(4) $0q_30 \rightarrow q_3$

$0q_31 \rightarrow q_3$

$1q_30 \rightarrow q_3$

$1q_31 \rightarrow q_3$

$0q_3 \rightarrow q_3$

$1q_3 \rightarrow q_3$

$q_30 \rightarrow q_3$

$q_31 \rightarrow q_3$

(5) $q_3\#\# \quad \#$

$q_101 \vdash 1q_21$

$\vdash 10q_1B$

$\vdash 1q_201$

$\vdash q_3101$

$\#q_101\#1q_21\#10q_1\#1q_201\#q_3101\#$

$q_301\#q_31\#q_3\#\#.$

and δ is defined by $\delta(q_1, 0) = (q_2, 1, R)$ and

④

$\delta(q_1, 1) = (q_2, 0, L)$ $w = 01$

$\delta(q_1, B) = (q_2, 1, L)$

$\delta(q_2, 0) = (q_3, 0, L)$

$\delta(q_2, 1) = (q_1, 0, R)$

$\delta(q_2, B) = (q_2, 0, R)$

		List A	List B
①		#	$\#q_1 01\#$
②		0	0
		1	1
		#	#
③	i)	$q_1 0$	$1 q_2$
	ii)	$0 q_1 1$	$q_2 00$
		$1 q_1 1$	$q_2 10$
	iii)	$0 q_1 \#$	$q_2 01\#$
		$1 q_1 \#$	$q_2 11\#$
	iv)	$0 q_2 0$	$q_3 00$
		$1 q_2 0$	$q_3 10$
	v)	$q_2 1$	$0 q_1$
	vi)	$q_2 \#$	$0 q_2 \#$

⑤

Eg. $\delta(q_0, B) = (P, 1, L)$

$0q_0B = 0q_0\# = P01\#$

$1q_0\# = P11\#$

④ for each q in F and X and Y in Γ,

LIST A	LIST B
XqY	q
Xq	q
qY	q

Eg.

$0q_30$	q_3
$0q_31$	q_3
$1q_30$	q_3
$1q_31$	q_3
$0q_3$	q_3
$1q_3$	q_3
q_30	q_3
q_31	q_3

⑤ $q\#\#$	$\#$ for each q in F

Eg: $\#q_010\#$.

$(x, y) = \#qw\#\alpha_1q_1\beta_1\# \ldots \#\alpha_{k-1}q_{k-1}\beta_{k-1}\#,$

$x(\#q_0w\#, \alpha_1q_1\beta_1\# \ldots)\#\alpha_kq_k\beta_k\#$

Final state

Problem:

Let $M = (\{q_1, q_2, q_3\}, \{0, 1\}, \{0, 1, B\}, \delta, q_1, B, \{q_3\})$

O/P

List C		List D
i	y_i	z_i
0	¢1¢	¢1¢1¢1
1	1¢	¢1¢1¢1
2	1¢0¢1¢	¢1¢0
3	1¢1¢	¢0
	1¢0¢	
4	$	¢$

THEOREM: PCP is undecidable

PROOF: MPCP

$\left\}\begin{array}{l} w = x \text{ (or)} \\ w = 10 \end{array}\right.$

The first pair is

	List A	List B	
①	#	$\#q_0w\#$	
②	X	X	for each X in Γ
	#	#	
③	qX	YP	if $\delta(q,X) = (P,Y,R)$
	zqX	PzY	if $\delta(q,X) = (P,Y,L)$
④	q#	YP#	if $\delta(q,B) = (P,Y,R)$; $\left.\begin{array}{l}0\\1\end{array}\right\} = z$
	zq#	PzY#	if $\delta(q,B) = (P,Y,L)$

Eg:

$0q_10 \Rightarrow q_201$

$1q_11 \Rightarrow q_211$

$\boxed{\delta(q_1,0) = (q_2,1,L)}$

(For each q in Q − F), P in Q, and X, Y, z in Γ.

