Lab 3: Intragalactic Radio Interferometry

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Abstract

The purpose of this lab was to experimentally investigate the interferometric fringe using the multiplying interferometer array on the roof of Campbell Hall. We observed the sun, the moon, and a point source in the 10.7 Gigahertz range for a large range of hour angles. We filtered the convolved signals, the "fringe", into the Hz-range, and used Fourier transform analysis to find the local fringe frequencies for each hour angle. This analysis, paired with least squares fitting using calculations of expected frequencies, allowed us to constrain the baseline between telescopes which is 1474 cm. These laboratory skills will be useful for future work in this lab, as well as in research and future careers.

1 Introduction

Interferometry is used for many scientific pursuits, from molecular scales to the cosmic. In radio astronomy, an interferometric array is used to increase the angular resolution of a telescope, which is useful for resolving very small or distant sources. Because an interferometric array with a larger baseline will create a larger signal delay between telescopes, understanding how to interpret the convolved signal, as is done in this lab, is a very important skill, and one that is required in order to get information about the observed source. Once this skill is mastered, the only limits to angular resolution are the precision of one's instruments, and the size of the baseline.

In this report, Section 2 covers the experimental design, as well as an interpretation of the relationship between the experimental design, which is in angular space and the measured signal, which is in frequency space. Section 3 gives an overview of our observations, not all of which were successful, but dives into an explanation of how the data we did gather can be interpreted into useful quantities using Fourier transforms locally, and over time. Finally, Section 4 covers the analysis of our data to least squares fit the error between measurements and calculations to constrain the baseline.

2 The Experiment: Interferometer inputs to Fringe outputs

2.1 The Interferometer

The interferometer on the roof of Campbell Hall has two telescopes oriented in the E-W directions, which observe signals E1 and E2, from radio sources emitting in the 10.7 GHz range. The telescopes are placed some physical distance apart (the baseline "b"), which will cause some delay/offset in our data (τ) when source signals reach the closest telescope before the furthest.

