Programming Language Concepts

Higher Order Functions

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Outline

Lambda Calculus

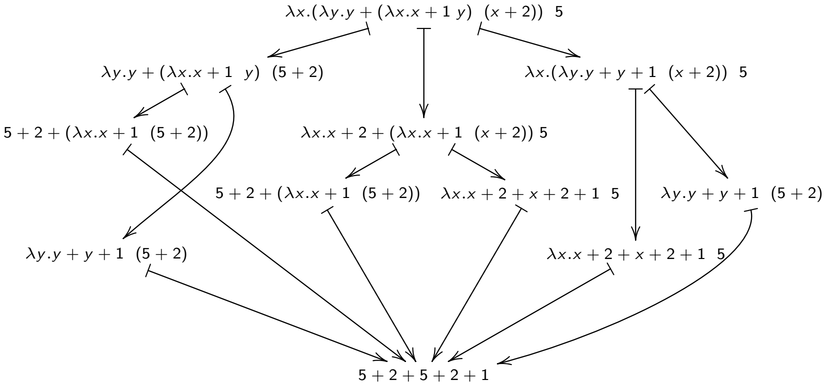
- 1930's by Alonso Church and Stephen Cole Kleene
- Mathematical foundation for computatibility and recursion
- Simplest functional paradigm language
- $\lambda var.expr$
  defines an anonymous function. Also called lambda abstraction
- *expr* can be any expression with other lambda abstractions and applications. Applications are one at a time.
- $(\lambda x.\lambda y.x + y)$ 3 4

- In '$\lambda var.expr$ ' all free occurences of *var* is bound by the λvar.
- Free variables of expression $FV(expr)$
  - $FV(name) = \{name\}$ if *name* is a variable
  - $FV(\lambda name.expr) = FV(expr) - \{name\}$
  - $FV(M\ N) = FV(M) \cup FV(N)$
- α conversion: expressions with all bound names changed to another name are equivalent:
  $\lambda f.f\ x \equiv_\alpha \lambda y.y\ x \equiv_\alpha \lambda z.z\ x$
  $\lambda x.x + (\lambda x.x + y) \equiv_\alpha \lambda t.t + (\lambda x.x + y) \equiv_\alpha \lambda t.t + (\lambda u.u + y)$
  $\lambda x.x + (\lambda x.x + y) \not\equiv_\alpha \lambda x.x + (\lambda x.x + t)$

β Reduction

- Basic computation step, function application in λ-calculus
- Based on substitution. All bound occurences of λ variable parameter is substituted by the actual parameter
- $(\lambda x.M)N \mapsto_\beta M[x/N]$ (all x's once bound by lambda are substituted with N).
- $(\lambda x.(\lambda y.y + (\lambda x.x + 1)\ y)(x + 2))\ 5$
- If no further β reduction is possible, it is called a normal form.
- There can be different reduction strategies but should reduce to same normal form. (Church Rosser property)

All possible reductions of a λ-expression. All reduce to the same normal form.



Introduction

- Mathematics:
  $(f \circ g)(x) = f(g(x))$, $(g \circ f)(x) = g(f(x))$
- "$\circ$" : Gets two unary functions and composes a new function.
  A function getting two functions and returning a new function.
- in Haskell:

```
f x = x+x
g x = x*x
compose func1 func2 x = func1 (func2 x)
t = compose f g
u = compose g f
```

- `t 3 = (3*3)+(3*3) = 18`
  `u 3 = (3+3)*(3+3) = 36`
- `compose`: $(\beta \to \gamma) \to (\alpha \to \beta) \to \alpha \to \gamma$

- "`compose`" function is a function getting two functions as parameters and returning a new function.
- Functions getting one or more functions as parameters are called Higher Order Functions.
- Many operations on functional languages are repetition of a basic task on data structures.
- Functions are first order values $\rightarrow$ new general purpose functions that uses other functions are possible.

