A BAYESIAN APPROACH TO COMPUTER MODEL CALIBRATION AND MODEL-ASSISTED DESIGN

A

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# ABSTRACT

Computer models of phenomena that are difficult or impossible to study directly are critical for enabling research and assisting design in many areas. In order to be effective, computer models must be calibrated so that they accurately represent the modeled phenomena. There exists a rich variety of methods for computer model calibration that have been developed in recent decades. Among the desiderata of such methods is a means of quantifying remaining uncertainty after calibration regarding both the values of the calibrated model inputs and the model outputs. Bayesian approaches to calibration have met this need in recent decades. However, limitations remain. Whereas in model calibration one finds point estimates or distributions of *calibration inputs* in order to induce the model to reflect reality accurately, interest in a computer model often centers primarily on its use for model-assisted design, in which the goal is to find values for *design inputs* to induce the modeled system to approximate some target outcome. Existing Bayesian approaches are limited to the first of these two tasks. The present work develops an approach adapting Bayesian methods for model calibration for application in model-assisted design. The approach retains the benefits of Bayesian calibration in accounting for and quantifying all sources of uncertainty. It is capable of generating a comprehensive assessment of the Pareto optimal inputs for a multi-objective optimization problem. The present work shows that this approach can apply as a method for model-assisted design using a previously calibrated system, and can also serve as a method for model-assisted design using a model that still requires calibration, accomplishing both ends simultaneously.

# DEDICATION

For my father, who made me see that this was possible, and for my wife, without whom it would not have been.

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# TABLE OF CONTENTS

Page

TITLE PAGE i

ABSTRACT ii

DEDICATION iii

ACKNOWLEDGMENTS iv

LIST OF TABLES vi

LIST OF FIGURES vii

CHAPTER

I. INTRODUCTION 1

Computer model calibration 1

Calibration versus model-assisted design 9

Model-assisted design 10

Gaussian process metamodels 11

Optimization 13

Applications 16

References 18

II. MULTI-OBJECTIVE ENGINEERING DESIGN VIA

COMPUTER MODEL CALIBRATION 29

Introduction 32

Counterfactual Bayes 37

Calibration for design 38

Simulated example 47

Wind turbine material design application 49

Discussion 59

References 63

Appendix: Turbine blade finite element model 72

Appendix: Surrogate model validation 73

III. A UNIFIED FRAMEWORK FOR COMPUTER MODEL

CALIBRATION AND ENGINEERING DESIGN 75

Table of Contents (Continued)

Page

Introduction 78

Dual calibration to target outcomes 85

Dependence of on 92

Case study application: vibration isolation design 103

Conclusion 116

References 118

Appendix: Validation of DCTO 124

IV. CONCLUSION 128

Benefits 128

Summary of chapter two 129

Summary of chapter three 132

Recommendations 134

Future directions 136

References 137

# LIST OF TABLES

Table Page

2.1 Turbine blade surrogate hyperparameter estimates 52

3.1 RMSEs for simulation calibration and design 101

3.2 Values of components in vibration isolation test rig 107

3.3 Vibration isolation experiment tests and results 109

3.4 Vibration isolation simulation inputs and results 113

# LIST OF FIGURES

Figure Page

2.1 Two choices of target outcomes for CTO 47

2.2 True two-dimensional profile outputs of the five-

dimensional simulated example model 48

2.3 Posterior draws from CTO in the simulated example 50

2.4 Estimated Pareto front with target outcome 54

2.5 Posterior distribution of 55

2.6 Approximate prior and posterior marginal predictive

densities for each of the three turbine outputs 55

2.7 Estimated Pareto front of the turbine system with

Quantified uncertainties and NSGA-II results 57

2.8 Estimated Pareto front for multiple turbine systems in

a variety of design spaces with NSGA-II results 47

2.9 CX-100 blade model 72

2.10 Results of 10-fold cross validation of the GP emulator

used for the turbine application 74

3.1 Example simulation model output 95

3.2 True value of the calibration parameter for each

value in the domain of 95

3.3 Simulation model output for all values of and 96

3.4 Simulation model outputs shown with various

model discrepancies 98

3.5 Noisy observations of the simulation model 99

3.6 Design input values for observations under the adaptive

sampling approach and space-filling design 100

List of Figures (Continued)

Figure Page

3.7 Prior and posterior distributions of and with true/

optimal values for various simulation discrepancies 102

3.8 Physical test rig for vibration isolation design experiment 104

3.9 Schematic diagram of the vibration isolation test rig 105

3.10 Schematic diagram of the applied impulse force in the

time domain 110

3.11 Five-times averaged acceleration response of the rigid

mass in the time domain 110

3.12 The dynamic vibration system 112

3.13 The posterior distributions of the calibration and design

inputs along with their priors 115

3.14 The posterior distributions of the model output at three

levels of the operational domain, with priors 117

3.15 Prior and posterior distributions of and for

validation of DCTO 125