



# Community detection in fine-grained dynamical networks

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Supervised by Rémy Cazabet

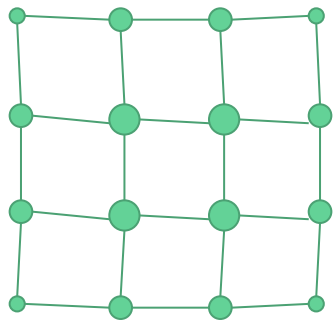
2020

# Outline

1. **Networks & communities**
2. Relevant algorithms
3. Static tests
4. Static method for dynamic results
5. Competitive random walks

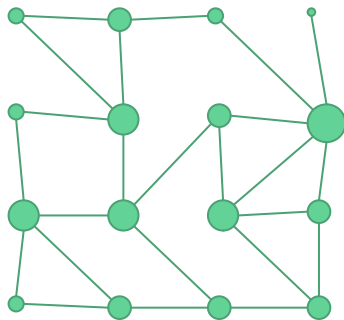


# From graphs to networks



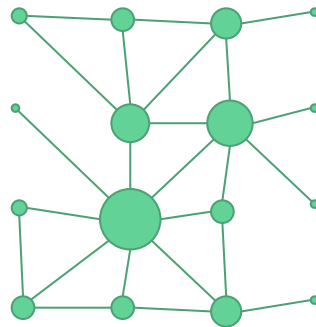
Regular graph

Structure  
High diameter  
Predefined degrees



Random graph

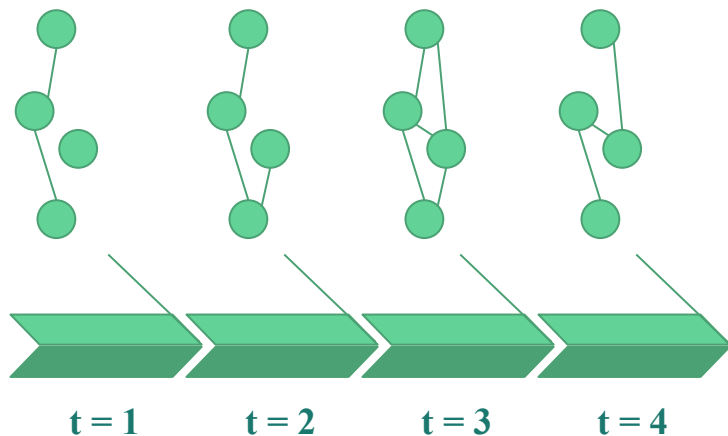
No structure  
Small diameter  
Poisson distribution



Real-world network

Hubs & clusters  
Small diameter  
Power-law distribution

# Dynamic networks



Snapshots

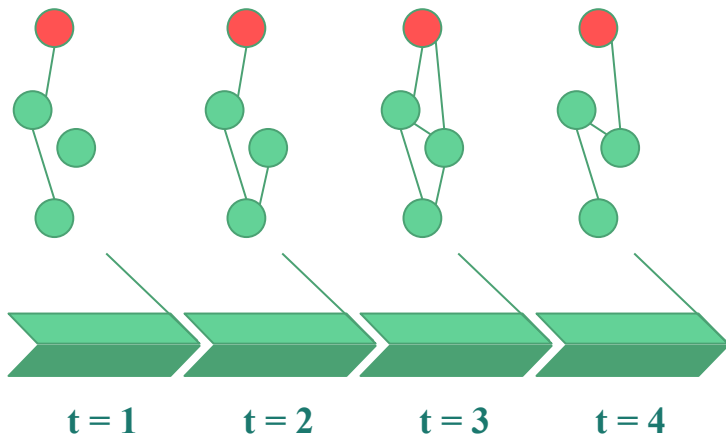
Network at each step  
Relations (eg friendships)



Link stream

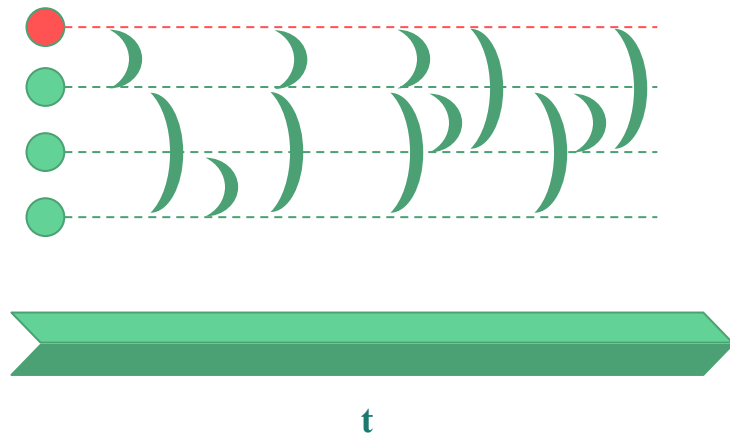
No network at a specific time  
Interactions (eg emails)

