ON THE THEORIES OF HYDRAULIC GEOMETRY

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ABSTRACT

Hydraulic geometry is of fundamental importance in planning, design, and management of river engineering and training works. Although some concepts of hydraulic geometry were proposed toward the end of the nineteenth century, the real impetus toward formulating a theory of hydraulic geometry was provided by the work of Leopold and Maddock (1953). A number of theories have since been proposed. Some of the theories are interrelated but others are based on quite different principles. All theories, however, assume that the river flow is steady and uniform and the river tends to attain a state of equilibrium or quasi-equilibrium. The differences are due to the differences in hydraulic mechanisms that the theories employ to explain the attainment of equilibrium by the river.

Key Words: Energy, Energy dissipation, Entropy, Equilibrium, Extremal hypothesis, Flow efficiency, Friction factor, Froude number, Sediment transport rate, Stream power, Tractive force

1 INTRODUCTION

The term “hydraulic geometry” connotes the relationships between the mean stream channel form and discharge both at-a-station and downstream along a stream network in a hydrologically homogeneous basin. The channel form includes the mean cross-section geometry (width, depth, cross-section, meander length), and the hydraulic variables which include the mean slope, mean friction, and mean velocity for a given influx of water and sediment to the channel and the specified channel boundary conditions. Leopold and Maddock (1953) expressed the hydraulic geometry relationships for a channel in the form of power functions of discharge as

$$B = aQ^b, \quad d = cQ^f, \quad V = kQ^m \tag{la}$$

where B is the channel width; d is the flow depth; V is the flow velocity; Q is the flow discharge; and a, b, c, f, k, and m are parameters. To equation (1a), also added are

$$n = NQ^p, \quad S = sQ^y \tag{1b}$$

where n is Manning’s roughness factor; S is slope; and N, p, s, and y are parameters. Exponents b, f, m, p and y represent, respectively, the rate of change of the hydraulic variables B, d, V, n and S as Q changes; and coefficients a, c, k, N and s are scale factors that define the values of B, d, V, n and S when Q = 1.

The hydraulic variables, width, depth and velocity, satisfy for rectangular channels the continuity equation:

$$Q = BdV \tag{2}$$

Therefore, the coefficients and exponents in equation (1a) satisfy:

$$ack = 1, \quad b + f + m = 1 \tag{3}$$

The at-a-site hydraulic geometry entails mean values over a certain period, such as a week, a month, a season, or a year. The concept of downstream hydraulic geometry involves spatial variation in channel form and process at a constant frequency of flow. Richards (1982) has noted that the downstream hydraulic geometry involving the channel process and form embodies two types of analyses both of which are expressed as power functions of the form (Rhoads, 1991) given by equations (1a, b). The first type of analysis is typified by the works of Leopold and Maddock (1953) and Wolman (1955) who

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formalized a set of relations, such as equations (1a, b), to relate the downstream changes in flow properties (width, mean depth, mean velocity, slope and friction) to mean discharge. This type of analysis describes the regulation of flow adjustments by channel form in response to increases in discharge downstream, and has been applied at particular cross-sections as well as in the downstream direction.

The second type of analysis is a modification of the original hydraulic geometry concept and entails variation of channel geometry for a particular reference discharge downstream with a given frequency. Implied in this analysis is an assumption of an appropriate discharge that is the dominant flow controlling channel dimensions (Knighton, 1987; Rhoads, 1991). For example, for perennial rivers in humid regions, the mean discharge or a discharge that approximates bankfull flow (Q_b), such as Q_2, $Q_{2.33}$, is often used in equations (1a, b). This concept is similar to that embodied in the regime theory (Blench, 1952, 1969). It should, however, be noted that the coefficients and exponents are not constrained by the continuity equation when the selected discharge substantially differs from the bankfull flow. On the other hand, Stall and Yang (1970) related hydraulic geometry to flow frequency and drainage area.

The hydraulic geometry relations are of great practical value in prediction of channel deformation; layout of river training works; design of stable canals and intakes, river flow control works, irrigation schemes, and river improvement works; and so on. Richards (1976) has reasoned that hydraulic geometry relations through their exponents can be employed to discriminate between different types of river sections. These relations can be used in planning for resource and impact assessment (Allen et al., 1994).

The hydraulic geometry relations of equations (1a, b) are derived using a variety of hypotheses. Each hypothesis leads to unique relations between channel form parameters and discharge, and the relations corresponding to one hypothesis are not necessarily identical to those corresponding to another hypothesis. The objective of this paper is to review various hypotheses that have been proposed for deriving at-a-site and downstream hydraulic geometry relations, put them in perspective, and comment on their usefulness in hydraulic design and river training works. The paper is organized as follows. Introducing hydraulic geometry relations in section 1, a brief review of literature related to various hydraulic geometry characteristics is given in section. Section 3 discusses various hypotheses of hydraulic geometry. The paper is concluded in section 4, followed by the cited literature.

2 CHARACTERISTICS OF HYDRAULIC GEOMETRY RELATIONS

Since the classic work of Leopold and Maddock (1953) and Wolman (1955), studies on hydraulic geometry have been voluminous. They have, however, focused on the following main issues: (1) the basis of hydraulic geometry relations, (2) the tendency of equilibrium state, (3) limitations of the equilibrium assumption, (4) the validity of power relations, (5) the stability of exponents in power relations, (6) the variability of exponents, (7) the effect of river channel patterns, (8) the variation of channel width with discharge, (9) the effect of stream size, (10) the variation in channel velocity, (11) the dependence of exponents on climatic, environmental factors and land use, (12) an extension of power relations to drainage basins, and (13) boundary conditions. A brief discussion of these issues pertaining to the characteristics of the hydraulic geometry relations of equations (1a, b) is in order.

2.1 Basis of Hydraulic Geometry Relations

According to Langbein (1964), Langbein and Leopold (1964), and Yang, et al. (1981) among others, the mean values of the hydraulic variables of equations (1a, b) are known to follow necessary hydraulic laws and the principle of the minimum energy dissipation rate. As a consequence, these mean values are functionally related and correspond to the equilibrium state of the channel. This state is regarded as the one corresponding to the maximum sediment transporting capacity. The implication is that an alluvial channel adjusts its width, depth, slope, velocity, and friction to achieve a stable condition in which it is capable of transporting a certain amount of water and sediment. Leopold and Maddock (1953) have stated that the average river system tends to develop in such a way as to produce an approximate equilibrium between the channel and the water and sediment it must transport.

2.2 Tendency for Equilibrium State

Knighton (1977a) observed that at cross-sections undergoing systematic change, the potential for adjustment toward some form of quasi-equilibrium in the short term is related to flow regime and channel boundary conditions. Marked changes in the channel form and associated hydraulic geometry can occur over a short period of time in the absence of exceptionally high flows and in a channel with high boundary resistance. This suggests that the approach to quasi-equilibrium or establishment of a new equilibrium position is relatively rapid.

Ponton (1972) found the hydraulic geometry of the Green and Birkenhood River basins in British Columbia, Canada, to significantly depart from the previous works, and attributed the departure to the recent glaciation in the area and the strong control which glacial features exercise on streams. He concluded that the equilibrium throughout the stream was not established. Many reaches within each system may have reached a quasi-equilibrium, as indicated by the at-a-station hydraulic geometry. However, these reaches were not yet adjusted to each other because of glacial features which separate them.

Heede (1972) investigated the influence of a forest on the hydraulic geometry of two mountain streams. He found that a dynamic equilibrium was attained in the streams. Sanitation cuts (removal of dead and dying trees) would not be permissible where a stream was in dynamic equilibrium and the bed material movement should be minimized.

2.3 Limitations of the Equilibrium Assumption

Rhodes (1978) emphasized geometric adjustments to discharge downstream in response to environmental history, bed load and climate. The power function relations have often been applied to small basins which in many cases have varying geology and climate, or human intervention disturbs the long-term equilibrium. Thus, the equilibrium assumption can only address “at-a-station” geometry, as each cross-section of the channel adjusts to the discharge of water and sediment in a unique surface and subsurface environment. Furthermore, for discharges deviating in the extreme from the mean discharge, i.e., extremely low and high flows, even the “at-a-station” interpretation of hydraulic exponents and coefficients will be less than meaningful. This is because the influence of geology, soil, widening and narrowing floodplains, upstream bog and marsh environments, or network topology on hydraulic geometry is not properly understood at the full range of space-time scales.

2.4 Vailidity of Power Relations

Dury (1976) confirmed the validity of power function relations for hydraulic geometry using extended sets of data at the 1.58-year mean annual discharge. Chong (1970) stated that hydraulic geometry relations of equations (1a, b) were similar over varying environments. Thus, it seems that the regional generalizations proposed in the literature are acceptable for rivers that have achieved “graded-time” equilibrium (Phillips and Harlan, 1984). However, Park (1977) suggests that simple power functions are not the best way to describe hydraulic geometry. Richards (1973, 1976) has reasoned that since the depth and velocity are functions of roughness, the power function model for depth and velocity will not reflect the true hydraulic nature, when the rate of change in roughness is not uniform. He then proposed a model for describing the nonuniform variation of roughness in relation to similar nonuniform changes of depth and velocity with discharge.

Betson (1979) developed procedures for predicting variations in geomorphic relationships in the Cumberland Plateau in Appalachia in the United States for use in streawmflow and sediment routing model components of a planning-level strip mining hydrology model. He found that at high flows the width and area could be predicted with 25% accuracy about 2/3 of the time. These results were comparable to or better than the results that would normally be obtained from the reconnaissance field surveys used in streamflow routing for planning-level applications.

Knighton (1974, 1975, 1977a) investigated the downstream and at-a-station variation in width-discharge relation and its implication for hydraulic geometry. The width of a channel with cohesive banks and no marked downstream variation in bank erodibility increased with discharge in the downstream direction and the rate of increase was principally a function of discharge. The channel width at a cross section was determined by flow exceeding the threshold of erosion and its magnitude increased regularly downstream with drainage area. The at-a-station rate of change was controlled by the bank material composition, especially silt-clay content. Thus, at a cross section, the rate of change of width can increase due to the deposition of noncohesive sediment in the form of point bars and central islands, suggesting that the b exponent can be used to distinguish meander and braided reaches from straight reaches. The effect of this adjustment is to decrease the mean velocity range. Arguing that the at-a-station variations of hydraulic geometry relations are due to variations in discharge and the downstream variations are caused by increasing discharge due to increasing drainage area, Stall and Yang (1970) expressed the hydraulic geometry relations in a general form, as was previously done by Stall and Fok (1968). The coefficients in the general relation reflect the influence of physiographic characteristics of the drainage basin.

Allen et al. (1994) employed downstream channel geometry for use in planning level models for resource and impact assessment. For the data on channel dimensions obtained from the literature, they found that over a large variety of stream types and physiographic provinces the channel width and depth was predicted with 86 percent efficiency.