Scanned by CamScanner

MODIFIED VERSION OF PCP (MPCP):

The MPCP of the following given list A and list B of k string each from input alphabet Σ^*.

$A = w_1, w_2, \dots, w_k$ $B = x_1, x_2 \dots x_k$

Thus there exists a sequence of integers $i_1, i_2, i_3 \dots i_r$ such that w_1

$$w_1 w_{i_1} w_{i_2} w_{i_3} \dots w_{i_r} = x_1 x_{i_1} x_{i_2} x_{i_3} \dots x_{i_r}$$

LEMMA:

If PCP were decidable then MPCP would be decidable i.e) MPCP reduces to PCP.

PROOF:

Let $A = w_1, w_{i_1}, w_{i_2} \dots w_{i_r}$ and $B = x_1 x_{i_1} x_{i_2} \dots x_{i_r}$

and $y_0 = ¢ y_1$, $z_0 = z_1$, $y_{k+1} = \$$, $z_{k+1} = ¢\$$.

Let $c = y_0 y_1 \dots y_{k+1}$ and $d = z_0 z_1 \dots z_{k+1}$

	List A	List B
i	w_i	x_i
1	1	111
2	10111	10
3	10	0

$i_1, \ldots, i_m$ is a solution to this instance of PCP.

Problem:

① Let input alphabet $\{0, 1\}$. Let A and B be list of three strings each. In this case, PCP has solution.

Let $M = 4$, $i_1 = 2$, $i_2 = 1$, $i_3 = 1$ and $i_4 = 3$.

	A	B
i	w_i	x_i
1	1	111
2	1011	10
3	10	0

An instance of PCP

Solution:

$$w_{i_1} w_{i_2} w_{i_3} w_{i_4} = x_{i_1} x_{i_2} x_{i_3} x_{i_4}$$

$$101111110 = 101111110$$

② Let input alphabet be $\{0, 1\}$.

	List A	List B
i	w_i	x_i
1	10	101
2	011	11
3	101	011

Let $m = 4$

$i_1 = 2$; $i_2 = 1$, $i_3 = 1$, $i_4 = 3$

Another PCP instance

$$01110101 01 = 01110110 1011$$

$q = 010100100100$

$111\ 010100100100\ 11\ C_2\ 11\ C_3\ 11 \ldots 11 C_n\ 111 w (0011)$

q_1 - code 1

$M = (\{q_1, q_2, q_3\}, \{0,1\}, \{0,1,B\}, \delta, q_1, B, \{q_2\})$

Rule

$\delta(q_1, 1) = (q_3, 0, R)$

$\delta(q_3, 0) = (q_1, 1, R)$

$\delta(q_3, 1) = (q_2, 0, R)$

$\delta(q_3, B) = (q_3, 1, L)$

$w = \{1101\}$

111 Code1 11 Code2 11 code3 11 code4 111 1101

$\underbrace{111\ 0100\ 1000100100}_{C_1}\ 11\ \underbrace{00010101000100}_{C_2}\ 11$

$\underbrace{000100100101100}_{C_3}\ 11\ \underbrace{0001000100010010}_{C_4}\ 111\ 1101$

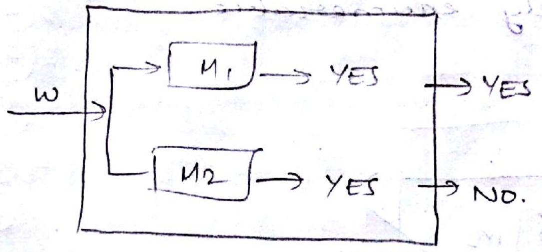
UNDECIDABILITY OF POST CORRESPONDENCE PROBLEM (PCP):

An instance of post correspondence problem consists of two lists

$A = w_1, w_2, \ldots, w_k$

$B = x_1, x_2, \ldots, x_k$ over some alphabet Σ.

