Spatiotemporal Stochastic Open-Channel Flow. II: Simulation Experiments

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Abstract: Spatiotemporal solutions for open-channel flow are obtained in a stochastic setting using field data on parameter variability. Statistical descriptions of the flow variables are estimated through Monte Carlo simulation using finite difference equations for a 10-km reach of the Columbia River. Results indicate considerable uncertainty in predicted flow behavior: ensemble coefficients of variation at different space-time locations ranged from 0.18–0.60 for flow velocity and from 0.04–0.13 for flow depth. The band widths between the 16% and 84% quantiles were typically 0.6–1.4 m/s and 5–7 m, respectively, for velocity and depth. Probability distributions for predicted velocities were found to be gamma, lognormal, or Weibull, whereas those for depth were normal, gamma, and, in a few cases, lognormal. The various quantiles of the predicted variables are associated with notions of risk, reliability, and variability that influence engineering decisions. Sensitivity of the level of uncertainty in predicted flow variables to the level of uncertainty in the parameters is investigated for a generalized stream system through fractional factorial analysis of coefficients of variation. Uncertainty in predicted flow velocity was most sensitive to the uncertainty in the channel cross-section geometry, particularly scale and shape parameters for flow area. Uncertainty in predicted flow depth was predominantly sensitive to the uncertainty in channel bed slope.

INTRODUCTION

Hydraulic engineers have long recognized the uncertainty inherent in estimating model parameters for use in predicting open-channel flow behavior. These parameters represent physical properties, boundary and initial conditions, and sink/source terms whose values are ambiguous due to significant spatial and temporal variability, measurement error, and limited sampling. Schumm and Winkley (1994), for example, describe the rich and complex variability that characterizes large alluvial channels and that poses formidable challenges to analysts.

Parameter uncertainty generates an uncertainty in predicted flow velocities and depths through the governing flow equations, resulting in a wide variety of possibilities for consideration in engineering decisions. Formal analysis of this uncertainty has been possible in recent years through the developing methodologies of stochastic hydraulics (Kuo and Lin 1992; Yen and Tung 1993). To date, however, no solution to one-dimensional (1D) unsteady open-channel flow has been presented in a complete spatiotemporal stochastic setting. This is due, in part, to computational limitations borne by methods used for approximating solutions to stochastic partial differential equations. A more significant hinderance, however, has been the absence of a comprehensive statistical description of model parameters derived from a large set of field data. In fact, most studies to date have made simplifying assumptions about the statistical structure of the parameters that are not well-supported by field data.

Building on the approach and field data described in a companion paper (Gates and Al-Zahrani 1996), spatiotemporal stochastic solutions for flow in a representative stream are presented. Using parameter statistics inferred from field data, unconditional solutions for probability distributions and statistics of the flow variables at fixed intervals along a reach of the Columbia River and at selected times are obtained, using Monte Carlo simulation with a finite-difference model. In addition, analysis is conducted to explore the sensitivity of the level of uncertainty in predicted flow variables to the level of uncertainty in model parameters for a generalized stream system. The results suggest implications for how stochastic modeling can be used to address notions of risk, reliability, and variability important to engineering decisions for open-channel systems.

UNCERTAINTY IN FLOW SIMULATION

Stochastic Flow Equations

As discussed in part I, the stochastic form of the equations that model one-dimensional open-channel flow assumes the parameters describing flow properties, boundary and initial conditions, and sink/source terms to be spatiotemporal random fields (STRFs). A STRF is a collection of random variables indexed by their locations in space and their positions in time, and dependent upon events in probability. The stochastic version of the familiar Saint-Venant formulation can be posed as

$$\frac{\partial h}{\partial t} + \left(\frac{A}{T_w}\right)\frac{\partial \bar{u}}{\partial x} + \bar{u}\,\frac{\partial h}{\partial x} = -\frac{q_s}{T_w} \tag{1}$$

and

$$S_f = S_0 - \left(\frac{1}{g}\right)\frac{\partial \bar{u}}{\partial t} - \left(\frac{\bar{u}}{g}\right)\frac{\partial \bar{u}}{\partial x} - \frac{\partial h}{\partial x} + \left(\frac{q_s}{gA}\right)(\bar{u} - u_s) \tag{2}$$

wherein the friction slope (m/m) is evaluated using the Manning equation

$$S_f = \bar{u}^2 n^2 / R^{4/3} \tag{3}$$

In (1)–(3) the parameters are spatiotemporal random fields: $A = A(x, t; \omega) = A[\boldsymbol{\Gamma}(x, t; \omega), h]$ = area of the channel flow cross section (m^2) dependent on the location, x, along the channel, the position, t, in time and an event, ω, in probability; $T_w = T_w(x, t; \omega) = T_w[\boldsymbol{\Gamma}(x, t; \omega), h]$ = top width of the cross section measured at the water surface (m); $R = R(x, t; \omega) = R[\boldsymbol{\Gamma}(x, t; \omega), h]$ = hydraulic radius of the flow cross section (m); $\boldsymbol{\Gamma}(x, t; \omega)$ = a vector of parameters that define the geometry of the channel cross section and that are used in evaluating A, T_w, and R; $n = n(x, t; \omega)$ = Manning hydraulic resistance (s/m$^{1/3}$); $q_s = q_s(x, t; \omega)$ = lateral outflow (+) or inflow (−) rate per unit length along the channel (a sink/source term) [(m^3/s)/m];

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Note. Discussion open until April 1, 1997. Separate discussions should be submitted for the individual papers in this symposium. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on July 28, 1995. This paper is part of the *Journal of Hydraulic Engineering*, Vol. 122, No. 11, November, 1996. ©ASCE, ISSN 0733-9429/96/0011-0652–0661/$4.00 + $.50 per page. Paper No. 11269.

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and $u_s = u_s(x, t; \omega)$ = velocity of the lateral flow in the x-direction. The dependent flow variables are $h = h(x, t; \omega)$ = the flow depth (m) and $\bar{u} = \bar{u}(x, t; \omega)$ = the cross-section averaged x-direction velocity.

To account for errors in estimation of momentum transfer introduced by the use of the cross-section averaged velocity, (2) can be amended to obtain

$$\frac{\partial(A\bar{u})}{\partial t} + \frac{\partial(\beta\bar{u}^2 A)}{\partial x} + gA\frac{\partial h}{\partial x} - gA(S_0 - S_f) + q_s u_s = 0 \quad (4)$$

wherein $\beta = \beta(x, t; \omega) = [(1/A)\int\int_A u^2\, dA]/(A\bar{u}^2)$ = momentum correction coefficient. In many cases, β can be assumed as unity without introducing significant error in solutions for flow depth and velocity (Liggett 1993; Xia and Yen 1994). Eq. (4), along with equations (1)–(3), can alternatively be expressed in terms of the channel flow rate, Q (m^3/s), with $\bar{u} = Q/A$.

Together with appropriate boundary and initial conditions (also treated as random fields) solutions to (1) and (2) or (4) are sought in the space-time domain of interest and in probability. Analytical solutions to these nonlinear stochastic partial differential equations (SPDEs) are not available for problems of general interest. Hence, the equations are approximated by a set of nonlinear algebraic difference equations applied to a discretization of the flow domain in space-time and in probability. That is, the difference equations are applied and solved in the continuous (x, t) domain at a discrete number of points in space and time and in the continuous probability space over a sample set of possible events (or realizations), ω. Gates and Al-Zahrani (1996) briefly describe how this can be accomplished using an implicit finite difference formulation in space-time in conjunction with finite-order methods or Monte Carlo simulation.