2.5 Stability of Hydraulic Geometry Relations and Exponents

The relations of equations (1a, b) have been calibrated for a range of environments, using both field observations and laboratory simulations. Chong (1970) stated, without a firm basis, that rivers over varying environments behaved in a similar manner. However, hydraulic geometry relations are stable for rivers that have achieved "graded equilibrium." Parker (1979) has stated that the scale factors, *a*, *c*, and *k*, vary from locality to locality but the exponents, *b*, *f*, and *m*, exhibit a remarkable degree of consistency, and seem independent of location and only weakly dependent on channel type. Knighton (1974) emphasized variations in exponents as opposed to mean values, as shown in Tables 1 and 2. Rhodes (1977, 1978) noted that the exponent values for high flow conditions can be vastly different from those for low flow conditions. From an analysis of a subalpine stream in a relatively homogeneous environment, Phillips and Harlin (1984) found that hydraulic exponents were not stable over space. Rhoads (1991) examined the factors that produce variations in hydraulic geometry parameters. He hypothesized that the parameters are functions of channel sediment characteristics and flood magnitude, and that the parameters vary continuously rather discretely. Phillips and Harlin (1984) found for a subalpine stream in a relatively homogeneous environment that hydraulic exponents were not stable over space. The interactions amongst channel form; discharge; and atmospheric, surface and subsurface environments in the system produce variables which even in the short run and at a station are neither consistently dependent nor independent. The exponents and coefficients of hydraulic geometry relations of equations (1a, b) vary from location to location on the same river and from river to river, as well as from the high flow range to the low flow range. This is because the influx of water and sediment and the boundary conditions that the river channel is subjected to vary from location to location as well as from river to river. This means that for a fixed influx of water and sediment a channel will exhibit a hierarchy of hydraulic geometry relations in response to the boundary conditions imposed on the channel. It is these boundary conditions that force the channel to adjust its allowable hydraulic variables. For example, if a river is leveed on both sides, then it cannot adjust its width and is, therefore, left to adjust other variables, such as depth, friction, slope, and velocity. Likewise, if a canal is lined, then it cannot adjust its friction. This aspect does not seem to have been fully explored in the literature.

2.6 Variability of Exponents

Kolberg and Howard (1995) examined the variability in exponents of hydraulic geometry relations for piedmont and midwestern streams. They analyzed active channel geometry and discharge relations using data from 318 alluvial channels in the Midwestern United States and 50 Piedmont sites. Their analysis showed that the discharge-width exponents were distinguishable, depending on the variations in materials forming the bed and banks of alluvial channels. For example, highly cohesive channels (high silt and clay beds), gravel and cobble streams, and noncohesive sand stream channels had statistically distinguishable exponents for midwestern streams. The estimated width-discharge exponents for high silt and clay, gravel, or cobble bed channels deviated significantly from those for alluvial beds with 30% or greater sand content. However, no significant trend was apparent for estimated exponents among other sand-silt-clay channel categories. In case of Piedmont data, no significant departure of the discharge-width relations was apparent for different groups of stream types based on sediment categories. Both the midwestern and piedmont data indicated that the width-discharge exponents ranged from 0.35 to 0.46 for groups of streams with width to depth ratios less than the 45 range. For groups of streams with the width to depth ratios greater than 45, the width-discharge exponents decreased to values below 0.15, suggesting a systematic variation in the exponents and a diminished influence of channel shape. These results are in agreement with the findings of Osterkamp and Hedman (1982).

Table 1 Average values of exponents, *b*, *f*, and *m*, gathered from literature: Downstream

Source	Exponent			Drainage area	Conditions
	b	f	m		
Lacey (1929-30)	--	0.33			
Leopold and Maddock (1953)	0.50	0.40	0.10		
Wolman (1955)	0.34	0.45	0.32	Duration = 50	Ephemeral streams (at *Q* equalled or exceeded at % of time)
	0.38	0.42	0.32	15	-do-
	0.45	0.43	0.17	2	-do-
	0.42	0.45	0.05	Bankful discharge	-do-
	0.57	0.40	0.03	50	Principal stations at Bradywine Creek and headwaters
	0.58	0.40	0.02	2	-do-
Leopold and Miller (1956)	0.29	0.15	0.58	Sedalia Gully near Sedalia, Colorado	
	0.31	0.20	0.49	Sowbelly Creek near Hat Creek, Nebraska	
Miller (1958)	0.38	0.25	0.39	High mountain streams	
Brush (1961)	0.55	0.36	0.09	Appalachian streams	
Ackers (1964)	0.42	0.43	0.15		
	0.43	0.43	0.14		
	0.53	0.35	0.12		
Langbein (1964)	0.53	0.37	0.10		
Scott (1966)	0.69	0.12	0.19	Perennial rivers	
	0.03	0.48	0.45	Ephemeral streams	
Kellerhals (1966)	0.50	0.40			Regime theory
Blench (1969)	0.50	0.33			Regime theory
Carlston (1969)	0.461	0.383	0.155	10 river basins	
	0.499	0.320	0.180	Yellow River	
Thornes (1970)	0.40	0.34	0.25	Suia-Missu and Araguaia basins, Mato Gross, Brazil	Minimization of error sum of squares
	0.47	0.41	0.04	-do-	*Q* greater than 1.94
	0.11	0.32	0.59	-do-	*Q* less than 1.94 cfs
	0.19	0.32	0.56	-do-	Maximization of explained variance *Q* greater than 5.02 cfs
	0.51	0.50	0.01	-do-	*Q* less than 15.02 cfs
	0.14	0.36	0.54	Smaller streams	
Ponton (1972)	0.60	0.40	-0.01	Green River	
	0.80	0.44	-0.23	Birkenhead River	
Knighton (1974)	0.61	0.31	0.08		
Smith (1974)	0.60	0.30	0.10		
	0.54	0.23	0.23		
	0.46	0.16	0.38		
Parker (1979)	0.50	0.415	0.085		
Lane and Foster (1980)	0.46	0.46	0.081		
Rhoads (1991)	0.46	0.46	--	Missouri River basin	106,155-1,358,000 sq. Km.
	0.49	0.30		James River basin	655-55810 sq. km
	0.51	0.37		Smokey Hill River	9207-49,880 sq. Km.
Allen, Arnold and Byers (1994)	0.557	0.341	0.104		674 cross sections from 15 states in the conterminous U. S.

Table 2 Average values of exponents *b*, *f*, and *m* gathered from literatures: Site specific

Source	Exponent			Drainage area	Conditions
	b	f	m		
	b	f	m		
Leopold and Maddock (1953)	0.26	0.40	0.34		
Leopold et al. (1964)	0.26	0.40	0.34	Midwestern U. S.	
	0.12	0.45	0.43	158 stations in the U. S. A.	
Wolman (1955)	0.04	0.41	0.55	Brandywine River, Pennsylvania	
Leopold and Miller (1956)	0.26	0.33	0.32		
Leopold and Langbein (1962)	0.23	0.42	0.36		Theoretical
Langbein (1964)	-	0.50	0.50	Circulating flume	
	0.23	0.42	0.35	Stable river section	
Scott (1966)	0.35	0.42	0.55		Ephemeral streams
	0.24	0.56	0.20		Perennial streams
Leopold and Skibitzke (1967)	0.16	0.30	0.52	Middlefork Salmon River near Cape Horn	138 sq. Mi.
	0.06	0.43	0.53	Bear Valley Creek near Cape Horn	180 sq. Mi.
	0.04	0.36	0.61	Middlefork Salmon River near Meyers Cave	2020 sq. Mi.
	0.08	0.41	0.52	Big Creek near Big Creek	470 sq. Mi.
	0.27	0.20	0.53	Salmon River at Salmon	2020 sq. Mi.
	0.10	0.40	0.49	Salmon River at Challis	3670 sq. Mi.
Coates (1969)	0.36	0.20	0.44	18 streams in the Appalachian Plateau	13-61 sq. Mi.
Church (1980)	0.22	0.31	0.48	Baffin Island Sandurs	
Stall and Yang (1970)	0.23	0.41	0.36	Big Sandy River, Kentucky	
Wilcox (1971)	0.09	0.36	0.53	Upper Holder Basin	
Heede (1972)	--	0.43	0.52	Fool Creek, Central Rocky Mountaimns	
	--	0.36	0.34	Ephemeral Streams in Semiarid U. S. A.	
Ponton (1972)	0.21	0.32	0.50	Coast mountain streams, B. C., Canada	
Knighton (1972)	0.29	0.40	0.31	River dean at Addington Hall	Braided reach
	0.11	0.56	0.33	Right distributory	Braided reach
	0.23	0.27	0.50	Left distributory	Braided reach
Richards (1973)	---	0.639	0.296	Rocester	
	---	0.44	0.474	Serlby Park	
	---	0.37	0.57	Yaxall	
	---	0.576	0.32	Draycott	
	---	0.342	0.603	Beedly	
	---		0.568		
	---		0.583		
Dury (1976)	0.543	0.344			
Riley (1978)	0.42	0.41	0.16	Gwydir River	
	0.35	0.48	0.17	Namoi River	
	0.35	0.52	0.13	Barwon River	
	0.38	0.46	0.16	Three rivers	
Williams (1978)	0.49	0.24	0.27		
Betson (1979)	0.245			Watersheds in Kentucky	

Table 2 Average values of exponents b, f, and m gathered from literatures: Site specific (to be continued)

Source	Exponent	Exponent	Exponent	Drainage area	Conditions
Lane and Foster (1980)	0.375	0.375	0.25		
	0.32	0.32	0.36		
Abrahams (1984)	0.419	-0.064	0.632	West channel, 8 observations	
	-.671	0.863	0.753	East channel: 8 observations	
Philips and Harlan (1984)	0.367	0.049	0.580	Blanca Meadow: 20 observations	
	0.419	-0.095	0.67	West channel: 80 observations	
	-0.706	0.912	0.792	East channel: 8 observations	
Rhoads (1991)	0.50	0.34	--	Missouri River basin	252 gaging stations

Rhoads (1991) has reasoned that variations among the three channel types are not discrete thresholds but can be viewed as continuous variations. These variations support the assertion of Howard (1980) that thresholds in the hydraulic geometry of alluvial sand and gravel streams exist. However, the implicit control of width-discharge relations by sediment characteristics was not strongly evident for the Virginia and North Carolina Piedmont streams investigated by Kolberg and Howard (1995). Despite the lack of strong relation between channel shape and sediment type for sand and gravel Piedmont streams (Schumm, 1960; Miller and Onesti, 1979; Nanson and Huang, 1999), Park (1977) argued that certain hydraulic exponents could be characteristic of different climatic and environmental regimes.

