# A Simple Method for Determining the Evaporation From Shallow Lakes and Ponds

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This study examines the summertime evaporation from a shallow lake in the Hudson Bay lowlands evaluated by the energy budget (Bowen ratio) and equilibrium model approaches. Energy budget calculations reveal that on the average, 55% of the daily net radiation is utilized in the evaporative process over the lake. Half-hourly and daily values of evaporation were approximated closely by the Priestley and Taylor (1972) model, where the ratio of actual to equilibrium evaporation equals 1.26. A simple model, expressed in terms of incoming solar radiation and screen height air temperature, is developed from the comparison of actual to equilibrium evaporation. Tests of the model at a different location indicate that the actual evaporation can be determined within 10% over periods of 2 weeks.

#### Introduction

An experiment in shallow lake evaporation was carried out in July 1972 in the vicinity of East Pen Island in the Hudson Bay lowlands of northern Ontario. This paper presents the results of actual evaporation determined by the energy balance approach. In addition, equilibrium model estimates of evaporation as developed by Slatyer and McIlroy [1961] are compared to energy balance measurements in order to examine the possibilities of estimating evaporation as a function of temperature and available radiant energy alone. The hypothesis, derived from this comparison, that the evaporation from shallow lakes and ponds can be accurately estimated as a function of temperature and incoming solar radiation is examined.

# **ENERGY BUDGET AND EQUILIBRIUM EVAPORATION**

The energy budget solution for evaporation which employs the Bowen ratio approach [Bowen, 1926] can be expressed in the form

$$LE = (Q^* - G)/(1 + \beta) = (Q^* - G) \cdot [1 - (\gamma)/(S + \gamma)(\Delta T/\Delta T_w)]$$
 (1)

where LE is the evaporative heat flux;  $Q^*$  and G are the net radiation and subsurface heat flow, respectively;  $\beta = H/LE$  is the Bowen ratio; H is the sensible heat flux; S is the slope of the saturation vapor pressure-temperature curve at the mean air temperature; and  $\Delta T$  and  $\Delta T_w$  are the vertical gradients of dry and wet bulb temperatures, respectively. Equation (1) yields accurate estimates of LE, as is shown in numerous experiments, providing the wet and dry bulb temperatures are measured very accurately and measurements are made within the boundary layer of the surface being investigated.

The theory of equilibrium evaporation as discussed by *Slat-yer and McIlroy* [1961] and *Monteith* [1965] is developed from the combination equation

$$LE = [S/(S + \gamma)](Q^* - G) + [(\rho Cp)/r_a](D_z - D_0)$$
 (2)

where  $\rho$  is the air density; Cp is the specific heat of air at constant pressure;  $D_z$  and  $D_0$  are the wet bulb depressions in the overlying air and at the surface, respectively; and  $r_a$  is the aerodynamic resistance to the diffusion of water vapor.

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The term  $\rho Cp(D_z - D_0)/r_a$  is the main factor which contributes to differences in LE between surfaces of a different wetness having similar available radiant energy. For example, in the case where the air in proximity to a moist surface remains unsaturated,  $D_0 = 0$ , and (2) reduces to the potential rate defined as

$$PLE = [S/(S + \gamma)](Q^* - G) + [(\rho Cp)/r_a]D_z$$
 (3)

where *PLE* is potential evaporation. Slatyer and McIlroy [1961] introduced the concept of equilibrium evaporation by taking the limited case  $D_z = D_0 = 0$ . For this instance, (2) reduces to

$$LE = LE_{EQ} = [S/(S + \gamma)](Q^* - G)$$
 (4)

where  $LE_{EQ}$  is the equilibrium evaporation rate. Under these conditions,  $LE_{EQ}$  has been interpreted as the lower limit of potential evaporation, since (4) represents the minimum possible evaporation from a saturated surface. In this instance the evaporation flux is independent of wind speed, depending solely on temperature and available radiant energy.

Recently, the evaporation from the saturated surfaces has been expressed as a function of the equilibrium rate. For example, *Priestley and Taylor* [1972] showed that potential evaporation on a daily basis was proportional to  $LE_{EQ}$  in the form

$$LE = \alpha [S/(S + \gamma)](Q^* - G)$$
 (5)

where  $\alpha = LE/LE_{EQ}$ . Using several sets of micrometeorological data from various surfaces with unlimited water supplies, they found an overall mean of  $\alpha = 1.26$ . Similar values have been shown to apply by *Davies and Allen* [1972] and *Ferguson and den Hartog* [1975] for a moist rye-grass surface and shallow lake, respectively. In this instance, providing  $\alpha$  remains constant, the evaporation can be calculated more simply from a form of (5) than from (3), since there is no need to specify a turbulent transfer mechanism.

