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STRESS INTENSITY FACTORS FOR CIRCUMFERENTIAL SURFACE CRACKS IN PIPES

M. BERGMAN
Swedish Plant Inspection Ltd., Box 49306, S-100 29 Stockholm, Sweden

Abstract—Stress intensity factors for circumferential surface cracks in pipes have been derived using the finite element method. Both cracks located at the in- and outside of the pipes have been analysed. The derived solutions cover a wide range of geometry and load configurations and are presented in a tabular form that defines influence functions for the stress intensity factor along the whole crack front. The solutions show good agreements in comparisons to other published solutions.

NOMENCLATURE

a = crack depth
A_j = coefficient describing the influence action
c = half crack length
E = Young's modulus
f_i = influence function
J = J-integral
K_I = stress intensity factor
r, θ = cyclindrical coordinates
R_i = internal radius
t = wall thickness
u = local radial coordinate
x, y = Cartesian coordinates
ϕ = parametric angle
ν = Poisson's ratio
σ_i = coefficients describing the elastic uncracked body stress state

INTRODUCTION

Stress intensity factors are used to describe the behaviour of cracks, both as regard to the risk of instantaneous failure of a cracked component and as to estimate how a crack grows due to fatigue or stress corrosion. A common crack geometry model utilised in these analyses, especially for the energy process industry, is the circumferential surface crack in a pipe. However, to the author's knowledge, no complete and accurate stress intensity factor solutions covering a wide range of geometry and load configurations exist for this geometry up to today.

The article presents a large number of solutions which have been calculated using the finite element method. Both cracks located at the in- and outside of the pipe are analysed for pipes with a thickness ratio (R_i/t) equal to 5 and 10. The crack depth ratio (a/t) ranges from 0.2 to 0.8 and the crack aspect ratio (c/a) ranges from unity to 16 in the analyses. A total of 80 different geometry configurations are considered and each configuration were subjected to six different load cases. The load cases are expressed as functions of the elastic stress state in an uncracked pipe and describe an axisymmetrical through thickness stress distribution up to a third degree polynomial and global bending stresses with respect to two perpendicular axes in the prospective crack plane. The load cases considered make it possible to analyse complex weld residual stress distributions as well as global bending arbitrarily oriented with respect to the location of the crack.

Both the number of geometry and load configurations considered means that a total number of

480 three dimensional and rather large finite element calculations have been conducted, something that would not have been easy to carry out without today's fast work stations. Also, an especially developed mesh generator that made the modelling more easy was utilised.

The derived solutions are to be implemented in a Swedish handbook and software [1] for fracture mechanics assessments.

GEOMETRICAL DESCRIPTION

The geometries considered are shown in Fig. 1. The pipe is described by its internal radius, R_i, and wall thickness, t, and the crack by its depth, a, and length, $2c$ along the in- and outside perimeter, respectively.

The crack front is described by a cylindrically transformed ellipse. A parametric description of the crack front equations yields

$$x = (R_i + a \sin\phi) \sin\frac{c\cos\phi}{R_i}, \quad y = (R_i + a\sin\phi)\cos\frac{c\cos\phi}{R_i}, \tag{1}$$

for the inside surface crack, and

$$x = (R_i + t - a\sin\phi)\sin\frac{c\cos\phi}{R_i + t}, \quad y = (R_i + t - a\sin\phi)\cos\frac{c\cos\phi}{R_i + t}, \tag{2}$$

for the outside surface crack. The parameter ϕ ranges from 0 to π and is used to give the position along the crack front, $2\phi/\pi$ equal to 0 or two gives the position at the intersections of the crack front with the free surface and $2\phi/\pi$ equal to unity gives the position at the deepest point of the crack front.

A local radial coordinate, u, is introduced which is equal to $r - R_i$ for the inside crack and $R_i + t - r$ for the outside crack, i.e. for the inside crack u/t is equal to 0 and unity at the in- and outside of the pipe respectively, and vice versa for the outside crack.

NUMERICAL TECHNIQUE

The finite element program ABAQUS [2] was used for the calculations. The program is especially suitable for this purpose due to its built in J-integral solver based on the domain integral technique, its capability to describe an arbitrary crack face pressure with the user subroutine DLOAD and the possibility to write specific programs to process the results.

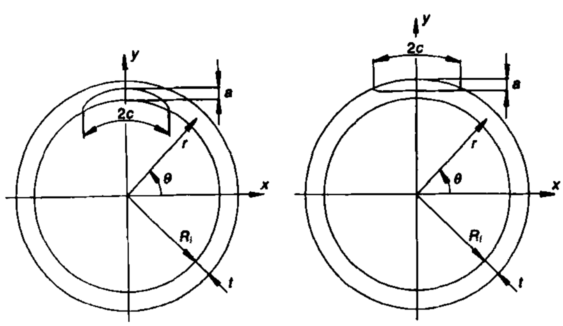

Fig. 1. Circumferential in- and outside surface crack in a pipe.

The finite element models were generated using a mesh generating program developed by Faleskog [3]. The program is able to model surface cracks in plates and pipes and generates complete input files to ABAQUS. Details of the mesh can be viewed by the program so that possible corrections can be made before a more time consuming mesh analysis is done using ABAQUS. Typical meshes for an in- and outside surface crack are shown in Figs 2 and 3, respectively. Due to symmetry conditions only a quarter of the pipe had to be modelled. The extension of the pipe was chosen to be 15 times the wall thickness of the pipe so that no significant end-effects would be present. The meshes consisted of 20 node brick elements, and the number of elements ranged from 1268 to 2342 depending on the length of the crack front. The crack tip elements were degenerated into wedges and mid-side nodes located perpendicular to the crack front were shifted to a quarter point location to obtain a $1/\sqrt{r}$ strain singularity. Multipoint constraint equations were used to coarsen the mesh far away from the crack tip region.

According to the principle of superposition, the load cases were applied as corresponding pressures on the crack face. This was realised with user subroutine DLOAD in ABAQUS which makes it possible to define arbitrary pressure distributions on element faces. The pressure is evaluated in the load integration points of the elements and converted to consistent nodal loads. All load cases were analysed in the same run in six consecutive steps. In order to have a fully opened crack during all steps, a constant pressure component with a large magnitude was applied in the first step and not removed in the following steps. This was corrected later on for in the post processing program by subtracting the stress intensity factor determined in the first step from the ones determined in the following steps.

A specific post processing program was developed which read the database generated by ABAQUS. The J-integral was evaluated in nodes along the crack front as the mean of the values determined in contour two to five around the crack tip. Within these contours, the J-integral showed almost no path dependency. The stress intensity factor K_I, was evaluated from the J-integral

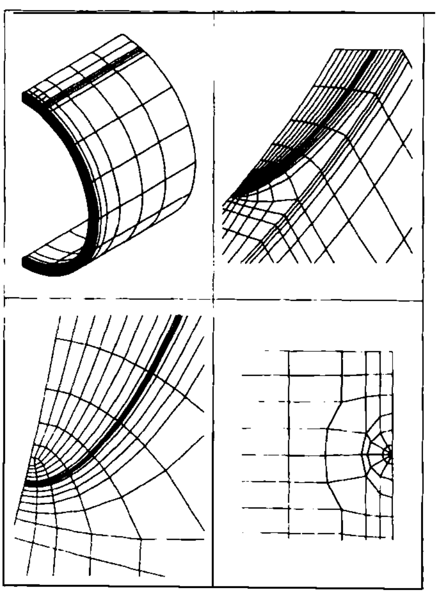

Fig. 2. Details of a typical finite element mesh for an inside surface crack.

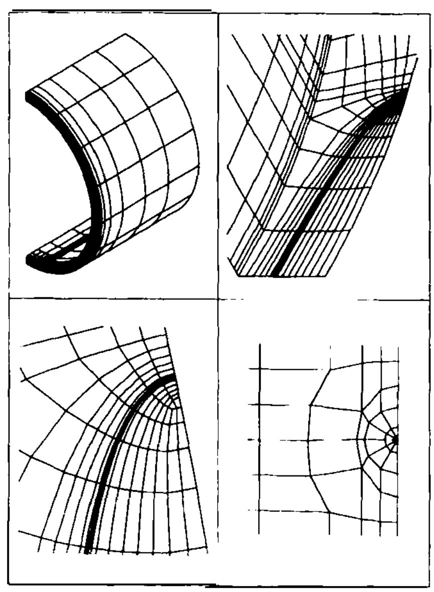

Fig. 3. Details of a typical finite element mesh for an outside surface crack.

under the assumption that plane strain conditions prevail along the whole crack front, i.e.

$$K_I = \sqrt{\frac{EJ}{(1-\nu^2)}}. \tag{3}$$

Poisson's ratio, ν, was set to 0.3 in the calculations. For each load case considered K_I was normalised according to Eq. (4) to give the influence function. The so obtained node values of the influence function were then fitted to a six degree polynomial of the parametric angle ϕ to give a closed form expression of the influence function along the crack front. For all cases, the fit correlation coefficient was almost identical to unity.

