Perturbations of dynamical dark energy cosmologies from w-models

I Mexican School on Cosmological Perturbation Theory
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Outline

- Science overview
- The dark side
- Theoretical background and analysis: DE parameterisations
- Perturbation analysis: recipes and techniques
- Conclusions

Curvature determined by content in the spacetime

Metric: way to describe distance/time

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

FRW: $$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2)$$

How is metric (distance measure) determinated?

Specify way in which curvature determine/ response to matter

$$G_{\mu\nu} = 8\pi G_0 T_{\mu\nu}$$

$G_{\mu\nu} \sim g, \partial g, \partial^2 g$ $\qquad$ $T_{\mu\nu} \sim \rho, P$

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- Einstein's General Relativity (GR) has been extensively tested succesfully

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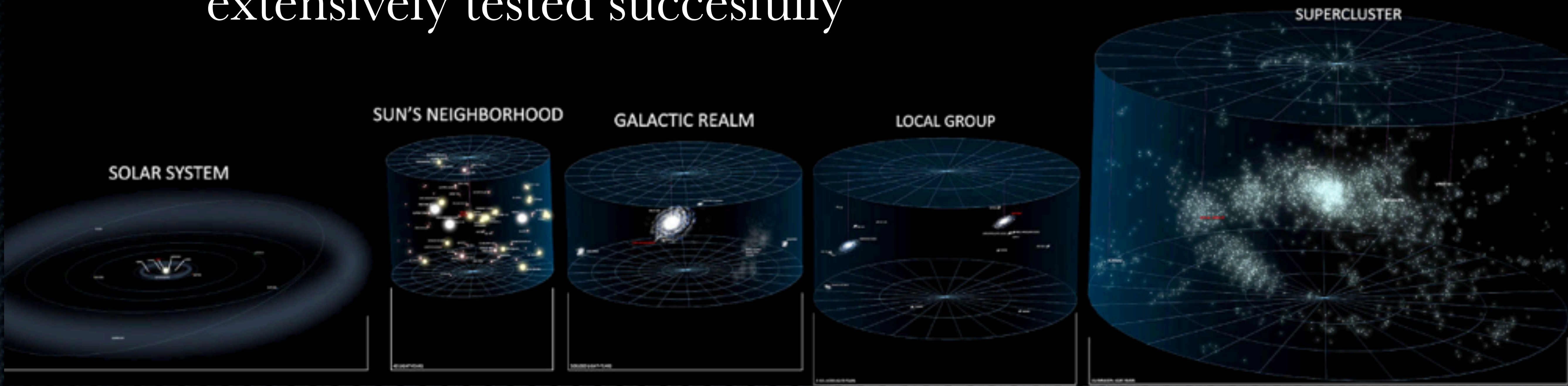

- General view

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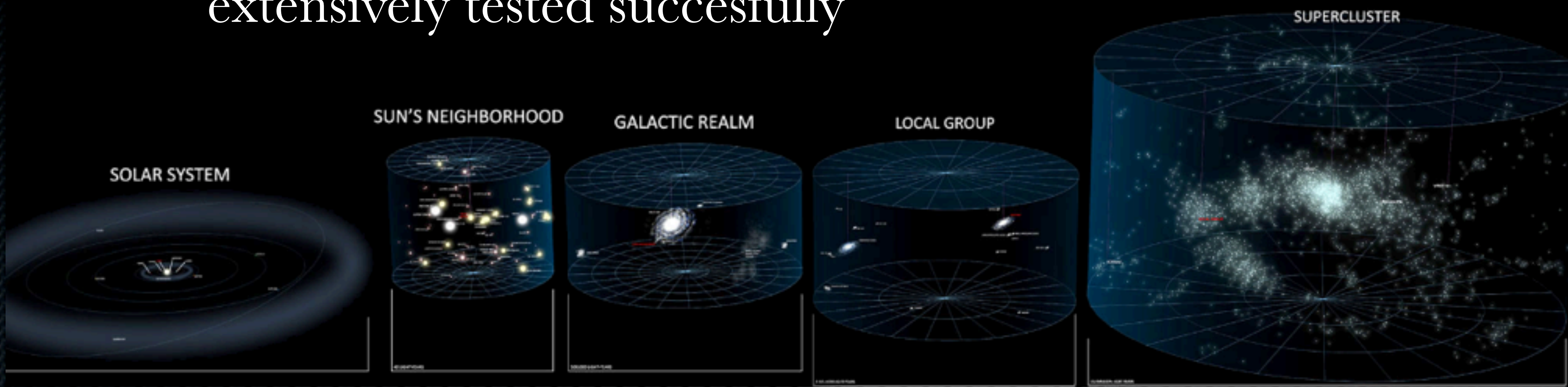

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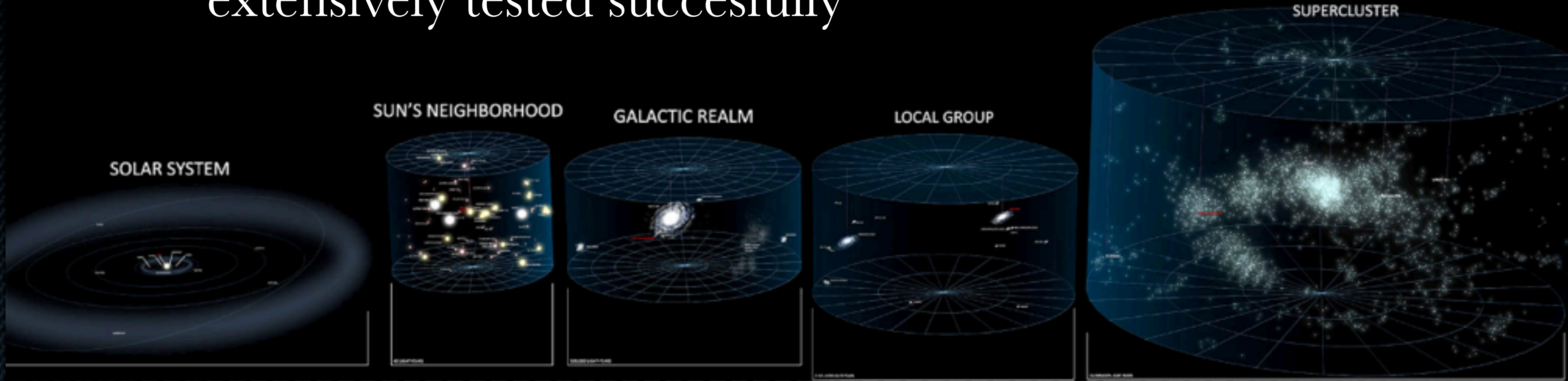