Monte Carlo Simulation with Finite-Difference Approximation

Field data indicate that the parameter STRFs of the stochastic 1D open-channel flow model have large relative variability, are statistically nonhomogeneous, have residual probability distributions that are non-normal, and have strong lag-dependent residual covariance structure (Gates and Al-Zahrani 1996). Governing equations with stochastic parameters of this type are best solved using Monte Carlo simulation. In this method, solutions of the governing difference equations are obtained for sample realizations of the parameter STRFs. Thereby, sample sets of the dependent-variable STRFs, $\bar{u}$ and h, are obtained that can be analyzed to describe probability distributions or statistics of interest (mean, coefficient of variation, skewness, quantiles, etc.). Two component models are needed to accomplish Monte Carlo simulation: a model for generating possible realizations of the parameter STRFs, and a deterministic finite-difference model for solving the flow equations for each generated realization.

In the present study, the approach developed by Der Kiureghian and Liu (1986) and described in Chang et al. (1994) was used to generate the realizations of the parameter STRFs. The method provides multivariate simulation that preserves the nonnormal marginal distributions of the random variables and the correlation structure without requiring a complete description of the joint distributions. This is accomplished by using a set of semiempirical formulas based on the Nataf bivariate distribution model. The correlation coefficient of each pair of nonnormal random variables is transformed to an equivalent correlation coefficient for a bivariate standard normal distribution using the semiempirical formulas. Multivariate realizations of the random variables can then be generated in a correlated standard normal space using any one of several available algorithms. Finally, the realization in the original space is obtained by an inverse transformation.

To solve the flow equations for $\bar{u}$ and h for each of the realizations of the parameter STRFs, the recent version of the NETWORK model (Fread 1985) was used. A modification of the widely used DWOPER model, NETWORK implements a Priessman four-point implicit finite-difference formulation of the governing (1) and (2) or (4), as described in part I. The Newton-Raphson method is used to solve the resulting system of nonlinear equations at each time step.

REPRESENTATIVE APPLICATION TO COLUMBIA RIVER

To illustrate a solution for spatiotemporal stochastic flow, a hypothetical engineering problem was posed for a 10-km reach of the Columbia River downstream of Grand Coulee Dam in the state of Washington (Fig. 1). It was assumed that installation of a cross-regulating barrage was contemplated for the downstream end of the reach. The barrage would be used to control upstream water levels for diversion of water to a large irrigation canal. The analysis concerns the backwater effect of a considered barrage design under conditions of a peak annual discharge from the upstream dam when diversion to the canal is shut off.

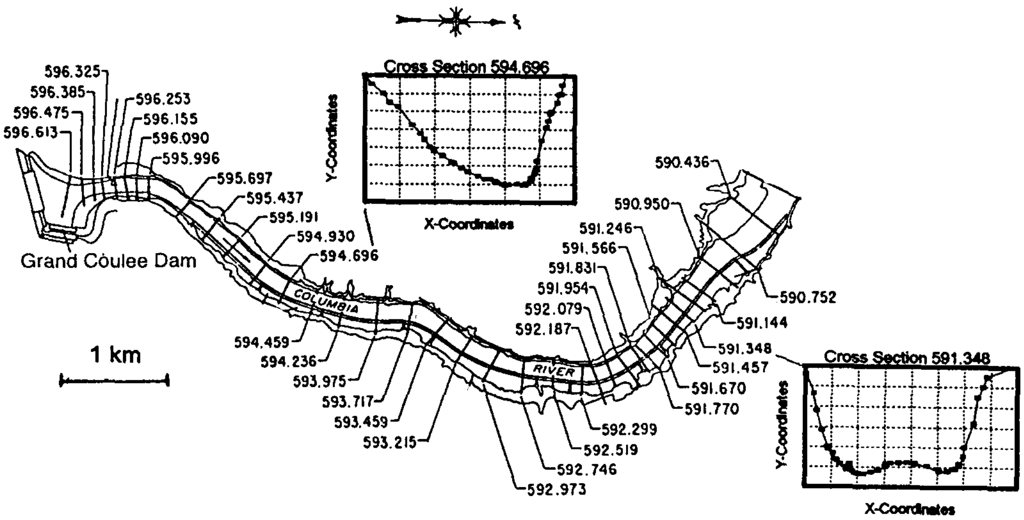

FIG. 1. Reach of Columbia River Used for Representative Stochastic Modeling Problem, Showing Locations of Surveyed Cross Sections along with Two Example Cross Sections

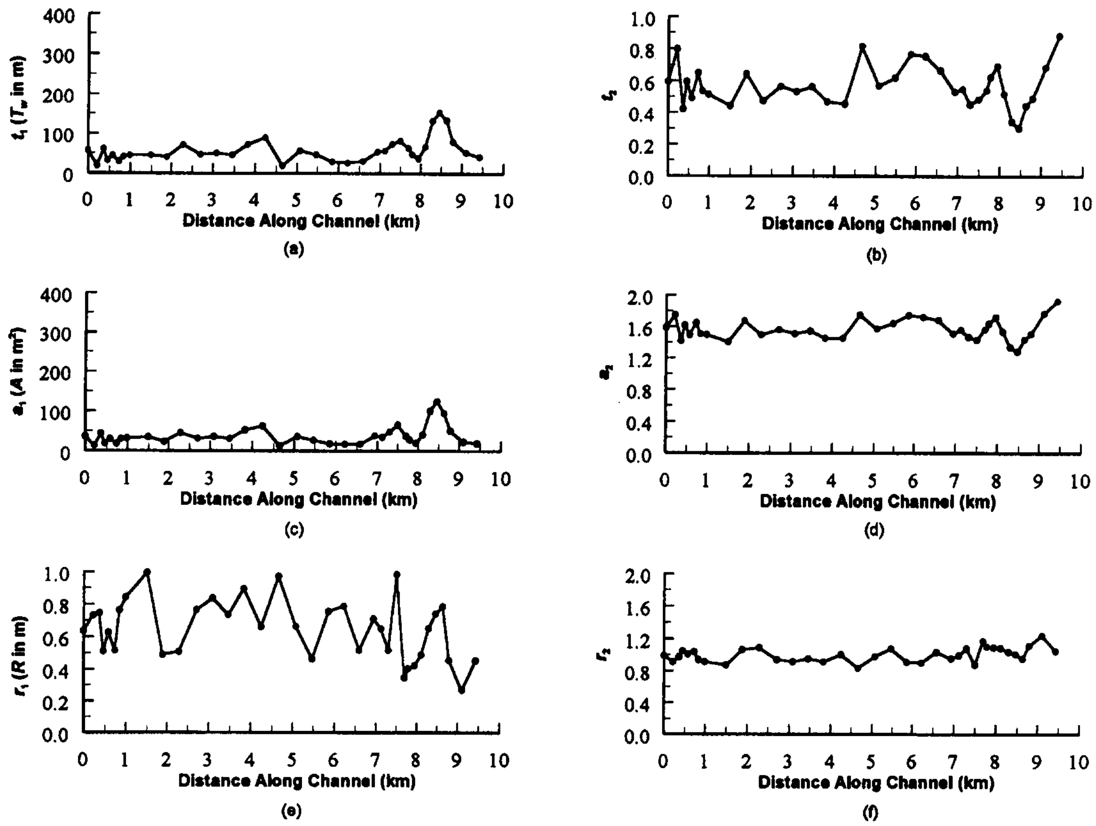

FIG. 2. Sample Data for Channel Cross-Section Parameters along Modeled Reach of Columbia River: (a) t_1; (b) t_2; (c) a_1; (d) a_2; (e) r_1; and (f) r_2

TABLE 1. Statistics of Parameter Data for Columbia River Downstream of Grand Coulee Dam