$$E_1 = cos(2\pi) \tag{1}$$

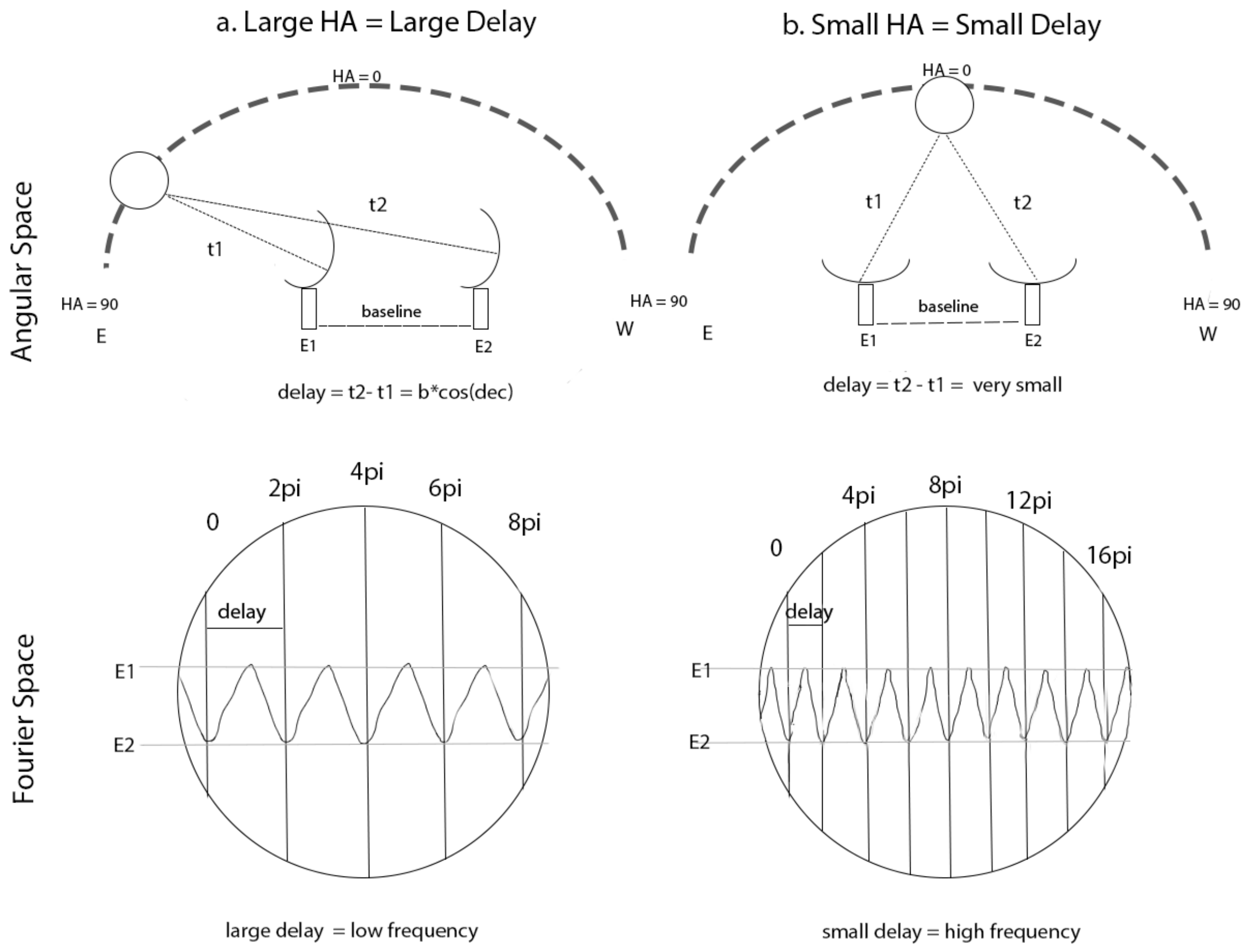

Figure 1: The "fringe" that is measured from our interferometer is a result of the time delay between telescopes. The signal is measured in Fourier space, and the delay is caused by changes in hour angle in angular space. Here, the fringe of both a large and a small hour angle are shown. The larger the hour angle, the closer the delay time is to the length of the baseline between telescopes, which in Fourier space, the delay is the wavelength of the fringe. A large wavelength will result in a low frequency. A small hour angle creates a small time delay, which creates a small wavelength in Fourier space, which leads to a high frequency.

$$E_2 = cos(2\pi\nu[t + \tau]) \tag{2}$$

This geometrical delay also depends on the source's position in the sky (measured in hour angle "ha"). The direct relationship between delay and hour angle and baseline is shown in Equation 3 and 4. Note that there may be an additional delay caused by electronic cable lengths (τ_c), but that analysis is beyond the scope of this report.

$$\tau = [\frac{b_{ew}}{c} * cos\delta] * sin(ha) \tag{3}$$

The geometric delay for every pixel is the baseline dotted into the position of each pixel

$$\tau = b * \frac{\sqrt{1 - sin\theta_x^2 - sin\theta_y^2}}{c} \tag{4}$$

2.2 The Receiving System

Each telescope sends it's signal down a cable of approximately equal length into a double heterodyne mixer. After each mixing stage is a filter, although in some cases the cable does the filtering for us. At the very end after it goes through amplifiers, mixers, and filters, the signals are multiplied together. When we're mixing one against the other, that's being done in a mixer, then each of the outputs of the last mixer goes into a low pass filter that filters each of the signals from the last single sideband mixer, and the final mixing is from the output of each antenna in a different mixer module. Then we do a time average using a super low pass filter that will let through signal in the Hz range.

The resulting output of our mixer receiving system will be the cross-correlation between signals, multiplied by the delay (Eq 5).

