

SYDE 556/750

Simulating Neurobiological Systems
Lecture 2: Neurons

Chris Eliasmith

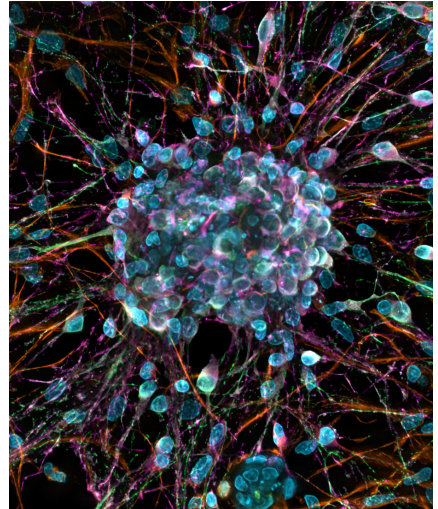
September 9, 2024

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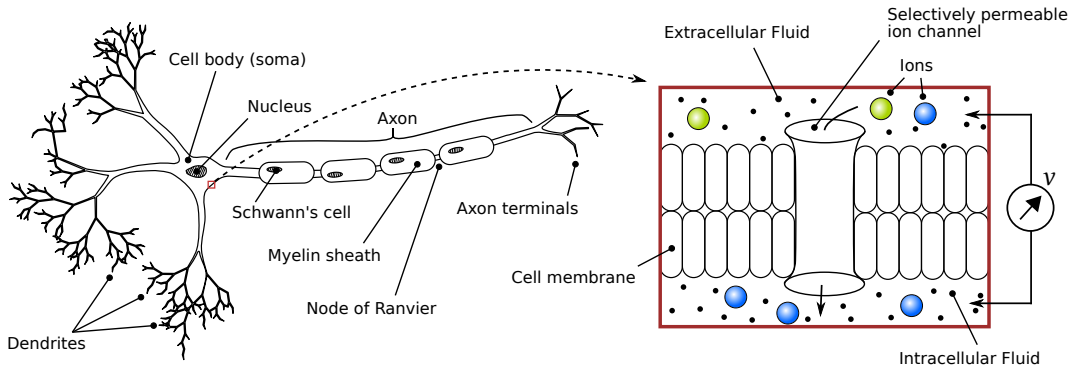


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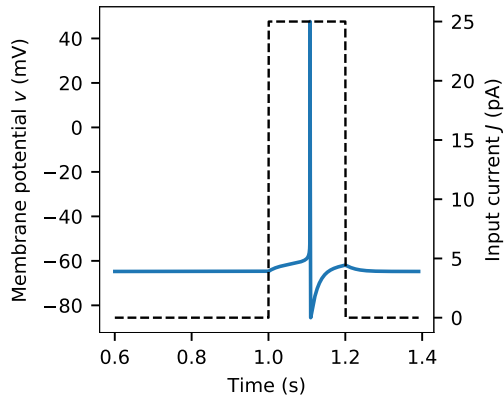
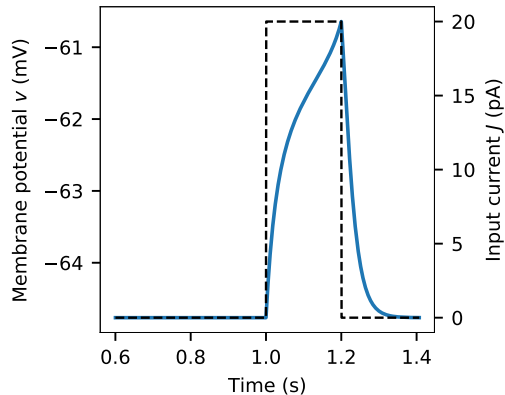
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Textbook Neuron and Cell Membrane



Injecting a Current Into a Detailed Neuron Model

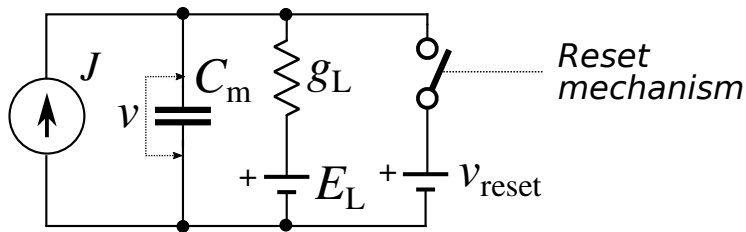


Computer simulation of an Hodgkin-Huxley type neuron with Traub kinematics (Roger D. Traub and Richard Miles, *Neuronal Networks of the Hippocampus*, Cambridge University Press, 1991)

Basic High-Level Details (Lapicque, 1907)

1. The cell acts like a *capacitor*, i.e., the voltage increases while we're injecting a current.
2. The capacitor is *leaky*. As soon as we stop injecting a current, the voltage collapses back to the resting potential E_L .
3. As soon as the voltage surpasses a certain value, the *threshold potential* v_{th} , the cell will generate a spike.
4. Shortly after the spike has been produced, the voltage drops below the resting potential. During this period, the *refractory period* of length τ_{ref} , we cannot get the neuron to spike again, even if we apply relatively large input currents J .

