SYDE 556/750

Simulating Neurobiological Systems
Lecture 4: Temporal Representations

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Sept 18 & 23, 2024

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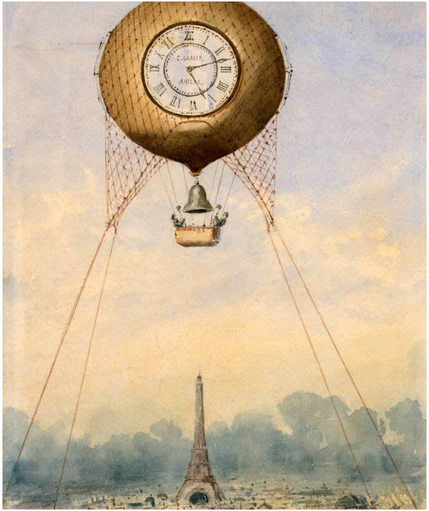
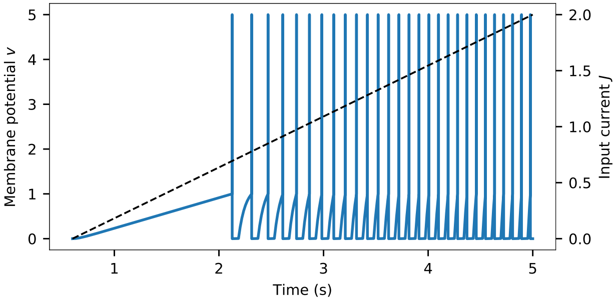
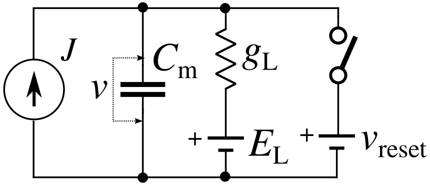
- Content: Terry Stewart, Andreas Stöckel, Chris Eliasmith

UNIVERSITY OF WATERLOO | FACULTY OF ENGINEERING



Reminder: The LIF Neuron





$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t} v(t) &= -\frac{1}{\tau_\mathrm{RC}} \big(v(t) - J \big) \,, && \text{if } v(t) < 1 \,, \\
v(t) &\leftarrow \delta(t - t_\mathrm{th}) \,, && \text{if } t = t_\mathrm{th} \,, \\
v(t) &\leftarrow 0 \,, && \text{if } t > t_\mathrm{th} \text{ and } t \geq t_\mathrm{th} + \tau_\mathrm{ref} \,,
\end{aligned}
$$

Temporal Decoding

- For population decoders, we needed to integrate their responses, $\mathbf{a}(\mathbf{x})$, over the represented variable, $\mathbf{x}$.

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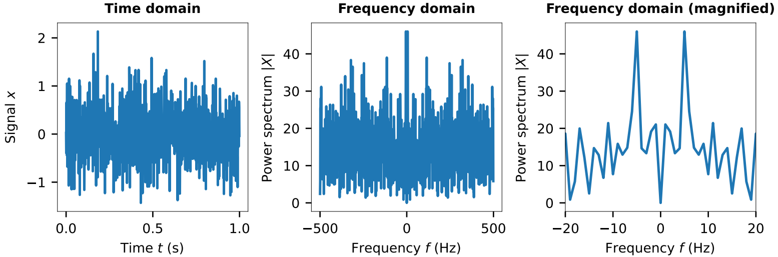
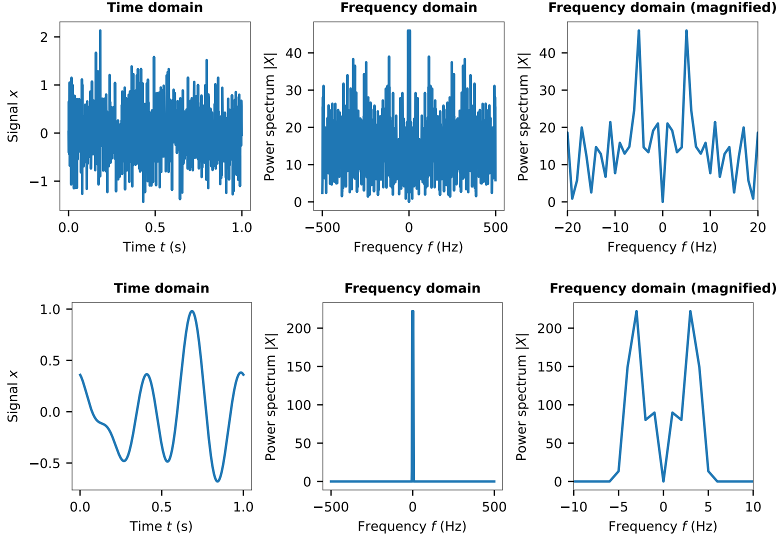
- For temporal decoders, we will likely want to integrate their responses, $\mathbf{a}(t)$, over the represented variable, $\mathbf{x}(t)$.
- What space do we want to sample to estimate the integrals?