# Dynamic networks



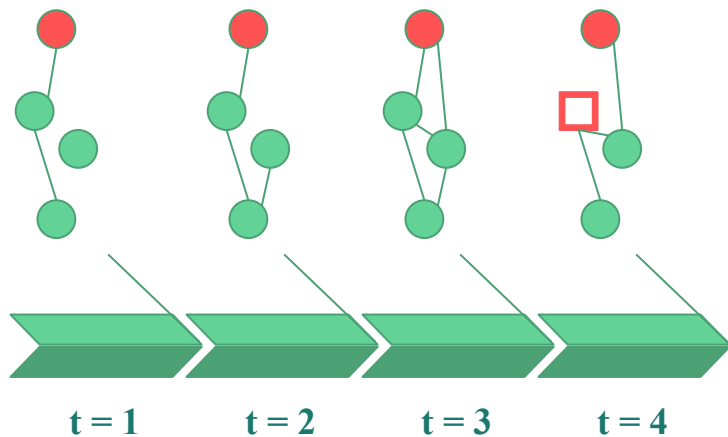
Snapshots

● node

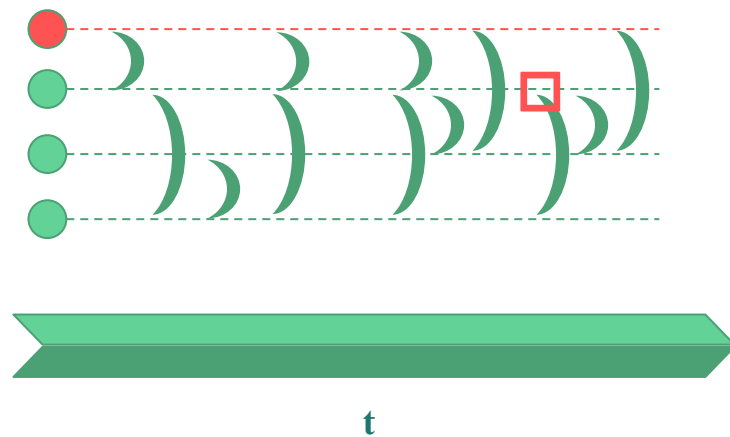


Link stream

# Dynamic networks



Snapshots

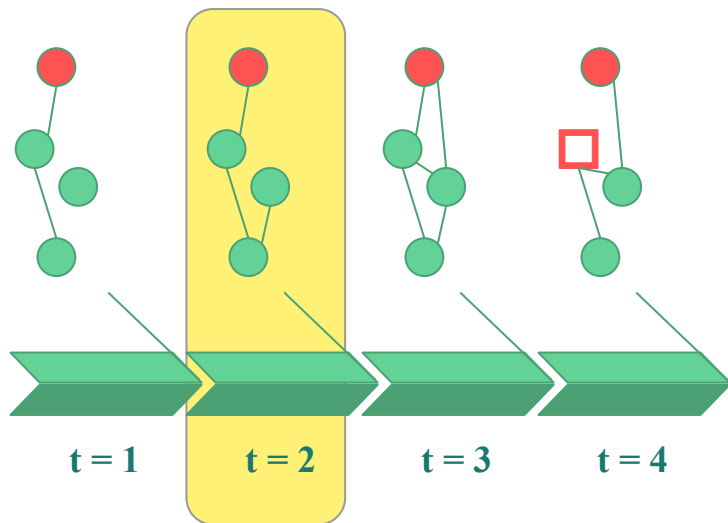


Link stream

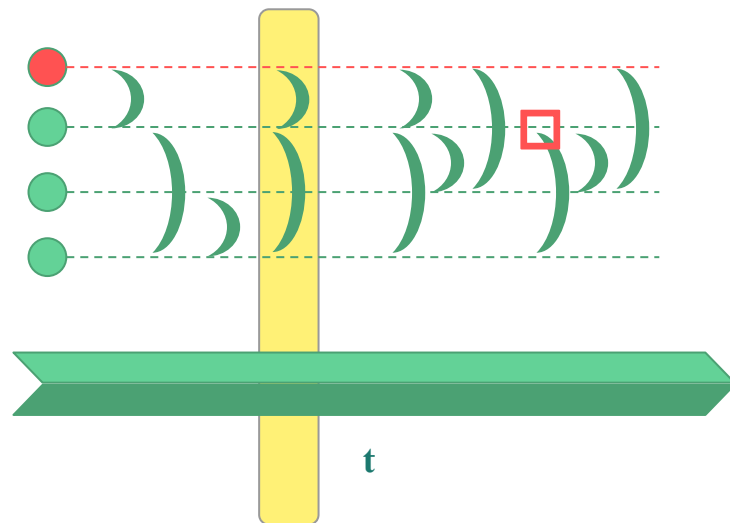
● node

□ tnode

# Dynamic networks



Snapshots



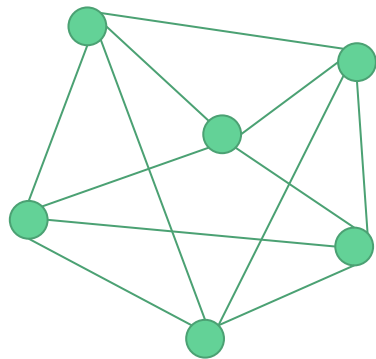
Link stream

● node

□ tnode

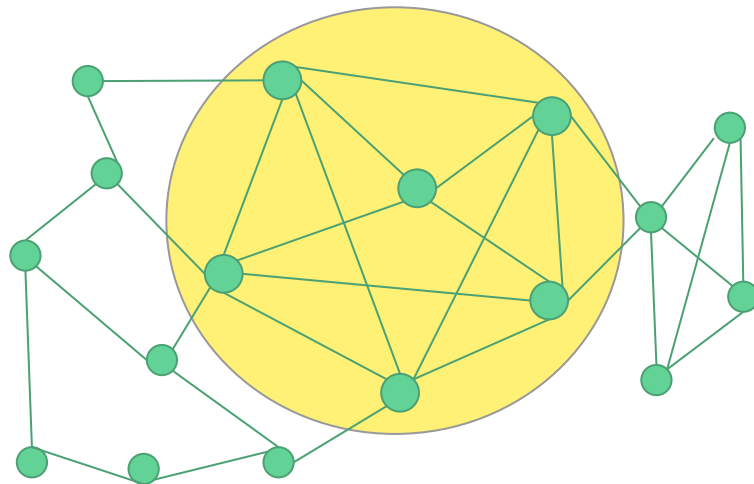
■ frame

# Communities



Dense subset...

at best: a clique

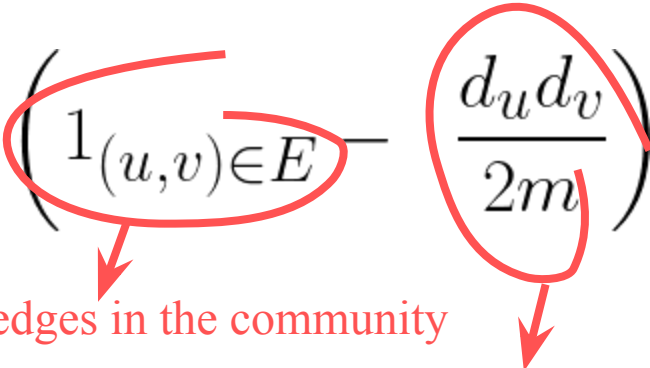


... with sparse cut

at best: isolated



## Quality functions: modularity

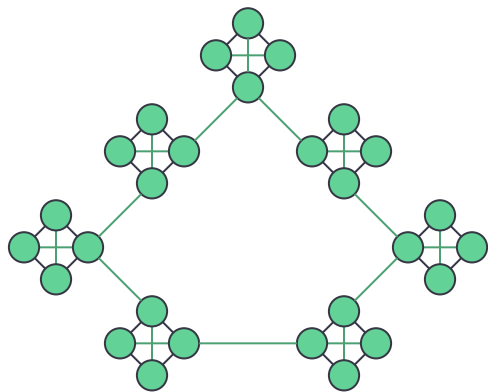
