Cosmodynamical issues in an accelerated universe

Celia Escamilla Rivera

[1] Department of Theoretical Physics and History of Science
University of Basque Country

[2] Astrophysics Department
University of Oxford

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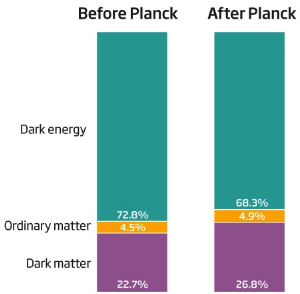
Our Universe's state of play

- The playground of the Universe: General Relativity

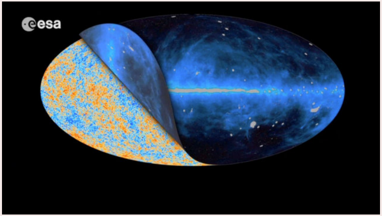
$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}. \tag{1}$$

- Cosmological solution $\rightarrow$ Describes an expanding universe.
- Dark energy $\rightarrow$ Required to explain the accelerating expansion.



Once upon a time in an early universe...

- The heart of Λ remains tied up with the story of the very early history of the universe.
- Measure CMB $\rightarrow$ Cosmological parameters.
- Supernovae and CMB results $\rightarrow$ Overall density of matter in the universe.



- From the Einstein equations in a FRW background, the cosmodynamics is established!

$$1 - \frac{\kappa\rho}{3H^2} - \frac{\Lambda}{3H^2} + \frac{kc^2}{a^2 H^2} = 0, \tag{2}$$

where $\kappa = 8\pi G$.

Outline

- Cosmodynamics with SNeIa and BAO probes
  - Mathematical settings
  - Ingredients
  - Methodology and Analysis
- Cosmodynamics in Eddington's theory of gravity
  - The theory
  - Tensor Modes News
  - Evolution of Tensor Modes
- Cosmodynamics at String level
  - A scenario made of strings
  - Building the model
  - Unifying the tachyon and Chaplygin gas
- BAO cosmography
  - Qualitative analysis
  - Forecasting

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Things we know:

Studying the nature of Dark Energy means → Studying the Equations of State

$$\Omega_X(a) \propto \exp\left[3\int_a^1 \frac{da'}{a'}(1+w(a'))\right], \tag{3}$$

which appears in the Hubble function as

$$H(a) = H_0\left[\Omega_m a^{-3} + \Omega_X(a)\right]^{1/2}, \tag{4}$$

$$a \doteq \frac{1}{(1+z)}. \tag{5}$$

To be useful, $w(z)$ must:

- be sufficiently sophisticated to be able to explain the data,
- and simple enough so as to provide reliable predictions.

Tension[a]

[a] S. Nesseris and L. Perivolaropoulos (2007).

Indicates that the EoS parameters values obtained by using the dataset can differ from those obtained from another dataset.

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Objective

- Study the cosmodynamics of dark energy using two representations of its possible evolution. Also, our main goal was to study the tension effects between SNeIa and BAO datasets[a].

[a] C. Escamilla-Rivera, R. Lazkoz, V. Salzano and I. Sendra. JCAP 1109 (2011) 003.

CPL parametrization[a]

[a] M. Chevallier, D. Polarski (2001) and E.V. Linder (2003).

$$w(a) = w_0 + (1-a)w_a. \tag{6}$$

$$E^2(z) = \Omega_m(1+z)^3 + (1-\Omega_m)(1+z)^{3(1+w_0+w_a)} \times \exp\left[\frac{-3w_a z}{1+z}\right]. \tag{7}$$

Low correlation parametrization (Wang's model)[a]

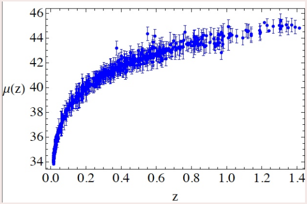
[a] Y. Wang (2008).

$$w(a) = \left(\frac{a_c - a}{a_c - 1}\right) w_0 + \left(\frac{a-1}{a_c-1}\right) w_c. \tag{8}$$

$$\begin{aligned} E^2(z) &= \Omega_m(1+z)^3 + (1-\Omega_m)(1+z)^{3\left[1+\left(\frac{a_c w_0 - w_c}{a_c - 1}\right)\right]} \\ &\times \exp\left\{3\left(\frac{w_c - w_0}{a_c - 1}\right)\frac{z}{1+z}\right\}. \end{aligned} \tag{9}$$

Current astrophysical data (2011):

- Supernovae
  Union2 sample[a]
  $N_{SNeIa} = 557$ events over $0.015 < z < 1.4$.



- BAO
  Spectroscopic Sloan Digital Sky Survey (SDSS) Data Release 7 (DR7) galaxy sample including both the Luminous Red Galaxy (LRG) and also the 2-degree Field Galaxy Redshift Survey (2dFGRS) data.[b]
  Total: 893 319 galaxies over 9100 deg^2.

[a] R. Amanullah, et al. (2010)
[b] W.J. Percival, et al. (2010).

Mock data via iCosmo:

SNeIa simulations.

- Taken into account the specifications of the Wide-Field Infrared Survey Telescope (WFIRST). [a]
- Analysis by using three fiducial models (WMAP7-year analysis):
  - Quiessence model derived by using only CMB data (CMB-oriented model): phantom model $w < -1$.
  - Quiessence model coming from combining CMB data with BAO (BAO-oriented model): regime is phantom (though only slightly).
  - Quiessence model coming from combining CMB data with SNeIa (SNeIa-oriented model): $w > -1$.

BAO simulations.

- Observational targets:

$$y(z) \equiv \frac{r(z)}{r_s(z_r)} \quad \text{and} \quad y'(z) \equiv \frac{r'(z)}{r_s(z_r)}\ , \tag{10}$$

- EUCLID survey[b]

[a] iCosmo: A. Refregier, et al. (2008). http://wfirst.gsfc.nasa.gov/
[b] M. Martinelli, et al. (2010), J.P. Beaulieu, et al. (2010), A. Refregier, et al., (2010).

