Author	Contact	Date
Riccardo Cecchini	rcecchini.ds[at]gmail.com	25 December 2025

Prime-Compound Phase-Lane Token Protocol (PCPL) for Symmetric Continuous Tokenizer Devices

Continuous symmetric encryption starting from asymmetric keys

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Abstract

I present the Prime-Compound Phase-Lane Token Protocol (PCPL), a no-handshake token system where a device emits one token per cycle and exactly one provider can validate it. PCPL combines (1) a public phase clock derived from coprime residues, (2) hidden prime-compound bouquets per provider, and (3) device-only state evolution that chains all lanes. I also introduce the symmetric continuous tokenizer device model, motivated by FPGA-based dynamic hash circuits and twin circuits for peer validation. A step-by-step algorithm description, correctness properties, and a deterministic simulation trace are provided. 1316

1. Symmetric continuous tokenizer devices

PCPL runs on a “symmetric continuous tokenizer” device designed for consumer computing. The device is envisioned as a reconfigurable hardware unit (for example, an FPGA-based key) that can:

- Acquire unique, device-specific hashing circuits or internal start variables. 15
- Continuously generate short-lived tokens or keys.
- Be validated only by its twin circuit(s), which share the same circuit family or seed lineage.

The symmetry comes from pairing: two devices can load the same dynamic hash circuit and evolve internal state in the same way, enabling mutual validation without exposing the evolving secrets.

1.1 Forks by variable alternation

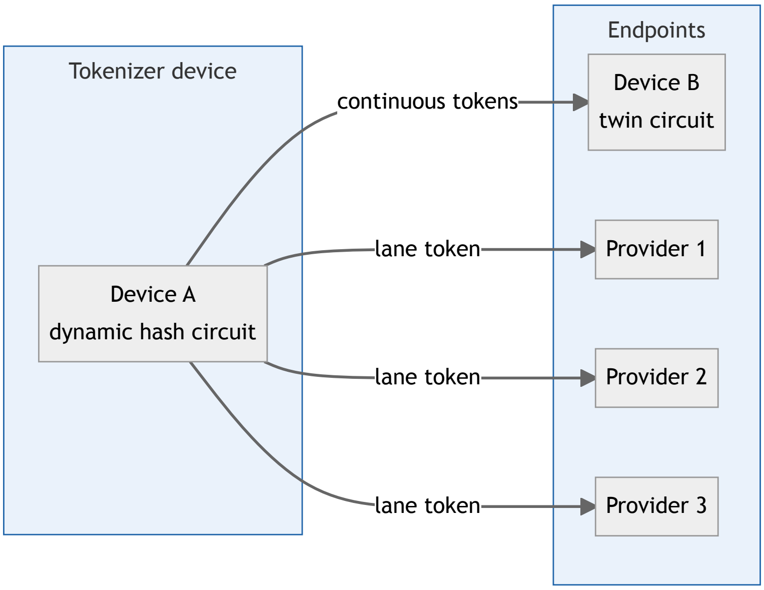
Beyond PCPL, the same circuit can be “forked” by alternating variable sets over time windows. Let a device maintain a base circuit C and a family of variables V_k selected by time window W_k. Each fork evolves as:

$$S_{t+1}^{(k)} = H\left(C,\, S_t^{(k)},\, V_k,\, t\right),$$
$$T^{(k)}(t) = \mathrm{Trunc}_k\left(H\left(K^{(k)}(t)\|\mathrm{enc\_t}(t)\|\Phi_t\|\mathrm{TOK}\right)\right), \quad t \in W_k.$$

This creates multiple parallel token streams sharing the same circuit but with distinct, time-delimited variable schedules. Such forks can be used for provider lanes (as in PCPL) or for isolated peer-to-peer sessions that are difficult to parallelize or replay.

1.2 Peer-to-peer continuity

The device model also targets in loco connections among peers. Two devices that share a circuit family and seed lineage can establish an isolated encryption context by evolving state in lockstep without querying a central provider.



2. System model and goals

PCPL is designed for:

- No runtime challenge/response or synchronization negotiation.
- One token per cycle, routed to exactly one provider out of x.
- Provider-side validation by local recomputation.

Threat model (minimal):

- A provider should not compute tokens for other providers.
- Observing accepted tokens should not reveal other lanes.
- Public time/phase information should not enable cross-lane forgery.

The “primes’ compounds” approach as differentiate hashing algorithm should be considered the simplest one. Even with certain vulnerabilities depending on chosen parameters, it’s good for working with integer-only circuits.

3. Notation and public parameters

Let:

- x be the number of providers (lanes).
- P, Q, R be pairwise coprime primes (also coprime with x).
- M be a prime modulus for multiplicative-group arithmetic.
- $H(\cdot)$ be a cryptographic hash (or a dynamic hash circuit). 13
- $\mathrm{Trunc}_k(\cdot)$ be truncation to k bits.
- t be the cycle counter.
- $||$ denote byte/bit-string concatenation.

Each provider i has three secret bouquets: $\mathrm{BouquetA}_i, \mathrm{BouquetB}_i, \mathrm{BouquetC}_i$, each a list of prime compounds.

3.0 Symbols and domain tags

To avoid accidental cross-use of hashes (“domain confusion”), **every hash that serves a distinct role appends a distinct domain tag**. 9

Glossary:

- **CRT clock:** the public schedule formed by the three residues mod P, Q, R. 20
- **Lane / provider:** one of x independent validators that each own distinct secrets.
- **Bouquet:** a per-lane list of modular bases (typically composite "prime compounds") used in the modular product.
- **QFT:** quantum Fourier transform (period finding 18) — optional analysis tool that can reveal *public* periods. 18

Domain tags (constants) used in this paper:

- `SEED` — derive the initial evolving state S_0
- `W` — derive per-lane memory words $W_i^{(0)}$
- `PRIME` — derive candidate primes for P, Q, R (and optionally M)
- `A0`, `B0`, `C0` — derive **public** phase offsets a_0, b_0, c_0
- `PERMKEY` — derive the device-only permutation key `perm_key`
- `PERMSEED` — derive the per-block shuffle seed used by π_B
- `PHASE` — domain tag for the phase digest Φ_t
- `EXP` — domain tag for bouquet exponent derivation e_j
- `KDF` — domain tag for per-lane key material $K_i(t)$ 10
- `TOK` — domain tag for the final emitted token $T_i(t)$
- `EVOLVE` — domain tag for state evolution S_{t+1}

3.1 Seed construction and coprime extraction

The device bootstraps a root seed Z from device-local entropy and context (for example: device secret, serial, provider list, and a boot nonce). In the demo, Z is produced by a deterministic RNG seeded with `--seed`, then bound to labels with $H(\cdot)$:

- $\text{perm\_key} = H(Z \| \text{PERMKEY})$
- $S_0 = H(Z \| \text{SEED})$
- $W_i = \text{Trunc}_k(H(Z \| \text{W} \| i))$

To extrapolate coprimes for P, Q, R (and optionally M), derive candidates from a seeded stream and select the first primes that are distinct and coprime with x:

1. $c_k \leftarrow \text{next\_prime}(H(Z \| \text{PRIME} \| k) \bmod 2^b)$
2. accept c_k if $\gcd(c_k, x) = 1$ and $c_k \notin \{P, Q, R, M\}$
3. continue until P, Q, R (and M if generated) are assigned

3.1.1 Public phase offsets and public parameter publication

The phase clock in §5.1 uses **public offsets** $a_0 \in [0, P)$, $b_0 \in [0, Q)$, $c_0 \in [0, R)$. They are **not** lane secrets: every validator must be able to recompute (a_t, b_t, c_t) from the same public schedule.

A simple deterministic derivation (used in the demo) is:

- $a_0 = H(Z \| \text{A0}) \bmod P$
- $b_0 = H(Z \| \text{B0}) \bmod Q$
- $c_0 = H(Z \| \text{C0}) \bmod R$

Even if derived from the device root seed Z, these offsets are treated as **published configuration**, together with x, P, Q, R (and M if used). Equivalently, you can derive them from a separate *public* setup seed Z_{pub}.

3.1.2 Provisioning contract (who knows what)

PCPL is "no-handshake" at runtime, but it still needs a provisioning step (manufacture, enrollment, or out-of-band setup). The clean separation is:

- **Public configuration (shared with everyone):** $x, P, Q, R, M, a_0, b_0, c_0$, the permutation algorithm description, and the canonical byte encoding rules (§5.0).
- **Device-only secrets:** Z, `perm_key`, S_t, the full bouquet set for all providers, and the lane-memory vector $W[0..x-1]$.