If you multiply two functions in the time domain, we're convolving them in the frequency domain.

$$signal = [E_1 * E_2](\tau) = fringe \tag{5}$$

If the delay is a delta function, then the Fourier transform of a delta function is a sine wave. The FT of the sum of two delta functions is the sum of two sine waves, so the Fourier Transforms add. The Fourier transform of the delay is the visibility.

$$V_{12}(\nu) = \frac{1}{2 * \pi}|(E_1)|^2 e^{2\pi i \tau_A \nu} + \frac{1}{2\pi}|(E_2)|^2 e^{2\pi i \tau_B \nu} \tag{6}$$

The modulating output function is called the "fringe", a Fourier transform of the source intensity distribution.

2.3 Visualizing the Fringe

What does this look like? Figure 1 shows two different sample delays caused by changes in hour angle. In 1a, in angular space, a low setting source with a large hour angle sends a signal that is observed by the first telescope (E1) a few seconds before the second telescope (E2). The difference in observing time will create a signal delay. The larger the hour angle, the closer the delay time is to the length of the baseline between telescopes. In Fourier space, which is what measure from our receiving system, this large delay is represented in the wavelength of the measured sine wave. The inverse of wavelength is frequency, so a large delay that causes a large wavelength will result in a low frequency. In 1b, in angular space, a high-rising source with a small hour angle sends a signal that reaches both telescopes at roughly the same time, so any time delay will be very very small. In Fourier space, this small delay causes a small wavelength, which leads to a high frequency.

3 Observations

Some of the brightest radio sources in the sky arc across the sky everyday. For our resolved sources, we chose to observe the Sun and Moon. The Moon's radio signals are emission from its blackbody spectrum, not solar reflection. For our unresolved sources, we were assigned Cygnus A, but ran into several technical pointing complications since it travels directly overhead. In its place, we were attempted to get a partial observation of the Crab Nebula for our point source, however, our sampling rate was too slow, resulting in aliased data. Ultimately, our sun data was the most informative, since the sun is an incredibly bright radio source which counteracted the setbacks from our low sampling rate.

Our team spent a lot of time in the observation-stage of this lab, trying to get the coordination between the pointing and the sampling aligned, so it is disappointing that we were unaware of the ideal sampling rate, because most of our observations were not used for analysis. Our team has learned the hard way to not make this mistake again, however I'm sure future students taking Astro 121 would appreciate having a ball-park sampling rate to begin with.