$$Q(C) = \frac{1}{2m} \sum_{u,v \in C} \left( 1_{(u,v) \in E} - \frac{d_u d_v}{2m} \right)$$


Existing edges in the community

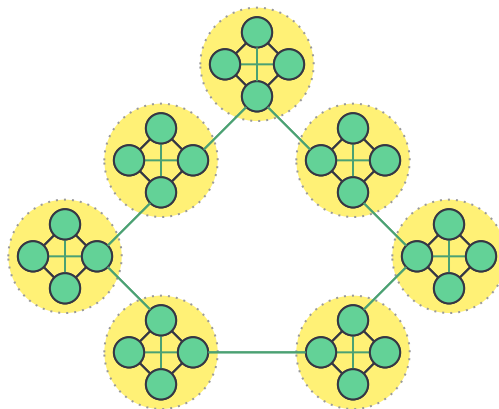
Expected edges in the configuration model

# Quality functions: modularity

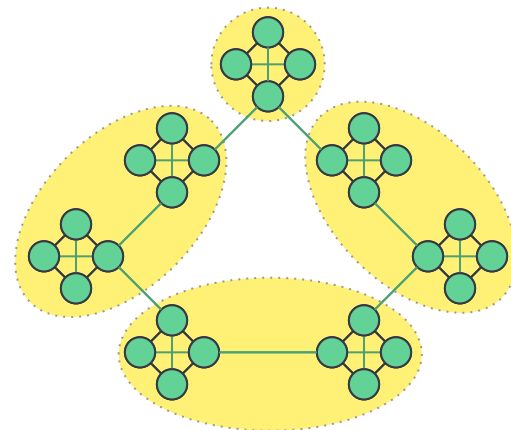
$$Q(C) = \frac{1}{2m} \sum_{u,v \in C} \left( 1_{(u,v) \in E} - \frac{d_u d_v}{2m} \right)$$



Ring of cliques



Intuitive communities

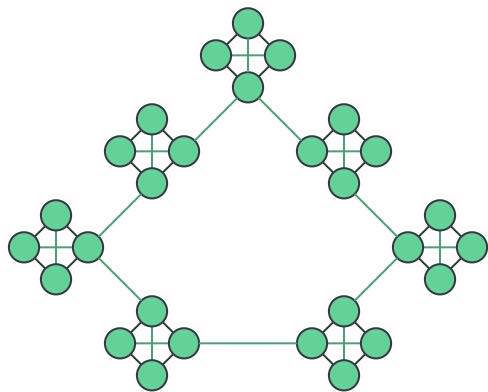


Resolution limit

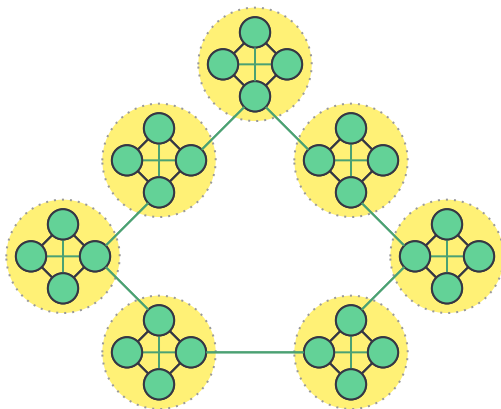
# Quality functions: modularity

$$Q(C) = \frac{1}{2m} \sum_{u,v \in C} \left( 1_{(u,v) \in E} - \lambda \frac{d_u d_v}{2m} \right)$$

