Asymptotic Symmetries from Celestial CFT (Celestial Holography)

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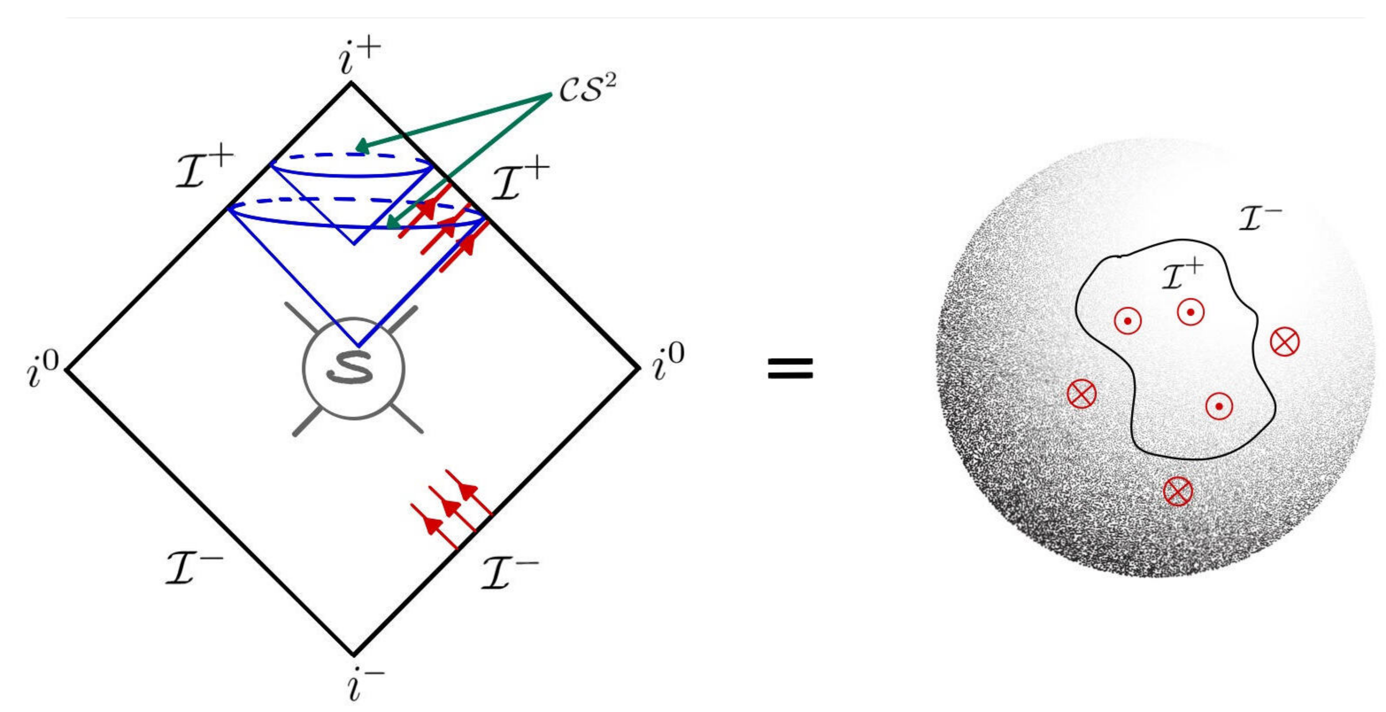
Abstract

Asymptotic symmetry plays an important role in determining physical observables of a theory. Recently it has been proposed that OPEs of appropriate celestial amplitudes can be used to find their asymptotic symmetries. This technique is very well-known these days in the context of celestial holography as Celestial CFT. In this work, we extend the construction to the maximally supersymmetric four dimensional $\mathcal{N}=8$ supergravity theory and construct the extended asymptotic symmetry algebra ($\mathcal{N}=8$ $\mathfrak{sbms}_4$). We find the appropriate currents for extensions of $\mathcal{N}=8$ super-Poincaré and $\mathrm{SU}(8)_R$ R-symmetry current algebra on the celestial sphere $\mathcal{CS}^2$. We generalise the definition of shadow transformations and show that there is *no* infinite dimensional extension of the global $\mathrm{SU}(8)_R$ algebra in the theory.

Zooming into the Sky ($\mathcal{I}^{\pm}$)



- Symmetries of asympototically flat spacetime:
  - → BMS Symmetries:
    - ∗ $4D$ Lorentz transformation (LT) acts as global $2D$ conformal transformation on $\mathcal{CS}^2$.
    - ∗ Angle dependent translations (supertranslation) of retarded time: $u \mapsto u + \alpha(z,\bar{z})$, where $\alpha(z,\bar{z})$ is any real function on $\mathcal{CS}^2$.
  - → Extended BMS Symmetry:
    - ∗ Enhancing global conformal transformation to local conformal transformations (superrotations), $z \mapsto f(z) + \mathcal{O}(1/r)$, where $f(z)$ is any function and this preserve the asymptotic behaviour of the metric.
  - → Extended super-BMS Symmetry:
    - ∗ Infinite dimensional extension of supersymmetry (Supersymmetric version of Extended BMS symmetry).

