Homework 2, Math. Modeling and Consulting 553.400/600, Spr. 2023

Rules: You may discuss ideas with other students but your answers, work, and MATLAB coding should be done by you. **Upload to Gradescope your answers and MATLAB code**.

Problem 1: Write a MATLAB program that solves SUDOKU, given a partial SUDOKU table. The input is a 9×9 matrix M which is partially filled with integers from $1, 2, \dots, 9$ (and with 0's indicating the missing entries that need to be filled in). The output is the completed 9×9 SUDOKU table $\mathcal{M}$ which has as many entries as possible agreeing with M. (In particular, it is possible that M won't have a completed SUDOKU table that agrees with M always, and we want a "best" solution agreeing with M in as many places as possible. The output should always have every integer $1, 2, \dots, 9$ exactly once in every row, column, and mini tic-tac-toe.)

a) Run your code on the following table, which is a SUDOKU puzzle of Arto Inkala:

8								
		3	6					
	7			9		2		
	5				7			
				4	5	7		
			1				3	
		1					6	8
		8	5				1	
	9					4		

In other words, the input to your code is

$$M = \begin{bmatrix} 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 0 & 0 & 9 & 0 & 2 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 5 & 7 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 6 & 8 \\ 0 & 0 & 8 & 5 & 0 & 0 & 0 & 1 & 0 \\ 0 & 9 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \end{bmatrix}$$

and the output is the solution $\mathcal{M}$.

b) Next, run your code on the following (for which the optimal solution $\mathcal{M}$ doesn't agree with all entries of M, but matches it as best possible):

8								3
		3	6					
	7			9		2		
	5				7			
				4	5	7		
9			1			6	3	
		1					6	8
		8	5				1	
5	9			8		4		

The output is the solution $\mathcal{M}$ (in which each digit appears exactly once in every row, column, and micro tic-tac-toe and, given this, $\mathcal{M}$ agrees with M in as many entries as possible).

Problem 2: Suppose you run your code from Problem 1 on a discrete-uniform random M, say M is generated by the MATLAB command `>>M=ceil(9*rand(9,9));` . The number of places where $\mathcal{M}$ disagrees with M, this number divided by 81, will be called the *error ratio*. Do 1000 repetitions of the experiment (generate random M, compute error ratio) and report the mean and standard deviation for the error ratios of these experiments.

Problem 3: Expand your MATLAB code for Problem 1 to work for Sudoku tables of order 4; this means that the tables are 16-by-16, with each mini tic-tac-toe being 4-by-4, arranged in a macro 4-by-4 tic-tac-toe. All rows, columns, and mini tic-tac-toe have each integer $1, 2, 3, \ldots, 16$ exactly once. Use your MATLAB code to make any two different Sudoku tables. (Eg, run your code on a matrix M of all zeros, and then on a matrix M where you set one element to be different than the first output.)

Problem 4: Recall that the Sudoku conditions included the following.

$$\forall i = 1,2,3 \;\; \forall j = 1,2,3 \;\; \forall m = 1,2,\ldots,9 \quad \sum_{k=1}^{3}\sum_{\ell=1}^{3} x_{i,j,k,\ell,m} = 1$$

$$\forall i = 1,2,3 \;\; \forall k = 1,2,3 \;\; \forall m = 1,2,\ldots,9 \quad \sum_{j=1}^{3}\sum_{\ell=1}^{3} x_{i,j,k,\ell,m} = 1$$

$$\forall j = 1,2,3 \;\; \forall \ell = 1,2,3 \;\; \forall m = 1,2,\ldots,9 \quad \sum_{i=1}^{3}\sum_{k=1}^{3} x_{i,j,k,\ell,m} = 1$$

In your code for Problem 1, now include the following additional constraints:

$$\forall i = 1,2,3 \;\; \forall \ell = 1,2,3 \;\; \forall m = 1,2,\ldots,9 \quad \sum_{j=1}^{3}\sum_{k=1}^{3} x_{i,j,k,\ell,m} = 1$$

$$\forall j = 1,2,3 \;\; \forall k = 1,2,3 \;\; \forall m = 1,2,\ldots,9 \quad \sum_{i=1}^{3}\sum_{\ell=1}^{3} x_{i,j,k,\ell,m} = 1$$

$$\forall k = 1,2,3 \;\; \forall \ell = 1,2,3 \;\; \forall m = 1,2,\ldots,9 \quad \sum_{i=1}^{3}\sum_{j=1}^{3} x_{i,j,k,\ell,m} = 1$$

Now run your code with the partial matrix:

$$M = \begin{bmatrix} 1 & 2 & 3 & 6 & 4 & 5 & 0 & 0 & 0 \\ 4 & 5 & 6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 7 & 8 & 9 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and give the output $\mathcal{M}$.