Parametric Uncertainty in Hydrologic Modeling

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ABSTRACT

IN most applications, parametric, hydrologic models are treated as deterministic. That is once the parameters are determined, the model always produces the same outputs for a given set of inputs. Parameters are generally treated as unknown constants. Once they are estimated, they are taken as constant. In fact, model parameters are generally estimated from observed data or from relationships that have been derived from observed data. The observed data generally includes some stochastic variables such as rainfall and runoff. Based on the premise that any function of a random variable is itself a random variable, this paper reviews theory and approaches enabling one to treat model parameters as uncertain random variables.

The report is a review of parametric uncertainty in hydrologic modeling. Several aspects of the topic are introduced but not treated in great mathematical detail. Many references to additional work are given that will enable the interested reader to pursue various aspects of the topic in detail.

INTRODUCTION

Hydrologic modeling has become commonplace over the past 25 years. Virtually all hydrologic design is based on the results of applying a hydrologic model. The ready availability of models and computers and the "user friendly" nature of many hydrologic models insures continued and virtual absolute reliance on such models.

The acceptance of hydrologic models has greatly improved our ability to perform complex, detailed hydrologic analyses. Many different designs can be evaluated at minimal cost once base line data are collected. Software is available today that not only does hydrologic analyses, but suggests appropriate model parameters in the form of "pop up" screens on microcomputers. Neat, professional looking reports can be prepared almost automatically.

Reliance on models has not been without cost. The ease of using models often reduces the time spent actually thinking about the situation being modeled. Routines are selected because they are readily available -not necessarily because they are the best for a given situation. Parameters are often selected with little thought to their actual validity and certainly with no thought to their variability. Model results are taken as "truth". Firms that have invested in getting a modeling system operational look for ways of recovering their investment by using the model as much as possible.

Certainly there is a reluctance to admit that model outputs are estimates - uncertain estimates. Even though point estimates of some quantity are generally provided, often considerable uncertainty exists. This uncertainty should be recognized and incorporated into the analysis of hydrologic systems. As concepts of risk analysis become more prevalent, it will increasingly be necessary to be able to quantify the probabilistic aspects of hydrologic model predictions. This report discusses some aspects of hydrologic uncertainty as reflected in model parameters and how this parametric uncertainty results in uncertain model outputs and hydrologic designs.

HYDROLOGIC MODELS

To limit the scope of this report somewhat it is necessary to define what is meant herein by a hydrologic model. Websters Dictionary (Guralnik, 1986) uses a shortened form of the Federal Council on Science and Technology's (1962) definition of hydrology to state that hydrology is:

> "the science dealing with the waters of the earth, their distribution on the surface and underground, and the cycle involving evaporation, precipitation, flow to the seas, etc.".

Admittedly the term "etc." needs to include a lot to satisfy everyone that this is a proper definition for hydrology. In any case most hydrologists have a pretty firm grasp of what hydrology is even if it can not be precisely defined.

The term "model", however, brings to mind many different things to hydrologists. Again appealing to Webster we find the following as possible definitions of model:

> "a generalized, hypothetical description, often based on an analogy, used in analyzing or explaining something"

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"a thing considered as a standard of excellence to be imitated".

If we take the first definition of model and substitute for the word "something" the word "hydrology", we have a reasonably good definition of a hydrologic model as used in this report.

Unfortunately it is becoming all too common to adopt the second definition of model and to use this "thing" to which we are applying the term "model" as a standard of excellence against which to judge all hydrologic estimates, all hydrologic procedures and to even teach "hydrology" in Universities, short courses and training programs. Many "hydrologists" have had little training in hydrology and a lot of training in modelology or

"the 'science' dealing with models".

Many "professionals" that deal in hydrology have had more detailed training in how to get a particular model or suite of models purporting to represent hydrology running on their particular computer system than they have in hydrology. This places great responsibility on model developers to insure that they clearly set forth the conditions for which their model is applicable, the limitations of the model and guidelines for estimating all of the parameters and inputs to the model.

Hydrologic models have been classified in a large number of ways. Some of the terms that have been used in model classification are deterministic, parametric, statistical, stochastic, physically based, empirical, blackbox, lumped, linear, nonlinear, distributed, theoretical, predictive, operational, research, design, similarity, iconic, analog, numerical, regression, event, continuous simulation, conceptual, etc.

I am going to limit this discussion to mathematical models. Mathematical models range from single prediction relationships to complex computer simulation algorithms. The mathematical basis for a model may be theoretical or empirical. A completely theoretical model would contain only relationships derived entirely from basic physical laws. Such laws are the conservation of mass, conservation of energy, laws of thermodynamics, etc. Empirical relationships are based on observations and/or experimentation. Manning's equation for uniform open channel flow is an empirical equation. Often empirical equations become so well accepted and entrenched in usage they are viewed as physical laws. The application of Darcy's equation for flow through a porous media is an example of this in hydrology.

There exist no completely theoretical operational models in hydrology. All hydrologic models contain empirical relationships. Thus we generally liberalize the definition of theoretical models to be models that include both a set of general or theoretical principles and a set of statements of empirical circumstances. A strictly empirical model is one that is based on no basic physical laws but contains only a representation of data (empirical results). Admittedly this is not a very clear distinction but such a distinction is only of academic interest anyway. Suffice it to say that no matter how sophisticated and detailed, *all* hydrologic models rely on empirical results to some extent.