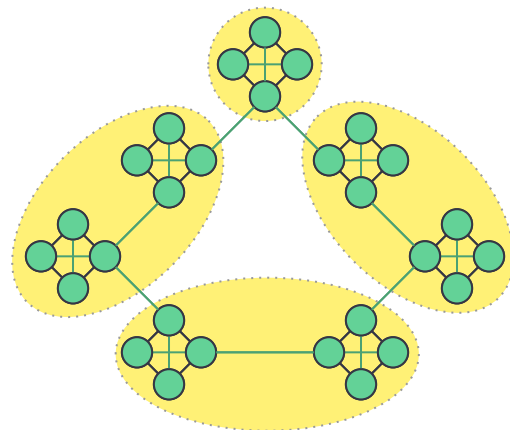
Resolution parameter



Ring of cliques



Intuitive communities



Resolution limit

# Quality functions: many more

Conductance

Surprise

Expansion

Clustering coefficient

Significance

Permanence

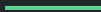
Separability



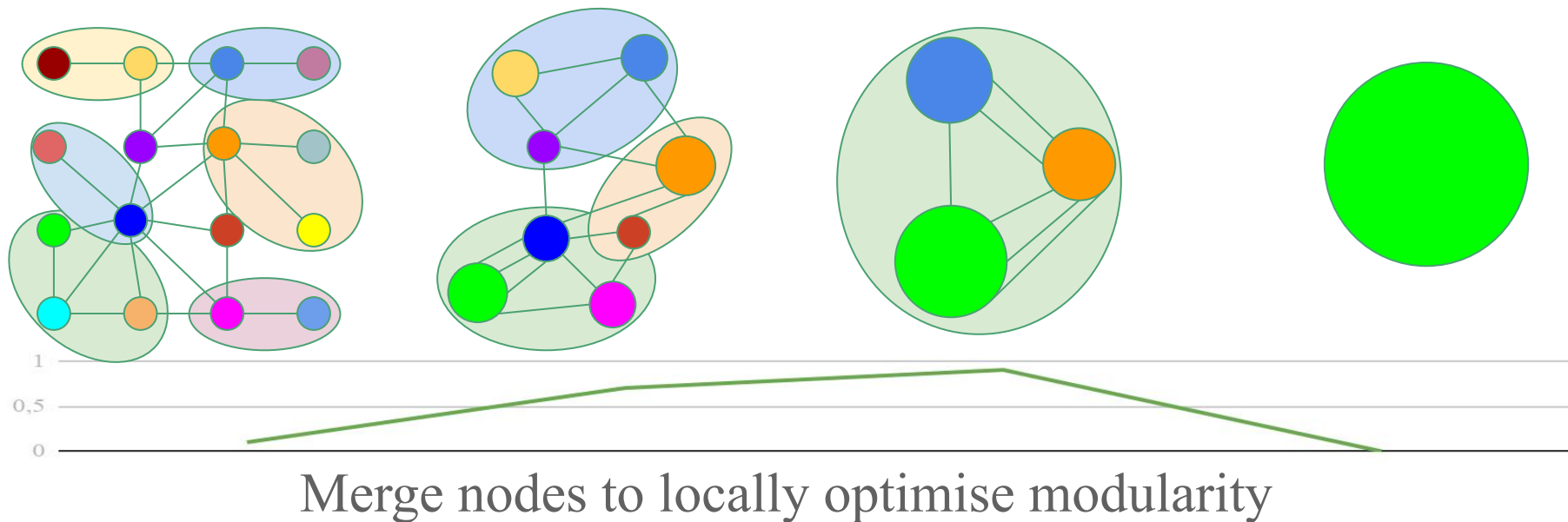
No unique standard definition

# Outline

1. Networks & communities
- 2. Relevant algorithms**
3. Static tests
4. Static method for dynamic results
5. Competitive random walks



