Lab 1: Radio Astronomy

Cee Gould, with Adam, Grace, and Scout

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1 Introduction

The purpose of this lab was to familiarize students with the process of experimental investigation as well as radio instrumentation that will be used throughout the 121 Radio Astronomy lab. This lab includes exercises to help gain familiarity with sampling electronic signals and converting them into digital signals using equipment such as a PicoScope 2206a, Oscilloscope, and Signal Generator.Students were also introduced to digital sampling, acquainted with aliasing and the Nyquist criterion, as well as proper use of Discrete Fourier Transforms, noise, the convolution theorem, frequency conversion, negative frequencies, and the mixing process, the basis of heterodyne spectroscopy. These laboratory skills will be useful for future work in this lab, as well as in research and future careers.

2 Nyquist Sampling

The first primary task of the lab was to experimentally explore the sampling frequency limits of a signal, ideally to produce the critical amount of data that will be enough to accurately represent the true signal, without over-compensating.

2.1 Digital signals

We needed to sample electronic signals from the signal generator and convert them into digital signals that we could plot using Python.

2.1.1 Sample Frequency

First we needed to choose a sample frequency to produce our first signal from the SRS synthesizer oscillator. The sampling frequency, V_{samp}, that our group chose was 62.5 MHz, because with our equipment we can set V_{samp} to only selected, quantized values of 62.5MHz/N where N is small.

We then connected this signal to an oscilloscope for viewing and to the PicoScope 2206a. Calling the module inside the ugradio package, ugradio. pico.capturedata, we were able to send the data from the Pico sampler and save it to our computers.

2.1.2 Signal Frequency

Once we had this starting frequency, we set out to sample the signal at a variety of smaller frequencies to see if we could find that critical limit where the true signal could be detected.

The signal frequency our group chose was a set range of V_{samp} * (0.1, 0.2..., 0.9), which gave us a final V_{sig} of (6.25, 12.5, 18.75, 25, 31.25, 37.5, 43.75, 50, 56.25)MHz. In Figure 1, each of our signal frequency outputs are plotted next to each other for comparison.