Random Signals



White Noise
(zero mean)

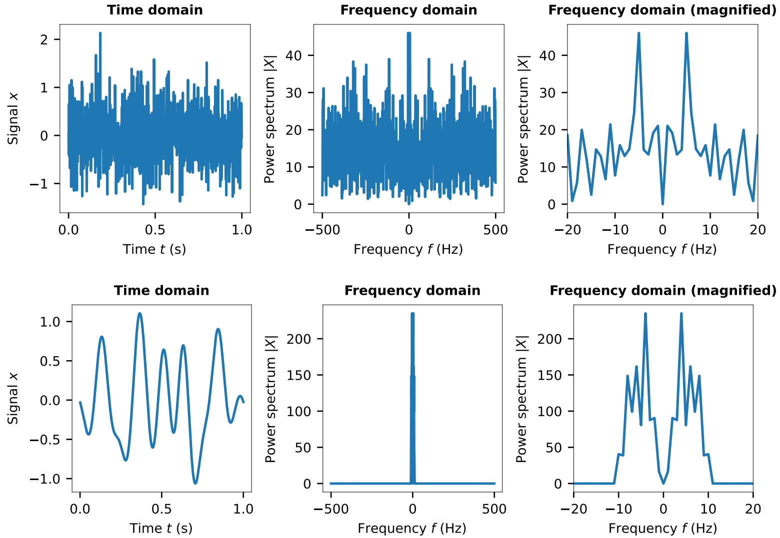
Random Signals



White Noise
(zero mean)

Bandlimited
White Noise
(zero mean,
$5\,$Hz bandwidth)

