2) $du - \frac{dp}{\rho a} = 0 \Rightarrow \frac{u_{wave}}{a_\infty} = \left(\frac{\gamma+1}{\gamma-1} \left({}^{p}/_{p_\infty} \right)^{\frac{(\gamma-1)}{2\gamma}} - \frac{2}{\gamma-1} \right) + \frac{u_\infty}{a_\infty}$

NLR wave compatability $\Rightarrow$ we have a right running wave meeting a region where this is unifor $\underset{= u_{wave}}{u+a}$

Need to relate ρ and dp then integrate $\Rightarrow$ assume CPIG?

Yes, assume CPIG

$a^2 = \frac{\partial p}{\partial \rho}\Big|_s$ $\qquad p/\rho^\gamma = C_1$ for isentropic (s=const)

$$a^2 = \left(\frac{\partial p}{\partial \rho}\Big|_s \right) = \gamma C_1 \rho^{\gamma-1}$$

$$a = \sqrt{\gamma C_1}\, \rho^{(\gamma-1)/2}$$

$$da = \sqrt{\gamma C_1}\, \frac{2}{\gamma-1}\, \rho^{\frac{(\gamma-1)}{2}-1}\, d\rho$$

$$\Rightarrow da = \frac{\gamma-1}{2} \frac{a}{\rho} d\rho$$

$$\frac{dp}{\rho a} = \frac{a^2 d\rho}{\rho a} \qquad d\rho = \frac{2}{\gamma-1} \frac{\rho}{a} da$$

$$= \frac{a^2}{\rho a} \cdot \frac{\rho}{a} \cdot \frac{2}{\gamma-1} da$$

$$\Rightarrow du - \frac{2}{\gamma-1} da = 0$$

integrate $\Rightarrow$

$$(u - u_\infty) - \frac{2}{\gamma-1}(a - a_\infty) = 0$$

$$u - \frac{2a}{\gamma-1} = u_\infty - \frac{2a_\infty}{\gamma-1}$$

$$u = u_\infty + \frac{2}{\gamma-1}(a - a_\infty)$$

$u_{wave} = u + a$ Because C_+ characteristics come from uniform region

$$u_{wave} = u_\infty + a\left(\frac{2}{\gamma-1} - 1\right) - \frac{2}{\gamma-1} a_\infty$$

$$u_{wave} = u_\infty + \frac{\gamma+1}{\gamma-1} a - \frac{2}{\gamma-1} a_\infty$$

$$\frac{u_{wave}}{a_\infty} = \left(\frac{\gamma+1}{\gamma-1} \frac{a}{a_\infty} - \frac{2}{\gamma-1}\right) + \frac{u_\infty}{a_\infty}$$

$$p = C a^{2\gamma/\gamma-1}$$

$$\frac{p}{p_\infty} = \left(\frac{a}{a_\infty}\right)^{2\gamma/\gamma-1}$$

$$\Rightarrow \quad \frac{u_{wave}}{a_\infty} = \left[\left(p/p_\infty\right)^{\frac{\gamma-1}{2\gamma}} \cdot \frac{\gamma+1}{\gamma-1} - \frac{2}{\gamma-1}\right] + \frac{u_\infty}{a_\infty}$$

All "particle velocity in same direction as wave propagation" means is right-running wave

$$u_{wave} = u_\infty + \frac{\gamma+1}{\gamma-1} a - \frac{2}{\gamma-1} a_\infty$$

$$\frac{du_{wave}}{dp} = \frac{\gamma+1}{\gamma-1} \frac{da}{dp} \qquad \frac{dp}{pa} = \frac{2}{\gamma-1} da \Rightarrow \frac{da}{dp} = \frac{a}{\gamma p} > 0$$

$$\frac{du_{wave}}{dp} = C \frac{da}{dp}$$

and da/dp is > 0 meaning u_{wave} increases with increasing pressure. In x-t wave diagram, dt/dx gets less steep because $dt/dx = \frac{1}{u+a}$