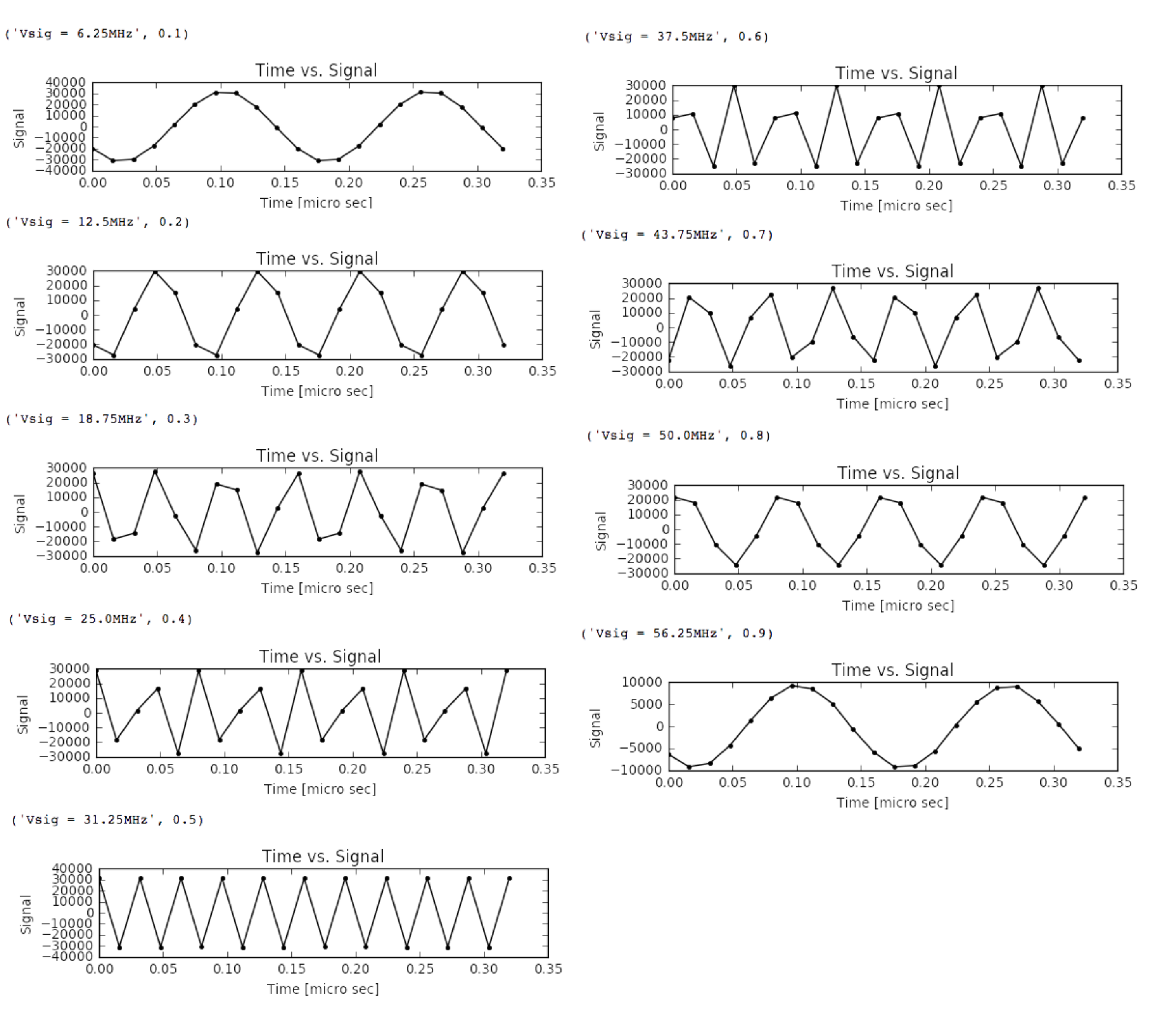

Figure 1: Output Signals, V_{sig} = (6.25, 12.5, 18.75, 25, 31.25, 37.5, 43.75, 50, 56.25)MHz. Our Nyquist rate is at the halfway point, in the 0.5 plot at 31.25MHz. Below the Nyquist rate are false sine-wave signals that are examples of aliasing.

2.2 Aliasing

What's going on here? We want to understand aliasing and the basic law of sampling: the Nyquist criterion.

According to the lab documentation, in order to find the Nyquist rate, one must sample at a rate that is more than twice the maximum frequency in the spectrum of the signal [1]. Since our signal frequencies are smaller than our sample frequency, we will be looking at the halfway point, rather than at the doubling point.

For example, our sample frequency was 62.5MHz, so at the halfway mark, 31.25MHz, we can see the Nyquist limit that will best preserve the sample frequency sine wave. Sampling below the Nyquist limit, like in our first plot at 6.25MHz, there is an aliased signal, a misleading false signal that is not the correct one produced from the original signal. If you sample your signal at, or above, the Nyquist rate, you can reconstruct the signal [1], however, slightly above the Nyquist limit, the amplitude information is not really preserved and there is a falsely modulated signal [2]. One must go much higher than the Nyquist frequency for the amplitude and the signal to be preserved accurately, which will actually sample better than at the Nyquist frequency. If our experiment had sampled higher than 62.5MHz, we may have been able to see this feature.

3 Fourier Transforms

3.1 DFTs

Any function can be expressed as a sum of a bunch of sine waves [3]. It does this by decomposing a function of time, like our signals, into the frequencies that make it up [4]. We can use Discrete Fourier Transforms (DFTs) to determine the frequency power spectrum of a time series. In Figure 2, we can see the power spectrum of the time series V_{sig} = 12.5MHz. Using the dft function in the ugradio module outputs a complex voltage vector that we can then multiply by its complex conjugate or square the length of, to create a power spectrum. We can see that the x-axis is no longer in the time domain, but in the frequency domain, in units of mega hertz.

According to Parseval's Theorem, Fourier transforms preserve power [3], so Figure 2 is correctly displaying the power information from that 12.5MHz time series, including the positive and negative frequencies, which are the same except the cosine either leads or trails the sine wave by $pi/2$ [3].

3.2 Leakage power and Frequency Resolution

When we gathered our data from the SRS Synthesizer Oscillator to create our time series and power spectrum plots (Figures 1 and 2), one could naively believe everything plotted is due to our signal. However, there is also a non-negligible amount of leakage power that contributes to our sampled our sine wave. Leakage power is described as a gradual loss of energy from a charged capacitor, even when turned off, that affects all power spectra calculated using Fourier techniques. [**?**]. To find the leakage power, in Figure 3 we turned up the vertical scale a lot and found non zero power at several frequencies other than Vsig.

