CURRICULUM HANDBOOK

Master of Science [M.Sc.]

in

PURE AND APPLIED MATHEMATICS

[CLASS OF 2025]

AFRICAN UNIVERSITY OF SCIENCE AND TECHNOLOGY

SUMMARY

#	COURSE NAME	CODE	CREDIT UNITS	CATEGORY
1	Foundations of Mathematical Analysis I	MTH 800	3	Foundation
2	Foundations of Mathematical Analysis II	MTH 801	3	Foundation
3	Foundations of Riemann Integral	MTH 802	3	Foundation
4	Linear Algebra	MTH 803	3	Foundation
5	Metric Spaces & Calculus in Rn	MTH 804	3	Foundation
6	Differential Equations (ODE)	MTH 805	3	Foundation
7	Linear Partial Differential Equations(Classical)	MTH 806	3	Foundation
8	Topology	MTH 811	3	Core
9	Measure Theory & Integration	MTH 812	3	Core
10	Lp Spaces & Related Topics	MTH 813	3	Core
11	Complex Analysis	MTH 814	3	Core
12	Linear Applicable Functional Analysis	MTH 815	3	Core
13	Sobolev Spaces & Elliptic PDEs	MTH 816	3	Core
14	Numerical Methods for PDEs I	MTH 818	3	Core
15	Master Dissertation	MTH 819	PASS/FAIL	Core
16	Iterative Algorithms for Nonlinear Equations	MTH 901	3	PhD
17	Evolution Equations, Semigroups, PDEs	MTH 902	3	PhD
18	Numerical Methods for PDEs II	MTH 903	3	PhD
19	Convex Analysis and Optimization	MTH 904	3	PhD
20	Hammerstein Integral Equations	MTH 905	3	PhD
21	Introduction to Probability Theory	MTH 906	3	PhD
22	Selected Topics	MTH 911	3	PhD
23	Ph.D. Research	MTH 999	PASS/FAIL	PhD

Note:

- General grade and examination requirements, calculation of GPA and CGPA, and other academic requirements can be found in Chapter 4 of the AUST Student Handbook.
- A minimum of 39 Credit Units is required for a Master of Science degree in Pure and Applied Mathematics.
- ALL *Foundation* and *Core* Courses are compulsory.
- There is no credit load for the *Master Dissertation*, with a grade of PASS/FAIL awarded for the course. A grade of a PASS is needed for an award of a Master of Science degree in Pure and Applied Mathematics.

COURSE NAME – MTH 800: Foundations of Mathematical Analysis I

References

a. (Text book, title, author, and year)

- C.E. Chidume and C.O. Chidume, Foundations of Mathematical Analysis, University of Ibadan Press, 2013.(TETFund Books Development Project).

b. (other supplemental materials): *Nil*

Specific course informationa. Brief description of the content of the course (catalog description)

a. Brief description of the content of the course (catalog description)

This course is meant as a first introduction to rigorous mathematics; understanding writing of proof and solution of exercises will be emphasized.

b. Course information

The real number system, Boundedness, Sequences, Convergence and divergence of sequences, limit superior and limit inferior of bounded sequences.

Specific goals for the course (in terms of outcomes by the student)

- The student will develop mathematical rigor and the ability to digest the details of condensed mathematical problems in the area of analysis.
- The student will be able to use concrete ideas to illustrate the concept of real numbers.
- The student will be able to establish the boundedness or unboundedness of a subset of real numbers.
- The student will develop the ability to prove convergence/divergence of a sequence of real numbers.
- The student will develop the ability to reorganize and make use of continuous functions
- The student will be abreast with the theory and intricacies of differentiation.
- The student will demonstrate the ability to prove convergence/divergence of a series of real numbers.
- The student will demonstrate a good understanding of aeries and easily derive certain properties of a series and apply them in problems.

Brief list of topics to be covered

- The real number system. Bounded subset of R: Bounded set, infimum, supremum, basic properties and proofs; examples and techniques for solving exercises.