#### EXPERIMENTAL METHODS

Site. The study was conducted during July 1972, near the Hudson Bay coastline adjacent to East Pen Island in northern Ontario, latitude 57°45'N and longitude 88°45'W (Figure 1).

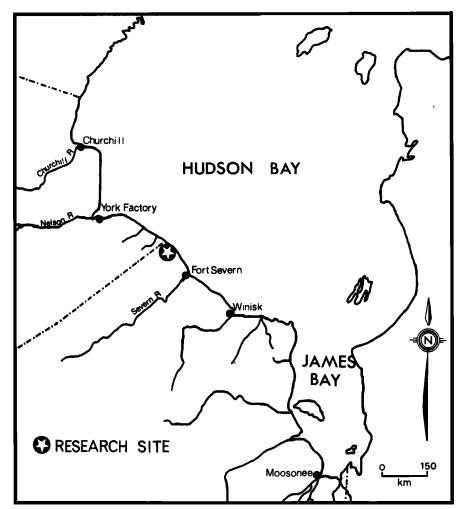


Fig. 1. Location of research site.

Observations were made over a shallow lake located a distance of 20 km from the tree line and 2 km inland from the coast.

The lake, which was roughly circular in shape with a diameter of 400 m, had an average depth of 0.6 m and a surface area of slightly more than 10<sup>5</sup> m<sup>2</sup>. A thick frozen mud layer formed the crust of the lake bottom. No surface drainage into or out of the lake was evident during the study.

Instrumentation. Net radiation was measured over the lake with a Swissteco (type S-1) net radiometer mounted 1 m above the surface. The heat flux into the lake was estimated indirectly by employing the expression

$$G = G_B + C_w \Delta T_L / \Delta t \tag{6}$$

where  $G_B$  is the heat flux through the lake bottom,  $C_W$  is the heat capacity of water, and  $\Delta T_L/\Delta t$  is the average temperature change in the lake over time. The average lake temperature was monitored with shielded five-junction thermopiles located in vertical profile, while  $G_B$ , a small term, was determined from the rate of melting of the frozen mud layer forming the lake bottom. Five-junction thermopile units, described by Wilson and Rouse [1972], were utilized to measure  $\Delta T$  and  $\Delta T_w$  at 0.25, 0.50, and 0.75 m above the surface. LE was computed half-hourly from Bowen ratio calculations with daily totals determined as the sum of the half-hourly values.

Additional measurements of wind speed, wind direction, precipitation, screen height air temperatures, and incoming solar radiation were recorded.

## RESULTS

Energy budget. Daily variations of the energy balance components are shown in Figure 2a. Component values are expressed as a percentage of  $Q^*$ . Although evaporation, on the average, is the dominant component comprising 55% of  $Q^*$ , G and H are large, accounting for 25 and 20%, respectively, of  $Q^*$ . The high variability in the component values from day-to-day is evident with heat storage dominating on the 3 days of July 4, 7, and 21.

Average values of the Bowen ratio and air temperature for daylight periods are shown in Figure 2b. The mean value for  $\beta$  was 0.365 for the 12 days of measurement and ranged between 0.083 and 0.634. The small magnitude of  $\beta$  is indicative of the absence of any surface control over evaporation. Comparison of Figures 2a and 2b shows the variation of  $\beta$  with temperature and the resultant changes in the energy balance components. In general,  $\beta$  shows a marked inverse relationship with air temperature.

Variations in  $\beta$  are accentuated in the energy balance components where the influence of the air temperature on the actual lake temperatures is apparent. A comparison of mean air temperatures with mean lake temperatures, given in Figure 3, [after *Rouse*, 1973], shows that changes in lake evaporation occur as rapidly as the day-to-day changes in atmospheric temperature. The effects of these changes are emphasized in the highly fluctuating changes in the heat storage of the lake