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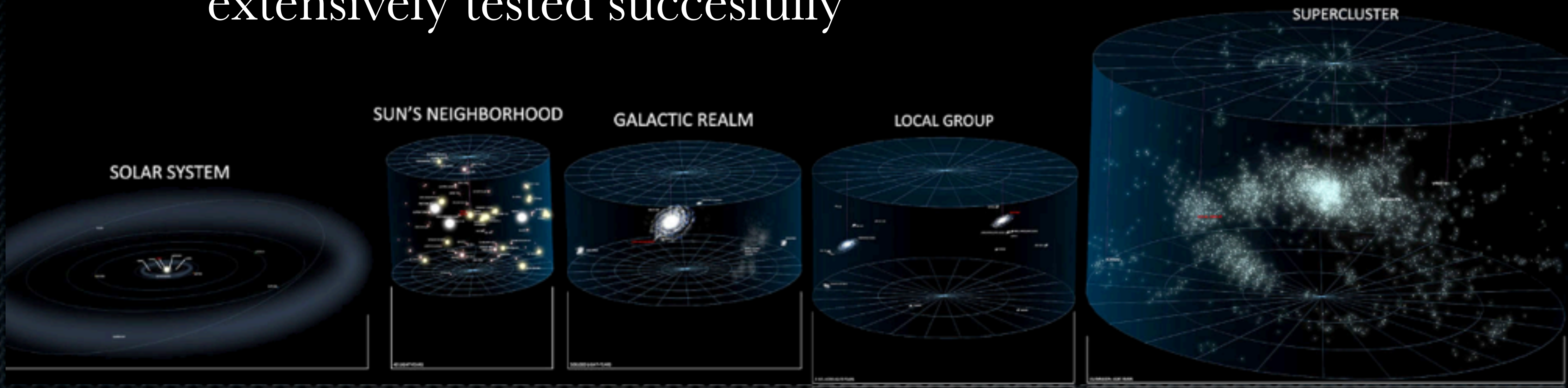

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- General view → Couple the vacuum theory to matter
- Assume a minimal coupling,
- solve the full Einstein equations and
- compare theoretical models with observations

Re-examine ingredients that went into the cosmological cookbook

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Gravity...?
General Relativity

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Maybe GR is not the gravitational theory on large scales?
Applying to cosmology is an extrapolation of the validity of GR as the gravitational theory.
Modify Gravity

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Structure formation and galaxy rotations...

Cosmic acceleration

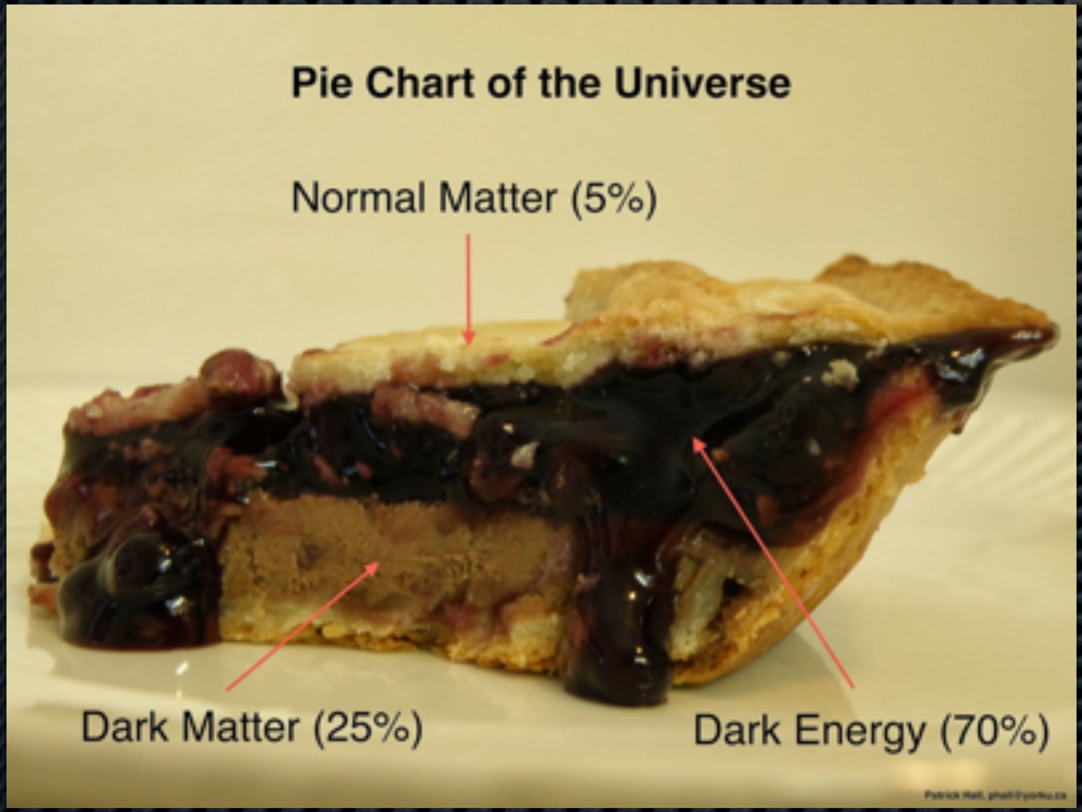
Gravity model
(GR+FRW)

+

Data
(CMB, structure formation, BAO,...)

=

Inconsistency $\longrightarrow$ Dark matter + Dark energy



Standard model: GR + LCDM

What is the dark energy?

T. Clifton, P. Ferreira, A. Padilla and C. Skordis
Physics Reports 513, 1, 1-189 (2012).

The models zoo

$$\mathcal{L} = R + 2\Lambda$$ Cosmological constant

$$\mathcal{L} = R + F(R)$$

$$\mathcal{L} = R + F(R, R_{\mu\nu}R^{\mu\nu}, R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta})$$

$$\mathcal{L} = R + \partial_\mu \phi \partial^\mu \phi + 2V(\phi)$$ Quintessence

$$\mathcal{L} = R + F(R) + \partial_\mu \partial^\mu \phi + 2V(\phi)$$

$$\mathcal{L} = R - 2f(-\frac{1}{2}\partial_\mu \partial^\mu \phi, \phi)$$ K-essence

$$\mathcal{L} = R - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \lambda(A^\mu A_\mu - 1)$$ Einstein-Aether

$$\vdots$$

Studying the nature of dark energy means: studying the equation of state

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0, \qquad \rho(z) = \rho_0(1+z)^{3(1+w(z))}$$

Evolution: Friedmann equation

$$h^2(z) \equiv \frac{H^2(z)}{H_0^2} = \Omega_m(1+z)^3 + (1-\Omega_m)e^{3\int_0^z \frac{1+w(z')}{1+z'}dz'}$$

Scalar field dynamics

The scalar field representation of dark energy parametrisations can be done by considering:

$$\rho_\phi = \frac{\dot{\phi}^2}{2} + V = \rho(a),$$

$$p_\phi = \frac{\dot{\phi}^2}{2} - V = p(a) = w(a)\rho(a),$$

$$\dot{\phi} = \sqrt{w+1}\sqrt{\rho}$$

$$V = \frac{\rho}{2}(1-w)$$

$$\frac{\rho_0}{H_0^2} = 3\Omega_0$$

Parameterisations and their scalar field representation

A. Cooray and D. Huterer
Astrophys.J. 513, L95 (1999)

Linear model

$$w(z) = w_0 - w_1 z$$

$$\dot{\phi} = \sqrt{1 + w_0 + w_1 - w_1/a}\sqrt{3\Omega_0} e^{\frac{3}{2}w_1\left(1-\frac{1}{a}\right)} a^{-\frac{3}{2}(1+w_0+w_1)}$$

$$V = \frac{3}{2}\left(1 - w_0 - w_1 + \frac{w_1}{a}\right)\Omega_0 e^{3w_1\left(1-\frac{1}{a}\right)} a^{-3(1+w_0+w_1)}$$

M. Chevallier, D. Polarski and E.V. Linder
Int.J.Mod.Phys. D10, 213 (2001)

Chevallier-Polarski-Linder model (CPL)

$$w(z) = w_0 + \frac{z}{1+z} w_1$$

$$\dot{\phi} = \sqrt{1 + w_0 + w_1 - a w_1}\sqrt{3\Omega_0} a^{-\frac{3}{2}(1+w_0+w_1)} e^{\frac{3}{2}w_1(a-1)}$$

$$V = \frac{3}{2}\left(1 - w_0 - w_1 + a w_1\right)\Omega_0 a^{-3(1+w_0+w_1)} e^{3w_1(a-1)}$$