Methodology

1. Study the parameter space of the model searching for the minimum χ^2.
2. Exploration of two dark energy parameters with Ω_m and Ω_b fixed $\rightarrow$ grid method.
3. Recipe to calculate the tension:

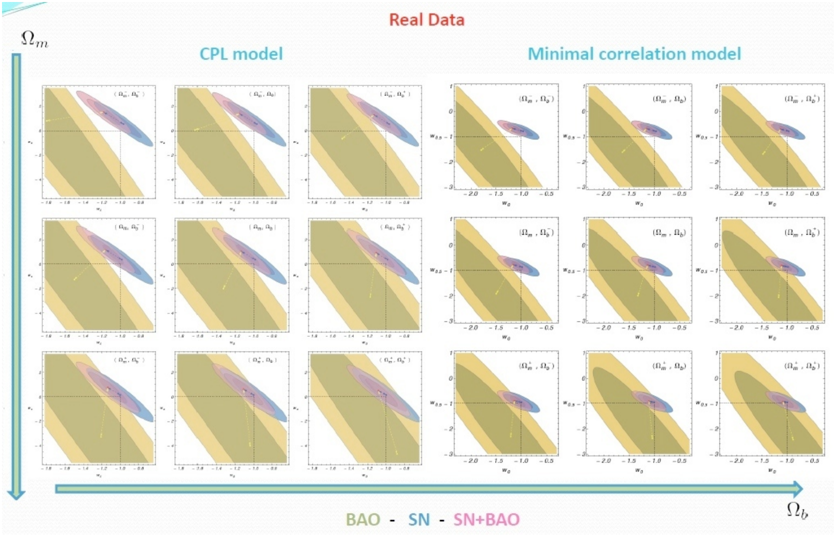
$$\chi^2_{tot} = \chi^2_{\rm SNeIa} + \chi^2_{\rm BAO}, \qquad (11)$$

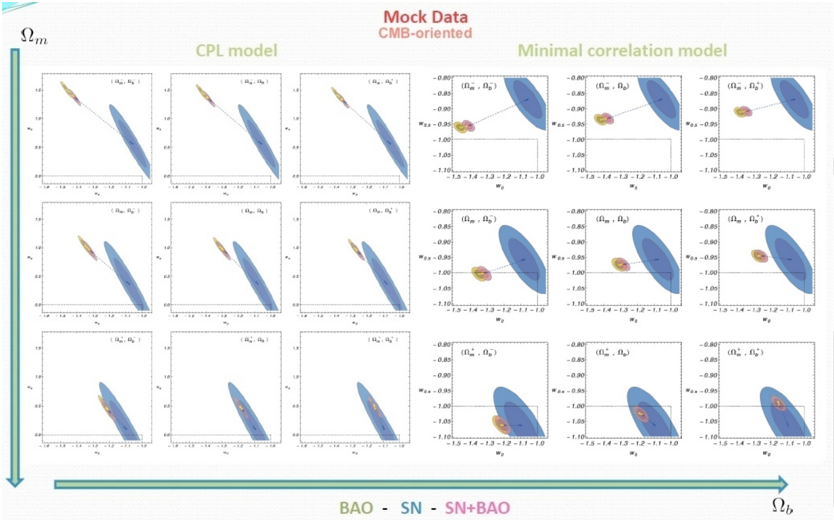
$$\Delta\chi^2_{\sigma} = \chi^2_{tot}(\theta_{\textbf{SNela+BAO}}) - \chi^2_{tot}(\theta_{\textbf{SNela}}), \qquad (12)$$

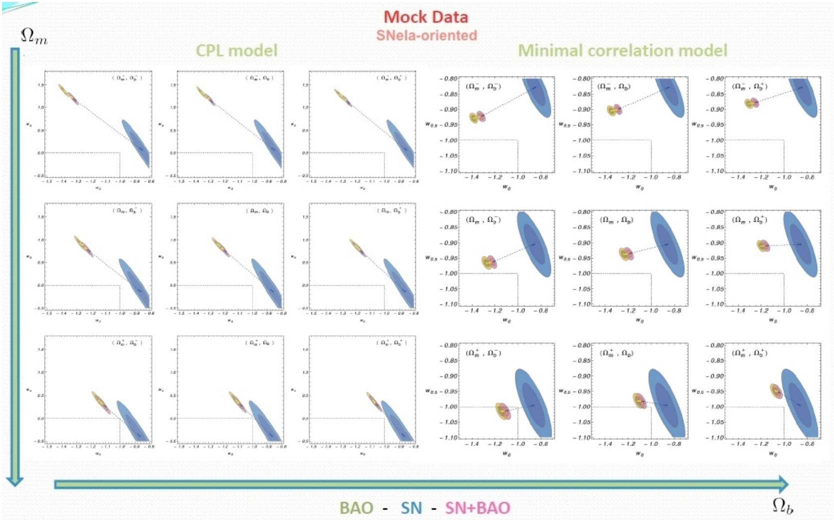
   where $\theta_{\textbf{SNela}}$ and $\theta_{\textbf{SNela+BAO}}$ are the best fit parameters vectors.
4. Chosen priors:

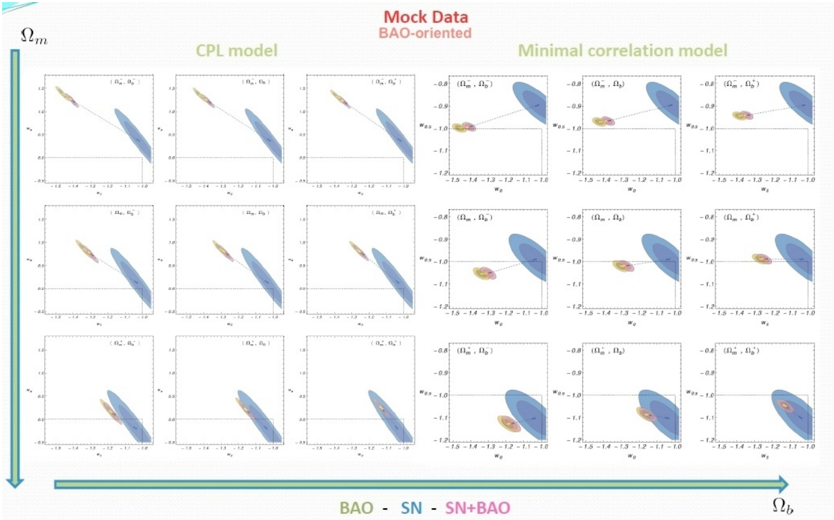
   $\Omega_m h^2 = 0.1308 \pm 0.0008 \rightarrow \Omega_m = 0.237 \pm 0.023$,
   $\Omega_b h^2 = 0.0223 \pm 0.0001 \rightarrow \Omega_b = 0.0405 \pm 0.0039$.

   We produce nine (Ω_m, Ω_b) couples.