2.7 Effect of River Channel Patterns

Wolman and Brush (1961), Chitale (1970, 1973), among others, investigated into river channel patterns and found that the coefficients and exponents in equations (1a, b) depended on the type of the pattern. He classified channels into three groups: (1) single, (2) multi-thread, and (3) transitional channels. Single channels were further subdivided into (a) meandering, (b) straight, (c) and transitional between meandering and straight. Meandering channels were distinguished as regular or irregular and simple or compound. Multithread channels were divided into (a) braided and (b) branching-out channels. For small streams, including straight, shoaled and meandered, the slope varied inversely with 0.12 power of discharge, whereas the power of discharge changed to - 0.21 for critical straight line water surface slope. For braided channels, the power changed to - 0.44. The meander length changed with discharge raised to the power of 0.5, but it changed with the range of discharge used. Similarly, the surface width changed with discharge raised to the power of 0.42.

Leopold and Wolman (1957), and Knighton (1972) investigated changes in a reach morphology and hydraulic geometry. The processes of erosion and deposition interact with channel hydraulics. A braid may gradually be transformed into a meandering reach by a slight modification to the flow pattern and without any change in the independent variables. In braided reaches, large differences in flow behavior exist between juxtaposed channels, and streamflow may be concentrated in that channel that offers the least resistance.

2.8 Variation of Channel Width with Discharge

Klein (1981) analyzed the variation of channel width with the downstream discharge and found that the value $b = 0.5$ was a good average. The low b values normally occur for small basins (in lower flows) and for very big basins (in very high flows). Thus, the $b = 0.5$ value, being a good average, tends to smooth out deviations from the average. The value of b ranged from 0.2 to 0.89. Klein (1981) argued that the simple power function for hydraulic geometry was valid for small basins and that did not hold over a wide range of discharges.

2.9 Variation in Channel Velocity

Mackin (1963) has noted that in individual channels there are just as many such segments with a downstream decrease in velocity as there are with a downstream increase in velocity. Carlson (1969) found in the Susquehanna River in the United States that the number of streams with a downstream velocity increase was balanced by an equal number of streams with either a constant velocity or a downstream decrease in velocity. The most common relationship on long segments of rivers is a nearly constant velocity; however, in many smaller streams the velocity may increase or decrease downstream because of geological influences present at the mean annual discharge. Large rivers, such as the Mississippi, accommodate a downriver increase in discharge principally through increase in depth, whereas smaller rivers generally accommodate the downriver increase in discharge principally through increase in width. Leopold (1953) has shown that large scale floods which move large quantities of sediment have nearly constant downstream velocity.

2.10 Effect of Stream Size

Thornes (1970) analyzed the differences in the explained variances of the power function geometry relations for streams of different sizes-smaller as well as bigger (e.g., Susia-Missu and Araguaia rivers in Brazil) undergoing significant changes. Smaller streams were unstable and out of phase with the steady state condition in the main stream. The difference was, therefore, between the minor stream and major tributary of the area, rather than between streams above and below a given discharge. Three possible explanations were advanced: (1) For the long term: Smaller channels undergo greater fluctuations from steady state conditions and are geomorphologically more active with strong slopes. (2) For the medium term: Small channels are substantially impacted by human activities, in the form of extensive land clearances. The width to depth ratios are substantially affected by suspended load. (3) For the short term: Seasonal changes affect most significantly the geometric relationships in smaller channels. In smaller streams, because of little or no baseflow, there is a marked difference between bankfull discharge and low discharge. This instability may reflect adjustments to the long-term erosion of upland areas, changes accommodating the impact of human activities on small basins or the effect of seasonal differences. The channel form adjusts to the wet season.

2.11 Effect of Land Use

Lane and Foster (1980) analyzed adjustments of stream channels due to changes in discharge and channel characteristics resulting from changing land use. Their results showed that channel widths increased as discharge and hydraulic resistance increased and that narrower channels resulted from a larger critical stress. Therefore, the land use that caused these changes caused a readjustment in streams. For example, changes in the amount of sediment eroded from the channel boundary as the boundary adjusted to changing discharge reflected changes in land use.

2.12 Hydraulic Geometry Relations for Drainage Basins

Singh and Broeren (1989), and McConkey and Singh (1992) found for subbasins of the Sangamon and Vermilion rivers in the United States that the power function relationship performed poorly for low flows. They observed that the variation in discharge was consistently dependent on the drainage area and annual flow duration. Because of the existence of riffles and pools in natural streams, the hydraulic geometry relations are rendered less reliable, especially at low flows which typically create more critical conditions for fish species and various aquatic life forms. The power function relations were reasonable for watersheds greater than 100 square miles but may not be reliable for smaller drainage area streams.

2.13 Boundary Conditions

The boundary conditions that a channel has to satisfy vary with the type of the channel. Based on their boundary conditions and hydrologic input of water and sediment, open channels can be classified (Yu and Wolman, 1987) into three groups: (a) channels with rigid boundaries and uniform flow, (b) canals with erodible beds and banks, and (c) natural rivers in equilibrium and regime. In the first case, there is only one flow depth above the critical depth associated with uniform flow. If the discharge is known, the geometry is determined. In the second case, the geometry, resistance, flow and sediment transport are interrelated. The channel geometry is determined by the laws of fluid flow, sediment transport, and bank stability. Assuming there is no appreciable scour or silting in a channel if the boundary conditions do not change over a long-term average or the channel is in equilibrium, the regime theory is invoked in engineering practice. In the third case, the hydraulic geometry is determined by the stability of channel banks, the availability of sediment (bed and bank material, and cohesive material) for transport, and vegetative cover, in addition to the magnitude and variability of flow. In analogy with regime in canals, the channel hydraulic geometry is assumed to be determined by discharge. Yu and Wolman (1987) investigated the effect of variable flow conditions, characteristic of natural rivers, on the channel hydraulic geometry.

3 HYPOTHESES FOR HYDRAULIC GEOMETRY RELATIONS

During the last 50 years, a multitude of approaches have been employed for deriving functional relationships among the aforementioned hydraulic variables. These approaches are based on the following theories: (1) regime theory, (2) power function theory, (3) tractive force theory and its variants-threshold channel theory and stability theory, (4) similarity theory, (5) hydrodynamic theory, (6) thermodynamic entropy theory, (7) energy-entropy theory, (8) minimum extremal theories (e.g., minimum channel mobility theory, minimum stream power theory, minimum energy degradation theory, minimum entropy production theory, and minimum variance theory), and (9) maximum extremal theories (maximum friction theory, maximum sediment discharge theory, maximum sediment discharge and Froude number theory, and maximum entropy theory). Some of these theories have been applied to only at-a-station hydraulic geometry, some to only downstream hydraulic geometry, and some to both. A short review of these theories is in order.

Looking at hydraulic geometry relations from a hydraulic point of view, the governing equations are formulated from (1) the conservation of mass of water, (2) conservation of sediment, (3) Newton's law of motion for water, (4) Newton's law for motion of sediment in suspension, (5) Newton's law for motion of sediment moving on the bed, and (6) Newton's law for motionless sediment in the banks (Stevens and Nordin, 1987). The first three equations have been adequately formulated. The last three equations are more difficult to formulate. The width of a river also depends on the type of the bank's soil and the stresses of flow on the banks. In practice, the continuity equation of water, a resistance equation, the sediment transport equation, and a morphological relation are invoked. These equations have four unknowns: width, depth, slope, and velocity. The morphological relation is derived in different ways, depending on the hypothesis to be employed. Often these hypotheses are extremal, either minimal or maximal. For example, the hypotheses of the least mobility (Dou, 1964), the minimum stream power (Chang, 1980, 1988; Yang, 1972, 1986), the minimum variance (Langbein, 1964; Williams, 1978), and the maximum efficiency principle (Davies and Sutherland, 1983; White et al., 1986) have been invoked to provide the morphological relation.

Ackers (1964) summarized the results of experiments conducted at the Hydraulics Research Station at Wallingford, England, on small streams that achieved stability at discharges between 0.4 and 0.5 cfs. Empirical correlations of stream geometry with discharge were found to be consistent with those deduced by the combination of three physical relationships: (1) the resistance formula, (2) sediment transport formula, and (3) the ratio of width to depth. The usefulness of these relationships depends on the quality of the data and should be applied in situations similar to those for which the data were collected.

While deriving hydraulic geometry relations, it is normally assumed that the flow in channels can be described adequately by steady, uniform and one-dimensional equations for water and sediment. For purposes of comparison, it is appropriate to briefly outline these equations. Since the water discharge Q and the bed-material load Q_S are constant within a channel reach, one can express that

$$Q = V\,A, \qquad C = \frac{Q_S}{Q} \tag{2}$$

where A is the cross-sectional area of the channel, V is the average velocity in the cross-section and C is the concentration of sediment in the channel.

The momentum equation for steady uniform flow is expressed as Chezy's equation or Manning's equation:

$$V = C_z \sqrt{RS}\ , \quad V = \frac{1}{n} R^{2/3} S^{1/2} \tag{3a, b}$$

where C_z is Chezy's roughness factor, and R is hydraulic radius. Also used is the Darcy-Weisback relation expressed as

$$\frac{f\,F^2}{8S}=1 \tag{4}$$

where f is the Darcy-Weisback friction factor, F is the Froude number. The left side of equation (4) is also known as the kinematic wave number for open channel flow. Depending on the size, gradation and density of sand, the density of the fluid, and the configuration of the send on the bed, the value of "f" covers a broad range because of the complexity of sand beds. Haque and Mahmood (1985) reported for one regime canal in Pakistan the value of f to vary from 0.031 to 0.049 for discharge varying from 1,840 to 3,060 cfs. Stevens and Nordin (1987) noted that the kinematic wave parameter is more basic parameter for canals than is the Froude number. If F is constant for a channel, then equation (4) shows that f/S is constant, an important condition in regime considerations.

There is a multitude of sediment transport formulas for a given sediment size, density and gradation, and concentration of wash load (Vanoni, 1975). The regime theory does not include an explicit expression for sediment transport but expresses it through the fundamental variables of velocity, hydraulic radius, and slope. Thus, one chooses an appropriate sediment transport formula. A governing equation for channel width for alluvial channels is not yet established. A morphological relation based on empirical considerations is, therefore, employed.

3.1 Regime Theory

Lindley (1919) defined the regime concept as the dimensions, width, depth and gradient, of a channel to carry a given supply (of water) loaded with a given silt that were all fixed by nature. The regime concept of Lindley (1919) constitutes the foundation for understanding alluvial channel behavior and designing canals in the Indian Subcontinent as well as in many other parts of the world. The concept can be summarized as: "Given the design discharge and an accompanying amount of sediment of known size, what will nature choose for the width, depth and bed slope of the channel to convey both the water and sediment from one point to another if the canal is to flow between banks and on a bed, all made of its own sediment?" One would also like to know the width, depth and bed slope for canals with rigid banks and alluvial beds, and alluvial beds and banks of residual soil, either disturbed or undisturbed, carrying various amounts of sediment.

