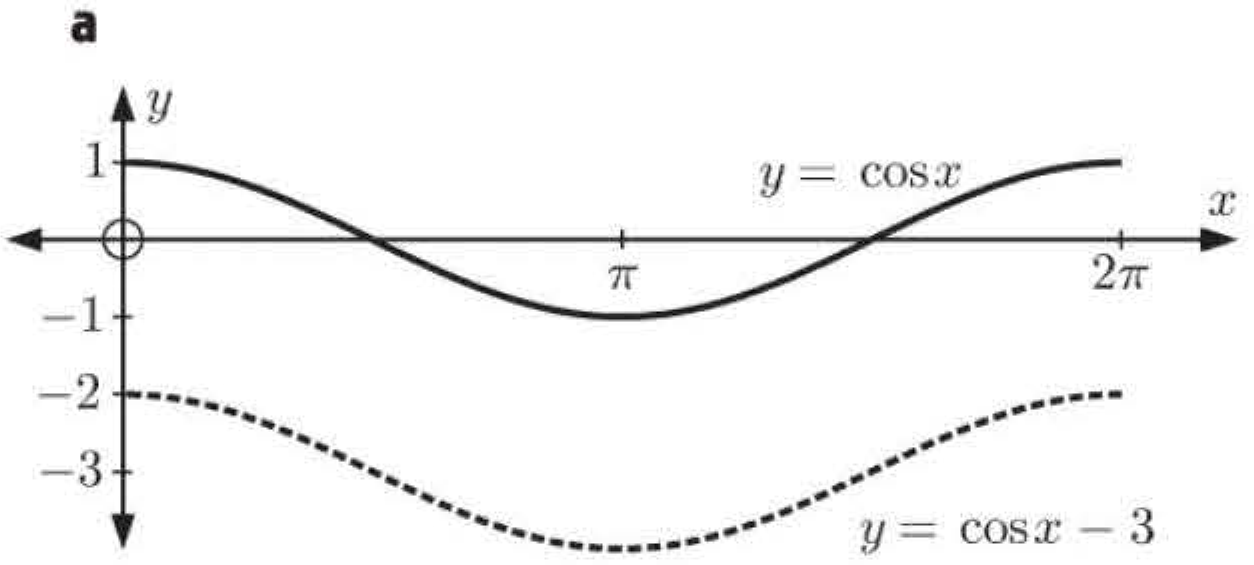


b $f(x)$ has minimum value $-1 + 2 = 1$ and maximum value $1 + 2 = 3$
$\therefore\ f(x) = k$ will have solutions for $1 \leqslant k \leqslant 3$

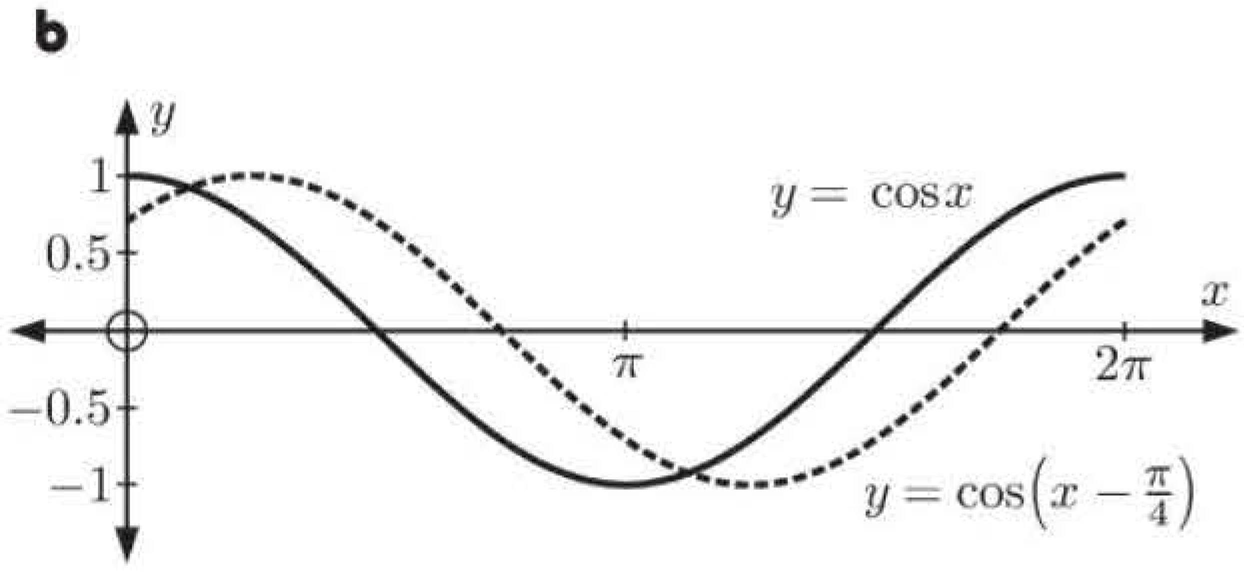
3 **a** The graph is periodic because it repeats itself over and over in a horizontal direction in intervals of the same length.