Parameterisations and their scalar field representation

E.M.Barboza, Jr and J.S.Alcaniz
Phys.Lett.B. 666, 415 (2008)

Barboza-Alcaniz model (BA)

$$w(z) = w_0 + w_1 \frac{z(1+z)}{1+z^2}$$

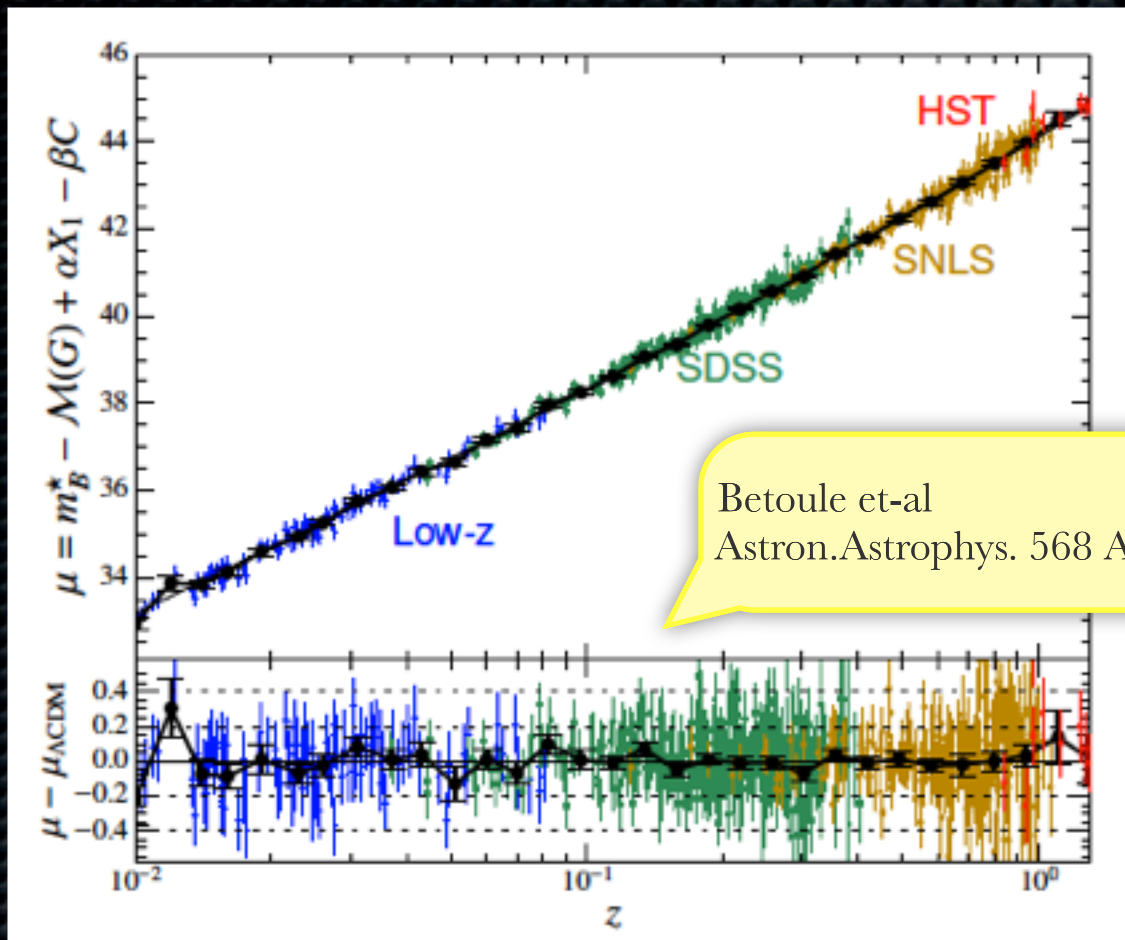
$$\dot{\phi} = \sqrt{1 + w_0 + w_1(1-a)/(2a^2 - 2a + 1)}\sqrt{3\Omega_0}a^{-\frac{3}{2}(1+w_0+w_1)} \times [1 + 2(a-1)a]^{\frac{3}{4}w_1}$$

$$V = \frac{3}{2}\Omega_0 a^{-\frac{3}{2}(1+w_0+w_1)}[1+2(a-1)a]^{\frac{3}{2}w_1} \times [1 + w_0 + w_1(1-a)/(2a^2 - 2a + 1)]$$

Step 1. Background analysis: testing the dark energy models with observations from SNe Ia and BAO

https://github.com/celia-escamilla-rivera/cosmo_tension

Astrophysical data



Betoule et-al
Astron.Astrophys. 568 A22 (2014)

Catalog SNe Ia JLA

$$740 \quad events \quad in \quad 0.01 < z < 1.3$$

Baryon Acoustic Oscillations

$$\mathbf{X}_{\mathbf{BAO}} = \begin{pmatrix} \frac{r_s(z_d)}{D_V(0.106,\Omega_m;w_0,w_1)} - 0.336 \\ \frac{r_s(z_d)}{D_V(0.35,\Omega_m;w_0,w_1)} - 0.1126 \\ \frac{r_s(z_d)}{D_V(0.57,\Omega_m;w_0,w_1)} - 0.07315 \end{pmatrix},$$

Beutler et al. Mon.Not.Roy.Astron.Soc. 416,3017 (2011)
X.Xu et al. Mon.Not.Roy.Astron.Soc. 427,2146 (2012)
L. Anderson et al. BOSS Collaboration. Mon.Not.Roy.Astron.Soc. 441, 24 (2014)