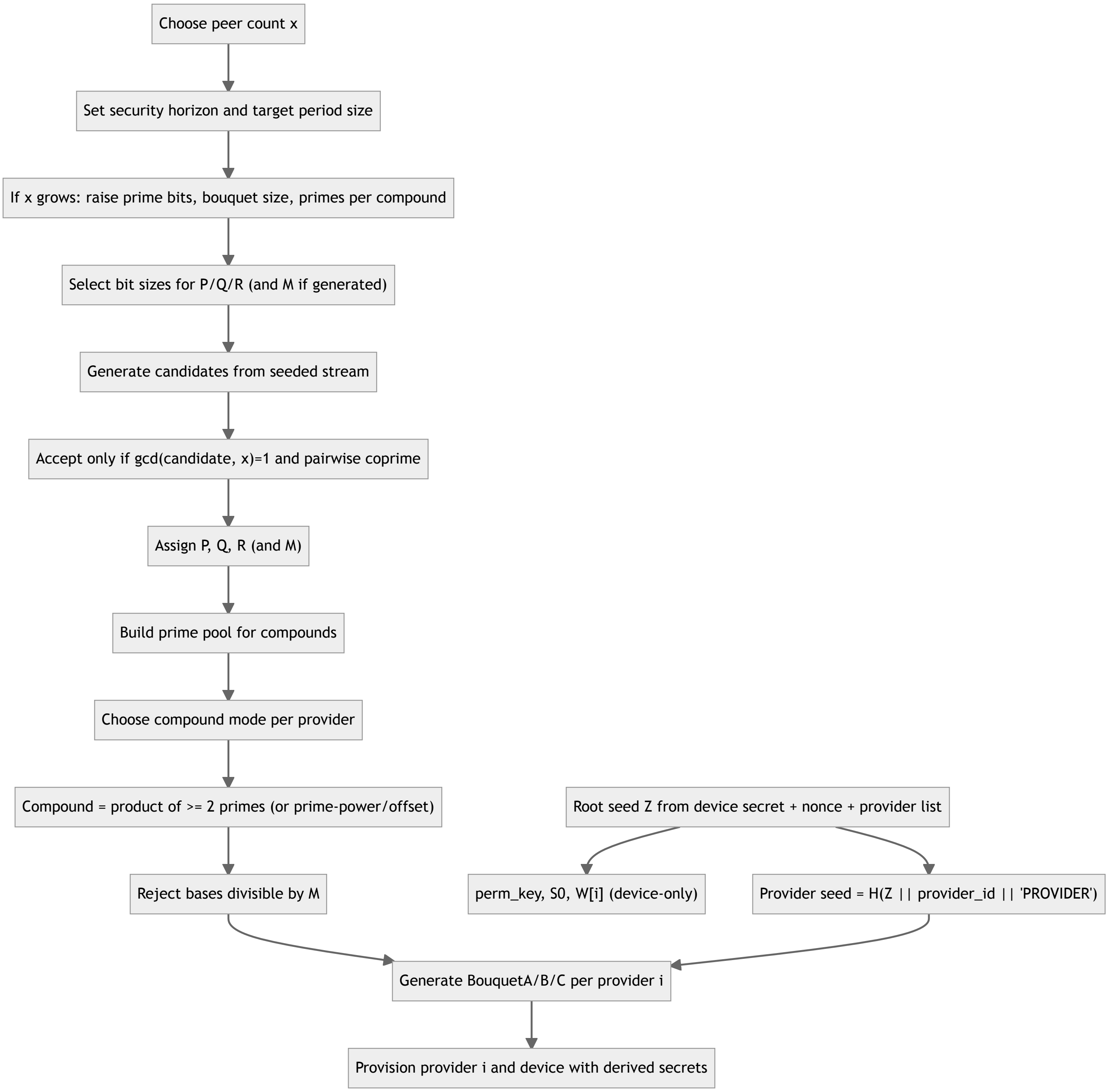
- **Provider-i secrets:** $(\mathrm{BouquetA}_i, \mathrm{BouquetB}_i, \mathrm{BouquetC}_i)$ and its stable identifier i (or `provider_id`).

The runtime cycle counter t must be common to device and providers. In practice, either:

1. t is derived from a shared epoch and fixed cycle duration, or
2. the device includes t alongside the emitted token (recommended for robustness).

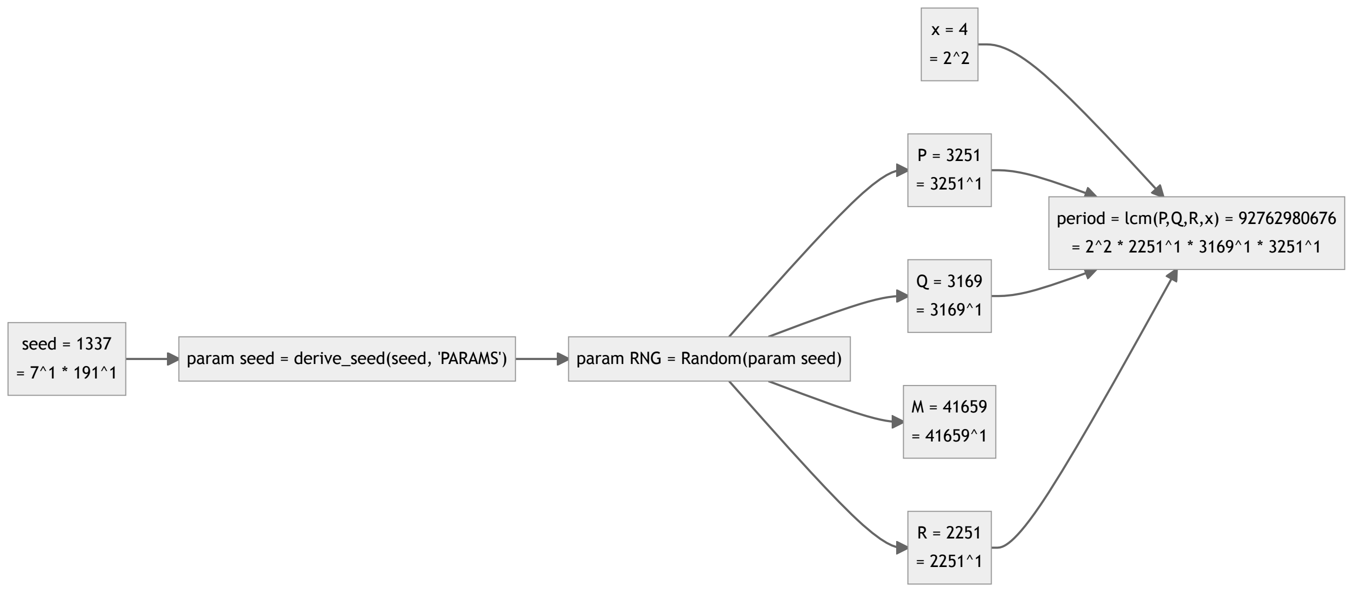
3.2 Best-practice coprimes, compounds, and key selection

Parameter and key selection should scale with the peer count and keep strict domain separation between device-only and provider-only secrets.

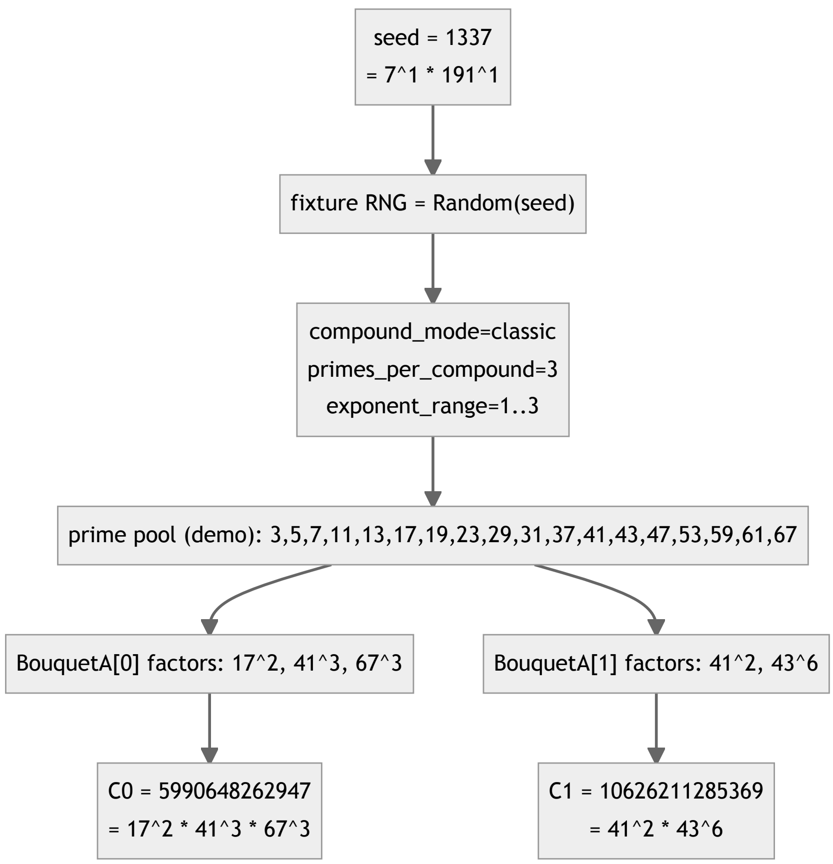


3.2.1 Seeded example flows (real values)

The demo can be run with small bit sizes so prime-factor detail fits on the page. The examples below use `prime_mode=generated`, `prime_bits=12`, `modulus_bits=16`, `compound_mode=classic`, `compound_primes=3`, `compound_count=4`, and the built-in prime pool; the compound example uses provider 0's BouquetA[0...1]. Each node shows the integer value and its prime-power factorization.