The Leaky Integrate-and-Fire Equivalent Circuit

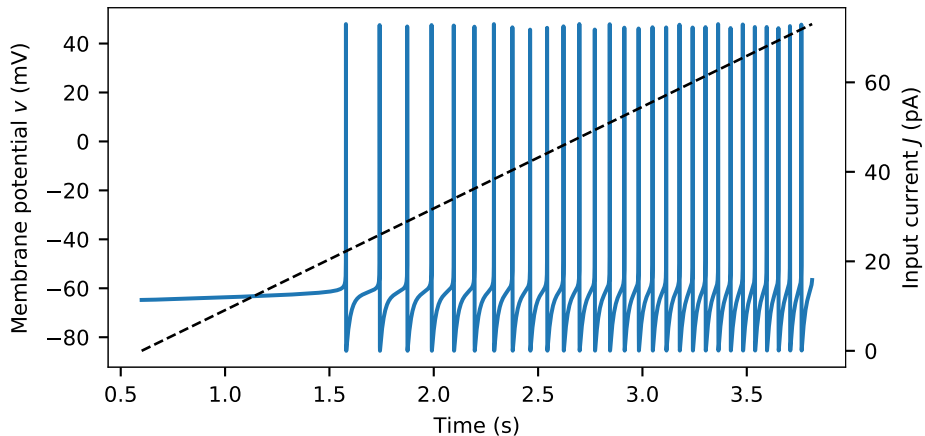


$$\frac{d}{dt}v(t) = \frac{1}{C_m} (g_L(E_L - v(t)) + J), \quad \text{if } v(t) < v_{\text{th}}.$$

if $v(t) = v_{\text{th}}$ at $t = t_{\text{th}}$, output a spike ($\delta(t - t_{\text{th}})$) and:

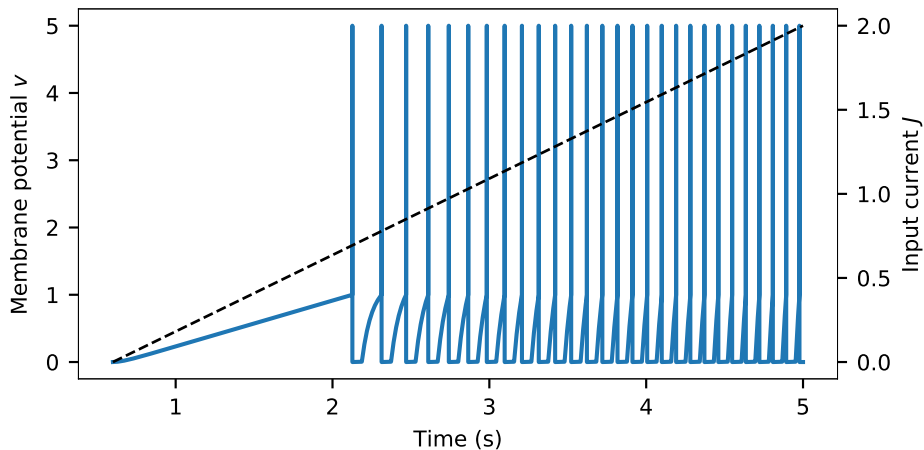
$$v(t) \leftarrow v_{\text{reset}}, \quad \text{if } t_{\text{th}} < t \leq t_{\text{th}} + \tau_{\text{ref}},$$

