Optimal progressivity of fiscal stimulus: a HANK approach

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Abstract

Literature regarding fiscal policy redistribution, often poses a trade-off between allocative welfare increases from progressive fiscal policy, and human capital investment. However, although that is a relevant trade-off in the long run, that might not be the case for short term fluctuactions. Fiscal policy does not only have benefits because of redistribution itself, but also because of reactivation matters in recessive episodes. Using an extension of the Aiyagari-Hugget-Bewley HANK model presented in Achdou, Lasry, Lions, & Moll (2017). I show that the countercyclical purpose of fiscal policy can be more or less effective, depending on how progressive the countercyclical policy is. The mechanism for progressivity to matter in terms of reactivation, is that individuals with lower level of income and wealth, tend to have larger Marginal Propensities to Consume (MPCs), and therefore, transfers to the lower part of the distribution have more direct effects on output. Nevertheless, lower savings induce a slower transition to the long run capital levels, that can harm aggregate welfare as well. In this paper, , to an environment in which, the government taxes income and redistributes income choosing an optimal progressivity rule.

Keywords: Fiscal Policy, Redistribution, Reactivation, Heterogeneous Agents.

JEL Codes: E21, E62, H12, H21, H3

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1 Introduction

Literature regarding fiscal policy redistribution, such as Bakış, Kaymak & Poschke (2015) and Heathcote, Storesletten & Violante (2017) often poses a trade-off between allocative welfare increases from progressive fiscal policy, and human capital investment. However, although that is a relevant trade-off in the long run, that might not be the case for short term fluctuactions. Fiscal policy does not only have benefits because of redistribution itself, but also because of reactivation matters in recessive episodes. Using an extension of the Aiyagari-Hugget-Bewley HANK model presented in Achdou, Lasry, Lions, & Moll (2017). I show that the countercyclical purpose of fiscal policy can be more or less effective, depending on how progressive the countercyclical policy is. The mechanism for progressivity to matter in terms of reactivation, is that individuals with lower level of income and wealth, tend to have larger Marginal Propensities to Consume (MPCs), and therefore, transfers to the lower part of the distribution have more direct effects on output. Nevertheless, lower savings induce a slower transition to the long run capital levels, that can harm aggregate welfare as well. In this paper, , to an environment in which, the government taxes income and redistributes income choosing an optimal progressivity rule. This paper is organized as follows: section (2) presents the model with which fiscal policy optimality can be analysed.

2 The Model

2.1 General Setting

2.1.1 Households

Households solve an optimal saving policy in an Aiyagari-Hugget-Bewley setting, as presented in Achdou et al. (2017), with two fundamental differences. The first fundamental difference, is that households have an additional source of income, which is a transfer T from the government, that depends on the the households productivity level z. The second fundamental difference, is unemployment; while Achdou et al. (2017) assume zero unemployment at each moment of time, we introduce the possibility of unemployment. The latter adds an employment state E to the wealth-productivity space (a, z).

$$\dot{a}_t = \mathbf{1}_{\{E_t=1\}} w_t \mathrm{e}^{z_t} + r_t a_t + T_t(z_t, \zeta_t) - c_t;\ \mathrm{d}z_t = -\eta\left(z_t - \bar{z}\right) + \sigma \mathrm{d}W_t \tag{1}$$

$$\Pr(E_{t+\mathrm{d}t} = 1 | E_t = 0) = \lambda_t^0 \mathrm{d}t;\ \Pr(E_{t+\mathrm{d}t} = 0 | E_t = 1) = \lambda_t^1 \mathrm{d}t \tag{2}$$