RESULTS AND DISCUSSION

The stress intensity factor, K_I, is given by

$$K_I = \sqrt{\pi a}\,\sigma_i f_i, \tag{4}$$

where f_i is the influence function to the stress component σ_i. A total of six ($i = 0, 1, \ldots, 5$) different load cases in form of elastic stress components in the axial direction of an uncracked pipe were considered. The first four components describe a third degree axisymmetrical stress distribution through the thickness according to

$$\sigma = \sigma_0 + \sigma_1 \frac{u}{a} + \sigma_2 \left(\frac{u}{a}\right)^2 + \sigma_3 \left(\frac{u}{a}\right)^3, \tag{5}$$

where u is a local radial coordinate as defined before, and the last two components describe global bending stresses with respect to the x and y axes (see Fig. 1) according to

$$\sigma = \sigma_4 \frac{y}{R_i + t} + \sigma_5 \frac{x}{R_i + t}. \tag{6}$$

The actual stress state normal to and within the prospective crack face is fitted to the above relations to determine σ_i. Then f_i is evaluated for each load case and the product $\sigma_i f_i$ summed to give the total value of K_I.

The influence function, f_i, is described by a six degree polynomial of the parametric angle ϕ according to

$$f_i = A_0 + A_1 \frac{2\phi}{\pi} + A_2 \left(\frac{2\phi}{\pi}\right)^2 + A_3 \left(\frac{2\phi}{\pi}\right)^3 + A_4 \left(\frac{2\phi}{\pi}\right)^4 + A_5 \left(\frac{2\phi}{\pi}\right)^5 + A_6 \left(\frac{2\phi}{\pi}\right)^6, \tag{7}$$

where A_j ($j = 0, 1, \ldots, 6$) are coefficients which are tabulated in Tables 1–10 for each load case. For different configurations of the ratios of a/t, c/a and R_i/t, Tables 1–5 give the influence coefficients for the inside surface cracks and Tables 6–10 the influence coefficients for the outside surface cracks. Note that Eq. (7) is only valid for the part of the crack front which is situated in the first quadrant, i.e. $0 \leqslant 2\phi/\pi \leqslant 1$. Due to the symmetry conditions, the influence functions for the second quadrant, $1 < 2\phi/\pi \leqslant 2$, are given by $f_i(\phi) = f_i(\pi - \phi)$ for the load cases $i = 0, 1, \ldots, 4$ and for the last load case by $f_5(\phi) = -f_5(\pi - \phi)$. As mentioned before, $2\phi/\pi$ equals 0 or two in the

Table 1. Influence function coefficients; inside surface crack, $c/a = 1$

		$R_i/t = 5$							$R_i/t = 10$						
a/t	Load	A_0	A_1	A_2	A_3	A_4	A_5	A_6	A_0	A_1	A_2	A_3	A_4	A_5	A_6
0.2	0	7.457e-1	-1.075e-1	-1.555	6.853	-1.182e1	9.393	-2.840	7.468e-1	-9.249e-2	-1.609	6.973	-1.203e1	9.581	-2.902
	1	1.250e-1	-1.067e-1	3.906	-1.162e1	1.747e1	-1.318e1	3.874	1.250e-1	-1.023e-1	3.894	-1.157e1	1.733e1	-1.305e1	3.830
	2	4.592e-2	9.340e-2	-2.639e-1	3.820	-7.436	6.069	-1.957	4.578e-2	9.584e-2	-2.767e-1	3.887	-7.587	6.204	-1.998
	3	2.327e-2	1.063e-1	-8.843e-1	4.584	-7.077	5.330	-1.765	2.312e-2	1.083e-1	-8.997e-1	4.658	-7.224	5.448	-1.798
	4	6.253e-1	-7.295e-2	-1.307	5.771	-9.975	7.922	-2.395	6.819e-1	-7.527e-2	-1.471	6.381	-1.102e1	8.776	-2.658
	5	1.607e-2	-9.437e-3	5.687e-3	-2.808e-2	2.586e-2	-1.791e-2	7.981e-3	8.755e-3	-5.362e-3	4.520e-3	-2.341e-2	3.288e-2	-2.852e-2	1.127e-2
0.4	0	7.740e-1	-1.397e-1	-1.726	7.678	-1.320e1	1.045e1	-3.154	7.780e-1	-1.111e-1	-1.838	7.925	-1.362e1	1.083e1	-3.281
	1	1.329e-1	6.979e-3	3.001	-8.745	1.303e1	-9.856	2.905	1.334e-1	1.553e-2	2.973	-8.634	1.276e1	-9.589	2.816
	2	5.079e-2	9.002e-2	-1.854e-1	3.451	-6.605	5.225	-1.645	5.090e-2	9.472e-2	-2.122e-1	3.587	-6.907	5.496	-1.729
	3	2.648e-2	9.311e-2	-7.567e-1	4.061	-5.909	4.136	-1.322	2.646e-2	9.695e-2	-7.871e-1	4.205	-6.195	4.369	-1.388
	4	6.524e-1	-8.372e-2	-1.461	6.520	-1.124e1	8.889	-2.681	7.124e-1	-8.359e-2	-1.686	7.283	-1.254e1	9.967	-3.018
	5	3.229e-2	-1.698e-2	4.380e-4	-6.238e-3	-5.040e-2	5.624e-2	-1.515e-2	1.756e-2	-9.685e-3	5.811e-5	-6.884e-3	-1.627e-2	2.032e-2	-4.969e-3
0.6	0	8.216e-1	-2.078e-1	-1.964	8.970	-1.549e1	1.227e1	-3.707	8.307e-1	-1.695e-1	-2.148	9.383	-1.618e1	1.288e1	-3.909
	1	1.465e-1	6.779e-2	2.371	-6.655	9.770	-7.405	2.188	1.481e-1	7.928e-2	2.326	-6.484	9.357	-6.998	2.051
	2	5.776e-2	8.196e-2	-1.438e-1	3.259	-6.125	4.714	-1.453	5.813e-2	8.837e-2	-1.842e-1	3.458	-6.568	5.113	-1.577
	3	3.059e-2	8.080e-2	-6.681e-1	3.699	-5.097	3.309	-1.017	3.066e-2	8.605e-2	-7.119e-1	3.906	-5.511	3.648	-1.114
	4	6.962e-1	-1.259e-1	-1.674	7.678	-1.329e1	1.051e1	-3.172	7.630e-1	-1.289e-1	-1.978	8.658	-1.495e1	1.191e1	-3.611
	5	4.884e-2	-2.367e-2	-7.370e-3	2.429e-2	-1.332e-1	1.221e-1	-3.058e-2	2.651e-2	-1.366e-2	-6.718e-3	1.647e-2	-7.205e-2	6.848e-2	-1.883e-2
0.8	0	8.762e-1	-2.897e-1	-2.199	1.048e1	-1.833e1	1.465e1	-4.457	8.904e-1	-2.483e-1	-2.465	1.109e1	-1.934e1	1.552e1	-4.738
	1	1.609e-1	1.056e-1	1.934	-5.127	7.349	-5.515	1.600	1.626e-1	1.191e-1	1.869	-4.897	6.807	-4.992	1.429
	2	6.396e-2	7.834e-2	-1.111e-1	3.129	-5.806	4.427	-1.371	6.396e-2	8.621e-2	-1.635e-1	3.382	-6.369	4.929	-1.524
	3	3.373e-2	7.526e-2	-6.002e-1	3.431	-4.516	2.775	-8.457e-1	3.339e-2	8.182e-2	-6.553e-1	3.690	-5.036	3.197	-9.634e-1
	4	7.461e-1	-1.804e-1	-1.884	9.028	-1.582e1	1.262e1	-3.836	8.204e-1	-1.933e-1	-2.276	1.027e1	-1.794e1	1.440e1	-4.392
	5	6.583e-2	-2.966e-2	-2.463e-2	1.058e-1	-3.174e-1	2.748e-1	-7.431e-2	3.567e-2	-1.759e-2	-1.685e-2	5.622e-2	-1.568e-1	1.385e-1	-3.887e-2