Important: the schedule/modulus primes (**P, Q, R** and **M**) are chosen for the public clock period and modular arithmetic. They are **not** meant to appear as factors inside bouquet compounds. Bouquets are built from a separate prime pool (small in the demo, larger in production) because the protocol only uses each compound as a *base modulo* **M** in `pow(compound % M, exponent, M)`. The only required constraint is `compound % M != 0` (equivalently `gcd(compound, M)=1`).



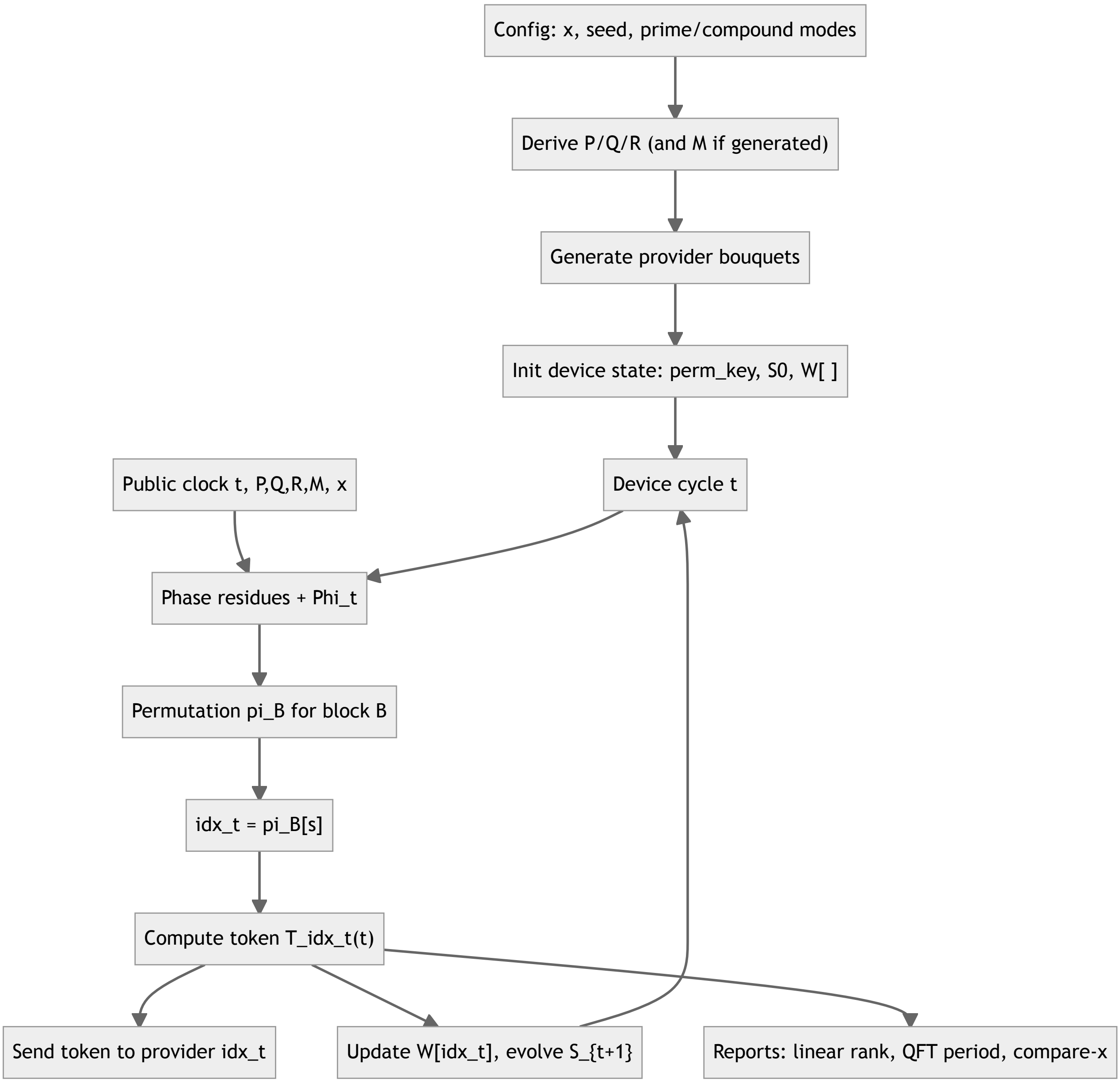
4. PCPL protocol overview

The protocol uses:

1. A public phase clock (CRT residues and coupled products).
2. A per-block permutation schedule to enforce “returns every x”.
3. Hidden bouquets to derive lane-specific tokens.
4. Device-only seed evolution that chains all lanes.

4.1 User device circuit (emitter)

The device knows the full schedule and all lane secrets, so it computes only the active lane per cycle and emits exactly one token.



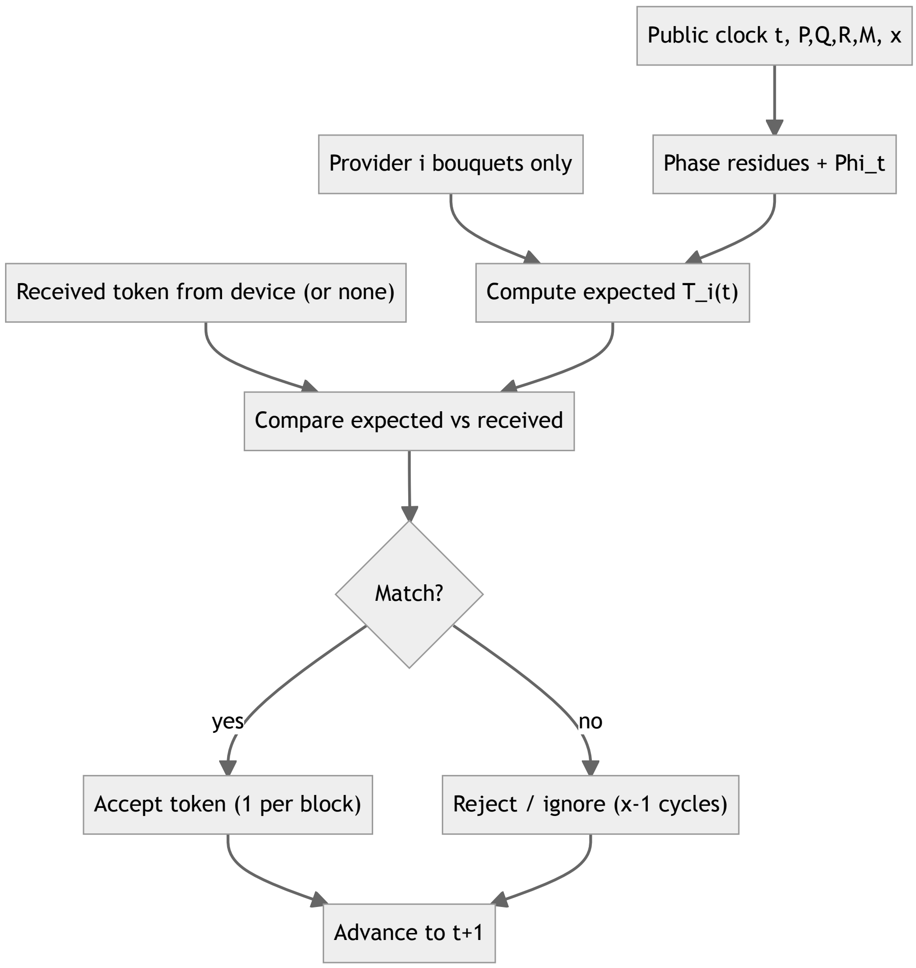
4.2 Blind provider circuit (validator)

Each provider only knows its own bouquets. It recomputes $T_i(t)$ every cycle, but the received token matches only once per block of x cycles. The other $x - 1$ cycles are expected mismatches because the device emitted a different lane.

