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A new stochastic approach for controlling point source river pollution

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Abstract

A stochastic optimization framework is presented for exploring cost-effective river basin water quality management strategies. The approach is based on a stochastic extension of the classical BOD-DO model which leads to a joint chance constraint on water quality. For solving the problem derived, a novel global optimization algorithm is combined with Monte Carlo simulation. The methodology is illustrated by numerical examples.

Introduction

The question of establishing efficient regional wastewater treatment strategies that meet preassigned water quality criteria has been the subject of investigations for several decades. In spite of the uncertainties related to this problem, most of the deterministic formulations were analyzed earlier. Recognizing, however, the significance of partially unknown, uncertain or statistically fluctuating processes in the environment, and making use of developments in modeling and numerical solution approaches, an increasing number of stochastic models have been introduced in recent years. Beck (1987) provides an overview of descriptive water quality studies, briefly referring also the the issue of decision making under uncertainty. Among the numerous recent studies in stochastic (river, lake or groundwater) quality modeling and management, one may refer, for example, to Beck and van Straten (1983); Brill et al. (1979), Chen and Papadopoulos (1988), Dorfman et al. (1972), Ellis (1987), Fujiwara et al. (1986, 1987), Lohani and Thanh (1979), Padgett and Papadopoulos (1979), Pintér and Somlyódy (1986), Somlyódy and van Straten (1986), Somlyódy and Wets (1985), Tung and Hathhorn (1988) or Wagner and Gorelick (1987), with many additional references therein. Without going into detail here on these works, we emphasize that, due to differences between the water quality problems analyzed and the modeling techniques used, they differ considerably with respect to their level of complexity, the background information needed, the solution methodology to be applied, and the details in which the results are presented.

In this paper, a stochastic extension of the classical BOD-DO model is presented. For its solution, a new global optimization algorithm is applied, in combination with stochastic simulation. Numerical examples illustrate the approach. For additional details, the reader is referred to Boon et al. (1989).

Modeling

The model-system applied here for river water quality managment consists of two main parts called descriptive and optimization models. First, deterministic (base) model variants will be briefed; this is followed by their stochastic extension.

Descriptive model. Water quality models are based on the physical and biochemical mass balance of the water system considered. Here we shall apply the classical Streeter-Phelps model (cf. Streeter and Phelps, 1925; or the treatise in Dobbins, 1964; Orlob, 1983) which approximates the co-evolution of dissolved oxygen deficit (D, mg/l) and BOD concentration (L, mg/l) on a river section as a function of their initial values D(0) and L(0), time (t, day), reaeration, oxidation and decomposition coefficients (k_1, k_2 and k_3, day^{-1}). Assuming steady-state conditions, the coupled differential equations

$$dL/dt = - k_3 L \quad ; \quad dD/dt = - k_1 D + k_2 L \tag{1}$$

by integration lead to

$$L(t) = L(0) \exp(-k_3 t)$$

$$D(t) = k_2 L(0)/(k_1 - k_2) [\exp(-k_2 t) - \exp(-k_1 t)] + D(0) \exp(-k_1 t) \tag{2}$$

Introducing streamflow velocity u (m/s) and longitudinal distance x = ut (m), one can obtain (by solving dD(x)/dx = 0 w.r.t. x) that the maximum (critical) oxygen deficit is reached at

$$x^c = u \ln A/[k_2(f-1)] \tag{3}$$

where

$$f = k_1/k_2 \ , \ A = f[1 - (f-1) D(0)/L(0)];$$

further, its value equals

$$D^c = k_2 L(0)/(k_1 - k_2) \{[A]^{-1/(f-1)} - [A]^{-B}\} + D(0)[A]^{-B} \tag{4}$$

where

$$B = f(f-1) \ .$$

Note that the values x^c and D^c are of primary importance in the context of water quality management.

As is well-known, the derivation of Equations (1)-(4) is based on a number of strong simplifying assumptions. This point will be considered to a certain extent later, when introducing stochastic model extensions.

Management model. Given the analytical solutions (2)-(4) and the DO saturation level D^s, the maximal deficit uniquely corresponds to the minimal DO level which occurs:

$$D^{min} = D^s - D^c$$

This way, assigning water quality (minimum DO) standards to all river sections of interest (each, by supposition, having a single significant point-source pollution discharge), the effects of site-specific wastewater treatment can be directly traced. Let us introduce index $i = 1,...,I$ for subsequent river sections and, simultaneously, for the river water quality monitoring sites. Denote by r_i the treatment efficiency (removal rate) to be chosen at each pollution discharge point. Further, let D_i^{min}, D_i^s and D_i^o be the minimum occurring DO level, the saturation level, and the standards for $i = 1,...,I$. Then, assuming simple linear additivity, the prescribed overall effect of wastewater treatment as observed at the monitoring stations can be expressed by:

$$L_{0i} = [L_{bi}Q_{ri} + (1-r_i)\, L_i q_i]/(Q_{ri} + q_i)$$

$$D_i^{\,c} = F_i(D_{0i},\, L_{0i},\, k_1,\, k_2) \qquad i = 1,\ldots,I \tag{5}$$

$$D_i^{min} = D_i^{\,s} - D_i^{\,c} \geq D_i^{\,o}$$

where:

D_{0i}, L_{0i}	=	initial DO deficit and BOD values on reach i;
L_{bi}, Q_{ri}	=	background BOD concentration and streamflow at the beginning of reach i;
L_i, q_i	=	input BOD concentration and discharge flow at section i;
F_i	=	critical deficit D_i^c as a function of D_{0i}, L_{0i}, k_1 smf k_2 (see 3 and 4).