Table 2. Influence function coefficients; inside surface crack, $c/a = 2$

		$R_i/t = 5$							$R_i/t = 10$						
a/t	Load	A_0	A_1	A_2	A_3	A_4	A_5	A_6	A_0	A_1	A_2	A_3	A_4	A_5	A_6
0.2	0	7.156e-1	-2.973e-1	9.910e-1	2.301	-8.244	8.201	-2.778	7.187e-1	-2.774e-1	9.143e-1	2.483	-8.520	8.428	-2.853
	1	1.078e-1	2.019e-1	1.066	-1.489e-1	-2.772	3.574	-1.472	1.086e-1	2.071e-1	1.053	-1.121e-1	-2.860	3.668	-1.507
	2	3.938e-2	1.320e-1	-1.011	7.971	-1.627e1	1.468e1	-5.119	3.970e-2	1.346e-1	-1.022	8.021	-1.639e1	1.480e1	-5.159
	3	1.944e-2	1.392e-1	-1.657	8.964	-1.700e1	1.548e1	-5.599	1.960e-2	1.411e-1	-1.670	9.022	-1.712e1	1.560e1	-5.636
	4	5.989e-1	-2.347e-1	8.285e-1	1.993	-7.030	6.953	-2.348	6.559e-1	-2.457e-1	8.339e-1	2.300	-7.843	7.740	-2.617
	5	3.611e-2	-2.231e-2	8.612e-2	-1.639e-1	3.547e-2	3.460e-2	-5.045e-3	1.970e-2	-1.208e-2	4.572e-2	-9.031e-2	2.634e-2	1.134e-2	-1.441e-4
0.4	0	7.679e-1	-3.185e-1	5.700e-1	4.504	-1.246e1	1.178e1	-3.910	7.807e-1	-2.827e-1	3.960e-1	4.970	-1.321e1	1.241e1	-4.121
	1	1.251e-1	2.551e-1	5.209e-1	1.780	-5.846	5.860	-2.118	1.288e-1	2.642e-1	4.875e-1	1.894	-6.104	6.125	-2.215
	2	4.867e-2	1.144e-1	-8.461e-1	7.258	-1.467e1	1.305e1	-4.514	5.037e-2	1.191e-1	-8.717e-1	7.379	-1.495e1	1.332e1	-4.609
	3	2.524e-2	1.148e-1	-1.409	7.869	-1.460e1	1.310e1	-4.738	2.621e-2	1.182e-1	-1.436	7.996	-1.487e1	1.335e1	-4.823
	4	6.431e-1	-2.385e-1	4.722e-1	3.936	-1.073e1	1.006e1	-3.324	7.138e-1	-2.435e-1	3.599e-1	4.621	-1.223e1	1.146e1	-3.796
	5	7.289e-2	-3.914e-2	1.230e-1	-1.086e-1	-3.475e-1	4.223e-1	-1.209e-1	3.977e-2	-2.087e-2	6.055e-2	-5.598e-2	-1.773e-1	2.172e-1	-6.226e-2
0.6	0	8.515e-1	-4.492e-1	6.537e-1	5.317	-1.457e1	1.373e1	-4.538	8.826e-1	-4.092e-1	3.557e-1	6.191	-1.606e1	1.501e1	-4.964
	1	1.517e-1	2.313e-1	4.887e-1	2.037	-6.148	5.902	-2.060	1.604e-1	2.414e-1	4.239e-1	2.273	-6.665	6.418	-2.246
	2	6.164e-2	8.360e-2	-6.362e-1	6.405	-1.278e1	1.111e1	-3.785	6.552e-2	8.904e-2	-6.820e-1	6.616	-1.326e1	1.157e1	-3.948
	3	3.286e-2	8.728e-2	-1.161	6.730	-1.201e1	1.045e1	-3.756	3.500e-2	9.155e-2	-1.204	6.936	-1.245e1	1.087e1	-3.895
	4	7.124e-1	-3.349e-1	5.386e-1	4.720	-1.266e1	1.177e1	-3.861	8.085e-1	-3.531e-1	3.230e-1	5.792	-1.494e1	1.390e1	-4.586
	5	1.123e-1	-6.235e-2	1.811e-1	-6.573e-2	-7.625e-1	8.500e-1	-2.497e-1	6.138e-2	-3.291e-2	8.333e-2	-2.675e-2	-3.920e-1	4.384e-1	-1.298e-1
0.8	0	9.444e-1	-4.598e-1	-5.505e-1	1.116e1	-2.570e1	2.327e1	-7.600	9.946e-1	-4.165e-1	-1.091	1.285e1	-2.868e1	2.581e1	-8.409
	1	1.792e-1	2.611e-1	-1.289e-2	4.015	-9.367	8.373	-2.796	1.914e-1	2.739e-1	-1.373e-1	4.447	-1.027e1	9.224	-3.083
	2	7.456e-2	6.156e-2	-4.541e-1	5.547	-1.063e1	8.831	-2.931	7.924e-2	6.923e-2	-5.289e-1	5.859	-1.131e1	9.472	-3.146
	3	4.010e-2	6.105e-2	-8.546e-1	5.187	-8.320	6.637	-2.339	4.228e-2	6.724e-2	-9.185e-1	5.460	-8.894	7.159	-2.509
	4	7.879e-1	-3.312e-1	-4.721e-1	9.716	-2.209e1	1.980e1	-6.432	9.125e-1	-3.536e-1	-1.005	1.196e1	-2.662e1	2.389e1	-7.770
	5	1.541e-1	-7.005e-2	5.403e-2	7.113e-1	-2.474	2.339	-7.110e-1	8.468e-2	-3.625e-2	-2.753e-3	4.241e-1	-1.352	1.277	-3.931e-1

Table 3. Influence function coefficients; inside surface crack, $c/a = 4$

		$R_i/t = 5$							$R_i/t = 10$						
a/t	Load	A_0	A_1	A_2	A_3	A_4	A_5	A_6	A_0	A_1	A_2	A_3	A_4	A_5	A_6
0.2	0	5.975e-1	-6.007e-1	8.229	-2.241e1	3.064e1	-2.137e1	5.959	6.021e-1	-5.860e-1	8.190	-2.226e1	3.035e1	-2.112e1	5.875
	1	7.085e-2	1.657e-1	1.583	-4.140e-1	-3.331	4.215	-1.664	7.218e-2	1.686e-1	1.584	-4.013e-1	-3.379	4.273	-1.687
	2	2.266e-2	8.073e-2	-5.587e-1	6.520	-1.299e1	1.121e1	-3.811	2.328e-2	8.184e-2	-5.590e-1	6.540	-1.305e1	1.127e1	-3.835
	3	1.000e-2	9.958e-2	-1.222	6.982	-1.202e1	1.034e1	-3.718	1.036e-2	1.003e-1	-1.225	6.920	-1.209e1	1.041e1	-3.744
	4	4.961e-1	-4.890e-1	6.855	-1.852e1	2.524e1	-1.762e1	4.917	5.486e-1	-5.289e-1	7.474	-2.025e1	2.757e1	-1.918e1	5.336
	5	6.842e-2	-7.537e-2	9.820e-1	-3.157	4.463	-3.265	9.864e-1	3.743e-2	-4.062e-2	5.312e-1	-1.717	2.435	-1.784	5.394e-1
0.4	0	6.548e-1	-6.946e-1	8.986	-2.424e1	3.317e1	-2.327e1	6.531	6.765e-1	-6.808e-1	9.032	-2.428e1	3.305e1	-2.305e1	6.435
	1	9.011e-2	1.424e-1	1.871	-1.348	-1.568	2.543	-1.064	9.691e-2	1.431e-1	1.912	-1.423	-1.544	2.591	-1.098
	2	3.240e-2	5.064e-2	-2.062e-1	5.153	-1.019e1	8.493	-2.832	3.571e-2	5.041e-2	-1.884e-1	5.146	-1.026e1	8.601	-2.880
	3	1.592e-2	7.171e-2	-8.851e-1	5.425	-8.818	7.152	-2.557	1.787e-2	7.177e-2	-8.809e-1	5.457	-8.934	7.286	-2.610
	4	5.309e-1	-5.439e-1	7.333	-1.937e1	2.640e1	-1.861e1	5.247	6.134e-1	-6.071e-1	8.214	-2.191e1	2.975e1	-2.077e1	5.804
	5	1.398e-1	-1.543e-1	1.998	-6.311	8.811	-6.425	1.947	7.728e-2	-8.288e-2	1.080	-3.452	4.855	-3.550	1.076
0.6	0	7.366e-1	-8.489e-1	1.015e1	-2.702e1	3.699e1	-2.618e1	7.448	7.878e-1	-8.554e-1	1.035e1	-2.736e1	3.699e1	-2.586e1	7.281
	1	1.159e-1	9.976e-2	2.270	-2.506	4.358e-1	7.473e-1	-4.438e-1	1.312e-1	9.372e-2	2.360	-2.655	4.328e-1	9.044e-1	-5.293e-1
	2	4.475e-2	1.800e-2	1.365e-1	3.925	-7.792	6.229	-2.034	5.194e-2	1.476e-2	1.716e-1	3.917	-7.950	6.464	-2.133
	3	2.312e-2	4.519e-2	-5.852e-1	4.201	-6.242	4.644	-1.659	2.721e-2	4.387e-2	-5.766e-1	4.258	-6.465	4.905	-1.760
	4	5.764e-1	-6.384e-1	8.017	-2.059e1	2.813e1	-2.018e1	5.814	7.101e-1	-7.567e-1	9.367	-2.446e1	3.298e1	-2.311e1	6.525
	5	2.181e-1	-2.499e-1	3.116	-9.603	1.321e1	-9.612	2.926	1.231e-1	-1.359e-1	1.698	-5.340	7.434	-5.424	1.649
0.8	0	8.463e-1	-1.006	1.123e1	-2.892e1	3.864e1	-2.694e1	7.610	9.267e-1	-1.015	1.128e1	-2.818e1	3.577e1	-2.387e1	6.569
	1	1.506e-1	5.425e-2	2.660	-3.447	1.762	-1.027e-1	-2.657e-1	1.721e-1	5.231e-2	2.663	-3.112	5.805e-1	1.166	-7.090e-1
	2	6.150e-2	-1.790e-2	4.826e-1	2.866	-6.093	5.040	-1.751	7.048e-2	-1.683e-2	4.546e-1	3.175	-6.977	5.950	-2.069
	3	3.290e-2	1.952e-2	-3.329e-1	3.397	-4.945	3.770	-1.469	3.748e-2	2.197e-2	-3.728e-1	3.685	-5.679	4.500	-1.723
	4	6.336e-1	-7.259e-1	8.527	-2.079e1	2.785e1	-1.994e1	5.751	8.292e-1	-8.905e-1	1.014e1	-2.488e1	3.143e1	-2.105e1	5 816
	5	3.019e-1	-3.399e-1	4.108	-1.215e1	1.605e1	-1.142e1	3.464	1.755e-1	-1.882e-1	2.248	-6.804	9.025	-6.347	1.898