For our purposes a hydrologic model will be defined as:

"a collection of physical laws and empirical observations written in mathematical terms and combined in such a way as to produce a set of results (outputs) based on a set of known and/or assumed conditions (inputs)".

There are many ways of "collecting" physical laws and empirical observations and of "combining" them to produce a model. Certain of these ways result in a formulation in which the use of a computer is desirable. Such models are called computer models. Computer models will be the focus of the remainder of the discussion in this paper.

Regardless of how models are classified, they can generally be represented as

$$\underline{O} = \underline{f}(\underline{I},\underline{P},t) + \underline{e} \quad \ldots\ldots [1]$$

where $\underline{O}$ is an nxk matrix of hydrologic responses to be modeled, $\underline{f}$ is a collection of l functional relationships, $\underline{I}$ is an nxm matrix of inputs, $\underline{P}$ is a vector of p parameters, t is time, e is an nxk matrix of errors, n is the number of data points, k is the number of responses, and m is the number of inputs.

Responses in $\underline{O}$ may range from a single number such as a peak flow or a runoff volume to a continuous record of flow, soil water content, evapotranspiration, and other quantities.

Model classification refers to the nature of $\underline{f}$.

The distinction between $\underline{I}$ and $\underline{P}$ is not always clear and not of extreme importance to the discussion here. Generally $\underline{I}$ represents inputs some of which are time varying such as rainfall, temperature, land use, etc. while $\underline{P}$ represents coefficients particular to a watershed that must be estimated from tables, charts, correlations, observed data, or some other means.

The error term $\underline{e}$, represents the difference between what actually occurs, $\underline{O}$, and what the model predicts, $\hat{\underline{O}}$.

$$\hat{\underline{O}} = \underline{f}(\underline{I},\underline{P},t) \quad \ldots\ldots [2]$$

$$\underline{e} = \underline{O} - \hat{\underline{O}} \quad \ldots\ldots [3]$$

A parametric model is a model having parameters that must be estimated in some fashion. The parameters may be estimated based on observed data (calibration), tables and/or charts (Manning's n or curve numbers), correlation type relations (regional analysis), analysis of site specific information (water holding capacities of soils, etc.), experience, an edict from an agency, or by some other means.

An empirical model is a model containing any empirical relationship. Empirical here means data based or based on observation. Even Darcy's law and Manning's equation are empirical equations.

A lumped model describes processes on a scale larger than a point. A completely distributed model would be one in which all processes are described at a point and then integrated over three dimensional space and time taking into account variations in space and time to produce the total watershed response.

Based on these somewhat restrictive definitions, *all* hydrologic models are to some degree parametric, empirical, and lumped.

A BRIEF LOOK BACK

It is not my intent to review the history of the developments in hydrologic modeling. I would like to remind the reader, however, that the basic foundations for hydrologic modeling have been in place for 30 years or so and what we are seeing today is heavily dependent on that early work. The advantage we have today is greatly enhanced convenience and speed in computations. Many of the current developments in hydrologic modeling are more heavily dependent on computational improvements than they are in improved representations of hydrologic processes when compared to models of 25 years ago.

The 1960's might be thought of as the golden years of hydrologic modeling. Digital computers were becoming widely available and hydrologic researchers began taking advantage of their power. The Stanford Project in Hydrologic Simulation was initiated in 1959 and under the general leadership of R. K. Linsley developed approaches to hydrologic modeling that continue to play a major role in the field to this day. An important development from this program was the initiative it provided in fostering research in the development of hydrologic models in several locations throughout the United States.

The most famous of the models developed at Stanford was the Stanford Watershed Model, SWM (Crawford and Linsley, 1962, 1966). In their 1966 report Crawford and Linsley stated:

> "The objective of the research is to develop a general system of quantitative analysis for hydrologic regimes. The most effective way for doing this has been to establish continuous mathematical relationships between elements of the hydrologic cycle. The operation of these mathematical relationships is observed and improved by using digital computers to carry the calculations forward in time. . . . As mathematical relationships are developed, every attempt is made to realistically reproduce physical processes in the model. Experimental results and analytical studies are used wherever possible to assist in defining the necessary relationships."

This statement of their research objective has been paraphrased and repeated by others in many locations throughout the world. It is significant that no criteria are given on which to judge the acceptability of the resulting model. This deficiency in statements of modeling objectiaves continues to this day in that it is rare for a model development research objective to include a statement concerning how good the model results must be before the model will be judged acceptable. Indeed there is no widely accepted criterion on which to base the evaluation of a model.

The SWM defined watershed flow and storage processes in mathematical terms and combined them so that a continuous record of estimated streamflow was generated in response to hourly rainfall and daily evapotranspiration. In developing the governing relationships, many model parameters were defined and a conceptual basis assigned to them. Estimation of the many parameters generally relied on a streamflow record and manually adjusting the parameters until a satisfactory estimate of streamflow was obtained.