Real Data
CPL model
Minimal correlation model
Ω_m
Ω_b
BAO - SN - SN+BAO


Mock Data
CMB-oriented
CPL model
Minimal correlation model
Ω_m
BAO - SN - SN+BAO
Ω_b
(Ω_m^-, Ω_b^-)
(Ω_m^-, Ω_b)
(Ω_m^-, Ω_b^+)
(Ω_m, Ω_b^-)
(Ω_m, Ω_b)
(Ω_m, Ω_b^+)
(Ω_m^+, Ω_b^-)
(Ω_m^+, Ω_b)
(Ω_m^+, Ω_b^+)
w_0
$w_{0.5}$


Mock Data
SNela-oriented
CPL model
Minimal correlation model
Ω_m
BAO - SN - SN+BAO
Ω_b


Mock Data
BAO-oriented
CPL model
Minimal correlation model
Ω_m
Ω_b
BAO - SN - SN+BAO

Summary 1

- Confirmed that there is tension in the two datasets, it must be stressed that ours are the first time simulations of BAO data from both radial and transverse directions in the literature.
- The tension depends strongly on the value of the cosmological priors.
- For most priors and for the SNeIa+BAO combinations best fit, the Universe is currently phantom-like, its dark energy EoS parameter has become more negative recently, and was dark matter dominated at early times.
- Future BAO data will improve constraints considerably making them far tighter.

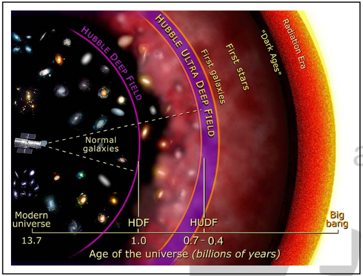
Things we know:

- Einstein's General Relativity (GR) has been extensively tested successfully in the solar system and beyond.
- In order to have a general view one needs to couple the vacuum theory to matter:
  - Assumes a minimal coupling,
  - solves the full Einstein equations and
  - finally, compares theoretical models with observations.

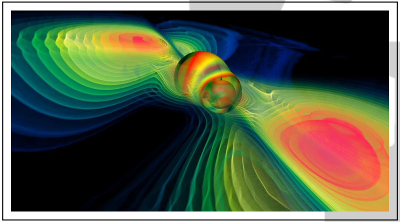
There are some problems:

- The way in which we coupling gravity-matter, e.g.
  - Dark Matter problem $\rightarrow$ Invoking new fundamental interactions rather than assuming exotic particle.
  - Dark Energy problem $\rightarrow$ (Cosmological acceleration) explained in term of more complicated interactions rather than postulating the existence of DE.

- Dynamical evolution of matter fields in GR → Formation of singularities seems unavoidable:
- Big Bang singularity
- Black Holes



- Not possible to perform measurements within GR on such singularities → We need an extending theory.



- A good candidate → Eddington inspired Born-Infeld Gravity (EiBI) [1]

[1] M. Banados and P. G. Ferreira, (2010).

Ingredients

- An Eddington action S_{Edd},

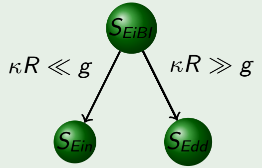
$$S_{Edd} = 2\kappa \int d^4x \sqrt{|R|}, \quad \text{where} \quad R_{\mu\nu} = R_{\mu\nu}(\Gamma). \tag{13}$$

Born-Infeld style + matter in the recipe

- Gravitational action

$$S_{EBI}[g, \Gamma, \Psi] = \frac{2}{\kappa} \int d^4x \left[\sqrt{|g_{\mu\nu} + \kappa R_{\mu\nu}(\Gamma)|} - \lambda \sqrt{g} \right] + S_m[g, \Psi]. \tag{14}$$

EiBI Theory with $S_{Mat} = 0$



Field Equations

- Varying with respect to $g_{\mu\nu}$:

$$\sqrt{\left|\frac{q}{g}\right|}(q^{-1})^{\mu\nu} - \lambda g^{\mu\nu} = -\kappa T^{\mu\nu}. \tag{15}$$

- Varying with respect to Γ:

$$q_{\mu\nu} = g_{\mu\nu} + \kappa R_{\mu\nu}. \tag{16}$$

- The auxiliary metric is compatible with the connection:

$$\Gamma^{\mu}_{\alpha\beta} = \frac{1}{2} q^{\mu\sigma}\left(q_{\sigma\alpha,\beta} + q_{\sigma\beta,\alpha} - q_{\alpha\beta,\sigma}\right). \tag{17}$$

- Eqs. (15) and (16) form a complete set of the Bañados and Ferreira theory.

Novel aspects

- In stars, compact objects and black holes $\rightarrow$ Eddington regime lead:
  - the avoidance of singularities,
  - significant modifications of the standard stellar astrophysics,
- Test of Eddington corrections to Newtonian gravity (Solar physics)[a] and around rotating sources[b].
- Existence of gravitational objects $\rightarrow$ constraints on the free parameter of the theory[c].
- Re-expression of the theory as a bigravity theory[d].

[a] J. Casanellas, et al. (2012).
[b] P. Pani, et al. (2011).
[c] P. Avelino, (2012).
[d] T. Delsate, et al. (2012). M. Banados, et al. (2009).

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Objective

- Study the cosmological behaviour of the Eddington regime $\rightarrow$ analysis of the structure and evolution of linear tensor mode perturbations or gravitational waves[a] [b]

[a] C. Escamilla-Rivera, M. Bañados, P. G. Ferreira. Phys.Rev. D85 (2012) 087302.
[b] C. Escamilla-Rivera, M. Bañados, P. G. Ferreira. C12-07-01.1 (2013). Marcel Grossmann Proceedings 2013.

We now split the dynamics of the problem in the background FLRW dynamics and their fluctuations about the background

$$g_{\mu\nu}dx^\mu dx^\nu = a^2\left[-d\eta^2 + (\delta_{ij} + h_{ij})\right]dx^i dx^j, \tag{18}$$

$$q_{\mu\nu}dx^\mu dx^\nu = -X^2 d\eta^2 + Y^2(\delta_{ij} + \gamma_{ij})\,dx^i dx^j, \tag{19}$$

Setting $h_{ii} = \gamma_{ii} = 0$, we can take the energy momentum tensor as $T^{\mu\nu} = \rho + a^{-2}(\delta^{ij} + h^{ij})P$.