3) $p_1 = 58.8 \cdot 10^3$ Pa $T_1 = 450K$ $T_4 = 450K$

a. Before the shock passes flow at pitot is quiescent, stationary

$p_{0,1} = 58.8 \cdot 10^3 \text{ Pa} = p_{static,1}$

b. $p_2/p_1 = 3.508$

$p_2 = p_{static} = 206.2704$

$\gamma = 1.4$ $R = 287$

$a_1 = (\gamma R T_1)^{1/2} = 425.22$ m/s

$p_2/p_1 = 1 + \frac{2\gamma}{\gamma+1}(M_1^2 - 1)$

$M_1 = 1.775$

$W_s = 1.775 \cdot a_1 = 754.66$ m/s

$\rho_2/\rho_1 = \dfrac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2} = 2.32$

$T_2/T_1 = \left(\frac{p_2}{p_1}\right) / \left(\rho_2/\rho_1\right) = 1.512$

$T_2 = 680.3K$

$a_2 = \sqrt{\gamma R T_2} = 522.8$ m/s

$u_2 = W_s(1 - \rho_1/\rho_2) = 429.2$ m/s

$M_2 = 429.2/522.8 = 0.821$

$p_{0,2} = p_2\left(1 + \frac{\gamma-1}{2}M_2^2\right)^{\gamma/\gamma-1} = 321 \cdot 10^3$ Pa

c. Contact surface has properties 3 and pressure, velocity equal to 2

$$p_4/p_2 = p_4/p_3 = \left(1 - \frac{\gamma-1}{2}\frac{u_2}{a_4}\right)^{-\frac{2\gamma}{\gamma-1}}$$

solves to $p_4/p_3 = 4.85$

$$T_3 = T_4 \left(p_3/p_4\right)^{\frac{\gamma-1}{\gamma}}$$

$$a_3 = \sqrt{\gamma R T_3} \qquad M_3 = u_3/a_3$$

$$\frac{p_{0,\,pitot}}{p_3} = \left(\frac{\gamma+1}{2}M_3^2\right)^{\gamma/\gamma-1}\left(\frac{\gamma+1}{2\gamma M_3^2-(\gamma-1)}\right)^{1/\gamma-1}$$

normal shock then isentropic to stagnation

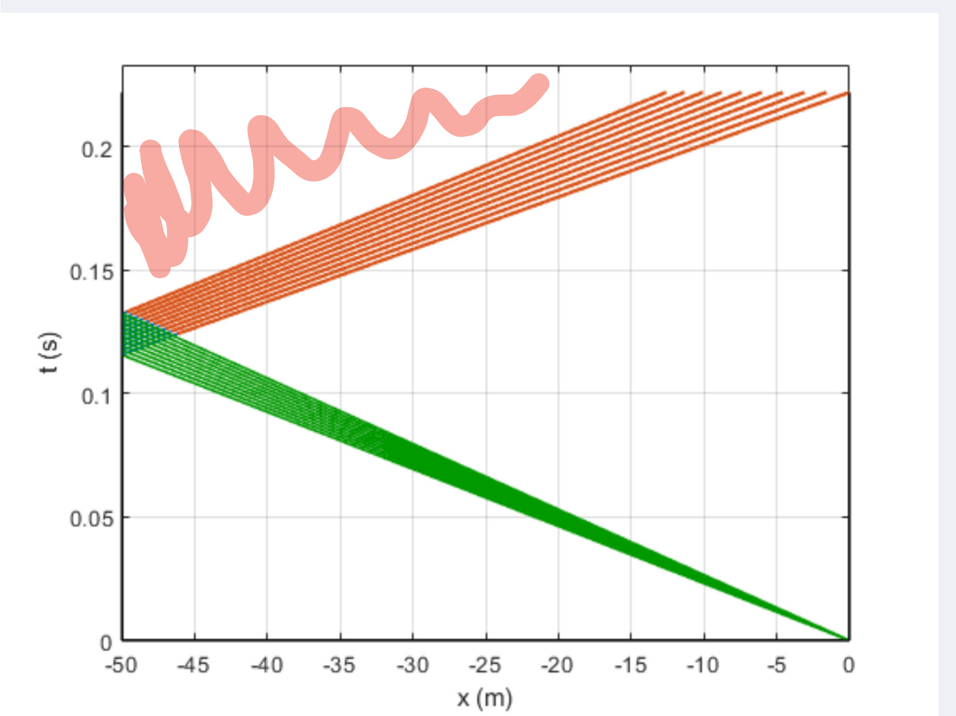
$\underline{p_3 = 206.27 \cdot 10^3\ Pa}$ $\quad$ $\underline{p_{0,\,pitot} = 536.8 \cdot 10^3\ Pa}$