In addition to the SWM, the Stanford program investigated the stochastic generation of rainfall (Pattison, 1965), the use of overland flow routing for small watersheds (Morgali, 1963), the use of queuing and storage theory (Bagley, 1964), and the impact of urbanization on flood peaks (James, 1965).

In 1966 Huggins and Monke (1966) presented the first developments of what has become known as ANSWERS (Beasley and Huggins, 1981). This model is representative of a class of models that breaks a watershed into a grid system, simulates the hydrology of each grid and the interaction of the grids with their neighbors, and integrates these results over the watershed to produce the total watershed response. Generally with models of this type the goal is to use physically based parameters since a set of parameters must be defined for each grid within the watershed.

In addition to general purpose models applicable to a wide range of watersheds, hydrologic models have been developed to address a specific question. The model developed at Iowa State University (Haan and Johnson, 1968a, 1968b; DeBoer and Johnson, 1971; Campbell et al., 1974) to evaluate the impact of subsurface drainage on flood flows in North Central Iowa represents such a model.

Since the 1960's there have been hundreds of papers written dealing with hydrologic modeling. ASAE has published a monograph entitled *"Hydrologic Modeling of Small Watersheds"* (Haan, Johnson and Brakensiek, 1982). Many new hydrology text books are appearing that have as a major point of emphasis the use of hydrologic models (Chow et al., 1988; Hromadka et al., 1987; Bedient and Huber, 1988). In the 1960's hydrologic modelers spent a great deal of time justifying why hydrologic models should be developed and their applicability to certain problems. Today the situation is reversed. Everyone wants to use models for every conceivable hydrologic problem and applications are frequently made well outside the verified domain of the model being used. Many practitioners would be unable to do hydrologic investigations without computer models.

In applying hydrologic models, two important problems must be considered: (a) Parameter estimation and (b) Model evaluation. These two considerations have been singled out for treatment in the remainder of this paper. They are not independent and are made more challenging by the multiobjectivity of hydrologic models and the stochastic nature of hydrologic events.

UNCERTAINTY

Before embarking on a discussion of parameter estimation, a brief discussion of uncertainty is in order. Vicens et al. (1975) have classified hydrologic uncertainty into three categories:

1. The inherent variability in natural processes.
2. Model uncertainty.
3. Parameter uncertainty.

The inherent variability in natural processes refers to variability in space and time of meteorologic factors such as rainfall, temperature, solar radiation, streamflow, etc. Often these processes are modeled as stochastic processes in deference to the strong apparently random component that is a part of their overall response. This source of variability also encompasses uncertainty in measured values of streamflow and other factors used in model evaluation and parameter estimation. When rainfall over a catchment is reported it is only an approximation to actual catchment rainfall based on a very small sample of the catchment area. Thus uncertainty exists as to the actual rainfall. The same may be said of other model inputs. Similarly, reported values of streamflow are estimates of actual streamflow and may be in error. This is especially true for estimates based on rating curves developed by inexperienced personnel. High flows are subject to error since they are rarely measured and must be based on an extrapolation of a rating curve. Often this extrapolation is into a region of overbank flows which might be quite different hydraulically from normal inbank channel flows used to develop the rating curve.

Model uncertainty arises because one can not be sure that a particular hydrologic process is completely and correctly modeled. For example, consider the infiltration and subsequent movement of water in a soil profile. The boundary and initial conditions may be correctly specified as well as the water content-water potential-hydraulic conductivity relationships. Certainly if we adopt a model such as the Green and Ampt (1911) model, we immediately recognize shortcomings in the model. The model only approximates the actual situation and thus introduces uncertainty into the analysis. But the same is true of a more detailed analysis. Richard's (1931) equation is generally well accepted as a model for describing this situation, yet questions remain. Are there conditions where non-Darcian flow occurs? What about macropore flow? What about the two-phase flow of water and air? Is one dimensional flow possible if spatial variability exists? What degree of approximation is introduced by the solution technique used in solving the flow equation for the conditions of interest? One might then conclude that even a solution based on Richard's equation is uncertain because of the uncertainty of the model itself and ones inability to precisely solve the governing equation in an actual problem of interest to a hydrologist.

Parameter uncertainty reflects incomplete models, incomplete information and inadequate parameter estimation techniques. Given a model and a set of circumstances, different parameters may be determined for the model by different individuals or by the same individual based on different sets of observed data. Given a "perfect" estimation technique, model parameters should asymptotically approach their "true" values as the amount of information (i.e. record length) used to estimate the parameters gets very large. True parameter values is used here in the sense of Kuczera (1983) as those values obtained using an objective estimation technique and an arbitrarily long sequence of data. Parameter estimates should be treated as random variables since their values depend on observed data which are themselves either stochastic or random variables or functions of random variables. Any variable that is a function of a random variable is itself a random variable (Haan, 1977).

Much of the perceived uncertainty in model parameters is the result of the approximate nature of the models containing the parameters. Models necessarily are incomplete in their description of hydrologic processes on a watershed scale. Many simplifying assumptions are made. Algorithms included in the model must represent all of the processes that are actually occurring on a watershed. When a particular process is not modeled or is incompletely modeled, other components of the model are forced to compensate for this model shortcoming. Consequently, physically based parameters lose their strict physical interpretation. These parameters are now reflecting processes they were not originally intended to represent. For example the SCS curve number model for predicting runoff volume from daily rainfall is a one parameter model with that parameter being the retention parameter, S, or its more familiar transform the curve number, CN. Obviously a model of this simplicity cannot represent all of the conditions and processes that combine to determine the volume of runoff resulting from a given volume of rainfall in a 24-h period.