This instance of PCP has a solution. If there is any sequence of integer $i_1, i_2, \ldots, i_m$ with $m \geq 1$ such that $w_{i_1}, w_{i_2}, \ldots, w_{i_m} = x_{i_1}, x_{i_2}, \ldots x_{i_m}$. The sequence recursive, the other is not recursively enumerable or recursive



UNIVERSAL TURING MACHINE AND UNDECIDABLE PROBLEMS

Rule

$Q \times \Gamma \rightarrow Q \times \Sigma \times \Gamma \times \qquad Q \times \Sigma \rightarrow Q \times \Gamma \times \{L, R\}$

$\delta(q_0, 0) \rightarrow (q_1, 1, R)$

TURING MACHINE CODING:

$111\, C_1\, 11\, C_2\, 11\, C_3 \ldots 11\, C_n \qquad 111\, w\, (0011)$

$C_1 = \delta(q_0, 0) = (q_1, 1, R)$

$C_1 = q_0\, 1\, 0\, 1\, q_1\, 1\, 1\, 1\, R$

$L = \{0^n 1^n \mid n \geq 1\}$

if $n = 2$

$w = \{0011\}$

$w \in L(RE)$

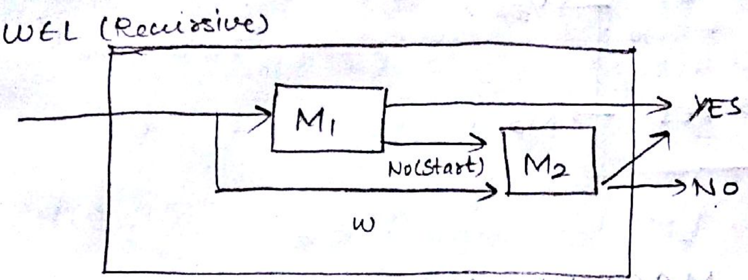
$w = \{0011\}$

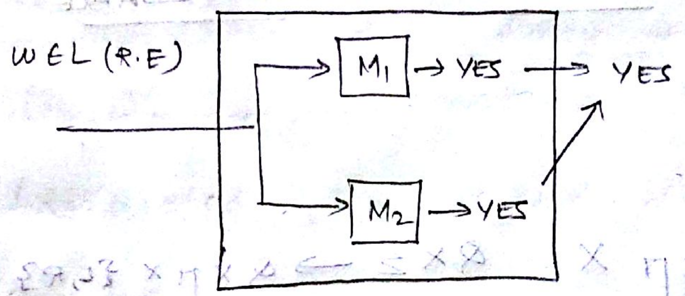
Assumption:

$q_0 = 0$	$0 = 0$	$L = 0$
$q_1 = 00$	$1 = 00$	$R = 00$
$q_2 = 000$	$B = 000$	
$q_3 = 0000$		

② The union of two recursive language is recursive.

③ The union of two recursively enumerable language is recursively enumerable.





If a language L and its complements L' are both recursively enumerable, then both L and L' are recursive

Assume that the system is accepted and hence L' is recursive.

Construct M. Simulate M_1 and M_2 simultaneously. M accepts W if M_1 accepts w and M rejects W if M_2 accepts w. M is recursive. Let L and L' be a pair of complementary languages, then

i) both L and L' are recursive.

ii) neither L nor L' are recursively enumerable.

iii) one of L or L' is recursively enumerable or

RECURSIVE ENUMERABLE LANGUAGE:

A language is recursive enumerable language if some turing machine accepts it.

L(RE)

① String w → TM → YES (Reaches the final state)

if w ∈ L(RE)

② if w ∉ L(RE) → T.M → No. (TM may not reach the final state).
(or)
loop forever.

RECURSIVE LANGUAGE:

① If w∈L (Recursive)

String w → TM → YES (TM reach the final state)

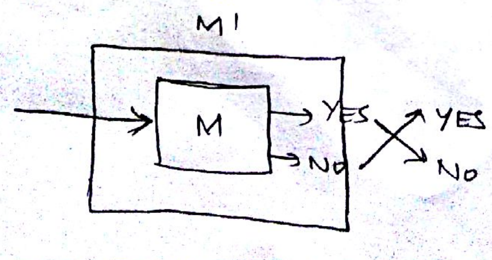
② if w∉L (Recursive)

→ TM → No (TM may not reach the final state).