3.3 Convolution and Correlation Theorem

The convolution theorem means that a convolution of two functions f g, is as if one function (f) is being "smoothed" by another (g). "Smoothing" describes a process where g slides along f, and at each step, sum f with weights drawn from the value of g at the point it is slid [6].

The correlation theorem does something different with similar results. "When it is combined with the expression of a time-shifted signal in Fourier domain, it shows that correlating a flat-spectrum signal with

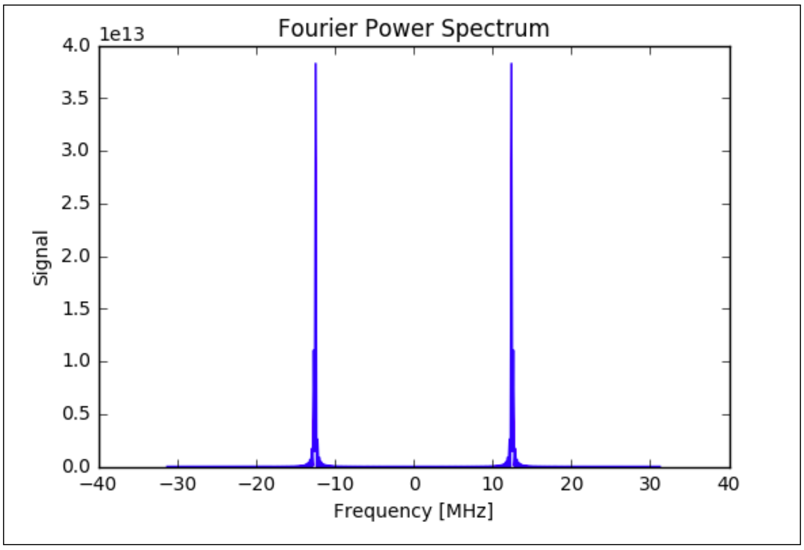

Figure 2: Power Spectrum for the time series $V_{sig} = 12.5$MHz, constructed by squaring the length of the complex voltage vector outputted from the discrete Fourier transform of the time series. The y-axis, labelled "Signal" is a voltage, measured in volts, and the x-axis is a frequency, measured in MHz.

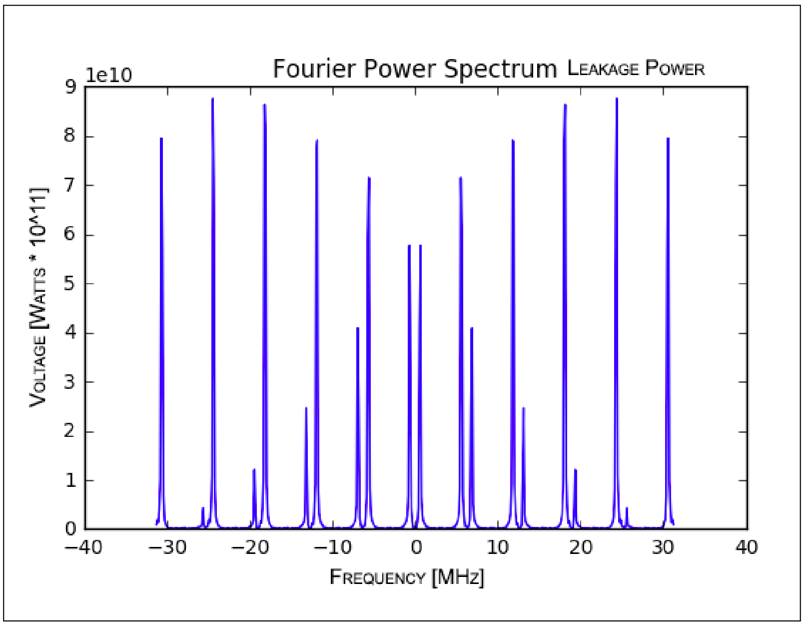

Figure 3: Leakage power visible from a power spectrum. Leakage power is caused by the machinery giving off excess power that contributes to the signal.