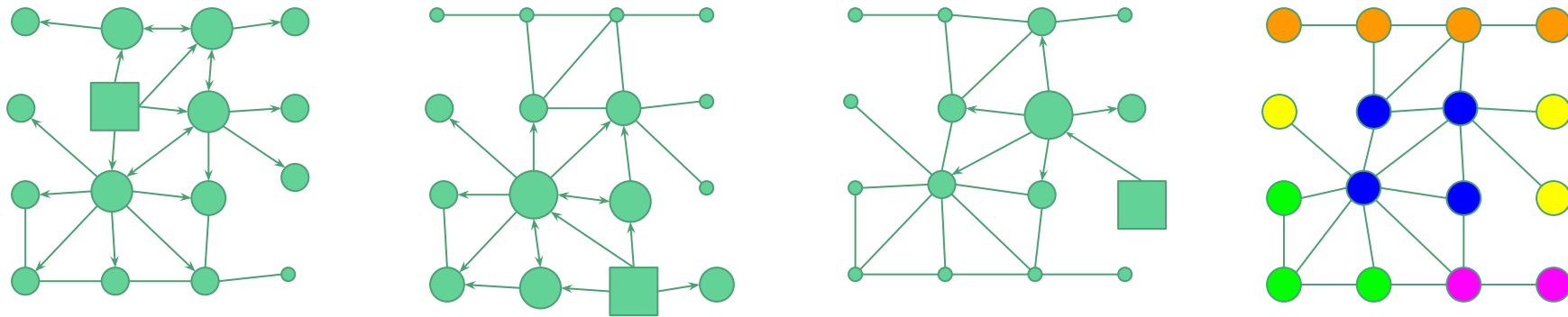
# Static algorithm: Louvain & Leiden



- + very fast (linear in  $m$ )
- + relevant results

- global modularity optimisation
- sparse communities

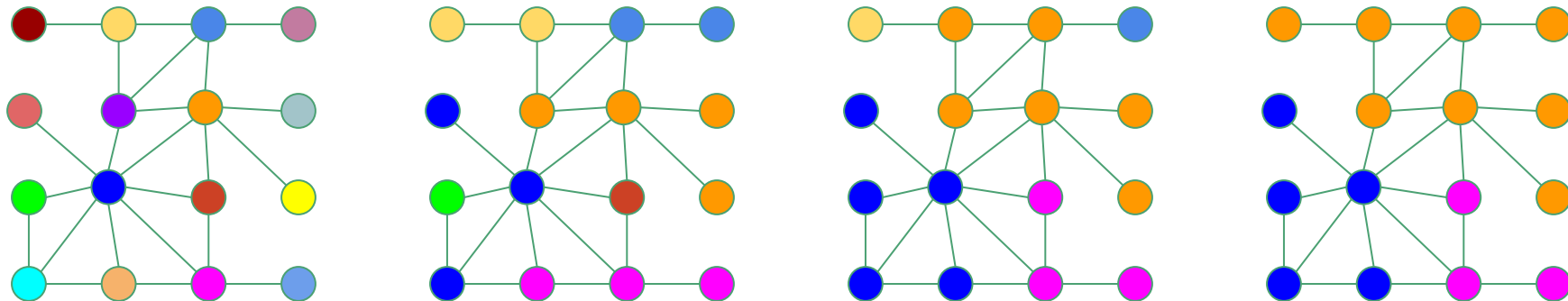
# Static algorithm: Walktrap



Merge nodes having similar results of short random walks

- + ideal for small diameters
- + local computation
- modularity optimisation
- sparsely connected communities

# Static algorithm: Label propagation



Update by taking neighbours' favourite label

- + matrix definition
- + local computation

- quadratic
- many different results



# Dynamic algorithms on snapshots

Aim: good instantaneous communities that evolve smoothly

- Dynamic Louvain: use degeneracy to smooth communities
- Multi-objective optimisation
- Detect and follow core nodes
- Match communities with similarity index

# Dynamic algorithms on link streams

Aim: local scalable algorithm with mathematical grounding

- Aggregate the stream on a sliding time-window
- Link tedges having consecutive tnodes
- Put a Gaussian weight around tedges
- Extend and prune an initial community

Problem: no unique definition in static — worse in dynamic

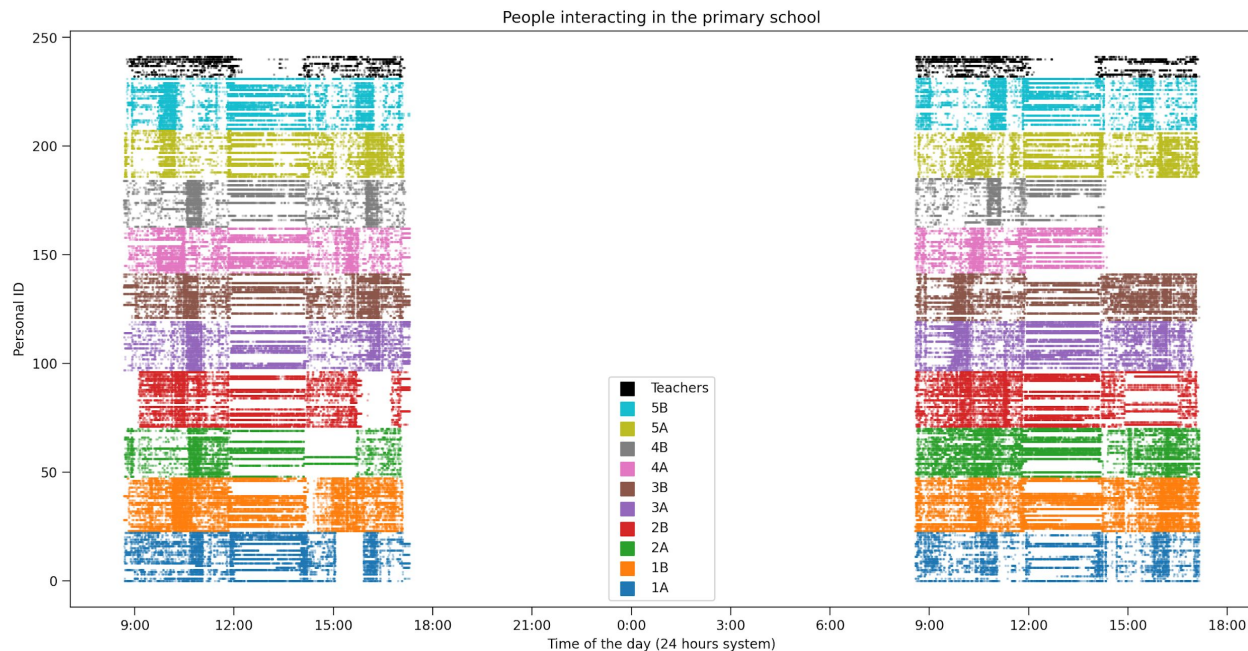
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# Link stream datasets: Sociopatterns

Interactions between people, recorded with RFID sensors



See also: hospital ward, traditional household, conference attendees, wild herd...

