Optimal progressivity of fiscal stimulus: a HANK approach

Carlos Eggers Prieto*

March 15, 2021

PRELIMINARY AND INCOMPLETE DRAFT - COMMENTS ARE GLADLY WELCOME

Check here for the most recent version.

Abstract

Literature regarding fiscal policy redistribution, often poses a trade-off between allocative welfare increases from progressive fiscal policy, and human capital investment. However, although that is a relevant trade-off in the long run, that might not be the case for short term fluctuactions. Fiscal policy does not only have benefits because of redistribution itself, but also because of reactivation matters in recessive episodes. Using an extension of the Aiyagari-Hugget-Bewley HANK model presented in Achdou, Lasry, Lions, & Moll (2017). I show that the countercyclical purpose of fiscal policy can be more or less effective, depending on how progressive the countercyclical policy is. The mechanism for progressivity to matter in terms of reactivation, is that individuals with lower level of income and wealth, tend to have larger Marginal Propensities to Consume (MPCs), and therefore, transfers to the lower part of the distribution have more direct effects on output. Nevertheless, lower savings induce a slower transition to the long run capital levels, that can harm aggregate welfare as well. In this paper, I adapt the ABH model to an environment with endogenous unemployment, in which, the government taxes income and redistributes income choosing an optimal progressivity rule.

Keywords: Fiscal Policy, Redistribution, Reactivation, Heterogeneous Agents.

JEL Codes: E21, E62, H12, H21, H3

*Economics Department, University of Chile: ceggers@fen.uchile.cl

1 Introduction

Literature regarding fiscal policy redistribution, such as Bakış, Kaymak & Poschke (2015) and Heathcote, Storesletten & Violante (2017) often poses a trade-off between allocative welfare increases from progressive fiscal policy, and human capital investment. However, although that is a relevant trade-off in the long run, that might not be the case for short term fluctuactions. Fiscal policy does not only have benefits because of redistribution itself, but also because of reactivation matters in recessive episodes. Using an extension of the Aiyagari-Hugget-Bewley (AHB) Heterogeneous Agents New Keynesian (HANK) model presented in Achdou, Lasry, Lions, & Moll (2017), I show that the countercyclical purpose of fiscal policy can be more or less effective, depending on how progressive the countercyclical policy is. The mechanism for progressivity to matter in terms of reactivation, is that individuals with lower level of income and wealth, tend to have larger Marginal Propensities to Consume (MPCs), and therefore, transfers to the lower part of the distribution have more direct effects on output[1]. Nevertheless, lower savings induce a slower transition to the long run capital levels, that can harm aggregate welfare as well. In this paper, I adapt the setting used in Achdou et al. (2017), to an environment in which, the government taxes income and redistributes income choosing an optimal progressivity rule, and labor markets adjust through the level of employment rather than salaries, meaning that consumption demand can actually induce output growth in recessive scenarios.

The Heathcote (2005) approach to fiscal policy with HAs, has shown that a non-ricardian consumption behaviour, which is consistent with microeconomic evidence. This implies that an expenditure increase, or a taxation decrease, even when the present value of future fiscal balances remains the same, can actually boost consumption if markets are incomplete and there are liquidity constraints. This is the first reason why thinking of fiscal policy in HAs environments, provides richer insights even about its effects on aggregate dynamics. In Werning (2007) a redistributive taxation policy and its aggregate effects on an HAs model is analysed. After the financial crisis, a similar approach was taken in Oh & Reis (2012) to analyse the effectiveness of targeted transfers in the U.S., arguing that larger MPCs in the lower part of the distribution boost output through larger consumption in comparison with non targeted schemes. More recently, Cantore & Freund (2021) has shown in a Two Agent New Keynesian (TANK) setting, that more redistributive schemes between capital and labor has a positive impact on fiscal multipliers due to the MPCs difference bewteen agents. In Le Grand & Ragot (2017), an AHB with HAs is used to get optimal fiscal policy dynamics and redistribution schemes into this dynamics, assuming however that government redistribution comes from choosing labor and capital taxes separately, and not different transfers across the agents distribution. It is concluded that, the tighter the credit constraints are, the larger the optimal capital taxes will be (implying a more progressive fiscal system as a whole). Previously, a similar approach in Bhandari, Evans, Golosov & Sargent (2013), got the conclusion that government debt levels did not affect the optimal taxing progressivity.