Table 4. Influence function coefficients; inside surface crack, $c/a = 8$

a/t	Load	$R_i/t = 5$							$R_i/t = 10$						
		A_0	A_1	A_2	A_3	A_4	A_5	A_6	A_0	A_1	A_2	A_3	A_4	A_5	A_6
0.2	0	4.465e-1	-2.557e-1	1.287e1	-4.560e1	7.476e1	-5.920e1	1.811e1	4.550e-1	-3.107e-1	1.332e1	-4.681e1	7.641e1	-6.031e1	1.839e1
	1	3.667e-2	1.315e-1	2.431	-2.956	6.499e-1	1.085	-7.135e-1	3.875e-2	1.139e-1	2.618	-3.611	1.825	5.762e-2	-3.702e-1
	2	9.101e-3	7.856e-2	-5.547e-1	6.723	-1.322e1	1.128e1	-3.828	1.001e-2	7.396e-2	-4.889e-1	6.420	-1.251e1	1.053e1	-3.541
	3	3.030e-3	1.047e-1	-1.291	7.100	-1.214e1	1.037e1	-3.750	3.581e-3	1.009e-1	-1.226	6.734	-1.123e1	9.386	-3.370
	4	3.566e-1	-1.987e-1	1.036e1	-3.624e1	5.941e1	-4.719e1	1.446e1	4.105e-1	-2.774e-1	1.205e1	-4.219e1	6.885e1	-5.437e1	1.659e1
	5	1.091e-1	-6.856e-2	3.189	-1.234e1	2.017e1	-1.611e1	5.059	6.091e-2	-4.593e-2	1.809	-6.969	1.138e1	-9.066	2.837
0.4	0	4.773e-1	-1.876e-1	1.295e1	-4.529e1	7.451e1	-5.949e1	1.833e1	4.969e-1	-1.715e-1	1.326e1	-4.616e1	7.577e1	-6.039e1	1.857e1
	1	4.675e-2	1.244e-1	2.885	-4.825	4.796	-3.138	8.441e-1	5.287e-2	1.274e-1	2.985	-5.071	5.115	-3.331	8.792e-1
	2	1.413e-2	4.597e-2	1.161e-2	4.161	-7.416	5.341	-1.620	1.708e-2	4.742e-2	5.221e-2	4.087	-7.362	5.353	-1.644
	3	6.088e-3	6.650e-2	-6.880e-1	4.102	-5.120	3.107	-1.039	7.811e-3	6.774e-2	-6.720e-1	4.098	-5.176	3.205	-1.088
	4	3.330e-1	-1.237e-1	9.234	-3.094e1	5.133e1	-4.171e1	1.300e1	4.348e-1	-1.449e-1	1.169e1	-4.020e1	6.604e1	-5.283e1	1.628e1
	5	2.161e-1	-8.885e-2	5.937	-2.250e1	3.641e1	-2.922e1	9.258	1.251e-1	-4.956e-2	3.401	-1.308e1	2.129e1	-1.708e1	5.403
0.6	0	5.282e-1	-2.379e-1	1.439e1	-4.945e1	8.189e1	-6.639e1	2.079e1	5.681e-1	-2.062e-1	1.488e1	-5.033e1	8.248e1	-6.648e1	2.074e1
	1	6.174e-2	1.161e-1	3.324	-6.268	7.770	-6.109	1.925	7.337e-2	1.247e-1	3.450	-6.374	7.583	-5.762	1.763
	2	2.110e-2	2.977e-2	3.554e-1	2.895	-4.781	2.785	-7.169e-1	2.638e-2	3.458e-2	3.922e-1	2.963	-5.129	3.215	-8.948e-1
	3	1.007e-2	5.124e-2	-4.229e-1	3.051	-2.877	9.325e-1	-2.760e-1	1.300e-2	5.495e-2	-4.188e-1	3.175	-3.266	1.374	-4.519e-1
	4	2.918e-1	-1.334e-1	8.363	-2.623e1	4.525e1	-3.876e1	1.254e1	4.753e-1	-1.668e-1	1.259e1	-4.162e1	6.853e1	-5.579e1	1.752e1
	5	3.180e-1	-1.234e-1	8.593	-3.114e1	4.943e1	-3.988e1	1.282e1	1.990e-1	-7.638e-2	5.294	-1.977e1	3.174e1	-2.554e1	8.166
0.8	0	6.002e-1	-2.498e-1	1.571e1	-5.183e1	8.416e1	-6.729e1	2.080e1	6.698e-1	-1.658e-1	1.610e1	-5.084e1	7.929e1	-6.239e1	1.931e1
	1	8.470e-2	9.597e-2	3.971	-8.154	1.098e1	-8.576	2.583	1.040e-1	1.245e-1	4.000	-7.406	8.603	-6.284	1.860
	2	3.237e-2	3.077e-3	8.643e-1	1.091	-1.382	-3.580e-2	1.175e-1	4.071e-2	1.929e-2	8.253e-1	1.686	-2.996	1.501	-3.819e-1
	3	1.687e-2	2.519e-2	6.837e-3	1.345	5.256e-1	-2.028	6.513e-1	2.130e-2	3.661e-2	-4.941e-2	1.839	-7.188e-1	-8.447e-1	2.578e-1
	4	2.355e-1	-1.234e-1	6.781	-1.855e1	3.417e1	-3.119e1	1.030e1	5.308e-1	-1.234e-1	1.290e1	-3.895e1	6.128e1	-4.927e1	1.548e1
	5	4.081e-1	-1.012e-1	1.035e1	-3.474e1	5.230e1	-4.155e1	1.335e1	2.810e-1	-6.964e-2	6.929	-2.443e1	3.694e1	-2.863e1	8.996

Table 5. Influence function coefficients; inside surface crack, $c/a = 16$

		$R_i/t = 5$							$R_i/t = 10$						
a/t	Load	A_0	A_1	A_2	A_3	A_4	A_5	A_6	A_0	A_1	A_2	A_3	A_4	A_5	A_6
0.2	0	2.944e-1	1.327	6.054	-3.004e1	5.606e1	-4.794e1	1.542e1	2.976e-1	1.351	6.067	-2.997e1	5.581e1	-4.764e1	1.529e1
	1	1.446e-2	1.674e-1	2.612	-3.955	3.180	-1.674	3.467e-1	1.545e-2	1.749e-1	2.598	-3.841	2.916	-1.400	2.381e-1
	2	2.395e-3	6.303e-2	-3.584e-1	5.588	-1.000e1	7.588	-2.365	2.872e-3	6.730e-2	-3.760e-1	5.692	-1.024e1	7.834	-2.461
	3	1.545e-4	8.153e-2	-9.692e-1	5.124	-6.997	4.802	-1.625	4.294e-4	8.458e-2	-9.875e-1	5.225	-7.227	5.044	-1.720
	4	1.995e-1	8.985e-1	4.311	-1.979e1	3.747e1	-3.277e1	1.068e1	2.582e-1	1.173	5.342	-2.585e1	4.823e1	-4.136e1	1.331e1
	5	1.428e-1	6.403e-1	2.940	-1.634e1	2.962e1	-2.535e1	8.369	8.237e-2	3.685e-1	1.693	-9.575	1.742e1	-1.488e1	4.893
0.4	0	3.117e-1	1.345	6.500	-3.072e1	5.818e1	-5.101e1	1.678e1	3.213e-1	1.417	6.404	-2.984e1	5.633e1	-4.925e1	1.611e1
	1	1.845e-2	1.723e-1	2.903	-5.212	6.564	-5.483	1.813	2.104e-2	1.947e-1	2.823	-4.679	5.441	-4.373	1.383
	2	4.218e-3	4.807e-2	-1.597e-2	3.986	-6.088	3.467	-8.365e-1	5.321e-3	5.988e-2	-8.248e-2	4.376	-6.929	4.329	-1.176
	3	1.247e-3	6.149e-2	-6.210e-1	3.385	-2.806	4.711e-1	-3.748e-2	1.814e-3	6.930e-2	-6.781e-1	3.707	-3.521	1.221	-3.340e-1
	4	8.707e-2	3.421e-1	2.630	-8.011	1.927e1	-2.032e1	7.187	2.398e-1	1.055	5.031	-2.170e1	4.193e1	-3.766e1	1.248e1
	5	2.417e-1	1.087	4.627	-2.271e1	4.017e1	-3.579e1	1.239e1	1.658e-1	7.265e-1	3.307	-1.749e1	3.158e1	-2.767e1	9.393
0.6	0	3.305e-1	1.372	6.926	-3.028e1	5.802e1	-5.228e1	1.762e1	3.518e-1	1.533	6.676	-2.799e1	5.324e1	-4.823e1	1.631e1
	1	2.262e-2	1.761e-1	3.199	-6.329	9.945	-9.517	3.404	2.841e-2	2.334e-1	2.914	-4.629	6.447	-6.328	2.294
	2	6.147e-3	2.420e-2	4.217e-1	1.998	-1.243	-1.596	1.026	8.612e-3	5.894e-2	1.627e-1	3.398	-4.200	1.236	2.001e-3
	3	2.523e-3	2.927e-2	-1.431e-1	1.101	2.537	-4.940	1.919	3.761e-3	5.647e-2	-3.970e-1	2.432	-3.560e-1	-2.090	8.712e-1
	4	5.614e-2	1.067	-5.928	-2.837	5.534e1	-7.678e1	3.052e1	2.001e-1	8.594e-1	4.306	-1.486e1	3.200e1	-3.205e1	1.128e1
	5	2.614e-1	1.139	5.302	-2.036e1	3.881e1	-4.199e1	1.688e1	2.449e-1	1.090	4.375	-2.038e1	3.505e1	-3.169e1	1.133e1
0.8	0	3.483e-1	1.360	8.075	-3.247e1	6.207e1	-5.556e1	1.840e1	3.888e-1	1.706	6.641	-2.211e1	3.898e1	-3.501e1	1.185e1
	1	2.561e-2	1.525e-1	3.653	-7.732	1.325e1	-1.234e1	4.124	3.757e-2	2.761e-1	2.940	-3.416	4.141	-4.409	1.618
	2	6.858e-3	-3.012e-3	7.762e-1	6.217e-1	1.882	-4.215	1.713	1.249e-2	6.418e-2	3.256e-1	3.121	-3.290	2.865e-1	2.796e-1
	3	2.615e-3	4.306e-3	1.613e-1	-2.087e-1	5.438	-7.355	2.566	5.888e-3	4.852e-2	-1.651e-1	1.518	1.870	-4.217	1.554
	4	2.674e-1	-2.665	5.035e1	-2.320e2	4.258e2	-3.335e2	9.345e1	1.391e-1	5.911e-1	3.001	-5.233	1.775e1	-2.266e1	8.631
	5	1.917e-1	6.880e-1	6.183	-1.872e1	4.928e1	-6.674e1	2.920e1	3.166e-1	1.449	4.848	-1.676e1	2.304e1	-2.116e1	8.285

