A tensor instability in Eddington-Born-Infeld theory of gravity

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in collaboration with

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Outline

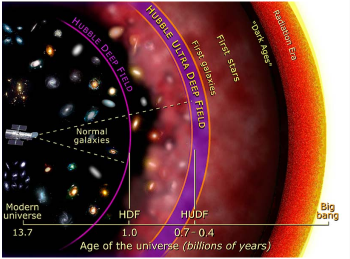
Things we know:

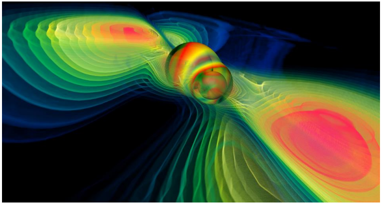
- Einstein's General Relativity (GR) has been extensively tested successfully in the solar system and beyond.
- In order to have a general view one needs to couple the vacuum theory to matter:
    - Assumes a minimal coupling,
    - solves the full Einstein equations and
    - finally, compares theoretical models with observations.

There are some problems:

- The way in which we coupling gravity-matter, e.g.
    - Dark Matter problem $\rightarrow$ Invoking new fundamental interactions rather than assuming exotic particle.
    - Dark Energy problem $\rightarrow$ (Cosmological acceleration) explained in term of more complicated interactions rather than postulating the existence of DE.

- Dynamical evolution of matter fields in GR $\rightarrow$ Formation of singularities seems unavoidable:
- Big Bang singularity
- Black Holes





- Not possible to perform measurements within GR on such singularities $\rightarrow$ We need an extending theory.

- A good candidate $\rightarrow$ Eddington inspired Born-Infeld Gravity (EiBI) [1]

[1] M. Banados, P. G. Ferreira, Phys. Rev. Lett. **105**, 011101 (2010).

Ingredients

- An Eddington action S_{Edd},

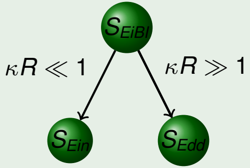
$$S_{Edd} = 2\kappa \int d^4x \sqrt{|R|}, \quad \text{where} \quad R_{\mu\nu} = R_{\mu\nu}(\Gamma). \qquad (1)$$

Born-Infeld style + matter in the recipe

- Gravitational action

$$S_{EBI}[g, \Gamma, \Psi] = \frac{2}{\kappa} \int d^4x \left[\sqrt{|g_{\mu\nu} + \kappa R_{\mu\nu}(\Gamma)|} - \lambda \sqrt{g} \right] + S_m[g, \Psi]. \qquad (2)$$

EiBI Theory with $S_{Mat} = 0$



Field Equations

- Varying with respect to $g_{\mu\nu}$:

$$\sqrt{\left|\frac{q}{g}\right|}(q^{-1})^{\mu\nu} - \lambda g^{\mu\nu} = -\kappa T^{\mu\nu}. \tag{3}$$

- Varying with respect to Γ:

$$q_{\mu\nu} = g_{\mu\nu} + \kappa R_{\mu\nu}. \tag{4}$$

- The auxiliary metric is compatible with the connection:

$$\Gamma^{\mu}_{\alpha\beta} = \frac{1}{2} q^{\mu\sigma} \left(q_{\sigma\alpha,\beta} + q_{\sigma\beta,\alpha} - q_{\alpha\beta,\sigma} \right). \tag{5}$$

- Eqs. (3) and (4) form a complete set of the theory.

Novel aspects

- In stars, compact objects and black holes $\rightarrow$ Eddington regime lead:
  - the avoidance of singularities,
  - significant modifications of the standard stellar astrophysics,
  - and in the very early universe $\rightarrow$ exist a minimum scale to avoid its collapse.
- Test of Eddington corrections to Newtonian gravity (Solar physics)[a] and around rotating sources[b].
- Existence of gravitational objects $\rightarrow$ constraints on the free parameter of the theory[c].
- Re-expression of the theory as a bigravity theory[d].