# Big available data

## Bitcoin

Blockchain is public by design

150 million transactions

7 years

175Gb of data

Track anonymous addresses

Communities may be recovered

## Twitter

Private but partially released

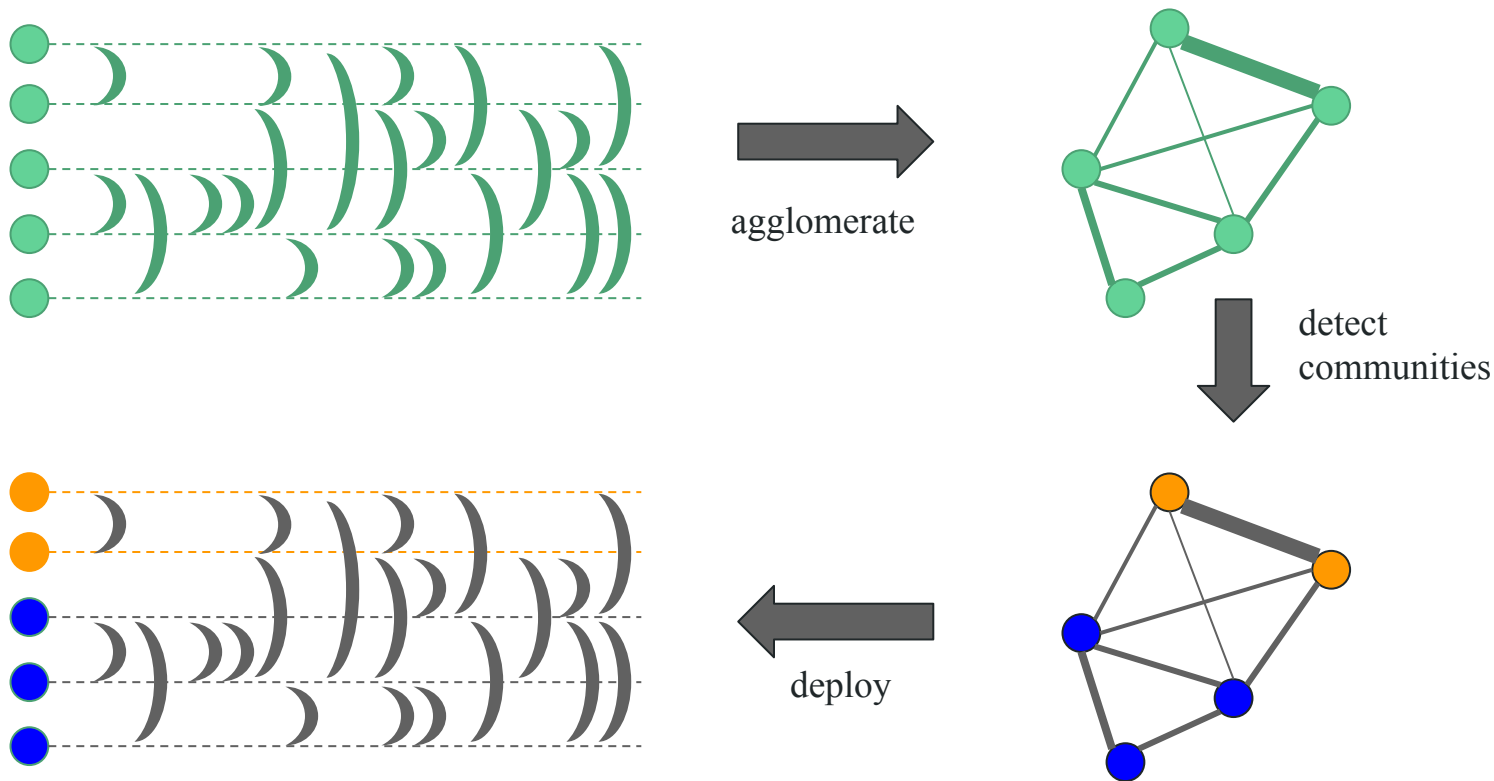
300 million users

500 million posts a day

Track posts of the same user

Communities may be recovered

# Contribution: test static algorithms on agglomerated graphs



# Contribution: test static algorithms on agglomerated graphs

Graph with 242 nodes and 2340 edges.