Injecting a Current Ramp into a Detailed Neuron Model



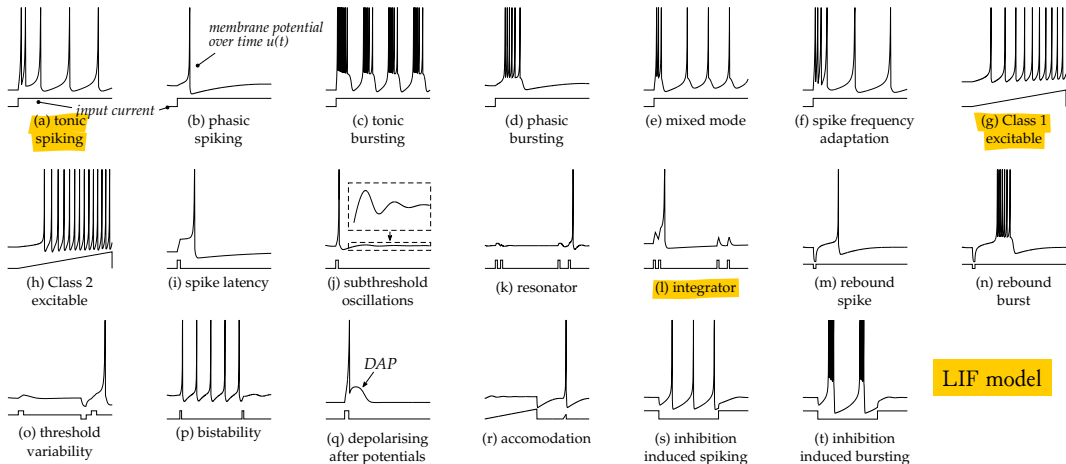
Computer simulation of an Hodgkin-Huxley type neuron with Traub kinematics (Roger D. Traub and Richard Miles, *Neuronal Networks of the Hippocampus*, Cambridge University Press, 1991)

Injecting a Current Ramp into a LIF Neuron Model



(note normalization to $v_{th} = 1$, $v_{reset} = E_L = 0$)

Limitations of the LIF Neuron Model



LIF Rate Approximation

- ▶ need to compute t_{th} (the time $v(t_{\text{th}}) = v_{\text{th}}$)
- ▶ assume: J is constant and $v(0) = 0$.
- ▶ also: $g_L = 1/R$ and $\tau_{RC} = RC_m$

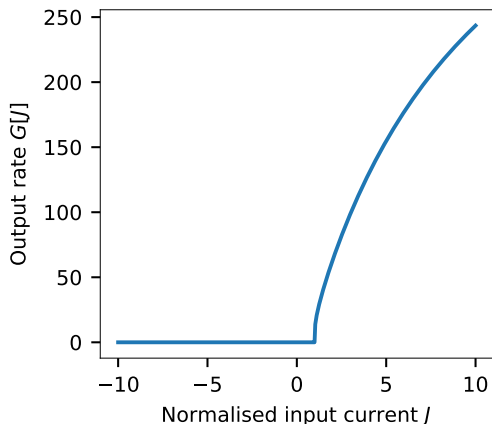
$$v(t) = - \int_0^t \frac{1}{\tau_{RC}} (v(t') - RJ) dt' = RJ \left(1 - e^{-\frac{t}{\tau_{RC}}} \right) .$$

$$v_{\text{th}} = RJ \left(1 - e^{-\frac{t_{\text{th}}}{\tau_{RC}}} \right) \Leftrightarrow 1 - \frac{v_{\text{th}}}{RJ} = e^{-\frac{t_{\text{th}}}{\tau_{RC}}} ,$$

$$t_{\text{th}} = -\tau_{RC} \log \left(1 - \frac{v_{\text{th}}}{RJ} \right)$$

$$G[J] = \begin{cases} \frac{1}{\tau_{\text{ref}} - \tau_{RC} \log \left(1 - \frac{v_{\text{th}}}{RJ} \right)} & \text{if } 1 - \frac{v_{\text{th}}}{RJ} > 0 , \\ 0 & \text{otherwise .} \end{cases}$$

Artificial Rate Neurons: LIF

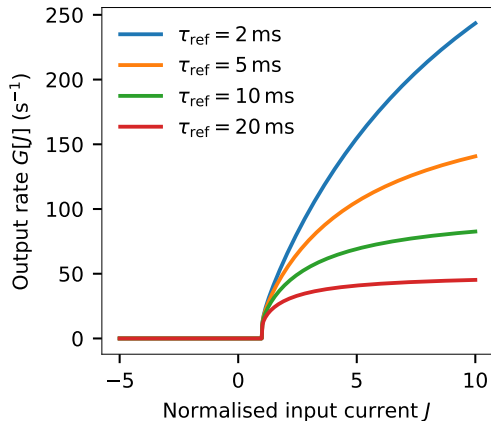
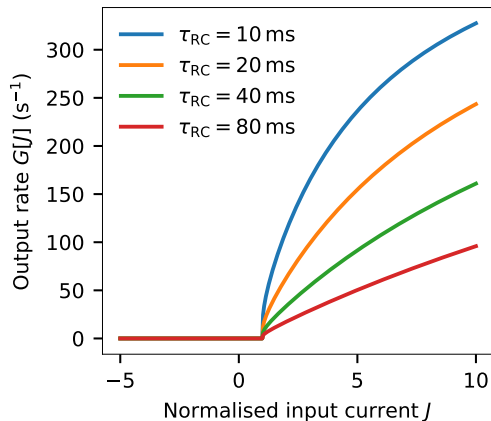


$$G[J] = \frac{1}{\tau_{\text{ref}} - \tau_{\text{RC}} \log \left(1 - \frac{1}{J}\right)}$$

