Programming Languages

Values and Types

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11. Summary

What are Value and Type?

- Value anything that exist, that can be computed, stored, take part in data structure.
  Constants, variable content, parameters, function return values, operator results...
- Type set of values of same kind.
  C types:
  - `int`, `char`, `long`,...
  - `float`, `double`
  - pointers
  - structures: `struct`, `union`
  - arrays

- Haskell types
  - `Bool, Int, Float, ...`
  - `Char, String`
  - tuples,(N-tuples), records
  - lists
  - functions
- Python types
  - `bool, int, float, complex`
  - `str, bytes, tuple, list, set, dict`
  - classes, functions
- Each type represents a set of values. Is that enough?
  What about the following set? Is it a type?
  {`"ahmet", 1 , 4 , 23.453, 2.32, 'b'`}
- Values should exhibit a similar behavior. The same group of operations should be defined on them.

Primitive vs Composite Types

- Primitive Types: Values that cannot be decomposed into other sub values.
  C: `int`, `float`, `double`, `char`, `long`, `short`, pointers
  Haskell: `Bool`, `Int`, `Float`, function values
  Python: `bool`, `int`, `float`, `str`, functions
- cardinality of a type: The number of distinct values that a datatype has. Denoted as: "#Type".
  $\#\text{Bool} = 2 \quad \#\text{char} = 256 \quad \#\text{short} = 2^{16}$
  $\#\text{int} = 2^{32} \quad \#\text{double} = 2^{32}, \ldots$
- What does cardinality mean? How many bits required to store the datatype?

User Defined Primitive Types

- enumerated types
  ```
  enum days {mon, tue, wed, thu, fri, sat, sun};
  enum months {jan, feb, mar, apr, ....  };
  ```
- ranges (Pascal and Ada)
  ```
   type Day = 1..31;
   var g:Day;
  ```
- Discrete Ordinal Primitive Types Datatypes values have one to one mapping to a range of integers.
  C: Every ordinal type is an alias for integers.
  Pascal, Ada: distinct types
- DOPT's are important as they
  i. can be array indices, `switch/case` labels
  ii. can be used as `for` loop variable (some languages like pascal)

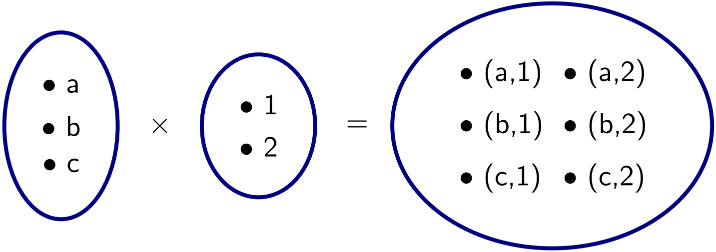
Composite Datatypes

User defined types with composition of one or more other datatypes. Depending on composition type:

- Cartesian Product (struct, tuples, records)
- Disjoint union (union (C), variant record (pascal), Data (haskell))
- Mapping (arrays, functions)
- Powerset (set datatype (Pascal))
- Recursive compositions (lists, trees, complex data structures)

Cartesian Product

- $S \times T = \{(x, y) \mid x \in S, y \in T\}$
- Example:
  $S = \{a, b, c\} \quad T = \{1, 2\}$
  $S \times T = \{(a, 1), (a, 2), (b, 1), (b, 2), (c, 1), (c, 2)\}$



- $\#(S \times T) = \#S \cdot \#T$

- C `struct`, Pascal `record`, functional languages tuple
- **in C:** string $\times$ int

```
struct Person {
     char name[20];
     int no;
} x = {"Osman␣Hamdi",23141};
```

- **in Haskell**: string $\times$ int

```
type People=(String,Int)
...
x = ("Osman␣Hamdi",23141)::People
```

- **in Python**: string $\times$ int

```
x = ( "Osman␣Hamdi", 23141)
type(x)
<type 'tuple'>
```

- Multiple Cartesian products:
  C: string $\times$ int $\times$ {MALE,FEMALE}

```
struct Person {
     char name[20];
     int no;
     enum Sex {MALE, FEMALE} sex;
} x = {"Osman␣Hamdi",23141,FEMALE};
```