Parameter (1)	Sample mean (2)	Sample CV (3)	Variance homogeneity (4)	Type of transformation (5)	Mean homogeneity (6)	Type of trend equation (7)	Fitted distribution of transformed and detrended data (8)
S_0 (m/m)	−0.000219	55.50	Yes	—	Yes	—	Normal
Bed elevation (m)	273.74	0.01	Yes	—	No	Cubic	Normal
t_1 (T_w in m)	57.04	0.57	No	Log	No	Cubic	Normal
t_2 (T_w in m)	0.57	0.23	Yes	—	Yes	—	Lognormal
a_1 (A in m^2)	38.12	0.64	No	Log	No	Cubic	Normal
a_2 (A in m^2)	1.57	0.09	Yes	—	Yes	—	Lognormal
r_1 (R in m)	0.65	0.28	Yes	—	No	Cubic	Gumbel
r_2 (R in m)	1.00	0.09	Yes	—	No	Cubic	Normal
q_l' (m/day)	0.09	0.83	No	Power	No	Linear	Normal
n	0.037	0.25	Yes	—	Yes	—	Lognormal
β	1.17	0.04	Yes	—	Yes	—	Normal

Uncertainty in Hydraulic Properties

Data on channel cross-section geometry were extracted from surveys of 36 cross sections reported by Blanton (1974). The spacing between surveyed cross sections along the sampled reach were variable but averaged about 260 m. These data were analyzed to describe A, T_w, and R as the following power functions of flow depth: $A = a_1h^{a_2}$, $T_w = t_1h^{t_2}$, and $R = r_1h^{r_2}$. The coefficient (scale) parameters a_1, t_1, and r_1 and the exponent (shape) parameters a_2, t_2, and r_2 were determined by least-squares nonlinear regression. The resulting average coefficients of determination, r^2, were very high: 0.99, 0.97 and 0.99, respectively, for the fitted equations for A, T_w, and R. Hence, in this model $\Gamma = [a_1, a_2, t_1, t_2, r_1, r_2]$. The sample data for each of the parameters are plotted in Fig. 2.

The statistics of the sample data for the parameters of Γ are summarized in Table 1. The relative variability in a_1 and t_1 was quite high, with coefficient of variation (CV) (ratio of standard deviation to mean), exceeding 0.50. The relative variability in r_1 and t_2 was moderate, with CV exceeding 0.20, and in a_2 and r_2 was low, with CV less than 0.10. In general, the parameters of Γ were not statistically homogeneous random fields. Analysis revealed trends in the variance of a_1 and t_1 along the channel reach. A ln transformation was used to remove the trend in both cases. Regression analysis was used to describe significant cubic trends in the mean values of $\ln(a_1)$, $\ln(t_1)$, r_1, and r_2 along the reach. Probability distribution functions were fit to the residuals (transformed and detrended values) of each parameter, revealing a mixture of normal and nonnormal marginal distributions, as indicated in Table 1. Autocorrelation and crosscorrelation coefficients were computed for different lag distances (separation distances, Δx, between cross sections) for the residuals. Exponential decay functions were fit to the data for use in modeling the lag-dependent correlation. Table 2 summarizes the autocorrelation and crosscorrelation lengths [approximate distances over which significant (5% significance level) correlation is detected] for the

TABLE 2. Matrix of Correlation Lengths (m) for Residuals of Geometric Parameters for Columbia River Downstream of Grand Coulee Dam

Parameter (1)	Bed elevation (2)	S_0 (3)	t_1 (4)	t_2 (5)	a_1 (6)	a_2 (7)	r_1 (8)	r_2 (9)
Bed elevation (m)	250	200	220	270	200	280	0	0
S_0 (m/m)	—	310	0	0	0	0	0	0
t_1 (T_w in m)	—	—	375	375	375	420	0	0
t_2 (T_w in m)	—	—	—	410	370	400	0	0
a_1 (A in m^2)	—	—	—	—	390	375	0	0
a_2 (A in m^2)	—	—	—	—	—	390	0	0
r_1 (R in m)	—	—	—	—	—	—	310	310
r_2 (R in m)	—	—	—	—	—	—	—	300

different parameters of Γ. Results show significant spatial correlation structure, with dependence among parameter values extending over distances as great as about 420 m, as estimated from the fitted exponential correlation functions (Al-Zahrani 1995).

Both channel bed elevation and bed slope, S_0, were analyzed to characterize channel longitudinal geometry. Values of S_0 were computed from changes in elevation of the thalweg, measured between surveyed cross sections. Descriptive statistics are given in Table 1. The CV for values of S_0 exceeded 50, indicating extremely high variability. No significant trend in the variance or mean of S_0 was detected; however, the bed elevation had a significant cubic trend in the mean along the channel reach. The residuals for both S_0 and bed elevation were fit to normal distributions. Autocorrelation and crosscorrelation lengths for the residuals are summarized in Table 2.

Data on spatiotemporal variability of Manning hydraulic resistance, n, were not available. However, based upon a study described by Blanton (1974), a value of 0.037 was used to estimate the mean of n along the channel reach. The CV value was assumed as 0.25 and the probability distribution was assumed lognormal, as inferred from studies of variability in n reported in part I. Values of n were assumed to be statistically homogeneous with no significant correlation over the separation distances (Δx = 200 m) under consideration.

Channel seepage, q_s, over a computational reach was assumed to be a random field dependent on average channel perimeter:

$$q_s = q_s'[(P_u + P_d)/2] \tag{5}$$

wherein $q_s' = q_s'(x; \omega)$ = seepage rate per unit wetted perimeter area along the reach (m/s); P_u = wetted perimeter at the upstream cross section bounding the reach (m); and P_d = wetted perimeter at the downstream cross section bounding the reach (m). The statistical characteristics assumed for q_s', based upon studies conducted by Haskell (1994), are summarized in Table 1.

Values of the momentum correction coefficient, β, were assumed to be normally distributed over the range 1.00 to 1.33, as indicated by Xia and Yen (1994). The end points of the range were assumed to correspond to the 0.5% and 99.5% quantiles, yielding a CV of 0.04. Values of β were assumed statistically homogeneous with no significant correlation structure.

Uncertainty in Boundary and Initial Conditions

The upstream boundary condition was taken as an hourly hydrograph for the peak annual flow. Records of hourly flow releases from the Grand Coulee Dam were obtained for the period 1969–94. The 24-h period corresponding to the peak flow rate was identified for each of the 26 years of record. A summary plot of the statistics is shown in Fig. 3. The mean values of the inflow, Q_I, ranged from a low of 3,665 m^3/s in the first hour to a high of 6,292 m^3/s in the sixth hour, with sample CV values ranging from a low of 0.23 in the seventh hour to a high of 0.58 in the second hour. No significant trends in mean were detected for any of the hourly flow rates. Trends in variance, however, were detected and a square root-transformation was used to detrend the data. Probability distributions of the detrended values of the hourly flows were either normal or lognormal. Correlation structure was found to be strong, with significant correlation (ranging between 0.55 and 1.0) extending over the entire 24-h period.