Regime represents a long-term average of river form rather than an instantaneously variable state. That means stable or "in regime" channels do not change over a period of one or several water years. It then expresses the natural tendency of channels carrying sediment within alluvial boundaries to seek a dynamic equilibrium (Hey, 1978; Hey and Thorne, 1986). The regime theory defines a regime channel as a nonsilting, nonscouring equilibrium channel carrying its normal suspended load (of a certain kind and quantity). The theory implies a unique solution for a stable channel, at a given steady discharge, transporting a known concentration of solids in alluvium of given character. The regime concept was developed, based on analyses of data from stable canals on the Indian Subcontinent (Lindley, 1919). Kennedy (1895), using data from stable canals, derived a relationship between mean velocity and depth of flow. Lacey (1929-30, 1946, 1957-58) proposed regime equations for wetted perimeter, hydraulic mean depth, and hydraulic gradient in terms of mean channel discharge and Lacey's factor. Blench (1957) derived hydraulic geometry relations which were similar to those of Lacey's but they distinguished between the influence of bed alluvium and bank material where these were dissimilar. Later, the regime concept was modified by including sediment concentration into geometric relationships (Inglis, 1946). Lacey (1957-58) suggested a group of nondimensional equations which, if sufficient data were available, would define a stable channel for alluvium of any diameter and sediment. These equations constituted what is called as Lacey's regime theory.

It may be appropriate to briefly review this theory. Lacey expressed nonsilting velocity V_0 as

$$V_0 = 1.17\,R^{1/2} \tag{5}$$

The canals had a range of discharges from 26 to 1,700 cfs and depths from 2.2 to 7.0 ft. All canals had the Froude number of 0.21. Reworking with the data, Lacey modified equation (5) as

$$V_0 = 1.17\,(f_L\,R)^{1/2} \tag{6}$$

where f_L is Lacey's silt factor defined as the square root of the ratio of the average flow velocity to the nonsilting velocity. The value of f was o.456 for discharges from 33 to 6,100 cfs and the hydraulic radii from 1.76 to 10.23 feet for his data. Equation (6) is the first regime equation.

Lacey obtained by fitting a wetted perimeter (ft)-discharge (cfs) relation:

$$P = 2.67\, Q^{1/2} \tag{7}$$

Equations (6) and (7) define the wetted perimeter and hydraulic radius. Lacey went on to employ his own form of Manning's equation:

$$V = \frac{1.3458}{n_a} R^{3/4}\, S^{1/2} \tag{8}$$

in which n_a was defined as the absolute rugosity. The relation between f and n_a was expressed as

$$n_a = 0.0225\, f^{1/4} \tag{9}$$

and between f and sediment size (in millimeters) defined by D_{50} as

$$f = 1.59\, D_{50}^{1/2} \tag{10}$$

For channel design, the silt factor is estimated by equation (10) and the width from equation (7). Employing equations (6), (7), (2) and P = AR, the hydraulic radius is expressed as

$$R = 0.47\, (\frac{Q}{f})^{1/3} \tag{11}$$

From equations (8), (9), and (11), the slope is found as

$$S = \frac{f^{5/3}}{1790\, Q^{1/6}} \tag{12}$$

These design equations do not explicitly require an expression for sediment transport. The implication is that the designed channel would carry the supplied sediment load.

Now it may be pertinent to examine Lacey's regime theory from the point view of basic hydraulic laws and equations. Equation (8) is a form of the momentum equation, for it contains the fundamental variables and expresses the velocity of flow to be proportional to the square root of (RS) and inversely proportional to the friction factor. Thus,

$$f = \frac{8\, g\, (n_a)^2}{1.81 R^{1/2}} \tag{13}$$

Combining equations (9) and (10), n_a can be determined as:

$$n_a = 0.0252\, (D_{50})^{1/8} \tag{14}$$

Lacey took n_a as 0.0225 in the Lower Bari Doab system. Equation (14) does not account for bedforms and other bed features and is quite simple.

For the same bed material size and concentration, equation (6) can be viewed as a sediment transport equation in a regime channel if

$$Q_S = K\, P V^3 \tag{15}$$

where K is a constant but would change for different sediment sizes or bed slopes. Equation (15) is valid for bed material transport if the transport is small, say below 5 tons/day/ft. For larger rates, Stevens and Nordin (1987) found that the transport was proportional to V to a much larger power. Then,

$$C = \frac{Q_S}{Q} = \frac{KPV^3}{PRV} = \frac{KV^2}{R} \tag{16}$$

Comparing equations (16) and (6),

$$f = 0.73 \frac{C}{K} \tag{17}$$

In China, the suspended sediment transport concentration is expressed as (Ding, 1985):

$$C^{1/n_a} = K_1 \frac{V^2}{gR} \frac{V}{V_a} \tag{18}$$

where n_a and K_1 are constants, and V_a is the fall velocity. Lacey's theory would require that V^2/R is constant and that V/V_a would be constant in a regime channel.

Steven and Nordin (1987) have given a critical account of the regime theory and Lacey's theory in particular. They have shown that Lacey's f as used is indeed a kind of roughness factor rather than a sediment concentration factor. They reasoned that a more definitive relation between f and the concentration of sediment carried by the channel and a better relation between n_a and the size and configuration of the bed forms on the beds of the channels were needed. Using the momentum equation, Blench (1957) derived the Darcy-Weisbach friction factor as

$$f = 2.2\left(\frac{v}{Vb}\right)^{1/4} \tag{19}$$

in which v is kinematic viscosity and b is channel bed width. It is difficult to explain the relation between f and b given by equation (19). Simons and Albertson (1963) found Lacey's equations to fill well into the sandbed and cohesive banks classification. Gill (1968) explained Lacey's equations using Darcy-Weisbach relation, Brown-Einstein equation for sediment transport, and continuity equation for flow. He, however, differentiated between f as a function of V and R and f as a function of R and S. Based on the measurements of velocity, depth, width, slope, and sediment load collected by the Alluvial Channel Observation Program (ACOP) in Pakistan (Tarar and Choudri, 1979), the following equations were proposed:

$$f_{VR} = 0.75\frac{V^2}{d} \tag{20}$$

$$f_{RS} = 192\,(dS^2)^{1/3} \tag{21}$$

$$f_{D50} = 1.76\,D_{50}^{1/2} \tag{22}$$

where d is hydraulic depth defined as A/W and W is the width of the water surface. The other equation for slope is

$$S = \frac{f_m^{5/3}}{1{,}830\,Q^{1/6}} \tag{23}$$

where

$$f_m = (f_{VR}\ f_{RS})^{1/2} \tag{24}$$

Since regime channels have different f_{RS} and f_{VR}, the use of f_m is not justified.

3.2 Power-Function Theory

The power-function theory was pioneered by Leopold and Maddock (1953) and Wolman (1955), is empirical and is based on measurements of channel geometry and the corresponding water and sediment discharge. Equations (1a, b) were derived as power function relations and can be obtained either graphically or using regression analyses. Such equations have been developed for both at-a-station geometry and downstream geometry. It is assumed that the data belong to rivers and canals in equilibrium or quasi-equilibrium, and this assumption forms the basis for deriving equations (1a) and (1b). Thus, it is essentially analogous to the regime theory. The regime theory is a concept and the regression analysis is a fitting technique to apply the concept to measured data. The discussion in the previous section is based on the power-function theory.

3.3 Tractive Force Theory

The tractive force theory, developed by Lane (1937, 1955), is based on the application of the principle of a limiting tractive force for a boundary of any given material under nonscouring conditions. A limiting force for an alluvial channel is defined by the force which is just sufficient to initiate the movement of particles which would otherwise remain on the bed and banks of the channel. Lane (1955) assumed the following: (1) At and above the water surface the side slope are at the angle of repose. (2) When the side slope is zero, the flow-wise tractive force alone suffices to cause the incipient motion. (3) Particles are held against the bed by the component of the submerged weight of the particles acting normal to the bed. (4) The tractive force acts in the direction and is equal to the weight of the water above the area on which the force acts. (5) Particles on the channel periphery are at a condition of incipient motion. The lift and drag forces of water on the particles and the downslope component of the gravity force are balanced by the friction force between the particles. The lift and drag forces are directly proportional to the tractive force. The force balance yields a relation between the flow depth, the maximum depth, side slope and the friction angle. From this relation, the top width, cross-sectional area, wetted perimeter, hydraulic depth, and hydraulic radius can be derived.

The concept of limiting tractive force can be applied to channel banks as well as bed, with proper allowance for the gravitational component of the stress of noncohesive materials with an inclined face. Evaluation of this limiting stress depends on experiments, and the Shields diagram for incipient motion is widely accepted to that end. It postulates a granular bed on the threshold of motion, and relates the cross-sectional geometry and weight of individual particles to the shearing force of the fluid. Thus, the design of a stable channel is based on the premise securing a distribution of the tractive force along the sides and bottom of channels such that the magnitude of this force at all points will be sufficiently large to prevent sediment deposits in unacceptable quantities and at the same time it will be small enough to prevent unacceptable scour. Then, limiting values of the tractive force are computed for various conditions that a channel must satisfy. This approach leads to a set of relations between hydraulic geometry parameters and discharge.

3.4 Threshold Channel Geometry Theory

Threshold conditions are produced over a long period of time and a large number of precipitation events. The alluvial material remaining on the bed and the banks of stream channels in a small watershed are subject to conditions just sufficient to initiate movement of this material. The channels are called threshold channels and the discharge forming the threshold shapes the threshold discharge. The overland flow area and the boundaries of the channel system will be in equilibrium until subjected to a precipitation event greater than which produced the threshold channel. Following the arguments presented in the tractive force theory, Li (1975) utilized the tractive force theory to derive the geometric relations for a threshold channel: trop width, cross-section area, wetted perimeter, hydraulic depth, hydraulic radius. The bankfull discharge can be employed as a good measure of the threshold discharge. For discharges less than this discharge, particles on the surfaces and boundaries of a channel do not move appreciably. The threshold discharge can be obtained from the continuity equation. The limiting tractive force was derived by the force balance wherein the resultant drag force and the downstream component of the submerged weight of the particles are balanced by the friction force between the particles. It was assumed that the local tractive force varied directly as the weight of the fluid above the area. Li (1975) derived the at-a-site and downstream hydraulic geometry relations for the threshold channel and tested the at-a-site relations using the data of Brush (1961) and the downstream relations using the data of Judd and Peterson (1969). Good agreement was reported.