Background Equations

$$-\frac{|XY^3|}{X^2a^4} + \frac{\lambda}{a^2} = -\kappa\frac{\rho}{a^2}, \tag{20}$$

$$\frac{|XY^3|}{Y^2a^4} - \frac{\lambda}{a^2} = -\kappa\frac{P}{a^2}, \tag{21}$$

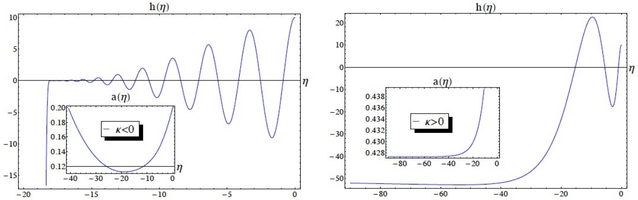
Tensor-Perturbed Equations

We start by taking the perturbed elements of the metrics (18), (19) and the energy-momentum tensor and construct the perturbed field equation for (15)

$$
\begin{aligned}
-\frac{XY^3}{a^4}\frac{1}{Y^2}\gamma^{ij} + \frac{\lambda}{a^2}h^{ij} &= \kappa\frac{P}{a^2}h^{ij}, \\
&= \left(-\frac{XY^3}{a^4 Y^2} + \frac{\lambda}{a^2}\right)h^{ij} \quad \rightarrow \quad \boxed{\gamma_{ij} = h_{ij}} \qquad (22)
\end{aligned}
$$

Perturbed Equation

$$
h''_{ij} + \left(3\frac{Y'}{Y} - \frac{X'}{X}\right)h'_{ij} + \left[4\left(\frac{Y'}{Y}\right)^2 + 2\frac{Y''}{Y} - 2\frac{X'}{X}\frac{Y'}{Y} - \frac{2}{\kappa}\left(\frac{X^2a^2}{Y^2} - X^2\right) + \left(\frac{X}{Y}\right)^2 k^2\right]h_{ij} = 0. \qquad (23)
$$



Evolution of tensor modes $h(\eta)$. Inside each figure we plot as well the behaviour of the scale factor for $\kappa < 0$ (left) and $\kappa > 0$ (right).

Perturbed Equation with the spatial background field equation

$$h_{ij}'' + \left(3\frac{Y'}{Y} - \frac{X'}{X}\right) h_{ij}' + \left(\frac{X}{Y}\right)^2 k^2 h_{ij} = 0. \tag{24}$$

How the system evolves in different regimes?

- Einstein regime.
  - Indistinguishable from Einstein gravity when $X = Y = a$.
  - Solution in the radiation era: [2]

$$h_{ij} \propto \frac{1}{\sqrt{\eta}}\mathcal{H}^{(1)}_{\frac{1}{2}}(k\eta), \frac{1}{\sqrt{\eta}}\mathcal{H}^{(2)}_{\frac{1}{2}}(k\eta) \tag{25}$$

where for

  - Late times: $k\eta \to$ Decaying oscillatory solutions
  - Early times:

$$h_{ij} = \frac{2}{\sqrt{\eta}}J_{\frac{1}{2}}(k\eta).$$

[2] J. E. Lidsey, et al. (1997). A. R. Liddle and D. H. Lyth, (2000).

- Eddington regime. Let us suppose the following approximation for the case $\kappa > 0$

$$\bar{a} \equiv \frac{a}{a_B} = 1 + e^{\left[\sqrt{\frac{8}{3\kappa}}(t-t_0)\right]},$$

$$V \equiv \left(\frac{Y}{a}\right)^2 = \sqrt{2}e^{\left[\frac{1}{2}\sqrt{\frac{8}{3\kappa}}(t-t_0)\right]},$$

$$U \equiv \left(\frac{X}{a}\right)^2 = \frac{V^3}{2},$$

where we assume that $q_{00} = -U$ and $q_{ij} = a^2 V \delta_{ij}$.

- Existence of $a_B \rightarrow$ Non-singular behavior.
- Auxiliary metric becomes singular as $t \rightarrow -\infty$. (Evolution of the tensor modes).

In conformal time:

$$\left(\frac{X}{Y}\right)^2 = \frac{e^{(\alpha\Delta\eta)}}{1-e^{(\alpha\Delta\eta)}} \quad \text{with} \quad \alpha = a_B\sqrt{8/(3\kappa)}$$

$$\left(3\frac{Y'}{Y}-\frac{X'}{X}\right) = \partial_\eta \ln(Y^3/X) = \partial_\eta \ln a^2\sqrt{V^3/U}$$

$$= 2\frac{a'}{a} + 2\alpha\frac{e^{(\alpha\Delta\eta)}}{1-e^{(\alpha\Delta\eta)}}$$

Evolution equation for the tensor mode ($\kappa > 0$)

$$h''_{ij} + 2\alpha\frac{e^{(\alpha\Delta\eta)}}{[1-e^{(\alpha\Delta\eta)}]}h'_{ij} + \frac{e^{(\alpha\Delta\eta)}}{[1-e^{(\alpha\Delta\eta)}]}k^2 h_{ij} = 0. \tag{26}$$

Consider $\Delta\eta \to -\infty$ then

$$h''_{ij} \simeq 0, \quad h_{ij} \propto A\eta + B. \tag{27}$$

- Now let us suppose the following approximation for the case $\kappa < 0$

$$\begin{aligned} a &= a_B\left(1 + \frac{2}{3|\kappa|}t^2\right) \\ X^2 &= \frac{4}{3}a^2\sqrt{\frac{|\kappa|}{2}}\frac{1}{|t|} \\ Y^2 &= \frac{4}{3}a^2\sqrt{\frac{2}{|\kappa|}}|t| \end{aligned}$$

In conformal time:

$$\begin{aligned} a &= a_B[1 + \tan^2(\beta\eta)] \quad \text{with} \quad \beta = a_B\sqrt{2/(3|\kappa|)} \\ X^2 &= a^2\frac{4}{3^{3/2}}\frac{1}{|\tan(\beta\eta)|} \\ Y^2 &= a^2\frac{4}{3^{1/2}}|\tan(\beta\eta)| \end{aligned}$$

Taylor expansion around $\eta = 0$:

$$\frac{X^2}{Y^2} = \frac{U}{V} \simeq \frac{1}{3\beta^2\eta^2}$$

$$\left(3\frac{Y'}{Y} - \frac{X'}{X}\right) \simeq \frac{2}{\eta}$$

Evolution equation for the tensor mode ($\kappa < 0$)

$$h''_{ij} + \frac{2}{\eta}h'_{ij} + \frac{k^2}{3\beta^2\eta^2}h_{ij} = 0 \tag{28}$$

with solution

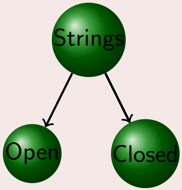
$$h_{ij} \propto \eta^{\left[-\frac{1}{2}\pm\frac{1}{2}\sqrt{1-(4k^2/3\beta^2)}\right]}. \tag{29}$$

Summary 2

- We found an instability in Eddington regime ($\kappa > 0$) and at the bounce ($\kappa < 0$).
- The singular behavior is induced by the evolution of the symmetric part of $q_{\mu\nu}$ (via X and Y) not present in Einstein gravity.
- These instabilities are present only in this particular form of the Eddington theory.
- Some work has been done to understand the process of gravitational collapse, in our study the tensor modes may play an unexpected role and can be included in these works.