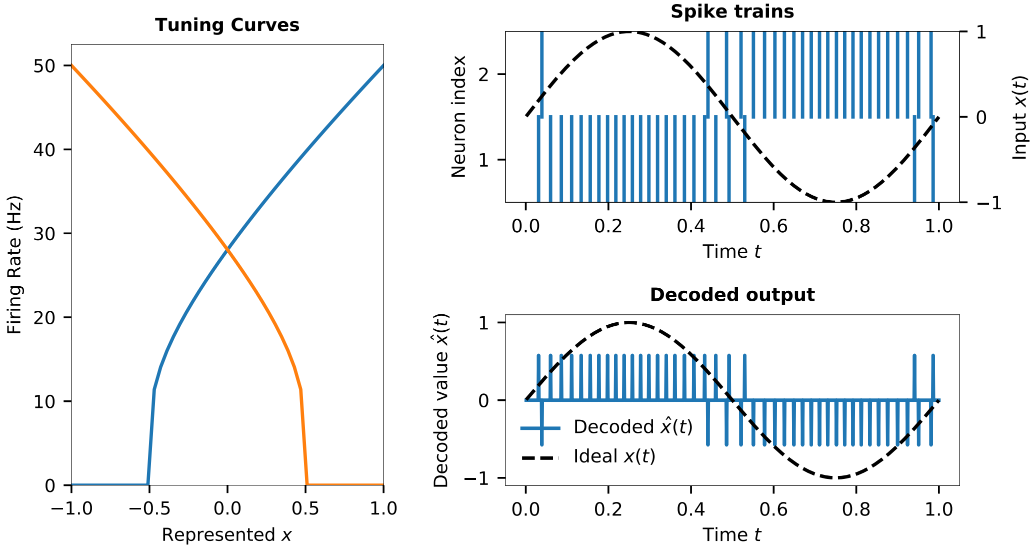
Random Signals



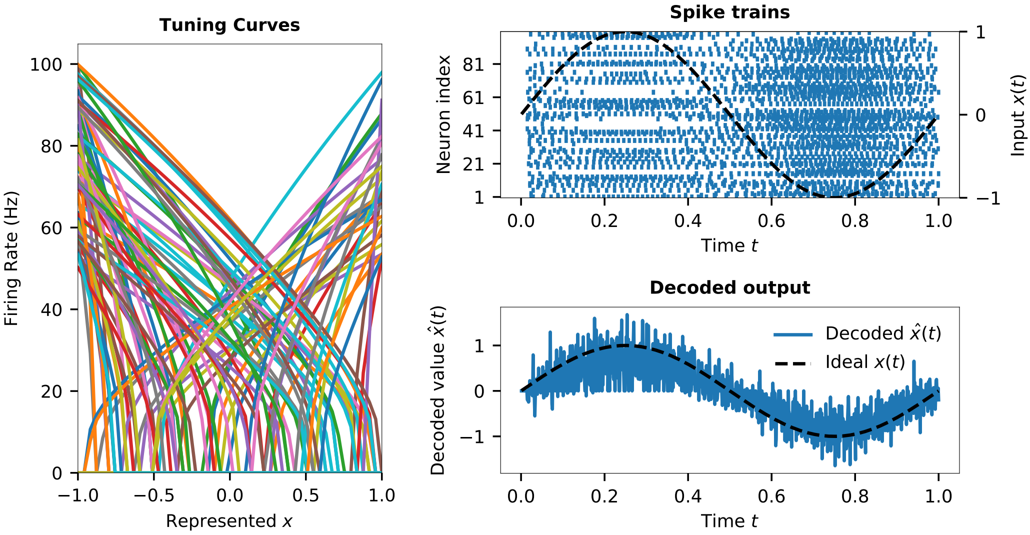
White Noise
(zero mean)

Bandlimited
White Noise
(zero mean,
10 Hz bandwidth)

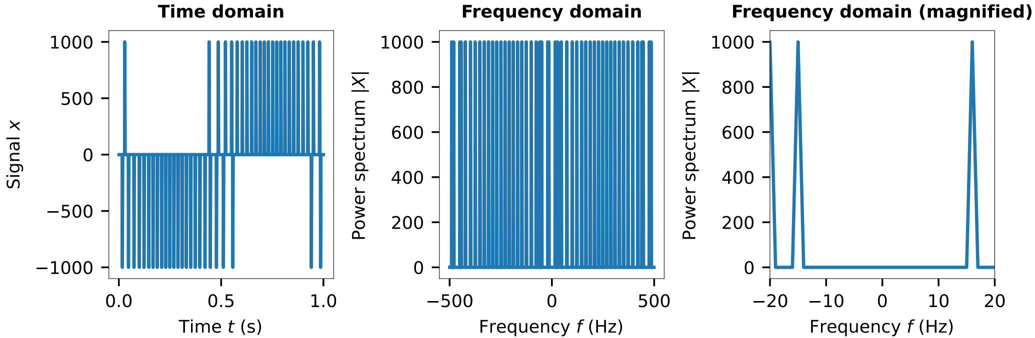
Temporal Decoding of Two Neurons - Weighted Spikes