d. $t_{shock} = 3/754.65 = 3.98 \cdot 10^{-3}\ s$

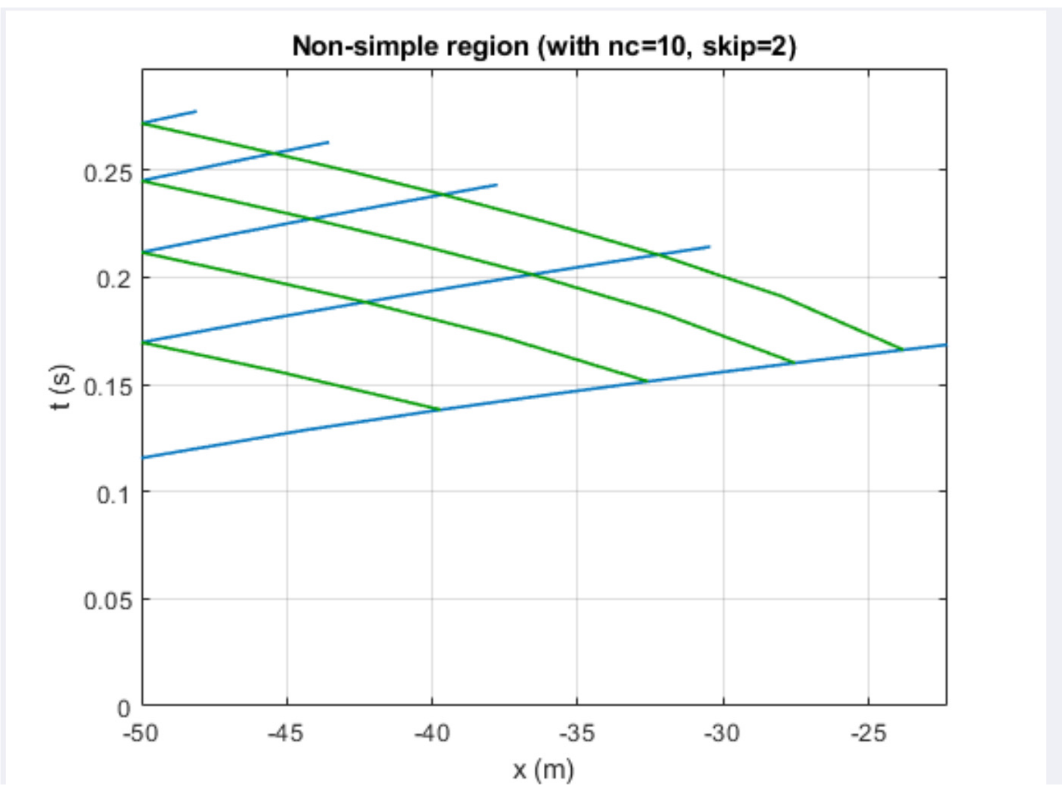
e. $\Delta t = 3\left(1/u_3 - 1/W_s\right) = 3.01 \cdot 10^{-3}\ s$