Haskell: string $\times$ int $\times$ float $\times$ string

```
x = ("Osman␣Hamdi",23141,3.98,"Yazar")
```

Python: str $\times$ int $\times$ float $\times$ str

```
x = ("Osman␣Hamdi",23141,3.98,"Yazar")
```

Homogeneous Cartesian Products

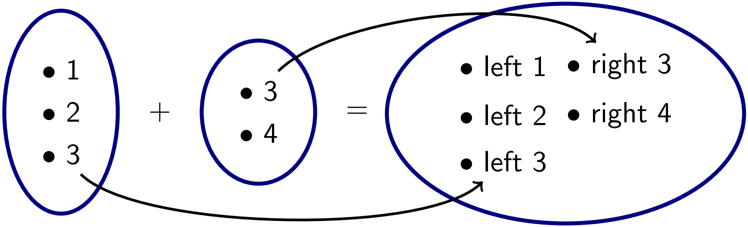
- $S^n = \overbrace{S \times S \times S \times \ldots \times S}^{n}$
  double^4 :

```
struct quad { double x,y,z,q; };
```

- $S^0 = \{()\}$ is 0-tuple.
- not empty set. A set with a single value.
- terminating value (nil) for functional language lists.
- C `void`. Means no value. Error on evaluation.
- Python: () . None used for no value.

Disjoint Union

- $S + T = \{\mathit{left}\ x \mid x \in S\} \cup \{\mathit{right}\ x \mid x \in T\}$
- Example:
  $S = \{1, 2, 3\} \quad T = \{3, 4\}$
  $S + T = \{\mathit{left}\ 1, \mathit{left}\ 2, \mathit{left}\ 3, \mathit{right}\ 3, \mathit{right}\ 4\}$



- $\#(S + T) = \#S + \#T$
- C `union`'s are disjoint union?

- **C:** int + double:

```
union number { double real; int integer; } x;
```

- C `union`'s are not safe! Same storage is shared. Valid field is unknown:

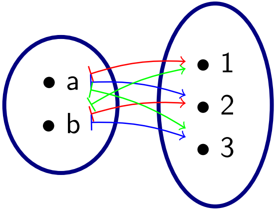
```
x.real=3.14; printf("%d\n",x.integer);
```

- **Haskel:** Float + Int + (Int × Int):

```
data Number = RealVal Float | IntVal Int | Rational (Int,Int)
x = Rational (3,4)
y = RealVal 3.14
z = IntVal 12    {-- You cannot access different values --}
```

Mappings

- The set of all possible mappings
- $S \mapsto T = \{V \mid \forall(x \in S)\exists(y \in T), (x \mapsto y) \in V\}$
- Example: $S = \{a, b\} \quad T = \{1, 2, 3\}$



Each color is a value in the mapping. Other 6 values are not drawn

$S \mapsto T = \{\{a \mapsto 1, b \mapsto 1\}, \{a \mapsto 1, b \mapsto 2\}, \{a \mapsto 1, b \mapsto 3\},$
$\{a \mapsto 2, b \mapsto 1\}, \{a \mapsto 2, b \mapsto 2\}, \{a \mapsto 2, b \mapsto 3\},$
$\{a \mapsto 3, b \mapsto 1\}, \{a \mapsto 3, b \mapsto 2\}, \{a \mapsto 3, b \mapsto 3\}\}$

- $\#(S \mapsto T) = \#T^{\#S}$

Arrays

- `double a[3]={1.2,2.4,-2.1};`
  $\mathtt{a} \in (\{0, 1, 2\} \mapsto \mathtt{double})$
  $\mathtt{a} = (0 \mapsto 1.2, 1 \mapsto 2.4, 2 \mapsto -2.1)$
- Arrays define a mapping from an integer range (or DOPT) to any other type
- **C:** $T\ \ \mathtt{x}[N] \ \Rightarrow\ \mathtt{x} \in (\{0, 1, ..., N-1\} \mapsto T)$
- Other array index types (Pascal):

```
type
   Day = (Mon,Tue,Wed,Thu,Fri,Sat,Sun);
   Month = (Jan,Feb,Mar,Apr,May,Jun,Jul,Aug,Sep,Oct,Nov,Dec);
var
   x : array Day of real;
   y : array Month of integer;
...
   x[Tue] := 2.4;
   y[Feb] := 28;
```