Based on the above discussion, it is apparent that even for models not normally thought of as stochastic, random variables abound. One may consider as random variables some of the model inputs ($\underline{I}$), model parameters ($\underline{P}$), model outputs ($\underline{O}$), and observed data used in parameter estimation and model evaluation. Thus it follows that both parameter and model evaluation must rely to some extent on statistical considerations. The question to be addressed is how to estimate model parameters and how to evaluate models in the face of this variability.

PARAMETER ESTIMATION

As the name implies, parameter estimation is the process by which the parameters of a hydrologic model are estimated for a particular application. Rational parameter estimation must be tied to some criterion if a unique parameter set is to be found. Some criteria that might be used include (a) personal judgement of goodness of fit of simulated hydrographs to observed hydrographs, (b) direct measurement of physical properties in the field or in the lab, (c) indirect measurement of physical properties through their relationships with other hydrologic processes and watershed characteristics, (d) optimization of some objective function either computationally or by trial and error, (e) satisfaction of agency requirements, and (f) compliance with published tables and charts.

Some of the things that make parameter estimation for hydrologic models difficult are (a) specification of appropriate criteria for parameter selection, (b) correlation among parameters, (c) amount of computations involved in many models, (d) restrictions on appropriate values for some of the parameters, (e) non uniqueness of parameter sets for certain objective functions, (f) thresholding in some of the model relationships, and (g) errors in data. Parameter estimation is made more difficult by increasing the number of parameters to be estimated, the lack of correspondence between individual parameters and measureable physical properties of the catchment, multiple objectives, limited data, and pronounced seasonality in hydrologic regimes.

Problems in parameter estimation have been recognized for some time. Dawdy and O'Donnell (1965) investigated the possibility of obtaining an efficient automatic procedure for finding numerical values of the various of the various parameters of an overall watershed model. Beard (1967) and DeCoursey and Snyder (1969) addressed computer procedures for finding optimal values of parameters for a hydrologic model. Jackson and Aron (1971) reviewed parameter estimation techniques in hydrology. Most of the earlier papers approached parameter estimation from a mathematical rather than statistical point of view. An objective function, generally a minimization of a sum of squares, was defined and search techniques were employed to find the parameter set that optimized the objective function.

More recently Sorooshian (1983) reviewed parameter estimation techniques for hydrologic models. In his review the shift from a deterministic, mathematical interpretation of parameters toward a stochastic, statistical interpretation can be noted. Increasingly causal models are being viewed as "somewhat structured empirical constructs whose elements are regression coefficients with physical sounding names" (Klemes, 1982). Adoption of this viewpoint leads to parameter estimation in a statistical framework and focuses attention on treating a parameter as a random variable (rv) with a probability density function (pdf). Troutman (1985) made an extensive investigation of parameter estimation using this approach.

To this point a vast majority of the research on parameter estimation in hydrology has been for the case where a single objective is to be met. For example, this objective might relate to prediction of peak flows, storm runoff volumes, *or* daily streamflow. Diskin and Simon (1977) investigated 12 such univariate objective functions. Attempts to find parameter sets that meet multiple objectives such as peak flows, runoff volumes, *and* daily streamflow in some optimal sense have been scarce.

The use of multiple objective criteria for parameter estimation permits more of the information contained in a data set to be used and distributes the importance of the parameter estimates among more components of the model. For example if a continuous flow simulation model is optimized based on peak flows, parameters related to evapotranspiration (ET) may be poorly estimated. If the estimation criteria included both peak flow and ET, it is likely that the precision of the ET parameters would be greatly improved without an adverse impact on the peak flow parameters.

Edwards (1988) reported an example of inproved parameter estimates using multiobjective optimization criteria when applied to the SCS runoff model. When model parameters were estimated based solely on a minimization of prediction error sum of squares for peak flows, the resulting parameters would do a good job of predicting peak flows but gave very poor estimates of runoff volume. Changing the estimation criteria to include a measure of error sum or squares on peaks and volumes and their interactions had no appreciable impact on predictions of peaks but greatly improved runoff volume estimates. If interest lies only in runoff peaks, then the choice of the simpler univariate objective function may be appropriate; however, potential users of the model or the estimated parameters may rightfully be skeptical of the univariate optimization and/or the model itself when the poorly estimated SCS curve number parameter (which governs runoff volume) is compared to more conventional estimates of this parameter as commonly found in tables. Including a measure of prediction errors on volumes in the objective function overcame this problem and resulted in curve number estimates that were in agreement with conventional estimates.

Runoff typically accounts for only 10 to 35% of annual rainfall. Estimation of all model parameters based solely on runoff ignores 65 to 90% of the processes accounting for water loss from a catchment. The assumption that if runoff can be predicted then all model parameters must have been adequately determined has no clear justification. Including some measure of performance in the optimization that reflects some of the flow and storage processes occurring on a watershed in addition to runoff should improve the stability (reduce the variance (reduce the absolute error) of the estimated parameters.