3.5 Channel Stability Theory

A stable channel must be in equilibrium, exhibiting no systematic erosion or deposition. Such a channel may, however, undergo short-term adjustments to random hydrologic fluctuations. Thus, equilibrium epitomizes the channel stability. Under the influence of gravity, a river tends to lower its potential energy and attain a higher degree of stability. This suggests that the lower the energy of the river the more stable it will be. For a unit length of a river, the potential energy relative to the adjacent downstream segment can be expressed in terms of the slope of the water surface. Under fluctuating natural conditions, the river will tend to lower its potential energy to move towards the minimum water slope and highest channel stability subject to the existing constraints. Thus, the result is similar to that implied by the theory of minimum stream power, that is, a tendency toward minimum water slope, given constant channel discharge. Since maximum stability leads to equilibrium, it might be plausible to identify the channel form under such a condition. As noted by Schumm (1981), all rivers have varied patterns and relative stabilities, when in equilibrium they all are at or near a state of probable maximum equilibrium.

The stability of a channel is related to the bed material movement. Greater movement of material results in lower stability, even if the sediment movement, on the average, is balanced by the influx of sediment coming from upstream. In alluvial rivers, the bed material movement and bed form development depend on the flow regime, such that a higher flow regime produces higher sediment movement and more developed bed forms. Since the flow regime can be expressed as proportional to the Froude number, slope, and the stream power per unit area (Simons and Senturk, 1977), the highest degree of stability of the river or the equilibrium state can be represented, under existing constraints, by the minimum values of these hydraulic factors. Under this condition, the channel will have the lowest bed material movement and highest stability. Thus, the channel stability theory is derived by minimization of the Froude number, slope and stream power. In the ensuing discussion, it is seen that several theories, such as the minimum Froude number theory, the maximum friction factor rate theory, and minimum stream power dissipation rate, are special cases of this general theory.

3.6 Similarity Principle

Neglecting the effect of viscous shear, Engelund and Hansen (1967) derived a hydraulic resistance equation for dune-covered beds in that the total loss of mechanical energy was divided into two parts: one due to flow expansion and other due to friction. It was then hypothesized that alluvial streams tend to adjust their bed roughness according to the similarity principle. For two different rivers or flow systems to be completely similar, the dimensionless effective stresses are equal, expressing dynamic similarity, slopes are equal and the ratio of flow depth to mean fall diameter is the same. Based on these considerations, equations for depth, width, slope, and meander length as functions of water and sediment discharge as well as grain size were derived. These equations were comparable with regime equations. More specifically, the exponents of discharge were almost the same but those of grain size were a little different. For depth and slope, the exponents of discharge were the same but they were different for width, however.

3.7 Hydrodynamic Theory

The hydrodynamic theory is based on the principles of conservation of mass and momentum for water and sediment (Einstein, 1942; Kalinske, 1947; Bagnold, 1956). Thus far, it has been applied to only downstream hydraulic geometry. Smith (1974) pioneered this theory. By applying the continuity equation for flow of water, (2) the Manning equation for water flux in the downstream direction, (3) sediment continuity equation, and (4) sediment flux relation in the longitudinal direction, Smith (1974) derived the downstream hydraulic geometry for steady-state channels, subject to three conditions: (1) Sediment mass is conserved during transport; (2) the channel has a form just sufficient to transport its total water discharge, and (3) the channel has a form just sufficient to transport its total sediment load. Three assumptions were made for deriving the form parameters of width, depth, velocity and slope: (a) The channel has a finite width, which is essential for specifying boundary conditions; (b) the channel is carved in noncohesive materials, with most of the sediment transport occurring close to the channel bed, which is approximately valid for a large class of channels; and (c) there is freedom to choose a time scale for which the channel has an essentially steady-state form (Schumm and Litchy, 1964). The governing equations were solved using a similarity principle. The basis of these equations is presented by Smith and Bretherton (1972). The hydraulic geometry relations derived by Smith (1974) corresponded well with those of Leopold and Maddock (1953). The analytical approach developed by Smith (1974) is promising and insightful but surprisingly it has received little attention. This theory has yet to be applied to the at-a-site hydraulic geometry.

3.8 Theory of Maximum Sediment Transport Rate

White et al. (1982) presented an approach which is variational in principle and which assumes that an alluvial channel adjusts its slope and geometry to maximize its transport capacity. In other words, for a given discharge and slope, the width of a channel adjusts itself to give a maximum transport rate. The maximization of transport rate is equivalent to minimization of slope. Six variables were considered to describe a channel system: the average velocity, the average depth, slope, discharge, sediment concentration, and channel width. The governing equations relating these variables are the continuity equation for water flow, sediment transport formulas, flow resistance equation, and the condition that the sediment transport should be maximized or equivalently the stream power should be minimized. Using the sediment transport formulas of Ackers and White (1973) and the friction relationships of White et a. (1982), together with the principle of maximum sediment transporting capacity and a variational principle, White et al. (1981, 1986) derived regime relationships for width, depth and slope of a channel in equilibrium for a wide range of practical applications. Comparisons with laboratory data showed that predictions of depth were excellent, except for very large sand channels and for meandering laboratory channels, and the prediction of slopes showed scatter when compared with observations.

3.9 Theory of Maximum Friction Factor

From an examination of published data on bed forms, channels with artificial roughness elements, meandering channels and bed armoring, Davies and Sutherland (1980, 1983) proposed the hypothesis of maximum friction factor: "If the flow of a fluid past an originally plane boundary is able to deform the boundary to a non-planar shape, it will do so in a such a way that the friction factor increases. The deformation will cease when the shape of the boundary is that which gives rise to a local maximum friction factor. Thus, the equilibrium shape of a non-planar self-deformed flow boundary or channel corresponds to a local maximum friction factor." Davies and Sutherland (1983) defined the friction factor by the Darcy-Weisbach relation whose right side involves width, depth, slope and discharge. Thus, maximization of the friction factor implies maximization of the right side of this equation. This hypothesis was found to be satisfactory for low flow regime bed forms by Davies (1980). Bridge (1981) reported maximum friction factors at bankfull stage in meander bends. Whittaker and Jaeggi (1982) observed a tendency toward maximizing of friction factors during development of step-pool structures. Under certain combinations of independent variables the maximization of friction factor hypothesis is equivalent to minimization hypotheses. Davies and Sutherland (1983) justified their hypothesis for determining the river behavior on the premise that it is compatible with the characteristics of turbulent flows and nonlinear processes.

3.10 Theory of Maximum Flow Efficiency

With the introduction of a channel form factor, Huang and Nanson (2000) illustrated the self-adjusting mechanism of alluvial channels with the basic flow relations of continuity, resistance, and sediment transport. Natural channels adjust their form and attain maximum flow efficiency. Under such conditions, rivers exhibit regular hydraulic geometry relations at dominant or bankfull stage. The maximum flow efficiency (MFE) was defined as the maximum sediment transporting capacity flow per unit available unit stream power. Thus, the theory of maximum flow efficiency combines the theory of maximum sediment transporting capacity and the theory of minimum stream power. They also reasoned that these three theories can also be explained in terms of the physical principle of least action.

Using the continuity equation, Lacey's flow equation (Lacey, 1957-58), DuBoys' sediment transport formula (Graf, 1971), and a non-dimensional channel shape factor defined by the width/depth ratio, Huang and Nason (2000) derived optimum channel geometry relations. They also provided an explanation of the physical evidence for maximum flow efficiency. The hydraulic geometry relations compared well with those of Lane (1955) and others.

3.11 Theory of Least Channel Mobility

For transporting a specific volume of water and sediment, a channel can have different widths and depths. Therefore, the discharge of water and sediment constitutes only a necessary but insufficient condition to determine channel morphology. Thus, for a given width, different values of depth and velocity can be obtained. Likewise, for a given depth different values of width and velocity can be obtained. In natural channels, the range of variations of cross-sectional form and width and depth is relatively small. For sections of stable channels having identical water and sediment characteristics as well as local boundary conditions, the cross-sections have similar forms. Dou (1964) hypothesized that for a given water discharge and sediment load and boundary conditions, channels with different cross-sections possess different mobility and in the process of deformation the channel tends to assume such a form that its mobility would be minimal. Such a tendency of the channel is called the principle of minimum channel mobility. It should be noted that channels existing under different conditions have different mobility.

A direct measure of channel mobility is the amplitude of the longitudinal and cross-sectional deformation of the channel. The amplitude depends on the magnitudes of fluctuations of water discharge and sediment load as well as the rate of channel erosion. For a given section, the channel mobility depends on the relative velocity and the ratio of the channel width to channel depth. Field observations indicate that the wider and shallower the channel, the greater its mobility. Also, the greater the stream velocity and consequently the greater the force acting on the channel, the less the channel mobility. Therefore, the ratio of the active force to the force representing the stability of the channel sediment particle load is another channel mobility. Using a generalized index of channel mobility, Dou (1964) derived hydraulic geometry relations which compared well with those of Leopold and Maddock (1953) as well as the regime equations (Blench, 1952, 1957; Lacey, 1929-30).

3.12 Theory of Least Resistance

Remette (1980) hypothesized that flow in a channel follows the line of the least resistance. This means that the potential energy spent is minimum and the kinetic energy recovered is maximum, and if possible the river slope is equal to the valley slope but not more. It was supposed that for a given discharge, the sediment discharge has to be maximum all along the rive bed. Thus, the river bed must organize itself according to the following: (1) Manning-Strickler relation for a bankfull discharge, (2) maximum value of water slope equal to the valley slope, (3) maximum bed load discharge, and (4) maximum energy yield: the ratio of potential energy (for excavating the bed to the final form) and the recovering kinetic energy. He employed the Meyer-Peter-Mueller bed load relation for sediment load. Following this hypothesis, Remette (1980) derived hydraulic geometry relations for straight and meandering single beds, and multiple meandering beds. His relations compared well with experimental data and those of Leopold and Wolman (1957).

3.13 Theory of Minimum Froude Number

Rivers adjust toward an equilibrium condition which can be measured by the channel stability. Since channel stability is closely related to the bed material movement, the bed material movement and bed form development depend on the flow regime, indicating that a higher flow regime produces higher sediment movement and more developed bed forms. The Froude number is recognized to be one of the key hydraulic factors governing the flow regime. Jia (1990) hypothesized that the minimum Froude number would lead to channel stability and equilibrium condition. He then went on to develop a theory comprising the water continuity equation, sediment concentration given by Yang (1972, 1987) and Manning-Strickler equation. Based on computer simulation, Jia (1990) found that Froude number was minimum at certain channel geometry under given constraints. Data from many alluvial rivers and stable canals were used to test the hypothesis. The results were in good agreement with data.