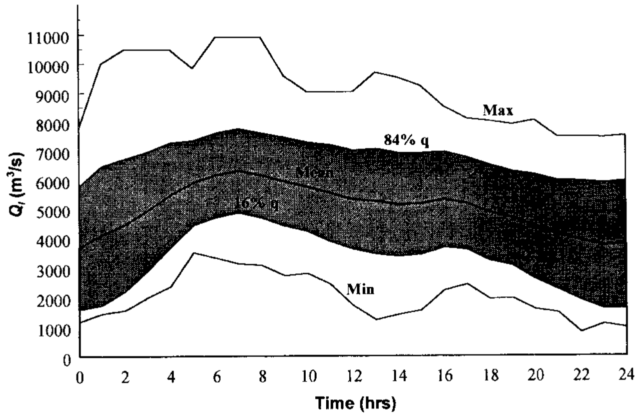

FIG. 3. Summary Plot of Statistics of Hourly Peak-Inflow Hydrograph for Modeled Reach of Columbia River

The downstream boundary condition was established by the barrage, assumed to be a broad-crested weir with a rectangular control section. Free flow was assumed to occur at all times over the weir according to the following rating equation:

$$Q = C_d C_v \frac{2}{3}\left(\frac{2g}{3}\right)^{0.5} b_c[(h_u + z_u) - e_c]^{1.5} \tag{6}$$

wherein C_d = discharge coefficient; C_v = velocity head correction coefficient, b_c = width of the crest at the control section (m); h_u = flow depth in the channel just upstream of the barrage (m); z_u = channel bed elevation just upstream of the barrage (m); and e_c = elevation of the crest (m). The values of C_d and C_v in (6) were modeled as independent normally distributed random variables. The values of C_d corresponding to the 2.5% and 97.5% quantiles were assumed to be 0.96 and 1.04 (Bos 1989), yielding a CV of 0.02. The 2.5% and 97.5% quantiles for C_v were taken as 1 and 1.2 (Bos 1989), respectively, yielding a CV of 0.05. The width of the weir control section was 250 m. The elevation of the weir crest was 278.5 m, a height of 15 m above the deterministic channel bed elevation just upstream of the barrage.

Initial conditions on $\bar{u}$ and h were computed by assuming steady flow along the channel reach for the initial flow rate of the peak hydrograph. Since this initial flow rate is stochastic, along with the stochastic channel properties, the initial conditions are random fields determined through the solution of the steady flow equations in NETWORK.

Uncertainty in Predicted Flow Variables

The procedure described previously for generating multivariate realizations of random fields was applied to each of the parameters of the application to the Columbia River. Tests were conducted to verify convergence of the distributions and statistics of each of the generated random fields to the assumed distributions and statistics. It was found that convergence within 0.2–1.5% difference for the mean and within 0.1–5% difference for the CV was achieved with a set of 700 realizations (Al-Zahrani 1995). Tests included those to insure preservation of trends and correlation structure also. The compu-

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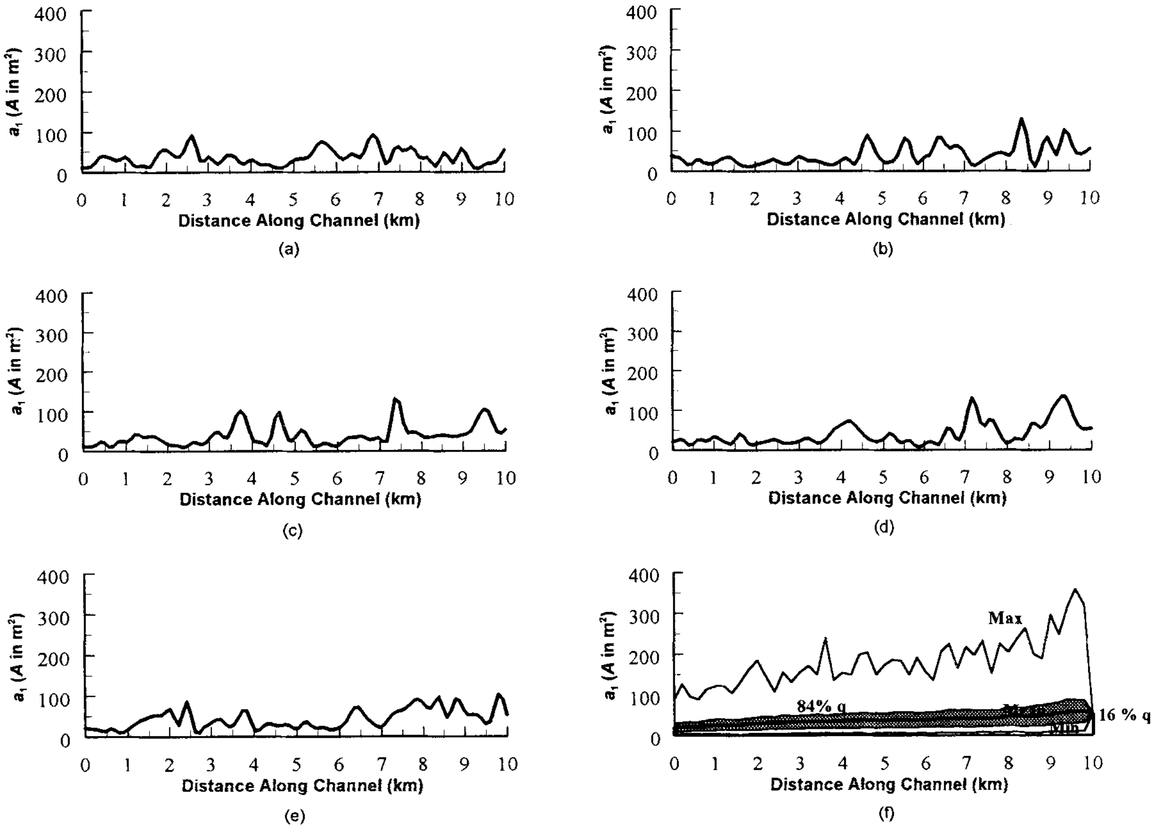

FIG. 4. Generated Values of Cross-Section Parameter a_1 for Representative Stochastic Modeling Problem: (a) Realization No. 157; (b) Realization No. 244; (c) Realization No. 307; (d) Realization No. 595; (e) Realization No. 679; and (f) Summary Statistics over Ensemble of 700 Realizations