Note that D_{0i} and L_{0i} can be iteratively determined by using (2), given the topographical location of all wastewater discharge sites and accepting a deterministic description. Under these assumptions, the water quality constraints implicitly depend on the r_i's. Observing that to each individual wastewater treatment option a yearly discounted operations and maintenance cost value can be associated, our basic management model can be formulated as

$$\text{minimize} \ \sum_i C_i(r_i) \tag{6}$$

under the feasibility constraints (5). Frequently, the cost functions C_i are convex, thus allowing piecewise linearization. Note that most deterministic model formulations are traditionally approximated via linear programming.

Let us note that model (5)-(6) can only be interpreted as an initial approximation of more realistic problem formulations. Namely, streamflow, temperature and related physical/biochemical processes are subject to random fluctuations; therefore (at least) the constraint set (5) should be replaced by some appropriate ("more realistic") stochastic model component.

Stochastic extensions. There is no room in the present frames to go into detail on stochastic modeling, for example a recent survey by Wets (1983) or the related earlier references. Therefore, only a part of our analysis carried out in Boon et al. (1989) is summed below.

All statistically varying quantities, referred to above, can be modeled by introducing a corresponding random variable (rv), with an associated probability distribution function (pdf). The pdf's were then estimated from actual observations (collected on the River Zala, Hungary) and using earlier literature data. Further, from available modeling tools of stochastic programming we chose the well-known chance constrained model. More precisely, we used the following probabilistic constraint:

> {the joint probability of satisfying given minimum DO level constraints, for all river sections, is sufficiently high} (7)

In (7), the minimum DO level can be analytically determined, assuming that given is a joint random realization of the river hydrologic conditions, temperature and load conditions (initial discharge loads and treatment efficiencies). This way, via Monte Carlo simulation, statistical performance estimates can be produced for each decision variant considered. We emphasize here that, although separate chance constraints are often used, mostly for numerical reaasons, in the present context we felt the use of a joint constraint more realistic. This, in turn, implies a significant increase of model complexity and generally may necessitate the application of nonconvex optimization techniques.

Solution Method

The optimization model, consisting of the objective function (6), constraints (7), and obvious technological limitations (lower/upper treatment efficiency bounds) can be formulated as:

$$\begin{aligned} &\text{minimize } \sum_i C_i(r_i) + w[\alpha - P\{\cdot\}]_+ \\ &P\{D_i^{min} \geq D_i^o \qquad i = 1,\ldots,I\} \geq \alpha \qquad (8) \\ &r_{i,min} \leq r_i \leq r_{i,max} \qquad i = 1,\ldots,I \end{aligned}$$

The notations D_i^{min} and D_i^o $i = 1,\ldots,I$ were introduced earlier; additionally, the symbol $P\{\cdot\}$ refers to the probability of the imbedded random event $\{\cdot\}$; α is a prescribed reliability level; $w > 0$ is a given multiplier for "penalizing" unfavorable deviations from the probability level prescribed; and $[h]$ equals by definition the maximum of h and zero.

As can be seen, the stochastic optimization model (8) has a fairly complicated structure: although its objective function can be approximated linearly and the treatment efficiency bounds define a simple rectangular region, the probabilistic constraint in (8) is highly nonlinear and its analytical form is, in general, unknown (recall that the probabilistic constraint involves the numerical evaluation of D_i^{min}). For this reason, a gradient-free (direct) optimization method was applied which is capable of finding the globally best solution (treatment configuration), even in the case when several different locally optimal decision variants may exist (note that this possibility cannot be a priori excluded, as a consequence of the unknown topology of the P constraint).

The optimization procedure is based on an adaptive partition and search of (generation of sample points in) the interval region. For details concerning theoretical features, implementation and some applications, see Pintér (1986a,b, 1988) and Pintér et al. (1986). For streamlining the optimization procedure, the interval feasible region (10) was first reduced on the basis of the following observation: if for all effluent discharges the maximal efficiency were taken at all previous plants, then this action obviously yields a lower bound of the removal rate to be selected there. This simple technique led in most calculations to significant reductions for the feasible region; following this, the global search procedure mentioned was carried out. In the numerical examples (four treatment plants to be chosen), our experience has shown that the best solution could well be approximated in a few hundred iterations (sequential selection of treatment configurations and their checking via Monte Carlo simulation). The results were finally checked via ex-post statistical performance analysis, in the course of which a more detailed stochastic simulation was accomplished, concerning the best solution found.