Functions

- C function:

```c
int f(int a) {
    if (a%2 == 0) return 0;
    else return 1;
}
```

- $\texttt{f} : \texttt{int} \mapsto \{0, 1\}$
  regardless of the function body: $\texttt{f} : \texttt{int} \mapsto \texttt{int}$
- Haskell:

```haskell
f a = if mod a 2 == 0 then 0 else 1
```

- in C, `f` expression is a pointer type `int (*)(int)`
  in Haskell it is a mapping: $\texttt{int}\mapsto\texttt{int}$

Array and Function Difference

Arrays:

- Values stored in memory
- Restricted: only integer domain
- `double`$\mapsto$`double` ?

Functions

- Defined by algorithms
- Efficiency, resource usage
- All types of mappings possible
- Side effect, output, error, termination problem.

Cartesian Mappings

- Cartesian mappings:
  `double a[3][4];`
  `double f(int m, int n);`
- Cartesian mapping versus mappings of mappings:
  `int`$\times$`int`$\mapsto$`double` and `int`$\mapsto$`(int`$\mapsto$`double)`
- For cartesian mapping, you need two or more values to get the value of the mapping
- In mapping of mappings, you get a new mapping as you supply a value.

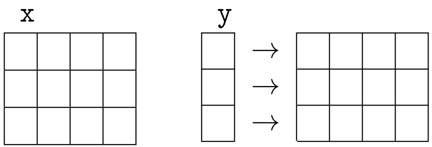
Cartesian Mapping vs Nested mapping

- Pascal arrays

```
var
   x : array [1..3,1..4] of double;
   y : array [1..3] of array [1..4] of double;
...
x[1,3] := x[2,3]+1;       y[1,3] := y[2,3]+1;
```

- Row operations:

```
y[1] := y[2] ;   √
x[1] := x[2] ;   ×
```



- Haskell functions:

```
f (x,y) = x+y
g x y = x+y
...
f (3+2)
g 3 2
```

- `g 3` √
  `f 3` ×
- Reuse the old definition to define a new function:
  `increment = g 1`
  `increment 1`
  `2`

Powerset

- $\mathcal{P}(S) = \{T \mid T \subseteq S\}$
- The set of all subsets
-

$S = $ • 1 • 2 • 3 $\quad \mathcal{P}(S) = $ •$\emptyset$ •$\{3\}$ •$\{2,3\}$ •$\{1\}$ •$\{1,2\}$ •$\{1,2,3\}$ •$\{2\}$ •$\{1,3\}$

- $\#\mathcal{P}(S) = 2^{\#S}$

- Set datatype is restricted and special datatype. Only exists in Pascal and special set languages like SetL
- set operations (Pascal)

```
type
   color = (red,green,blue,white,black);
   colorset = set of color;
var
   a,b : colorset;
...
a := [red,blue];
b := a*b;                (* intersection *)
b := a+[green,red];      (* union *)
b := a-[blue];           (* difference *)
if (green in b) then ... (* element test *)
if (a = []) then ...     (* set equality *)
```

- in C++ and Python implemented as class.

Recursive Types

- $S = ...S...$
- Types including themselves in composition.

Lists

- $S = Int \times S + \{null\}$

$$\begin{aligned} S = \quad & \{right\ empty\} \cup \{left\ (x, empty) \mid x \in Int\} \cup \\ & \{left\ (x, left\ (y, empty)) \mid x, y \in Int\} \cup \\ & \{left\ (x, left\ (y, left\ (z, empty))) \mid x, y, z \in Int\} \cup \ldots \end{aligned}$$

- $S =$
  $\{right\ empty, left(1, empty), left(2, empty), left(3, empty), ...,$
  $left(1, left(1, empty)), left(1, left(2, empty)), left(1, left(3, empty), ...$
  $left(1, left(1, left(1, empty))), left(1, left(1, left(2, empty))), ...\}$

- C lists: pointer based. Not actual recursion.

```
struct List {
     int x;
     List *next;
} a;
```

- Haskell lists.

```
data List = Left (Int,List) | Empty

x = Left (1, Left(2, Left(3,Empty)))  {-- [1,2,3] list --}
y = Empty                             {-- empty list, [] --}
```