Exploring CCFT on the Celestial Sphere ($\mathcal{CS}^2$)

Celestial Holography: 4d physics $\mapsto$ 2d *celestial* CFT

- Observables of CCFT is the Mellin transformed flat space scattering amplitude.

$$\langle out|S|in\rangle \xrightarrow[\text{on } \mathcal{CS}^2]{\text{LT as Global Conformal Transformation}} \langle \mathcal{O}_1 \cdots \mathcal{O}_n\rangle$$

- u-direction on $\mathcal{I}^{\pm} \mapsto$ spectrum of weights$\big(\{p_k, h_k\} \mapsto \{(z_k,\bar{z}_k), \Delta_k, J_k\}\big)$
- Under Mellin transformation, $\mathcal{A}_{J_1\ldots J_n}\big(\Delta_i, z_i, \bar{z}_i\big) = \big(\prod_{i=1}^n g\big(\Delta_i\big)\int d\omega_i\ \omega_i^{\Delta_i-1}\big)\delta^{(4)}\big(\sum_i \epsilon_i\omega_i q_i\big)\mathcal{M}_{\ell_1\ldots\ell_n}\big(\omega_i, z_i, \bar{z}_i\big)$

Steps to find symmetry algebra via CCFT:

- Construct the conserved currents via shadow transformation of the conformal primaries in the theory.
- Compute the OPEs (Operator Product Expansions) of various currents using collinear and/or soft limits.
- Construct the algebra of quantized modes of the currents from OPEs using standard methods of $2d$ CFT.

Asymptotic symmetry algebra computation reduces to the computation of appropriate OPEs which depends on *soft* and *collinear* limits in the bulk.

Soft and Collinear limits in $\mathcal{N}=8$ Supergravity

$\mathcal{N}=8$ supergravity amplitude can be related to the amplitudes in $\mathcal{N}=4$ super Yang-Mills,

$$\mathcal{N}=8\ \ \text{Supergravity} \sim (\mathcal{N}=4\ \ \text{Super Yang-Mills}) \otimes (\mathcal{N}=4\ \ \text{Super Yang-Mills}).$$

The double copy relation of collinear limits in component formalism is given by

$$M_n(1^{h_1}, 2^{h_2}, \ldots, n) \xrightarrow{1||2} \sum_h \text{Split}^{\text{SG}}_{-h}(z, 1^{h_1}, 2^{h_2}) M_{n-1}(p^h, \ldots, n)$$

where,

$$\text{Split}^{\text{SG}}_{-(h+\tilde{h})}\left(z, 1^{h_1+\tilde{h}_1}, 2^{h_2+\tilde{h}_2}\right) = -s_{12} \times \text{Split}^{\text{SYM}}_{-h}\left(z, 1^{h_1}, 2^{h_2}\right) \times \text{Split}^{\text{SYM}}_{-\tilde{h}}\left(z, 2^{\tilde{h}_2}, 1^{\tilde{h}_1}\right)$$

Similarly we have the double copy factorisation relations for soft limits.

Symmetry Generators in $\mathcal{N}=8$ Supergravity

The conformal soft operators in $\mathcal{N}=8$ SUGRA are given by,

- $J_1(z,\bar{z}) = \lim_{\Delta\to 1}(\Delta-1)\mathcal{O}_{\Delta,+2}(z,\bar{z})$, $\quad \bar{J}_1(z,\bar{z}) = \lim_{\Delta\to 1}(\Delta-1)\mathcal{O}_{\Delta,-2}(z,\bar{z})$ (Leading Soft Gravitons)
- $J_0(z,\bar{z}) = \lim_{\Delta\to 0}\Delta\mathcal{O}_{\Delta,+2}(z,\bar{z})$, $\quad \bar{J}_0(z,\bar{z}) = \lim_{\Delta\to 0}\Delta\mathcal{O}_{\Delta,-2}(z,\bar{z})$ (Subleading soft Gravitons)
- $J^A_{1/2}(z,\bar{z}) = \lim_{\Delta\to\frac{1}{2}}\left(\Delta-\frac{1}{2}\right)\mathcal{O}^A_{\Delta,+\frac{3}{2}}(z,\bar{z})$, $\quad \bar{J}_{1/2\ A}(z,\bar{z}) = \lim_{\Delta\to\frac{1}{2}}\left(\Delta-\frac{1}{2}\right)\mathcal{O}_{\Delta,-\frac{3}{2},A}(z,\bar{z})$ (Leading soft Gravitinos)

The $\mathfrak{bms}_4$ part of the $\mathcal{N}=8$ $\mathfrak{sbms}_4$ algebra is known to be generated by the shadow transform of the $\Delta=0$ graviton operator,

$$T_0(z,\bar{z}) = \lim_{\Delta\to 0}\frac{3!\Delta}{2\pi}\int d^2z'\frac{1}{(z-z')^4}\mathcal{O}_{\Delta,-2}\left(z',\bar{z}'\right) \tag{1}$$