Traditionally parameter estimation criteria have been tied to some measure of how well the model predicted streamflow agreed with observed streamflow. Especially for continuous flow simulation, these criteria are difficult to apply. An alternative measure of performance and thus a basis for parameter estimation is how well the model performs in a design situation. For example one might select the parameters of a daily flow model so that the design capacity of a reservoir to meet some water demand would be as close as possible to the capacity estimated based on observed streamflow data using the same capacity estimation algorithm.

Thus two issues central to parameter estimation are the criteria to be used and the stochastic properties of the estimates.

Parameters as Random Variables

Treating model parameters as rvs with pdfs is familiar in terms of the multiple regression model (Haan, 1977)

$$\underline{Y} = \underline{X}\underline{\beta} + \underline{\epsilon} \quad \ldots\ldots\ldots\ldots\ldots\ldots [4]$$

where $\underline{Y}$ is a nx1 vector of dependent variables, $\underline{X}$ is a nxp matrix of independent variables, $\underline{\beta}$ is a px1 vector of regression coefficients and $\underline{\varepsilon}$ is a nx1 vector of errors. Making the usual assumptions, $\underline{\beta}$ is estimated as $\hat{\underline{\beta}}$. $\hat{\underline{\beta}}$ has a p-variate normal distribution (Draper and Smith, 1966) with a mean vector given by

$$\hat{\underline{\beta}} = (\underline{X}'\underline{X})^{-1}\underline{X}'\underline{Y} \quad \ldots\ldots\ldots\ldots\ldots\ldots [5]$$

and a covariance matrix given by

$$\mathrm{cov}(\hat{\underline{\beta}}) = \sigma^2(\underline{X}'\underline{X})^{-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots [6]$$

where σ^2 is the variance of ε. The variance of a predicted value of Y at $\underline{X}_h$, $\hat{Y}_h$, is given by

$$\mathrm{var}(\hat{Y}_h) = \sigma^2 \; \underline{X}_h(\underline{X}'\underline{X})^{-1}\underline{X}'_h \mathrm{cov}(\hat{\underline{\beta}})\underline{X}'_h \quad \ldots\ldots\ldots [7]$$

Confidence limits on $\hat{Y}_h$ are given by

$$L = \underline{X}_h\hat{\underline{\beta}} - t_{1-\alpha/2,n-p}[\mathrm{var}(\hat{Y}_h)]^{1/2}$$

$$U = \underline{X}_h\hat{\underline{\beta}} + t_{1-\alpha/2,n-p}[\mathrm{var}(\hat{Y}_h)]^{1/2} \quad \ldots\ldots\ldots\ldots\ldots [8]$$

where t can be determined from a standard statistical table of the t distribution.

Thus when a regression model is used, it is possible to not only compute a point estimate for the dependent variable Y, but to investigate the probabilistic behavior of $\hat{Y}$ as well. $\hat{Y}$ may be the best estimate for Y or the expected value of Y, yet there is a 50% chance that $\hat{Y}$ underestimates and a 50% chance that it overestimates Y. From the above equations it can be seen that the probabilistic behavior of $\hat{Y}$ depends directly on the probabilistic behavior of the parameter estimates $\hat{\underline{\beta}}$.

When we quote point estimates for Y from a regression model, we are communicating only part of the information available concerning that estimate. Much more information is imparted if statements regarding confidence intervals on $\hat{Y}$ are provided as well. Admittedly these statistical confidence intervals may detract from the "coinfidence" a user might place in the estimate since generally for hydroloigic work confidence intervals are quite wide. To state authoritatively that the estimate for the 100-year flood based on a regional regression relationship is 50 m^3/s is somewhat more comforting than to state that there is a 95% probability that the values 25 to 80 m^3/s encompass the true 100-year flow. Yet the latter statement is a more accurate representation of our state of knowledge than the statement based on the single point estimate. The single point estimate implies unjustified accuracy in the estimate.

It is a natural extension of the above results to recognize that parameters estimated for hydrologic models depend to some extent on observed data which are random variables. Thus the parameters are random variables and estimates made with these parameters must also be random variables having a probabilistic structure. In essence we should be striving for ways to put confidence intervals on estimates made with hydrologic models just as we do with regression relationships. These confidence intervals will obviously depend on the probabilistic nature of the model parameters.

To further illustrate the concept of model parameters as rvs, consider a 24-h rainfall volume of P. If P were to occur on a watershed on 2 successive days, the volume of runoff, Q, attributable to P would be different for the two events. If P occurred on 2 days when all of the conditions on the watershed were identical except the time distribution of P during the 24-h period, the Q would again be different for the two rainfalls. The reasons that Q varies for a given P are many and need not be enumerated here. What is important is to recognize that Q for a given P may vary over a considerable range and may in fact be treated as a random variable having a pdf. Traditional rainfall-runoff models generally do not allow for a random variation in Q given P. For example the SCS curve number model

$$Q = (P - 0.2S)^2/(P + 0.8S) \text{ for } P > 0.2S \quad \ldots\ldots\ldots [9]$$