b **i** period $= 8$ **ii** maximum value $= 5$ **iii** minimum value $= -1$

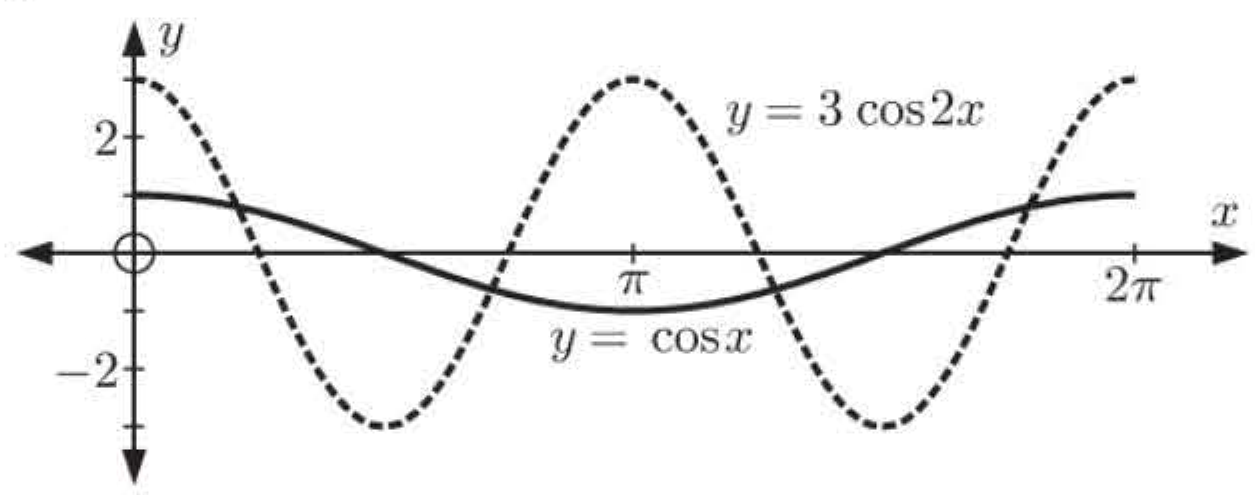
4 **a**



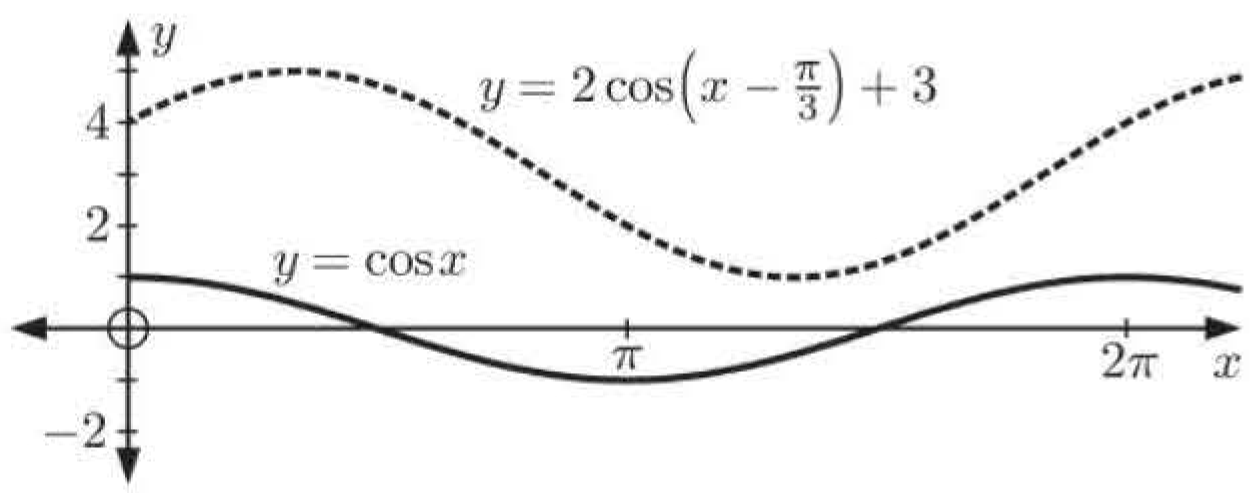
b



c

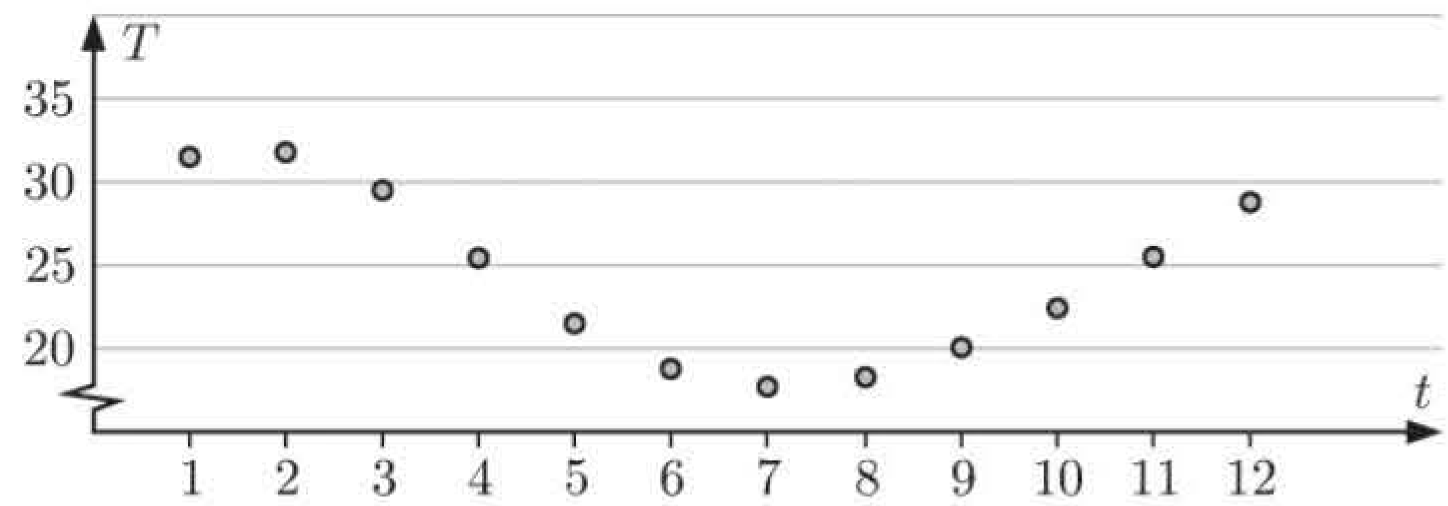


d



5

Month	1	2	3	4	5	6	7	8	9	10	11	12
Temp	31.5	31.8	29.5	25.4	21.5	18.8	17.7	18.3	20.1	22.4	25.5	28.8