Table 6. Influence function coefficients; outside surface crack, $c/a = 1$

		$R_i/t = 5$							$R_i/t = 10$						
a/t	Load	A_0	A_1	A_2	A_3	A_4	A_5	A_6	A_0	A_1	A_2	A_3	A_4	A_5	A_6
0.2	0	7.484e-1	-5.062e-2	-1.753	7.299	-1.261e1	1.010e1	-3.072	7.483e-1	-6.267e-2	-1.713	7.209	-1.245e1	9.957	-3.025
	1	1.246e-1	-9.029e-2	3.861	-1.143e1	1.697e1	-1.270e1	3.715	1.248e-1	-9.376e-2	3.871	-1.147e1	1.708e1	-1.280e1	3.748
	2	4.545e-2	1.024e-1	-3.119e-1	4.074	-8.004	6.571	-2.108	4.556e-2	1.005e-1	-3.016e-1	4.020	-7.884	6.466	-2.077
	3	2.285e-2	1.137e-1	-9.418e-1	4.861	-7.621	5.762	-1.883	2.292e-2	1.122e-1	-9.296e-1	4.803	-7.507	5.673	-1.859
	4	7.440e-1	-6.740e-2	-1.741	7.234	-1.248e1	9.993	-3.039	7.452e-1	-7.159e-2	-1.704	7.166	-1.237e1	9.888	-3.004
	5	1.591e-2	-1.028e-2	7.864e-3	-4.597e-2	7.139e-2	-6.236e-2	2.369e-2	8.723e-3	-5.659e-3	5.923e-3	-3.228e-2	5.391e-2	-4.910e-2	1.865e-2
0.4	0	7.810e-1	-2.233e-2	-2.135	8.581	-1.478e1	1.187e1	-3.620	7.825e-1	-4.977e-2	-2.055	8.411	-1.448e1	1.159e1	-3.531
	1	1.326e-1	4.143e-2	2.897	-8.321	1.199e1	-8.851	2.577	1.334e-1	3.354e-2	2.918	-8.409	1.220e1	-9.057	2.642
	2	5.003e-2	1.088e-1	-2.873e-1	3.969	-7.742	6.221	-1.945	5.052e-2	1.045e-1	-2.657e-1	3.859	-7.504	6.018	-1.886
	3	2.577e-2	1.081e-1	-8.706e-1	4.599	-6.959	4.965	-1.546	2.606e-2	1.048e-1	-8.466e-1	4.486	-6.744	4.802	-1.505
	4	7.714e-1	-5.566e-2	-2.106	8.439	-1.449e1	1.163e1	-3.547	7.771e-1	-6.771e-2	-2.039	8.331	-1.431e1	1.146e1	-3.491
	5	3.174e-2	-1.968e-2	-2.335e-4	-3.184e-2	3.078e-2	-2.036e-2	9.936e-3	1.744e-2	-1.059e-2	1.659e-3	-2.294e-2	2.587e-2	-2.052e-2	9.289e-3
0.6	0	8.371e-1	-2.569e-2	-2.640	1.046e1	-1.803e1	1.453e1	-4.443	8.412e-1	-7.473e-2	-2.511	1.019e1	-1.757e1	1.412e1	-4.310
	1	1.471e-1	1.205e-1	2.199	-5.990	8.175	-5.876	1.691	1.491e-1	1.068e-1	2.231	-6.118	8.493	-6.174	1.784
	2	5.697e-2	1.103e-1	-3.004e-1	4.031	-7.809	6.182	-1.892	5.811e-2	1.030e-1	-2.682e-1	3.871	-7.466	5.896	-1.812
	3	2.971e-2	1.027e-1	-8.348e-1	4.479	-6.614	4.499	-1.336	3.043e-2	9.741e-2	-8.005e-1	4.320	-6.316	4.281	-1.284
	4	8.205e-1	-7.460e-2	-2.585	1.020e1	-1.750e1	1.409e1	-4.310	8.318e-1	-1.005e-1	-2.480	1.005e1	-1.728e1	1.388e1	-4.239
	5	4.757e-2	-2.887e-2	-1.869e-2	1.341e-2	-5.339e-2	5.373e-2	-1.327e-2	2.619e-2	-1.526e-2	-7.732e-3	2.628e-3	-2.611e-2	2.572e-2	-5.153e-3
0.8	0	9.052e-1	-4.910e-2	-3.210	1.273e1	-2.213e1	1.793e1	-5.490	9.116e-1	-1.258e-1	-3.020	1.235e1	-2.147e1	1.736e1	-5.315
	1	1.629e-1	1.738e-1	1.678	-4.187	5.177	-3.498	9.697e-1	1.655e-1	1.535e-1	1.726	-4.370	5.607	-3.882	1.082
	2	6.320e-2	1.139e-1	-3.241e-1	4.147	-7.994	6.287	-1.906	6.449e-2	1.041e-1	-2.800e-1	3.934	-7.549	5.932	-1.814
	3	3.261e-2	1.021e-1	-8.158e-1	4.427	-6.431	4.235	-1.215	3.336e-2	9.539e-2	-7.715e-1	4.225	-6.063	3.981	-1.163
	4	8.799e-1	-1.129e-1	-3.119	1.231e1	-2.129e1	1.722e1	-5.274	8.978e-1	-1.591e-1	-2.972	1.213e1	-2.102e1	1.699e1	-5.202
	5	6.351e-2	-3.850e-2	-4.787e-2	9.870e-2	-2.028e-1	1.810e-1	-5.340e-2	3.506e-2	-2.011e-2	-2.230e-2	4.861e-2	-1.123e-1	9.965e-2	-2.827e-2