3.14 Theory of Minimum Energy Dissipation Rate

Yang et al. (1981), Yang and Song (1986), and Yang (1972 1987) employed the theory of minimum energy dissipation rate to derive hydraulic geometry relations. According to the theory, when a channel is equilibrium, its rate of energy dissipation is at its minimum and this minimum value depends on the boundary conditions the channel has to satisfy. The governing equations include (1) an equation expressing the total rate of energy dissipation required to transport water and sediment through a river reach, (2) the Manning–Strickler equation, and (3) an equation for sediment concentration in terms of the dimensionless unit stream power (Yang, 1973). Restricting their analysis to approximately rectangular channels and minimizing the total rate of energy dissipation, they were able to derive width and depth relations with discharge which agreed with laboratory experiments of Barr et al. (1980), and the exponents of these relations were within the range of variations of measured data from river gaging stations. The theoretical values of hydraulic exponents for channel depth and slope derived from the theory of minimum energy dissipation rate were 9/22 and –2/11, respectively (Yang and Song, 1981). These values are very close, respectively, to the usually mentioned values of 0.41 for gaging stations in the United States and –1/6 for the regime formula used in India and Pakistan.

3.15 Theory of Minimum Stream Power

Chang (1979a, b) hypothesized the concept of minimum stream power which states that “for an alluvial channel, the necessary and sufficient condition of equilibrium occurs when the stream power per unit channel length is a minimum subject to given constraints. Hence, an alluvial channel with water discharge and sediment load as the independent variables tends to establish its width, depth and slope such that the stream power is a minimum. Since the water discharge is a given parameter, the minimum stream power means minimum channel slope." For a stream in equilibrium, the channel geometry, slope and velocity are governed by the water discharge and sediment load. For three unknowns, namely width, depth and slope, three independent relations are needed which included (1) a resistance relation, (2) sediment transport formula, and (3) stream power relation.

Employing the flow resistance equation developed by Engelund and Hansen (1967), the bed load formulas of DuBoys (Graf, 11971) and Einstein and Brown (Simons and Senturk, 1977), Chang (1979a, b, 1980, 1988) derived hydraulic geometry relations and channel patterns. He showed that a stable channel configuration corresponds to a minimum stream power per unit channel length and for small values of water discharge and sediment load, there exists a unique minimum, indicating a unique stable channel configuration. When the slope of this unique stable channel equals the valley slope, the channel pattern is straight. Above a certain threshold valley slope, the stream power has two minima, indicating two possible channel configurations and slopes. Whenever multiple channel slopes exist on a unique valley slope, the river becomes sinuous. Highly sinuous rivers which have small width/depth ratio occur on flatter slopes. As the valley slope increases, rivers become more braided and less sinuous. The reason for development of meandering is the expenditure of the minimum stream power per unit channel length. With its multiple channel configurations and slopes, a meandering river minimizes its stream power expenditure as well as its sediment load, subject to physical constraints.

For design of stable channels, a unique solution of slope, width and flow depth is needed. With reasonably reliable methods available for prediction of form and grain roughness, it is possible to predict flow velocity and depth with sufficient accuracy for a specified bed width. The bed slope for uniform flow in alluvial channels depends on the quantity and quality (i.e., size) of sediment being transported. Thus, sediment transport and friction equations are sufficient to predict the slope and flow depth for a specified water discharge, sediment load and bed width. To bring a closure to the problem of determining a unique solution, another equation is needed for determining the channel width. To that end, by analogy with the theory of least work, Chang (1980) hypothesized that the dependent variables, width, depth and slope, must have such values that the total rate at which work is done upon the water and sediment mixture by external forces must be minimum. In this way, with an assumed width, the depth and slope are computed for a given water discharge, sediment discharge and sediment size. The channel dimensions leading to the minimum slope are supposed to be the regime dimensions.

3.16 Theory of Minimum Energy Degradation Rate

Brebner and Wilson (1967) employed the principle of minimum rate of energy degradation, which is a special case of the principle of minimum entropy production rate used in irreversible thermodynamics, to derive a generalized excess energy gradient equation which then yielded a set of hydraulic geometry relations. This principle is also similar to the principle of least work. According to this principle, for an isothermal system in equilibrium the rate of energy degradation is minimum permitted by the boundary conditions and the applicable phenomenological laws. Considering two-phase flow for water and sediment, they expressed the generalized excess energy gradient equation as a Durand-type equation and substituted into the expression for energy-dissipation rate and thus derived equations for the portion of the flow associated with the bed. Then, by minimizing the energy dissipation rate they derived hydraulic geometry relations which were compatible with the regime relations.

3.17 Thermodynamic Entropy Theory

Yalin and Da Silva (1999) explained the meaning of regime channels and the regime channel formation criterion on the basis of the second law of thermodynamics (entropy increase law) and Gibbs' equation. They characterized an alluvial channel by three characteristics: width, flow depth, and slope. A channel tends to acquire a regime state when one of the energy-related characteristics tends to acquire a minimum value. Thus, these three characteristics can be derived by solving (1) the resistance equation, (2) transport equation, and (3) minimum energy-characteristic equation which has been defined variously. Using thermodynamic principles, Yalin and Da Silva (1999) derived a final equilibrium state equation in terms of the uniform flow velocity or the minimum Froude number. Then they derived expressions for flow depth and width. Thus, they presented a methodology for computing a regime channel for a given constant discharge.

3.18 Theory of Minimum Variance

Employing the theory of minimum variance, Langbein (1964) derived power relationships between a hydraulic variable, such as velocity, depth, width, or slope and bankfull discharge. His fundamental postulate is that a change in stream power is accommodated by the channel change encompassing as equal a change of each component of power as possible, the components of power being velocity, depth, width, and slope. This condition of equal change is met when the sum of the variances of the components of power is a minimum. Langbein (1964) discussed three examples which are meaningful in discussion of hydraulic geometry relationships: (1) Response to changes in flow over a sand bed between the fixed walls of a circulating flume at constant discharge: This example has one degree of freedom, i.e., the liberty to change roughness. (2) The accommodation of a river channel, at a given cross-section, to changing discharge. This example also has one degree of freedom. (3) A river in a humid region has the liberty to adjust its profile, velocity, depth and width to accommodate the downstream increase in discharge. There are three degrees of freedom in this case. Although Kennedy et al. (1964) severely criticized Langbein's approach, the basic tenets are insightful indeed.

Knighton (1977b) presented an alternative derivation of the minimum variance hypothesis. His main argument was that the stream would approach or converge to a limit state or quasi-equilibrium state through a sequence of channel adjustments. By arguing in Euclidean space terms, the minimum variance hypothesis was shown to be a special case of the more general problem. Knighton showed how the variance hypothesis might be used to examine transitory states in natural streams and how a stream might approach a more probable state through adjustment of its channel form.

Williams (1978) examined the theory of minimum variance in regard to its validity and its ability to predict observed hydraulic exponents. He identified velocity, depth, width, bed shear stress, friction factor, slope (energy gradient), and stream power for use of the theory. If the slope is constant for a particular station, then only the first five of these variables need to be considered. To that end, he considered five cases: (1) cross-sections where both width and slope are approximately constant; (2) stations with cohesive but non-vertical banks where water surface slope is constant; (3) streams in which the slope at a station is constant but the entire flow boundary is loose and readily eroded; (4) stations having one firm and one loose bank; and (5) channels in loose, readily erodible material as in case (3) but with water surface slope varying with discharge.

Dozier (1976) examined the validity of the variance minimization principle by studying changes in the longitudinal distribution of the bed shear-stress components over a two-month period in two reaches, one meandering and one straight, of a rapidly eroding supraglacial stream. The coefficients of variation of bed shear stress, skin resistance, and form resistance decreased in both reaches. Except for skin resistance, the values were lower for the curved reach than for the straight reach.

Riley (1978) examined the role of the minimum variance theory in defining the regime characteristics of the lower Namoi-Gwydir drainage basin. The mean condition of a stream results from minimization of the variation of certain free and constrained variables within the channel system. He noted that the theory had a great value in predicting possible range of channel shapes, given a range of hydraulic conditions, but was of limited value in predicting a hydraulic condition with any degree of certainty and geometry.

3.19 Theory of Maximum Entropy

Langbein (1964), Scheidegger and Langbein (1966), and Yang et al. (1981) amongst others emphasized that equations (1a, b) correspond to the case when the channel is in equilibrium state. By hypothesizing that corresponding to this state, when a channel adjusts its hydraulic variables, the adjustment is shared equally among the hydraulic variables, Deng and Zhang (1994) employed the principle of maximum entropy and derived morphological equations. Their equations were based on the assumption that for a given discharge the flow depth and width were independent variables among five hydraulic variables. They did not advance any justification for this key assumption. However, in practice the channel is seldom in an equilibrium state and this means that the adjustment among hydraulic variables will be unequal. It is not clear as to the exact proportion in which the adjustment will be shared among variables. Nevertheless, two points can be made. First, there will be a hierarchy of hydraulic geometry relations, depending on the adjustment of hydraulic variables. Second, the adjustment can explain the variability in the parameters (scale and exponents) of these relations. Cheema et al. (1997) applied the theory of maximum entropy to determine the stable width of a channel.

3.20 Theory of Maximum Entropy-Minimum Energy

Singh et al. (2003) coupled the principle of maximum entropy and the principle of minimum energy dissipation rate for deriving hydraulic geometry relations. Following the principle of minimum energy dissipation rate, the spatial variation of the stream power of a channel for a given discharge is accomplished by the spatial variation in channel form (flow depth and channel width) and hydraulic variables, including energy slope, flow velocity and friction. Thus, it is possible to proportion the adjustment of stream power by friction, channel width, and flow depth. Furthermore, it is hypothesized that the change in stream power is distributed among the changes in flow depth, channel width, flow velocity, slope, and friction, depending on the boundary conditions the channel has to satisfy. Williams (1967, 1978) found from an analysis of data from 165 gaging stations that a channel adjusted all its hydraulic parameters in response to changes in the influx of water and sediment and that self-adjustments were realized in an evenly distributed manner among factors.

According to the principle of maximum entropy (Jaynes, 1957), any system in equilibrium state under steady constraints tends to maximize its entropy. When a river reaches a dynamic (or quasi-dynamic) equilibrium, the entropy should attain its maximum value. The principle of maximum entropy (POME) states that the entropy of a system is maximum when all probabilities are equal, i.e., the probability distribution is uniform or rectangular. Applying this principle in conjunction with the principle of minimum energy dissipation rate to a river in its dynamic equilibrium leads to a hierarchy of downstream hydraulic geometry relations.