Functions/Curry

- Cartesian form vs curried form:
  $\alpha \times \beta \rightarrow \gamma$ vs $\alpha \rightarrow \beta \rightarrow \gamma$
- Curry function gets a
  binary function in cartesian form and converts it to curried form.

```
curry f x y = f(x,y)
add (x,y) = x+y
increment = curry add 1
---
increment 5
6
```

- curry: $(\alpha \times \beta \rightarrow \gamma) \rightarrow \alpha \rightarrow \beta \rightarrow \gamma$
- Haskell library includes it as `curry`.

Functions/Map

```
square x = x*x
day no = case no of 1 -> "mon" ; 2 -> "tue" ; 3 -> "wed";
        4 -> "thu" ; 5 -> "fri" ; 6 -> "sat" ; 7 -> "sun"
map func [] = []
map func (el:rest) = (func el):(map func rest)
----
map square [1,3,4,6]
[1,9,6,36]
map day [1,3,4,6]
["mon","wed","thu","sat"]
```

- `map`:$(\alpha \to \beta) \to [\alpha] \to [\beta]$
- Gets a function and a list. Applies the function to all elements and returns a new list of results.
- Haskell library includes it as `map`.

Functions/Filter

```
iseven x = if mod x 2 == 0 then True else False
isgreater x = x>5

filter func [] = []
filter func (el:rest) = if func el then
                            el:(filter func rest)
                        else (filter func rest)
----
filter iseven [1,2,3,4,5,6,7]
[2,4,6]
filter isgreater [1,2,3,4,5,6,7]
[6,7]
```

- `filter`:$(\alpha \to \mathit{Bool}) \to [\alpha] \to [\alpha]$
- Gets a boolean function and a list. Returns a list with only members evaluated to `True` by the boolean function.
- Haskell library includes it as `filter`.

Functions/Reduce (Fold Right)

```
sum x y = x+y
product x y = x*y

reduce func s [] = s
reduce func s (el:rest) = func el (reduce func s rest)
----
reduce sum 0 [1,2,3,4]
10                               // 1+2+3+4+0
reduce product 1 [1,2,3,4]
24                               // 1*2*3*4*1
```

- `reduce`:$(\alpha \to \beta \to \beta) \to \beta \to [\alpha] \to \beta$
- Gets a binary function, a list and a seed element. Applies function to all elements right to left with a single value.
  $reduce\ f\ s\ [a_1, a_2, ..., a_n] = f\ a_1\ (f\ a_2\ (....\ (f\ a_n\ s)))$
- Haskell library includes it as `foldr`.

- Sum of a numbers in a list:
  `listsum = reduce sum 0`
- Product of a numbers in a list:
  `listproduct = reduce product 1`
- Sum of squares of a list:
  `squaresum x = reduce sum 0 (map square x)`

Functions/Fold Left

```
subtract x y = x - y
foldl func s [] = s
foldl func s (el:rest) =
          foldl func (func s el) rest
----
reduce subtract 0 [1,2,3,4]
-2                               // 1-(2-(3-(4-0)))
foldl subtract 0 [1,2,3,4]
-10                              // ((((0-1)-2)-3)-4)
```

- `foldl`:$(\alpha \rightarrow \beta \rightarrow \alpha) \rightarrow \alpha \rightarrow [\beta] \rightarrow \alpha$
- Reduce operation, left associative.:
  $reduce\ f\ s\ [a_1, a_2, ..., a_n] = f\ (f\ (f\ ...(f\ s\ a_1)\ a_2\ ...))\ a_n$
- Haskell library includes it as `foldl`.