Two relevant works that consider the relationship between the business cycle and fiscal policy welfare effects with HAs are Aguirre (2015) and Aguirre (2020). These papers show how different levels of fiscal cyclicality have distributional impacts, in addition to the aggregate impacts that are commonly known. The distributional effects of fiscal cyclicality, arise from the fact that fiscal policy acts as a *social insurance* for the idiosyncratic risks faced by households (which are correlated with the aggregate risk of the economy). This social insurance function of fiscal policy is less determinant for welfare in wealthier households due to

[1] The heterogeneity of MPCs is a relevant theoretical result of Heterogeneous Agents (HAs) models that has been largely exploited to take policy insights. There is also empirical evidence of this heterogeneity (see Japelli & Pistafarri (2014)).

their larger stock of assets and consumption smoothing capacity. We argue that the social insurance effect of fiscal policy is the reason why a more progressive than normal fiscal policy is better for welfare in recessive periods, even ignoring the larger effectiveness of output stimulus.

In this paper, a new theoretical result is explored, regarding the transition from an optimal level of progressivity in a recessive context, and the optimal level of progressivity in the long run. Also, the progressivity whose impacts we aim to explore, is transfers progressivity, rather taxes progressivity, as the second can be imposed directly from either previous theoretical works, or simply by observing real data on labor and capital taxes. It is known that larger MPCs on the lower part of the distribution tend to increase fiscal multipliers as fiscal expenditure becomes more progressive. The policy implication of that fact is that is that on bust periods, a more progressive than normal fiscal policy is preferable. However, two questions remain unanswered: the first one, is how different the level of progressivity of the fiscal policy should be during the shock; the second one, how persistent does this difference should be. The most relevant tradeoff is on persistence, as more persistant progressivity shifts, would discourage savings and therefore make the transition to the long run level of output actually slower.

This paper is organized as follows: section (2) presents the model with which fiscal policy optimality can be analysed, section (3) evaluates fiscal responses from different countries to the COVID-19 crisis, showing how the optimal progressivity implied by the model differs or not with the actions governments took.

2 The Model

2.1 General Setting

2.1.1 Households

Households solve an optimal saving policy in an Aiyagari-Hugget-Bewley setting, as presented in Achdou et al. (2017), with two fundamental differences. The first fundamental difference, is that households have an additional source of income, which is a transfer T from the government, that depends on the the households productivity level z. The second fundamental difference, is unemployment; while Achdou et al. (2017) assume zero unemployment at each moment of time, we introduce the possibility of unemployment. The latter adds an employment state E to the wealth-productivity space (a, z).

$$v(a_0, z_0, E_0) = \max_{\{c_t\}_{t\geq 0}} \mathbb{E}_0 \left[\int_0^\infty \mathrm{e}^{-\rho t} u\left(c_t\right) \mathrm{d}t \right] \tag{1}$$

$$\dot{a}_t = \mathbf{1}_{\{E_t=1\}}(1-\tau_l)w_t \mathrm{e}^{z_t} + (1-\tau_a)r_t a_t + T_t(z_t, \zeta_t) - c_t;\ \mathrm{d}z_t = -\eta\left(z_t - \bar{z}\right) + \sigma \mathrm{d}W_t \tag{2}$$

$$\Pr(E_{t+\mathrm{d}t} = 1 | E_t = 0) = \lambda_t^0 \mathrm{d}t;\ \Pr(E_{t+\mathrm{d}t} = 0 | E_t = 1) = \lambda_t^1 \mathrm{d}t \tag{3}$$