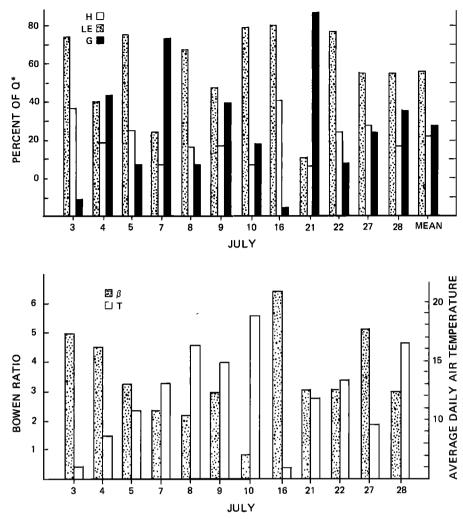


Fig. 2. Daily patterns of the energy balance components, Bowen ratio, and average air temperature.

during daylight hours. Much of the energy utilized in warming the lake is later released as evaporation during the night when the lake is cooling. This is seen in Figure 4, which compares the hourly variation in  $\beta$  and temperature during daylight periods with variations at night. The line through the latter, in Figure 4, represents the average daylight relationship of  $\beta$  to T for the lake, while the point plots represent the nighttime hourly values for July 3-4, 5-6, and 27-28. The comparison shows that the percentage distribution of nocturnal and daylight available radiant energy into H and LE is constant.

Comparison of actual to equilibrium evaporation. As is shown in (5), evaporation from a surface over any time period can be expressed as a function of equilibrium evaporation. The  $\alpha$  parameter expresses the ratio of actual to equilibrium evaporation  $LE/LE_{EQ}$ , where  $0 < \alpha < 1.26$  (although values of  $\alpha > 1.26$  are possible). The lower limit represents the case of no evaporation, while the upper limit is the limit that *Priestley and Taylor* [1972] consider to represent potential evaporation.

It is hypothesized that since there are no moisture restrictions,  $\alpha$  should remain relatively constant. Hence if  $\alpha$  is known, variations in the evaporative flux can be calculated solely from a knowledge of temperature and available energy. This hypothesis was tested for the shallow lake by comparing both half-hourly and daily totals of equilibrium estimates of evaporation with those determined from the energy budget.

The good agreement between both the half-hourly and daily

relationships of LE and  $LE_{EQ}$  is shown in Figures 5 and 6. In both cases, correlation coefficients are high, and standard errors are low. For the lake a line representing  $\alpha = 1.26$  fits the data well. Since the lake represents potential conditions, the findings agree with Priestley and Taylor's determination of  $\alpha = 1.26$  for potential evaporation.

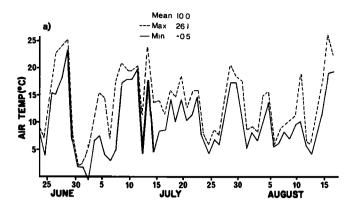
Because surface control over evaporation is relatively constant for the shallow lake, as is shown in Figures 5 and 6, a simple model based on the concept of the equilibrium model can be utilized to estimate the actual evaporation as a function of temperature and available radiant energy. For the lake, evaporation can be closely approximated by

$$LE = 1.26[S/(S + \gamma)](Q^* - G)$$
 (7)

All that is required are the measurements of air temperature  $Q^*$  and G. If these are known, the evaporation estimates should be accurate to within 5%. The difficulty in applying this equation to shallow lakes in general arises from the unavailability of  $Q^*$  and G data, particularly the latter. Hence the need for a model that incorporates a few variables that are readily obtainable is evident.

# DERIVATION AND TEST OF A SIMPLE EVAPORATION MODEL

As is shown by Stewart [1975] and Stewart and Rouse [1976], the equilibrium model format can be reexpressed as a function of temperature and incoming solar radiation  $K \downarrow$  by



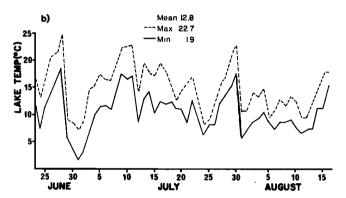


Fig. 3. Seasonal variation in the daily maximum and minimum air temperature and lake water temperatures.

replacing  $Q^* - G$  with a linear expression of  $K \downarrow$ . In this case it is assumed that the magnitude of G is small enough to be neglected or that  $G/Q^*$  remains relatively constant over time. Hence (7) can be reformulated as

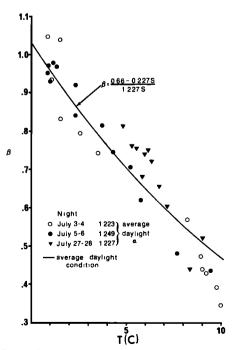


Fig. 4. Comparison of hourly nighttime Bowen ratio and temperature to average daylight conditions.