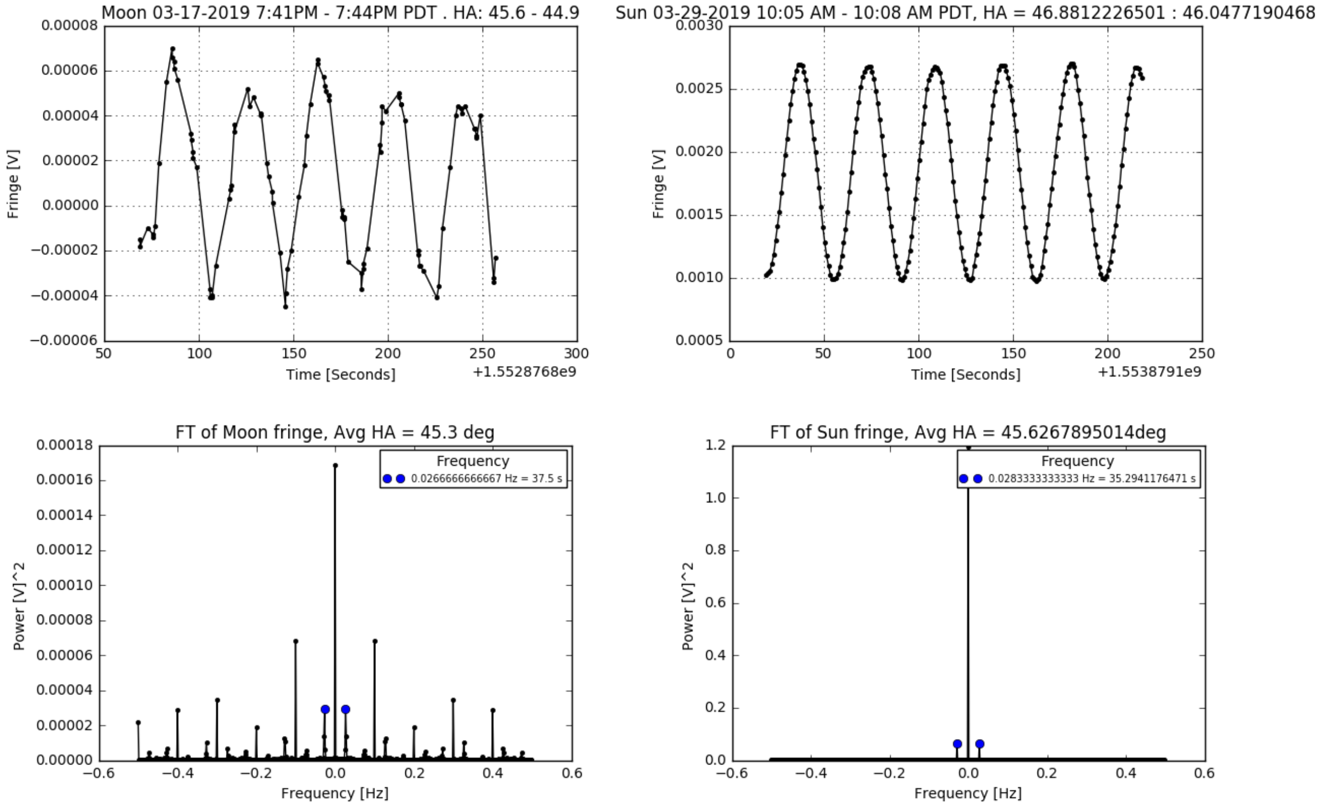

Figure 2: Observation of both a 3-minute period of the Moon from March 17th starting at 7:41pm, and a 3-minute period of the SUn from March 29th starting at 10:05am. The wavelength distance between periods of the oscillating fringe corresponds to the delay between the interferometric telescopes. To measure the time delay, we took the Fourier transform of the fringe, and measured the frequency of the off-center peak to get the local fringe frequency. The moon data is much more jagged in the time series, and more "peaky" in frequency space largely because we had a low sampling rate, causing aliasing. We also somehow had non-evenly spaced samples. For both the moon and point source data, all of our data is sampled too low, so we use the Sun data from March 29th for analysis in the rest of this report.

3.1 Fourier Space - Local Fringe Frequency

The Fourier transform of a delay is represented as a sine wave in Fourier/frequency space. This sine wave is called the "fringe". The wavelength of the fringe is the delay time between the telescopes. When we take a Fourier transform of the fringe over a selected time interval, the local fringe frequency will appear as a spike, which we can convert into the delay time. Figure 2 is an example of finding the frequency of a fringe over a 3-minute period by taking the Fourier transform. The top graph of Figure 2a shows the time series of the fringe of the Moon on March 17th, 7:41pm. This data is non-ideal, because the data was not evenly sampled, and not sampled as frequently per minute as it could have been–leading to a jagged "sine wave". A Fourier transform will be most informative if it has a large number of points to synthesize, so before taking the FT I increased the range of the timeseries to include several more periods. The Fourier transform of this data is graphed below. Figure 2b shows a similar period, but has a higher sampling rate, so the signal matches the expected sine wave and the Fourier transform only has one sideband peak.

We can visually check if the local frequency is accurate by converting the frequency into seconds (1/f), and then "eye-balling" the wavelength of the period in the time series graph. Quantitatively checking our measurement using Equation 9 will be more accurate. This quantitative checking in explored in further detail in Section 4.1.