$$\mathbf{C}_{\mathbf{BAO}}^{-1} = \mathrm{diag}(4444, 215156, 721487).$$

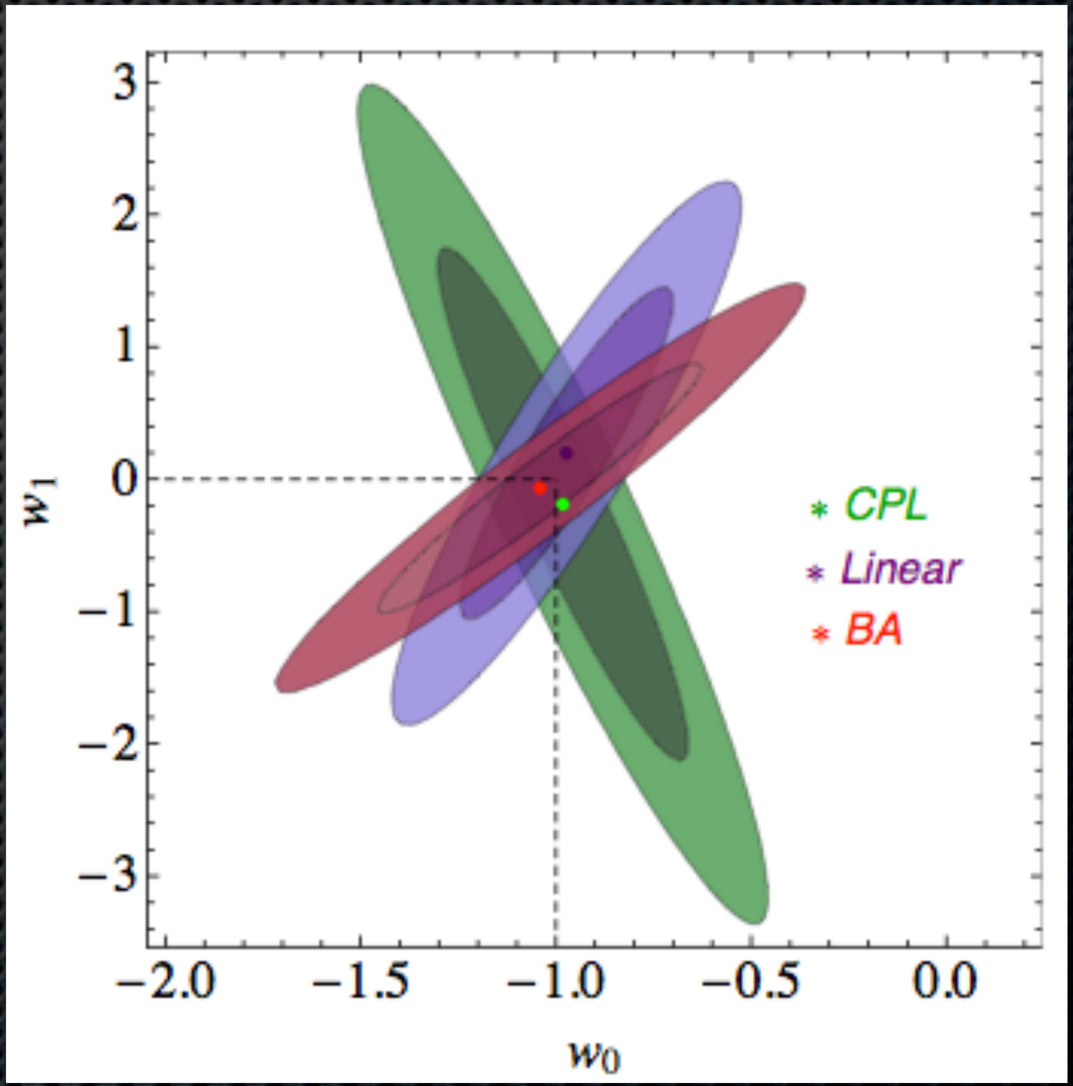
Background analysis

Sigma-distance

$$\chi^2_{tot} = \chi^2_{\rm SNeIa} + \chi^2_{\rm BAO},$$

$$\Delta\chi^2_\sigma = \chi^2_{tot}(\theta_{\bf SNeIa+BAO}) - \chi^2_{tot}(\theta_{\bf SNeIa})$$

C. Escamilla-Rivera, R. Lazkoz
JCAP 1109 003 (2011)



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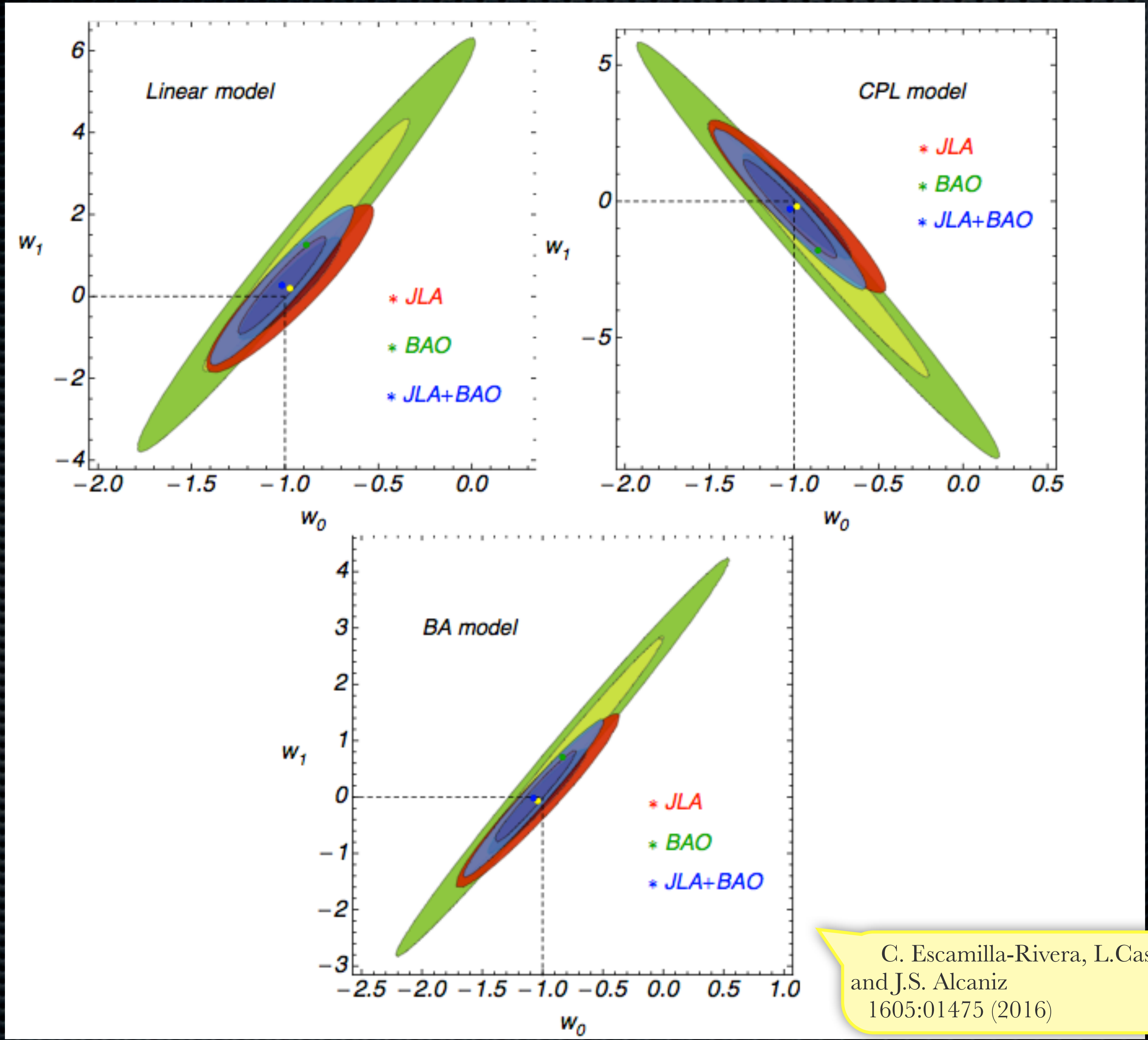
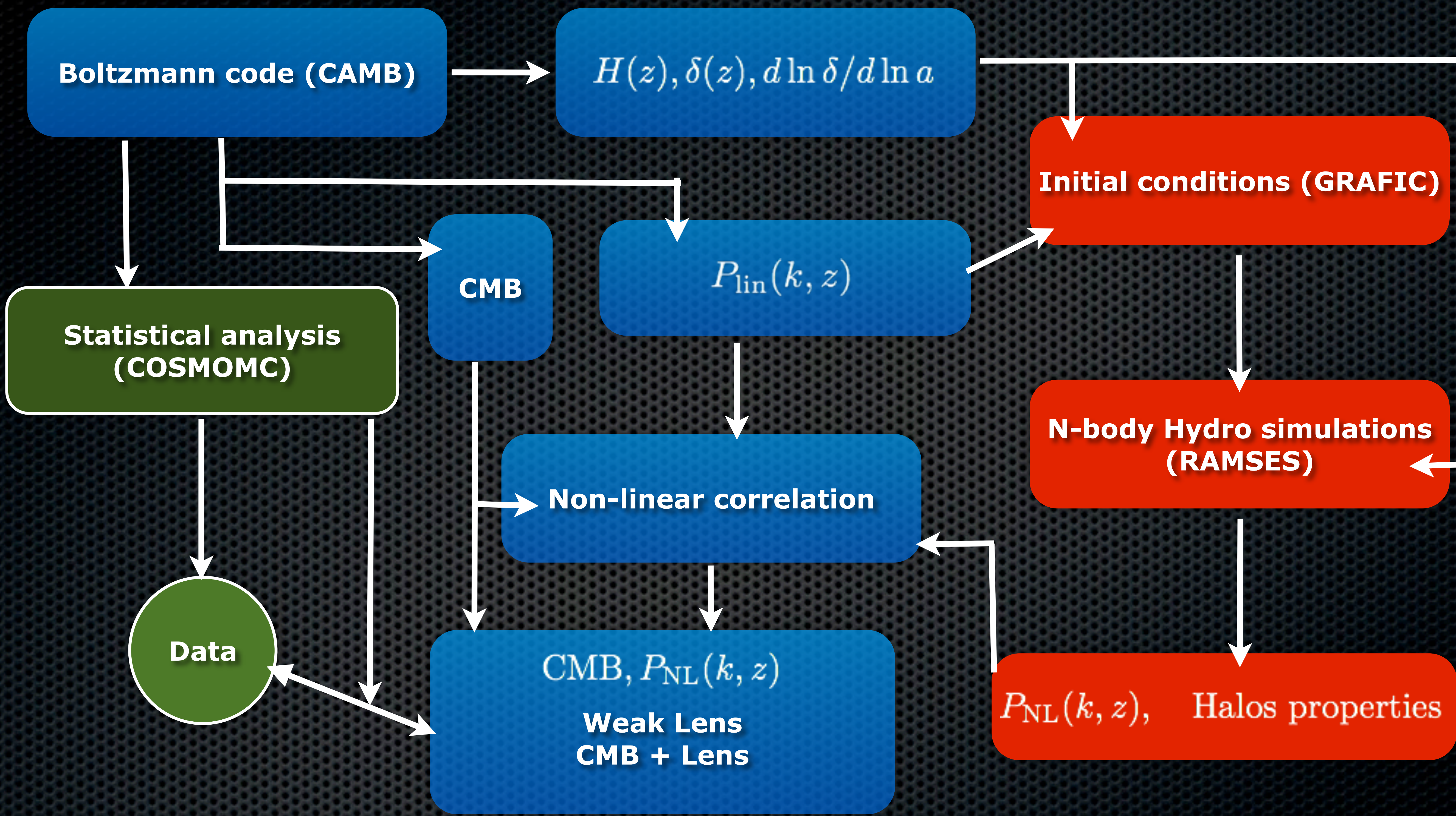