2.1.2 Firms

A representative firm produces according to the equation $Y_t = A_t F(K_t, L_t) = A_t K_t^{\alpha} L_t^{1-\alpha}$. The total amount of capital K in the economy is equal to the amount of wealth that is held in the asset a. This means that $K = \int_0^{\infty}\int_0^{\infty} a g(a, z) \mathrm{d}a \mathrm{d}z$. The government taxes firms profits at a rate τ, so the firm maximization problem is:

$$\max_{\{K_t, L_t\}} \left[(1-\tau) A K_t^{\alpha} L_t^{1-\alpha} - r_t K_t - w_t L_t\right] \tag{3}$$

As opposed to Achdou et al. (2017), where $L_t = 1\ \forall\ t$, the labor market adjusts through the employment level, while wages are rigid ($w_t = \bar{w}$). From an empirical point of view, this is a rather realistic assumption, and will allow fiscal policy to have a multiplier effect, as aggregate demand for consumption can induce output increases and therefore labor demand. Firms choose a level of employment L_t that is given by $L_t = \frac{\lambda_t^0}{\lambda_t^0+\lambda_t^1}$. For simplicity, $(\lambda_t^0, \lambda_t^1)$ is assumed to be $(L_t, 1-L_t)$, so the firm only chooses L_t. Equation (2) now becomes:

$$\Pr(E_{t+\mathrm{d}t} = 1|E_t = 0) = L_t \mathrm{d}t;\ \Pr(E_{t+\mathrm{d}t} = 0|E_t = 1) = (1 - L_t)\mathrm{d}t \tag{4}$$

In equilibrium, the solution of the maximization problem implies $r_t = A_t K_t^{\alpha-1} L_t^{1-\alpha}$, and $L_t = \left(\frac{A_t K_t^\alpha}{\bar{w}}\right)^{\frac{1}{1-\alpha}}$. Assuming that wage rigidities do not hold on the long run, $\bar{w}$ is determined in such a way that $L_\infty = 1$. $\bar{w}$ now becomes $\bar{w} = A_\infty K_\infty^\alpha$, and therefore:

$$L_t = \left(\frac{A_t K_t^\alpha}{A_\infty K_\infty^\alpha}\right)^{\frac{1}{1-\alpha}};\ r_t = \frac{A_t^2 K_t^{2\alpha-1}}{A_\infty K_\infty^\alpha} \tag{5}$$

2.1.3 Government

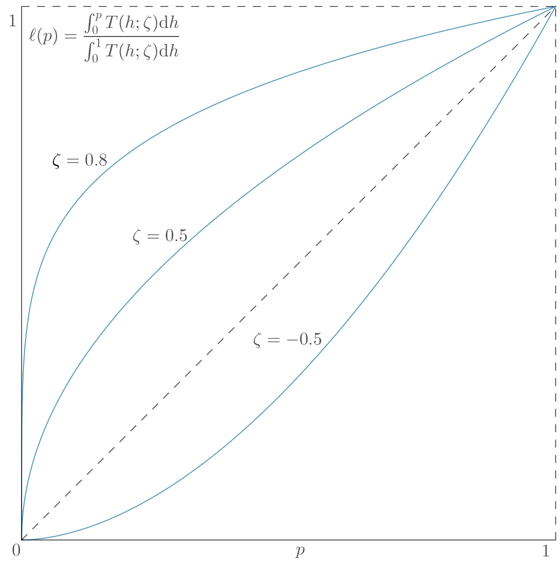
The government has a non-neutral transfer policy, which means that each individual receives a transfer $T \geq 0$ that is a function of its labor productivity z. From a distributional point of view, this means that the goverment makes the transfer using a function of the percentile p of z. This function's *progressivity* can be analysed using a Lorenz Concentration Curve (LCC) over the percentile p. The LCC function is:

$$\ell(p) = \frac{\int_0^p T(h)\mathrm{d}h}{\int_0^1 T(h)\mathrm{d}h}$$

Notice that, although $\ell'(p) \geq 0\ \forall\ p \in [0,1]$ independently from which kind of $T(p)$ is chosen, the sign of its second derivative, is actually ambiguous, and depends on the T policy. A concave T is interpreted as a *progressive* policy, as concentrates more resources on the lower part of the distribution, while a convex T is interpreted as regressive.