[a] J. Casanellas, P. Pani, I. Lopes, V. Cardoso, (2012).
[b] P. Pani, E. Berti, V. Cardoso, J. Read, (2011).
[c] P. Avelino, (2012).
[d] T. Delsate, J. Steinhoff, (2012). M. Banados, A. Gomberoff, D. Rodrigues and C. Skordis, (2009).

Objective

- Study the cosmological behaviour of the Eddington regime $\rightarrow$ analysis of the structure and evolution of linear tensor mode perturbations or gravitational waves[a].

[a] C. Escamilla-Rivera, M. Bañados, P. G. Ferreira, (2012).

We now split the dynamics of the problem in the background FLRW dynamics and their fluctuations about the background

$$g_{\mu\nu}dx^\mu dx^\nu = a^2\left[-d\eta^2 + (\delta_{ij} + h_{ij})\right] dx^i dx^j, \tag{6}$$

$$q_{\mu\nu}dx^\mu dx^\nu = -X^2 d\eta^2 + Y^2\left(\delta_{ij} + \gamma_{ij}\right) dx^i dx^j, \tag{7}$$

Setting $h_{ii} = \gamma_{ii} = 0$, we can take the energy momentum tensor as $T^{\mu\nu} = \rho + a^{-2}(\delta^{ij} + h^{ij})P$.

Background Equations

$$-\frac{|XY^3|}{X^2 a^4} + \frac{\lambda}{a^2} = -\kappa\frac{\rho}{a^2}, \tag{8}$$

$$\frac{|XY^3|}{Y^2 a^4} - \frac{\lambda}{a^2} = -\kappa\frac{P}{a^2}, \tag{9}$$

Tensor-Perturbed Equations

We start by taking the perturbed elements of the metrics (6), (7) and the energy-momentum tensor and construct the perturbed field equation for (3)

$$
\begin{aligned}
-\frac{XY^3}{a^4}\frac{1}{Y^2}\gamma^{ij} + \frac{\lambda}{a^2}h^{ij} &= \kappa\frac{P}{a^2}h^{ij}, \\
&= \left(-\frac{XY^3}{a^4Y^2} + \frac{\lambda}{a^2}\right)h^{ij} \quad \rightarrow \quad \boxed{\gamma_{ij} = h_{ij}}
\end{aligned}
\qquad (10)
$$

Perturbed Equation

$$
h''_{ij} + \left(3\frac{Y'}{Y} - \frac{X'}{X}\right)h'_{ij} + \left(\frac{X}{Y}\right)^2 k^2 h_{ij} = 0. \qquad (11)
$$

How the system evolves in different regimes?

- Einstein regime.
  - Indistinguible from Einstein gravity when $X = Y = a$.
  - Solution in the radiation era: [2]

$$h_{ij} \propto \frac{1}{\sqrt{\eta}}\mathcal{H}^{(1)}_{\frac{1}{2}}(k\eta), \frac{1}{\sqrt{\eta}}\mathcal{H}^{(2)}_{\frac{1}{2}}(k\eta) \tag{12}$$

where for

  - Late times: $k\eta \rightarrow$ Decaying oscillatory solutions
  - Early times:

$$h_{ij} = \frac{2}{\sqrt{\eta}} J_{\frac{1}{2}}(k\eta).$$

[2] J. E. Lidsey, A. R. Liddle, E. W. Kolb, E. J. Copeland, T. Barreiro and M. Abney, (1997). A. R. Liddle, D. H. Lyth, (2000).

- Eddington regime. Let us suppose the following approximation[3] for the case $\kappa > 0$

$$\begin{aligned}
\bar{a} &\equiv \frac{a}{a_B} = 1 + e^{\left[\sqrt{\frac{8}{3\kappa}}(t-t_0)\right]}, \\
V &\equiv \left(\frac{Y}{a}\right)^2 = \sqrt{2}e^{\left[\frac{1}{2}\sqrt{\frac{8}{3\kappa}}(t-t_0)\right]}, \\
U &\equiv \left(\frac{X}{a}\right)^2 = \frac{V^3}{2},
\end{aligned}$$

where we assume that $q_{00} = -U$ and $q_{ij} = a^2 V\delta_{ij}$.