Illustrative Results

Consider the following numerical example, based on actual data collected weekly on the River Zala in Hungary in the period 1975-1986. Note that a hypothetical sewage discharge configuration is examined under the given natural conditions.

Stochastic model input

-- daily streamflow (m^3/s): a lognormal distribution is fitted (mean value = 7.63; standard deviation = 5.45);

-- water temperature (C^o): streamflow-dependent, resulting from regression analysis;

-- background DO: water temperature-dependent, resulting from regression analysis;

-- stream velocity, river depth (m), parameters k_1, k_2 and saturation DO level: all of them being streamflow- and/or temperature-dependent, as described by known hydrological and empirical equations (Boon et al., 1989).

Deterministic model parameters and background information

-- joint probability bound of meeting the critical deficit standards for all river sections: varying between 0.5 and 0.977;

-- initial conditions (based on a hypothetical set of municipal wastewater discharges):

reach	BOD-load (kg/d) (untreated)	discharge flow (m^3/s)	DO (mg/l) standard	initial r bounds
1	15000.	0.434	4.0	[0, 0.97]
2	15000.	0.434	6.0	[0, 0.98]
3	6000.	0.170	6.0	[0, 0.97]
4	21000.	0.608	6.0	[0, 0.97]

here again, r refers to the treatment efficiency (see 8);

-- wastewater treatment costs: piecewise linear approximation is applied (corresponding to treatment phases of 30, 85, 95, 98 and 99 percent removal rate), only investment costs are considered.

The stochastic optimization runs were performed, systematically increasing the probability bound in meeting the joint chance constraint. For comparison with these results, corresponding deterministic computations were completed. In these runs, that streamflow value was taken (as main input) which is exceeded with the probability given in the joint chance constraint.

For illustration, we first present a single example in detail:

	Deterministic Results			Stochastic Results		
	Probability of Exceedence: 0.9			Reliability Level: 0.9		
Reach	Lower Bound Vector r	Optimal Vector r	Total Costs	Lower Bound Vector r	Optimal Vector r	Total Costs
1	0.623	0.846	2356.3	0.620	0.850	2356.4
2	0.639	0.779		0.684	0.777	
3	0.127	0.689		0.105	0.656	
4	0.717	0.830		0.711	0.837	

Note that costs are given in million Hungarian forints (Ft, 1 Ft ≈ 0.02 US Dollar). Next we present some more results, in summarized form:

Deterministic Results		Stochastic Results	
Probability of Exceedence	Total Costs	Joint Probability Bound	Total Costs
0.500	2163.69	0.500	2230.89
0.700	2286.76	0.700	2259.62
0.800	2300.65	0.800	2301.97
0.900	2356.33	0.900	2356.44
0.950	2417.76	0.950	2503.80
0.970	2482.82	0.970	2624.83
0.975	2511.72	0.975	2700.94
0.980	2515.69	0.977*	4341.69**

* the maximal reliability level which can be satisfied
** the corresponding maximal summed treatment cost value

From this table one see that, in our specific example, the deterministic results approximate very well the stochastic ones up to a certain threshold (probability) level; then the results tend to deviate in a fastly increasing manner. This phenomenon seems to be the consequence of the interdependent structure of randomly varying streamflow rates, water temperature, and background DO level. (In our example, we studied a case when Q and T were loosely related.) It is expected that the difference between deterministic and stochastic solutions increases if the dependence becomes stronger. A more comprehensive sensitivity study is under elaboration.

Summing up the results of the two approaches, one may say that, on one hand, easier and faster obtained deterministic results may provide very realistic information under problems-specific circumstances. On the other hand, assuming that proper, statistically sound background information is available, the stochastic model approach yields a far more detailed and realistic description. This in several cases (that may typically include the most important ones) will lead to significantly different results from the deterministic ones. This point is illustrated in detail by most of the references related to stochastic modeling.

The stochastic water quality modeling and management approach presented in this paper can be developed further into several directions, viz.

-- consideration of more refined descriptive submodels (e.g. hydrometeorological model, water quality model, etc.);

-- corresponding generalization of the base management model;

-- formulation, solution and comparison of different stochastic optimization model variants.

For more details on these issues, refer to Boon et al. (1989).

Closing the Gap Between Theory and Practice

This study was initiated within VITUKI, a research and advisory Institute for Water Resources Development of the Hungarian Government. It is a first survey of the potential and limitations of the proposed techniques and approaches. Although the results have not been used in practice, mostly realistic field data and cost functions have been applied to attain realistic results. This does not, however, imply that at this stage the approach can be directly used for decision making and planning; the primary goal has been the development of the tool. However, as VITUKI has a task in providing advice and technical bases for the kind of decisions discussed in this paper, the approach is not meant to be academic, but to result in a practical instrument for water quality management.

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