Why only 1-of-x is correct: the provider computes the *same* lane token formula as the device, but with a fixed lane index i. The device emits $T_{\mathrm{idx}_t}(t)$ where $\mathrm{idx}_t = \pi_B[s]$ is hidden by $\mathrm{perm\_key}$. Therefore the provider is correct iff $i = \mathrm{idx}_t$.
Because π_B is a permutation, this happens exactly once per block of x cycles. The device is "always right" because it emits the scheduled lane token; the provider is "blind" because it does not know $\mathrm{perm\_key}$ and cannot predict which cycle is its match.

Provider-side token generation (every cycle):

$$\begin{aligned} EA_i(t) &= \mathrm{Eval}(\mathrm{BouquetA}_i, a_t, u_1), \\ EB_i(t) &= \mathrm{Eval}(\mathrm{BouquetB}_i, b_t, u_2), \\ EC_i(t) &= \mathrm{Eval}(\mathrm{BouquetC}_i, c_t, u_3), \\ K_i(t) &= H\big(\mathrm{enc\_i}(i) \| \mathrm{encM}(EA_i(t)) \| \mathrm{encM}(EB_i(t)) \| \mathrm{encM}(EC_i(t)) \| \Phi_t \| \mathrm{KDF}\big)\,, \\ T_i(t) &= \mathrm{Trunc}_k\big(H\big(K_i(t) \| \mathrm{enc\_t}(t) \| \Phi_t \| \mathrm{TOK}\big)\big)\,. \end{aligned}$$



4.3 Shared vs distinct per-cycle logic

The device and each provider run a synchronized per-cycle hash pipeline. They differ in which lane index is used and whether device-only state is updated.

- **Shared per-cycle hash pipeline (device + provider):** for a given lane index i and cycle t, compute Φ_t, then $EA_i(t), EB_i(t), EC_i(t)$, then $K_i(t)$ and $T_i(t)$ using the canonical encoding and domain tags. This runs every cycle.
- **Device-only additions:** compute idx_t from `perm_key`, evaluate only that lane, emit $T_{\mathrm{idx}_t}(t)$, update $W[\mathrm{idx}_t]$, and evolve S_{t+1} from all W and chain products. This makes every emitted token influence future cycles.
- **Provider-only behavior:** for its fixed lane i, compute $T_i(t)$ every cycle and compare against any received token. Exactly 1-of-x cycles match because the device selects each lane once per block. Providers do not know `perm_key` and do not maintain S_t or $W[]$.

5. Step-by-step algorithm

5.0 Canonical encoding and concatenation (unambiguous hash inputs)

The operator `||` / $\|$ in the formulas means **byte-string concatenation**. Implementations **MUST** use a canonical serialization so that different tuples cannot map to the same byte string (classic "`1|23` vs `12|3`" bug).

Use fixed-length big-endian integer encoding (**I2OSP** [8]) with lengths derived from the public moduli:

- $\ell_P = \lceil \log_2(P)/8 \rceil$, ℓ_Q, ℓ_R, ℓ_M similarly
- lane identifier: $\ell_i = 4$ bytes (`enc_i(i) = I2OSP(i,4)`) unless you need a larger ID space
- cycle counter: $\ell_t = 8$ bytes (`enc_t(t) = I2OSP(t,8)`) unless you expect more than 2^{64} cycles
- small indices (bouquet element index j, prime-candidate index k, ...): `encU32(n) = I2OSP(n,4)`

Encoding functions:

- `encP(a) = I2OSP(a, ℓP)` and similarly `encQ`, `encR`, `encM`
- `enc_i(i) = I2OSP(i, ℓi)`, `enc_t(t) = I2OSP(t, ℓt)`

Domain tags (`PHASE`, `EXP`, `KDF`, `TOK`, ...) are fixed byte strings as defined in §3.0 and are appended **as-is**. If you prefer numeric tags, serialize the numeric constant with `I2OSP(tag, 4)` (or any fixed width) — the only requirement is uniqueness.

With this convention, the core digests become:

$$\Phi_t = H(\mathrm{encP}(a_t)\|\mathrm{encQ}(b_t)\|\mathrm{encR}(c_t)\|\mathrm{encM}(u_1)\|\mathrm{encM}(u_2)\|\mathrm{encM}(u_3)\|\mathrm{PHASE})\,.$$

$$e_j = H(\mathrm{encRes}(x_{\mathrm{res}})\|\mathrm{encM}(u)\|\mathrm{encU32}(j)\|\mathrm{EXP}) \bmod (M-1),$$

where `encRes` is the residue encoder for a_t / b_t / c_t (use `encP`, `encQ`, or `encR` depending on which residue is in scope).

$$K_i(t) = H\big(\mathrm{enc\_i}(i)\|\mathrm{encM}(EA_i(t))\|\mathrm{encM}(EB_i(t))\|\mathrm{encM}(EC_i(t))\|\Phi_t\|\mathrm{KDF}\big)\,,$$

$$T_i(t) = \mathrm{Trunc}_k\big(H\big(K_i(t)\|\mathrm{enc\_t}(t)\|\Phi_t\|\mathrm{TOK}\big)\big)\,.$$

`Trunc_k` can mean "take the first k bits" (most common), or "interpret as an integer and reduce mod 2^k". The demo uses byte truncation.

Rule of thumb: never concatenate decimal strings, and never concatenate variable-length integers without either fixed widths or length-prefixes. [9]

Demo note: the Python demo uses BLAKE2b [3] and a typed, length-prefixed encoding for each hash part (tag + 4-byte length) instead of fixed-width I2OSP. Older demo runs omitted `enc_i(i)` in the KDF; the current demo includes it to match the spec, so token traces differ from earlier runs.

5.1 Phase clock

Public offsets a_0, b_0, c_0 are part of the public configuration (§3.1.1).

For cycle t:

$$\begin{aligned} a_t &= (a_0 + t) \bmod P, \\ b_t &= (b_0 + t) \bmod Q, \\ c_t &= (c_0 + t) \bmod R. \end{aligned}$$

Coupled products:

$$\begin{aligned} u_1 &= (a_t\, b_t) \bmod M, \\ u_2 &= (b_t\, c_t) \bmod M, \\ u_3 &= (c_t\, a_t) \bmod M. \end{aligned}$$

Phase digest:

$$\Phi_t = H(\mathrm{encP}(a_t)\|\mathrm{encQ}(b_t)\|\mathrm{encR}(c_t)\|\mathrm{encM}(u_1)\|\mathrm{encM}(u_2)\|\mathrm{encM}(u_3)\|\mathrm{PHASE})\,.$$

5.2 Permutation schedule (“returns every x”)

The **schedule** (which provider is selected at each cycle) must be independent from the hashed token bytes.
It is driven only by the public cycle counter and the device’s private permutation key.

Define a block index and a slot inside that block:

$$B = \left\lfloor \frac{t}{x} \right\rfloor, \qquad s = t \bmod x.$$

For each block B, the device computes a permutation π_B of $\{0, \ldots, x-1\}$ using a deterministic PRNG derived from:

- the device-only `perm_key`
- the block index B
- the public digest $\Phi_{B \cdot x}$ (fixed for the block start)
- the domain tag `PERMSEED`

Then the selected lane for cycle t is:

$$\mathrm{idx}_t = \pi_B[s].$$

Because π_B is a permutation, each lane appears **exactly once** per block of length x (“returns every x”).
Hashing and truncation happen *after* this selection and therefore cannot break the 1-of-x property.

5.2.1 Device-side destination selection

Using only public phase data and its private permutation key, the device computes:

$$\begin{aligned} B &= \left\lfloor \frac{t}{x} \right\rfloor, \quad s = t \bmod x, \\ \pi_B &= \mathrm{PermuteBlock}\left(\mathrm{perm\_key},\ B,\ \Phi_{B \cdot x},\ x\right), \\ \mathrm{idx}_t &= \pi_B[s]. \end{aligned}$$

Providers do **not** know `perm_key`, so they cannot predict idx_t (even though t and Φ_t are public).

5.3 Bouquet evaluation

Each bouquet is a list of compounds C_j, each a modular base (typically a product of primes). For a residue x_{res} and coupling u, define the per-element exponent:

$$e_j = H(\mathrm{encRes}(x_{\mathrm{res}}) \| \mathrm{encM}(u) \| \mathrm{encU32}(j) \| \mathrm{EXP}) \bmod (M-1).$$

Bouquet evaluation is the modular product:

$$\mathrm{Eval}(\mathrm{Bouquet}, x_{\mathrm{res}}, u) = \prod_j C_j^{e_j} \bmod M.$$

For provider i:

$$\begin{aligned} EA_i(t) &= \mathrm{Eval}(\mathrm{BouquetA}_i, a_t, u_1), \\ EB_i(t) &= \mathrm{Eval}(\mathrm{BouquetB}_i, b_t, u_2), \\ EC_i(t) &= \mathrm{Eval}(\mathrm{BouquetC}_i, c_t, u_3). \end{aligned}$$

5.3.1 Prime-compound construction variants

Compounds do not need to be prime: any base coprime with M is valid. Here, “prime compound” means a composite base built from two or more primes (a compound prime). This expands the base space and lets you tune complexity by increasing the number of factors and exponents, while preserving continuity.