With the below modification we can have the proper stress tensor in our theory,

$$T_{\text{mod}} := T_0 + \frac{1}{2}\partial^3\epsilon_{\bar{J}_0} \quad \text{where} \quad \epsilon_{\bar{J}_0} := \int_{\bar{z}_0}^{\bar{z}} d\bar{w}\bar{J}_0(z,\bar{w}) \tag{2}$$

The supertranslation generator $P(z)$ is defined as:

$$P(z) = \lim_{\Delta\to 1}\frac{(\Delta-1)}{4}\partial_{\bar{z}}\mathcal{O}_{\Delta,+2}(z,\bar{z}) \tag{3}$$

The supercurrents are the shadow transform of the $\Delta=\frac{1}{2}$ gravitino operator:

$$S_A(z) = \lim_{\Delta\to\frac{1}{2}}\frac{\Delta-\frac{1}{2}}{\pi}\int d^2z'\frac{1}{(z-z')^3}\mathcal{O}_{A;\Delta,-\frac{3}{2}}(z',\bar{z}') \tag{4}$$

Possible R-symmetry generators in $\mathcal{N}=8$ Supergravity

We can construct a current $\widetilde{\mathcal{G}}^A_B(z,\bar{z})$ whose modes will extend the generators $\left(T^A_B\right)^C$ of R-symmetry. The most general integral transform corresponding to negative and positive helicity soft graviphotons respectively as,

$$\begin{aligned} G_{AB}(z,\bar{z}) &= \lim_{\Delta\to 0}\frac{\Delta}{\pi}\int d^2z'\frac{1}{(z-z')^a}\frac{1}{(\bar{z}-\bar{z}')^b}\mathcal{O}_{AB;\Delta,-1}(z',\bar{z}') \\ \overline{G}^{CD}(z,\bar{z}) &= \lim_{\Delta\to 0}\frac{\Delta}{\pi}\int d^2z'\frac{1}{(\bar{z}-\bar{z}')^{a'}}\frac{1}{(z-z')^{b'}}\mathcal{O}^{CD}_{\Delta,+1}(z',\bar{z}'). \end{aligned} \tag{5}$$

To construct a suitable current for R-symmetry, we need to use double soft limits of the graviphoton operators for which we construct a normalised local operator,

$$\mathcal{G}^{CD}_{AB}(z,\bar{z}) =: \mathcal{G}^{CD}_{AB}(z,\bar{z};z,\bar{z}) :=: G_{AB}(z)\overline{G}^{CD}(\bar{z}) - \overline{G}^{CD}(\bar{z})G^{AB}(z) :\equiv: \left[G_{AB}(z), \overline{G}^{CD}(\bar{z})\right]: . \tag{6}$$

We conclude that the OPEs between the R-symmetry currents and the supercurrents are trivial

$$\left(\widetilde{\mathcal{G}}^C_A\right)^D_B(z,\bar{z})S_D(w) \sim \text{regular}, \quad \left(\widetilde{\mathcal{G}}^C_A\right)^D_B(z,\bar{z})\overline{S}^B(\bar{w}) \sim \text{regular}. \tag{7}$$

This implies the possible non-extension of the global $\mathrm{SU}(8)_R$ R-symmetry algebra.

Extended $\mathcal{N}=8$ Asymptotic Symmetry Algebra

From a direct asymptotic symmetry analysis of the supergravity theory using CCFT prescription, we have confirmed that indeed supergravity does not result in R-symmetry extension. The rest of the symmetry algebra is as expected,

$$\begin{aligned} \mathfrak{bms}_4 :\quad & \left[L_m, L_n\right] = (m-n)L_{m+n}, \quad \left[\bar{L}_m, \bar{L}_n\right] = (m-n)\bar{L}_{m+n} \\ & \left[L_n, P_{kl}\right] = \big(\tfrac{1}{2}n-k\big)P_{n+k,l}, \quad \left[\bar{L}_n, P_{kl}\right] = \big(\tfrac{1}{2}n-l\big)P_{k,n+l} \\ \mathfrak{sbms}_4 :\quad & \left[L_n, (S_A)_m\right] = \big(\tfrac{n}{2}-m\big)(S_A)_{m+n}, \quad \left[\bar{L}_n, (S_A)_m\right] = 0 \\ & \left[L_n, \overline{S}^A_m\right] = 0, \quad \left[\bar{L}_n, \overline{S}^A_m\right] = \big(\tfrac{n}{2}-m\big)\overline{S}^A_{m+n}, \quad \left\{(S_B)_m, \overline{S}^A_n\right\} = \delta^A_B P_{mn}. \end{aligned} \tag{8}$$

RESULTS in QR Code

arXiv:2212.12133
Asymptotic Symmetries in N=8 SUGRA

arXiv:2212.11480
Soft and Collinear limits in N=8 SUGRA

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