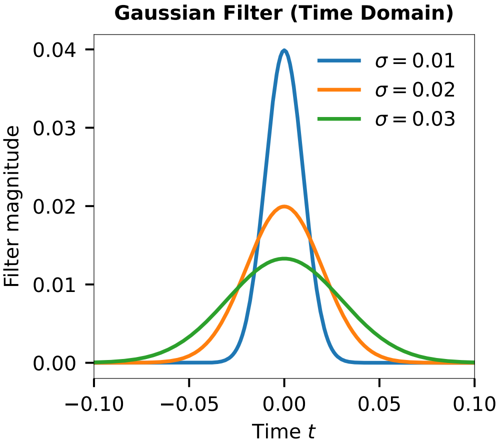
Temporal Decoding of One Hundred Neurons - Weighted Spikes



Frequency Response of Two Neurons



Filtering by Convolution



Gaussian Filter

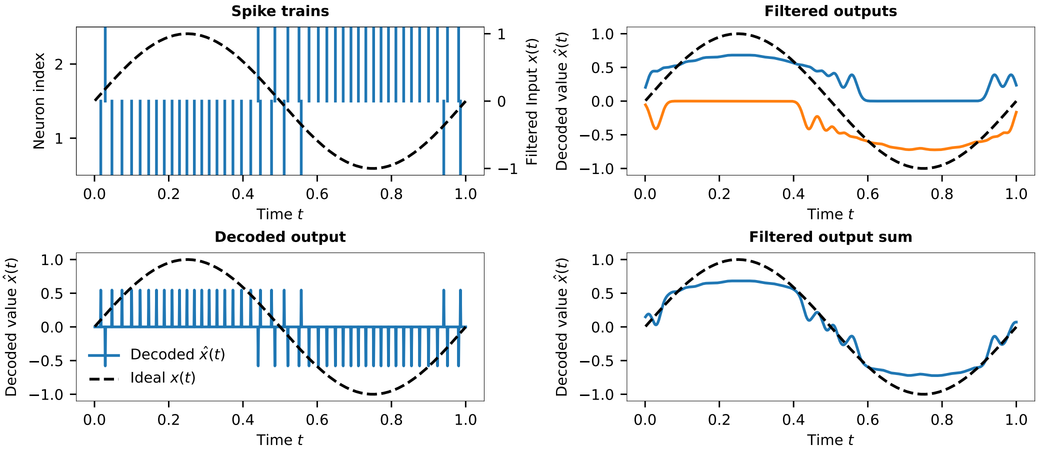
$$h(t) = c \exp\left(\frac{-t^2}{\sigma^2}\right)$$