The only hard requirement is $\gcd(C, M) = 1$ (no factor of M). This coprimality is **with respect to the modulus** M, not with respect to P, Q, R or x: compounds may share factors with each other, but they must not share factors with M to stay in $\mathbb{F}_M^*$.

- **Multi-prime compounds:** $C = \prod_{i=1}^{r} p_i^{e_i}$ with $r \geq 2$ (the general case).
- **Prime powers:** $C = p^k$ (smooth but non-prime bases).
- **Semiprimes:** $C = pq$ (a 2-prime special case).
- **Offset compounds:** $C = (\prod p_i^{e_i}) + \delta$ with small δ to create a quasi-continuous family.
- **Quantized reals:** map a real parameter ρ to $C = \lfloor \alpha\rho \rfloor$ for fixed scale α, then ensure $\gcd(C, M) = 1$.

The demo exposes these families via compound generation modes while keeping the exponent schedule unchanged; the "blend" mode just mixes these families and does not change the phase periodicity, which is driven solely by P, Q, R and x.

5.4 Token derivation

Key derivation (domain-separated by lane identifier i):

$$K_i(t) = H\left(\text{enc\_i}(i) \| \text{encM}(EA_i(t)) \| \text{encM}(EB_i(t)) \| \text{encM}(EC_i(t)) \| \Phi_t \| \text{KDF}\right).$$

Token:

$$T_i(t) = \text{Trunc}_k\left(H\left(K_i(t) \| \text{enc\_t}(t) \| \Phi_t \| \text{TOK}\right)\right).$$

Implementation notes:

- In code, $K_i(t)$ is the **hash digest bytes** (not an integer).
- When concatenating integers, always use the canonical fixed-length encoding (§5.0).
- Including i inside the KDF provides explicit lane domain-separation even if two providers were accidentally provisioned with identical bouquets.
- If you prefer a standardized PRF/KDF wrapper, use HMAC or HKDF with explicit context labels. 410

5.4.1 Worked example with real integers (toy parameters + SHA-256 1)

This example is **not** meant to be secure (the primes are tiny); it exists only to show the math and key composition end-to-end with concrete numbers.

Parameters:

- $x = 4$
- $P = 11, Q = 13, R = 17$ (pairwise coprime and coprime with x)
- $M = 19$ (prime modulus, so $|\mathbb{F}_M^*| = M - 1 = 18$)
- public offsets: $a_0 = 2, b_0 = 3, c_0 = 5$
- lane: $i = 2$
- cycle: $t = 7$
- encoding: fixed-length big-endian as in §5.0 (here every modulus fits in 1 byte)
- hash: SHA-256, $k = 64$ (token is first 64 bits of the final hash)

Provider $i = 2$ bouquets (each base is coprime with M):

- $\text{BouquetA}_2 = [15,\ 77]$ (compounds: $3 \cdot 5$ and $7 \cdot 11$)
- $\text{BouquetB}_2 = [91,\ 143]$ (compounds: $7 \cdot 13$ and $11 \cdot 13$)
- $\text{BouquetC}_2 = [85,\ 187]$ (compounds: $5 \cdot 17$ and $11 \cdot 17$)

Computed public phase values:

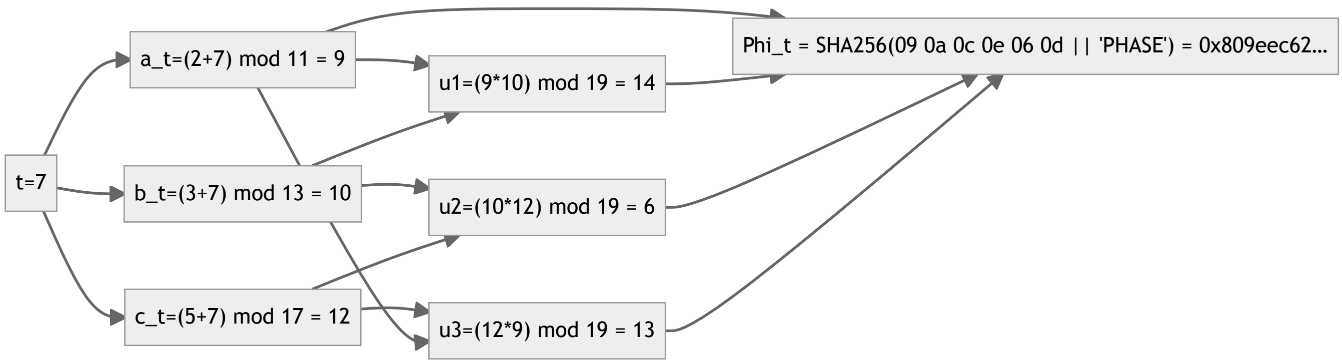
- $a_t = 9, b_t = 10, c_t = 12$
- $u_1 = 14, u_2 = 6, u_3 = 13$
- $\Phi_t = \text{SHA256}(\texttt{09 0a 0c 0e 06 0d} \| \text{PHASE}) = \texttt{0x809eec62}\ldots$

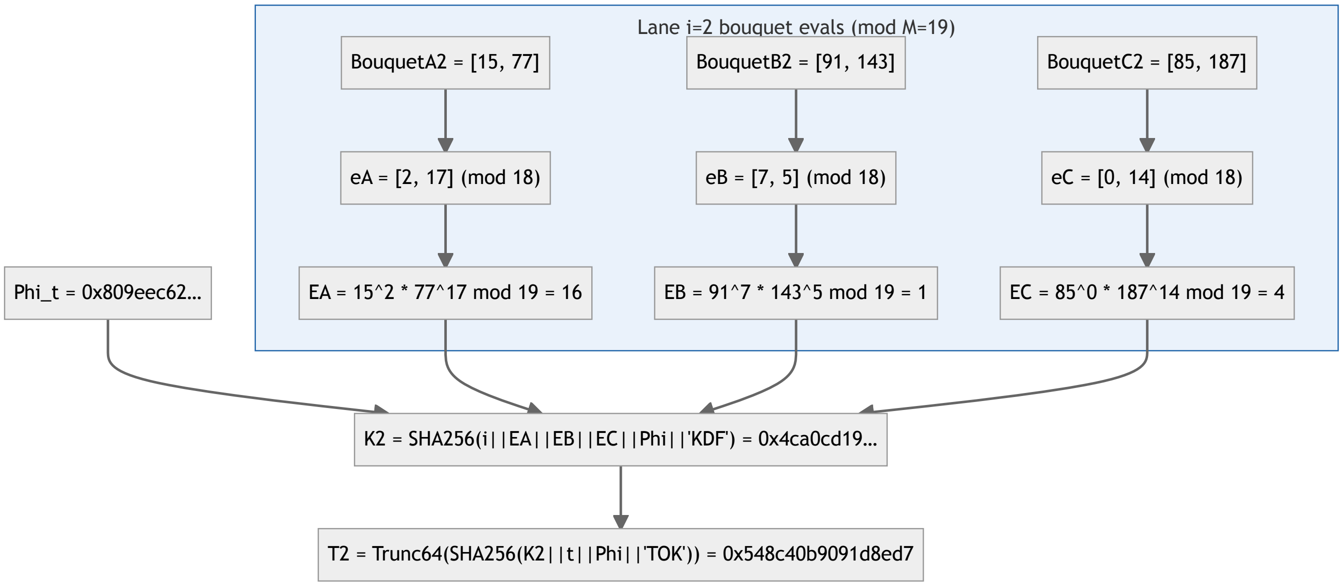
Computed bouquet exponents and evaluations (all exponents reduced mod 18):

- $e^A = [2, 17] \Rightarrow EA_2(t) = 16$
- $e^B = [7, 5] \Rightarrow EB_2(t) = 1$
- $e^C = [0, 14] \Rightarrow EC_2(t) = 4$

Final key and token:

- $K_2(t) = \mathrm{SHA256}(i \| EA \| EB \| EC \| \Phi_t \| \mathrm{KDF}) = \texttt{0x4ca0cd19...}$
- $T_2(t) = \mathrm{Trunc}_{64}(\mathrm{SHA256}(K_2(t) \| t \| \Phi_t \| \mathrm{TOK})) = \texttt{0x548c40b9091d8ed7}$





This is the exact structure the providers use: even though they cannot predict **which** lane the device will emit on a given cycle, they can still recompute their own lane's expected $T_i(t)$ and accept only when it matches.

5.4.2 Why SHA-256 and truncation still respect the permutation rule (and enable async validation)

PCPL has two *separate* mechanisms that people often conflate:

1. **Permutation rule ("returns every x")** — decides **which lane** is active at time t.
2. **Token derivation** — decides **what value** the device emits for that lane at time t.