- Sequences in R. Definition and examples, order relation; monotone sequences and basic properties, a fundamental theorem, the Euler number, sequences defined by recurrence relations, sandwich theorem and applications, uniqueness of limit, limit theorem and proofs, sequences converging to $+\infty$ and $-\infty$. Techniques for solving numerous types of exercises on sequences in R.
- Bolzano-Weierstrass theorem; lim sup and lim inf; convergence sequence revisited; Cauchy sequences, completeness of R.

COURSE NAME – MTH 801: Foundations of Mathematical Analysis II

References

a. (Text book, title, author, and year)

- C.E. Chidume and C.O. Chidume, Foundations of Mathematical Analysis, University of Ibadan Press, 2013.(TETFund Books Development Project).

b. (other supplemental materials): *Nil*

Specific course information

a. Brief description of the content of the course (catalog description)

This course is meant as a first introduction to rigorous mathematics; understanding writing of proof and solution of exercises will be emphasized.

b. Course information

Continuity of real valued functions. Differentiability, Series of real numbers, Intermediate value theorem, Topological notions, Series of non-negative real numbers.

c. Prerequisites or Co-requisites

- MTH800

Specific goals for the course (in terms of outcomes by the student)

- The student will develop mathematical rigor and the ability to digest the details of condensed mathematical problems in the area of analysis.
- The student will be able to use concrete ideas to illustrate the concept of real numbers
- The student will be able to establish the boundedness or unboundedness of a subset of real numbers
- The student will develop the ability to prove convergence/divergence of a sequence of real numbers
- The student will develop the ability to reorganize and make use of continuous functions
- The student will be abreast with the theory and intricacies of differentiation
- The student will demonstrate the ability to prove convergence/divergence of a series of real numbers
- The student will demonstrate a good understanding of series and easily derive certain properties of a series and apply them in problems

Brief list of topics to be covered

- Continuity: Limits of functions, topological notions, one-side continuity; continuity theorems. Problem's solving techniques.
- Intermediate value theorem; continuous real valued maps f on [a, b]; consequences boundedness of f, existences of minimizers and maximizers; uniform continuity, etc. Comparison with continuous maps on (a, b).
- Topological notions: open sets, closed sets, compact sets, compact sets and continuity; compactness in terms of open covers.
- Differentiability: the derivative, examples; Rolle's theorem, Mean value theorem, L'Hospital's rule. Applications
- Series of non-negative real numbers: Definition, examples and basic properties; the integral test, the comparison test, the limit comparison test, D'Alembert's ratio test, the Cauchy's root test, alternating series, absolute and conditional convergence, re-arrangement of series.

COURSE NAME – MTH 802: Foundations of Riemann Integral

References

a. (Text book, title, author, and year)

- C.E. Chidume and C.O. Chidume, Foundations of Mathematical Analysis, University of Ibadan Press, 2013.(TETFund Books Development Project).

b. (Other supplemental materials): Nil

Specific course information

a. Brief description of the content of the course (catalog description)

This course is meant as a first introduction to rigorous mathematics; understanding writing of proof and solution of exercises will be emphasized.

b. Course information

The Riemann integral. Convergence of sequence of real-valued functions, point-wise and uniform convergence series of real-valued functions. Power series.

c. Prerequisites or Co-requisites

- MTH 800, MTH 801

Specific goals for the course (in terms of outcomes by the student)

- The student will understand the idea behind the concept of integration in general and ba able to make meaning of other concepts of Integration he/she will meet in the future. In addition the student will be able to manipulate sequence of real valued functions.
- At the end of the course, the student will have totally mastered the subject by knowing the summary and by doing lots of exercises
- The student will demonstrate the ability to expose the course materials to others
- The student will demonstrate the ability to apply the results in new contexts
- The student will demonstrate the ability to read and understand advanced books

Brief list of topics to be covered

- The Riemann integral: definition and examples; basic properties, two basic theorems, the fundamental theorem of calculus, integration by parts formula.
- Point-wise convergence of a sequences of real-valued functions, questions of interest; point-wise convergence a disaster for these questions.