C. Escamilla-Rivera, L.Casarini, J. Fabris and J.S. Alcaniz
1605:01475 (2016)



: **Theoretical predictions**

: **Statistical analysis**

: **N-body/Hydro simulations**

- Cosmological parameter estimation

Halo model

Seljak 2000, Peacock & Smith 2000, Cooray & Sheth 2000

Total power: $\Delta^2(k) = \Delta^2_{2H} + \Delta^2_{1H}$

1-halo term: $\Delta^2_{2H} \approx \Delta^2_{\rm lin}(k)$

2-halo term: $\Delta^2_{1H} = 4\pi(k/2\pi)^3\bar{\rho}^{-2}\int_0^\infty M^2W^2(k,M)F(M)dM$

Fourier transform of density profile

Mass function (e.g Sheth & Tormen)

Mass - concentration relationship

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Available power matter recipes

HALOFIT (Smith et al. 2003, Takahashi et al. 2012)
formula based on Halo model, fit on N-body

1. Smith et al. 2003, w=-1 models, only 20-30% precision
2. Takahashi et al. 2012, w=const models, 5-10% precision
   expanded to include:
   (a) massive neutrino (Bird 2012)
   (b) f(R) models (Zhao 2014)

COSMIC EMU (Heitmann et al. 2014)
interpolation of N-body Coyote simulations, w=const models, high accuracy, cosmo parameter range limited, approx 1% precision.

used by WMAP also for $w = \mathrm{const}$ and $w = w_0 + w_a(1 - a)$

used by Planck also for $w = w_0 + w_a(1 - a)$ models

PKequal algorithm

Tarjet model $\mathcal{M}$

DE model with (w_0, w_a)

$w_{eq} = ?$

$\sigma_{8,eq} = ?$

Auxiliar model $\mathcal{M}_{eq}$

DE model with $w_{eq} = \text{const}$

$\sigma_{8,eq}$

Coyote emulator

Impose the following conditions

(1) Same distance to the LSS

$$\rightarrow \int_z^{z_{lss}} \frac{dz'}{E_{\mathcal{M}_{eq}}(z')} = \int_z^{z_{lss}} \frac{dz'}{E_{\mathcal{M}}(z')}$$

$$E^2_{\mathcal{M}} = \Omega_m(1+z)^3 + \Omega_r(1+z)^4 + (1-\Omega_m-\Omega_r) f_{\mathcal{M}},$$

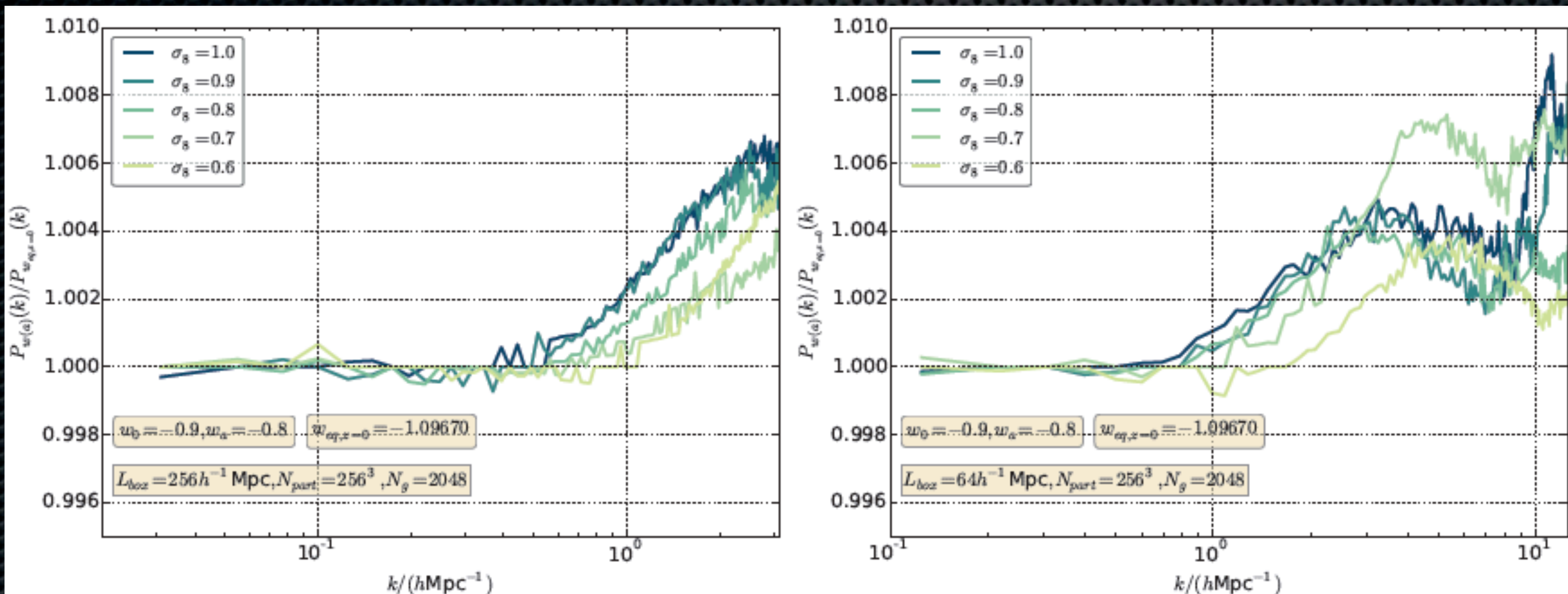
$$f_{\mathcal{M}} = \begin{cases} (1+z)^{3(1+w_{eq})} \\ \text{specific DE model} \end{cases}$$