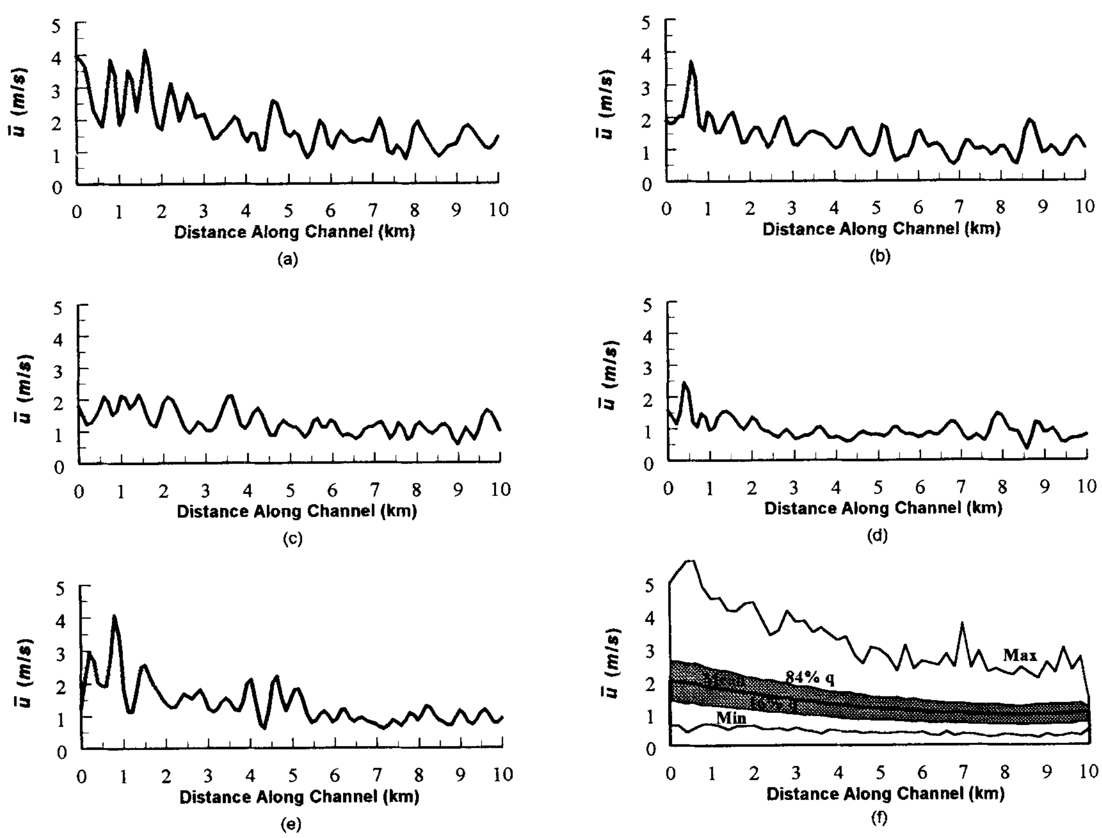

FIG. 5. Stochastic Solutions for Flow Velocity for Time t = 6 h for Representative Stochastic Modeling Problem: (a) Realization No. 157; (b) Realization No. 244; (c) Realization No. 307; (d) Realization No. 595; (e) Realization No. 679; and (f) Summary Statistics over Ensemble of 700 Realizations

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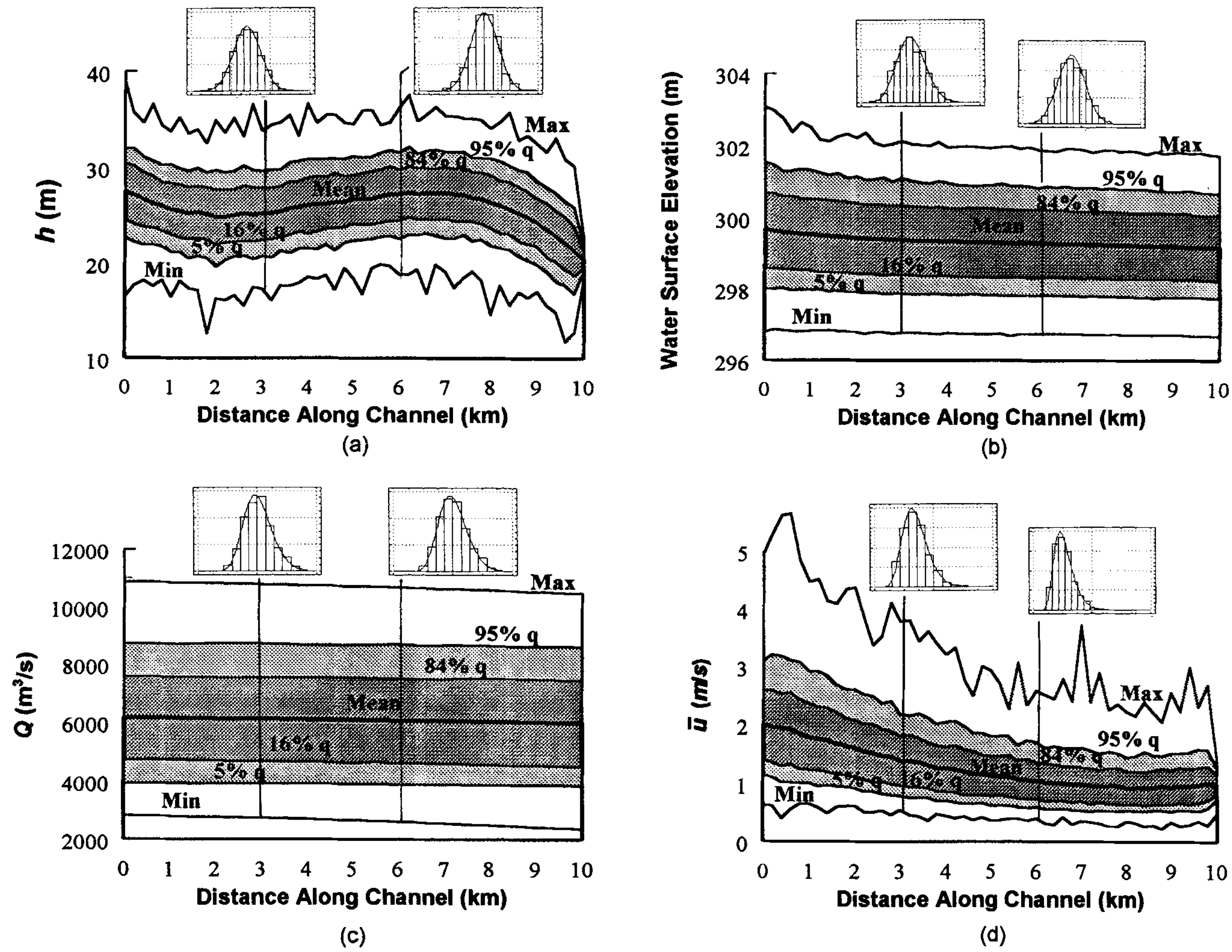

FIG. 6. Summary Statistics over Ensemble of 700 Realizations at Time $t = 6$ h, Showing Example Frequency Histograms and Fitted Probability Distributions at Two Locations for: (a) Flow Depth; (b) Water Surface Elevation; (c) Flow Rate; and (d) Flow Velocity

tational nodes consisted of 51 cross sections, equally spaced at 200 m intervals. The locations of the computational nodes did not correspond to the locations of available field data. Effects of conditioning the parameter realizations on available measurements, taken at variable intervals along the channel, were not considered in this study but will be addressed in a sequel paper. However, the impact of such conditioning on the overall uncertainty in predicted flow may not be substantial (Gates and Al-Zahrani 1996). The channel geometry was treated as deterministic at the downstream cross section corresponding to the location of the barrage. As an example, five of the 700 realizations of a_1 along the channel were randomly selected and plotted in Fig. 4, along with a summary plot of the ensemble statistics over the reach.

Experiments with Monte Carlo simulation revealed that 700 realizations resulted in stable predictions of the distributions and statistics of the flow variables. Computational time steps of 10 min were used in the finite difference solutions. Hence, each realization required the solution of 102 nonlinear equations [finite-difference forms of (1) and (4) over each of the 50 channel finite-difference grids plus the upstream and downstream boundary equations] over 144 time steps. About 1 h of central processing unit (CPU) time was required to complete solutions for the 700 realizations on a personal computer with an Intel Pentium/90 microprocessor.

Stochastic solutions of the flow variables at a number of space-time points are described in detail in Al-Zahrani (1995). Solutions for $\bar{u}$ along the channel reach at $t = 6$ h (at the peak of the mean inflow hydrograph) for the five randomly selected realizations are shown in Fig. 5, along with a summary plot of the statistics. These results illustrate the substantial variability in the collection of possible flow conditions due to parametric uncertainty. The statistical summary plot in Fig. 5 shows the mean, the 84% and 16% quantiles (would correspond to the mean plus and minus one standard deviation, respectively, if the distribution were normal), and the maximum (99.9% quantile), and minimum (0.1% quantile) values over the entire set of 700 realizations. The CV of $\bar{u}$ over the ensemble of realizations at the computational nodes ranged from 0.18 to 0.36, averaging about 0.33, at $t = 6$ h. Over the entire 24-h period, the ensemble CV of $\bar{u}$ at the computational nodes ranged from a low of 0.18 to a high of 0.60. The average ensemble CV along the channel reach and over the entire period was about 0.42, indicating high relative variability in predicted values of $\bar{u}$. A statistical summary plot of flow velocity along the channel at $t = 6$ h that includes the 5% and 95% quantiles is shown in Fig. 6. The 95% quantile plot indicates the values of $\bar{u}$ along the channel reach having a probability of exceedance of 5% = (100−95%). Probability distributions fit to predicted values of $\bar{u}$ at the computational nodes were found to be gamma, lognormal, or Weibull. Example frequency histograms and their fitted distributions are illustrated for two selected cross sections in Fig. 6.