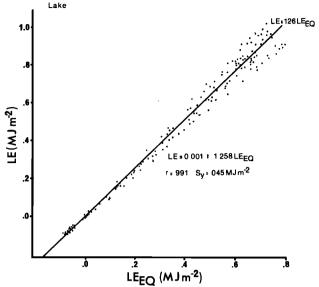


Fig. 5. Comparison of half-hourly LE with  $LE_{EQ}$  for the lake.

$$LE_L = 1.26 [S/(S + \gamma)](a + bK\downarrow)$$
  
=  $[S/(S + \gamma)](A + BK\downarrow)$  (8)

where 'a' is a regression constant, b is a regression coefficient, and A = 1.26a and B = 1.26b. The advantage of using (8) lies in the requirement of only two variables which are  $K \downarrow$  and screen height air temperature, the latter of which, as is shown by Wilson and Rouse [1972], is closely related to  $S/(S + \gamma)$ . Both are more readily available than either  $O^*$  or G.

In developing an evaporation model for the shallow lake the problem of handling the large and variable heat storage term within the model framework had to be overcome. This was accomplished by eliminating the heat storage term  $G_s$  from the evaporation calculations by extending the time period over which the model is applied. For shallow tundra lakes,  $G_s \to 0$  over periods of a few days because of rapid day-to-day fluctuations. If an appropriate time interval for which  $G_s \to 0$  can be determined, a model similar to that expressed in (8) can be formulated.

An estimate of the fluctuation in the daily heat content of

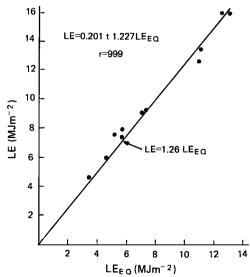


Fig. 6. Comparison of daily totals of LE with  $LE_{EQ}$  for the lake.

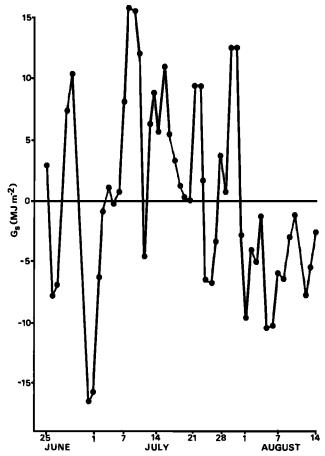


Fig. 7. Seasonal variation in the daily heat storage content of the

the lake, used in this study, in relation to the mean seasonal heat storage was obtained by using the maximum and minimum temperature data shown in Figure 3 and the expression

$$G_s = \rho C_w d(T_{mw} - T_B) \tag{9}$$

in which d is the average depth of the lake,  $T_{mw}$  is the mean daily lake temperature calculated as  $(T_{max} + T_{min})/2$ , and  $T_B$  is the mean temperature for the lake (12.8°C) obtained over 55 days of measurement. Figure 7 shows that  $G_s \rightarrow 0$  approximately 13 times between June 25 and August 14 for an average time interval of 4.5 days. This suggests that an evaporation model in the form of (8) should be applicable over periods as short as 4.5 days for a shallow lake. However, because of the large range in the time interval for which  $G_s \rightarrow 0$  of from 1-9 days, the use of periods of 1-2 weeks for the evaporation calculation would be more appropriate. A time interval of this length is necessary, since  $G_s$  must be approximately zero in order to obtain accurate LE estimates. This extension of the calculation period forces the ratio  $G/Q^*$  to zero.

As is shown in Figure 6, the daily evaporation from the lake can be closely approximated by (7). Simplification was undertaken by expressing  $\dot{Q}^* - G$  as a linear function of  $K \downarrow$ . Remembering that G for the lake was determined as the sum of the heat storage term  $(G_s)$  and the flux of heat through the lake bottom  $(G_B)$ ,  $G_s$  was set to zero. Linear regression gives

$$(Q^* - G_B)_L = 0.9225 + 0.7353K \downarrow \tag{10}$$

where all units are in MJ m<sup>-2</sup>. Although the relationship shown in Figure 8 is based on 12 days of data, comparison of

(10) with the linear regression obtained by using data collected by Weaver [1970] for the months of July, August, and September over a shallow lake at Barrow, Alaska, shows similar results.

From (7) and (10)

$$LE_L^{1} = [S/(S+\gamma)](1.624+0.9265K)$$
 (11)

where  $LE_L^1$  is the evaporation for the lake for a single day,  $G_s = 0$  being assumed. By summing  $LE_L$  over time intervals for which  $G_s \to 0$  the evaporation can be computed as

$$LE_{L}^{1} = \sum_{i=7}^{n} LE_{Li}^{1}$$
 (12)

where n > 7 days and  $LE_{Li}$  is the evaporation for day i.