Table 7. Influence function coefficients; outside surface crack, $c/a = 2$

		$R_i/t = 5$							$R_i/t = 10$						
a/t	Load	A_0	A_1	A_2	A_3	A_4	A_5	A_6	A_0	A_1	A_2	A_3	A_4	A_5	A_6
0.2	0	7.237e-1	-2.197e-1	7.072e-1	2.949	-9.205	8.986	-3.036	7.231e-1	-2.369e-1	7.665e-1	2.821	-9.024	8.840	-2.988
	1	1.096e-1	2.222e-1	1.018	-1.361e-2	-3.093	3.915	-1.598	1.096e-1	2.176e-1	1.029	-4.150e-2	-3.028	3.847	-1.573
	2	4.010e-2	1.422e-1	-1.052	8.157	-1.670e1	1.511e1	-5.265	4.012e-2	1.399e-1	-1.043	8.118	-1.661e1	1.502e1	-5.237
	3	1.980e-2	1.466e-1	-1.702	9.178	-1.745e1	1.590e1	-5.735	1.982e-2	1.449e-1	-1.692	9.133	-1.736e1	1.581e1	-5.708
	4	7.186e-1	-2.317e-1	7.009e-1	2.885	-9.043	8.848	-2.993	7.199e-1	-2.432e-1	7.629e-1	2.782	-8.925	8.757	-2.962
	5	3.588e-2	-2.120e-2	7.274e-2	-1.450e-1	2.639e-2	4.256e-2	-1.041e-2	1.966e-2	-1.168e-2	4.073e-2	-7.853e-2	8.565e-3	2.972e-2	-7.980e-3
0.4	0	7.943e-1	-1.539e-1	-8.061e-2	6.090	-1.490e1	1.380e1	-4.571	7.972e-1	-1.971e-1	5.450e-2	5.816	-1.453e1	1.351e1	-4.479
	1	1.318e-1	2.988e-1	3.968e-1	2.161	-6.693	6.724	-2.433	1.331e-1	2.869e-1	4.216e-1	2.101	-6.564	6.593	-2.385
	2	5.158e-2	1.365e-1	-9.420e-1	7.687	-1.564e1	1.399e1	-4.837	5.228e-2	1.306e-1	-9.226e-1	7.608	-1.546e1	1.382e1	-4.781
	3	2.682e-2	1.303e-1	-1.506	8.320	-1.555e1	1.397e1	-5.023	2.723e-2	1.263e-1	-1.487	8.234	-1.537e1	1.381e1	-4.973
	4	7.808e-1	-1.774e-1	-8.353e-2	5.920	-1.448e1	1.343e1	-4.454	7.905e-1	-2.100e-1	5.313e-2	5.718	-1.430e1	1.330e1	-4.417
	5	7.202e-2	-3.487e-2	7.487e-2	-6.332e-2	-3.146e-1	3.822e-1	-1.143e-1	3.960e-2	-1.961e-2	4.639e-2	-4.005e-2	-1.756e-1	2.138e-1	-6.346e-2
0.6	0	9.151e-1	-1.907e-1	-4.779e-1	8.232	-1.920e1	1.756e1	-5.781	9.253e-1	-2.777e-1	-2.459e-1	7.784	-1.863e1	1.716e1	-5.659
	1	1.684e-1	3.013e-1	2.477e-1	2.802	-7.775	7.502	-2.629	1.722e-1	2.767e-1	2.946e-1	2.696	-7.566	7.302	-2.559
	2	6.894e-2	1.189e-1	-8.058e-1	7.138	-1.439e1	1.265e1	-4.315	7.087e-2	1.068e-1	-7.724e-1	7.012	-1.413e1	1.241e1	-4.233
	3	3.684e-2	1.116e-1	-1.319	7.449	-1.351e1	1.182e1	-4.205	3.798e-2	1.039e-1	-1.288	7.320	-1.325e1	1.159e1	-4.135
	4	8.876e-1	-2.223e-1	-4.752e-1	7.926	-1.844e1	1.688e1	-5.562	9.120e-1	-2.945e-1	-2.455e-1	7.613	-1.822e1	1.680e1	-5.546
	5	1.105e-1	-5.196e-2	6.933e-2	4.150e-2	-6.959e-1	7.672e-1	-2.379e-1	6.092e-2	-2.979e-2	5.109e-2	6.457e-3	-3.794e-1	4.209e-1	-1.286e-1
0.8	0	1.059	-7.114e-2	-2.542	1.671e1	-3.486e1	3.075e1	-9.918	1.081	-2.281e-1	-2.202	1.609e1	-3.415e1	3.033e1	-9.815
	1	2.081e-1	3.682e-1	-4.645e-1	5.453	-1.226e1	1.102e1	-3.669	2.147e-1	3.245e-1	-3.937e-1	5.303	-1.199e1	1.080e1	-3.600
	2	8.655e-2	1.148e-1	-7.294e-1	6.656	-1.298e1	1.097e1	-3.626	8.947e-2	9.431e-2	-6.825e-1	6.491	-1.264e1	1.068e1	-3.539
	3	4.630e-2	9.647e-2	-1.084	6.164	-1.031e1	8.377	-2.883	4.784e-2	8.407e-2	-1.044	6.002	-1.000e1	8.125	-2.812
	4	1.012	-1.160e-1	-2.451	1.594e1	-3.316e1	2.924e1	-9.429	1.058	-2.506e-1	-2.163	1.570e1	-3.329e1	2.957e1	-9.573
	5	1.519e-1	-4.991e-2	-1.709e-1	1.014	-2.609	2.449	-7.825e-1	8.408e-2	-3.017e-2	-6.774e-2	5.144e-1	-1.397	1.313	-4.149e-1

Table 8. Influence function coefficients; outside surface crack, $c/a = 4$

		$R_i/t = 5$							$R_i/t = 10$						
a/t	Load	A_0	A_1	A_2	A_3	A_4	A_5	A_6	A_0	A_1	A_2	A_3	A_4	A_5	A_6
0.2	0	6.097e-1	-5.834e-1	8.371	-2.290e1	3.142e1	-2.199e1	6.142	6.099e-1	-5.993e-1	8.440	-2.311e1	3.175e1	-2.224e1	6.216
	1	7.425e-2	1.504e-1	1.825	-1.291	-1.763	2.857	-1.215	7.449e-2	1.467e-1	1.836	-1.330	-1.698	2.806	-1.198
	2	2.410e-2	7.908e-2	-4.932e-1	6.234	-1.233e1	1.050e1	-3.538	2.425e-2	7.749e-2	-4.871e-1	6.208	-1.228e1	1.046e1	-3.523
	3	1.082e-2	9.813e-2	-1.167	6.585	-1.122e1	9.451	-3.366	1.093e-2	9.710e-2	-1.161	6.558	-1.117e1	9.406	-3.352
	4	6.027e-1	-5.852e-1	8.253	-2.262e1	3.111e1	-2.180e1	6.094	6.065e-1	-6.007e-1	8.381	-2.299e1	3.162e1	-2.216e1	6.196
	5	6.842e-2	-7.573e-2	9.761e-1	-3.192	4.578	-3.373	1.020	3.751e-2	-4.203e-2	5.399e-1	-1.760	2.521	-1.855	5.606e-1
0.4	0	6.933e-1	-5.719e-1	8.706	-2.316e1	3.123e1	-2.167e1	6.024	7.023e-1	-6.213e-1	8.952	-2.391e1	3.232e1	-2.244e1	6.234
	1	1.014e-1	1.673e-1	1.889	-1.285	-1.830	2.843	-1.184	1.047e-1	1.545e-1	1.945	-1.459	-1.578	2.670	-1.136
	2	3.774e-2	6.014e-2	-1.994e-1	5.262	-1.054e1	8.866	-2.973	3.948e-2	5.449e-2	-1.727e-1	5.167	-1.039e1	8.753	-2.940
	3	1.902e-2	7.769e-2	-8.997e-1	5.593	-9.238	7.568	-2.706	2.008e-2	7.435e-2	-8.811e-1	5.521	-9.118	7.476	-2.679
	4	6.687e-1	-5.671e-1	8.356	-2.222e1	3.009e1	-2.096e1	5.842	6.925e-1	-6.218e-1	8.804	-2.356e1	3.192e1	-2.219e-1	6.169
	5	1.406e-1	-1.364e-1	1.837	-5.998	8.529	-6.274	1.908	7.751e-2	-7.759e-2	1.033	-3.358	4.765	-3.500	1.063
0.6	0	8.178e-1	-6.098e-1	9.105	-2.138e1	2.505e1	-1.526e1	3.773	8.558e-1	-7.790e-1	1.047e1	-2.729e1	3.649e1	-2.540e1	7.145
	1	1.389e-1	1.859e-1	1.603	1.493	-8.540	9.337	-3.427	1.518e-1	1.074e-1	2.478	-2.840	5.251e-1	9.085e-1	-5.447e-1
	2	5.517e-2	7.283e-2	-4.517e-1	7.339	-1.538e1	1.348e1	-4.547	6.171e-2	1.948e-2	2.264e-1	3.891	-8.077	6.641	-2.203
	3	2.892e-2	8.643e-2	-1.083	6.987	-1.234e1	1.042e1	-3.650	3.287e-2	4.694e-2	-5.586e-1	4.322	-6.702	5.154	-1.849
	4	7.620e-1	-5.878e-1	8.407	-1.949e1	2.283e1	-1.397e1	3.469	8.341e-1	-7.723e-1	1.017e1	-2.653e1	3.556e1	-2.482e1	6.990
	5	2.246e-1	-2.180e-1	2.891	-9.559	1.390e1	-1.048e1	3.247	1.249e-1	-1.243e-1	1.600	-5.145	7.241	-5.308	1.617
0.8	0	9.719e-1	-9.699e-1	1.437e1	-3.973e1	5.625e1	-4.140e1	1.228e1	1.060	-8.946e-1	1.141e1	-2.700e1	3.220e1	-2.083e1	5.771
	1	1.852e-1	-7.681e-2	5.287	-1.303e1	1.777e1	-1.293e1	3.689	2.106e-1	7.950e-2	2.783	-2.900	-4.163e-1	2.087	-9.634e-1
	2	7.670e-2	-1.130e-1	2.072	-2.848	3.310	-2.371	4.992e-1	8.808e-2	-4.940e-3	4.970e-1	3.404	-7.757	6.673	-2.284
	3	4.099e-2	-3.526e-2	5.561e-1	2.995e-1	2.004e-2	-7.449e-2	-3.166e-1	4.734e-2	2.980e-2	-3.707e-1	3.935	-6.364	5.117	-1.910
	4	8.683e-1	-8.984e-1	1.287e1	-3.523e1	5.018e1	-3.733e1	1.117e1	1.019	-8.769e-1	1.092e1	-2.577e1	3.080e1	-2.002e1	5.571
	5	3.239e-1	-2.860e-1	3.671	-1.141e1	1.511e1	-1.049e1	3.099	1.822e-1	-1.670e-1	2.055	-6.317	8.277	-5.703	1.681