where c chosen s.t. $\int_{-\infty}^{\infty} h(t)\,\mathrm{d}t = 1$

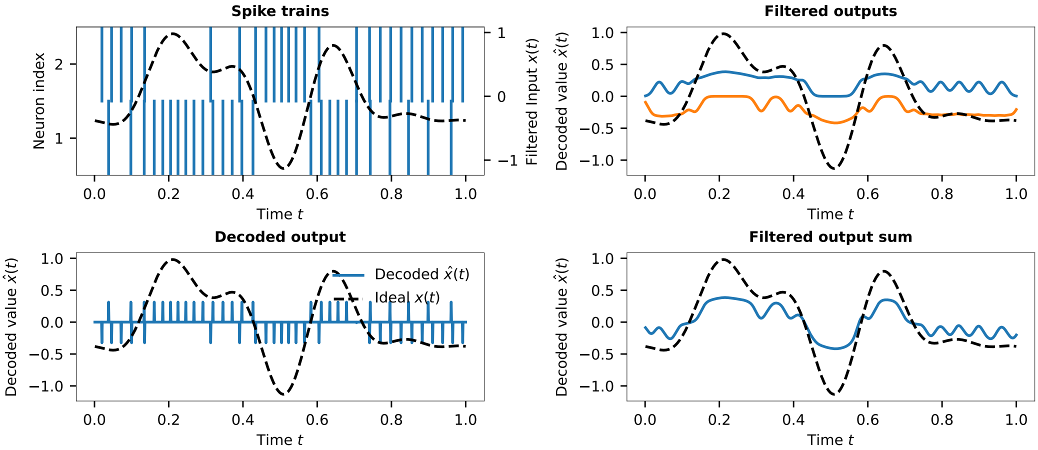
Convolution

$$(f * h)(t) = \int_{-\infty}^{\infty} f(t - t')h(t')\,\mathrm{d}t'$$

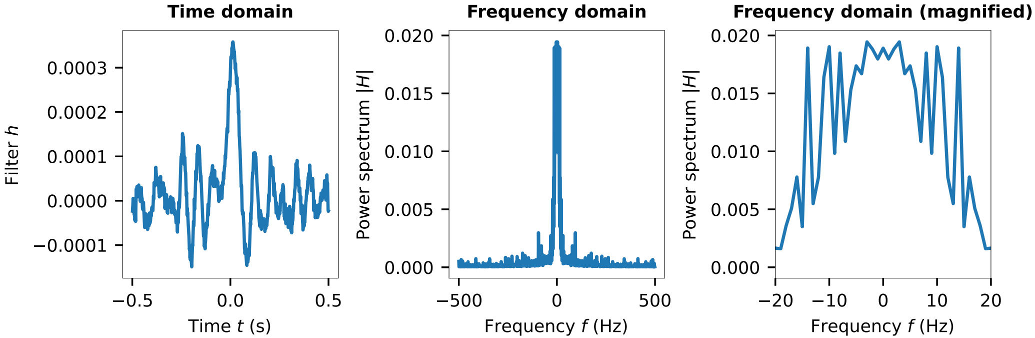
Filtering a Spike Train



Filtering a Spike Train for a Random Signal

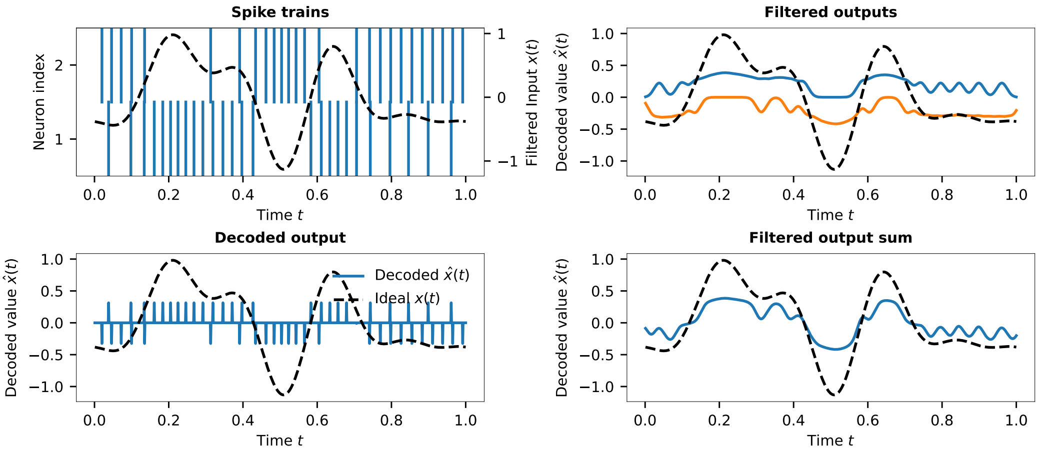


Optimal Filter



$$H(\omega) = \frac{X(\omega)\overline{R}(\omega)}{|R(\omega)|^2}$$

Filtering a Spike Train for a Random Signal (Optimal Filter)