In this model, the government chooses a transfer function $T(p)$ such that $\ell(p) = p^{1-\zeta}$, where $\zeta \in (-\infty, 1]$ is the progressivity parameter (see Figure (1)). To achieve $\ell(p) = p^{1-\zeta}$, we take advantage from the fact that, in each instant of time, $z \sim \mathcal{N}(\bar{z}, \frac{\sigma^2}{2\eta})$, which implies that percentiles can be obtained with the analytical expression $p(z) = \Phi\left[(z-\bar{z})\sqrt{\frac{2\eta}{\sigma^2}}\right]$, where $\Phi(\ )$ is the cumulative standard normal density function. Given the above, T can be defined directly in the z domain, rather than p.

Figure 1: Lorenz Concentration Curves for $T(p;\zeta)$



The cumulative transfer over the z density function, then is:

$$T_{\mathrm{ac}}(z) = \int_{-\infty}^{\infty} T(z,\zeta)\mathrm{d}\phi\left[(z-\bar{z})\sqrt{\frac{2\eta}{\sigma^2}}\right] = Q\left(\Phi\left[(z-\bar{z})\sqrt{\frac{2\eta}{\sigma^2}}\right]\right)^{1-\zeta} \tag{6}$$

While ζ gives a shape to the LCC, Q gives the necessary *scale* to satisfy the government restriction:

$$\int_0^{\infty}\int_0^{\infty} T\left(z,\zeta\right)g(a,z)\mathrm{d}a\mathrm{d}z = \tau A\left[\int_0^{\infty}\int_0^{\infty} a g(a,z)\mathrm{d}a\mathrm{d}z\right]^{\alpha} \tag{7}$$

Finally, considering that $T(z,\zeta) = \frac{\partial T(z,\zeta)}{\partial z}$, we have that:

$$T(z,\zeta) = Q(1-\zeta)\left(\Phi\left[(z-\bar{z})\sqrt{\frac{2\eta}{\sigma^2}}\right]\right)^{-\zeta}\sqrt{\frac{2\eta}{\sigma^2}}\phi\left[(z-\bar{z})\sqrt{\frac{2\eta}{\sigma^2}}\right] \tag{8}$$

Now, the optimization problem of the government is:

$$v^G(A_0,K_0) = \max_{\{\zeta_t\}_{t\geq 0}} \int_0^{\infty}\int_{\bar{a}}^{\infty}\int_{-\infty}^{\infty} \mathrm{e}^{-\rho t}u[c_t(a,z)]g(a,z,t)\mathrm{d}a\mathrm{d}z\mathrm{d}t \tag{9}$$

2.1.4 Equilibrium

Finding the equilibrium, requires solving the households problem, and attaching it to the evolution of the distribution. For this, a Partial Differential Equations (PDEs) system known as *Backward-Forward Mean Field Games* has to be solved. On one hand, the Hamilton-Jacobi-Bellman (HJB) equation takes a state (a,z) and gets an optimal evolution policy for it. On the other hand, the Kolmogorov-Fokker-Plank (KFP) equation, takes the optimal evolution of the state in each point of the distribution $g(a,z)$, and gets the evolution of the whole distribution over time. With this, equilibrium can be found for each point in time $t\geq 0$. In general, given a state $\mathbf{x}$, an instantaneous return function $\pi(\mathbf{x}_t)$, a distribution $f(\mathbf{x},t)$, and a processes $\mathrm{d}\mathbf{x}_t = \vec{\mu}\,(\mathbf{x}_t, t)\mathrm{d}t + \Sigma(\mathbf{x}_t)\mathrm{d}\mathbf{w}_t$, the sistem of equations can be written as:

$$\text{HJB} : \rho v(\mathbf{x}_t, t) = \pi(\mathbf{x}_t) + \sum_{i=1}^{n} \mu_i(\mathbf{x}_t)\frac{\partial v(\mathbf{x}_t, t)}{\partial x_{i,t}} + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{i,j}^2(\mathbf{x}_t)\frac{\partial^2 v(\mathbf{x}_t, t)}{\partial x_i \partial x_j} + \mathbb{E}_t[\dot{v}(\mathbf{x}_t, t)]$$

$$\text{KFP} : \frac{\partial f(\mathbf{x}_t, t)}{\partial t} = -\sum_{i=1}^{n}\frac{\partial[\mu_i(\mathbf{x}_t) f(\mathbf{x}_t, t)]}{\partial x_i} + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial^2[\sigma_{i,j}^2(\mathbf{x}_t) f(\mathbf{x}_t, t)]}{\partial x_i \partial x_j}$$

Hamilton-Jacobi-Bellman Equation (HJB):

When introducing equation (8) into the a_t process, and recalling that $w_t = \bar{w}$ the following process is obtained:

$$\mathrm{d}a_t = \left[\mathbf{1}_{\{E=1\}}\bar{w}\mathrm{e}^{z_t} + r_t a_t - c_t + Q_t(1-\zeta_t)\left(\Phi\left[(z_t - \bar{z})\sqrt{\frac{2\eta}{\sigma^2}}\right]\right)^{-\zeta_t}\sqrt{\frac{2\eta}{\sigma^2}}\phi\left[(z_t - \bar{z})\sqrt{\frac{2\eta}{\sigma^2}}\right]\right]\mathrm{d}t \tag{10}$$

Having this in consideration:

$$\begin{aligned}
\text{HJB} : v(a_t, z_t, t, E_t = j) = \max_{c_t}\{u(c_t) & \\
& + \left[\mathbf{1}_{\{j=1\}}\bar{w}\mathrm{e}^{z_t} + r_t a_t + T_t(z_t, \zeta_t) - c_t\right]\partial_a v(a_t, z_t, t, E_t = j) \\
& - \eta(z_t - \bar{z})\partial_{z_t} v(a, z_t, t, E_t = j) \\
& + \frac{1}{2}\sigma^2\partial_{z_t, z_t} v(a_t, z_t, t, E_t = j) \\
& + \mathbb{E}_t[\dot{v}(a_t, z_t, t, E_t = j)] \\
& + \lambda_t^j\left[v(a_t, z_t, t, E_t = i) - v(a_t, z_t, t, E_t = j)\right]\}
\end{aligned} \tag{11}$$

Kolmogorov-Fokker-Plank Equation (KFP):

$$\begin{aligned}
\text{KFP} : \dot{g}(a_t, z_t, t, E_t = j) = & \\
& - \partial_{a_t}\left(\left[\mathbf{1}_{\{j=1\}}\bar{w}\mathrm{e}^{z_t} + r_t a_t + T(z_t, \zeta_t) - c_t\right] g(a_t, z_t, t, E_t = j)\right) \\
& + \partial_{z_t}\left[\eta(z_t - \bar{x}) g(a_t, z_t, t, E_t = j)\right] \\
& + \frac{1}{2}\sigma^2\partial_{z_t, z_t} g(a_t, z_t, E_t = j) \\
& + \lambda_t^j\left[g(a_t, z_t, t, E_t = i) - g(a_t, z_t, t, E_t = j)\right]
\end{aligned} \tag{12}$$

2.2 Steady state optimal rule

Solving the steady state is useful for two purposes. First, it is needed to know the level of $\bar{w}$, and then compute L_t to solve de PDEs for the dynamic case. The second reason to solve the steady state, is to have a benchmark to then compare the optimal ζ_t with ζ_∞ and see if the difference for the recessive context is actually significant. The equilibrium in steady state, is found by imposing $\dot{g}(a_t, z_t, t) = \dot{v}(a_t, z_t, t) = 0$, and therefore suppressing the time dependence of variables. Also, as $L_\infty = 1$, the Poisson part is suppressed too.

