Optimal progressivity of fiscal stimulus: a HANK approach

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Outline

Motivation

- Literature regarding fiscal policy redistribution, such as [Bakış, Kaymak & Poschke, 2015] and [Heathcote, Storesletten & Violante, 2017] often poses a trade-off between allocative welfare increases from progressive fiscal policy, and human capital investment. However, although that is a relevant trade-off in the long run, that might not be the case for short term fluctuactions.
- Literature about the short term effects of redistributive fiscal schemes has concluded that, heterogeneous agents (HAs) models give realistic insights about fiscal multipliers, because of higher marginal propensities to consume (MPCs) in the lower part of the distribution. However, this analysis has not been made for the government expenditure, or has not been able to design a framework from which a transition rule of progressivity arises.
- I propose an extension to the traditional Aiyagari-Huggett-Bewley model used in [Achdou, Lasry, Lions, & Moll, 2017], to explore an optimal progressivity dynamic in a negative productivity shock.

Outline

General setting: Households

- Households solve an optimal saving policy in an Aiyagari-Hugget-Bewley setting, as presented in [Achdou et al., 2017], with two fundamental differences: government transfers income, and employed-unemployed state variable.

$$v(a_0, z_0, E_0) = \max_{\{c_t\}_{t\geq 0}} \mathbb{E}_0 \left[\int_0^\infty \mathrm{e}^{-\rho t} u\left(c_t\right) \mathrm{d}t \right] \tag{1}$$

$$\dot{a}_t = \mathbf{1}_{\{E_t=1\}} w_t \mathrm{e}^{z_t} + r_t a_t + T_t(z_t, \zeta_t) - c_t;\ \mathrm{d}z_t = -\eta\left(z_t - \bar{z}\right) + \sigma \mathrm{d}W_t \tag{2}$$

$$\Pr(E_{t+\mathrm{d}t} = 1 | E_t = 0) = \lambda_t^0 \mathrm{d}t;\ \Pr(E_{t+\mathrm{d}t} = 0 | E_t = 1) = \lambda_t^1 \mathrm{d}t \tag{3}$$

General setting: Firms

- A representative firm produces $Y_t = A_t F(K_t, L_t) = A_t K_t^{\alpha} L_t^{1-\alpha}$; $K = \int_0^{\infty} \int_0^{\infty} a g(a, z) \mathrm{d}a \mathrm{d}z$
- The government taxes profits at a rate τ, so the maximization problem is:

$$\max_{\{K_t, L_t\}} \left[(1-\tau) A K_t^{\alpha} L_t^{1-\alpha} - r_t K_t - w_t L_t \right] \tag{4}$$

- In [Achdou et al., 2017], $L_t = 1 \; \forall \; t$. Here the labor market adjusts through the employment level, while wages are rigid ($w_t = \bar{w}$). Firms choose a level of employment L_t that is given by $L_t = \frac{\lambda_t^0}{\lambda_t^0 + \lambda_t^1}$. For simplicity, $(\lambda_t^0, \lambda_t^1)$ is assumed to be $(L_t, 1 - L_t)$, so the firm only chooses $L_t \in [0, 1]$.

General setting: Firms

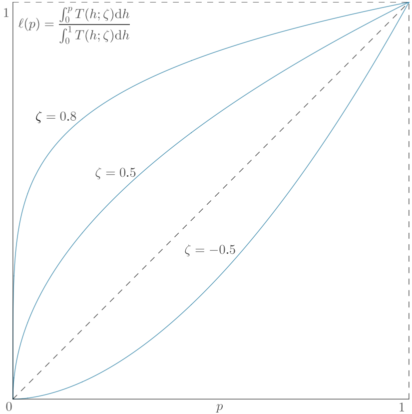
- In equilibrium, the solution of the maximization problem implies $r_t = (1-\tau)\alpha A_t K_t^{\alpha-1} L_t^{1-\alpha}$, and $L_t = \left[\frac{(1-\tau)(1-\alpha)A_t K_t^{\alpha}}{\bar{w}}\right]^{\frac{1}{\alpha}}$. Assuming that wage rigidities do not hold on the long run, $\bar{w}$ is determined in such a way that $L_\infty = 1$. $\bar{w}$ now becomes $\bar{w} = (1-\tau)(1-\alpha)A_\infty K_\infty^{\alpha}$, and therefore:

$$L_t = \left(\frac{A_t K_t^{\alpha}}{A_\infty K_\infty^{\alpha}}\right)^{\frac{1}{\alpha}};\ r_t = (1-\tau)\alpha\left(\frac{A_t}{A_\infty^{1-\alpha}}\right)^{\frac{1}{\alpha}} K_\infty^{\alpha-1};\ r_\infty = (1-\tau)\alpha A_\infty K_\infty^{\alpha-1} \tag{5}$$

General setting: Government

- The transfer policy is a function $T(z)$ of labor productivity. From a distributional point of view, it is a function of the percentile p of z. *Progressivity* is analysed with a Lorenz Curve $\ell(p)$.The government chooses a transfer function $T(p)$ such that $\ell(p) = p^{1-\zeta}$, where $\zeta \in (-\infty, 1]$.

