Aadi Shah, Joel Mathew, Kabir Sahni

Coin Cascade

Description

This is a 6-round game using a single coin. The objective is to get the highest score possible. Your score starts at zero. If you land on heads, your score changes to c * n + n, where c is the current score and n is the turn number. If you land on tails, your score changes to c * n - n. You have six turns.

Probability Distribution

Let X = Score; top row = X, and bottom row = P(X)

1956	1944	1896	1884	1716	1704	1656	1644	1236	1224	1176	1164
0.015625	0.015625	0.015625	0.015625	0.015625	0.015625	0.015625	0.015625	0.015625	0.015625	0.015625	0.015625

996	984	936	924	516	504	456	444	276	264	216	204
0.015625	0.015625	0.015625	0.015625	0.03125	0.03125	0.03125	0.03125	0.03125	0.03125	0.03125	0.03125

-204	-216	-264	-276	-444	-456	-504	-516	-924	-936	-984	-996
0.03125	0.03125	0.03125	0.03125	0.03125	0.03125	0.03125	0.03125	0.015625	0.015625	0.015625	0.015625

-1164	-1176	-1224	-1236	-1644	-1656	-1704	-1716	-1884	-1896	-1944	-1956
0.015625	0.015625	0.015625	0.015625	0.015625	0.015625	0.015625	0.015625	0.015625	0.015625	0.015625	0.015625

Expected Value and Standard Deviation

The expected value of this distribution is calculated as follows:

$E(X) = 1956(0.015625) + 1944(0.015625) + 1896(0.015625) + \cdots + \; -1944(0.015625) = 0$

This means that if you repeatedly play the game long enough, the sum of the player's scores at the end of each round is expected to be 0.

The standard deviation of this distribution is calculated as follows:

$$\sigma_x = \sqrt{(1956 - 0)^2 \cdot 0.015625 \; + \; (1944 - 0)^2 \cdot 0.015625 \; + \cdots + \; (-1944 - 0)^2 \cdot 0.015625} \simeq 1096$$

The typical deviation of winnings from the expected score of 0 is 1096 points.

Simulation

To run this simulation, we created a computer program that flips a coin 6 times for each of the 64 trials. It records the score from each round and adds them up to calculate the total score. Note: top row = Score, bottom row = Count

1956	1944	1896	1884	1716	1704	1656	1644	1236	1224	1176	1164	996	984	936	924
2	1	1	0	2	1	0	0	0	2	0	0	2	0	1	1

516	504	456	444	276	264	216	204	-204	-216	-264	-276	-444	-456	-504	-516
1	3	2	3	4	3	1	4	3	2	2	4	1	2	3	2

-924	-936	-984	-996	-1164	-1176	-1224	-1236	-1644	-1656	-1704	-1716	-1884	-1896	-1944	-1956
1	0	0	0	2	1	1	0	1	1	2	0	0	2	0	0

Expected Result?

In this simulation, after 64 times playing this game, and summing their scores, the player gains 3,660 points, or approximately 57.19 points per game. This is not unexpected, because the expected value of 0 points is after infinitely many trials, and an approximate 57.19 points won per game is well within the standard deviation value of 1096.