- Uniform convergence: definition and examples; questions of interest; some useful answers.
- Series of real-valued functions; Weierstrass M -test, term-by-term differentiation and integration.
- Power series: definition and examples, fundamental theorem, uniform convergence of power series; uniform convergence at end points.
- Equi-continuity; definition and examples; the Arzela-Ascoli theorem.
- Miscellaneous examples, exercises; techniques for solving problems.

COURSE NAME – MTH 803: Linear Algebra

References

a. (Text book, title, author, and year)

- D. C. lay, Linear Algebra and its Applications, Addison-Wesley, (2012).

b. (Other supplemental materials):

- S. Axler, Linear Algebra Done Right, Springer, (1997).
- K. Ezzinbi, Linear Algebra, AUST, (2016).

Specific course information

a. Brief description of the content of the course (catalog description)

Vector spaces, Linear maps, matrices, eigenvalues and eigen vectors

b. Prerequisites or Co-requisites

MTH 800, MTH 801 & MTH 802

Specific goals for the course (in terms of outcomes by the student)

- Specific outcomes of instruction, ex. The student will be able to solve linear systems using tools from linear algebra.
- The student will demonstrate the ability to the complexity of matrices using reduction methods.

Brief list of topics to be covered

- Motivations Systems of Linear Equations (Definition, Elementary transformations, Methods for solving Homogeneous or Non homogeneous Systems Linear Equations, Cramer Systems).
- Vector Spaces (Fields, Vector spaces, Linear combination, Generating systems of vectors, Linearly independent systems of vectors, Basis, Dimension, Vector Subspaces.
- Linear Maps (Definitions, Properties, Operations on Linear maps, Monomorphisms, Epimorphisms,Isomorphisms, Rank. Linear maps between finite dimensional spaces, Dimension Formula, Algebraic dual, and introduction to matrix theory)
- Matrices (Definitions, Properties, Matrix Operations, Rank of a matrix, Matrices associated to Linear maps defined between finite dimensional linear spaces, Determinant of a square matrix, Special classes of Matrices, Inversion of nonsingular square matrices, Eigenvalues, Eigenvectors, Diagonalization, Properties of Symmetric matrices, Nilpotent matrices,Jordan canonical forms of matrices.) Notion of Bilinear and Quadratic Forms Inner products. Euclidean

spaces Definitions, Geometry of inner product spaces, Orthogonal transformation. Notion of Prehilbertian spaces.

COURSE NAME – MTH 804: Metric Spaces & Calculus in Rn

References

a. (Text book, title, author, and year)

- C.E. Chidume, Functional Analysis, An introduction to metric spaces, Longman Nigeria Ltd. (1989), ISBN 978-139-7764.
- C.E. Chidume, An introduction to metric spaces (New Edition)
- E. Zeidler, Nonlinear functional analysis and its applications, Springer, (1986).
- L. Berge, Topological spaces, Dover books, (1997).
- C. E. Chidume & K. Ezzinbi, Lecture notes in metric spaces and differentiability in Rn, AUST, (2012).

b. (Other supplemental materials): Nil

Specific course information

a. Brief description of the content of the course (catalog description)

Metric spaces, differentiability, mean value theorem, local inverse mapping theorem, Taylor expansion

b. Prerequisites or Co-requisites

MTH 800, MTH 801, MTH 802, & MTH 803.

Specific goals for the course (in terms of outcomes by the student)

- a. specific outcomes of instruction, ex. The student will be able to understand to the topology of metric spaces and especially topology of Rn and then to learn how to differentiate in Rn.
- The student will demonstrate the ability to use theorems from metric spaces and differentiability to solve problems in ordinary differential equations and partial differential equations.
- explicitly indicate which of the student outcomes listed in Criterion 3 or any other outcomes are addressed by the course.