2.1.2 Firms

A representative firm produces according to the equation $Y_t = A_t F(K_t, L_t) = A_t K_t^\alpha L_t^{1-\alpha}$. The total amount of capital K in the economy is equal to the amount of wealth that is held in the asset a. This means that

$K = \int_0^\infty \int_0^\infty a g(a,z) \mathrm{d}a \mathrm{d}z$. The government taxes firms profits at a rate τ, so the firm maximization problem is:

$$\max_{\{K_t, L_t\}} \left(A K_t^\alpha L_t^{1-\alpha} - r_t K_t - w_t L_t \right) \tag{4}$$

As opposed to Achdou et al. (2017), where $L_t = 1 \; \forall \; t$, the labor market adjusts through the employment level, while wages are rigid ($w_t = \bar{w}$). From an empirical point of view, this is a rather realistic assumption, and will allow fiscal policy to have a multiplier effect, as aggregate demand for consumption can induce output increases and therefore labor demand. Firms choose a level of employment L_t that is given by $L_t = \frac{\lambda_t^0}{\lambda_t^0 + \lambda_t^1}$. For simplicity, $(\lambda_t^0, \lambda_t^1)$ is assumed to be $(L_t, 1 - L_t)$, so the firm only chooses L_t. Equation (3) now becomes:

$$\Pr(E_{t+\mathrm{d}t} = 1 | E_t = 0) = L_t \mathrm{d}t; \; \Pr(E_{t+\mathrm{d}t} = 0 | E_t = 1) = (1 - L_t) \mathrm{d}t \tag{5}$$

In equilibrium, the solution of the maximization problem implies $r_t = A_t K_t^{\alpha-1} L_t^{1-\alpha}$, and $L_t = \left(\frac{A_t K_t^\alpha}{\bar{w}} \right)^{\frac{1}{1-\alpha}}$. Assuming that wage rigidities do not hold on the long run, $\bar{w}$ is determined in such a way that $L_\infty = 1$. $\bar{w}$ now becomes $\bar{w} = A_\infty K_\infty^\alpha$, and therefore:

$$L_t = \left(\frac{A_t K_t^\alpha}{A_\infty K_\infty^\alpha} \right)^{\frac{1}{1-\alpha}} \; ; \; r_t = \frac{A_t^2 K_t^{2\alpha - 1}}{A_\infty K_\infty^\alpha} \tag{6}$$

2.1.3 Government

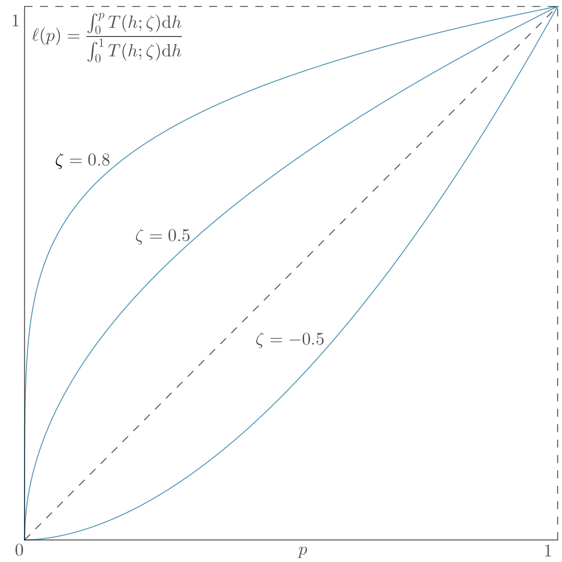
The government has a non-neutral transfer policy, which means that each individual receives a transfer $T \geq 0$ that is a function of its labor productivity z. From a distributional point of view, this means that the goverment makes the transfer using a function of the percentile p of z. This function's *progressivity* can be analysed using a Lorenz Concentration Curve (LCC) over the percentile p. The LCC function is:

$$\ell(p) = \frac{\int_0^p T(h) \mathrm{d}h}{\int_0^1 T(h) \mathrm{d}h}$$

Notice that, although $\ell'(p) \geq 0 \; \forall \; p \in [0,1]$ independently from which kind of $T(p)$ is chosen, the sign of its second derivative, is actually ambiguous, and depends on the T policy. A concave T is interpreted as a *progressive* policy, as concentrates more resources on the lower part of the distribution, while a convex T is interpreted as regressive.