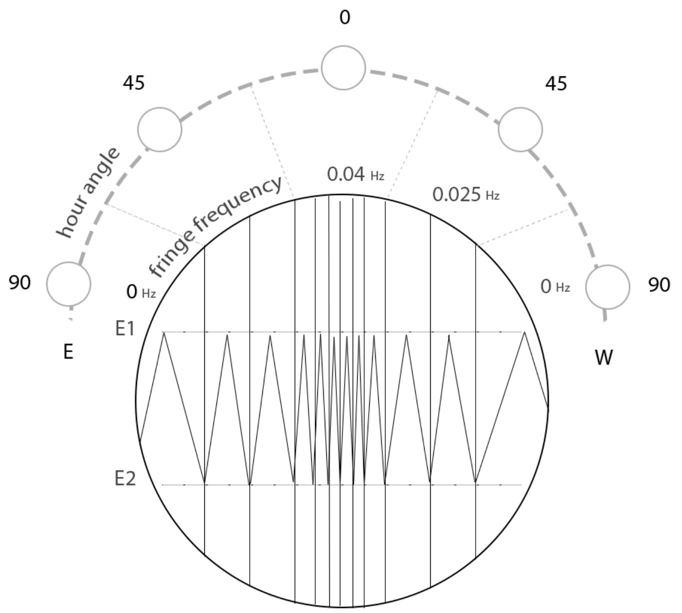

Figure 3: Model of the fringe pattern increasing and decreasing in frequency over time, mapped onto a circular sky as an orthographic projection

3.2 Fringe pattern over time

Different delays (τ) will create different frequencies of the sine wave. Since frequency corresponds to hour angle, which changes with time, we can model a time series of the fringe frequencies throughout the day, as shown in Figure 3. As an object rises in the sky, the fringe frequencies go from low to high, then high to low as the object sets.

If we took a Fourier transform of Figure 3, our model of oscillating fringe frequencies throughout the day, we would expect to observe a series of spikes with different sideband values, which correspond to frequencies. As an object rises in the sky, the fringe frequencies start small, so sidebands would be close together, and then the frequencies get larger as the source passes overhead, so the sidebands would be far apart, then this pattern would reverse. This phenomenon can be seen in Figure 4.

In our horizon-to-horizon observations, we expect point-sources to follow this pattern because the angular resolution of point sources are by-definition unresolved, meaning that the object is so small and far away that the dimensions of the object are negligible in our delay calculations. However, for our horizon-to-horizon measurements of resolved sources like the Sun and Moon, their angular size is non-negligible. As these objects move across the sky, if their angular size is larger than the period of the fringe, which happens at smaller hour angles, then some of our frequencies will be obstructed and cancel out. We were not able to gather point-source data, but we can observe the cancelling-out effect of the Sun in Figure 7, where the overall shape flattens out in the middle as it reaches zenith.

If we plot the delay time vs hour angle, as in Figure 6a, we can see that at 0 Degrees, our time delay should minimize to near-zero, but the angular resolution of the sun is large enough to overlap the small wavelengths caused by the high frequencies of this region, so the resolution of the minimum time delay is limited by the angular diameter of the sun. The overall shape of cancelling or not-cancelling out the fringe pattern is referred to as the source's "envelope", which shaped like a Bessel function.