4 CONCLUDING REMARKS

It is clear from the above discussion that although a number of theories of at-a-site and downstream hydraulic geometry have been developed, it is not clear how these theories compare. Comparison of these theories using the same data is lacking and should be pursued. Furthermore, it is also not clear which theory should be applied where and under what conditions? Some of the theories require more data than others. The survey of literature made in this study suggests that there is plenty of data available for different rivers in different countries but efforts to archive these data and organize them into a database have not been reported. Despite all the work done and new theories developed during the past half a century, the classic work of Leopold and Maddock (1953) still remains the benchmark contribution. This then raises a question if real progress has indeed been achieved in the area of hydraulic geometry. Another area that needs greater attention is the watershed geometry and evolution of channel networks. The work on hydraulic geometry of channels serves as an excellent starting point to move on to the development of a theory of drainage basin geometry and channel network evolution. This will permit integration of channel hydraulics and drainage basin hydrology and geomorphology.

REFERENCES

Abrahams, A. D., 1984, Channel networks: A geomorphological perspective. Water Resources Research, Vol. 20, pp. 161-188.

Ackers, P., 1964, Experiments on small streams in alluvium. Journal of the Hydraulics Division, Proc. ASCE, Vol. 90, No. HY4, pp. 1-37.

Ackers, P. and White, W. R., 1973, Sediment transport: New approach and analysis. Journal of the Hydraulics Division, ASCE, Vol. 99, No. HY11, pp. 2041-2060.

Allen, P. M., Arnold, J. G. and Byars, B. W., 1994, Downstream channel geometry for use of in planning-level models. Water Resources Bulletin, Vol. 30, No. 4, pp. 663-671.

Bagnold, R. A., 1956, The flow of cohesionless grains in fluids. Philosophical Transactions, Royal Society of London, Series A, Vol. 249, No. 964, pp. 235-297.

Barr, D. I. H., Alan, M. K. and Nishat, A., 1980, A contribution to regime theory relating principally to channel geometry. Proceedings, Institution of Civil Engineers, Vol. 69, pp. 651-670.

Blench, T., 1952, Regime theory for self-formed sediment bearing channels. Transactions, American Society of Civil Engineers, Vol. 117, Vol. 383-408.

Blench, T., 1957, Regime Behavior of Canals and Rivers. 138 pp., Butterworths, London.

Blench, T., 1969, Coordination in mobile-bed hydraulics. Journal of Hydraulics Division, Proc. ASCE, Vol. 95, Nol. HY6, pp. 1871-1988.

Betson, R. P., 1979, A geomorphic model for use in streamflow routing. Water Resources Research, Vol. 15, No. 1, pp. 95-101.

Brebner, A. and Wilson, K. C., 1967, Derivation of the regime equations from relationships for pressurized flow by use of the principle of energy-degradation rate. Proceedings, Institution of Civil Engineers, Vol. 36, pp. 47-62.

Bridge, J. S., 1981, A summary of flow and sedimentary process studies on the River South Esk, Glen Clova, Scotland. Absttr. Mod. Ancient Flow Systems, p. 19, University of Keele, U. K.

Brush, L. M., 1961, Drainage basins, channels, and flow characteristics of selected streams in central Pennsylvania. U. S. Geological Survey Professional Paper 282-F, pp. 145-175.

Carlston, C. W., 1969, Downstream variations in the hydraulic geometry of streams: special emphasis on mean velocity. American Journal of Science, Vol. 267, pp. 499-509.

Chang, H. H., 1979a, Geometry of rivers in regime. Journal of the Hydraulics Division, ASCE, Vol. 105, No. HY6, pp. 691-706.

Chang, H. H., 1979b, Minimum stream power and river channel patterns. Journal of Hydrology, Vol. 41, pp. 303-327.

Chang, H.H., 1980, Geometry of gravel stream. Journal of the Hydraulics Division, ASCE, Vol. 106, No HY9, pp. 1443-1456.

Chang, H.H., 1988, Fluvial Processes in River Engineering. John Wiley & Sons, New York.

Cheema, M. N., Marino, M. A. and de Vries, J. J., 1997. Stable width of an alluvial channel. Journal of Irrigation and Drainage Engineering, Vol. 123, No. 1, pp. 55-61.

Chitale, S. V., 1970, River channel patterns. Proceedings of Hydraulics Division, ASCE, Vol. 96, No. HY1, pp. 201-222.

Chitale, S. V., 1973, Theories and relationships of river channel patterns. Journal of Hydrology, Vol. 19, pp. 285-308.

Chong, S. E., 1970, The width, depth and velocity of Sungei Kimla, Perak. Geographica, Vol. 6, pp. 72-63.

Church, M., 1980, On the equation of hydraulic geometry. Department of Geography, University of British Columbia, Vancouver, Canada.

Coates, D. R., 1969, Hydraulic geometry in a glaciated region. Transactions, American Geophysical Union, Vol. 50, pp. 149.

Davies, T. R. H., 1980, Bed form spacing and flow resistance. Journal of Hydraulis Division, ASCE, Vol. 106, No. HY3, pp. 423-433.

Davies, T. R. H. and Sutherland, A. J., 1980, Resistance to flow past deformable boundaries. Earth Surface Processes, Vol. 5, pp. 175-179.

Davies, T.R.H. and Sutherland, A.J., 1983, Extremal hypotheses for river behavior. Water Resources Research, Vol. 19, No.1, pp. 141-148.

Deng, Z. and Zhang, K., 1994, Morphologic equations based on the principle of maximum entropy. International Journal of Sediment Research, Vol.9, No.1, pp. 31-46.

Ding, L. 1985, Computation of deposition and scouring in reservoirs. Lecture Notes of the Training Course on Reservoir Sedimentation, Int. Res. and Train. Centre on Erosion and Sedimentation, Beijing, China, IV-6.

Dou, G. R., 1964, Hydraulic geometry of plain alluvial rivers and tidal river mouth. Journal of Hydraulic Engineering (in Chinese), No. 2, pp. 1-13.

Dozier, J., 1976, An examination of the variance minimization tendencies of a supraglacial stream. Journal of Hydrology, Vol. 31, pp. 359-380.

Dury, G. H., 1976, Discharge prediction, present and former, from channel dimensions. Journal of Hydrology, Vol. 30, pp. 219-245.

Einstein, H. A., 1942, Formulas for transportation of bed-load. Transactions, ASCE, Vol. 107, pp. 561-575.

Engelund, F. and Hansen, E., 1967, A monograph on sediment transport in alluvial streams. Teknisk Vorlag, Copenhagen, Denmark,

Gill, M. A., 1968, Rationalization of Lacey's regime flow equations. Journal of Hydraulic Division, ASCE, Vol. 94, No. HY4, pp. 983-995.

Graf, W. H., 1971, Hydraulics of Sediment Transport. McGraw-Hill Book Company, New York.

Haque, M. I. and Mahmood, K., 1985. Form friction factor in sandbed canals. Third U. S.-Pakistan Binational Symposium on Mechanics of Alluvial Channels, Lahore, Pakistan.

Hey, R. D., 1978, Determinant hydraulic geometry of river channels. Journal of Hydraulics Division, ASCE, Vol. 104, No. HY 6, pp. 869-885.

Hey, R.D. and Thorne, C.K., 1986, Stable channels with moble gravel beds. Journal of Hydraulic Engineering, ASCE, Vol.112, No.8., pp.

Heede, B. D., 1972, Influences of a forest on the hydraulic geometry of two mountain streams. Water Resources Bulletin, Vol. 8, No. 3, pp. 523-530.

Howard, A. D., 1980, Thresholds in river regimes. Thresholds in Geomorphology, pp. 227-258, Allen and Unwin, Winchester, Massachusetts.

Huang, H. Q. and Nason, G. C., 2000, Hydraulic geometry and maximum flow efficiency as products of the principle of least action. Earth Surface Processes and Landforms, Vol. 25, pp. 1-16.

Inglis, C. C., 1946, Meanders and their bearing in river training. Maritime Paper No. 7, Institution of Civil Engineers, London.

Jaynes, E.T., 1957, Information theory and statistical mechanics, I. Physical Review, Vol.106, pp. 620-630.
Jia, Y., 1990, Minimum Froude number and the equilibrium of alluvial sand rivers. Earth Surface Processes and Land Forms, Vol. 15, pp. 199-209.
Judd, H. E. and Peterson, D. F., 1969, Hydraulics of large bed element channels. Report No. PRWG 17-6, 115 pp., Utah Water Resources Research Laboratory, Logan, Utah.
Kalinske, A. A., 1947, Movement of sediment as bed load in rivers. Transactions, American Geophysical Union, Vol. 28, pp. 615-620.
Kellerhals, R., 1966, Stable channels with gravel-paved beds. ASCE Water Resources Engineering Conference, Reprint 330, 38 pp., Denver, Colorado.
Kennedy, J. F., Richardson, P. D. and Sutera, S. P., 1964, Discussion of "Geometry of River Channels by W. B. Langbein." Journal of Hydraulics Division, Proc. ASCE, Vol. 90, No. HY , pp. 332-341.
Kennedy, R. G., 1895, On the prevention of silting in irrigation canals. Proceedings, Institution of Civil Engineers, Vol. 119, pp. 281-290.
Klein, M., 1981, Drainage area and the variation of channel geometry downstream. Earth Surface Processes and Landforms, Vol. 6, pp. 589-593.
Knighton, A. D., 1972, Changes in braided reach. Geological Society of America Bulletin, Vol. 83, pp. 3813-3922.
Knighton, A. D., 1974, Variation in width-discharge relation and some implications for hydraulic geometry. Geological Society of America Bulletin, Vol. 85, pp. 1069-1076.
Knighton, A. D., 1975, Variations in at-a-station hydraulic geometry. American Journal of Science, Vol. 275, pp. 186-218.
Knighton, A. D., 1977a, Short-term changes in hydraulic geometry. River Channel Changes, New York, pp. 101-119.
Knighton, A. D., 1977b, Alternative derivation of the minimum variance hydpothesis. Eological Society of America Bulletin, Vol. 83, pp. 3813-3822.
Knighton, A. D., 1987, River channel adjustment-The downstream dimension. River Channels: Environment and Process, pp. 95-128, Basil Blackwell, Oxford.
Kolberg, F. J. and Howard, A. D., 1995, Active channel geometry and discharge relations of U. S. piedmont and midwestern streams: The variable exponent model revisited. Water Resources Research, Vol. 31, No. 9, pp. 2353-2365.
Lacey, G., 1929-1930, Stable channels in alluvium. Proceedings, Institution of Civil Engineers, Vol. 229, pp. 259-384.
Lacey, G., 1946, A general theory of flow-in alluvium. Journal of the Institution of Civil engineering. London, Vol.27, 16-47, Vol.28, pp. 425-451.
Lacey, G., 1957-1958, Flow in alluvial channels with sand mobile beds. Proceedings, Institution of Civil Engineers, London, Vol. 9, pp. 146-164.
Lane, E. W., 1937, Stable channels in erodible materials. Transactions, ASCE, Vol. 102, pp. 123-142.
Lane, E. W., 1955, Design of stable canals. Transactions, ASCE, Vol. 120, pp. 1234-1260.
Lane, L. J. and Foster, G. R., 1980, Modeling channel processes with changing land use. Proceedings, ASCE Symposium on Watershed Management, Vol. 1, pp. 200-214.
Langbein, W.B., 1964, Geometry of river channels. Journal of the Hydraulics Division, ASCE, Vol. 90, No. HY2, pp. 301-311.
Langbein, W. B. and Leopold, L. B., 1964, Quasi-equilibrium states in channel morphology. American Journal of Science, Vol. 262, pp. 782-792.
Leopold, L. H., 1953, Downstream change of velocity in rivers. American Journal of Science, Vol. 25, pp. 606-624.
Leopold, L. H. and Langbein, W. B., 1962, The concept of entropy in landscape evolution. U. S. Geological Survey Professional Paper 500-A, 20 pp.
Leopold, L. B. and Maddock, T. J., 1953, Hydraulic geometry of stream channels and some physiographic implications. U. S. Geological Survey Professional Paper 252, 55 p.
Leopold, L. B. and Miller, J. P., 1956, Ephemeral streams-Hydraulic factors and their relation to drainage net. U. S. Geological Survey Professional paper 282-A, 38 p.
Leoplod, L. B. and Skibitzke, H. E., 1967, Observations on measured rivers. Geografika Annales, Vol. 49A, No. 2-4, pp. 247-255.
Leopold, L. B. and Wolman, L. B., 1957, River channel patterns: braided, meandering and straight. U. S. Geological Survey Professional Paper 282-B, U. S. Government Printing Office, Washington, D. C.
Leopold, L. B., Wolman, M. G. and Miller, J. P., 1964, Fluvial Processes and Geomorphology. Freeman and Co., San Francisco, California.
Li, R. M., 1975, Mathematical modeling of response from small watershed. Unpublished Ph. D. dissertation, 212 pp., Colorado State University, Fort Collins, Colorado.
Lindley, E. S., 1919, Regime channels. Proceedings, Punjab Engineering Congress, Vol. 7, p. 63.
McConkey, S. A. and Singh, K. P., 1992, Alternative approach to the formulation of basin hydraulic geometry equations. Water Resources Bulletin, Vol. 28, No. 2, pp. 305-312.