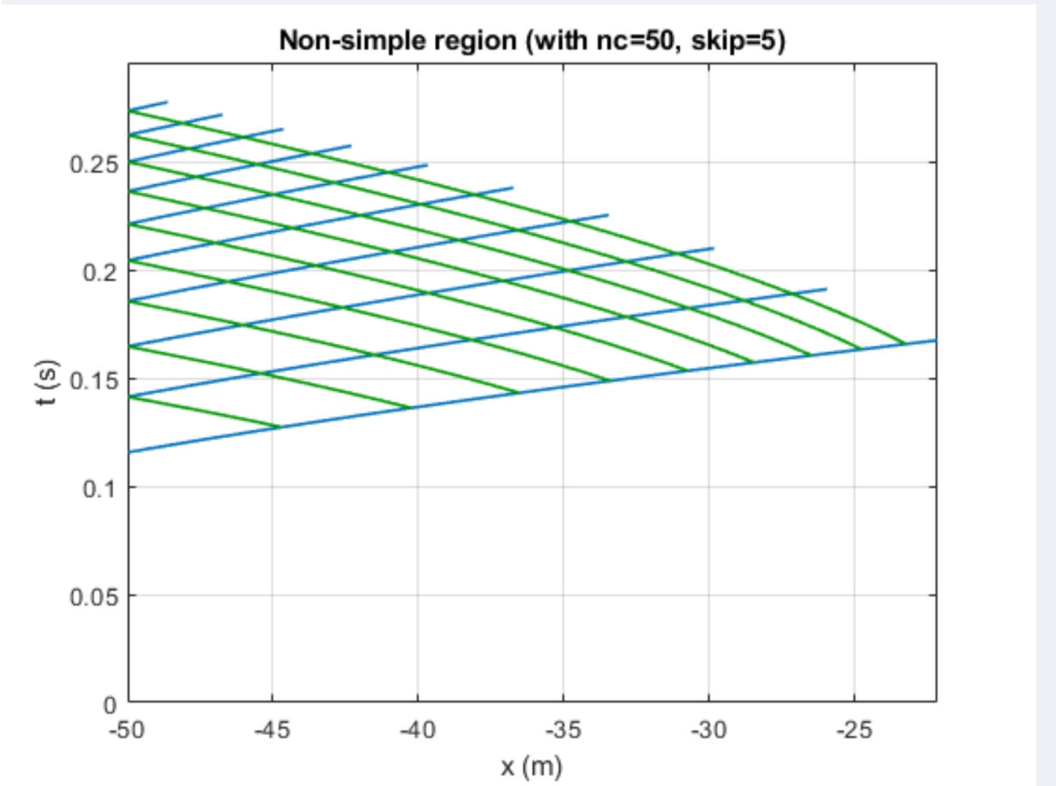
1)