Brief list of topics to be covered

- Metric spaces
- Open sets- closed sets
- Convergence of sequences in metric spaces
- Compactness, connectedness, completeness

- Continuity in metric spaces and applications
- Differentiability in Rn.
- Mean value theorem
- Local inverse mapping theorem and implicit functions theorem
- Taylor expansion
- Applications

COURSE NAME – MTH 805: Differential Equations (ODE)

References

a. (Text book, title, author, and year)

- M.W. Hirsh and S. Smale, Differential equations and dynamical systemss and linear algebra, Academic Press, (1974).
- K. Ezzinbi, Ordinary differential equations, AUST, (2016).

b. (other supplemental materials): Nil

Specific course information

a. Brief description of the content of the course (catalog description)

Ordinary differential equations, stability and long time behavior and applications

b. Prerequisites or Co-requisites

MTH 800, MTH 801, MTH 802, MTH 803

Specific goals for the course (in terms of outcomes by the student)

- a. specific outcomes of instruction, ex. The student will be able to solve ordinary differential equations.
- The student will demonstrate the ability to study the asymptotic behavior of solutions.

Brief list of topics to be covered

- Scalar ordinary differential equations
- Ordinary differential equations, basic theory
- Linear systems and resolvent operator
- Stability and asymptotic behavior
- Lyapunov methods
- Applications

COURSE NAME – MTH 806: Linear Partial Differential Equations (Classical)

References

a. *(Text book, title, author, and year)*

- Tyn Myint-U, Lokenath Debnath, Linear Partial Differential Equations for Scientists and Engineers.

b. (Other supplemental materials): Nil

Specific course information

a. Brief description of the content of the course (catalog description)

Fourier Series, Piece-wise continuous functions, Convergence results , Applications for Ordinary Differential Equations, One-Dimensional Partial Differential Equations, Basic definitions, Classification, Separation of variables Method, Heat Equation with Homogeneous Boundary conditions, Heat Equations with homogeneous Neumann boundary conditions, Heat Equations with Robin's boundary conditions, Solving PDE using Fourier Series, Wave Equations with variant Boundary Conditions

b. Prerequisites or Co-requisites

MTH 800, MTH 801, & MTH 802

Specific goals for the course (in terms of outcomes by the student)

- The student will be able to find the fourier series of a given function
- The student will be able to classify Partial differential Equations
- The student will be able to classify the different boundary conditions
- The student will able to use separation variables

COURSE NAME – MTH 811: Topology

References

a. *(Text book, title, author, and year)*

- J.R. Munkres, Topology-A First course, Prentice-Hall, New Jersey.(and the second revised edition: Topology, Prentice Hall 2000)
- Lynn, J. Steen, Jr. Arthur, Counter example in topology
- N. Djitte, Lecture notes on topological spaces, AUST lecture note series, 2013.

b. (Other supplemental materials): Nil

Specific course information

a. Brief description of the content of the course (catalog description)

Topological spaces and continuous functions. Topological spaces; basis and subbasis for a topology; the order topology; the product and box topology; the subspaces topology;closed sets and limit points; continuous functions; the metric topology; the quotient topology. Connectedness and Compactness: connected and path connected spaces; connected and compact sets in the real line; limit point compactness; compactness in metric spaces. Countability and separation axioms: The *countability axioms; the separation axioms; Uryshon lemma; Uryshon metrization theorem; Tietze extension theorem.*

b. Prerequisites or Co-requisites
MTH 800, MTH 801 & MTH 802

Specific goals for the course (in terms of outcomes by the student)

- The student will be able to manipulate different topological concepts
- The student will be able to characterize continuity in different forms
- The student will become free with connecteness and compactness

Brief list of topics to be covered

- Topological spaces and continuous functions. Topological spaces; basis and subbasis for a topology; the order topology; the product and box topology; the subspaces topology;closed sets and limit points; continuous functions; the metric topology; the quotient topology.
- Connectedness and Compactness: connected and path connected spaces; connected and compact sets in the real line; limit point compactness; compactness in metric spaces.
- Countability and separation axioms: the countability axioms; the separation axioms; Uryshon lemma; Uryshon metrization theorem; Tietze extension theorem.