may be thought of as simply a transformation between the expected value of Q given P and P. The fact that Q varies randomly for a given P must be reflected in the retention parameter, S. This parameter can be calculated from

$$S = 5P + 10Q - 10\sqrt{Q^2+1.25PQ} \quad \ldots\ldots\ldots\ldots [10]$$

If calculated from a number of observed rainfall-runoff events, S is found to vary over a considerable range and may also be treated as a rv having a pdf. Exercises such as this have been carried out (Hjelmfelt et al., 1981; Haan and Schulze, 1987) and in several instances the lognormal distribution has been found to be a good descriptor of the pdf of S. For a given P the pdf of S and the pdf of Q are related through the curve number equation. A transformation from one pdf to the other is possible based on the method of derived distributions (Haan, 1977):

$$p_Q(q) = p_S(q)\left|\frac{ds}{dq}\right| \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots [11]$$

where the notation $p_X(x)$ refers to the pdf of the random variable X evaluated at X = x. If P is considered as a random variable as well

$$p_Q(q) = \int_u p_{S,P}(q,u)\left|J\left[\frac{s,p}{q,u}\right]\right| du \quad \ldots\ldots\ldots [12]$$

where u is a conveniently defined auxiliary variable. If S and P are independent in a probability sense, equation [12] is simplified since in this case $p_{S,P}(s,p) = p_S(s)p_P(p)$. Equations [11] and [12] can be used to get the pdf of Q and to place confidence intervals on Q (Haan and Schulze, 1987; Haan and Edwards, 1988).

Extending this reasoning to a multiparameter model, each of the parameters could be treated as a random variable with a pdf. In general the parameters may have a correlation structure among themselves making it necessary to treat them in the form of a multivariate pdf rather than a collection of univariate pdfs. Extension of equations [11] and [12] to multivariate models is theoretically straight forward but computationally intense.

The ability to evaluate the risk associated with a particular hydrologic decision requires the ability to state the probabilistic behavior of variables used in making that decision. Mays (1987) reviewed work related to determining the risk and reliability associated with hydraulic structures. Model prediction uncertainty was approximated based on a Taylor series expansion of the model about the mean parameter vector. Model prediction uncertainty arises from the combined effects of using uncertain parameters (where the uncertainty is expressed in terms of a variance) in a predictive model. Benjamin and Cornell (1970) set forth the equations used to approximate model prediction means and variances. A second order approximation for the mean is

$$\bar{O} = f(\bar{p}_1, \bar{p}_2, \ldots, \bar{p}_k) + \frac{1}{2}\sum_{i=1}^{k}\sum_{j=1}^{k}\left[\frac{\partial^2 f}{\partial p_i \partial p_j}\right]_{\underline{P}} \mathrm{cov}(p_i, p_j) \quad \ldots\ldots\ldots\ldots\ldots\ldots [13]$$

and a first order approximation for the variance is

$$\mathrm{var}(O) = \sum_{i=1}^{k} \sum_{j=1}^{k} \left[\frac{\partial f}{\partial p_i}\right]_{\bar{P}} \left[\frac{\partial f}{\partial p_j}\right]_{\bar{P}} \mathrm{cov}(p_i, p_j) \quad [14]$$

In these expressions the partial derivatives are evaluated at $\bar{P}$, the mean parameter vector.

Prasher et al. (1984) used first and second order analysis to evaluate uncertainty in drainage design. They presented a figure showing the expected water table height midway between drains and the expected height plus and minus one and two standard deviations.

If one is willing to make a distributional assumption using a two-parameter pdf for model output, O, the method of moments can be used to estimates the parameters of this pdf based on the mean and variance (as given by equations [13] and [14] for example). The pdf can then be used to make probabilistic statements about O. Again these probabilistic statements, even though approximate, impart a great deal more information than a simple estimate of the mean response.

Often parameter means, variances and even pdfs can be estimated from observed data. Basically all that is required is to estimate model parameters from several sets of data and then to look at the statistical properties of these estimates. A consistent criterion for parameter estimation must be used for this procedure to be meaningful. An example of this is finding the SCS runoff model retention parameter S to be approximately lognormally distributed (Hjelmfeldt et al., 1981; Haan and Schulze, 1987). Ben Jamaa (1988) found that the parameters of the van Genuchten (1980) equations for describing soil hydraulic properties could be described by a bivariate lognormal distribution.

Knowledge of parameter uncertainty provides considerable insight into model predictions and prediction uncertainty. Equations [11] and [12] and [13] and [14] suggest ways of transferring knowledge of parameter uncertainty to estimates of uncertainty in model predictions.

Parameter Estimation Criteria

There are several criteria that are commonly used in parameter estimation. I will limit this discussion to objective procedures as opposed to those procedures relying primarily on user judgement. Objective parameter optimization generally deals with some function of the error term, $\underline{e}$, of equation [1]. Generally a pdf for $\underline{e}$ must be assumed as well as other properties such as independence, constant variance and zero mean. Equations [1] to [3] show that $\underline{e}$ is a function of the parameters, $\underline{P}$.

Method of moments: The method of moments requires equating the first p sample moments of $\hat{e}$ with the first p population moments of the pdf of $\underline{e}$. In general the population moments will be a function of the p model parameters. Thus p equations in p unknowns result which must be solved for values of the p unknown parameters.