THEOREM:

① The complement of recursive language is recursive.

w∈L (Recursive)



8/10/2015

MULTITRACK TURING MACHINE:

47 = 101111

2 | 47
2 | 23 – 1
2 | 11 – 1
2 | 5 – 1
2 | 2 – 1
1 – 0

¢	1	0	1	1	1	1	B	B	
B	B	B	B	1	0	1	B	B	......
B	1	0	1	0	1	0	B	B	

$2\overline{)47}$ = 23, 46 ; $3\overline{)47}$ = 15, 45, 2 ; $5\overline{)47}$ = 9, 45, 2

Recursive enumerable language

The turing machine reaches the final state, it is called determinable / decidable.

UNDECIDABILITY:

A problem whose language is recursive is said to be decidable. Otherwise it is undecidable. The problem is undecidable if there is no algorithm that takes as input an instance of the problem and determines whether the answer to that instance is yes or no.

No algorithm - infinite loop.

FERMAT'S CONJECTURE:

$\boxed{x^i + y^i = z^i}$ if $i \geq 3$.

M moves left until it encounters a blank (B) and the repeats the cycle. The repetition ends if the first searching right for a '0' M encounters a blank, then the n zeroes in $0^m 1 0^n$ have all been changed to '1' and n+1 of the n zeroes have been changed into blank. M replaces the n+1 1's by a zero and n blank (B's), leaving m-n zeroes on its tape.

2) Beginning a cycle m cannot find a zero to change to a blank, because first m has already been changed, then $n \geq m$, so $m-n=0$. m replaces all remaining 1's and 0's by blank (B).

MODIFIED VE

for m<n.

3,8,90,

halts

places

write for

0' to '1'.

1/10/2015

DFA : $Q \times \Sigma \to Q$

NFA-ϵ : $Q \times \Sigma \cup \{\epsilon\} \to 2^Q$

PDA : $Q \times \Sigma \times \Gamma \to Q \times \Gamma$

TM : $Q \times \Sigma \to Q \times \Gamma \times \{L, R\}$

PROPER SUBTRACTION : ($\dot{-}$)

$a-b$ (or) $m \dot{-} n$

① $m-n$ for $m \geq n$

② $m-n = 0$ for $m<n$

$m \dot{-} n$ is defined to be $m-n$ for $m \geq n$ & 0 for $m<n$.

eg. $0^m 1 0^n$ If $m=2, n=1$

$= 0^2 1 0^1$

$= 0010$

Input :

0	0	1	0	B	B

(or)

Output

B	0	B	B	B.

TM : $M = (\{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}, \{0, 1\}, \{0, 1, B\}, \delta, q_0, B, \{\})$

PROCEDURE:

If started with $0^m 1 0^n$ on its tape, halts with 0^{m-n} on its tape m repeatedly replaces its leading zero by blank (B). then such a write for a 1 followed by a '0' and changes the '0' to '1'.

Rule

$\delta(q, \alpha_2) = (P, \beta, L)$

Design a turing machine that accepts the language $L = \{0^n 1^n \mid n \geq 1\}$

Design:

TM $M = (\{q_0, q_1, q_2, q_3, q_4\}, \{0, 1\}, \{0, 1, X, Y, B\}, \delta, q_0, B, \{q_4\})$

States	Symbols				
	0	1	X	Y	B
q_0	(q_1, X, R)	–	–	(q_3, Y, R)	–
q_1	$(q_1, 0, R)$	(q_2, Y, L)	–	(q_1, Y, R)	–
q_2	$(q_2, 0, L)$	–	(q_0, X, R)	(q_2, Y, L)	–
q_3	–	–	–	(q_3, Y, R)	(q_4, B, R)
q_4	–	–	–	–	–