COURSE NAME – MTH 812: Measure Theory & Integration

References

a. (Text book, title, author, and year)

- Measure Theory and Integration for and By the Learner (2017). Gane Samb LO. SPAS Text Book Series.

b. (Other supplemental materials):

- Halmos P.(1950). Measure Theory. Van Nostrand. New York.
- Lo`eve Michel (1997). Probability Theory I, Springer Verlag, 4-th Ed, 19975.
- Bogachev, Vladimir I.(2017). Measure Theory I. Springer-Verlag.
- Parthasarathy, K. R. (2005). Introduction to Probabality and Measure. Hindustan Book Agency. India, 2005.

Specific course information

a. Brief description of the content of the course (catalog description)

- Measure Theory and Integration is one of the basis of Modern Mathematics. Its main objective is to endow the learner with a deep and definitive basis on the modern theory on integrations and all its applications. The core theory is measure theory and measurability which is the largest generalization of the continuity concept. In Real Analysis I, the learner is require to master all aspects of measurability, countably additive applications on sets, in particulars measures. Constructions of measures and their properties are central in this part. Hahn-Jordan decomposition theorem.
- Measure Theory and Integration is devoted to Integration and applications : Modern integration with respect to a measure, Lebesgue-Stieljes Measures on Rk, Main Convergence Theorems: Monotone Convergence Theorem, Fatou-Lebesgue Dominated Theorem. Full comparison between Riemann and Lebesgue integration on R is central here. The applications cover; the product measure (finite and infinite), the Fubini and Tonelli theorems, Radon Nikodym Theorem.
- Measure Theory and Integration is ended by a large summary of Probability Theory which consists in adapting the main results of Measure Theory with the terminology of Probability Measure.

b. Prerequisites or Co-requisites

MTH 804 & MTH 811

Specific goals for the course (in terms of outcomes by the student)

- Reach excellence levels for each main part of the courses

- Perfectly master the subject by knowing the summary and by doing all the theoretical exercises.
- Reading the technical parts.
- Being able to expose the course materials to others
- Being able to apply the results in new contexts through series of exercises.
- Being able to read advanced book

Brief list of topics to be covered

- Sets and Sets operations (sets operations, limits inferior and superior of sequences of subsets, inverse images)
- Product space, class of rectangles, projections and their inverses images. Relation between inverses images of projections and class of rectangles.
- Measurability of sets (Sigma-algebra, algebra, semi-algebra, monotone class, Dynkin class, pi-class. Measurable sets)
- Generation of sigma-algebras, of algebras, of monotone classes, of Dynkin classes. The monotone class generation theorem, the pi-lambda theorem. The algebra generated by a semi-algebra.
- Specific sigma-algebra. Borel sigma-algebra and its multiple generation on Rk . Semi-algebra of the intervals class, of the class of intervals open at left and closed at right.
- Borel sigma-algebra on topological spaces.
- Product sigma-algebra. Semi-algebra of measurable rectangles. Infinite case : semi-algebra of cylinders with finite bases. Measurability of Sections. The point of view of inverse images.
- Measurable mappings (General definitions. Composition of Measurable mappings. Measurability and continuity in Borel sigma-algebra)
- Measurability of real-valued Borel mappings. Different criteria. Operations of real valued measurable mapping. Limits inferior and superior and limits, if any, of sequences of real-valued mappings.
- Measurable applications mapping with values in a product space. Measurability of the projections.
- Special case of measurable applications of mappings from R to R. A measurable : monotone function (countability of the set of discontinuity points), left or right continuity functions, functions left (right) continuous anywhere expect, at most, on a countable sets of points such that left limit (right) limit exists, semi-continuous functions, convex functions, etc.
- Class of simple real-value functions from R to R (denoted E). Density in the class of real-valued measurable function.
- Measures (Additive sets applications. sigma-additivity, sigma-finiteness. subadditivity sigma-sub-additivity. Relation between additivity and sigma-sub-additivity or, additivity and continuity at the empty set, and sigma-additivity)