a $T = a\sin b(t-c)+d$ $\qquad$ period $= \dfrac{2\pi}{b} = 12, \quad \therefore \quad b = \dfrac{2\pi}{12} = \dfrac{\pi}{6}$

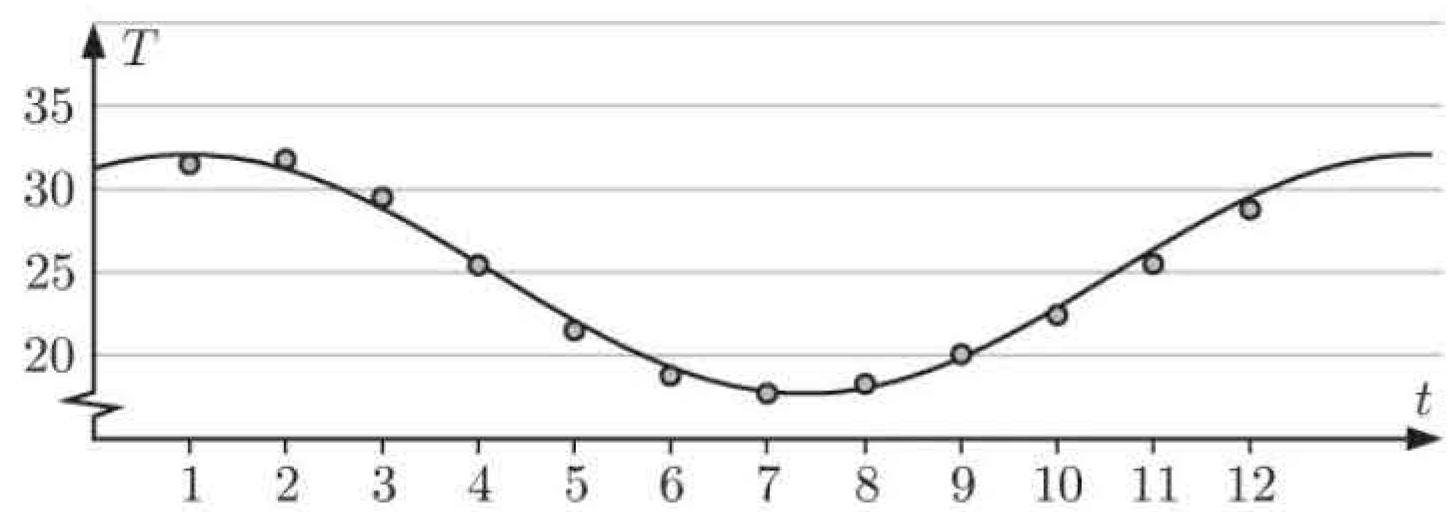
max. $= 31.8$ $\quad \therefore \quad a = \dfrac{\text{max.} - \text{min.}}{2} \approx \dfrac{31.8 - 17.7}{2} \approx 7.05$