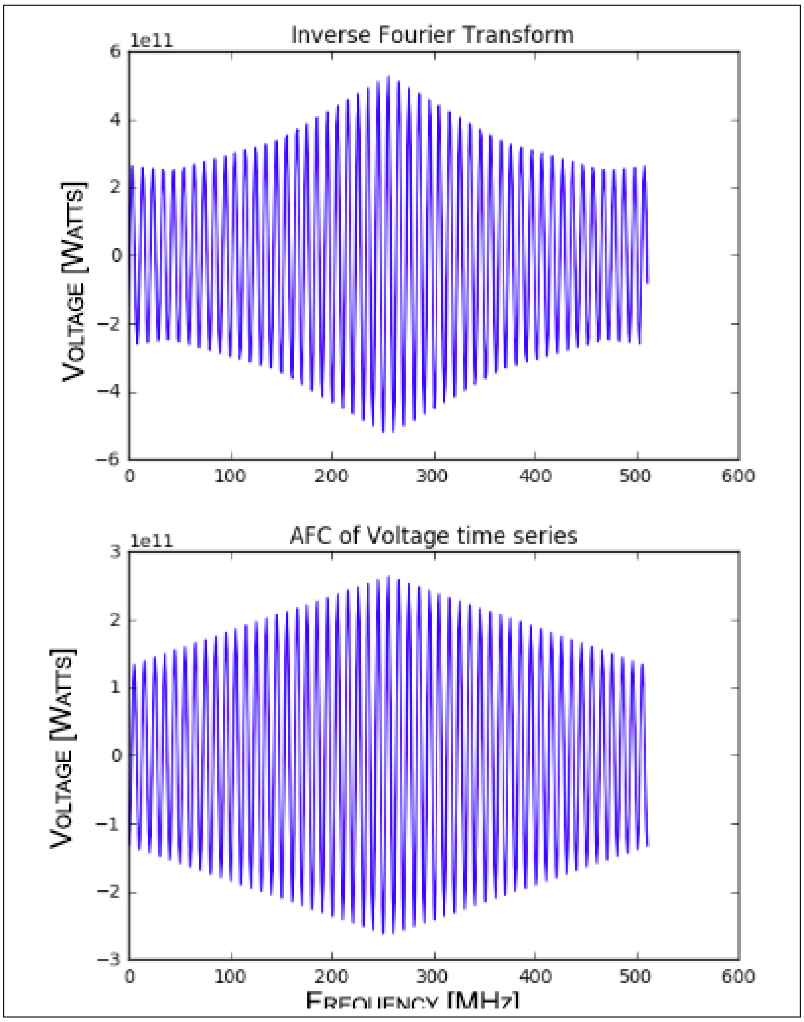

Figure 4: According to the correlation theorem, the FT of the power spectrum should equal the AFC. Our plot shows a striking similar appearance, although the shape of the inverse Fourier transform has been constricted by another unknown source, impacting the exact replication of AFC amplitude.

a time-shifted version of itself yields a measure of the power of the signal at the delay corresponding to the time shift" [6].

This similarity between theorems can be shown if we take one of the power spectra and take its inverse Fourier transform, and separately calculate the auto correlation function (AFC) directly from the voltage time series, and compare, as in Figure 4. We can see from the similarities in wavelength in each of the two graphs shown that multiplying in the time domain is indeed the same as convolving in the frequency domain [8].

4 Heterodyne Mixers

Our last task of the lab was to construct a double-sideband (DSB) mixer and explore the mixing process.

To mix two frequencies together, or to "heterodyne", is to combine a large frequency with another large frequency to ultimately shift from one frequency range into a new smaller one [7].

To experiment with this, we used two SRS synthesizer oscillators as inputs to a ZAD mixer, one represented a local oscillator LO with frequency Vlo = 20MHz, the other will be the "signal" with frequencies, Vsig = Vlo +- dv. The lab asked us to choose frequency difference dv somewhat small compared to Vlo, like 5%. So our dv = 1MHz, so Vsig = 21MHz and 19MHz. The resulting power spectrum for Vsig = 21MHz is shown in 5. In the figure, we can see that our new signal hovers around zero, much smaller than the 21MHz and 20MHz inputs that were combined. In addition, we can see sidebands on either side of the signal. Our experimentally-produced signal is different than the theoretical process because ideally we would like the cosine and sine waves to be phase shifted by exactly $\pi/2$, but in reality we see false “images” of tones at corresponding positive and negative frequencies on either side of zero [8].