Table 9. Influence function coefficients; outside surface crack, $c/a = 8$

		$R_i/t = 5$							$R_i/t = 10$						
a/t	Load	A_0	A_1	A_2	A_3	A_4	A_5	A_6	A_0	A_1	A_2	A_3	A_4	A_5	A_6
0.2	0	4.587e-1	-2.811e-1	1.330e1	-4.657e1	7.591e1	-5.991e1	1.827e1	4.605e-1	-2.918e-1	1.338e1	-4.690e1	7.647e1	-6.033e1	1.839e1
	1	3.955e-2	1.197e-1	2.622	-3.561	1.742	1.075e-1	-3.834e-1	4.032e-2	1.170e-1	2.649	-3.674	1.917	-1.448e-2	-3.508e-1
	2	1.033e-2	7.617e-2	-4.892e-1	6.452	-1.256e1	1.058e1	-3.556	1.076e-2	7.501e-2	-4.743e-1	6.393	-1.247e1	1.051e1	-3.540
	3	3.744e-3	1.023e-1	-1.230	6.772	-1.130e1	9.453	-3.396	4.010e-3	1.016e-1	-1.220	6.734	-1.124e1	9.412	-3.384
	4	4.429e-1	-2.758e-1	1.280e1	-4.472e1	7.303e1	-5.775e1	1.764e1	4.549e-1	-2.909e-1	1.319e1	-4.625e1	7.547e1	-5.957e1	1.817e1
	5	1.107e-1	-7.871e-2	3.237	-1.251e1	2.047e1	-1.633e1	5.119	6.105e-2	-4.422e-2	1.793	-6.933	1.134e1	-9.043	2.832
0.4	0	4.930e-1	3.307e-2	1.209e1	-4.213e1	7.005e1	-5.669e1	1.763e1	5.125e-1	3.180e-2	1.157e1	-3.888e1	6.288e1	-5.001e1	1.539e1
	1	5.014e-2	1.967e-1	2.563	-3.710	3.388	-2.369	6.791e-1	5.659e-2	2.419e-1	1.786	-7.201e-2	-3.724	3.840	-1.329
	2	1.564e-2	6.901e-2	-7.714e-2	4.524	-7.861	5.578	-1.677	1.869e-2	1.094e-1	-6.205e-1	6.877	-1.229e1	9.352	-2.879
	3	6.950e-3	7.495e-2	-7.160e-1	4.272	-5.364	3.280	-1.098	8.769e-3	1.010e-1	-1.032	5.601	-7.830	5.367	-1.759
	4	4.321e-1	2.186e-2	1.057e1	-3.616e1	6.067e1	-4.969e1	1.557e1	4.925e-1	2.682e-2	1.106e1	-3.702e1	6.004e1	-4.792e1	1.478e1
	5	2.248e-1	-1.626e-2	5.535	-2.168e1	3.562e1	-2.892e1	9.252	1.273e-1	-2.360e-2	3.149	-1.217e1	1.977e1	-1.589e1	5.049
0.6	0	5.293e-1	-2.133e-2	1.353e1	-4.424e1	7.393e1	-6.140e1	1.958e1	5.888e-1	-2.648e-2	1.511e1	-5.182e1	8.829e1	-7.390e1	2.370e1
	1	5.770e-2	1.709e-1	3.041	-4.525	5.247	-4.601	1.561	7.776e-2	1.798e-1	3.595	-7.374	1.080e1	-9.513	3.196
	2	1.793e-2	4.754e-2	2.557e-1	3.606	-5.737	3.304	-8.394e-1	2.807e-2	5.508e-2	4.681e-1	2.539	-3.747	1.598	-2.820e-1
	3	7.691e-3	5.951e-2	-4.795e-1	3.465	-3.432	1.251	-3.632e-1	1.383e-2	6.430e-2	-3.853e-1	3.038	-2.729	7.067e-1	-1.971e-1
	4	3.904e-1	-2.860e-2	1.018e1	-3.141e1	5.428e1	-4.696e1	1.533e1	5.419e-1	-3.200e-2	1.389e1	-4.718e1	8.095e1	-6.830e1	2.201e1
	5	3.530e-1	-4.674e-2	8.575	-3.165e1	5.079e1	-4.154e1	1.354e1	2.087e-1	-3.246e-2	5.285	-2.036e1	3.357e1	-2.765e1	8.993
0.8	0	5.424e-1	1.091e-1	1.344e1	-3.821e1	6.270e1	-5.421e1	1.810e1	6.705e-1	4.157e-2	1.559e1	-4.445e1	6.944e1	-5.788e1	1.908e1
	1	5.673e-2	1.893e-1	3.213	-3.709	4.442	-5.015	1.987	9.887e-2	1.790e-1	3.825	-5.333	5.673	-5.234	1.957
	2	1.589e-2	4.446e-2	4.577e-1	3.362	-4.581	1.596	-1.192e-1	3.661e-2	4.341e-2	7.223e-1	2.746	-4.415	1.937	-2.971e-1
	3	5.941e-3	4.919e-2	-2.589e-1	2.758	-1.415	-1.069	5.208e-1	1.819e-2	5.054e-2	-1.235e-1	2.502	-1.580	-5.954e-1	3.158e-1
	4	2.939e-1	6.915e-2	7.851	-1.821e1	3.433e1	-3.472e1	1.261e1	5.823e-1	3.211e-2	1.352e1	-3.722e1	5.903e1	-5.056e1	1.698e1
	5	4.821e-1	6.039e-2	1.025e1	-3.284e1	4.625e1	-3.583e1	1.164e1	3.093e-1	-1.681e-2	6.899	-2.366e1	3.469e1	-2.665e1	8.449

Table 10. Influence function coefficients; outside surface crack, $c/a = 16$

		$R_i/t = 5$							$R_i/t = 10$						
a/t	Load	A_0	A_1	A_2	A_3	A_4	A_5	A_6	A_0	A_1	A_2	A_3	A_4	A_5	A_6
0.2	0	3.058e-1	1.140	7.826	-3.564e1	6.517e1	-5.518e1	1.763e1	3.089e-1	1.151	7.837	-3.578e1	6.538e1	-5.529e1	1.764e1
	1	1.557e-2	1.595e-1	2.675	-3.900	2.878	-1.276	1.647e-1	1.671e-2	1.653e-1	2.672	-3.908	2.840	-1.194	1.245e-1
	2	2.588e-3	7.174e-2	-4.700e-1	6.213	-1.139e1	9.003	-2.903	3.187e-3	7.537e-2	-4.770e-1	6.240	-1.148e1	9.111	-2.946
	3	1.110e-4	9.368e-2	-1.127	5.959	-8.899	6.765	-2.368	4.804e-4	9.619e-2	-1.135	5.991	-8.984	6.856	-2.403
	4	2.650e-1	9.824e-1	6.818	-3.013e1	5.568e1	-4.776e1	1.537e1	2.961e-1	1.101	7.498	-3.401e1	6.235e1	-5.292e1	1.692e1
	5	1.511e-1	5.514e-1	3.814	-1.963e1	3.533e1	-3.006e1	9.859	8.544e-2	3.108e-1	2.167	-1.123e1	2.023e1	-1.717e1	5.616
0.4	0	3.045e-1	1.343	6.550	-2.959e1	5.685e1	-5.057e1	1.672e1	3.186e-1	1.494	5.751	-2.661e1	5.136e1	-4.578e1	1.512e1
	1	1.409e-2	1.512e-1	2.935	-4.940	6.488	-5.648	1.853	1.914e-2	2.033e-1	2.680	-3.916	4.465	-3.807	1.226
	2	1.479e-3	3.294e-2	4.140e-3	4.057	-5.979	3.321	-8.297e-1	4.095e-3	5.533e-2	-8.311e-2	4.456	-6.850	4.156	-1.123
	3	-6.112e-4	5.029e-2	-6.049e-1	3.407	-2.742	4.604e-1	-8.702e-2	9.577e-4	6.441e-2	-6.638e-1	3.692	-3.373	1.065	-2.985e-1
	4	1.508e-1	6.281e-1	3.959	-1.420e1	3.122e1	-3.085e1	1.062e1	2.686e-1	1.250	4.966	-2.180e1	4.328e1	-3.955e1	1.320e1
	5	2.707e-1	1.201	5.198	-2.652e1	4.855e1	-4.413e1	1.546e1	1.716e-1	7.941e-1	2.944	-1.630e1	2.990e1	-2.672e1	9.228
0.6	0	2.989e-1	1.337	6.507	-2.649e1	5.406e1	-5.147e1	1.782e1	3.219e-1	1.442	6.579	-2.599e1	5.321e1	-5.109e1	1.777e1
	1	8.378e-3	1.234e-1	3.148	-5.761	1.046e1	-1.084e1	3.897	1.578e-2	1.712e-1	3.045	-4.881	8.597	-9.257	3.398
	2	-1.968e-3	-1.476e-2	4.500e-1	1.893	-1.396e-1	-2.873	1.400	1.547e-3	1.453e-2	3.302e-1	2.712	-1.885	-1.345	9.085e-1
	3	-2.767e-3	5.831e-4	-9.638e-2	8.534e-1	3.518	-5.849	2.131	-7.511e-4	2.163e-2	-2.145e-1	1.588	1.964	-4.482	1.691
	4	2.434e-2	2.599e-2	-2.904	1.781e1	-1.792e1	5.388	-5.002e-1	2.078e-1	8.983e-1	4.935	-1.716e1	4.000e1	-4.151e1	1.479e1
	5	3.197e-1	1.529	4.980	-1.943e1	3.742e1	-4.202e1	1.723e1	2.580e-1	1.173	4.314	-1.999e1	3.620e1	-3.511e1	1.318e1
0.8	0	2.923e-1	1.340	6.445	-2.180e1	4.742e1	-4.805e1	1.709e1	3.050e-1	1.466	5.820	-1.645e1	3.869e1	-4.313e1	1.630e1
	1	2.910e-3	9.020e-2	3.341	-6.356	1.405e1	-1.524e1	5.418	5.168e-3	1.510e-1	2.863	-3.085	8.111	-1.105e1	4.390
	2	-5.214e-3	-5.373e-2	7.803e-1	1.096e-1	5.032	-8.069	3.081	-4.943e-3	-1.427e-2	3.980e-1	2.509	5.289e-1	-4.696	2.183
	3	-4.796e-3	-3.612e-2	2.663e-1	-1.180	8.707	-1.078e1	3.693	-5.038e-3	-6.263e-3	-5.595e-2	7.369e-1	5.062	-7.974	2.921
	4	2.554e-1	-3.075	5.999e1	-3.085e2	6.374e2	-5.627e2	1.791e2	1.027e-1	4.500e-1	3.760	-8.843	3.518e1	-4.519e1	1.738e1
	5	2.629e-1	1.288	5.138	-1.213e1	4.018e1	-6.654e1	3.188e1	3.232e-1	1.652	3.324	-7.142	8.310	-1.480e1	8.356