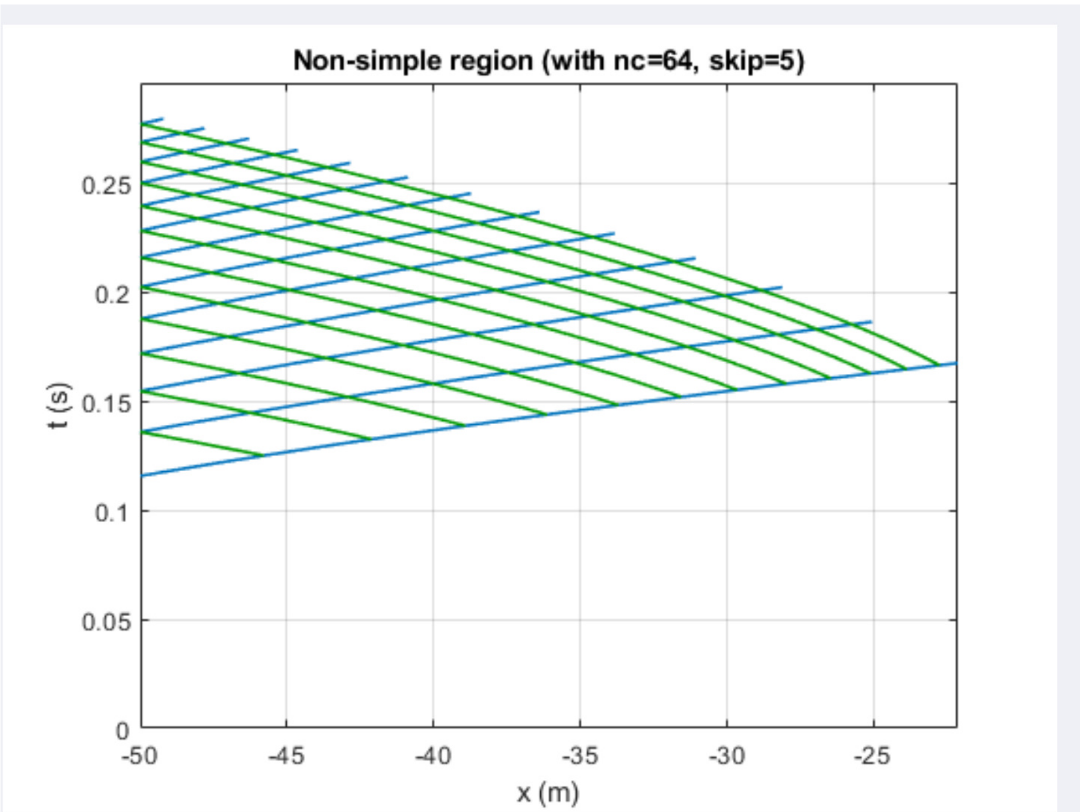
Part 1

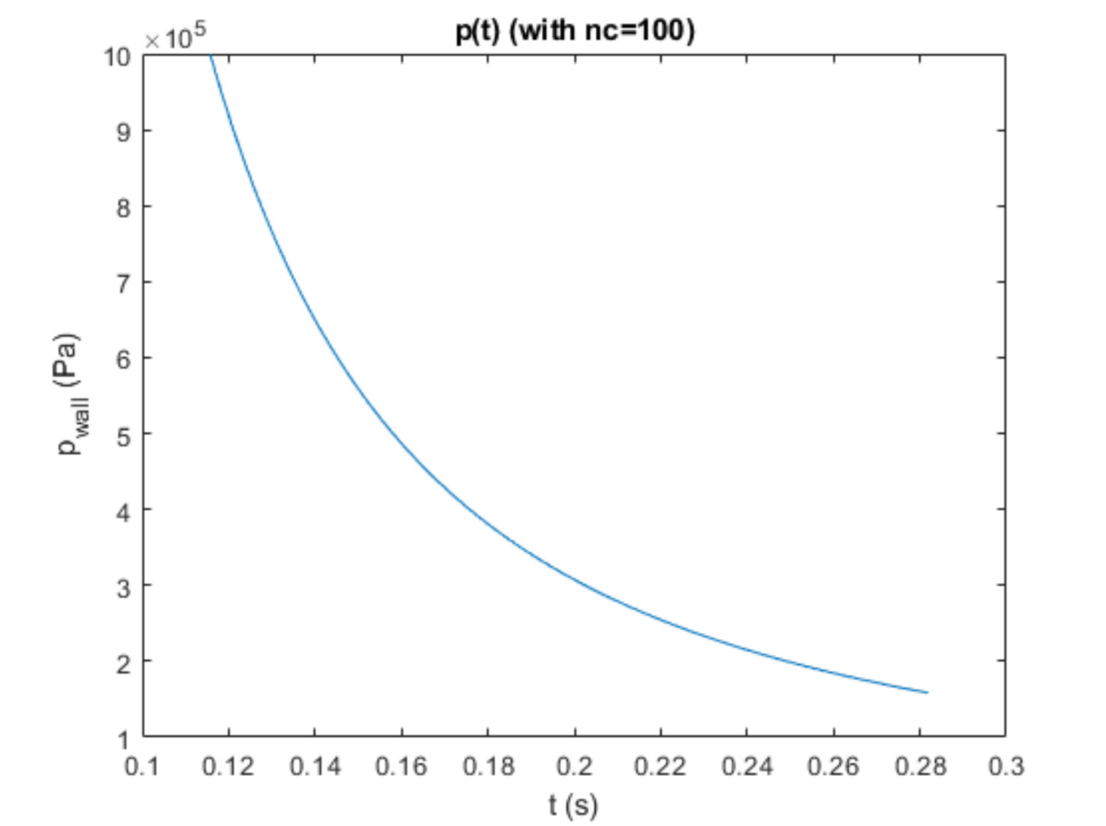


Red highlighted region has constant properties and it makes up 72% of the tube at this time




Non-simple region (with nc=50, skip=5)
t (s)
x (m)
0.25
0.2
0.15
0.1
0.05
0
-50
-45
-40
-35
-30
-25


Non-simple region (with nc=64, skip=5)
t (s)
x (m)
0.25
0.2
0.15
0.1
0.05
0
-50
-45
-40
-35
-30
-25



Code says N=64 issufficient for convergence using tail-only criterion (comparing tail-shape)