1) Reservoir pressure 1.8 atm (p_{o1}) test section $M_t = 2.6$

$p_e = 1$ atm $\qquad M_e = 0.1$

assume

Find η_d

real p_{o2}

$$\eta_d = \frac{(p_{o2}/p_{o,1})_{real}}{1} = (1.0/1.8) = 0.56$$

assuming at $M=0.1$ $p_2 = p_{o2}$

which $p_{o2} = 1.007$? is a pretty good assumption

2) $M_1 = 2.5$ $T_1 = 310\,K$ $p_1 = 70\,kPa$ $l = 64\,cm$ $D = 2cm$

$\bar{f} = 0.005$ $= 0.64m$ $= 0.02m$

Shock at $M = 2$

a. length where shock occurs

$$\int_{x_1}^{x_2} \frac{4\bar{f}}{D}\,dx = \left(-\frac{1}{\gamma M^2} - \frac{\gamma+1}{2\gamma}\ln\left(\frac{M^2}{1+\frac{\gamma-1}{2}M^2}\right)\right)\Bigg|_{M_1}^{M_2}$$

$$\int_{x_1}^{x_2} \frac{4\bar{f}\,dx}{D} = \frac{4\bar{f}}{D}(x_2 - x_1)$$

$$x_1 = 0$$

$$\frac{4\bar{f}}{D} = \frac{4 \cdot 0.005}{0.02} = 1$$

$$x_2 = -\frac{1}{1.4(2)^2} - \frac{2.4}{2.8}\ln\left(\frac{4}{1+0.2(4)}\right) + \frac{1}{1.4(2.5)^2} + \frac{2.4}{2.8}\ln\left(\frac{2.5^2}{1+0.2(2.5)^2}\right)$$

$$= -0.179 - 0.684 + 0.114 - 0.8757$$

$$x_2 = 0.127\,m = 12.7\,cm$$

b. Fanno flow assumptions & normal shock (1D)

→ Use fanno relations to see how flow is just prior to shock

$$\frac{T_2}{T_1} = \frac{2+(\gamma-1)M_1^2}{2+(\gamma-1)M_2^2} = 1.25 \;\&\; T_2 = 310(1.25)K$$

$$T_2 = 387.5\,K$$

$$\frac{p_2}{p_1} = \frac{M_1}{M_2}\left(\frac{T_2}{T_1}\right)^{1/2}$$

$$p_2 = \frac{2.5}{2}(1.25)^{1/2} \cdot p_1$$

$$p_2 = 97.83\,kPa$$

2) b. Cont $P_{o2} = \left(1 + \frac{\gamma-1}{2} M_2^2\right)^{\gamma/\gamma-1} \cdot P_2$

$$\frac{P_3}{P_2} = \frac{2\gamma M_2^2 - (\gamma-1)}{\gamma+1} = 4.5$$

$$P_3 = 440.235 \text{ kPa}$$

$$S(x_2 - x_1) = F(M_2) - F(M_1)$$

$$x_2 = x_1 +$$

c. $$M_3^2 = \frac{(\gamma-1)M_2^2 + 2}{2\gamma M_2^2 - (\gamma-1)}$$

$$M_3 = (0.333)^{1/2}$$

$$M_3 = 0.577$$

Will it choke?

$x_3 = x_2 = 12.7 \text{ cm}$

Find $M_4 = M_*$ and correlate length → compare to known L

$M_4 = 1$

$$\int_{x_3}^{x_4} \frac{4\bar{f}}{D} dx = \left(-\frac{1}{\gamma M^2} - \frac{\gamma+1}{2\gamma} \ln\left(\frac{M}{1 + \frac{\gamma-1}{2} M^2} \right) \right) \Bigg|_{M_3}^{M_4}$$