Things we know:

- From early days of String Theory $\rightarrow$ spectrum does contain a negative squared mass: Tachyon.
- Tachyonic mode: signals the fact that we are perturbing strings in a false vacuum.
- Tachyon condensation:
  1. Tachyon rolls down to the minimum of the potential.
  2. Then it disappears by acquiring a vev that cancels the original negative squared mass.
- At tree level:



- OS-tachyon drives the decay of branes $\rightarrow$ D-branes can be thought of as being made of tachyons.
- CS-tachyon drives the disappearance of the space-time.

Open strings news

- The study of its condensation leaves important insights as the identification of the tachyon field as a proper time in Quantum Cosmology[a],
- and the fact that this relationship is broken in the presence of the Electromagnetic fields (EM)[b].

[a] A. Sen (2002-2003).
[b] H.García Compeán, et al. (2005), C. Escamilla-Rivera, et al. (2010).

Closed strings news

- Some interesting studies in these lines have been performed on which the classical closed string tachyon plays an important role in the evolution of the universe[a].
- Two interesting questions in this matter[b]:
  1. Can such a closed string tachyon drive the inflation?
  2. Can this tachyon field be used to create a crunch scenario of the universe?

[a] H. Yang and B. Zwiebach (2005), J. McGreevy and E. Silverstein (2005).
[b] C. Escamilla-Rivera, et al. (2011).

Building the Closed String Tachyon model

We start by considering critical bosonic string theory in 26-dimensional space-time with a constant dilation and without dynamics related to a constant B-field[a]

$$S = \frac{1}{2\kappa_0^2} \int d^{26}x \sqrt{-g_{26}}\; e^{-2\Phi_0} \left[R - (\nabla T)^2 - 2V(T) \right]. \qquad (30)$$

Let us now choose the tachyon field to be a function only of time, i.e. $T = T(t)$, also we consider a spatially flat $3+1$ dimensional FRW background:

$$ds^2 = -dt^2 + e^{2\alpha(t)} \left(dr^2 + r^2 d\Omega^2 \right), \qquad (31)$$

where $a(t) = e^{\alpha(t)}$ is the scale factor. Substituting this metric in the above action, the effective four-dimensional action can be written as

$$S = m_p^2 \int \left[-3\dot{\alpha}^2(t) + \frac{1}{2}\dot{T}^2 - \mathcal{V}(T) \right] e^{3\alpha(t)} dt. \qquad (32)$$

[a] H. Yang and B. Zwiebach, (2005).

Hamiltonian formalism

The corresponding differential equation is then given by

$$-\frac{1}{2}(\partial_T W)^2 + \frac{3}{4}W^2 = \mathcal{V}(T). \tag{33}$$

Using the $W(T) = A + BT^2$ proposal we obtain the effective scalar potential

$$\mathcal{V}(T) = \frac{3}{4}(A + BT^2)^2 - 2B^2T^2. \tag{34}$$

Non-singular, expansion and contraction phenomenas.[a]

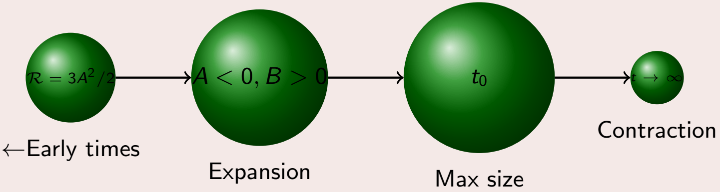
[a] C.Escamilla-Rivera, G. Garcia, O. Loaiza and O. Obregon (2013).

- Consider the form of $W(T)$ and using the Hamiltonian momenta we compute the tachyon field

$$T(t) = e^{2Bt}, \tag{35}$$

and the scale factor

$$a(t) = e^{\left(-\frac{1}{2}At - \frac{1}{8}e^{4Bt}\right)}. \tag{36}$$



- The potential $\mathcal{V}(T)$ has a minimum in $T_0 = \sqrt{(4B - 3A)/3B}$ where

$$\mathcal{V}(T_0) = \frac{2}{3}B(3A - 2B), \quad \text{at} \quad t_T = \frac{1}{4B}\ln\left(-\frac{A}{B} + \frac{4}{3}\right). \tag{37}$$

- We see that an internal flat space

$$\mathcal{V}(T)|_{A=0} = -2B^2T^2 + \frac{3}{4}B^2T^4, \tag{38}$$

where, when $a(-\infty) = 1 \to$ the universe starts without the presence of a singularity, avoiding a big bang scenario.

- The time at which the scale factor reaches a maximum is

$$t_0 = \frac{1}{4B}\ln\left(-\frac{A}{B}\right). \tag{39}$$

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Objective

- A proposal to make the closed string tachyon and a Chaplygin gas interact, model which is capable to describe the current cosmic acceleration[a] [b] [c].

[a] A. R. Amani, C. Escamilla-Rivera and H.R. Faghani. Phys.Rev. D88 (2013) 124008.
[b] C. Escamilla-Rivera, O. Loaiza-Brito and O. Obregon. Class.Quant.Grav. 30 (2013).
[c] C. Escamilla-Rivera. C12-07-01.1 (2013). Marcel Grossmann Proceedings 2013.

In this case, we extended the work [C.Escamilla-Rivera, G. Garcia, O. Loaiza and O. Obregon (2013)] by

- first, taking the function S in a non-flat universe and computing the effective tachyon potential

$$V(T) = \frac{3}{4}D^2(1+\beta k)^2 T^4 + D\left(\frac{3}{2}C - 2D\right)(1+\beta k)^2 T^2 + 3ke^{-2\alpha}. \quad (40)$$

- Second, we add a modified Chaplygin gas[a] $p_{MCG} = A\rho_{MCG} - B/\rho^{\gamma}_{MCG}$. Therefore, the total energy density and pressure are:

$$\rho_{tot} = \rho_T + \rho_{MCG}, \quad (41)$$

$$p_{tot} = p_T + p_{MCG}, \quad (42)$$

[a] Chimento, L. (2009)

where:

Closed String Tachyon density and pressure

$$\rho_T = \frac{3}{4}(1+\beta\, k)^2 \left[C + D\, e^{4D(1+\beta k)t}\right]^2 + 3k \exp\left[C(1+\beta k)t + \frac{1}{4}e^{4D(1+\beta k)t}\right] - \left(\frac{B}{\eta} + \exp\left[\frac{3}{2}C\eta(\gamma+1)(1+\beta k)t + \frac{3}{8}\eta(\gamma+1)e^{4D(1+\beta k)t}\right]\right)^{\frac{1}{\gamma+1}}, \quad (43)$$

$$p_T = (1+\beta\, k)^2\left[4D^2 e^{4D(1+\beta k)t} - \frac{3}{4}\left(C + De^{4D(1+\beta k)t}\right)^2\right] - k\exp\left[C(1+\beta k)t + \frac{1}{4}e^{4D(1+\beta k)t}\right] - A\left(\frac{B}{\eta} + \exp\left[\frac{3}{2}C\eta(\gamma+1)(1+\beta k)t + \frac{3}{8}\eta(\gamma+1)e^{4D(1+\beta k)t}\right]\right)^{\frac{1}{\gamma+1}} + \frac{B}{\left[\frac{B}{\eta} + \exp\left[\frac{3}{2}C\eta(\gamma+1)(1+\beta k)t + \frac{3}{8}\eta(\gamma+1)e^{4D(1+\beta k)t}\right]\right]^{\frac{\gamma}{\gamma+1}}}. \quad (44)$$

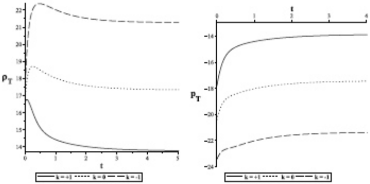
and:

Chaplygin gas density and pressure

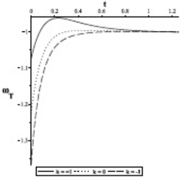
$$\rho_{MCG} = \left[\frac{B}{\eta} + c_0\, a^{-3\eta(\gamma+1)}\right]^{\frac{1}{\gamma+1}}, \tag{45}$$

$$p_{MCG} = A\left[\frac{B}{\eta} + a^{-3\eta(\gamma+1)}\right]^{\frac{1}{\gamma+1}} - \frac{B}{\left[\frac{B}{\eta} + a^{-3\eta(\gamma+1)}\right]^{\frac{\gamma}{\gamma+1}}}, \tag{46}$$

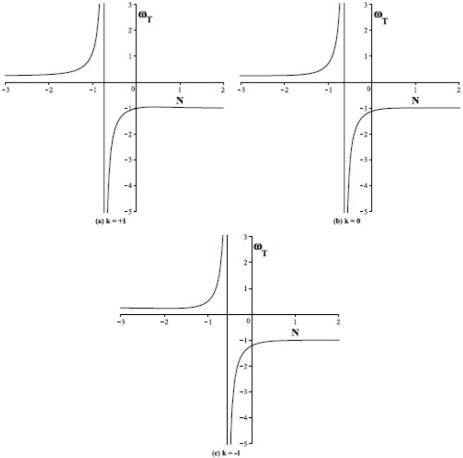
with $\eta = 1 - b^2 + A$.



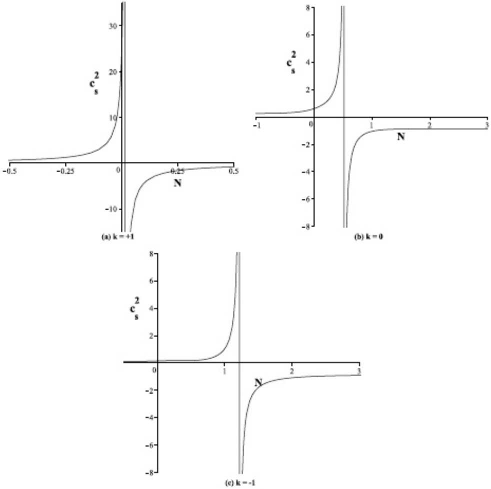
The energy density and pressure of the closed string tachyon in terms of time evolution for $B = 2, C = -5, D = -0.25, b = 0.25, \beta = -0.1, \gamma = 0.5, c_0 = 3.25$ and $A = 0.25$ in different cases $k = \pm 1, 0$.



The EoS of the closed string tachyon in terms of time evolution for $B = 2, C = -5, D = -0.25, b = 0.25, \beta = -0.1, \gamma = 0.5, c_0 = 3.25$ and $A = 0.25$ in different cases $k = \pm 1, 0$.



The EoS of the closed string tachyon in terms of time evolution for $B = 2, C = -5, D = -0.25, b = 0.25, \beta = -0.1, \gamma = 0.5, c_0 = 3.25$ and $A = 0.25$ in different cases $k = \pm 1, 0$



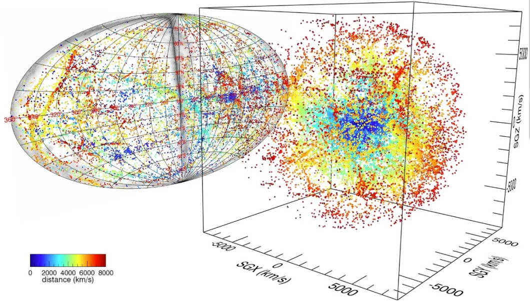
Graphs of the c_s^2 in terms of time evolution for $B = 2, C = -5, D = -0.25, b = 0.25, \beta = -0.1, \gamma = 0.5, c_0 = 3.25$ and $A = 0.25$ in geometries $k = \pm 1, 0$.

Summary 3

- We obtained a set of Einstein and field equations in a non-flat internal 22d for the interaction between the closed string tachyon with modified Chaplygin gas.
- The EoS obtained has showed an accelerating universe and crossing over the phantom-divided line.
- In this model a stability stage is assured at late times.

Things we know:

- Cosmology: Sub-sector 1 → Cosmodynamics.
- Cosmology: Sub-sector 2 → Cosmography.
  1. Extracting the maximum amount of information from measured distance.
  2. Assuming Cosmological Principle.
  3. Studies without ever having to address how much dark energy and dark matter are needed.



Objective

- Discuss the expected constraints of a cosmographic and, therefore, model-independent analysis of future BAO observations[a].

[a] R. Lazkoz, J. Alcaniz, C. Escamilla-Rivera, V. Salzano and I. Sendra. JCAP 1312 (2013) 005.