In this model, the government chooses a transfer function $T(p)$ such that $\ell(p) = p^{1-\zeta}$, where $\zeta \in (-\infty, 1]$ is the progressivity parameter (see Figure (1)). To achieve $\ell(p) = p^{1-\zeta}$, we take advantage from the fact that, in each instant of time, $z \sim \mathcal{N}(\bar{z}, \frac{\sigma^2}{2\eta})$, which implies that percentiles can be obtained with the analytical expression $p(z) = \Phi \left[(z - \bar{z}) \sqrt{\frac{2\eta}{\sigma^2}} \right]$, where $\Phi(\;)$ is the cumulative standard normal density function. Given the above, T can be defined directly in the z domain, rather than p.

Figure 1: Lorenz Concentration Curves for $T(p;\zeta)$



The cumulative transfer over the z density function, then is:

$$T_{\mathrm{ac}}(z) = \int_{-\infty}^{\infty} T(z,\zeta)\mathrm{d}\phi\left[(z-\bar{z})\sqrt{\frac{2\eta}{\sigma^2}}\right] = Q\left(\Phi\left[(z-\bar{z})\sqrt{\frac{2\eta}{\sigma^2}}\right]\right)^{1-\zeta} \tag{7}$$

While ζ gives a shape to the LCC, Q gives the necessary *scale* to satisfy the government restriction:

$$\int_0^{\infty}\int_0^{\infty} T\left(z,\zeta\right) g(a,z)\mathrm{d}a\mathrm{d}z = \tau A\left[\int_0^{\infty}\int_0^{\infty} a g(a,z)\mathrm{d}a\mathrm{d}z\right]^{\alpha} \tag{8}$$

Finally, considering that $T(z,\zeta) = \frac{\partial T_{\mathrm{ac}}(z,\zeta)}{\partial z}$, we have that:

$$T(z,\zeta) = Q(1-\zeta)\left(\Phi\left[(z-\bar{z})\sqrt{\frac{2\eta}{\sigma^2}}\right]\right)^{-\zeta}\sqrt{\frac{2\eta}{\sigma^2}}\phi\left[(z-\bar{z})\sqrt{\frac{2\eta}{\sigma^2}}\right] \tag{9}$$

Now, the optimization problem of the government is:

$$v^G(A_0,K_0) = \max_{\{\zeta_t\}_{t\geq 0}}\int_0^{\infty}\int_{\bar{a}}^{\infty}\int_{-\infty}^{\infty} \mathrm{e}^{-\rho t}u[c_t(a,z)]g(a,z,t)\mathrm{d}a\mathrm{d}z\mathrm{d}t \tag{10}$$

2.1.4 Equilibrium

Finding the equilibrium, requires solving the households problem, and attaching it to the evolution of the distribution. For this, a Partial Differential Equations (PDEs) system known as *Backward-Forward Mean Field Games* has to be solved. On one hand, the Hamilton-Jacobi-Bellman (HJB) equation takes a state (a,z) and gets an optimal evolution policy for it. On the other hand, the Kolmogorov-Fokker-Plank (KFP) equation, takes the optimal evolution of the state in each point of the distribution $g(a,z)$, and gets the evolution of the whole distribution over time. With this, equilibrium can be found for each point in time $t \geq 0$. In general, given a state $\mathbf{x}$, an instantaneous return function $\pi(\mathbf{x}_t)$, a distribution $f(\mathbf{x},t)$, and a processes $\mathrm{d}\mathbf{x}_t = \vec{\mu}\,(\mathbf{x}_t, t)\mathrm{d}t + \Sigma(\mathbf{x}_t)\mathrm{d}\mathbf{w}_t$, the sistem of equations can be written as:

$$\text{HJB} : \rho v(\mathbf{x}_t, t) = \pi(\mathbf{x}_t) + \sum_{i=1}^{n} \mu_i(\mathbf{x}_t)\frac{\partial v(\mathbf{x}_t, t)}{\partial x_{i,t}} + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{i,j}^2(\mathbf{x}_t)\frac{\partial^2 v(\mathbf{x}_t, t)}{\partial x_i \partial x_j} + \mathbb{E}_t[\dot{v}(\mathbf{x}_t, t)]$$

$$\text{KFP} : \frac{\partial f(\mathbf{x}_t, t)}{\partial t} = -\sum_{i=1}^{n}\frac{\partial[\mu_i(\mathbf{x}_t)f(\mathbf{x}_t, t)]}{\partial x_i} + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial^2[\sigma_{i,j}^2(\mathbf{x}_t)f(\mathbf{x}_t, t)]}{\partial x_i \partial x_j}$$

Hamilton-Jacobi-Bellman Equation (HJB):

When introducing equation (9) into the a_t process, and recalling that $w_t = \bar{w}$ the following process is obtained:

$$\mathrm{d}a_t = \left[\mathbf{1}_{\{E=1\}}\bar{w}\mathrm{e}^{z_t} + r_t a_t - c_t + Q_t(1-\zeta_t)\left(\Phi\left[(z_t - \bar{z})\sqrt{\frac{2\eta}{\sigma^2}}\right]\right)^{-\zeta_t}\sqrt{\frac{2\eta}{\sigma^2}}\phi\left[(z_t - \bar{z})\sqrt{\frac{2\eta}{\sigma^2}}\right]\right]\mathrm{d}t \tag{11}$$

Having this in consideration:

$$\begin{aligned}
\text{HJB} : v(a_t, z_t, t, E_t = j) = \max_{c_t}\{&u(c_t) \\
&+ \left[\mathbf{1}_{\{j=1\}}\bar{w}\mathrm{e}^{z_t} + r_t a_t + T_t(z_t, \zeta_t) - c_t\right]\partial_a v(a_t, z_t, t, E_t = j) \\
&- \eta(z_t - \bar{z})\partial_{z_t} v(a, z_t, t, E_t = j) \\
&+ \frac{1}{2}\sigma^2\partial_{z_t, z_t} v(a_t, z_t, t, E_t = j) \\
&+ \mathbb{E}_t[\dot{v}(a_t, z_t, t, E_t = j)] \\
&+ \lambda_t^j\left[v(a_t, z_t, t, E_t = i) - v(a_t, z_t, t, E_t = j)\right]\}
\end{aligned} \tag{12}$$

Kolmogorov-Fokker-Plank Equation (KFP):

$$\begin{aligned}
\text{KFP} : \dot{g}(a_t, z_t, t, E_t = j) = &\\
&- \partial_{a_t}\left(\left[\mathbf{1}_{\{j=1\}}\bar{w}\mathrm{e}^{z_t} + r_t a_t + T(z_t, \zeta_t) - c_t\right]g(a_t, z_t, t, E_t = j)\right) \\
&+ \partial_{z_t}\left[\eta(z_t - \bar{x})g(a_t, z_t, t, E_t = j)\right] \\
&+ \frac{1}{2}\sigma^2\partial_{z_t, z_t} g(a_t, z_t, E_t = j) \\
&+ \lambda_t^j\left[g(a_t, z_t, t, E_t = i) - g(a_t, z_t, t, E_t = j)\right]
\end{aligned} \tag{13}$$

2.2 Steady state optimal rule

Solving the steady state is useful for two purposes. First, it is needed to know the level of $\bar{w}$, and then compute L_t to solve de PDEs for the dynamic case. The second reason to solve the steady state, is to have a benchmark to then compare the optimal ζ_t with ζ_∞ and see if the difference for the recessive context is actually significant. The equilibrium in steady state, is found by imposing $\dot{g}(a_t, z_t, t) = \dot{v}(a_t, z_t, t) = 0$, and therefore suppressing the time dependence of variables. Also, as $L_\infty = 1$, the Poisson part is suppressed too.