- Polymorphic lists: a single definition defines lists of many types.
- $List\ \alpha = \alpha \times (List\ \alpha) + \{empty\}$

```
data List alpha = Left (alpha,List alpha) | Empty

x = Left (1, Left(2, Left(3,Empty)))     {-- [1,2,3] list --}
y = Left ("ali",Left("ahmet",Empty))    {-- ["ali","ahmet"] --
z = Left(23.1,Left(32.2,Left(1.0,Empty))){-- [23.1,32.2,1.0] -
```

- $Left(1, Left(\text{``}ali\text{''}, Left(15.23, Empty) \in List\ \alpha$? No. Most languages only permits homogeneous lists.

Haskell Lists

- binary operator ":" for list construction:
  `data [alpha] = (alpha :  [alpha]) | []`
- `x = (1:(2:(3:[])))`
- Syntactic sugar:
  `[1,2,3]` $\equiv$ `(1:(2:(3:[])))`
  `["ali"]` $\equiv$ `("ali":[])`

General Recursive Types

- $T = \ldots T \ldots$
- Formula requires a minimal solution to be representable:
  $S = Int \times S$
  Is it possible to write a single value? No minimum solution here!
- List example:
  `x = Left(1,Left(2,x))`
  $x \in S$? Yes
  can we process `[1,2,1,2,1,2,...]` value?
- Some languages like Haskell lets user define such values. All iterations go infinite. Useful in some domains though.
- Most languages allow only a subset of S, the subset of finite values.

- $Tree\ \alpha = empty + node\ \alpha \times Tree\alpha \times Tree\alpha$

$$\begin{aligned} Tree\ \alpha = \quad & \{empty\} \cup \{node(x, empty, empty) \mid x \in \alpha\} \cup \\ & \{node(x, node(y, empty, empty), empty) \mid x, y \in \alpha\} \cup \\ & \{node(x, empty, node(y, empty, empty)) \mid x, y \in \alpha\} \cup \\ & \{node(x, node(y, empty, empty), node(z, empty, empty)) \mid x, y, z \in \alpha\} \cup .. \end{aligned}$$

- C++ (pointers and template definition)

```
template<class Alpha>
struct Tree {
     Alpha x;
     Tree *left ,*right;
} root;
```

- Haskell

```
data Tree alpha = Empty |
                        Node (alpha,Tree alpha, Tree alpha)

x = Node (1,Node (2,Empty,Empty),Node(3,Empty,Empty))
y = Node(3,Empty,Empty)
```

Strings

- Language design choice:
  1. Primitive type (ML, Python):
     Language keeps an internal table of strings
  2. Character array (C, Pascal, ...)
  3. Character list (Haskell, Prolog, Lisp)
- Design choice affects the complexity and efficiency of: concatenation, assignment, equality, lexical order, decomposition

Type Systems

- Types are required to provide data processing, integrity checking, efficiency, access controls. Type compatibility on operators is essential.
- Simple bugs can be avoided at compile time.
- Irrelevant operations:

```
y=true * 12;
x=12; x[1]=6;
y=5; x.a = 4;
```

- When to do type checking? Latest time is before the operation. Two options:
  1. Compile time → static type checking
  2. Run time → dynamic type checking

Static Type Checking

- Compile time type information is used to do type checking.
- All incompatibilities are resolved at compile time. Variables have a fixed time during their lifetime.
- Most languages do static type checking
- User defined constants, variable and function types:
  - Strict type checking. User has to declare all types (C, C++, Fortran,...)
  - Languages with type inference (Haskell, ML, Scheme...)
- No type operations after compilation. All issues are resolved. Direct machine code instructions.

Dynamic Type Checking

- Run-time type checking. No checking until the operation is to be executed.
- Interpreted languages like Lisp, Prolog, PHP, Perl, Python.
- Python:

```
def whichmonth(inp):
    if isinstance(inp, int):
        return inp
    elif isinstance(inp, str):
        if inp == "January":
            return 1
        elif inp == "February":
            return 2
        ....
        elif inp == "December":
            return 12
...
inp = input()     /* user input at run time? */
month=whichmonth(inp)
```

- Run time decision based on users choice is possible.
- Has to carry type information along with variable at run time.
- Type of a variable can change at run-time (depends on the language).