Functions/Iterate

```
twice x = 2*x

iterate func s 0 = s
iterate func s n = func (iterate func s (n-1))
----
iterate twice 1 4
16                          // twice (twice ( twice (twice 1))
iterate square 3 3
6561                        // square (square (square 3))
```

- `iterate`:$(\alpha \to \alpha) \to \alpha \to int \to \alpha$
- Applies same function for given number of times, starting with the initial seed value. $iterate\ f\ s\ n = f^n\ s = \underbrace{f\ (f\ (f\ ...(f\ s))}_{n}$

Functions/Value Iteration (for)

```
for func s m n =
      if m>n then s
      else for func (func s m) (m+1) n

----
for sum 0 1 4
10        // sum (sum (sum (sum 0 1) 2) 3) 4
for product 1 1 4
24        // product (product (product (product 1 1) 2) 3) 4
```

- `for`: $(\alpha \rightarrow int \rightarrow \alpha) \rightarrow \alpha \rightarrow int \rightarrow int \rightarrow \alpha$
- Applies a binary integer function to a range of integers in order.
  $for\ f\ s\ m\ n = f(f\ (f\ (f\ (f\ s\ m)\ (m+1))\ (m+2))\ ...)\ n$

- multiply (with summation):
  `multiply x = iterate (sum x) x`
- integer power operation (Haskell '^'):
  `power x = iterate (product x) x`
- sum of values in range 1 to n:
  `seriessum = for sum 0 1`
- Factorial operation:
  `factorial = for product 1 1`

Higher Order Functions in C

C allows similar definitions based on function pointers. Example: bsearch() and qsort() funtions in C library.

```
typedef struct Person { char name[30]; int no;} person;
int cmpnmbs(void *a, void *b) {
    person *ka=(person *)a; person *kb=(person *)b;
    return ka->no - kb->no;
}
int cmpnames(void *a, void *b) {
    person *ka=(person *)a; person *kb=(person *)b;
    return strncmp(ka->name,kb->name,30);
}
int main() {        int i;
    person list[]={{"veli",4},{"ali",12},{"ayse",8},
                   {"osman",6},{"fatma",1},{"mehmet",3}};
    qsort(list,6,sizeof(person),cmpnmbs);
    ...
    qsort(list,6,sizeof(person),cmpnames);
    ...
}
```

Fibonacci

Fibonacci series: 1 1 2 3 5 8 13 21 ..

$fib(0) = 1\ ;\ fib(1) = 1\ ;\ fib(n) = fib(n-1) + fib(n-2)$

```
fib n = let f (x,y) = (y,x+y)
            (a,b) = iterate f (0,1) n
        in b
----
fib 5       // f(f(f(f(0,1))))
8           //(0,1)->(1,1)->(1,2)->(2,3)->(3,5)->(5,8)
```

Sorting

Quicksort:

1. First element of the list is `x` and rest is `xs`
2. select smaller elements of `xs` from `x`, sort them and put before `x`.
3. select greater elements of `xs` from `x`, sort them and put after `x`.

```
notfunc f x y = not (f x y)

sort _ [] = []
sort func (x:xs) = (sort func (filter (func x) xs)) ++
                   (x: (sort func (filter ((notfunc func) x) xs)))
---
sort (>) [5,3,7,8,9,3,2,6,1]
[1,2,3,3,5,6,7,8,9]
sort (<) [5,3,7,8,9,3,2,6,1]
[9,8,7,6,5,3,3,2,1]
```

List Reverse

- Taking the reverse
    - First element is `x` rest is `xs`
    - Reverse the `xs`, append `x` at the end

  Loose time for appending `x` at the end at each step (N times append of size N).
- Fast version, extra parameter (initially empty list) added:
    - Take the first element, insert at the beginning of the extra parameter.
    - Recurse rest of the list with the new extra parameter.
    - When recursion at the deepest, return the extra parameter.

  Inserts to the beginning of the list at each step. Faster (N times insertion)

```
reverse1 [] = []
reverse1 (x:xs) = (reverse1 xs) ++ [x]

reverse2 x = reverse2' x []  where
        reverse2' [] x = x
        reverse2' (x:xs) y = reverse2' xs (x:y)
---
reverse1 [1..10000]          // slow
reverse2 [1..10000]          // fast
```