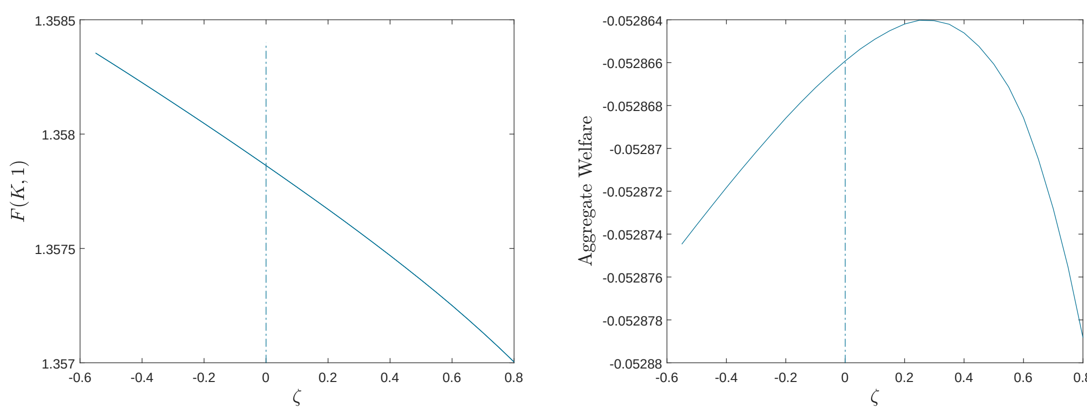
Simply by imposing the latter, the system to be solved is pretty straightforward:

$$
\begin{aligned}
\text{HJB} : v(a,z) = \max_c \{ & u(c) \\
& + [w e^z + ra + T(z,\zeta) - c]\, \partial_a v(a,z) \\
& - \eta(z - \bar{z}) \partial_z v(a,z) \\
& + \frac{1}{2}\sigma^2 \partial_{z,z} v(a,z) \}
\end{aligned}
\tag{14}
$$

$$
\begin{aligned}
\text{KFP} : 0 = & \\
& - \partial_a \left([w e^z + ra + T(z,\zeta) - c]\, g(a,z) \right) \\
& + \partial_z \left[\eta(z - \bar{z}) g(a,z) \right] \\
& + \frac{1}{2}\sigma^2 \partial_{z,z} g(a,z)
\end{aligned}
\tag{15}
$$

The result, when solving the system, is a distribution of income and wealth, and a saving policy for each point of the distribution. As shown in Achdou et al. (2017), households with low levels of wealth or income, have large marginal propensities to consume (MPCs), that are actually equal to 1 near the liquidity constraint. As aggregate capital comes from savings, a larger concentration of government transfers at the lower part of the income distribution, tends to give a lower aggregate capital level at equilibrium, which is in turn associated with lower wages and therefore utility. However as utility functions are concave, a less progressive transfer policy is not efficient either. The trade-off between aggregate capital level, and reallocation of resources implies, then, that some optimal level of progressivity must exist. Figure (2) shows the tradeoff between aggregate capital and progressivity, as well as the existence of an optimal ζ[2].