min. $= 17.7$

$$d = \frac{\text{max.} + \text{min.}}{2} \approx \frac{31.8 + 17.7}{2} \approx 24.75$$

$$c = \frac{7+14}{2} = 10.5 \quad \{\text{values of } t \text{ at min. and max.}\}$$

So, $\quad T \approx 7.05\sin(\frac{\pi}{6}(t-10.5)) + 24.75$



b From technology, $\quad T \approx 7.21\sin(0.488t + 1.082) + 24.75$

$\qquad \approx 7.21\sin(0.488(t+2.22)) + 24.75$

The model fits reasonably well.

6 **a** translation through $\begin{pmatrix} \frac{\pi}{3} \\ 1 \end{pmatrix}$

b vertical stretch with scale factor 2, followed by a reflection in the x-axis

c horizontal stretch with scale factor $\frac{1}{3}$

7 **a** If $y = a\sin(bx - c) + d$

then $a = \dfrac{\text{max.} - \text{min.}}{2} = \dfrac{1 - -\frac{1}{2}}{2} = \frac{3}{4}$,

$\dfrac{2\pi}{b} = \frac{\pi}{2} \quad \therefore \quad b = 4$,

$d = \dfrac{\text{max.} + \text{min.}}{2} = \dfrac{1 - \frac{1}{2}}{2} = \frac{1}{4}$

So, $y = \frac{3}{4}\sin(4x - c) + \frac{1}{4}$

and passes through $(0, 0)$

$\therefore \ \frac{3}{4}\sin(0 - c) + \frac{1}{4} = 0$

$\therefore \ \sin(-c) = -\frac{1}{3}$

$\therefore \ c = \arcsin(\frac{1}{3})$

$\therefore \ c \approx 0.340$

So, $y = \frac{3}{4}\sin(4x - 0.340) + \frac{1}{4}$.

b If $y = a\tan(b(x - c)) + d$

then principal axis $= 0 \quad \therefore \quad d = 0$,

$\dfrac{\pi}{b} = \pi \quad \therefore \quad b = 1, \ c = \frac{\pi}{2}$

So, $y = a\tan(x - \frac{\pi}{2})$

and passes through $(\frac{\pi}{4}, -1)$

$\therefore \ a\tan(\frac{\pi}{4} - \frac{\pi}{2}) = -1$

$\therefore \ a\tan(-\frac{\pi}{4}) = -1$

$\therefore \ a(-1) = -1$

$\therefore \ a = 1$

So, $y = \tan(x - \frac{\pi}{2})$.

8 **a**
$$\begin{aligned} &\csc x \tan x \\ &= \frac{1}{\sin x}\frac{\sin x}{\cos x} \\ &= \sec x \end{aligned}$$

b
$$\begin{aligned} \frac{\tan x}{\sec x} &= \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}} \\ &= \sin x \end{aligned}$$

c
$$\begin{aligned} &\sec x - \tan x \sin x \\ &= \frac{1}{\cos x} - \frac{\sin x}{\cos x}\sin x \\ &= \frac{1 - \sin^2 x}{\cos x} \\ &= \frac{\cos^2 x}{\cos x} \\ &= \cos x \end{aligned}$$

9 **a** $y = \arctan x$ is the inverse function of $y = \tan x$ for the restricted domain $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

b