- Measure (non-negative measures). Definitions. Properties, examples and manipulations.
- Construction of Measures. From semi-algebras to algebras. From algebras to sigma-algebra. Construction of Carath´eodory. Exterior Measures.
- Construction of the Lebesgue-Stieljes Measures of Rk. Distribution function. Lebesgue Measure. Counting measures. Construction of Vittali of non measurable sets on R.
- Product Measure : Statement and manipulations (General proof in the real Analysis II, application of Monotone Convergence Theorem). Hahn-Jordan decomposition.

COURSE NAME – MTH 813: Lp Spaces & Related Topics

References

a. *(Text book, title, author, and year)*

- *Measure Theory and Integration for and By the Learner (2017).* Gane Samb LO. SPAS Text Book Series.

b. (Other supplemental materials):

- Halmos P.(1950). Measure Theory. Van Nostrand. New York.
- Lo`eve Michel (1997). Probability Theory I, Springer Verlag, 4-th Ed, 1997
- Bogachev, Vladimir I.(2017). Measure Theory I. Springer-Verlag.
- *Parthasarathy, K. R. (2005). Introduction to Probabality and Measure. Hindustan Book Agency. India, 2005.*

Specific course information

a. Brief description of the content of the course (catalog description)

- Measure Theory and Integration is one of the basis of Modern Mathematics. Its main objective is to endow the learner with a deep and definitive basis on the modern theory on integrations and all its applications. The core theory is measure theory and measurability which is the largest generalization of the continuity concept. In Real Analysis I, the learner is require to master all aspects of measurability, countably additive applications on sets, in particulars measures. Constructions of measures and their properties are central in this part. Hahn-Jordan decomposition theorem.
- Lp spaces & Related Topics is devoted to Integration and applications : Modern integration with respect to a measure, Lebesgue-Stieljes Measures on Rk, Main Convergence Theorems : Monotone Convergence Theorem, Fatou-Lebesgue Dominated Theorem. Full comparison between Riemann and Lebesgue integration on R is central here. The applications cover : the product measure (finite and infinite), the Fubini and Tonelli theorems, Radon Nikodym Theorem.
- Lp spaces & Related Topics is ended by a large summary of Probability Theory which consists in adapting the main results of Measure Theory with the terminology of Probability Measure.

b. Prerequisites or Co-requisites

MTH 804, MTH 811 & MTH 812

Specific goals for the course (in terms of outcomes by the student)

- Reach excellence levels for each main part of the courses.

- Perfectly master the subject by knowing the summary and by doing all the theoretical exercises.
- Reading the technical parts.
- Being able to expose the course materials to others
- Being able to apply the results in new contexts through series of exercises.
- Being able to read advanced book

Contents

- **Integration with respect to a measure.**
  Construction with the four classical steps. Properties of the class E. Extension to non-negative real-valued functions, and next to arbitrary functions. Integration with respect to the counting measure. Comparison between series and integrals with respect to counting measures. Integration with respect to the Lebesgue measure on R. Easy comparison with Riemann Integration of bounded functions on compact sets.

- **Convergence Theorem.**
  Type of convergence : almost-everywhere, in measure. Almost-everywhere Cauchy sequences and Cauchy sequence in measure. Comparison theorem. Convergence Theorem : Monotone Convergence Theorem (Beppo-Levy). Fatou-Lebesgue Dominated Theorem and Young Theorem.

- **Applications.**
  (1) Inversion between the summation sign and the limits sign, Inversion between the summation sign and the differentiation sign
  (2) Differentiation of series of functions, inversion of double summation (counting measures)
  (3) Existence of the product measure with any elements of topology. Integration with respect to a finite product measure. Fubini's and Tonelli's Formula.
  (4) Full comparison between Riemann and L ebesgue Integrals on compact sets. Lebesque Integral on R and improper Riemann integral.
  (5) Radon Nikodym Theorem. Statement and proof.