(2) Amplitude of the density fluctuations at the scale of 8 Mpc/h is the same in both models

$$\sigma_{8,eq} \frac{D_{\mathcal{M}_{eq}}(z)}{D_{\mathcal{M}_{eq}}(0)} = \sigma_8 \frac{D_{\mathcal{M}}(z)}{D_{\mathcal{M}}(0)}$$

For a set $(w_0, w_a) \rightarrow$ unique value of $(w_{eq}, \sigma_{8,eq})$

varying $\sigma_8 : 0.6 < \sigma_8 < 1.0$



Left: $L = 256\mathrm{Mpch}^{-1}$. Right: $L = 64\mathrm{Mpch}^{-1}$, both $z = 0$.

L.Casarini, et-al 2016

Step 2. Treatment of the linear perturbations

(a) Solve the perturbative equations for different modes

https://github.com/celia-escamilla-rivera/DE_perturbations

(b) Modify CAMB to implement PPF (Parametrized Post-Friedmann)

http://camb.info/ppf/

In order to avoid the crossing instability problem at the phantom divided line, i.e. w=-1.

Linear perturbations

$$R_{\mu\nu} = 8\pi G \left(T^b_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T^b \right) + \phi_{,\mu}\phi_{,\nu} - g_{\mu\nu} V,$$

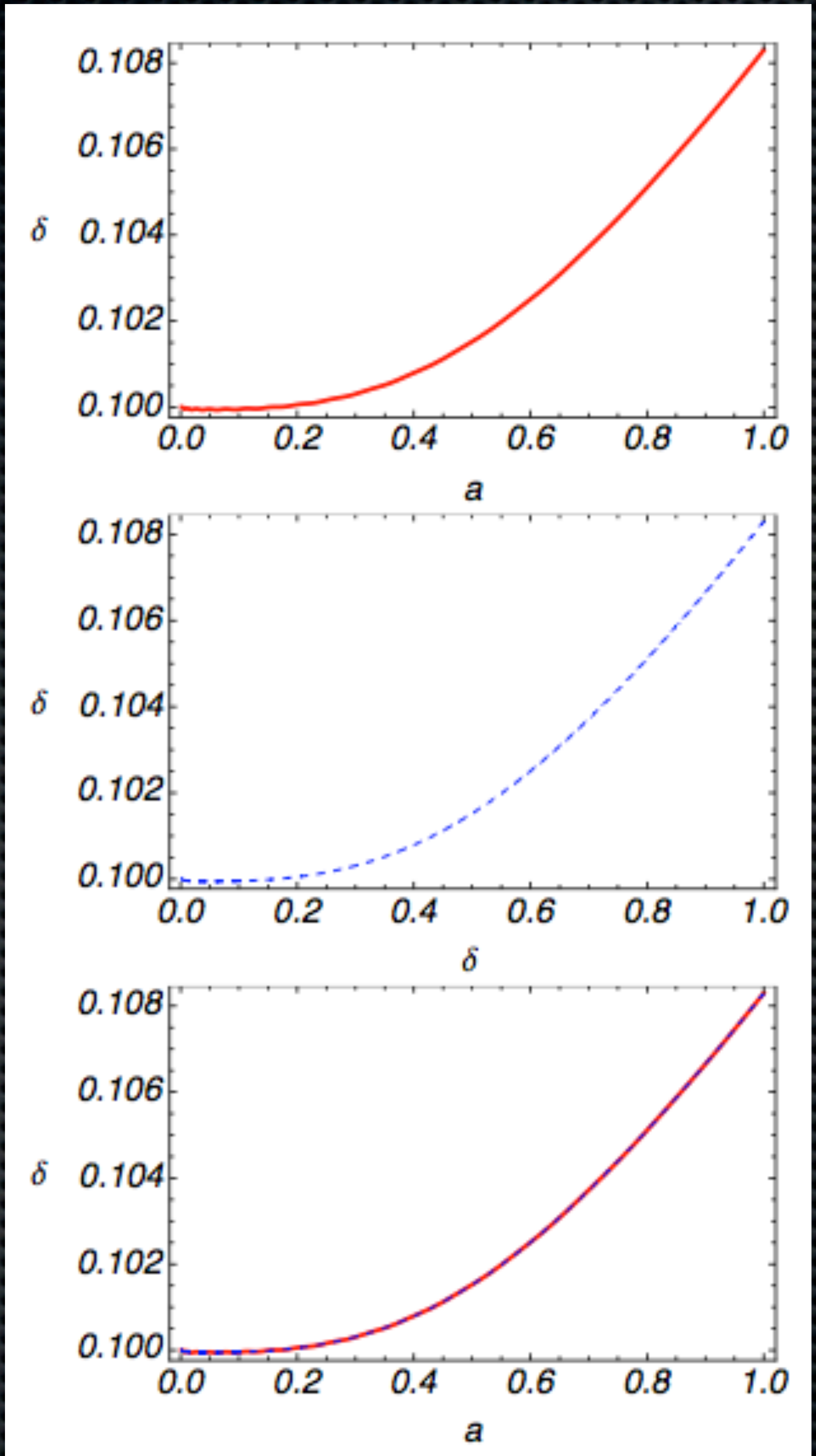
$$\delta R_{\mu\nu} = 8\pi G \left(\delta T^b_{\mu\nu} - \frac{1}{2} h_{\mu\nu} T^b - \frac{1}{2} g_{\mu\nu} \delta T^b \right) + (\delta\phi_{,\mu}\phi_{,\nu} + \phi_{,\mu}\delta\phi_{,\nu}) - h_{\mu\nu} V - g_{\mu\nu}\delta V.$$

$$\delta'' + \left(\frac{2}{a} + \frac{f'}{f} \right) \delta' - \frac{3}{2}\frac{\Omega}{f^2}\delta = 2\phi'\lambda' - \frac{V_\phi}{f^2}\lambda$$

$$\lambda'' + \left(\frac{3}{a} + \frac{f'}{f} \right) \lambda' + \left[\left(\frac{k l_0}{a} \right)^2 + V_{\phi\phi} \right] \frac{\lambda}{f^2} = \phi'\delta'$$

$$f \equiv \dot{a} \quad \lambda = \delta\phi \quad and \quad \phi' = \dot{\phi}/f.$$