Five realizations of the computed flow depth along the channel at $t = 6$ h are shown in Fig. 7, along with a statistical summary plot. The relative variability in predicted values of h was considerably less than that for $\bar{u}$: ensemble CV of h at the computational nodes ranged from 0.04 to 0.13, averaging

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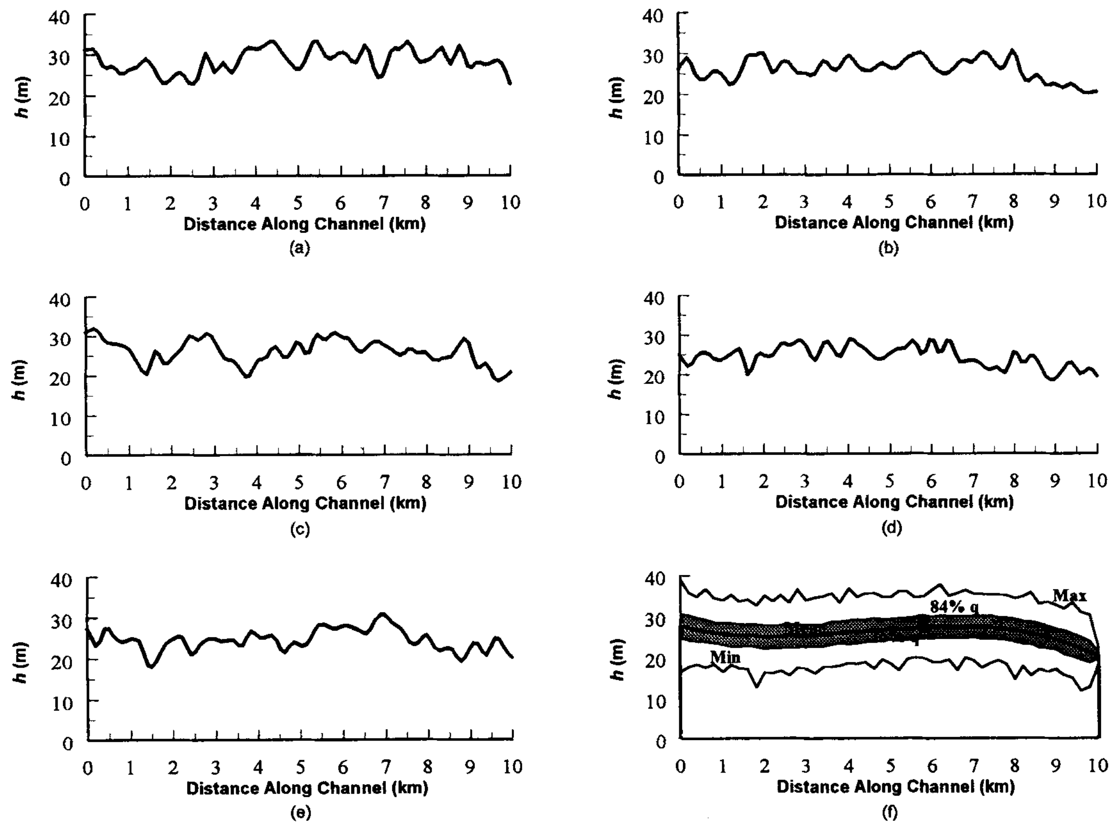

FIG. 7. Stochastic Solutions for Flow Depth for Time $t = 6$ h for Representative Stochastic Modeling Problem: (a) Realization No. 157; (b) Realization No. 244; (c) Realization No. 307; (d) Realization No. 595; (e) Realization No. 679; and (f) Summary Statistics over Ensemble of 700 Realizations

about 0.11, at $t = 6$ h. The average ensemble CV along the channel reach and over the entire 24-h period was about 0.11. However, in absolute terms, the variability was substantial: the range between the maximum and minimum predicted values was as great as 25 m, and the range between the 16% and 84% quantiles was typically 5–7 m. Probability distributions for h were found to be typically normal or gamma, but in a few cases were lognormal. A complete statistical summary is plotted in Fig. 6 for $t = 6$ h.

Uncertainty in predicted values of channel flow rate and water surface elevation also was explored. Statistical summary plots are shown in Fig. 6 for $t = 6$ h. The significant variability is evident from these plots. The range of predicted flow rates between the 16% and 84% quantiles was typically about 3,000–4,000 m^3/s, while the corresponding range for predicted water surface elevations was 2.5–4 m. Probability distributions for channel flow rate were typically lognormal or gamma, while those for water surface elevation were normal or gamma.

SENSITIVITY TO PARAMETER UNCERTAINTY IN GENERALIZED STREAM SYSTEMS

The sensitivity of the uncertainty in predicted values of $\bar{u}$ and h to model parameter uncertainty was explored by systematically analyzing simulation results over ranges of CV values of selected parameters for a generalized stream system. The generalized system was constructed to have mean parameter characteristics, probability distributions, trends, and correlation structure reflective of the average channel conditions observed in the large field data set described in part I. The considered ranges of CV were within the ranges observed in the field data set.

Considered Ranges of Parameter Uncertainty

A baseline generalized stream system for use in sensitivity analysis was defined to be reflective of typical stream conditions. Investigation of the field data set described in part I indicated that the statistics of the channel geometric parameters for reach 2 of the Colorado River were representative of conditions over the data set (Al-Zahrani 1995). Thus, the generalized stream system was constructed to reflect the average characteristics of this river reach as closely as possible. The mean values, probability distributions, trends, and correlation structure of the geometric parameters for the generalized stream were assumed the same as those for reach 2 of the Colorado River. The only exception was in the case of the mean channel bed slope. Computational experiments revealed that the mean bed slope of the generalized stream should be reduced by about 50% from that for the Colorado River. This would allow a wider range of CV values to be considered in the sensitivity analysis while minimizing physically unreasonable realizations in Monte Carlo simulation. The statistics of the model parameters that were used to define the baseline generalized stream system for use in sensitivity analysis are summarized in Table 3.