$q_0, 0011 \vdash X q_1 011 \vdash X 0 q_1 11 \vdash \cancel{X q_0 X 1}_{q_2}\ X q_2 0 Y 1$

Rule:

$\delta(q_0, 0) = (q_1, X, R)$

$\delta(q_1, 0) = (q_1, 0, R)$

$\delta(q_1, 1) = (q_2, Y, L)$

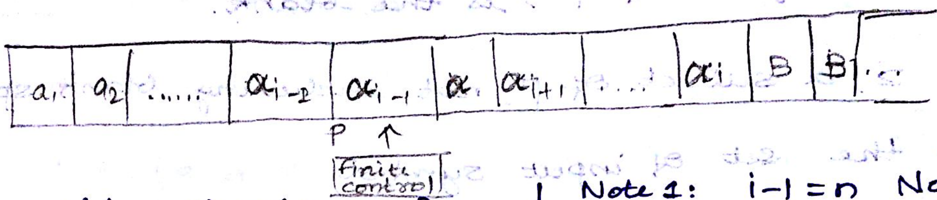
In one move, the turing machine depending upon the symbol scanned by the tape head and the state of finite control.

i) It may change the state

ii) Prints a symbol on the tape cell scanned.

iii) Replacing what was written there and move its head left or right one cell.

$$\delta(q, a_i) = (P, X, L)$$

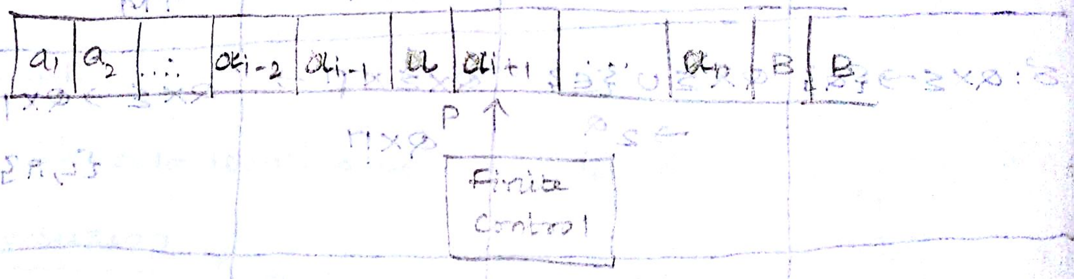


$$\delta(q, a_i) = (P, X, L)$$

Note 1: $i-1 = n$ No R Move

Note 2: $i = 1$ No L Move

$$\delta(q, a_i) = (P, X, R)$$



$$\delta(q, a_i) = (P, X, R)$$

Instantaneous Description ID:

① $\alpha_1 q \alpha_2 \underset{M}{\overset{R}{\vdash}} \alpha_1 \beta P B$

Rule:

$$f(q_1, \alpha_2) = (P, \beta, R)$$

② $\alpha_1 q \alpha_2 \underset{M}{\overset{L}{\vdash}} P \alpha_1 \beta$

16/9/2015

TURING MACHINE (TM)

* In 1936, Alan Turing - Turing machine
* Turing machine (TM)

$$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$$

where

Q is the finite set of states.

Γ is the finite set of tape symbol

∴ B, a symbol of Γ, is the blank.

∴ Σ, a subset of Γ, not including blank space B, is the set of input symbol.

$\delta : Q \times \Sigma \rightarrow Q \times \Gamma \times \{L, R\}$

DFA	NFA	PDA	TM
$\delta : Q \times \Sigma \rightarrow \{Q\}$	$Q \times \Sigma \cup \{\epsilon\} \rightarrow 2^Q$	$Q \times \Sigma \times \Gamma \rightarrow Q \times \Gamma$	$Q \times \Sigma \rightarrow Q \times \Gamma \times \{L, R\}$

q_0 is the start state

$F \subseteq Q$, is the final state.