Mackin, J. H., 1963, Rational and empirical methods of investigation in geology. The Fabric of Geology, Wesley Publishing Co, Reading Massachusetts, pp. 135-163.
Miller, J. P., 1958, High mountain streams: Effect of geology on channel characteristics and bed material. New Mexico Bureau of Mines and Mineral Resources, Memo 4, Socorro, New Mexico.
Miller, T. K. and Onesti, L. J., 1979, The relationship between channel shape and sediment characteristics in the channel perimeter. Geological Society of America Bulletin, Vol. 90, pp. 310-304.
Nanson, G. C. and Huang, H. Q., 1999, Anabranching rivers: Divided efficiency leading to fluvial diversity. Varieties of Fluvial Form, Wiley, Chichester, Sussex, England.
Osterkamp, W. R. and Hedman, E. R., 1982, Perennial-streamflow characteristics related to channel geometry and sediment in Missouri River basins. U. S. Geological Survey Professional Paper 1242, 37 pp., Washington, D. C.
Park, C. C., 1977, World-wide variations in hydraulic geometry exponents of stream channels: An analysis and some observations. Journal of Hydrology, Vol. 33, pp. 133-146.
Parker, G., 1979, Hydraulic geometry of active gravel rivers. Journal of Hydraulic Division, Proc. ASCE, Vol. 105, No. HY9, pp. 1185-1201.
Phillips, P. J. and Harlin, J. M., 1984, Spatial dependency of hydraulic geometry exponents in a subalpine stream. Journal of Hydrology, Vol. 71, pp. 277-283.
Ponton, J. R., 1972, Hydraulic geometry in the Green and Birkenhead river basins, British Columbia. Mountain Geomorphology: Geomorphological Processes in the Canadian Zcordillera, pp. 151-160, Tantalus Research Limited, Vancouver, Canada.
Remette, M., 1980, A theoretical approach on fluvial processes. Proceedings of International Symposium on River Sedimentation, Theme No. 3, Beijing, China, pp. C16-1 to C-16-22.
Rhodes, D. D., 1977, The b-f-m diagram: Graphical representation and interpretation of at-a-station hydraulic geometry. American Journal of Science, Vol. 277, pp. 73-96.
Rhodes, D. D., 1978, World wide variations in hydraulic geometry exponents of stream channels: an analysis and some observations-Comments. Journal of Hydrology, Vol. 33, pp. 133-146.
Rhoads, B. L., 1991, A continuously varying parameter model of downstream hydraulic geometry. Water Resources Research, Vol. 27, No. 8, pp. 1865-1872.
Richards, K. S., 1973, Hydraulic geometry and channel roughness-a nonlinear system. American Journal of Science, Vol. 273, pp. 877-896.
Richards, K. S., 1976, Complex width-discharge relations in natural river sections. Geological Society of America Bulletin, Vol. 87, pp. 199-206.
Richards, K. S., 1982, Rivers: Form and Process in Alluvial Channels. Metheun, London.
Riley, S. J., 1978, The role of minimum variance theory in defining the regime characteristics of the lower Namoi-Gwydir basin. Water Resources Bulletin, Vol. 14, pp. 1-11.
Scheidegger, A. E. and Langbein, W. B., 1966, Probability concepts in geomorphology. U. S. Geological Survey Professional Paper 500-C, C-14 p.
Schumm, S. A., 1960, The shape of alluvial channels in relation to sediment type. U. S. Geological Survey Professional Paper 352B, pp. 17-30.
Schumm, S. A., 1981, Evolution and response of fluvial systems: Sedimentologic implications. Society of Economic Paleontologists and Mineralogists Special Publications 31.
Schumm, S. A. and Litchy, R. W., 1964, Time, space and causalty in geomorphology. American Journal of Science, Vol. 263, pp. 110-119.
Scott, C. H., 1966, Suspended sediment and the hydraulic geometry of channels. Erosion and Deposition in the Loess-Mantled Great Plains, Madison Creek Drainage Basin, Nebraska, Geological Survey Professional paper 352-H, Washington, D. C..
Simons, D. B. and Albertson, M. L., 1963, Uniform water conveyance channels in alluvial material. Transactions, American Society of Civil Engineers, Vol. 128, pp. 65-107.
Simons, D. B. and Senturk, F., 1977, Sediment Transport Technology. Water Resources Publications, Littleton, Colorado.
Singh, K. P. and Broeren, S. M., 1989, Hydraulic geometry of streams and streams habitat assessment. Journal of Water Resources Planning and Management, Vol. 115, No. 5, pp. 583-597.
Singh, V. P., Deng, Z. Q. and Yang, C. T., 2003, Downstream hydraulic geometry relations using the principles of minimum energy dissipation rate and maximum Entropy, Part I and Part II. Water Resources Research, under print.
Smith, T. R., 1974, A derivation of the hydraulic geometry of steady-state channels from conservation principles and sediment transport laws. Journal of Geology, Vol. 82, pp. 98-104.
Smith, T. R. and Bretherton, F. P., 1972, Stability and conservation of mass in drainage basin evolution. Water Resources Research, Vol. , pp. 1506-1524.
Stall, J. B. and Fok, Y.-S., 1968, Hydraulic geometry of Illinois streams. University of Illinois Water Resources Center, Research Report No. 15.

Stall, J. B. and Yang, C. T., 1970, Hydraulic geometry of 12 selected stream systems of the United States. University of Illinois Water Resources Research Center, Research Report No. 32,

Stevens, M. A. and Nordin, C. F., 1987, Critique of the regime theory for alluvial channels. Journal of Hydraulic Engineering, Vol. 113, No. 11, pp. 1359-1380.

Tarar, R. N. and Choudri, A. M., 1979, Behavioral evaluation of some Pakistan canals, part 1. Int. Symp. On Mechanics of Alluvial Channels, Lahore, Pakistan, June 26-29.

Thornes, J. B., 1970, The hydraulic geometry of stream channels in the Xingu-Araguaia headwaters. The Geographical Journals, Vol. 136, pp. 376-382.

Vanoni, V. A., Ed. 1975, Sedimentation Engineering. American Society of Civil Engineers, New York.

White, W. R., Bettess, R. and Paris, E., 1982, Analytical approach to river regime. Journal of the Hydraulics Division, Proceedings of ASCE, Vol. 108, No. HY10, pp. 1179-1193.

White, W.R., Bettes, R. and Wang, S.Q., 1986, A study on river regime. Hydraulics Research Station, Wallingford, England.

White, W. R., Paris, E. and Bettess, R., 1981, River regime based on sediment transport concepts. Report IT 201, Hydraulics Research Station, Wallingford, England.

Whittaker, J. B. and Jaeggi, M. N. R., 1982, On the origin of step pool systems in mountain streams. Journal of Hydraulics Division, ASCE, Vol. 108, No. HY6, pp. 758-773.

Williams, G.P., 1967, Flume experiments on the transport of a coarse sand. U.S. Geological Servey professional paper 562-B, Washington, D.C.

Williams, G.P., 1978, Hydraulic geometry of river cross-sections- Theory of minimum variance. Professional paper 1029, U.S. Geological Survey, Washington, D.C.

Wilcox, D. N., 1971, Investigation into the relations between bedload transport and channel shape. Geological Society of America Bulletin, Vol. 82, pp. 2159-2176.

Wolman, M. G., 1955, The natural channel of Brandywine Creek, Pennsylvania. U. S. Geological Survey Professional Paper 271, Washington, D. C.

Yang, C. T., 1972, Unit stream power and sediment transport. Journal of the Hydraulics Division, ASCE, Vol. 98, No. HY10, pp. 1805-1826.

Yang, C. T., 1987, Energy dissipation rate approach in river mechanics. Sediment Transport in Gravel-Bed Rivers, pp 735-766, John Wiley &sons, New York.

Yang, C. T. and Song, C. C. S., 1981, Discussion of “A contribution to regime theory relating principally to channel geometry,” by D. I. H. Barr, M. K. Allan and A. Nishat. Proceedings, Institution of Civil Engineers, Part 2, Vol. 71, pp. 961-965.

Yang, C. T. And Song, C. C. S., 1986, Theory of minimum energy and energy dissipation rate. Chapter 11 in Encyclopedia of Fluid Mechanics, Gulf Publishing Company.

Yang, C. T., Song, C. C. and Woldenberg, M. T., 1981, Hydraulic geometry and minimum rate of energy dissipation. Water Resources Research, Vol. 17, pp. 877-896.

Yu, B. and Wolman, M. G., 1987, Some dynamic aspects of river geometry. Water Resources Research, Vol. 23, No. 3, pp. 501-509.