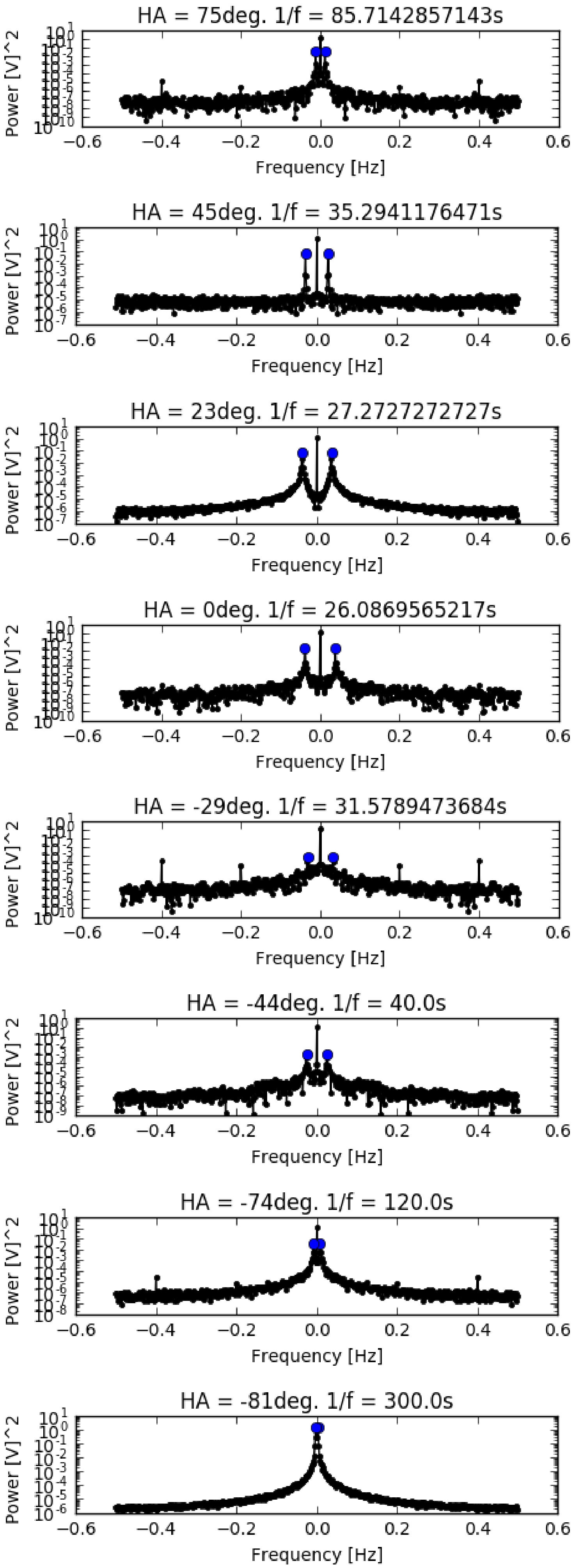

Figure 4: Plot of local fringe frequencies of Sun data from March 29th, 2019. As the Sun rises in the sky, hour angle and the time delay between telescopes decreases, and as it sets, the hour angle and time delay increases.

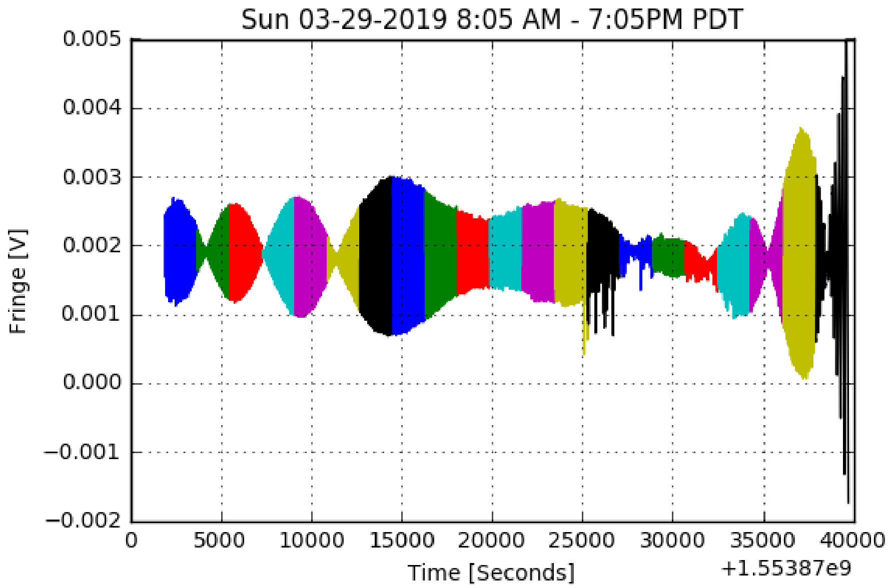

Figure 5: Horizon-to-horizon observations of the Sun on March 29th, from 8:05am to 7:05pm. We can see that the fringe pattern modulates across the sky.