Truncation affects only (2), never (1).

A. The permutation rule is independent of hashing and truncation

Within a block $B = \lfloor t/x \rfloor$, the device computes a deterministic permutation π_B of the lane indices and selects:

$$\mathrm{idx}_t = \pi_B[t \bmod x].$$

Because π_B is a **permutation**, every lane appears exactly once per block, regardless of how you compute $K_i(t)$ or $T_i(t)$.
So "returns every x" is preserved as long as `Permute(...)` is deterministic.

B. Determinism is what makes re-derivation possible

For any fixed lane i and cycle t, the derivation is a pure function:

$$K_i(t) = H(\mathrm{enc}_i(i) \| \mathrm{encM}(EA_i(t)) \| \mathrm{encM}(EB_i(t)) \| \mathrm{encM}(EC_i(t)) \| \Phi_t \| \mathrm{KDF})$$

$$T_i(t) = \mathrm{Trunc}_k\Big(H\big(K_i(t) \| \mathrm{enc}_t(t) \| \Phi_t \| \mathrm{TOK}\big)\Big).$$

There are no hidden "external variables" beyond:

- public parameters ($P, Q, R, M, x, a_0, b_0, c_0$, and the definition of H and truncation),
- the cycle counter t (sent in the message, or derived from a shared epoch),
- and the lane's provisioned secrets (bouquets).

So a provider can recompute its expected $T_i(t)$ **for any** t, even if it has not seen recent traffic.

C. Truncation preserves correctness (it only trades bandwidth for collision probability)

Both device and provider compute the same 256-bit hash output, then apply the same deterministic truncation rule.
Therefore truncation cannot "break synchronization" — it can only increase the chance that two different inputs collide in k bits.
Choose k large enough for your threat model (e.g., 64 bits is already far beyond typical OTP sizes).

D. Why this is "async": providers don't need the permutation key (but they still run a synchronized circuit)

Providers do **not** need `perm_key` or π_B to validate, because the expected token for lane i at cycle t is a pure function of:

- public data: t (or its epoch mapping) and Φ_t
- provider-i secrets: $\mathrm{BouquetA}_i, \mathrm{BouquetB}_i, \mathrm{BouquetC}_i$
- the agreed hash/KDF rules and domain tags

What "async" means here is: **no extra coordination channel** is required to tell a provider *when* it will be selected.
The provider simply runs the same per-cycle hash pipeline as the device (for its own lane only) and compares if/when a message arrives.

Operationally:

- Every cycle, provider i advances its local counter to the current t (using the same public epoch mapping as the device) and computes $T_i(t)$.
- Most cycles there is no message; the computed token is discarded.
- When a device message arrives claiming (t, i, T), provider i compares T against its locally generated $T_i(t)$ (optionally checking a small $\pm\Delta$ window for clock skew / network jitter).

Because the device contacts each provider **exactly once per block of length** x, a given provider sees a *matching* token only **1 time out of** x **cycles** (and rejects/mismatches the other $x - 1$).

```
%%{init: {"theme":"neutral","flowchart":{"curve":"basis"}} }%%
flowchart LR
  subgraph Device
    t["cycle t"] --> Phi["Φ_t (public)"]
    Phi --> Perm["idx_t = π_B[t mod x] (device-only)"]
    Perm --> Lane["choose lane i = idx_t"]
    Lane --> Ki["K_i(t) = H(i||EA||EB||EC||Φ_t||KDF)"]
    Ki --> Tok["T = Trunc_k(H(K_i(t)||t||Φ_t||TOK))"]
    Tok --> Send["send (t,i,T)"]
  end

  subgraph Provider_i["Provider i (continuous validator)"]
```

```
    Clock["public epoch → local t"] --> Loop["every cycle: compute Φ_t, EA/EB/EC, K_i(t), T_i(t)"]
    Loop --> Buf["buffer T_i(t) (±Δ window)"]
    Rx["receive (t,i,T)"] --> Cmp["constant-time compare"] [17]
    Buf --> Cmp
    Cmp --> Match{"match & unused?"}
    Match -->|yes| Accept["accept (≈1/x cycles)"]
    Match -->|no| Reject["reject / ignore (≈x-1/x cycles)"]
  end

  Send --> Rx
```

5.5 Device emission and state evolution

The device computes only $T_{\text{idx}_t}(t)$ for the scheduled lane and updates internal state:

- $W[i]$ is a per-lane **memory word**. Initialize as $W_i^{(0)}$ (§3.1), then update only when lane i is active (e.g., store `Int(T) mod M` or store raw token bytes — choose one representation and encode it canonically in the hash below).
- The seed S_t evolves using **all lanes** and adjacent products, so "inactive" lanes still influence the future through their last stored $W[i]$.

For x lanes, define (non-cyclic adjacency):

$$m_\ell = (W_\ell \cdot W_{\ell+1}) \bmod M, \quad \ell = 0, \ldots, x-2.$$

Seed evolution:

$$S_{t+1} = H(S_t \| W_0 \| \cdots \| W_{x-1} \| m_0 \| \cdots \| m_{x-2} \| \Phi_t \| \text{EVOLVE}) \,.$$

Implementation note: if $W[i]$ and m_ℓ are stored as integers, serialize them with `encM(·)` (or another fixed-width encoder) before hashing.

5.6 Provider verification (continuous hashing circuit)

A provider is **not** a passive "checker that wakes up at the right permutation time".
To be able to validate in constant time (and to match the intended hardware/circuit model), provider i runs a synchronized per-cycle pipeline that continuously derives its current expected token.

Minimal runtime behavior:

1. **Clock discipline / epoch mapping.** Maintain a local view of the public cycle counter t (e.g., from NTP[13]/GPS time, a block height, or any agreed public epoch-to-t mapping).; conceptually similar to the moving factor in HOTP/TOTP) [11]
2. **Per-cycle update.** For each cycle t, compute Φ_t, then evaluate bouquets and derive: $EA_i(t), EB_i(t), EC_i(t) \to K_i(t) \to T_i(t)$.
3. **Small validation window (optional).** Keep $T_i(t)$ plus a small $\pm\Delta$ window (e.g., previous/next few cycles) to tolerate network delay and small clock skew.
4. **On receive.** When a message arrives with (t, i, T):
    - reject if i is not this provider's identifier
    - reject if t is outside the allowed window
    - constant-time compare T with the locally buffered expected token(s)
    - accept at most once per cycle (track recently accepted (i, t) to prevent trivial replay inside the skew window)

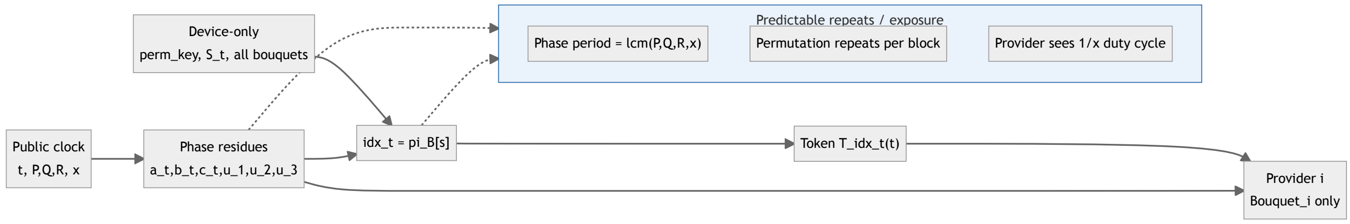
Acceptance frequency: because the device selects each provider exactly once per block of length x, provider i will see a valid match only about **1 time in x cycles**. All other cycles either have no message or produce a mismatch by construction.

5.7 Device-side vs provider-side variables

The protocol deliberately separates what the device computes from what providers can infer:

- **Public inputs:** t (or its epoch mapping), x, P, Q, R, M, the public offsets a_0, b_0, c_0, the permutation algorithm (but not `perm_key`), and the canonical encoding rules.
- **Device-only state:** $\mathrm{perm\_key}$, S_t, all lane secrets, and the lane-memory vector $W[0..x-1]$.
- **Provider i secrets:** $\mathrm{BouquetA}_i, \mathrm{BouquetB}_i, \mathrm{BouquetC}_i$.
- **Ignored by providers:** $\mathrm{perm\_key}$, S_t, other providers' bouquets, and the full W vector.

The device computes only $T_{\mathrm{idx}_t}(t)$ for the current lane; the provider computes only its own lane token and does not need the device seed.