Method	#	aNMI%	Modul%	Conduc%	Surp	Stab%	Time (s)
groundtruth	11.0	100	68	33	2274	100	2.004000000965789e-06
louvain	7.2	83	72	18	2375	98	0.05116954990000124
leidenW	6.6	81	72	16	2432	96	0.04283884670000191
leiden	6.9	83	72	16	2535	99	0.040756389099999527
label	8.0	85	70	19	2468	100	0.015577493500001083
walktrap	8.0	86	72	18	2650	100	0.04235497800000019
infomap	8.0	87	72	18	2657	100	0.01987894130000143

Decent results compared to groundtruth

Quality functions are not the truth

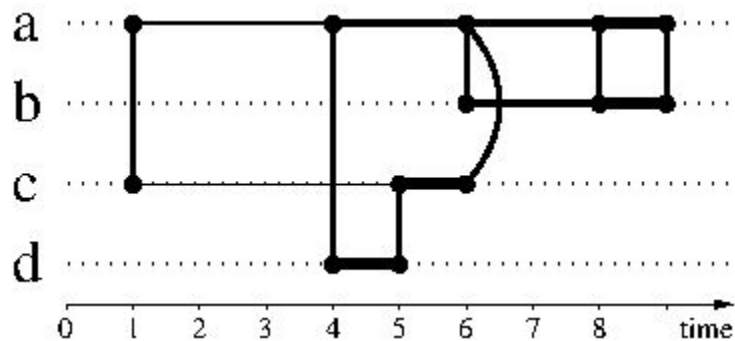
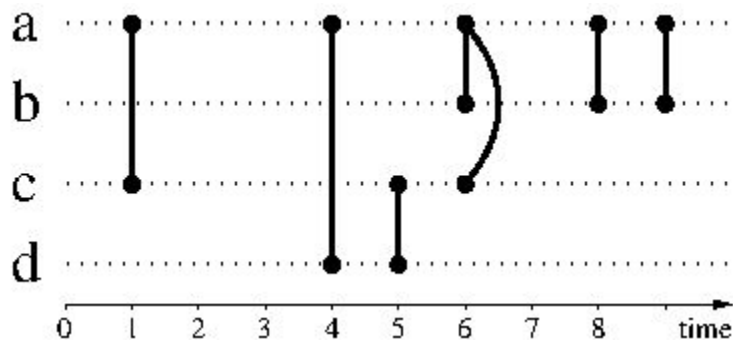
All very fast and stable on small networks

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dynamic results**
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-



# Contribution: topochrone graphs



Build chronological edges

Apply time decay

Detect communities using static algorithms

$$\omega_{(t,u),(t+\delta t,u)}^{\text{const}} = \alpha$$

$$\omega_{(t,u),(t+\delta t,u)}^{\text{lin}} = \frac{\alpha}{\frac{\delta t}{\tau} + 1}$$

$$\omega_{(t,u),(t+\delta t,u)}^{\text{exp}} = \alpha \exp -\frac{\delta t}{\tau}$$

# Contribution: topochrone graphs

175'000 nodes, 300'000 edges

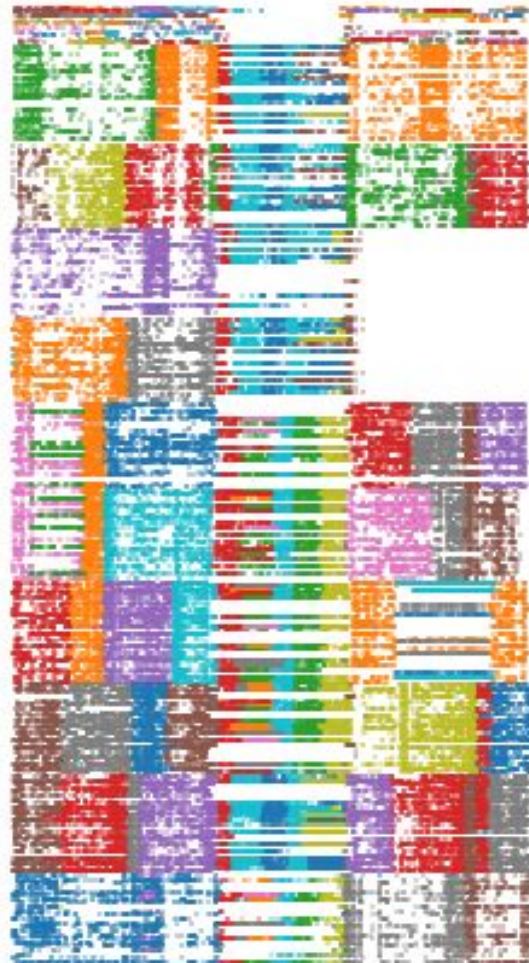
Label propagation is too slow (30 min)

Walktrap is 30x slower than Leiden (9 sec)

≈100 communities found

Pupils together during class time

School-wide blending during breaks



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# Contribution: competitive random walks

Ants of different species navigate:

- jump  $k$  times (topology)
- survive  $t$  frames (chronology)

Community accepts **remarkable** tnodes:

$$\mathcal{R}_{b|a} = \frac{|\mathcal{D}_a| \cdot p_{b|a}}{|\mathcal{D}| \cdot p_b} > 1$$

$p_{b|a}$  = portion of pheromones from species  $a$

$p_b$  = portion of total pheromones

**Input:** link stream  $L$ , depth  $k$ ,  
walk duration  $\tau$ , precision  $\pi$

**Output:** Non-overlapping farms

Create  $f = \pi \frac{m}{\tau}$  farms of 1 random tnode

**for**  $k$  times **do**

**for**  $\omega = \pi \frac{T \cdot n}{\tau}$  times **do**

        Choose  $F$  proportionally to  $|F|$

        Ant starts at random in  $F$

        Ant walks  $\tau$  frames and  $k$  jumps, laying  
        down  $F$ -pheromones