The CV can be interpreted as a measure of the degree of uncertainty in prescribing model parameters. Ranges of CV of the fitted probability distributions of the transformed residual data considered in this study are summarized in Table 4 along with the corresponding ranges of sample CVs of the generated data. These ranges were used in fractional factorial analysis to describe sensitivity to parameter uncertainty and were specified to fulfill two conditions: (1) to allow a wide range of CV to be considered while minimizing physically unreasonable realizations in Monte Carlo simulation, given the defined statistics of the generalized stream system; and (2) to fall within

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TABLE 3. Statistics of Parameter Data used in Generalized Stream for Sensitivity Analysis

Parameter (1)	Sample mean (2)	Variance homogeneity (3)	Type of transformation (4)	Mean homogeneity (5)	Type of trend equation (6)	SPATIAL CORRELATION STRUCTURE OF TRANSFORMED AND DETRENDED DATA: Autocorrelation length (m) (7)	Crosscorrelation Length (m): Min (8)	Crosscorrelation Length (m): Max (9)	Fitted distribution of transformed and detrended data (10)
S_0 (m/m)	0.00075	Yes	—	Yes	—	270	0	0	Normal
t_1 (T_w in m)	127.0	Yes	—	No	Cubic	870	0	850	Weibull
t_2 (T_w in m)	0.9	No	Log	No	Cubic	310	0	350	Weibull
a_1 (A in m^2)	65.7	Yes	—	No	Cubic	370	0	850	Gumbel
a_2 (A in m^2)	1.9	No	Log	No	Cubic	330	0	390	Weibull
r_1 (R in m)	0.7	Yes	—	Yes	—	420	0	290	Weibull
r_2 (R in m)	0.9	No	Power	No	Cubic	350	0	350	Weibull
q_l' (m/d)	0.0	—	—	—	—	—	—	—	—
n	0.04	Yes	—	Yes	—	—	—	—	Lognormal
β	1.2	Yes	—	Yes	—	—	—	—	Normal

TABLE 4. Range of CV Values Considered for Sensitivity Analysis in Generalized Stream

Parameter (1)	CV of Fitted Transformed Residual Data: Lower bound (2)	CV of Fitted Transformed Residual Data: Upper bound (3)	CV of Generated Data: Lower bound (4)	CV of Generated Data: Upper bound (5)
S_0	0.1	1.00	0.10	1.00
t_1	0.05	0.50	0.09	0.43
t_2	0.025	0.25	0.23	0.52
a_1	0.05	0.50	0.10	0.41
a_2	0.02	0.20	0.11	0.22
r_1	0.035	0.35	0.04	0.34
r_2	0.025	0.25	0.05	0.25
Q_l	0.04	0.40	0.04	0.40
n	0.04	0.40	0.04	0.40

bounds on CV that were similar to those reflected in the field data set described in part I.

Fractional Factorial Analysis of Coefficients of Variation

In a complete two-level factorial design, an investigator selects two levels (usually an upper and lower bound) of a number of factors and runs experiments with all possible combinations to examine the effect on the response variables. For N factors, the total number of runs required in a two-level experiment is 2^N (Box et al. 1978). While such an approach does not allow a full exploration of the factor space, it can indicate major trends in sensitivity.

In the sensitivity analysis presented here, the main factors are the CV values of nine stochastic parameters: the six parameters of **Γ**, S_0, n, and the hourly inflow. The two levels considered for each factor are the upper and lower bounds on the CV range given in Table 4. The response variables of interest are the average ensemble CV values of $\bar{u}$ and h computed over all space-time points. If a complete two-level factorial design were implemented, $2^9 = 512$ sets of Monte Carlo simulation experiments would need to be conducted. In this problem, an ensemble of about 500 realizations were to be used in each Monte Carlo experiment, requiring a total of 256,000 solutions of the unsteady flow problem. A computational problem of this magnitude was deemed infeasible; hence, a fractional factorial design was adopted.

For problems where the number of factors is substantial (say 5 or more), the total number of runs can be greatly reduced. This is because the higher-order interactions among the factors tend to be redundant and the effects on the response variables often are negligible. Box et al. (1978) describe fractional designs of resolution R_F that implement only a fraction of the total 2^N required by a complete two-level design. The higher the resolution is, the higher is the order of factor interactions considered in the design, and the greater is the number of runs required. In this problem, a high resolution of R_F = VI was used, requiring a total of 128 sets of Monte Carlo simulation experiments.

The effect of the CV of an individual stochastic parameter or of the combinations of CVs of two or more parameters was calculated as the average change in the CV of the response variable of interest, $\bar{u}$ or h, in moving from the lower to the upper bounds of the considered parameter CVs over the 128 sets of experiments. Specifically, Yates's algorithm was used to compute the effects and the F-test was applied to the residuals to verify the statistical significance of the effects (Box et al. 1978).

Results of Sensitivity Analysis

Over the 128 sets of simulation experiments, representing different combinations of upper and lower bounds of CVs of the input parameters, the highest CV values obtained for $\bar{u}$, h, water surface elevation, and Q were 0.97, 0.13, 0.002, and 0.39, respectively. The corresponding lowest values of CV obtained were 0.07, 0.01, 0.0002, and 0.04. Analysis indicated that five main effects associated with individual parameter CV values and 10 effects associated with interactions among parameter CV values were significant (statistically significant, as determined by F-test, and contributing at least 1% of the total effects) in influencing the CV of the flow velocity. For the CV of the flow depth, there were two significant main parameter CV effects and 11 significant interaction effects. There were two significant main effects and five interaction effects for the CV of water surface elevation, and two significant main effects and no interaction effects for the CV of channel flow rate. The significant effects, relative effects (ratio of CV of flow variable to CV of factor), and percentage of total effect, are listed in rank order in Table 5. These results indicate the relative degree to which the uncertainties in the stochastic response variables are sensitive to the uncertainties of the stochastic parameters.

Typically, sensitivity was greatest to main effects and two-factor interactions of the considered parameter CVs. The CV of the predicted flow velocity was most sensitive to the main effect of the CV of a_2 (contributing 30% of the total effects), moderately sensitive to the CVs of Q_l and S_0 and slightly sensitive to the CVs of t_2 and a_1. Effects of six two-factor interactions and four three-factor interactions were associated entirely with the CVs of parameters of channel geometry and Q_l. The strongest two-factor effect was created by the flow area

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TABLE 5. Summary of Significant Effects of Parameter CVs on CVs of Predicted Variables

Factors (parameters) whose CVs contribute effect (1)	Effect (2)	Relative effect (3)	Percent total effect (4)
(*a*) Effect on CV of $\bar{u}$			
a_2	0.37	3.35	30
$a_1 - a_2$	−0.22	−1.06	18
Q_I	0.12	0.35	10
S_0	0.05	0.06	4
$t_2 - a_2$	0.05	0.23	4
$t_2 - a_1$	−0.04	−0.12	3
t_2	0.03	0.10	3
$a_2 - S_0$	−0.02	−0.05	2
$a_2 - Q_I$	−0.02	−0.09	2
$a_1 - Q_I$	−0.02	−0.06	2
$t_1 - r_2 - S_0$	0.02	0.04	2
$t_2 - a_1 - a_2$	−0.02	−0.07	1
$a_1 - a_2 - S_0$	0.02	0.04	1
$t_1 - a_1 - r_2$	0.01	0.05	1
a_1	0.01	0.04	1
(*b*) Effect on CV of *h*			
S_0	0.07	0.08	39
Q_I	0.02	0.06	12
$S_0 - Q_I$	−0.01	−0.02	6
$t_2 - a_2$	−0.01	−0.04	4
$t_2 - a_2 - S_0$	0.01	0.01	3
$a_1 - a_2 - S_0$	0.01	0.01	3
$t_2 - a_1 - S_0$	−0.005	−0.01	3
$a_1 - a_2$	−0.004	−0.02	2
$t_2 - a_1$	0.004	0.01	2
$t_1 - a_2 - S_0$	−0.003	−0.01	2
$t_1 - t_2 - S_0$	0.002	0.00	1
$t_2 - a_1 - a_2 - S_0$	−0.002	−0.01	1
$a_1 - S_0$	0.002	0.00	1
(*c*) Effect on CV of Water Surface Elevation			
Q_I	0.0009	0.003	46
$t_2 - a_2$	−0.0004	−0.002	18
$a_1 - a_2$	−0.0003	−0.001	14
$t_2 - a_1$	0.0002	0.001	12
$t_1 - a_2$	0.0001	0	4
t_2	0.0001	0	4
$t_1 - t_2$	−0.0001	0	3
(*d*) Effect on CV of *Q*			
Q_I	0.31	0.86	97
t_2	0.01	0.04	3

scale and shape factors, a_1-a_2 (18% of total effects). It is noteworthy that the CV of *n* had no significant main or interaction effect on the CV of $\bar{u}$. This result appears in general concordance with the findings of Lai et al. (1992) who reported that predicted deterministic flow rates were insensitive to a ±10% variation in the value of *n*, assumed uniform over a 5.2-km-long channel with a prescribed upstream discharge.