- **Lp spaces.**
  Lp spaces as Banach spaces. L 2 Hilbert spaces. Spaces of sequences as Lp spaces with respect to the counting measure. Equi-integrability or uniform continuity and comparison between Lp-convergence and convergence in probability. (E) Introduction to probability Theory
  (1) Probabilistic terminology of Measure Theory: probability measure theory, events, random variables, integration with respect to a probability measure including mathematical expectations, probability laws (measure image), cumulative distribution functions (Lebesgues-

Stieljes measures), moments, characteristic moment (using Stone-Weierstrass approximation Theorem), independence (product measure)

(2) Kolmogorov Foundamental Theorem (Infinite product measure)

(3) Application of Radon-Nikodym theorem to probability theory : probability density function with respect to a given measure Mathematical expectation.

Brief list of topics to be covered

- Measure Theory and Integration is devoted to Integration and applications : Modern integration with respect to a measure, Lebesgue-Stieljes Measures on Rk, Main Convergence Theorems: Monotone Convergence Theorem, Fatou-Lebesgue Dominaited Theorem. Full comparison between Riemann and Lebesgue integration on R is central here. The applications cover: the product measure (finite and infinite), the Fubini and Tonelli Measures, Radon Nikodym Theorem. Introduction the Banach spaces Lp. Inequality de Markov, Holder, Minkowski, cr-inequality.
- Measure Theory and Integration is ended by a large summary of Probability Theory which consists in adapting the main results of Measure Theory with the terminology of Probability Measure.
- Mapping Convergence. Types of convergence: almost-everywhere , in measure. Almost-everywhere Cauchy sequences and Cauchy sequence in measure. Comparison theorem. Convergence Theorem. Monotone Convergence Theorem (Beppo-Levy). Fatou-Lebesgue Dominated Theorem and Young Theorem.
- Applications. Inversion between the summation sign and the limits sign, Inversion between the summation sign and the differentiation sign. Differentiation of series of functions, inversion of double summation (counting measures). Existence of the product measure with any elements of topology. Integration with respect to a finite product measure. Fubini's and Tonelli's Formula. Full comparison between Riemann and Lebesgue Integrals on compact sets. Lebesque Integral on R and improper Riemann integral.
- Radon Nikodym Theorem. Statement and proof.
- Lp spaces (Lp spaces as Banach spaces. L2 Hilbert spaces. Spaces of sequences as Lp spaces with respect to the counting measure. Equi-integrability or uniform continuity and comparison between Lp-convergence and convergence in probability)
- Introduction to probability Theory. Probabilistic terminology of Measure Theory : probability measure theory, events, random variables, integration with respect to a probability measure including mathematical expectations, probability laws (measure image), cumulative distribution functions (Lebesgue-Stieljes measures), moments, characteristic moment (using Stone-Weierstrass approximation Theorem), independence (product measure). Kolmogorov Foundamental Theorem (Infinite product measure). Application of Radon-Nikodym theorem to

probability theory : probability density function with respect to a given measure, mathematical expectation.

- Integration with respect to a measure. Construction with the four classical steps. Properties of the class E. Extension to non-negative real-valued functions, and next to arbitrary functions. Integration with respect to the counting measure. Comparison between series and integrals with respect to counting measures. Integration with respect to the Lebesgue measure on R. Easy comparison with Riemann Integration of bounded functions on compact sets.

COURSE NAME – MTH 814: Complex Analysis

References

a. (Text book, title, author, and year)

- J.B. Conway, Functions of one complex variables (Springer Verger)

b. (Other supplemental materials):

Brief list of topics to be covered

- Analytic functions: Complex numbers and their properties. The extended plane. Compactification of complex plane. Stereographic projection.
- Functions of complex variables, limits, continuity and complex differentiation. Cauchy Riemann Equations. Analytic functions. Conformal mappings.
- Elementary functions(some, rational functions, exponent, trigonometric functions, logarithm).
- Properties of Analytic functions
- Integral
- Curvilinear integral: main properties;
- Cauchy's integral theorem
- Taylor series
- Expansion in Laurent series
- Classification of isolated singularities of analytic functions, entire and meromorphic functions
- Residues and application to calculation of integrals, Jordan lemma
- Argument principle. Rouche's theorem. Fundamental theorem of algebra. Open mapping principle; maximum modulus principle.