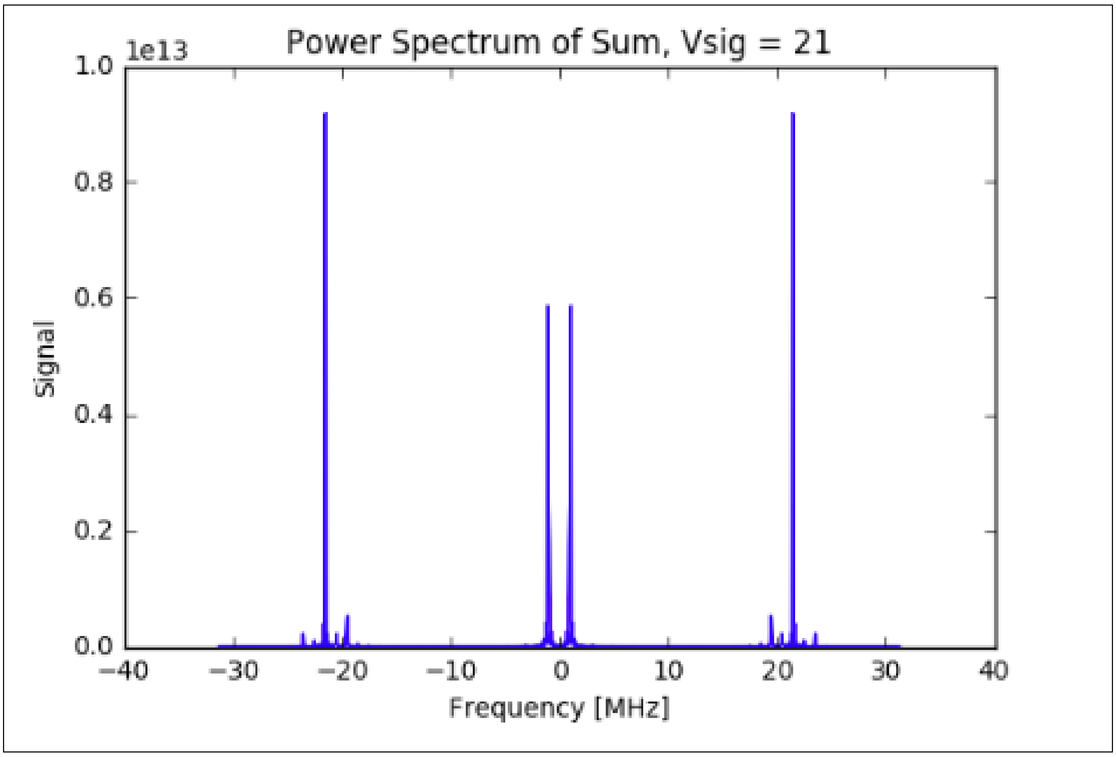

Figure 5: Power spectrum vs frequency for heterodyned DSB Mixer data where Vlo = 20 MHz and Vsig = 21MHz. Sidebands on either side of the signal are produced as by-products of signal mixing.

4.1 Conclusion

In this lab, our group focused on gathering generated radio signals, digitally analyzing them using Fourier transforms, understanding the beauty and complexity of Fourier transforms, and beginning to experimentally adjust our incoming radio signal to adjust for higher frequencies, using techniques such as the DSB mixer. All of these skills are building our foundational knowledge of experimental radio astronomy, as well as our appreciation for the theoretical and experimental complexity that comes along with each new discovery of the radio universe.

References

[1] Parsons, A. (2014). Nyquist Sampling. [online] AstroBaki. [Accessed 12 Feb. 2019].

[2] Dave, D. (2009). Sampling Rate, Nyquist Rate, and Aliasing (data acquisition effects). [online] YouTube. Available at: https://www.youtube.com/watch?v=Fy9dJgGCWZI [Accessed 12 Feb. 2019].

[3] Parsons, A. (2018). Fourier Transform. [online] AstroBaki. [Accessed 12 Feb. 2019].

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[8] Parsons, A. (2014). Heterodyne Mixers. [online] AstroBaki. [Accessed 12 Feb. 2019].