$$\frac{4\bar{f}(x_4 - x_3)}{D} = \frac{4(0.005)}{0.02}(x_4 - 0.127)$$

RHS: $M_4 = 1$, $M_3 = 0.577$

$$RHS = -0.558 + 1.6189$$

$$RHS = 1.061$$

$$x_4 - 0.127 = 1.061$$

$x_4 = 1.188 \text{ m}$ which is longer than the duct, will not choke

→ So, compare choked condition to duct exit length and back out M_5

$$\int_{x_5}^{x_4} \frac{4f}{D}\,dx = \left(-\frac{1}{\gamma M^2} - \frac{\gamma+1}{2\gamma}\ln\left(\frac{M}{1+\frac{\gamma-1}{2}M^2}\right)\right)\Bigg|_{M_5}^{M_4}$$

$$x_4 - x_5 = F(M_4{=}1) - F(M_5)$$

→ In retrospect I should have coded this up.

$$x_4 - L = F_M(1) - F(M_e)$$

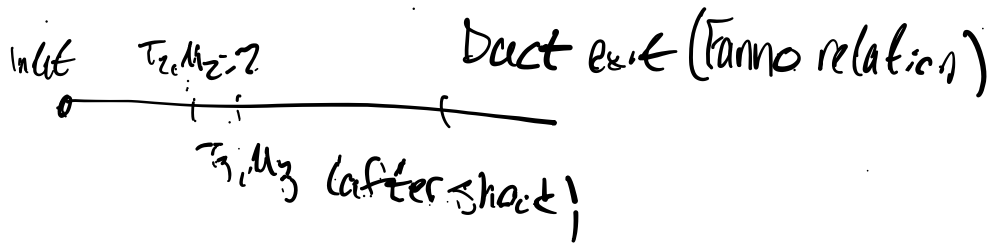
$$-x_4 + L + F(1) = F(M_e)$$

now I need to solve M from this → Bisection

Appended code shows

$M_e = 0.797$

d. T_{exit}: T_0 const throughout (adiabatic process)



$$T_e = \left(\frac{T_2}{T_1}\cdot T_1\right)\left(\frac{T_3}{T_2}\right)\left(\frac{T_e}{T_3}\right) = 98.6\,K$$

Fanno relation   normal shock   Fanno relation

3) $r = e^{1/(x+1)}$ radius is @ at duct inlet

$f = 0.005$, $\frac{\partial f}{\partial x} = 0$

$M_1 = 6$ $P_a = 200$ kPa $T_{01} = 600$K

a. i. $$\frac{dM}{M} = \frac{\left(1+\frac{\gamma-1}{2}M^2\right)}{M^2-1}\frac{dA}{A} - \left(\frac{1+\frac{\gamma-1}{2}M^2}{M^2-1}\right)\frac{\gamma M^2}{2}\frac{4f\,dx}{D(x)}$$

$$r = e^{1/(x+1)} = e^{(x+1)^{-1}}$$

$$D(x) = 2\,e^{(x+1)^{-1}}$$

$$A(x) = \pi\,e^{(x+1)^{-1}}\cdot e^{(x+1)^{-1}} = \pi e^{\frac{2}{x+1}} \qquad A(0) = \pi e^2$$

$$dA = dx\frac{dA}{dx} = dx\cdot\left(-\frac{2}{(x+1)^2}\right)\cdot\pi e^{(2/x+1)}$$

$$\frac{dA}{A} = dx\cdot\left(-\frac{2}{(x+1)^2}\right)$$

$$\frac{dM}{M} = \frac{\left(1+\frac{\gamma-1}{2}M^2\right)}{M^2-1}\left(\frac{-2}{(x+1)^2}\right)dx - \left(\frac{1+\frac{\gamma-1}{2}M^2}{M^2-1}\right)\frac{\gamma M^2}{2}\frac{4f\,dx}{D(x)}$$

$$\frac{\partial M}{M} = dx\left(\frac{1+\frac{\gamma-1}{2}M^2}{M^2-1}\right)\left(\frac{-2}{(x+1)^2} - \frac{\gamma M^2 f}{e^{1/(x+1)}}\right)$$