Static vs Dynamic Type Checking

- Static type checking is faster. Dynamic type checking does type checking before each operation at run time. Also uses extra memory to keep run-time type information.
- Static type checking is more restrictive meaning safer. Bugs avoided at compile time, earlier is better.
- Dynamic type checking is less restrictive meaning more flexible. Operations working on dynamic run-time type information can be defined.

Type Equality

- $S \stackrel{?}{\equiv} T$ How to decide?
  - Name Equivalence: Types should be defined at the same exact place.
  - Structural Equivalence: Types should have same value set. (mathematical set equality).
- Most languages use name equivalence.
- C example:

```
typedef struct Comp { double x, y;}  Complex;
struct COMP { double x,y; };

struct Comp a;
Complex b;
struct COMP c;

/* ... */
a=b;    /* Valid, equal types */
a=c;    /* Compile error, incompatible types */
```

Structural Equality

$S \equiv T$ if and only if:

1. S and T are primitive types and $S = T$ (same type),
2. if $S = A \times B$, $T = A' \times B'$, $A \equiv A'$, and $B \equiv B'$,
3. if $S = A + B$, $T = A' + B'$, and ($A \equiv A'$ and $B \equiv B'$) or ($A \equiv B'$ and $B \equiv A'$),
4. if $S = A \mapsto B$, $T = A' \mapsto B'$, $A \equiv A'$ and $B \equiv B'$,
5. if $S = \mathcal{P}(A)$, $T = \mathcal{P}(A')$, and $A \equiv A'$.

Otherwise $S \not\equiv T$

- Harder to implement structural equality. Especially recursive cases.
- $T = \{nil\} + A \times T \ , \quad T' = \{nil\} + A \times T'$
  $T = \{nil\} + A \times T' \ , \quad T' = \{nil\} + A \times T$
- `struct Circle { double x,y,a;};`
  `struct Square { double x,y,a;};`
  Two types have a semantical difference. User errors may need less tolerance in such cases.
- Automated type conversion is a different concept. Does not necessarily conflicts with name equivalence.

```
enum Day {Mon, Tue, Wed, Thu, Fri, Sat, Sun} x;
x=3;
```

Type Completeness

- First order values:
  - Assignment
  - Function parameter
  - Take part in compositions
  - Return value from a function
- Most imperative languages (Pascal, Fortran) classify functions as second order value. (C represents function names as pointers)
- Functions are first order values in most functional languages like Haskell and Scheme .
- Arrays, structures (records)?
- Type completeness principle: First order values should take part in all operations above, no arbitrary restrictions should exist.

C Types:

	Primitive	Array	Struct	Func.
Assignment	✓	×	✓	×
Function parameter	✓	×	✓	×
Function return	✓	×	✓	×
In compositions	✓	✓	✓	×

Haskell Types:

	Primitive	Array	Struct	Func.
Variable definition	✓	✓	✓	✓
Function parameter	✓	✓	✓	✓
Function return	✓	✓	✓	✓
In compositions	✓	✓	✓	✓

Pascal Types:

	Primitive	Array	Struct.	Func.
Assignment	✓	✓	✓	×
Function parameter	✓	✓	✓	×
Function return	✓	×	×	×
In compositions	✓	✓	✓	×

Expressions

Program segments that gives a value when evaluated:

- Literals
- Variable and constant access
- Aggregates
- Variable references
- Function calls
- Conditional expressions
- Iterative expressions (Haskell)

Literals/Variable and Constant Access

- Literals: Constants with same value with their notation
  `123, 0755, 0xa12, 12451233L, -123.342,`
  ` -1.23342e-2, 'c', '\021', "ayse", True, False`
- Variable and constant access: User defined constants and variables give their content when evaluated.

```
int x;
#define pi 3.1416
x=pi*r*r
```

Aggregates

- Used to construct composite values without any declaration/definition. Haskell:

```
x=(12,"ali",True)                {-- 3 Tuple --}
y={name="ali", no=12}            {-- record   --}
f=\x -> x*x                      {-- function --}
l=[1,2,3,4]                      {-- recursive type, list --}
```