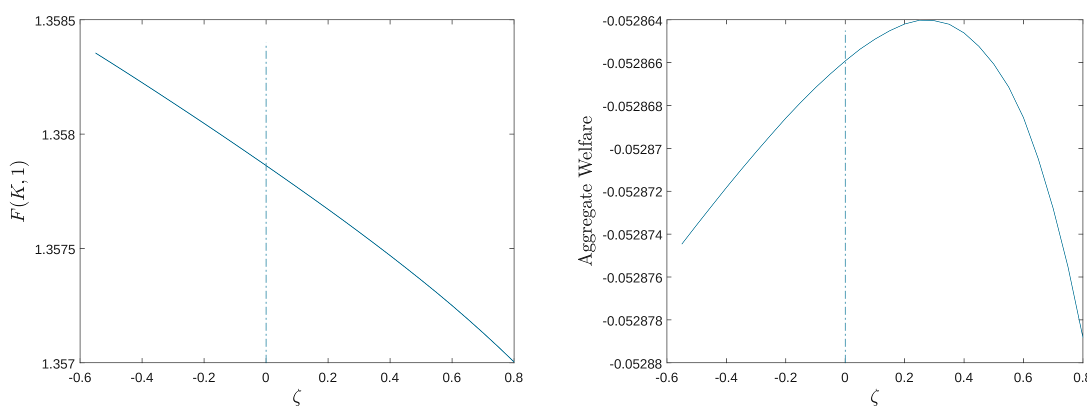
Simply by imposing the latter, the system to be solved is pretty straightforward:

$$
\begin{aligned}
\text{HJB} : v(a,z) = \max_{c}\{u(c) & \\
& + [w e^{z} + ra + T(z,\zeta) - c]\, \partial_a v(a,z) \\
& - \eta(z-\bar{z})\partial_z v(a,z) \\
& + \frac{1}{2}\sigma^2 \partial_{z,z} v(a,z)\}
\end{aligned} \tag{13}
$$

$$
\begin{aligned}
\text{KFP} : 0 = & \\
& - \partial_a \left([w e^{z} + ra + T(z,\zeta) - c]\, g(a,z) \right) \\
& + \partial_z \left[\eta(z-\bar{z}) g(a,z) \right] \\
& + \frac{1}{2}\sigma^2 \partial_{z,z} g(a,z)
\end{aligned} \tag{14}
$$

The result, when solving the system, is a distribution of income and wealth, and a saving policy for each point of the distribution. As shown in Achdou et al. (2017), households with low levels of wealth or income, have large marginal propensities to consume (MPCs), that are actually equal to 1 near the liquidity constraint. As aggregate capital comes from savings, a larger concentration of government transfers at the lower part of the income distribution, tends to give a lower aggregate capital level at equilibrium, which is in turn associated with lower wages and therefore utility. However as utility functions are concave, a less progressive transfer policy is not efficient either. The trade-off between aggregate capital level, and reallocation of resources implies, then, that some optimal level of progressivity must exist. Figure (2) shows the tradeoff between aggregate capital and progressivity, as well as the existence of an optimal ζ[1].

Figure 2: Production level y for each ζ



It is important to notice that this trade-off is not new. As mentioned in section (1), Heathcote, Storesletten & Violante (2017) has shown that larger levels of progressivity tend to reduce investment on human capital. Whether human or physical capital accumulation is modelled, output and welfare effects are pretty alike. Moreover, K accumulation implies a larger output Y for the long run, but also a higher wage, as well as in models with human capital. The important thing to have in mind, is that the optimal steady state

[1] For this *pre calibration* example, the parameters of Achdou et al. (2017) are assumed, with $\tau = 0.25$

policy ζ_∞ might not be the same as ζ_t, and in such a case, the question is how would ζ_t evolve over time until convergence.

2.3 *MIT* shock and dynamic optimal rule

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