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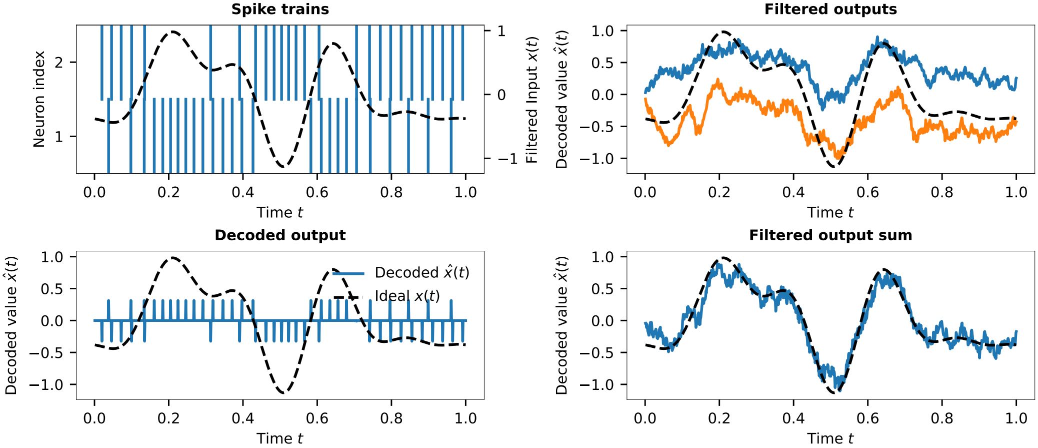

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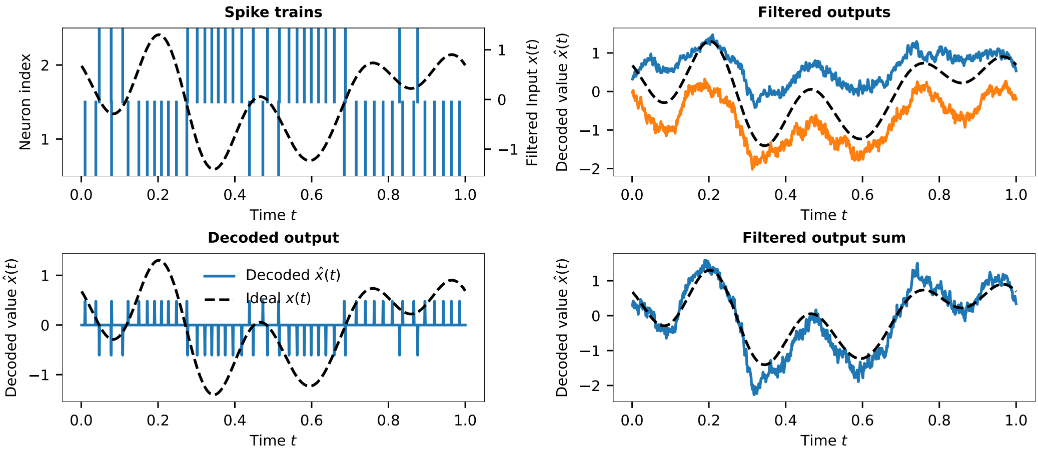
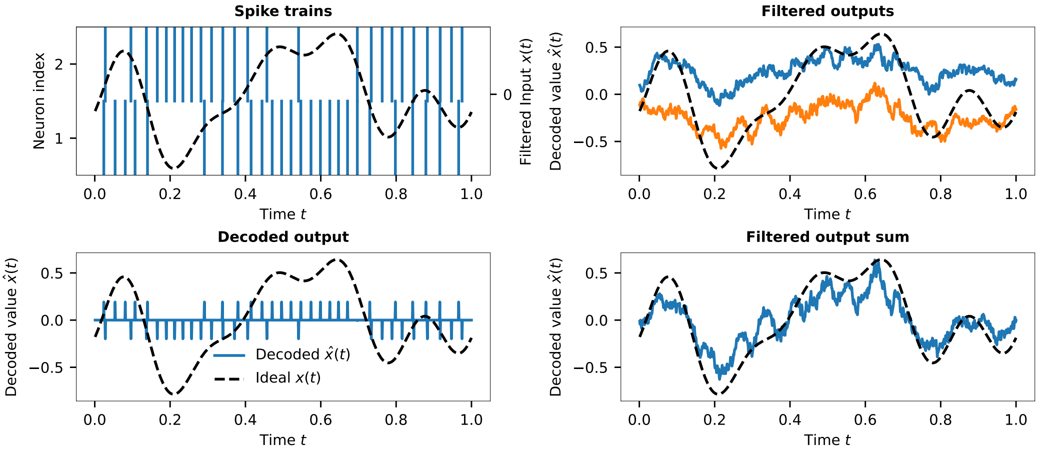
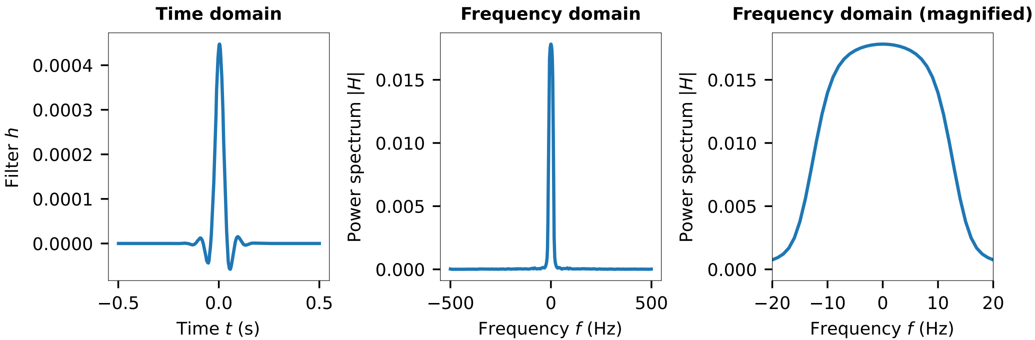