The CV of the predicted flow depth was highly sensitive to the CV of S_0 (39% of total effects) and moderately sensitive to the CV of Q_I. Small to moderate sensitivity was displayed toward the five two-factor, five three-factor, and one four-factor interactions associated with the CVs of parameters of cross-section geometry (flow area and top width), S_0 and Q_I. As in the case for the flow velocity, the uncertainty in *n* did not contribute significantly to the uncertainty in predicted flow depth.

For the predicted water surface elevation, the CV responded primarily to the main effect of the CV of Q_I (46% of total effects). There was a small sensitivity to the CV of t_2. Several two-factor interactions of the CVs of flow area and top width parameters contributed small to moderate effects.

The CV of the predicted channel flow rate was dominated by the effect of the CV of Q_I (97% of total effects). Only a slight sensitivity was revealed toward the CV of t_2.

CONCLUSIONS AND IMPLICATIONS

Consideration of spatiotemporal parameter uncertainty in the open-channel flow equations allows statistical measures to be associated with predictions of the flow variables. A representative solution for the Columbia River revealed statistical characteristics (probability density function, mean, CV, quantiles) that were nonhomogeneous in space and time for the ensemble of possible flow scenarios. Uncertainty was high in predicted values of flow velocity (spatiotemporal average ensemble CV = 0.42) and was low to moderate in the case of flow depth (spatiotemporal average ensemble CV = 0.11). Band widths between the 16% and 84% quantiles of predicted flow variables were found to be substantial. Such measures frame the confidence that is ascribed to possible system behavior.

For a generalized example, sensitivity analysis showed that uncertainty in predicted flow variables is most sensitive to uncertainty in channel cross-section geometry (particularly parameters for flow area and top width), bed slope, and inflow rate. Generally, the parameters in decreasing order of the size of contributing effects were: cross-section geometry/inflow rate/bed slope for flow velocity, bed slope/inflow rate/cross-section geometry for flow depth, and inflow rate/cross-section geometry for both water surface elevation and channel flow rate. Results indicated a general insensitivity to the uncertainty in Manning's *n*. Such findings contribute to an understanding of the fundamental structure of the stochastic flow problem and give guidance in the relative value of field data on parameter variability.

The merit of the stochastic approach is its contribution to the need to grapple with notions of risk, reliability, and variability in engineering decisions for complex systems. For example, an engineer may want to estimate the risk of flooding associated with alternative specified bank elevations along the reach of a river. Since risk can be defined by probability of exceedance, the estimated 95% quantile water surface elevation in Fig. 6 would define a bank elevation line associated with a 5% risk of flooding along that reach of the Columbia River at a time $t = 6$ h into the peak hydrograph. On the other hand, this same line defines the level required to contain the flood with a reliability of 95%. Greater reliabilities of flood containment, and the higher bank elevations (and associated higher costs) required to achieve them, can be similarly estimated. Selected quantiles of predicted water levels and flow depth also can be associated with risk and reliability related to space-time dependent impacts on bridges, regulating structures, riparian vegetation, fish habitat, recreational activities, and so forth. Applications to engineering of man-made channels for irrigation water distribution are also pertinent. Similar notions associated with erosion, sedimentation, and other factors can be attached to probabilities of predicted flow velocities. For example, results like those presented here can be used to compute probabilities of exceedence of erosive velocities along a selected river reach.

This study represents a launch into the comprehensive stochastic solution of Saint-Venant open-channel flow; the uncharted waters, however, are deep and much exploration remains to be done. Understanding of the significance and implications of modeling uncertainty will grow as the number and extent of available field data sets grow. Future modeling studies will need to consider affects of conditioning simulations on measured values, alternative correlation structures and trends among flow parameters, alternative boundary conditions, higher-order correlation among statistical parameters, and sensitivity to statistics other than the CVs alone. As more

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extensive applications in hydraulic engineering problems are explored, the following paradox is expected to be reconfirmed: as uncertainty in a problem is acknowledged and exposed, the path to solution often becomes more clear.

ACKNOWLEDGMENTS

The research conducted in this study was funded by the Kingdom of Saudi Arabia and by Colorado State University. Data on hourly flows in the Columbia River downstream of Grand Coulee Dam were provided by the United States Geological Survey and the United States Bureau of Reclamation. The writers extend their appreciation.

APPENDIX. REFERENCES

Al-Zahrani, M. (1995). "Stochastic modeling of unsteady open-channel flow," PhD thesis, Civ. Engrg. Dept., Colorado State Univ., Fort Collins, Colo.

Blanton, J. O. (1974). "Channel hydraulic studies for Columbia riverbank stabilization below Grand Coulee Dam." *Proc. Hydr. Div. Spec. Conf.*, ASCE, New York, N.Y.

Bos, M. G., ed. (1989). *Discharge measurement structures*, 3rd Ed., *Int. Inst. for Land Reclamation and Improvement*, Wageningen, The Netherlands.

Box, G. E. P., Hunter, W. G., and Hunter, J. S. (1978). *Statistics for experimenters: an introduction to design, data analysis, and model building*. John Wiley and Sons, New York, N.Y.

Chang, C.-H., Tung, Y.-K., and Yang, J.-C. (1994). "Monte Carlo simulation for correlated variables with marginal distributions." *J. Hydr. Engrg.*, ASCE, 120(3), 313–331.

Der Kiureghian, A., and Liu, P.-L. (1986). "Structural reliability under incomplete probability information." *J. Engrg. Mech.*, ASCE, 112(1), 85–104.

Fread, D. L. (1985). "Channel routing." *Hydrological forecasting*, M. G. Anderson and T. P. Burt, eds., J. Wiley and Sons, New York, N.Y., 437–503.

Gates, T. K., and Al-Zahrani, M. A. (1996). "Spatiotemporal stochastic open-channel flow. I: model and its parameter data." *J. Hydr. Engrg.*, ASCE, 122(11), 641–651.

Haskell, W. C. (1994). "Statistical characterization of seepage losses from open channels," MS Thesis Civ. Engrg. Dept., Colorado State Univ., Fort Collins, Colo.

Kuo, J.-T., and Lin, G.-F., eds. (1992). *Stochastic Hydraulics '92. Proc., 6th IAHR Int. Symp. on Stochastic Hydr.* Dept. Civil Engrg., National Taiwan Univ., Taipei, Taiwan.

Lai, C., Schaffranek, W., and Baltzer, R. A. (1992). "Frictional resistance treatment in unsteady open-channel flow simulation." *Channel flow resistance: centennial of Manning's formula*, B. C. Yen, ed., Water Resources Pub., Littleton, Colo.

Liggett, J. A. (1993). "Critical depth, velocity profiles, and averaging." *J. Irrig. and Drain. Engrg.*, ASCE, 119(2), 416–422.

Schumm, S. A., and Winkley, B. R., eds. (1994). *The variability of large alluvial rivers*. ASCE Press, New York, N.Y.

Xia, R., and Yen, B. C. (1994). "Significance of averaging coefficients in open-channel flow equations." *J. Hydr. Engrg.*, ASCE, 120(2), 169–191.

Yen, B. C., and Tung, Y.-K., eds. (1993). *Reliability and uncertainty analyses in hydraulic design*. ASCE, New York, N.Y.

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