$$\frac{\partial M}{dx} = M\left(\frac{1+\frac{\gamma-1}{2}M^2}{M^2-1}\right)\left(\frac{-2}{(x+1)^2} - \frac{\gamma M^2 f}{e^{1/(x+1)}}\right)$$

$$M + dM = M + \frac{\partial M}{dx}\cdot dx$$

→ So, step thru and solve numerically with $dx = 1$mm

$L_{choke} = 59.185$ m via appended code

ii. $u_i = M_i \cdot \sqrt{\gamma R T_i}$

T_0 is const (adiabatic)

$T_0 = 600K$

$$\frac{T_0}{T_i} = \left(1 + \frac{\gamma-1}{2} M_i\right)$$

$$T_i = \frac{1}{1 + \frac{\gamma-1}{2} M^2} \cdot T_0$$

If I take

$$\sum_{i=1}^{n} \left(\frac{dx}{u_i} = t_i\right)$$ where t_i is time spent in each dx

I will have residence time.

$\rightarrow$ Code outputs $\boxed{t = 0.0839}$

iii. Static pressure:

$p = \rho R T$

$\rho u A = \text{Const} = \rho_{x=0} u_{x=0} A_{x=0}$

$\hookrightarrow$ u_i is known A_i is known

$$\rho_i = \frac{\text{Const}}{u_i A_i}$$ $A(x=0) = \pi e^2$

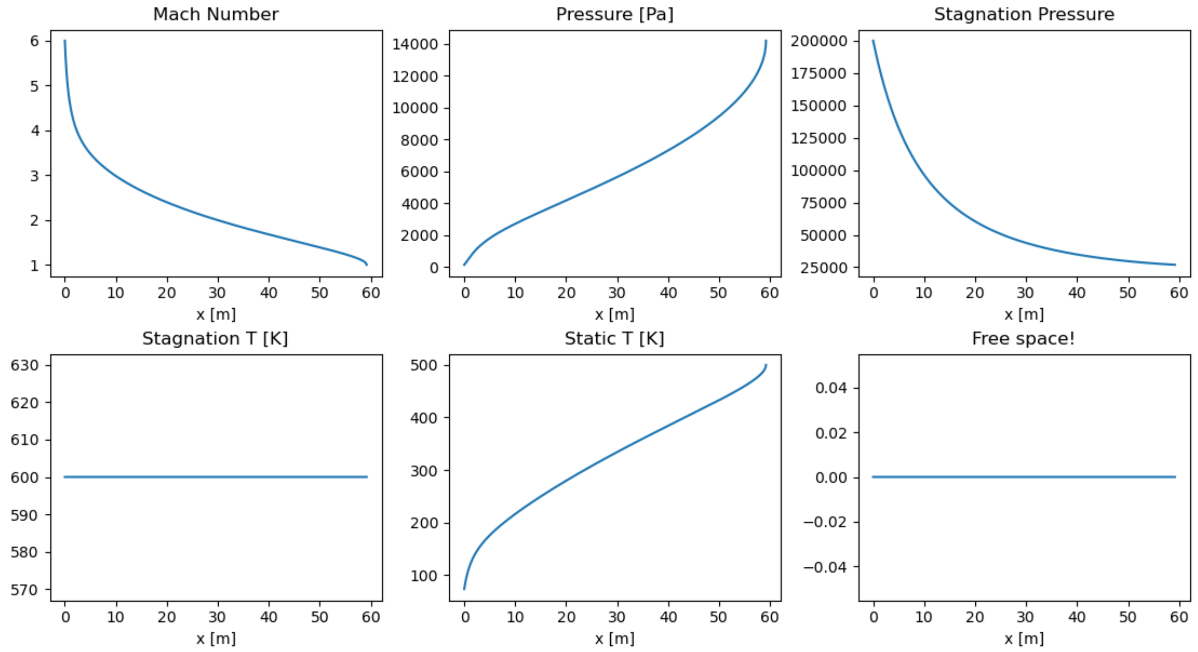
$p_i = \rho_i R T_i$

$$p(x=0) = \left(1 + \frac{\gamma-1}{2} M^2\right)^{-\frac{\gamma}{\gamma-1}} \cdot p_0$$