Filtering a Spike Train for a Random Signal (Optimal Filter)

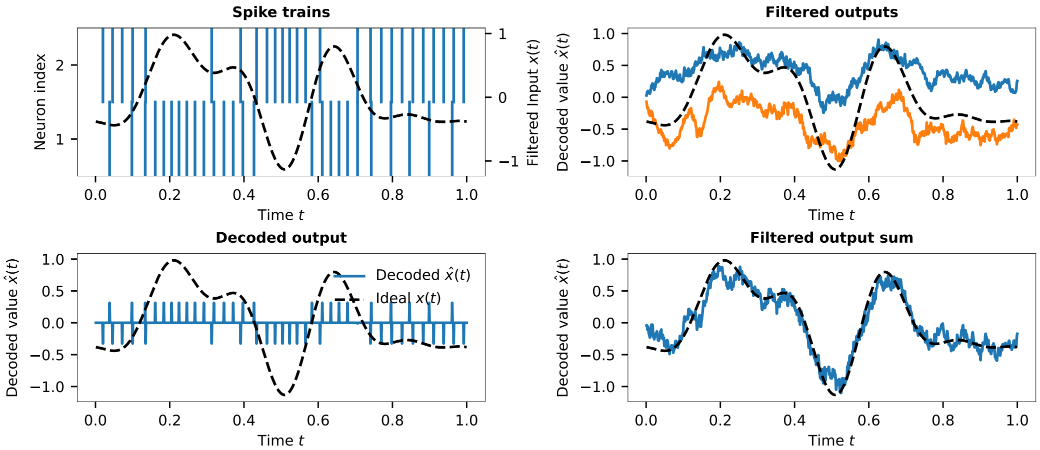


Optimal Filter (Improved)



$$H(\omega) = \frac{X(\omega)\overline{R}(\omega) * W(\omega)}{|R(\omega)|^2 * W(\omega)}$$

Filtering a Spike Train for a Random Signal (Improved Optimal Filter)