**end for**

    Tnodes choose most remarkable farm

    Delete small farms when  $|F| < \tau$

**end for**

# Contribution: competitive random walks

Classes during lessons are still discovered  
80 to 100 communities

Some nodes have no community  
Easy to find overlapping communities

Noisy results (nodes with few nodes)  
Unstable results (different at every run)



# Contribution: competitive random walks

Complexity:

- runs:  $k \approx 5$
- walkers:  $w = \text{tnodes} / t \approx 2m / t$
- walk duration:  $t$

Overall:  $O(k \cdot m)$

Assign tnodes to most remarkable community

- |  |                                       |
|--|---------------------------------------|
| + quite fast (linear in $m$ )          | – unstable and noisy                  |
| + detects several scales               | – based on emergence instead of maths |
| + overlapping / incomplete communities | – sensitivity to constant $t$         |

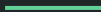
# Conclusion

Link streams = high precision temporal graphs.  
Community detection has standard algorithms for the static case; not for link streams.

Data of any size can be found.  
We propose 2 methods with linear complexity.

Further work:

- extensive tests
- mathematical grounding
- optimisation





Any question?

Fabrice Lécuyer  
Supervised by Rémy Cazabet

2020



# Synthetic datasets

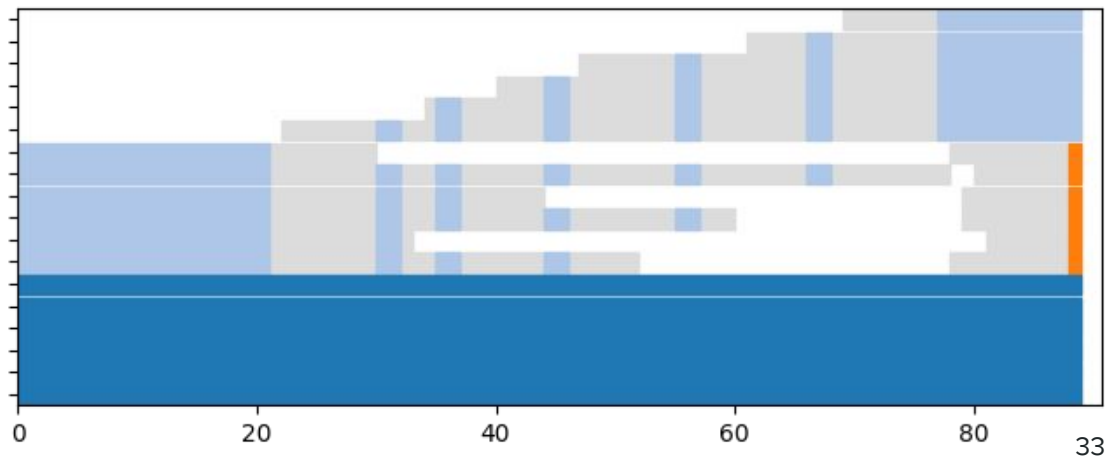
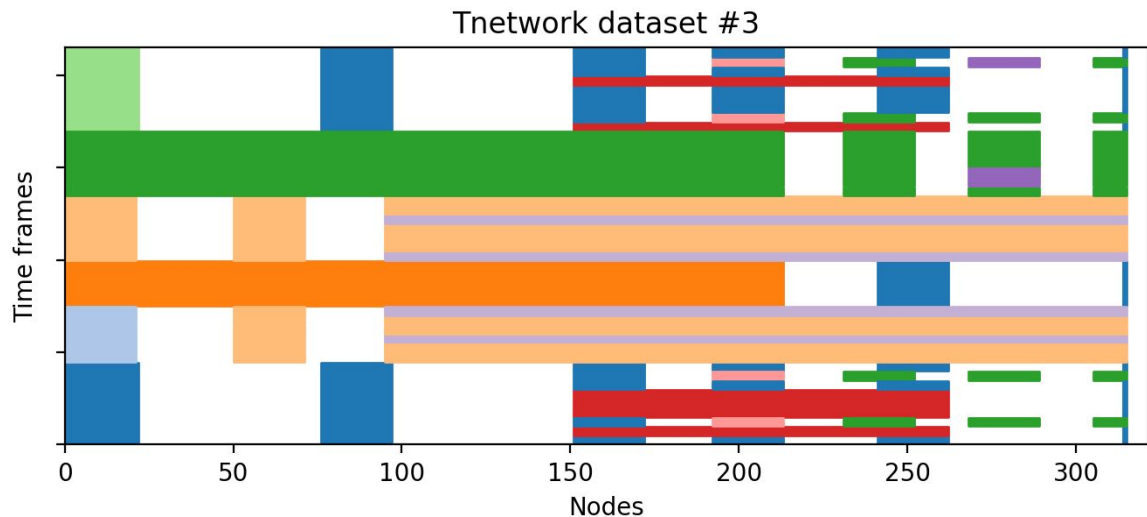
Python library *tnetwork*

Create predefined scenarios

Create random events

Test tricky situations

Compare average situations



## Quality functions: modularity

$$\mathcal{Q}(C) = \frac{1}{2m} \sum_{u,v \in C} \left( 1_{(u,v) \in E} - \frac{d_u d_v}{2m} \right)$$