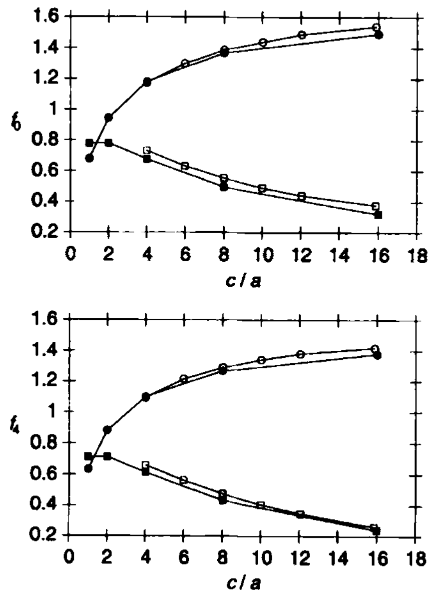

Fig. 4. Comparisons of influence function solutions as a function of c/a. Filled marks show the presented solutions for $a/t = 0.4$ and $R_i/t = 10$, and opened marks show the solutions by Poette and Albaladejo for $a/t = 0.417$ and $R_i/t = 9.5$. Circles symbolise the deepest point of the crack front and boxes symbolise the intersections of the crack front with the free surface.

position at the intersections of the crack front with the free surface and unity in the position at the deepest point of the crack front. This implies that A_0 gives the value of the influence function at the intersections of the crack front with the free surface and the sum of A_j gives the value of influence function at the deepest point of the crack front.

Figures 4 and 5 show a part of a comparison for the inside surface cracks between the presented solutions and the ones by Poette and Albaldejo [4]. The presentation is made for load cases 0 and 4, and for geometry configurations that fall close since the two solutions cannot be compared directly with respect to the geometry. Poette and Albaladejo also used the finite element method and solid elements to derive their solutions. The only significant difference between the two analyses is that a larger number of elements was used in the present investigation, especially in order to resolve the behaviour of K_I at the intersections of the crack front with the free surface where it decreases. This explains why the presented solutions fall somewhat below the ones by Poette and Albaladejo at that position. However, the solutions agree well in all cases studied.

The solutions for the outside surface cracks have been spot checked against the solutions by Delale and Erdogan [5] and showed a fairly good agreement. Delale and Erdogan used the finite element method with shell and line-spring elements to derive their solutions. This technique is not as accurate as the one used here, especially where the crack is shallow as is the case in the vicinity of the intersections of the crack front with the free surface. The solutions by Delale and Erdogan were restricted to the deepest point of crack and to load cases 0 and 1.

As can be expected, the influence functions for the outside surface cracks are of the same order as for the inside surface cracks which indicates that no basic errors are present between the solutions. This also implies that in an engineering sense it is possible to use the stress intensity factor solutions for an inside surface crack to assess an outside surface crack.

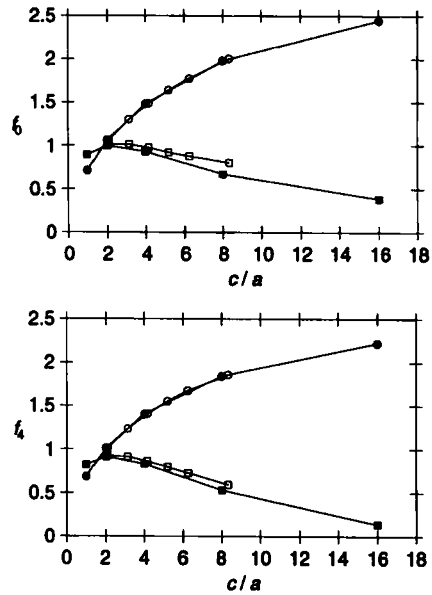

Fig. 5. Comparisons of influence function solutions as a function of c/a. Filled marks show the presented solutions for $a/t = 0.8$ and $R_i/t = 10$, and opened marks show the solutions by Poette and Albaladejo for $a/t = 0.8$ and $R_i/t = 9.5$. See text to Fig. 4.

NUMERICAL EXAMPLE

According to EPRI [6] the following axial residual stress distribution is recommended for austenitic pipe welds when the pipe thickness exceeds one inch.

$$\sigma = 207\left[1 - 6.91\frac{u}{t} + 8.69\left(\frac{u}{t}\right)^2 - 0.48\left(\frac{u}{t}\right)^3 - 2.03\left(\frac{u}{t}\right)^4\right]\text{MPa}, \tag{8}$$

where u is a local radial coordinate as defined before for the inside surface cracks. The stress distribution is assumed to be axisymmetrical.

The target here is to determine K_I for a circumferential surface crack situated in the weld region at the inside of the pipe if no external loads are present. The characteristic dimension ratios are $a/t = 0.6$, $c/a = 2$ and $R_i/t = 5$, and the pipe is 30 mm thick. Only K_I at the deepest point of the crack front and at the intersections of the crack front with the free surface are required.

If Eq. (8) would have a cubic variation through the thickness an easy way to determine the stress components would be to arrange the expression in accordance with Eq. (5). Since this is not the case, a least square fit of Eq. (8) to a third degree polynomial with respect to u/a is instead performed for close spaced values situated within the prospective crack region, i.e. $0 \leqslant u/t \leqslant 0.6$, as illustrated in Fig. 6. The fit results in the polynomial

$$\sigma = 207\left[1 - 4.22\frac{u}{a} + 3.46\left(\frac{u}{a}\right)^2 - 0.63\left(\frac{u}{a}\right)^3\right]\text{MPa}, \tag{9}$$

and hence $\sigma_0 = 207$, $\sigma_1 = -207 \times 4.22$, $\sigma_2 = 207 \times 3.46$ and $\sigma_3 = -207 \times 0.63$ MPa. For each stress component the influence function is then evaluated at the deepest point of the crack front ($2\phi/\pi = 1$) and at the intersections of the crack front with the free surface ($2\phi/\pi = 0$). The influence functions

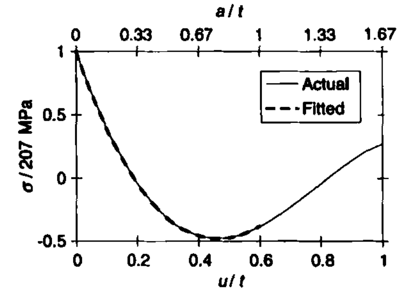

Fig. 6. Actual and fitted weld residual stress distribution.

are given in Table 2 and according to Eq. (7), A_0 gives the value of the influence function at the intersections of the crack front with the free surface and the sum of A_j the value of influence function at the deepest point of the crack front. Hence,

$$K_{\mathrm{I}}(2\phi/\pi = 1) = \sqrt{\pi 0.018} \times 207(1 \times 0.99 - 4.22 \times 0.60 + 3.46 \times 0.46 - 0.63 \times 0.03)\ \mathrm{MPa}\sqrt{\mathrm{m}}$$
$$= -9.9\ \mathrm{MPa}\sqrt{\mathrm{m}}, \tag{10}$$

$$K_{\mathrm{I}}(2\phi/\pi = 0) = \sqrt{\pi 0.018} \times 207(1 \times 0.85 - 4.22 \times 0.15 + 3.46 \times 0.06 - 0.63 \times 0.38)\ \mathrm{MPa}\sqrt{\mathrm{m}}$$
$$= 19.9\ \mathrm{MPa}\sqrt{\mathrm{m}}. \tag{11}$$

Due to the distribution of the weld residual stresses, a higher stress intensity factor value is obtained at the intersections of the crack front with the free surface. K_{I} becomes negative at the deepest point of the crack indicating that the crack is partly closed if no external loads are present.

CONCLUSIONS

(1) Stress intensity factor solutions for circumferential surface cracks in pipes have been derived which consider both cracks located at the in- and outside of the pipes. The solutions for the in- and outside surface crack are of the same order and show in an engineering sense no significant difference.
(2) The solutions cover a wide range of geometry and load configurations. The geometry configurations analysed were $a/t = 0.2, 0.4, \ldots, 0.8$, $c/a = 1, 2, 4, \ldots, 16$ and $R_i/t = 5, 10$. Six different load cases with respect to the elastic stress state in an uncracked pipe were considered. The first four cases describe an axisymmetrical stress distribution through the thickness and the two last cases describe global bending stresses, which make it possible to analyse complex weld residual distributions as well as bending arbitrarily oriented with respect to the location of the crack.
(3) The solutions present the stress intensity factor along the whole crack front.
(4) The solutions are accurate due to the used finite element technique. Comparisons to other solutions where so were possible showed good agreement.

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