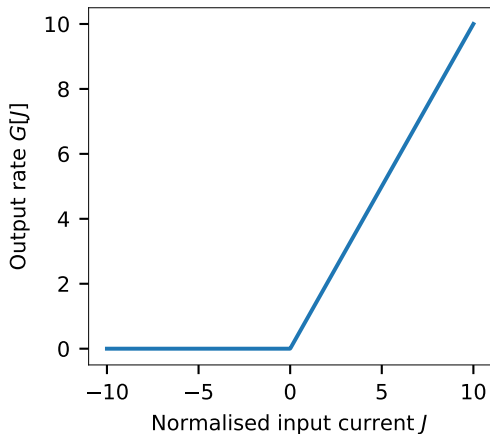
Usefulness to neurobiological systems modellers:

- ⊕ Biologically motivated
- ⊕ Captures saturation effects
- Relatively slow to evaluate numerically (for machine-learning people)
- ⊖ Spike onset is smooth in noisy systems

Exploring the LIF Rate Approximation



Artificial Rate Neurons: ReLU

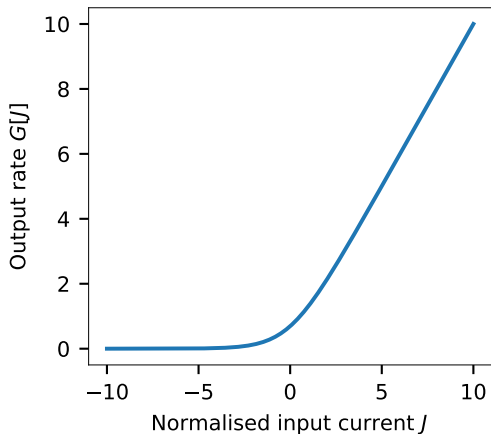


$$G[J] = \max\{0, J\}$$

Usefulness to neurobiological systems modellers:

- ⊕ Fast to evaluate
- Rough approximation of the LIF response curve
- ⊖ Does not capture saturation effects
- ⊖ Spike onset is smooth in noisy systems

Artificial Rate Neurons: Smooth ReLU (Softplus)

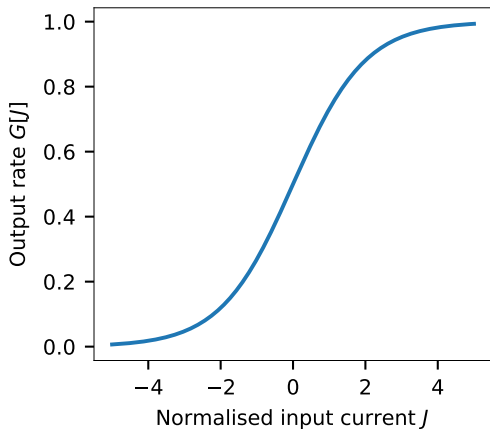


$$G[J] = \log(1 + \exp(J))$$

Usefulness to neurobiological systems modellers:

- ⊕ Models smooth spike onset
- Rough approximation of the LIF response curve
- ⊖ Does not capture saturation effects

Artificial Rate Neurons: Logistic Function

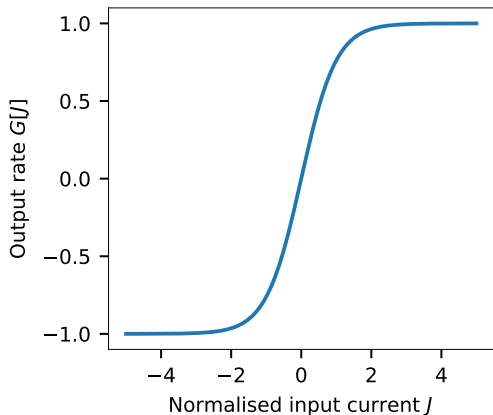


$$G[J] = \frac{1}{1 + e^{-J}}$$

Usefulness to neurobiological systems modellers:

- Models smooth spike onset and saturation (?)

Artificial Rate Neurons: Hyperbolic Tangent



$$G[J] = \tanh(J) = \frac{e^J - e^{-J}}{e^J + e^{-J}}$$

Usefulness to neurobiological systems modellers:

- Models smooth spike onset and saturation (?)
- Negative rates

Image sources

Title slide

Image of rat primary cortical neurons in culture.

Author: ZEISS Microscopy, <http://www.zeiss.com/celldiscoverer>.

From Wikimedia.