$\boxed{p(x=L) = 14.211 \text{ kPa}}$

iv. $\boxed{T(x=L) = 500.105 \text{ K}}$

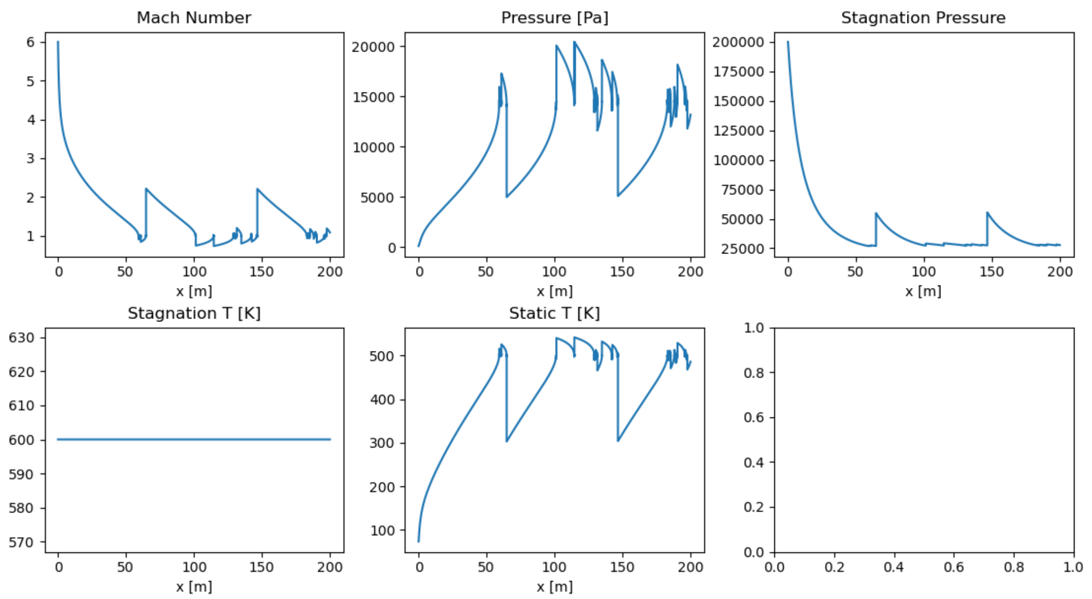
b. See plots


Mach Number
Pressure [Pa]
Stagnation Pressure
Stagnation T [K]
Static T [K]
Free space!
x [m]

c. See plots below.

What's happening is we jump subsonic, but the friction drives it to supersonic, drives it to sub...

The changes in the response of these flow properties to friction at the sonic condition drive the up-down response seen after choking.



d. $\mu = \mu_0 \left(\frac{T}{T_0}\right)^{1.5} \left(\frac{T_0 + 110.4}{T + 110.4}\right)$ $\mu_0 = 1.716 \cdot 10^{-5}$

$T_0 = 273.15$

$$f = \frac{0.0625}{\log\left(\frac{0.001}{3.7 D(x)} + \frac{5.74}{Re_D^{0.9}}\right)}$$

$\Rightarrow$ assume constant over small lengths such as dx to avoid crazy calculus

$$1/Re_D = \frac{\mu(x)}{\rho(x)\, u(x)\, D(x)}$$

$\rightarrow$ Now I have to march this along with every other property...

- $\rightarrow$ Assume everything that is a function of x is constant over each dx and update according to M as previously afterwards

$\rightarrow$ Code shows

$f\_avg = 0.00412 \Rightarrow$ about 20% off

$L_{choke} = 71.12m$