Least squares: In the least squares procedure, a sum, S, is defined as the sum of squares of the e_i

$$S = \Sigma\, e_i^2 \quad [15]$$

Values of the p parameters that minimize S are sought. For hydrologic models, numerical search techniqiues are generally employed in p-dimensional space.

Maximum likelihood: The likelihood function, L, of $\underline{e}$ is written as

$$L = \prod_{i=1}^{n} p(e_i|\underline{P}) \quad [16]$$

where $p(e_i|\underline{P})$ is the pdf of $\underline{e}$ given $\underline{P}$. Values of $\underline{P}$ are sought that maximize L. Again for hydrologic models, numerical search techniques are generally required.

Arbitrary objective functions: Any objective function or criterion function, C, can be used to find $\underline{P}$. In general

$$C = G[f_1(\underline{O}), f_2(\hat{\underline{O}})] = G[f_1(\underline{O}), f_3(\underline{P})] \quad [17]$$

where G is the arbitrary function, f_1 is a function of the observed values of O and f_2 is a function of the estimated values of O. Note that f_2 may be transformed to a function of the parameters, f_3, through equation [2]. Using numerical search techniques, $\underline{P}$ is sought that optimizes C. An example of this approach might be setting $f_1(O) = \ln(O)$, $f_2 = \ln(\hat{O})$, and $G = \Sigma[f_1(O)-f_2(\hat{O})]^2$ and finding $\underline{P}$ that minimizes G. This would be a minimization of the sum of squares of the differences in the logarithms of the observed and predicted outputs.

For all of the above four estimation procedures, $\underline{e}$ is a nx1 vector, point estimates for the parameters are obtained and a single objective is used. With these methods, several independent sets of data would be required to estimate the variance of the parameters and/or a pdf for the parameters.

Bayesian estimation: Bayesian estimation is fundamentally different from the above procedures in that it evolves from probabilistic considerations rather than some arbitrarily specified objective function. Bayesian estimation is concerned with the pdf of the parameters rather than point estimates. Point estimates, however, may be derived as the mode of the resulting parameter distribution.

Some references to Bayesian estimation are Box and Tiao (1973), Kuczera (1983), Vicens et al. (1975), Edwards (1988), Edwards and Haan (1988, 1989). Bayesian estimation has the advantage that multiple objectives may be incorporated into the analysis. Edwards (1988) shows that the point estimates for $\underline{P}$ may be found by minimizing the determinant of $\underline{S}(\underline{P})$, $|\underline{S}(\underline{P})|$, with respect to $\underline{P}$ where

$$\underline{S}(\underline{P}) = \underline{e}'\underline{e} \quad [18]$$

for the case where $\underline{e}$ is a nxk matrix of errors as in equation [1]. $\underline{S}(\underline{P})$ is a kxk matrix of sums of squares and cross products of errors. Since $|\underline{S}(\underline{P})|$ is simply a number (the determinant of $\underline{e}'\underline{e}$), numerical search procedures can be used to find the $\underline{P}$ that minimizes $|\underline{S}(\underline{P})|$. Note that if $\underline{e}$ is a nx1 vector, equations [15] and [18] are identical. In this case the Bayesian approach provides a statistical justification for the least squares procedure. Box and Tiao (1973) show that the posterior pdf of $\underline{P}$ is given by

$$p_{\underline{P}|\underline{O}}(\underline{p}|\underline{O}) \propto |\underline{S}(\underline{P})|^{-n/2} \qquad [19]$$

where the constant of proportionality is defined to insure that the pdf integrates to unity.

Other criteria: James and Burges (1982) present an excellent discussion of parameter estimation and estimation criteria. The points made there will not be repeated here.

If a model structure is such that only a subset of parameters impact one particular aspect of model output, those parameters may be estimated based on an optimization criteria related to that particular aspect of model output. For example a p-parameter model may be structured so that a subset p_1 of the p parameters governs runoff volumes and a second nonoverlapping subset p_2 governs peak flows. The parameters in p_1 could then be estimated based on a criterion related to volumes while the parameters in p_2 could be estimated based on a criterion related to peaks.

If in this case the parameters in p_1 and p_2 are all estimated based on peaks, the parameters in p_1 will be poorly determined and likely will have a large variance. While strict division of influence between parameter sets and outputs generally does not occur, it is common for some parameters to have relatively little influence on optimization criteria and thus be poorly determined. James and Burges (1982) define sensitivity coefficients that can help to identify this possibility. If the model is then used in a situation where the aspect of the model that is highly dependent on the poorly defined parameters is important, a questionable design may result. From the preceding situation, designing a storage reservoir with a model whose parameters were optimized on peaks would be a situation where a potentially poor design could result. Use of equation [18] partially overcomes this problem since prediction errors on both peaks and volumes can be incorporated into parameter estimation.