4 Analysis

4.1 Observed vs Calculated Fringe frequencies

We can use the difference between the observed fringe frequency and the calculated fringe frequency to constrain our value for the baseline between the telescopes. Figure 6b shows these differences, including error bars deviations on the measured frequencies from the calculated frequencies. The errors bars are sigma, calculated using Equation 7.

$$\sigma = |\sqrt{\sum(data_i - calculated_i)^2)|/N/\sqrt{2})} \tag{7}$$

The chi-square of difference between calculated and observed is 0.77037224278, which ideally we would like to be closer to 1.

4.2 Finding the Baseline

Aside from re-taking our data or trying to otherwise improve our observed frequencies, we can also improve upon the model calculation.

The baseline, $Bcos(\delta)$ depends on the baseline orientation, baseline length, declination, and hour angle, as in Equation 9.

$$f = \frac{b_x}{\lambda} cos(\delta) * cos(ha) - \frac{b_y}{\lambda} * sin(latitude) * cos(\delta) * sin(ha) * \omega \tag{8}$$

In our original model, we assumed the baseline was close to 50 feet, however, we can improve upon this estimation by testing a range of possible baselines and seeing which baseline gives us the lowest chi-square of the least squares residual between calculated and observed fringe frequencies.

$$ChiSquare = \sum(|(model - data)|^2/\sigma^2) \tag{9}$$

We found that the minimized chi-square for the baseline is at 1474cm, which is 48.35 feet.