COURSE NAME – MTH 815: Linear Applicable Functional Analysis

References

a. (Text book, title, author, and year)

- C.E. Chidume, Applicable Functional Analysis, Ibadan University Press Publishing House, 2014, University of Ibadan, Ibadan, Nigeria. ISBN: 978-978-8456-31-5.

b. (Other supplemental materials): Nil

Brief list of topics to be covered

- Motivations
- Normed Linear Spaces;
- Bounded Linear Maps;
- Hahn-Banach Theorem: Analytic and Geometric Forms;
- Uniform Boundedness Principle;
- Open mapping Theorem;
- Hilbert spaces;
- Operators on Hilbert spaces;
- Reflexive Spaces;
- Weak and Weak* Topologies;
- Kakutani’s Theorem;
- Banach Alaoglu Theorem;
- Milman-Pettis Theorem;
- Ebertein-Smul’yan Theorem;
- Uniformly Convex spaces.
- Applications of weak topology in Optimization Theory and in Fixed Point Theory.
- Memory Management
- File System
- Case studies of various OS

COURSE NAME – MTH 816: Sobolev Spaces & Elliptic PDEs

References

a. (Text book, title, author, and year)

- H. Brezis, Functional Analysis, Sobolev spaces and partial differential equations, Universitext, Springer, (2010). K. Ezzinbi, Partial Differential Equations, AUST, (2015).

b. (Other supplemental materials)

Specific course information

a. Prerequisites or Co-requisites

MTH 800, MTH 801, MTH 802, MTH 803, MTH 811, MTH 812, MTH 813

Specific goals for the course (in terms of outcomes by the student)

- Specific outcomes of instruction, ex. The student will be able to solve partial differential equations using distributions theory and sobolev spaces.
- The student will demonstrate the ability to solve elliptic problems using variational methods.

Brief list of topics to be covered

- Lp spaces
- Distributions and properties
- Fourier analysis
- Tempered distributions
- Sobolev spaces
- Green Fromula
- Lax-Migram and Stampachia Theorems
- Varational methods
- Elliptic problems

COURSE NAME – MTH 818: Numerical Methods for PDEs I

References

a. (Text book, title, author, and year)

- H. Brezis., Functional Analysis, Sobolev Spaces and Partial Differential Equations, Springer Verlag, New York, 2010.

b. (Other supplemental materials):

- B. Lucquin and O. Pironneau., Introduction to scientific computing, Wiley, 1998.
- J. C. Strikwerda, Wadsworth 1989. Finite difference schemes and partial differential equations,
- T. Winther, Introduction to differential equations: A computational approach, Springer, 1998.

Specific course information

a. Brief description of the content of the course (catalog description)

This course deals with introduction to Modern Numerical Analysis of Partial Differential Equations. In particular, we are concern with Finite difference approximation of Partial Differential Equations. The course mainly divided into two parts;

- Numerical approximation of Ordinary differential equations (ODE).
- Numerical approximation of Partial differential equations (PDE).

b. Prerequisites or Co-requisites

MTH 803, MTH 811, MTH 815, MTH 816

Specific goals for the course (in terms of outcomes by the student)

As far as PDE are concerned, we focus on traditional equations that are: heat equation, wave equation, Poisson's equation, but we also discussed some nonlinear equations to illustrate the difficulty to carry on with the linear theory. Implicit, explicit and multi-steps schemes are presented and thoroughly analyzed. After the Mathematical analysis of the approximate problem, we reformulate the problem in the standard form AX = F, and we introduce the students to coding if time permit.