5.8 Reference pseudocode (implementation-oriented)

The following pseudocode matches the specification above (not optimized).

```
# Public parameters (shared):
#   x, P, Q, R, M, a0, b0, c0
# Hash function H(·) and Trunc_k(·) agreed by all parties
# Canonical encoders: encP, encQ, encR, encM, enc_i, enc_t, encU32
# Domain tags: PHASE, EXP, KDF, TOK, EVOLVE, PERMSEED

function Phase(t):
    a = (a0 + t) mod P
    b = (b0 + t) mod Q
    c = (c0 + t) mod R
    u1 = (a * b) mod M
    u2 = (b * c) mod M
    u3 = (c * a) mod M

    Phi = H( encP(a) || encQ(b) || encR(c) ||
             encM(u1) || encM(u2) || encM(u3) ||
             PHASE )
    return (a,b,c,u1,u2,u3,Phi)

function PermuteBlock(perm_key, B, Phi_block, x):
    # Deterministic Fisher–Yates [14] using hash-derived bytes as a PRNG stream. [6][7]
    # Stable for the whole block B.
    seed = H( perm_key || encU32(B) || Phi_block || PERMSEED )
    L = [0,1,2,...,x-1]
    stream = Expand(seed)      # e.g., seed || H(seed||0) || H(seed||1) || ...
    for i from x-1 downto 1:
        r = NextU32(stream)    # pull 32 bits from stream
        j = r mod (i+1)
        swap L[i], L[j]
    return L   # permutation π_B

function EvalBouquet(bouquet, res_encoder, x_res, u, M):
    acc = 1 mod M
    for j from 0 to len(bouquet)-1:
        Cj = bouquet[j] mod M
        ej = H( res_encoder(x_res) || encM(u) || encU32(j) || EXP ) mod (M-1)
        acc = (acc * pow(Cj, ej, M)) mod M
```

```
    return acc

function LaneToken(i, t, bouquets_i):
    (a,b,c,u1,u2,u3,Phi) = Phase(t)
    EA = EvalBouquet(bouquets_i.A, encP, a, u1, M)
    EB = EvalBouquet(bouquets_i.B, encQ, b, u2, M)
    EC = EvalBouquet(bouquets_i.C, encR, c, u3, M)

    K  = H( enc_i(i) || encM(EA) || encM(EB) || encM(EC) || Phi || KDF )
    T  = Trunc_k( H( K || enc_t(t) || Phi || TOK ) )
    return T

# DEVICE (emitter)
state: perm_key (secret), S, W[0..x-1] (secret per-lane memory), bouquets_all
for each cycle t = 0,1,2,...:
    (a,b,c,u1,u2,u3,Phi) = Phase(t)

    B = floor(t / x)
    s = t mod x
    Phi_block = Phase(B*x).Phi
    pi = PermuteBlock(perm_key, B, Phi_block, x)
    idx = pi[s]

    T = LaneToken(idx, t, bouquets_all[idx])
    send (t, idx, T) to provider idx

    W[idx] = Int(T) mod M   # or store raw bytes; if bytes, encode consistently in EVOLVE
    m[ℓ] = (W[ℓ] * W[ℓ+1]) mod M for ℓ = 0..x-2
    S = H( S || W[0] || ... || W[x-1] || m[0] || ... || m[x-2] || Phi || EVOLVE )

# PROVIDER i (validator)
state: bouquets_i (secret), i (public identifier)
on receive (t, i, T_rx):
    T_exp = LaneToken(i, t, bouquets_i)
    accept iff (T_rx == T_exp)
```

Notes:

- `Expand(seed)` is any deterministic method to obtain enough pseudorandom bytes from `seed` (e.g., `H(seed||0)`, `H(seed||1)`, ...). Both device and any party that needs to recompute `π_B` must use the **same** expansion.
- Providers do **not** need `perm_key` and do not need to know `π_B` to validate. In practice they run the per-cycle hash pipeline continuously and compare against the current (or ±Δ window) expected token when contacted.

6. Correctness and periodicity

6.1 Exact 1-of-x matching

Within each block of length x, the device computes a permutation π_B of $\{0, \dots, x-1\}$ and selects:

$$\mathrm{idx}_t = \pi_B[t \bmod x].$$

As the slot $s = t \bmod x$ runs through $0, 1, \dots, x-1$ inside the same block B, the permutation property guarantees that each lane identifier appears **exactly once**.
Therefore:

- in every block of x cycles, the device contacts each provider exactly once
- each provider i will see a *matching* token only on its single scheduled cycle in that block (≈ **1 time out of** x)

Hashing and truncation do not affect this property because they happen after lane selection.

6.2 Phase and schedule periodicity

The public phase clock is defined by three modular counters:

$$a_t = (a_0 + t) \bmod P, \quad b_t = (b_0 + t) \bmod Q, \quad c_t = (c_0 + t) \bmod R.$$

The triple (a_t, b_t, c_t) repeats with period:

$$L = \mathrm{lcm}(P, Q, R).$$

If P, Q, R are pairwise coprime, then $L = PQR$.
The derived values u_1, u_2, u_3 and the phase digest Φ_t are deterministic functions of (a_t, b_t, c_t), so they repeat with the same period L.

The lane-selection schedule adds the block structure of length x.
If the permutation were fixed, combining phase period and block slotting would yield an overall cycle-period of $\mathrm{lcm}(L, x)$.
In PCPL the permutation is *re-derived per block* from $(\mathrm{perm\_key}, B, \Phi_{B\cdot x})$, so practical repetition is pushed out further and is dominated by:

- the phase period L (public)
- the block counter wrap-around implied by the chosen encoding length for B (e.g., 2^{32} blocks if `encU32(B)` is used)

6.3 Modular exponent correctness

The `Eval(·)` step uses modular exponentiation and products modulo M.
To keep these operations well-defined and avoid degenerate values, enforce:

- **Coprime bases:** each base used in a product must be coprime with M (in particular, not divisible by M), otherwise terms collapse (e.g., $C \equiv 0 \pmod{M}$).
- **Group arithmetic:** if M is prime, the multiplicative group $\mathbb{F}_M^*$ has order $M-1$, so exponents can be reduced modulo $M-1$ without changing the result.
  (If M is composite, use a group where the reduction rule is explicit, or keep full-width exponents.)

These checks belong in provisioning and bouquet generation, not at verification time.

6.4 Peer-count variations (x=2,3,4 and composite counts)

Changing x changes the block size, the number of permutations, and the chain width:

x	block length	permutations	chain products	note
2	2	2	1	twin pairing (2 lanes)
3	3	6	2	prime lane count
4	4	24	3	2^2 prime power
6	6	720	5	composite $(2 \cdot 3)$

In general: block length $= x$, permutation space $= x!$, chain width $= x - 1$.
The public phase repeats with period $\mathrm{lcm}(P, Q, R)$; the x-cycle block structure is an additional factor, and re-deriving π_B per block pushes repetition out further (bounded in practice by the counter wrap-around of the chosen block encoding).
For composite x (e.g., $6 = 2 \cdot 3$), choose P, Q, R coprime with all prime factors of x to avoid shrinking the phase/block interaction period.

7. Security intuition (informal)

- **Lane isolation:** each provider uses distinct secret bouquets, so observing one lane does not reveal others.

- **Phase coupling:** public residues are mixed and hashed, preventing linear predictability from the CRT clock alone.
- **Device chaining:** even stale lanes influence future state, reinforcing the requirement that "every token matters".
- **Quantum period-finding:** QFT can reveal the public period $\mathrm{lcm}(P, Q, R, x)$ but not the hidden bouquets or $\mathrm{perm\_key}$; use large coprimes and device-only chaining to avoid exploitable structure.

8. Experimental validation (deterministic simulation)

A simulator was implemented cycle-by-cycle to validate correctness. The demo verifies:

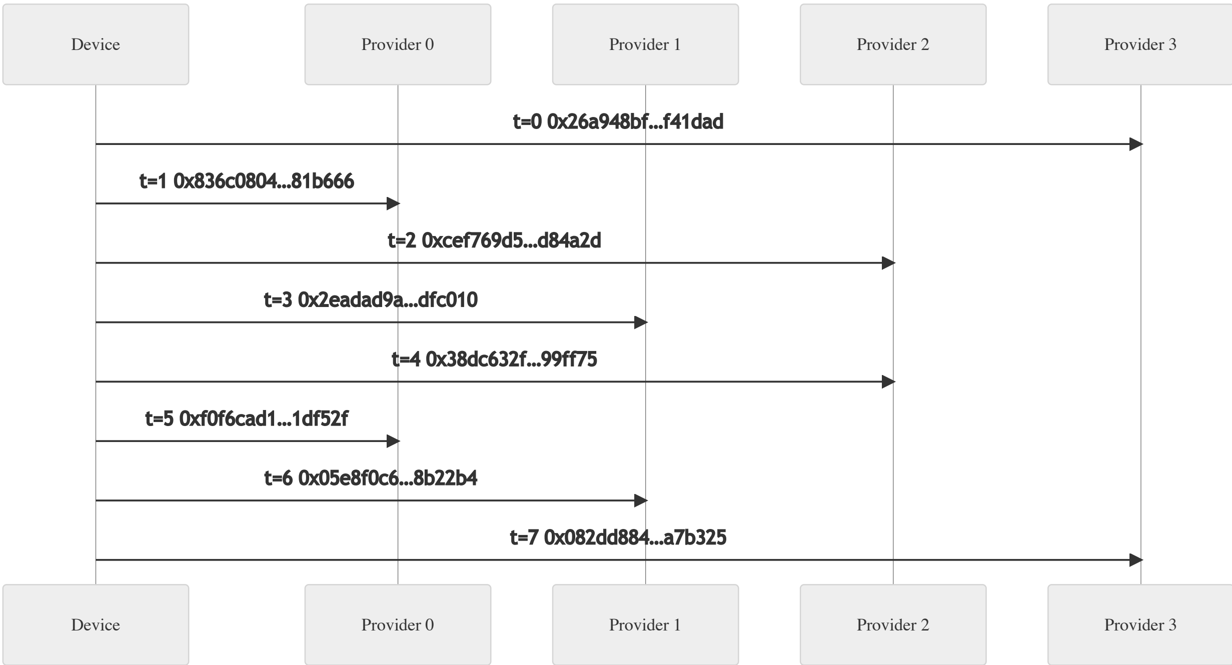
- Each block yields a valid permutation.
- Exactly one provider matches each cycle.
- Each provider appears once per block.
- Optional pre-hash difficulty metrics and QFT-visible period reports.
- Optional prime/compound generation modes for non-arbitrary parameter testing.

Repository: cekkr/phaselane-algorithm@github.com.

8.1 Sample token trace (x=4, seed=1337)

For PDF export, the original wide table was replaced with an A4-friendly summary table and a sequence diagram (tokens truncated for readability; the matched provider's recomputed token equals the device token by construction).

t	block	slot	idx_t	device token (truncated)	matched provider
0	0	0	3	`0x26a948bf…f41dad`	3
1	0	1	0	`0x836c0804…81b666`	0
2	0	2	2	`0xcef769d5…d84a2d`	2
3	0	3	1	`0x2eadad9a…dfc010`	1
4	1	0	2	`0x38dc632f…99ff75`	2
5	1	1	0	`0xf0f6cad1…1df52f`	0
6	1	2	1	`0x05e8f0c6…8b22b4`	1
7	1	3	3	`0x082dd884…a7b325`	3



8.2 Full token trace (verbatim values)

The full deterministic trace (block permutations, schedule, device tokens, and per-lane tokens) is exported to a separate, auto-generated file to keep the paper A4-friendly. See `papers/token-trace.md`, generated by `demo/export_token_trace.py`.

Regenerate with:
`python3 demo/export_token_trace.py --blocks 4 --out papers/token-trace.md`

8.3 Pre-hash difficulty and period reporting

The demo can emit a linear pre-hash difficulty report (rank of exponent vectors modulo 2 and 65537) and the QFT-visible public period:

- `python3 demo/pcpl_cycle_test.py --linear-report --analysis-window 64`
- `python3 demo/pcpl_cycle_test.py --qft-report`
- `python3 demo/pcpl_cycle_test.py --compare-x 2,3,4,5,6`
- `python3 demo/pcpl_cycle_test.py --prime-mode generated --prime-bits 31 --compound-mode blend --compound-prime-bits 12`

8.4 Multi-configuration results snapshot

All runs below completed the full correctness checks (permutation, 1-of-x matching, chaining).

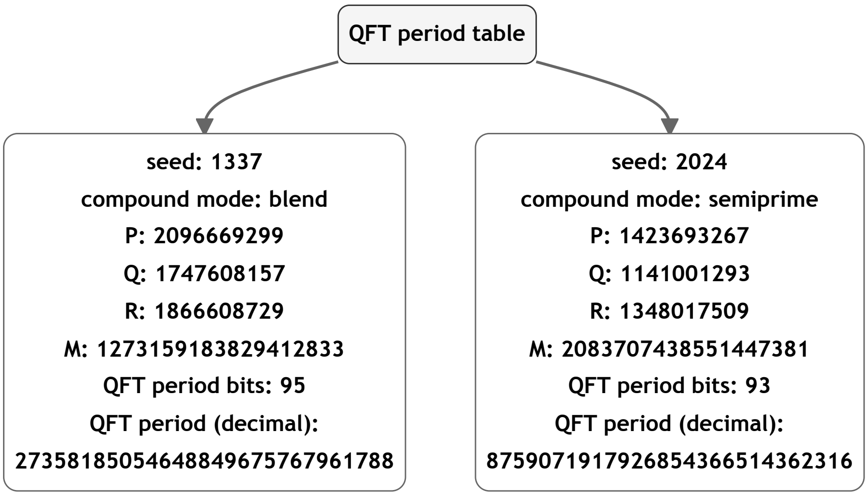
Fixed primes (P/Q/R near 1e6, seed=1337) with compare-x and 64-cycle linear window:

x	chain width (x-1)	QFT period bits	QFT period (decimal)
2	1	61	2000146002862007326
3	2	62	3000219004293010989
4	3	62	4000292005724014652
5	4	63	5000365007155018315
6	5	63	6000438008586021978

Across all x above, the pre-hash exponent vectors reached full rank (4/4) modulo 2 and 65537, with 64/64 unique rows for A/B/C over the sample window.

For $x = 6$ (composite $2 \cdot 3$), the schedule still yields exactly one match per cycle, but the duty cycle per provider is $1/6$ and the permutation space grows to $6! = 720$. Ensure P, Q, R are coprime with both 2 and 3 to keep the public period large.

Generated primes (x=4, 64 cycles, 12-bit compound primes):



Full multi-configuration outputs (additional compound modes and seeds) are in `papers/pcpl-results.md`.

9. Discussion and limitations

- Parameter choice matters; P, Q, R, M must be prime and pairwise coprime.
- The permutation schedule is device-only; leakage of the permutation key can reveal lane order, but not lane tokens.
- The security of the scheme relies on the strength of $H(\cdot)$ and the secrecy of bouquets, not on the hardness of factoring revealed integers.
- The public period $\mathrm{lcm}(P, Q, R, x)$ is visible (and QFT-recoverable), so period size should be chosen large enough for the deployment horizon.
- For testing, primes and compound bases can be generated from a seeded stream to avoid arbitrary constants. [6]
- This paper was developed and formatted with the help of OpenAI models.

10. Conclusion

PCPL provides a deterministic, no-handshake token protocol with exact 1-of-x matching and a device-only chaining mechanism. Combined with symmetric continuous tokenizer devices, it supports both provider validation and peer-to-peer isolation with dynamic, evolving secrets. The included simulation and trace demonstrate the protocol's behavior cycle by cycle.

References

1. NIST FIPS 180-4 (Update 1), *Secure Hash Standard (SHS)*
2. NIST FIPS 202, *SHA-3 Standard: Permutation-Based Hash and Extendable-Output Functions*
3. RFC 7693, *The BLAKE2 Cryptographic Hash and Message Authentication Code*
4. RFC 2104, *HMAC: Keyed-Hashing for Message Authentication*
5. RFC 5869, *HMAC-based Extract-and-Expand Key Derivation Function (HKDF)*
6. NIST SP 800-90A Rev. 1, *Recommendation for Random Number Generation Using Deterministic RBGs*

7. RFC 4086, *Randomness Requirements for Security*
8. RFC 8017, *PKCS #1: RSA Cryptography Specifications Version 2.2* (I2OSP/OS2IP)
9. NIST SP 800-185, *SHA-3 Derived Functions: cSHAKE, KMAC, TupleHash, and ParallelHash*
10. NIST SP 800-56C Rev. 2, *Recommendation for Key-Derivation Methods in Key-Establishment Schemes*
11. RFC 4226, *HOTP: An HMAC-Based One-Time Password Algorithm*
12. RFC 6238, *TOTP: Time-Based One-Time Password Algorithm*
13. RFC 5905, *Network Time Protocol Version 4 (NTPv4)*
14. R. Durstenfeld (1964), "Algorithm 235: Random permutation", *Communications of the ACM* 7(7)
15. B. Gassend et al. (2002), "Controlled Physical Random Functions", (PUFs)
16. G. E. Suh & S. Devadas (2007), "Physical Unclonable Functions for Device Authentication and Secret Key Generation", DAC '07
17. P. C. Kocher (1996), "Timing Attacks on Implementations of Diffie-Hellman, RSA, DSS, and Other Systems", CRYPTO '96
18. P. W. Shor (1994), "Algorithms for Quantum Computation: Discrete Logarithms and Factoring", FOCS '94
19. M. A. Nielsen & I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press)
20. A. Menezes, P. van Oorschot, S. Vanstone, *Handbook of Applied Cryptography* (CRC Press; online edition)