- Python:

```
x = (12, "ali", True)
y = [ 1, 2, [2, 3], "a"]
z = { 'name':'ali', 'no':'12'}
f = lambda x:x+1
```

- Ansi C has aggregates
  only at the definition. There is no aggregates in the statements!

```
struct Person { char name[20], int no };
struct Person p = {"Ali␣Cin", 332314};
double arr[3][2] = {{0,1}, {1.2,4}, {12, 1.4}};
p={"Veli␣Cin",123412}; × /* not possible in ANSI C!*/
```

- C99 Compound literals allow array and structure aggragates

```
int (*arr)[2];
arr = {{0, 1}, {1.2,4}, {12, 1.4}}; √
p = (struct person) {"Veli␣Cin",123412}; √ /* C99 */
```

- C++11 has function aggregetes (lambda)

```
void sort(int a[], int n, (*f)(int,int)) {
        ...
}
auto f = [](int a) { return a+1;} ;
...
sort(arr, n, [](int a, int b) { return a-b;});
n = f(n)
```

Variable References

- Variable access vs variable reference
- value vs l-value
- pointers are not references! You can use pointers as references with special operators.
- Some languages regard references like first order values (Java, C++ partially)
- Some languages distinguish the reference from the content of the variable (Unix shells, ML)

Function Calls

- $F(Gp_1, Gp_2, ..., Gp_n)$
- Function name followed by actual parameter list. Function is called, executed and the returned value is substituted in the expression position.
- Actual parameters: parameters send in the call
- Formal parameters: parameter names used in function definition
- Operators can be considered as function calls. The difference is the infix notation.
- $\oplus(a, b)$ vs $a \oplus b$
- languages has built-in mechanisms for operators. Some languages allow user defined operators (operator overloading): C++, Haskell.

Conditional Expressions

- Evaluate to different values based on a condition.
- Haskell: `if` *condition* `then` *exp1* `else` *exp2* .
  `case` *value* `of` *p1* `->` *exp1* `;` *p2* `->` *exp2* ...
- C: `(`*condition*`)?`*exp1*`:`*exp2* `;`

```
x = (a>b)?a:b;
y = ((a>b)?sin:cos)(x);      /* Does it work? try yourself... *
```

- Python: *exp1* `if` *condition* `else` *exp2*
- `if .. else` in C is not conditional expression but conditional statement. No value when evaluated!

- Haskell:

```
x = if (a>b) then a else b
y = (if (a>b) then (+) else (*)) x y
data Day = Mon | Tue | Wed | Thu | Fri | Sat | Sun
convert a = case a of
          Left (x,rest) -> x : (convert rest)
          Empty -> []
daynumber g = case g of
          Mon -> 1
          Tue -> 2
          ...
          Sun -> 7
```

- case checks for a pattern and evaluate the RHS expression with substituting variables according to pattern at LHS.

Iterative Expressions

- Expressions that do a group of operations on elements of a list or data structure, and returns a value.
- `[ expr | variable <- list , condition]`
- Similar to set notation in math: $\{expr | var \in list, condition\}$
- Haskell:

```
x = [1,2,3,4,5,6,7,8,9,10,11,12]
y = [ a*2 | a <- x ]                    {-- [2,4,6,8,...24 ] --}
z = [ a | a <- x, mod a 3 == 1 ]        {-- [1,4,7,10] --}
```

- Python:

```
x = [1,2,3,4,5,6,7,8,9,10,11,12]
y = [ a*2  for a in x ]                  #  [2,4,6,8,...24 ]
z = [ a for a in x if a % 3 == 1 ]       #  [1,4,7,10]
z = { a:a*a for a in x}                  #  {1:1,4:16,7:49,10:100}
```

Block Expressions

- Some languages allow multiple/statements in a block to calculate a value.
- GCC extension for compound statement expressions:

```
double s, i, arr[10];
s = ( { double t = 0;
        for (i = 0; i < 10; i++)
            t += arr[i];
        t;}) + 1;
```

Value of the last expression is the value of the block.

- ML has similar block expression syntax.
- This allows arbitrary computation for evaluation of the expression.

Summary

- Value and type
- Primitive types
- Composite types
- Recursive types
- When to type check
- How to type check
- Expressions