As an alternative to equation [18] to include multiple objectives, a weighted criterion function could be used. Such a function might be

$$C = w_1 \Sigma e_1^2 + w_2 \Sigma e_2^2 \qquad [20]$$

where the subscripts refer to different objectives (i.e. peaks and volumes), w is a weight and e is the error. The selection of the weights is arbitrary and may be done to reflect the relative importance of the two objectives. Equation [20] differs from equation [18] in that interaction between the two objectives is not included. For the case where k=2, equation [18] becomes

$$C = \Sigma e_1^2 + \Sigma e_2^2 - 2\Sigma e_1 e_2 \qquad [21]$$

which is similar to equation [20] with $w_1 = w_2 = 1$ and an interaction or covariance term added. Equation [20] can be generalized to any number of objectives. It is not necessary that all e_i used in an equation like equations [18] or [20] be related to flow. For example e_2 might refer to some measure of soil water content if observed data were available and the model provided estimates of soil water.

INCORPORATING UNCERTAINTY IN MODEL RESULTS

If we are able to estimate the mean and variance of model parameters, equations [13] and [14] can be used to estimate the mean and variance of models outputs. Assuming a pdf for these outputs, say a normal distribution, enables one to make estimates of the probabilities of model outputs within various limits. This information can also be transformed to probabilistic statements on the performance of a system designed based on the model outputs.

If a pdf for model parameters is available, equations [11] and [12] can be used to compute the pdf of a model output. In many situations, equations [11] and [12] may be difficult to apply since the functional relationship between the model output and model parameters may be in the form of a simulation model rather than an analytic function. In such a case, Monte Carlo sampling of the multivariate parameter distribution can be used to generate many sets of parameters. Each of these parameter sets can be used in the hydrologic model to generate a model output. The probabilistic behavior of the model output or some design based on the model output can be examined. Confidence intervals can be constructed or probabilistic statements can be made regarding the model outputs or hydrologic designs.

Edwards and Haan (1988, 1989) have used Bayesian estimation and Monte Carlo sampling of the parameter distributions to place confidence intervals on peak flow estimates for various return periods. As a result of this approach they were able to state, for example, that they were 90% confident that the 100-year flood was between two limits, q_l and q_u, with an expected value of q_{100} rather than simply stating that the 100-year peak flow is q_{100}. The width of the confidence intervals is a reflection of parameter uncertainty. Inclusion of confidence limits in statements of model predictions gives the user of the prediction an indication of the reliability of the estimate. The width of the confidence intervals is a function of parameter uncertainty which in turn depends on the ability of the model to simulate observed responses. Thus the width of the confidence intervals can be used as a measure of the validity or usability of a model.

Ben Salem (1988) has used Monte Carlo sampling of the pdf of 24-h rainfall extremes and the S parameter of the SCS runoff model to make probabilistic statements regarding the flood storage required to meet a given design criterion for a small flood water retarding structure. As a result of using this approach, he was able to state, for example, that he was 95% confident that a flood storage height h_{95} would contain the 10-year flood. Likewise he was 50% confident that h_{50}, the expected 10-year flood storage height, would contain the flood. The value of h_{50} is the conventional storage height that would be assigned to the 10-year flood. When viewed in a probabilistic sense, however, this value can be seen to be quite risky.

Risk is the probability of an adverse outcome given a particular course of action. The assessment of risk requires the ability to define the probability of an adverse outcome. In terms of a load, L, and a capacity, C, risk is the probability that athe load will exceed the capacity. Both the load imposed on a system and the capacity of a system to handle a load are rvs with pdfs. Risk, R, is given by

$$R = \text{prob}(L > C) = \text{prob}\ (L - C > 0) \qquad [22]$$

In terms of pdfs

$$R = \int_0^\infty p_C(c) \int_C^\infty p_L(l) dl\ dc \qquad [23]$$

Thus one must be able to evaluate the pdf of L and C. In a hydrologic context, the load would generally be a peak flow into or a water demand placed on a system or some water quality parameter. The capacity would be the ability of the system to handle the flow or meet the demand or a water quality criterion.

In conventional design we often simply attempt to have the expected value of C exceed the expected value of L. In such a situation it is not possible to evaluate risk. If one accepts the premise that L and C are rvs whose pdfs have unbounded upper tails, there is always a finite probability that L will exceed C or that $R \neq 0$. The magnitude of this probability depends on the variance of L and C, the pdfs of L and C and the magnitude of the difference in the expected values of L and C. Implying that a structure is safe if the expected value of C exceeds that of L depends on the definition of "safe". Certainly if safe means that the capacity will not be exceeded by the load, such an implication is not only misleading but is false! Once an acceptable risk is determined, equation [23] can be used to determine the capacity required for the system.

Risk analysis is becoming more prevalent especially in connection with water quality issues. To properly address risk, it will be necessary for our models to be able to project not only point estimates, but probabilistic estimates as well.

SUMMARY

Hydrologic modeling has become commonplace. Currently most hydrologic models provide point estimates. The current trend in modeling is to address problems of parameter estimation and uncertainty in a manner that enables the prediction of pdfs of model outputs. Specifying these pdfs enables one to evaluate system performance in a probabilistic manner.

This approach provides decision makers with considerably more information and makes it possible to estimate the risk associated with a particular decision. Pdfs of model results can also be used as a model evaluation and selection criteria since models with smaller error variances would generally be preferred all other things constant.

Several techniques are currently available that enable a modeler to estimate the pdfs of model parameters and model predictions. The techniques will see increasing application over the next several years.

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