Figure 2: Production level y for each ζ



It is important to notice that this trade-off is not new. As mentioned in section (1), Heathcote, Storesletten & Violante (2017) has shown that larger levels of progressivity tend to reduce investment on human capital. Whether human or physical capital accumulation is modelled, output and welfare effects are pretty alike. Moreover, K accumulation implies a larger output Y for the long run, but also a higher wage, as well as in models with human capital. The important thing to have in mind, is that the optimal steady state

[2]For this *pre calibration* example, the parameters of Achdou et al. (2017) are assumed, with $\tau = 0.25$

policy ζ_∞ might not be the same as ζ_t, and in such a case, the question is how would ζ_t evolve over time until convergence.

2.3 *MIT* shock and dynamic optimal rule

Things get hard here.

3 Evaluating COVID-19 fiscal responses

References

Achdou, Y., Han, J., Lasry, J. M., Lions, P. L., & Moll, B. (2017). *Income and wealth distribution in macroeconomics: A continuous-time approach* (No. w23732). National Bureau of Economic Research.

Aguirre, A. (2015). *Welfare Effects of Fiscal Procyclicality: Who Wins with a Structural Balance Fiscal Rule?.* Central Bank of Chile Working Paper.

Aguirre, A. (2020). *Welfare Effects of Fiscal Procyclicality: Public Insurance with Heterogeneous Agents.* Banco Central de Chile.

Alvaredo, F., Atkinson, A. B., Piketty, T., & Saez, E. (2013). The top 1 percent in international and historical perspective. *Journal of Economic perspectives*, 27(3), 3-20.

Bakış, O., Kaymak, B., & Poschke, M. (2015). Transitional dynamics and the optimal progressivity of income redistribution. *Review of Economic Dynamics*, 18(3), 679-693.

Bassetto, M. (2014). Optimal fiscal policy with heterogeneous agents. *Quantitative Economics, 5(3), 675-704.*

Benabou, R. (2002). Tax and education policy in a heterogeneous-agent economy: What levels of redistribution maximize growth and efficiency?. *Econometrica*, 70(2), 481-517.

Bhandari, A., Evans, D., Golosov, M., & Sargent, T. J. (2013). *Taxes, debts, and redistributions with aggregate shocks* (No. w19470). National Bureau of Economic Research.

Brinca, P., Holter, H. A., Krusell, P., & Malafry, L. (2016). Fiscal multipliers in the 21st century. *Journal of Monetary Economics*, 77, 53-69.

Broer, T., Krusell, P., & Öberg, E. (2021). *Fiscal Multipliers: A Heterogenous-Agent Perspective* (No. w28366). National Bureau of Economic Research.

Cantore, C., & Freund, L. B. (2021). Workers, capitalists, and the government: Fiscal policy and income (re) distribution. *Journal of Monetary Economics.*

Gabaix, X., Lasry, J. M., Lions, P. L., & Moll, B. (2016). The dynamics of inequality. *Econometrica*, 84(6), 2071-2111.

Heathcote, J. (2005). Fiscal policy with heterogeneous agents and incomplete markets. *The Review of Economic Studies*, 72(1), 161-188.

Heathcote, J., Storesletten, K., Violante, G. L. (2017). Optimal tax progressivity: An analytical framework. *The Quarterly Journal of Economics*, 132(4), 1693-1754.

Jappelli, T., & Pistaferri, L. (2014). Fiscal policy and MPC heterogeneity. *American Economic Journal: Macroeconomics*, 6(4), 107-36.

Kolosova, K. (2013). *On the relationship between government spending multiplier and welfare.* Bocconi University, mimeo.

Le Grand, F., & Ragot, X. (2017). *Optimal fiscal policy with heterogeneous agents and aggregate shocks.* Document de travail.

Oh, H., & Reis, R. (2012). Targeted transfers and the fiscal response to the great recession. *Journal of Monetary Economics*, 59, S50-S64.

Salgado, S., Guvenen, F., & Bloom, N. (2019). *Skewed business cycles* (No. w26565). National Bureau of Economic Research.

Werning, I. (2007). Optimal fiscal policy with redistribution. *The Quarterly Journal of Economics*, 122(3), 925-967.