e.j. CPL model



$k = 0.01$

$k = 1.0$

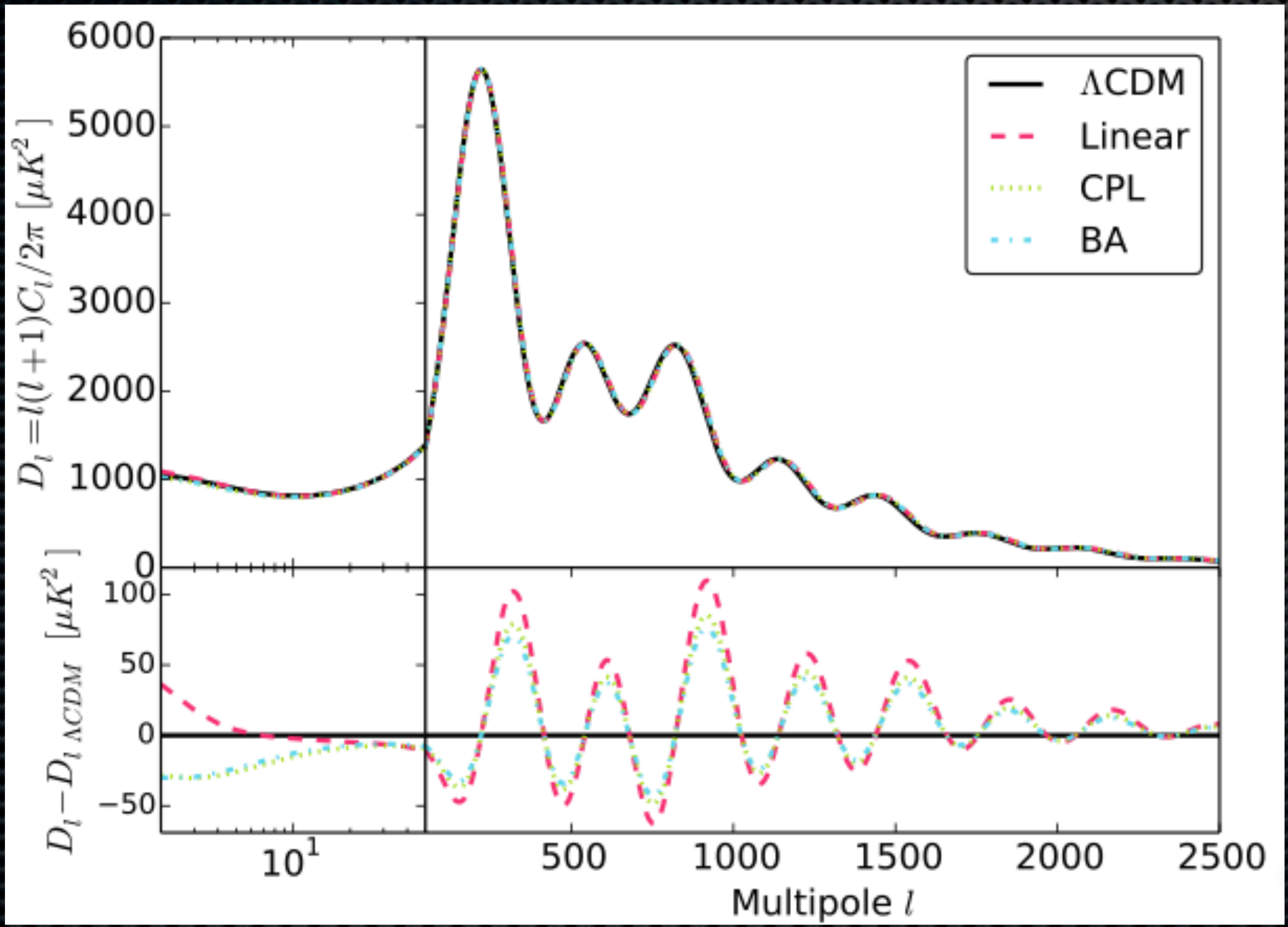
the $\delta(a)$ evolution is a scale invariant

C. Escamilla-Rivera, L.Casarini, J. Fabris and J.S. Alcaniz 1605:01475 (2016)

Linear perturbations (complete treatment)

$$\delta_i' + 3H(\hat{c}_{s,i}^2 - w_i)(\delta_i + 3H(1+w_i)v_i/k) + (1+w_i)kv_i = -3(1+w_i)h'$$

$$v_i' + H(1-3\hat{c}_{s,i}^2)v_i + kA = k\hat{c}_{s,i}^2\ \delta_i/(1+w_i)$$



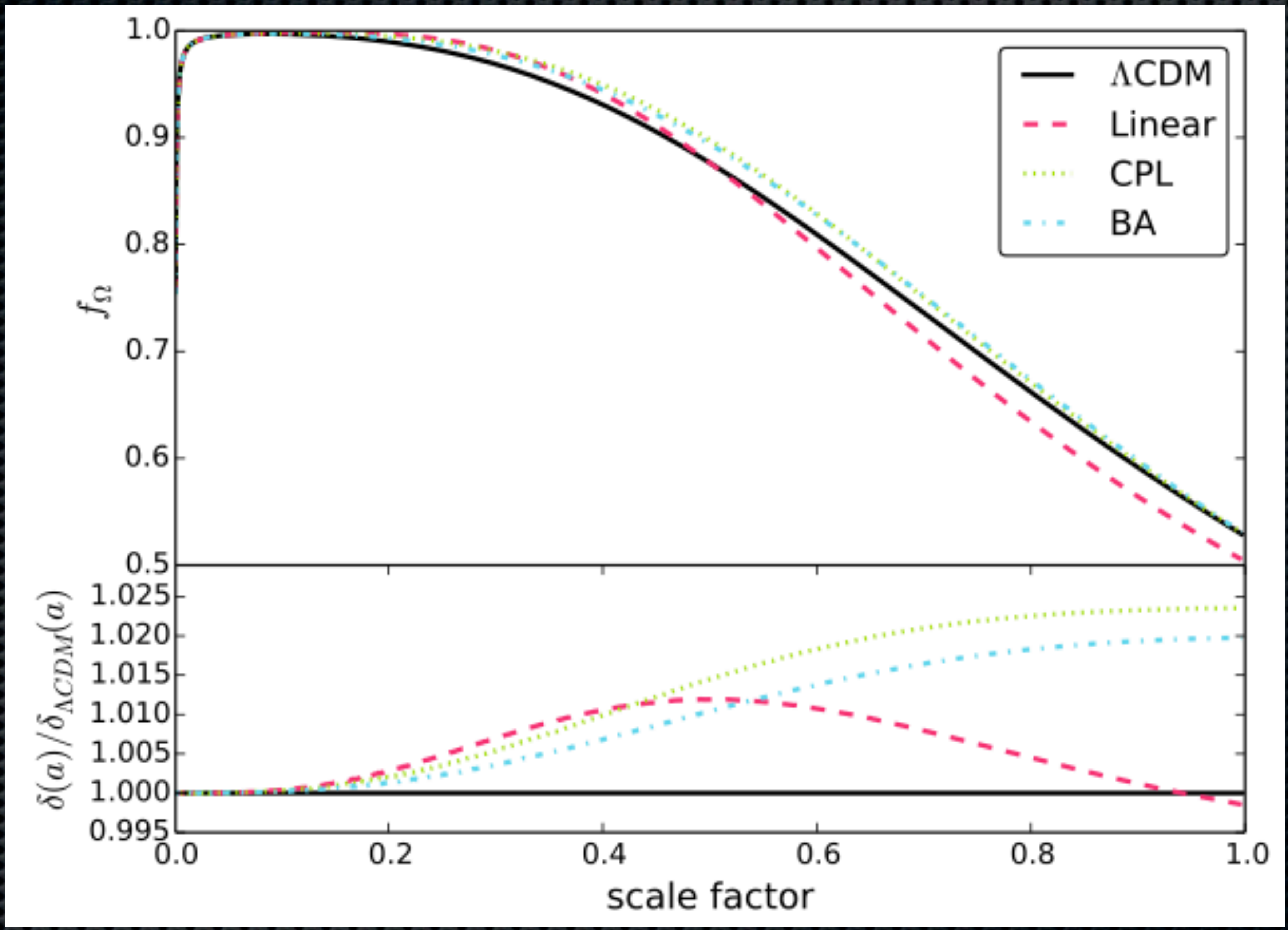
$ln(10^{10}A_s) = 3.09, \quad n_s = 0.966$