BAO cosmography scenario

- Key relation $\rightarrow$ a Taylor expansion:

$$\begin{aligned}\frac{a(t)}{a(t_0)} &= 1 + H_0(t-t_0) - \frac{q_0}{2}H_0^2(t-t_0)^2 + \frac{j_0}{3!}H_0^3(t-t_0)^3 \\ &+ \frac{s_0}{4!}H_0^4(t-t_0)^4 + \frac{l_0}{5!}H_0^5(t-t_0)^5 + O[(t-t_0)^6], \end{aligned} \tag{47}$$

where:

$$\begin{gathered} H(t) = \frac{\dot{a}}{a}, \quad q(t) = -\frac{1}{H^2}\frac{\ddot{a}}{a}, \quad j(t) = \frac{1}{H^3}\frac{\dddot{a}}{a}, \\ s(t) = \frac{1}{H^4}\frac{\ddddot{a}}{a}, \quad l(t) = \frac{1}{H^5}\frac{1}{a}\frac{\mathrm{d}^5 a}{\mathrm{d}t^5}, \end{gathered} \tag{48}$$

and[a]

$$\zeta = \frac{z}{(1+z)} . \tag{49}$$

[a] Cattoen, C and Visser, M. (2007).

BAO cosmography scenario

- Hubble parameter:

$$H(\zeta, q_0, j_0, s_0, l_0) \propto \mathcal{H}_0^\zeta + \mathcal{H}_1^\zeta \zeta + \mathcal{H}_2^\zeta \zeta^2 + \mathcal{H}_3^\zeta \zeta^3 + \mathcal{H}_4^\zeta \zeta^4, \tag{50}$$

$$\mathcal{H}_0^\zeta = 1,$$

$$\mathcal{H}_1^\zeta = 1 + q_0,$$

$$\mathcal{H}_2^\zeta = \frac{1}{2}(2 + 2q_0 - q_0^2 + j_0),$$

$$\mathcal{H}_3^\zeta = \frac{1}{6}(6 + 3q_0^3 - 3q_0^2 + 6q_0 + 3j_0 - 4j_0 q_0 - s_0),$$

$$\mathcal{H}_4^\zeta = \frac{1}{24}(24 - 15q_0^4 + 12q_0^3 - 12q_0^2 + 24q_0 - 4j_0^2 + 12j_0 - 16j_0 q_0$$

$$+25 j_0 q_0^2 - 4s_0 + 7q_0 s_0 + l_0);$$

BAO cosmography scenario

- Transverse mode $y(z) \equiv r(z)/r_s(z_r)$:

$$
\begin{aligned}
r(\zeta, q_0, j_0, s_0, l_0) &= \mathcal{R}_1^\zeta \zeta + \mathcal{R}_2^\zeta \zeta^2 + \mathcal{R}_3^\zeta \zeta^3 + \mathcal{R}_4^\zeta \zeta^4 + \mathcal{R}_5^\zeta \zeta^5, \qquad (51)\\
\mathcal{R}_1^\zeta &= 1,\\
\mathcal{R}_2^\zeta &= \frac{1}{2}(1 - q_0),\\
\mathcal{R}_3^\zeta &= \frac{1}{6}(2 - 2q_0 + 3q_0^2 - j_0),\\
\mathcal{R}_4^\zeta &= \frac{1}{24}(6 - 6q_0 + 9q_0^2 - 15q_0^3 - 3j_0 + 10q_0 j_0 + s_0),\\
\mathcal{R}_5^\zeta &= \frac{1}{120}(24 - 24q_0 + 36q_0^2 - 60q_0^3 + 105q_0^4 - 12j_0 + 10j_0^2\\
&+40q_0 j_0 - 105q_0^2 j_0 + 4s_0 - 15q_0 s_0 - l_0);
\end{aligned}
$$

BAO cosmography scenario

- Dilation scale:

$$D_V(\zeta, q_0, j_0, s_0, l_0) = \mathcal{D}_1^\zeta \zeta + \mathcal{D}_2^\zeta \zeta^2 + \mathcal{D}_3^\zeta \zeta^3 + \mathcal{D}_4^\zeta \zeta^4 + \mathcal{D}_5^\zeta \zeta^5, \tag{52}$$

$$\mathcal{D}_1^\zeta = 1,$$

$$\mathcal{D}_2^\zeta = \frac{1}{3}(1 - 2q_0),$$

$$\mathcal{D}_3^\zeta = \frac{1}{36}(7 - 10q_0 + 29q_0^2 - 10j_0),$$

$$\mathcal{D}_4^\zeta = \frac{1}{324}(44 - 57q_0 + 117q_0^2 - 376q_0^3 - 39j_0 + 258q_0j_0 + 27s_0),$$

$$\mathcal{D}_5^\zeta = \frac{1}{19440}(2017 - 2492q_0 + 4638q_0^2 - 10460q_0^3 + 35395q_0^4 - 1536j_0$$

$$+3540j_0^2 + 6990q_0j_0 - 36300q_0^2j_0 + 702s_0 - 5400q_0s_0);$$

Feature 1: Qualitative analysis

- We calculated the exact cosmographic parameters expressions assuming a flat quiessence model with $\Omega_m = 0.259^{+0.099}_{-0.095}$ and $w = -1.12^{+0.42}_{-0.43}$[a]:

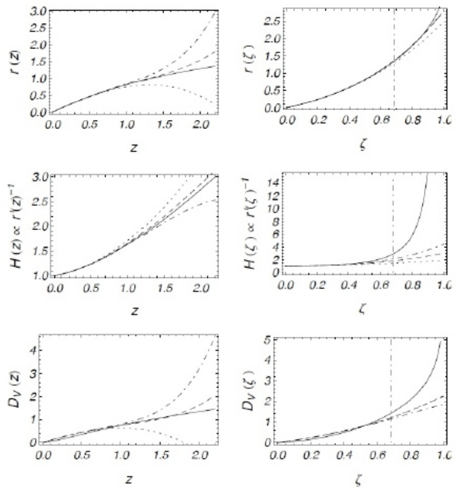
$$q(z) = \frac{(1+z)}{2H^2(z)}\frac{dH^2(z)}{dz} - 1, \tag{53}$$

$$j(z) = \frac{(1+z)^2}{2H^2(z)}\frac{d^2H^2(z)}{dz^2} - \frac{(1+z)}{2H^2(z)}\frac{dH^2(z)}{dz} + 1\,. \tag{54}$$

- We found the cosmographic parameters values:

$$\begin{aligned} q_0 &= -0.755^{+0.495}_{-0.504}, \quad j_0 = 1.448^{+1.738}_{-1.779}, \\ s_0 &= 0.730^{+4.143}_{-4.235}, \quad l_0 = 4.152^{+8.478}_{-8.678}. \end{aligned} \tag{55}$$

[a] WMAP7-year analysis.



Curves of the exact expressions for BAO quantities with cosmographic series for the fiducial cosmological model used with $\Omega_m = 0.259$ and $w = -1.12$.