Figure: Lorenz Concentration Curves for $T(p;\zeta)$



General setting: Government

- In each instant of time, $z \sim \mathcal{N}(\bar{z}, \frac{\sigma^2}{2\eta})$, which implies that percentiles can be obtained with the analytical expression $p(z) = \Phi\left[(z - \bar{z})\sqrt{\frac{2\eta}{\sigma^2}}\right]$. Now T can be defined directly in the z domain, rather than p. The cumulative transfer over the z density function, then is:

$$T_{\mathrm{ac}}(z) = \int_{-\infty}^{\infty} T(z,\zeta)\phi\left[(z - \bar{z})\sqrt{\frac{2\eta}{\sigma^2}}\right]\mathrm{d}z = Q\left(\Phi\left[(z - \bar{z})\sqrt{\frac{2\eta}{\sigma^2}}\right]\right)^{1-\zeta} \tag{6}$$

While ζ gives a shape to the LCC, Q gives the necessary *scale* to satisfy the government restriction:

$$\int_0^{\infty}\int_0^{\infty} T\left(z,\zeta\right) g(a,z)\mathrm{d}a\mathrm{d}z = \tau A\left[\int_0^{\infty}\int_0^{\infty} a g(a,z)\mathrm{d}a\mathrm{d}z\right]^{\alpha} \tag{7}$$

General setting: Government

- Finally, considering that $T(z,\zeta) = \frac{\partial T(z,\zeta)}{\partial z}$, we have that:

$$T(z,\zeta) = Q(1-\zeta)\left(\Phi\left[(z-\bar{z})\sqrt{\frac{2\eta}{\sigma^2}}\right]\right)^{-\zeta}\sqrt{\frac{2\eta}{\sigma^2}}\phi\left[(z-\bar{z})\sqrt{\frac{2\eta}{\sigma^2}}\right] \tag{8}$$

- Now, the optimization problem of the government is:

$$v^G(A_0,K_0) = \max_{\{\zeta_t\}_{t\geq 0}} \int_0^\infty \int_{\bar{a}}^\infty \int_{-\infty}^\infty \mathrm{e}^{-\rho t} u[c_t(a,z)] g(a,z,t) \mathrm{d}a \mathrm{d}z \mathrm{d}t \tag{9}$$

General setting: Equilibrium

- Finding the equilibrium, requires solving the households problem, and attaching it to the evolution of the distribution. For this, a Partial Differential Equations (PDEs) system known as *Backward-Forward Mean Field Games* has to be solved.
- In general, given a state $\mathbf{x}$, an instantaneous return function $\pi(\mathbf{x}_t)$, a distribution $f(\mathbf{x}, t)$, and a processes $\mathrm{d}\mathbf{x}_t = \vec{\mu}(\mathbf{x}_t, t)\mathrm{d}t + \Sigma(\mathbf{x}_t)\mathrm{d}\mathbf{w}_t$, the sistem of equations can be written as:

$$\text{HJB} : \rho v(\mathbf{x}_t, t) = \pi(\mathbf{x}_t) + \sum_{i=1}^{n} \mu_i(\mathbf{x}_t) \frac{\partial v(\mathbf{x}_t, t)}{\partial x_{i,t}} + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{i,j}^2(\mathbf{x}_t) \frac{\partial^2 v(\mathbf{x}_t, t)}{\partial x_i \partial x_j} + \dot{v}(\mathbf{x}_t, t)$$

$$\text{KFP} : \frac{\partial f(\mathbf{x}_t, t)}{\partial t} = -\sum_{i=1}^{n} \frac{\partial[\mu_i(\mathbf{x}_t) f(\mathbf{x}_t, t)]}{\partial x_i} + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial^2[\sigma_{i,j}^2(\mathbf{x}_t) f(\mathbf{x}_t, t)]}{\partial x_i \partial x_j}$$