To test the general applicability of (11) as a model for estimating evaporation from shallow lakes, data were obtained from Ferguson and den Hartog [1975] for Perch Lake, a shallow lake, located 2 km southwest of the Chalk River Atomic Energy of Canada Laboratories (latitude of 46°03'N and longitude of 77°23'W). Perch Lake is similar in size and depth to the lake at Pen Island, being circular with a surface area of 4.5  $\times$  10<sup>5</sup> m<sup>2</sup> and having a mean depth of 2 m. Dense forest averaging 15–20 m in height encircled the lake, starting at a distance from shore of 30 to 40 m.

Evaporation, heat storage, and radiation data were available for June through September for 1970, 1971, and 1972. The data record was not complete, however, since several weeks of measurement were missing owing to instrument failure. Available measurements included daily totals of  $G_s$ ,  $Q^*$ , LE by the energy budget method, average daily water surface temperature, and air temperature. Daily totals of  $K \downarrow$ , required for use in  $LE_L$ , were not measured. This created a problem, since the nearest station measuring  $K \downarrow$  was located at a distance of 200 km at Ottawa. Since differences between daily values of  $Q^*$  for a grass surface of Chalk River and one at Ottawa were as large as 60% the Ottawa data were unusable. In lieu of  $K \downarrow$  measurements an accurate empirical expression, derived by Robinson et al. [1972], was used in the derivation of  $K \downarrow$ , where

$$Q^* = 0.368 + 0.823K^*(MJ \text{ m}^{-2} \text{ day}^{-1})$$
 (13)

in which  $K^*$  is the net shortwave radiation. By substituting a surface albedo a = 0.08, a value which is similar to that

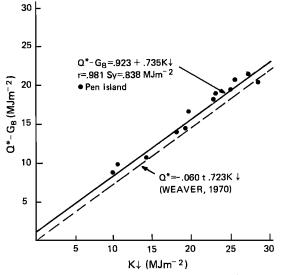


Fig. 8. Comparison of daily totals of  $Q^* - G$  with  $K \downarrow$  for the lake.

recorded for Lake Ontario from June through September [Davies and Schertzer, 1974] and recorded for a shallow lake in Alaska [Weaver, 1970], and by replacing  $K^*$  with (1-a)K, daily values of K for Perch Lake were computed from

$$K| = 0.486 + 1.32Q^* \tag{14}$$

These values for  $K \downarrow$  were then used in (11) to derive values for IF.

Figure 9a shows the relationship between two-weekly totals of  $LE_L$  and measured values of LE for Perch Lake. It is apparent that the model performs satisfactorily. Furthermore, when the calculation period is lengthened to a month, as is shown in Figure 9b, the model performance is improved. These results indicate that the generalized evaporation model expressed in (11) is reliable in estimating the two-weekly and monthly totals of evaporation from shallow lakes.

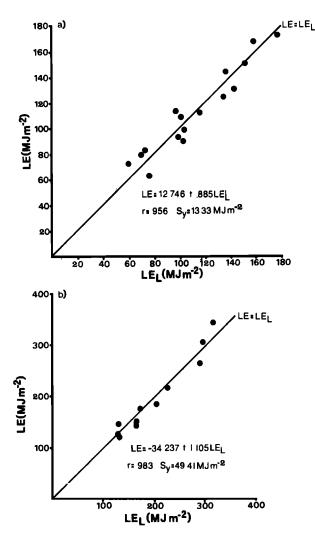


Fig. 9. Comparison of two-weekly and monthly totals of LE with  $LE_L$  at Perch Lake, Ontario.

#### Conclusions

The results of this study show that daily evaporation can be estimated as a function of equilibrium evaporation within  $\pm 5\%$  for a shallow lake. The ratio of actual to equilibrium evaporation can be closely approximated by 1.26, a value further supporting the *Priestley and Taylor* [1972] value  $\alpha = 1.26$ , which is considered representative of potential evaporation conditions. For shallow lakes the Priestley and Taylor model can be utilized to estimate the daily evaporation within 5%

The results further show that the evaporation for shallow lakes and ponds can be accurately estimated as a function of temperature and incoming solar radiation. A test of a model, based on this concept, shows that for lakes with depths between 0.5 and 2 m the evaporation can be estimated within 10% for periods of 2 weeks to a month.

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