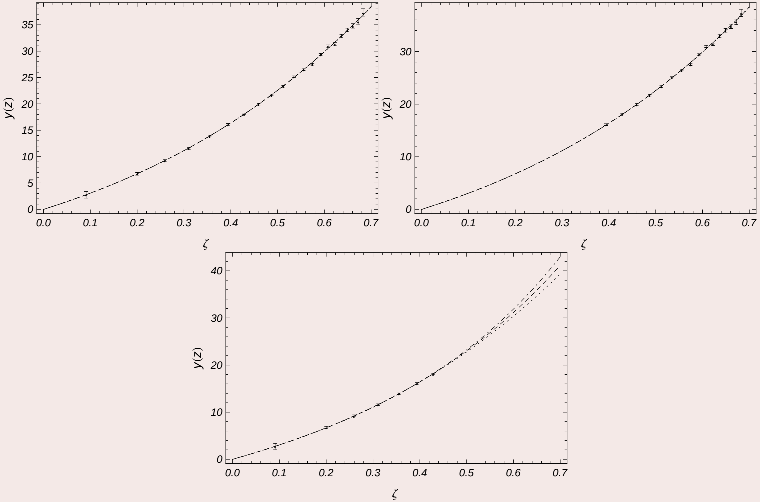
Feature 2: Forecasting

- Generated mock data using iCosmo tool and the quiessence model as a fiducial input $\rightarrow \{y, \sigma_y\}, \{y', \sigma'_y\}$.
- We have simulated Euclid-like survey[a]: 6.1×10^7 galaxies in the redshift range $0.6 < z < 2.1$,
- and used the distribution of galaxies in [J. Geach, et al. (2009)].
- Point taken into account $\rightarrow$ not too close to the fiducial model, so:
  - we realized $N = 200$ simulations.
  - Randomly, extracting the fiducial model parameters.
  - Extracting the mean and the dispersion of y and y' distributions.
- To determine the best fit cosmography parameters we used a MCMC method.

[a] M. Martinelli, et al. (2010)

Redshift	y	σ_y	y'	$\sigma_{y'}$
0.1	2.758	0.616	27.153	3.676
0.25	6.742	0.250	25.449	1.477
0.35	9.214	0.200	24.877	0.892
0.45	11.578	0.180	23.147	0.617
0.55	13.904	0.169	22.347	0.462
0.65	16.107	0.162	20.915	0.364
0.75	18.105	0.158	19.681	0.299
0.85	19.938	0.156	18.496	0.252
0.95	21.699	0.156	17.347	0.218
1.05	23.341	0.157	16.583	0.191
1.15	25.138	0.158	15.434	0.171
1.25	26.481	0.160	14.744	0.154
1.35	27.515	0.169	13.815	0.147
1.45	29.381	0.185	13.207	0.145
1.55	30.963	0.209	12.481	0.149
1.65	31.371	0.240	11.904	0.156
1.75	32.904	0.281	11.217	0.168
1.85	34.028	0.338	10.899	0.186
1.95	34.790	0.417	10.294	0.212
2.05	35.645	0.529	9.752	0.250
2.15	37.341	0.693	9.344	0.303

Euclid mock data distribution of BAO.



Curves of y versus Euclid simulated data.

We compare:

$$q_0 = -0.755^{+0.495}_{-0.504}, \quad j_0 = 1.448^{+1.738}_{-1.779},$$

$$s_0 = 0.730^{+4.143}_{-4.235}, \quad l_0 = 4.152^{+8.478}_{-8.678}.$$

z−range		q_0	j_0	s_0	l_0	q_0^{min}	j_0^{min}	s_0^{min}	l_0^{min}	$\mathrm{FoM}_{q_0 j_0}$	$\ln \mathcal{B}_{ij}$
0.1 − 2.15	$y-2D$	$-0.696^{+0.086}_{-0.084}$	$0.462^{+0.987}_{-0.966}$	–	–	−0.697	0.467	–	–	525.57	0
	$y-3D$	$-0.764^{+0.051}_{-0.046}$	$1.774^{+0.296}_{-0.321}$	$0.006^{+0.233}_{-0.149}$	–	−0.758	1.595	−2.550	–	489.19	0.21
	$y-4D$	$-0.748^{+0.035}_{-0.039}$	$1.588^{+0.210}_{-0.200}$	$0.004^{+0.544}_{-0.346}$	$-0.050^{+0.183}_{-0.692}$	−0.774	1.939	2.232	−0.127	98.84	0.23
0.65 − 2.15	$y-2D$	$-0.711^{+0.101}_{-0.101}$	$0.627^{+1.194}_{-1.134}$	–	–	−0.710	0.614	–	–	408.53	0
	$y-3D$	$-0.769^{+0.057}_{-0.052}$	$1.807^{+0.326}_{-0.365}$	$0.005^{+0.179}_{-0.207}$	–	−0.785	1.954	0.824	–	351.68	0.15
	$y-4D$	$-0.740^{+0.092}_{-0.047}$	$1.535^{+0.239}_{-1.322}$	$-0.044^{+0.352}_{-7.710}$	$-0.010^{+0.108}_{-0.193}$	−0.758	1.711	1.022	−0.113	14.89	0.20
0.1 − 0.75	$y-2D$	$-0.565^{+0.324}_{-0.322}$	$-1.434^{+4.502}_{-3.913}$	–	–	−0.565	−1.438	–	–	33.23	0
	$y-3D$	$-0.565^{+0.033}_{-0.040}$	$0.016^{+0.293}_{-0.177}$	$0.009^{+0.327}_{-0.220}$	–	−0.634	0.030	−11.36	–	–	0.06
	$y-4D$	$-0.536^{+0.032}_{-0.051}$	$0.020^{+0.310}_{-0.149}$	$-0.0004^{+0.2084}_{-0.2368}$	$-0.004^{+0.210}_{-0.232}$	−0.617	−0.018	−7.759	0.065	–	0.27

Summary 4

- We provide general expressions for the transversal and radial BAO modes.
- We have shown that BAO data can place tight constraints on the deceleration and jerk parameters which allows a
    - Model-independent check of the cosmic acceleration ($q_0 < 0$) and,
    - a discrimination between the ΛCDM model ($j_0 = 1$) and alternative mechanism of cosmic acceleration.

And finally...

It is standard practice in cosmology to say *"all of our results can be tested using current observations"*, and of course this is valid and should be made.

At present, the idea of a method with the ability to discriminate between dark energy models and show a more accurate cosmodynamics using the underlying structure of the astronomical data is already presented in the literature. [a]

[a] A. Montiel, R. Lazkoz, I. Sendra, C. Escamilla-Rivera and V. Salzano. Accepted in Phys. Rev. D (2014).