with $\quad k_0 = 0.05\mathrm{Mpc}^{-1}$

C. Escamilla-Rivera, L.Casarini, J. Fabris and J.S. Alcaniz 1605:01475 (2016)

Linear perturbations (complete treatment)

$$f_\Omega = \frac{d \ln \delta}{d \ln a}$$



$\approx 2.5\%$

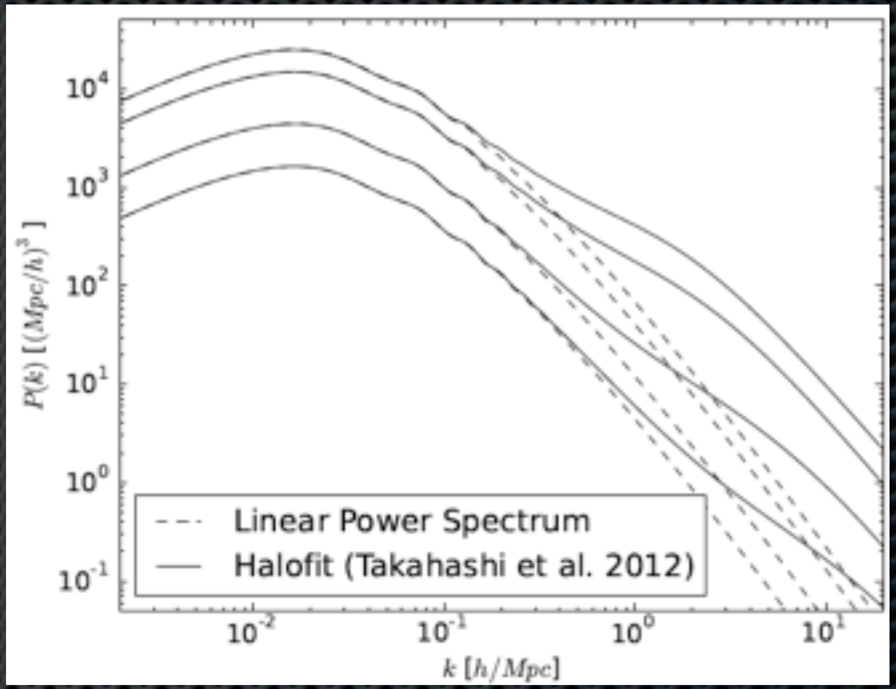
C. Escamilla-Rivera, L.Casarini, J. Fabris and J.S. Alcaniz
1605:01475 (2016)

Step 3. Estimate the non-linear perturbations

Smith et al. MNRAS 341 (2003)

(a) Extending the Halofit routine in Coyote suite.

Heitmann et al. ApJ 780,111 (2014)



(b) Introduction and modification of PKequal algorithm

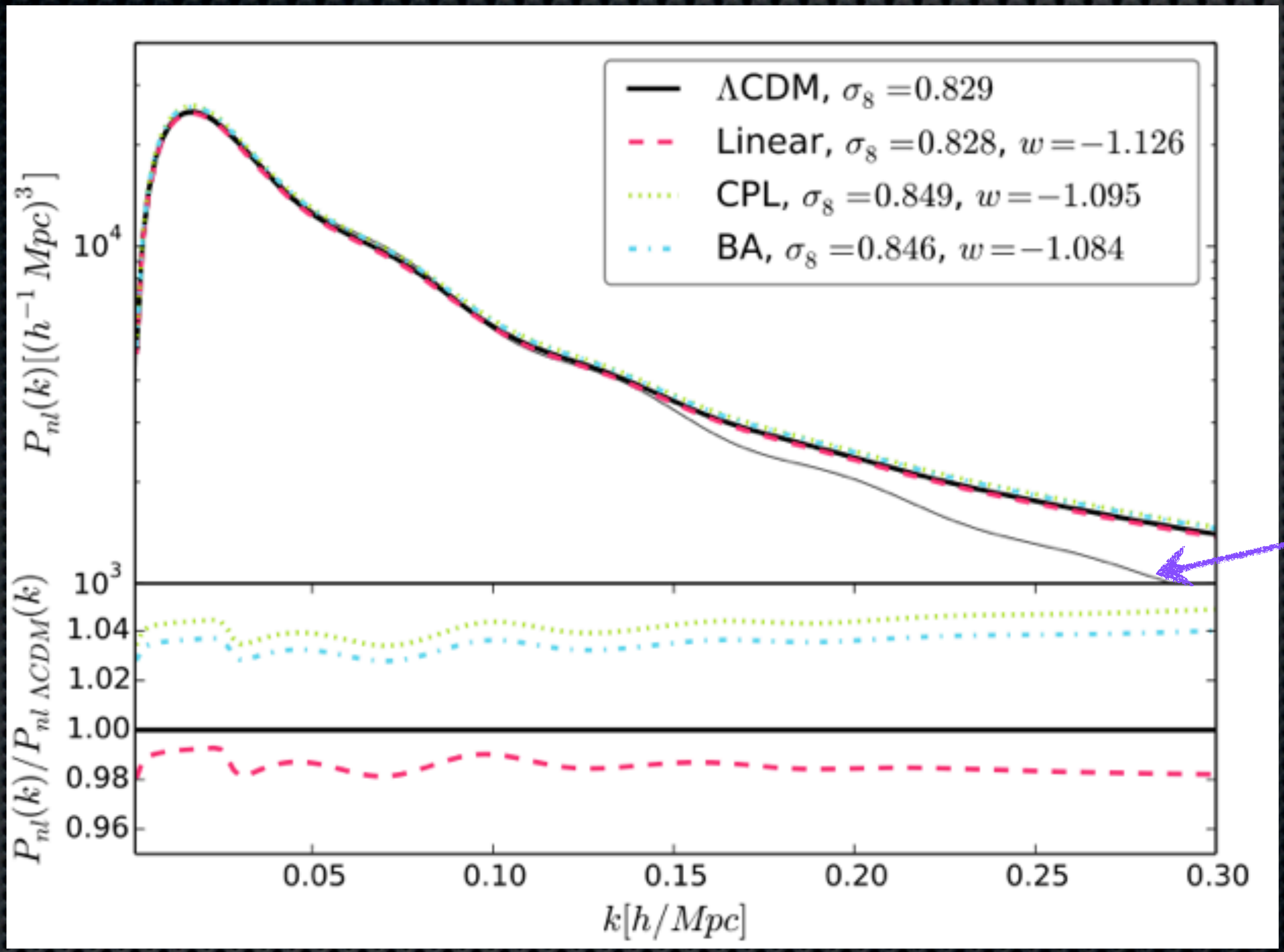
https://github.com/luciano-casarini/PKequal

L. Casarini et al 1601.07230v2

Allows to extend $w = \text{const}$ to $w = w(a)$

i.e $w_{eq} = \text{const}$ with the same $P(k)$

Non-linear perturbations



C. Escamilla-Rivera, L.Casarini, J. Fabris and J.S. Alcaniz
1605:01475 (2016)

For 3 dark energy models (Linear, CPL, BA):

- We compute the dynamics in a scalar field representation,
- We perform cosmological tests - Tension between data analysis,
- Complete treatment of linear perturbations: differences of the growth factors for extreme values of k are almost negligible,
- Non-linear contributions: matter power spectra is ‘almost’ the same for all scales of interest.

We expect the next generation of cosmology experiments to be able to distinguish between time-dependent w(z) and the standard LCDM cosmology.