General setting: Equilibrium

Hamilton-Jacobi-Bellman Equation (HJB):

$$
\begin{aligned}
\mathrm{HJB}: v(a_t, z_t, t, E_t = j) = \max_{c_t}\{u(c_t) & \\
& + \left[\mathbf{1}_{\{j=1\}}\bar{w}\mathrm{e}^{z_t} + r_t a_t + T_t(z_t, \zeta_t) - c_t\right] \partial_a v(a_t, z_t, t, E_t = j) \\
& - \eta(z_t - \bar{z})\partial_{z_t} v(a, z_t, t, E_t = j) \\
& + \frac{1}{2}\sigma^2 \partial_{z_t, z_t} v(a_t, z_t, t, E_t = j) \\
& + \mathbb{E}_t[\dot{v}(a_t, z_t, t, E_t = j)] \\
& + \lambda_t^j \left[v(a_t, z_t, t, E_t = i) - v(a_t, z_t, t, E_t = j)\right]\}
\end{aligned}
\tag{10}
$$

General setting: Equilibrium

Kolmogorov-Fokker-Plank Equation (KFP):

$$\begin{aligned}
\mathrm{KFP} : \dot{g}(a_t, z_t, t, E_t = j) = \\
& - \partial_{a_t} \left(\left[\mathbf{1}_{\{j=1\}} \bar{w} \mathrm{e}^{z_t} + r_t a_t + T(z_t, \zeta_t) - c_t \right] g(a_t, z_t, t, E_t = j) \right) \\
& + \partial_{z_t} \left[\eta (z_t - \bar{x}) g(a_t, z_t, t, E_t = j) \right] \\
& + \frac{1}{2} \sigma^2 \partial_{z_t, z_t} g(a_t, z_t, E_t = j) \\
& + \lambda_t^j \left[g(a_t, z_t, t, E_t = i) - g(a_t, z_t, t, E_t = j) \right]
\end{aligned} \tag{11}$$

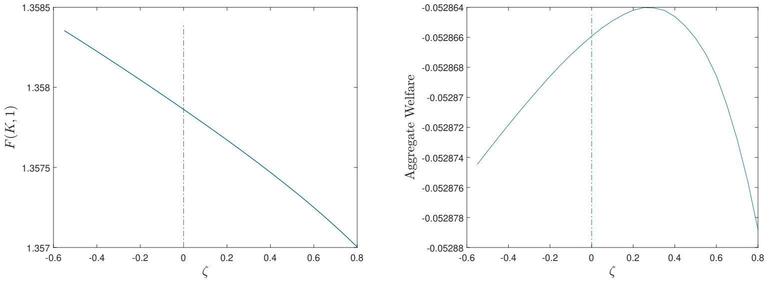
Steady state optimal rule: Solving the system

- Solving the steady state is needed to know the level of $\bar{w}$, and then compute L_t to solve de PDEs for the dynamic case. Also gives a benchmark to then compare the optimal ζ_t with ζ_∞.
- The equilibrium in steady state, is found by imposing $\dot{g}(a_t, z_t, t) = \dot{v}(a_t, z_t, t) = 0$, and therefore suppressing the time dependence of variables. Also, as $L_\infty = 1$, the Poisson part is suppressed too. Simply by imposing the latter, the system to be solved is much *easier*.

Steady state optimal rule: Finding ζ_∞

As aggregate capital comes from savings, a larger concentration of government transfers at the lower part of the income distribution, tends to give a lower aggregate capital level at equilibrium, which is in turn associated with lower wages and therefore utility.

Figure: Production level y and aggregate welfare for each ζ



MIT shock and dynamic optimal rule: Forthcoming homework

- Simulate a transition after a shock of aggregate productivity ($A_0 < A_\infty$) following:

$$\mathrm{d}A_t = -\theta(A_t - A_\infty)\mathrm{d}t$$

- Compute L_t and r_t $\forall$ $t \geq 0$.
- Find the function ζ_t over time that maximizes government objective.
  - The *easy* way: a deterministic transition $\mathrm{d}\zeta_t = -\psi(\zeta_t - \zeta_\infty)\mathrm{d}t$. Government chooses only ζ_0 and ψ.
  - The (very) hard way: find a non-parametric ζ_t with numerical algorithm. Although it is not theoretically impossible, to my knowledge the fastest algorithm would take hours per iteration.
  - The first of the two above options has the benefit of computational tractability, but also an explicit interpretation in terms of intensity and persistence of the policy.

Outline

Once the model is solved: Interesting application

- Ideally, the model would be calibrated, and an optimal ζ_t could be compared with fiscal policies aimed to counter COVID-19.
- Counterfactual policy analysis (what could have happened if?).
- It is **a lot** of work, but not impossible. I actually did some distributional analysis for fiscal policies in Central America while working at the IDB (see chapters 1 and 2 for detail in this publication).

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