- Existence of $a_B \rightarrow$ Non-singular behavior $\rightarrow$ Einstein static universe.
- Auxiliary metric become singular as $t \rightarrow -\infty$. (Evolution of the tensor modes).

[3] J. Scargill, M. Bañados and P. G. Ferreira. (2012).

In conformal time:

$$\left(\frac{X}{Y}\right)^2 = \frac{e^{(\alpha\Delta\eta)}}{1-e^{(\alpha\Delta\eta)}} \quad \text{with} \quad \alpha = a_B\sqrt{8/(3\kappa)}$$

$$\left(3\frac{Y'}{Y} - \frac{X'}{X}\right) = \partial_\eta \ln(Y^3/X) = \partial_\eta \ln a^2\sqrt{V^3/U}$$

$$= 2\frac{a'}{a} + 2\alpha\frac{e^{(\alpha\Delta\eta)}}{1-e^{(\alpha\Delta\eta)}}$$

Evolution equation for the tensor mode ($\kappa > 0$)

$$h''_{ij} + 2\alpha\frac{e^{(\alpha\Delta\eta)}}{[1-e^{(\alpha\Delta\eta)}]}h'_{ij} + \frac{e^{(\alpha\Delta\eta)}}{[1-e^{(\alpha\Delta\eta)}]}k^2 h_{ij} = 0. \tag{13}$$

Consider $\Delta\eta \to -\infty$ then

$$h''_{ij} \simeq 0, \quad h_{ij} \propto A\eta + B. \tag{14}$$

- Now let us suppose the following approximation for the case $\kappa < 0$

$$
\begin{aligned}
a &= a_B\left(1 + \frac{2}{3|\kappa|}t^2\right) \\
X^2 &= \frac{4}{3}a^2\sqrt{\frac{|\kappa|}{2}}\frac{1}{|t|} \\
Y^2 &= \frac{4}{3}a^2\sqrt{\frac{2}{|\kappa|}}|t|
\end{aligned}
$$

In conformal time:

$$
\begin{aligned}
a &= a_B[1 + \tan^2(\beta\eta)] \quad \text{with} \quad \beta = a_B\sqrt{2/(3|\kappa|)} \\
X^2 &= a^2\frac{4}{3^{3/2}}\frac{1}{|\tan(\beta\eta)|} \\
Y^2 &= a^2\frac{4}{3^{1/2}}|\tan(\beta\eta)|
\end{aligned}
$$

Taylor expansion around $\eta = 0$:

$$\frac{X^2}{Y^2} = \frac{U}{V} \simeq \frac{1}{3\beta^2\eta^2}$$

$$\left(3\frac{Y'}{Y} - \frac{X'}{X}\right) \simeq \frac{2}{\eta}$$

Evolution equation for the tensor mode ($\kappa < 0$)

$$h_{ij}'' + \frac{2}{\eta}h_{ij}' + \frac{k^2}{3\beta^2\eta^2}h_{ij} = 0 \tag{15}$$

with solution

$$h_{ij} \propto \eta^{\left[-\frac{1}{2}\pm\frac{1}{2}\sqrt{1-(4k^2/3\beta^2)}\right]}. \tag{16}$$

Summary

- We found an instability in Eddington regime ($\kappa > 0$) and at the bounce ($\kappa < 0$).
- The singular behavior is induced by the evolution of the homogeneous part of $q_{\mu\nu}$ (via X and Y) not present in Einstein gravity.
- These instabilities are present only in this particular form of the Eddington theory.
- Some work has been done to understand the process of gravitational collapse, in our study the tensor modes may play an unexpected role and can be included in these works.
- Our analysis gives the possibility of interesting effects in regions of density and curvature.