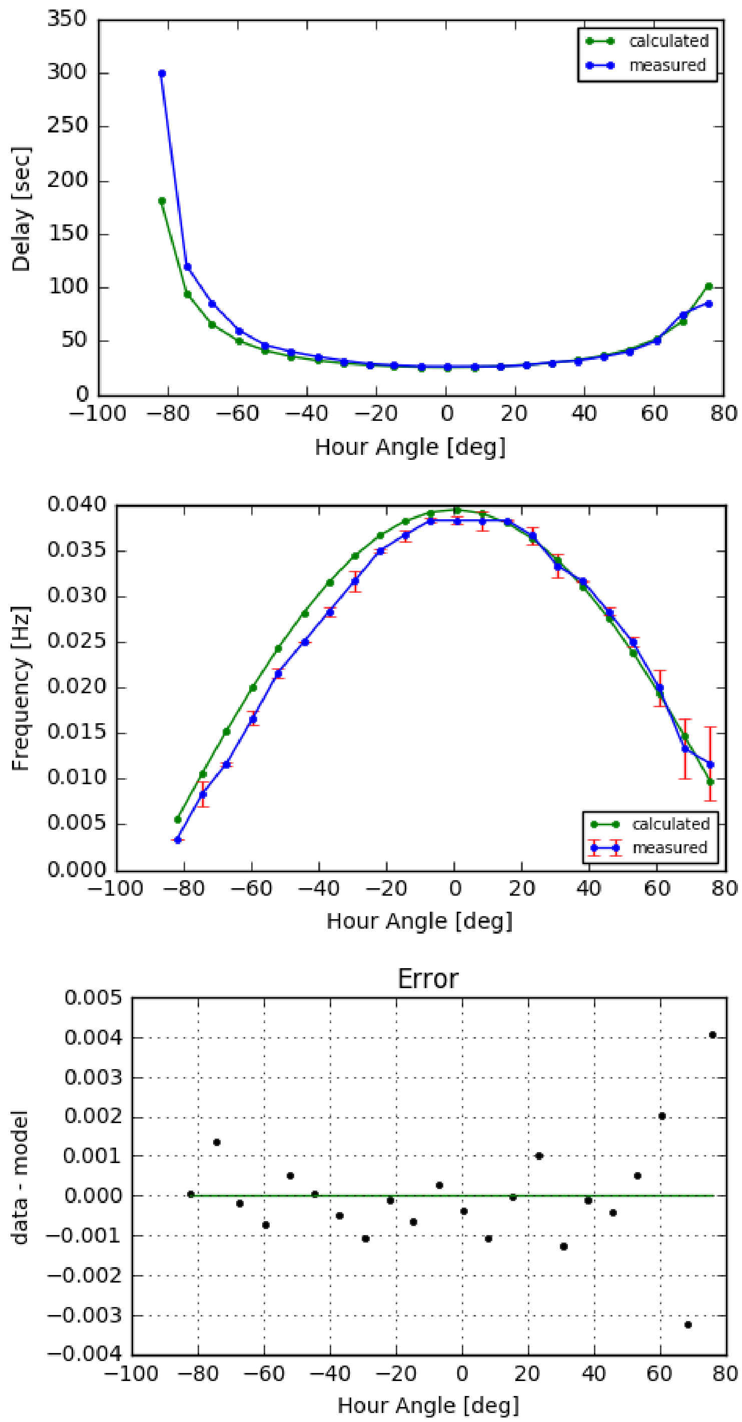

Figure 6: Comparison of observed and calculated delay times and frequencies vs hour angle for the Sun observation from Fig 4. Negative hour angles are past zenith, positive hour angles and pre-zenith, so these plots are in reverse-chronological order. We can find the error on our observation measurements by subtracting each calculated frequency from the observed frequency, taking the standard deviation, and dividing by $\sqrt{2}$. Our largest error is at 75.66 degrees E: 0.0117Hz 0.004. The largest error is at 44.45 degrees W: 0.025 3.9e-5. The high error bars near the horizon may be due to difficulties in finding the local fringe frequency from the Fourier transform in those regions.

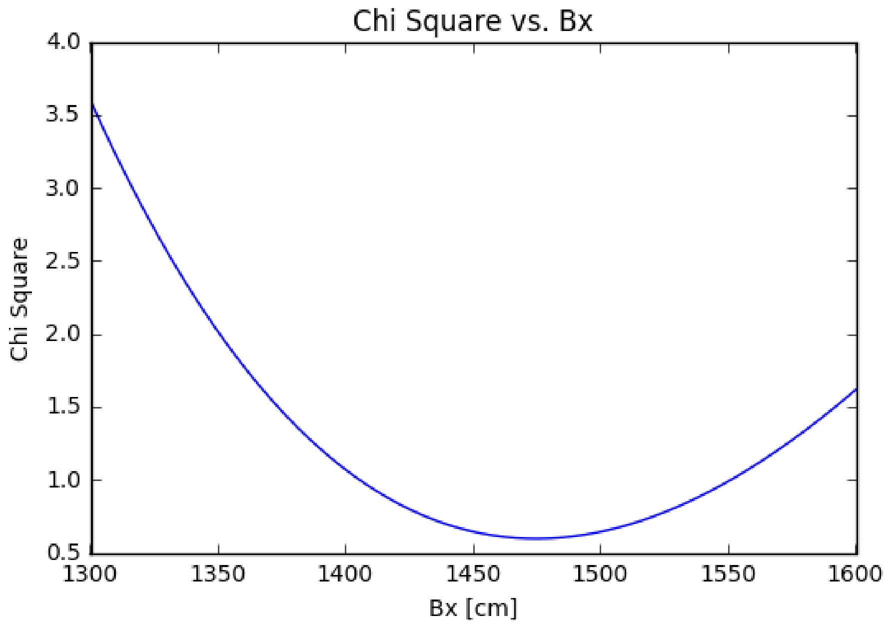

Figure 7: The minimum of the Chi-Square well, where the Baseline (EW) is minimized is 1474 cm. We tested a range of baselines from 1300-1600, and inserted these baselines to create a model for fringe frequency. The model that was best fit to the observations has the lowest chi square.

5 Conclusion

In this report we observed the solar fringe frequencies at a range of hour angles, and used this range of observed fringes to compare to a calculated range of fringes using the least-squares method. We then used the residuals to improve upon the calculation model, where we found that our estimation of baseline was incorrect, and the optimized baseline(E-W) value is 1474 cm 0.5. This report contains only a sample of the vast uses of interferometry, and it is a tool that the world will undoubtedly be using more in the future.

References

[1] Parsons, A. (2019). Radio Interferometry at X Band. [online] AstroBaki. [Accessed 8 April. 2019].