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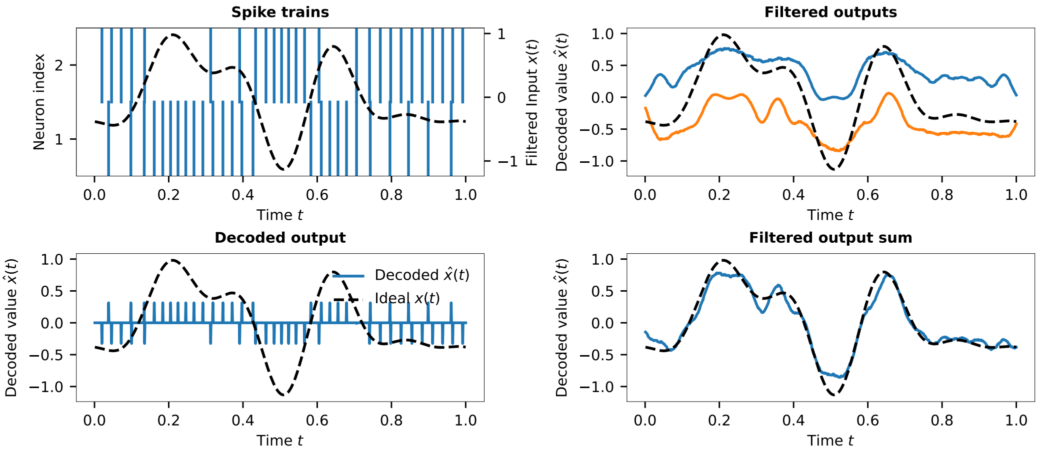
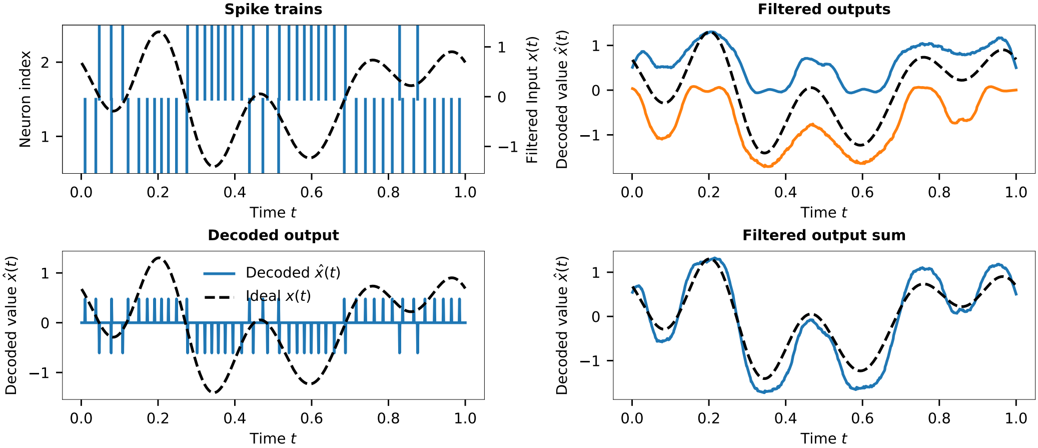
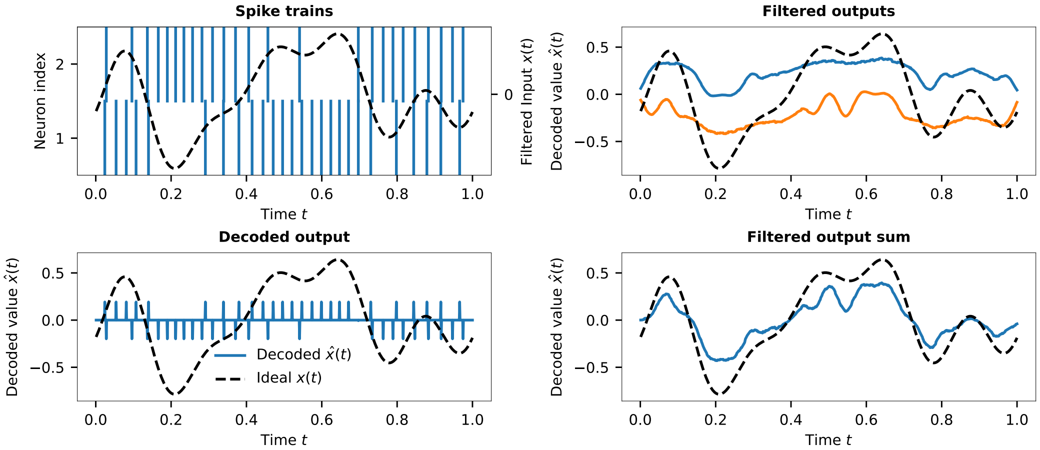


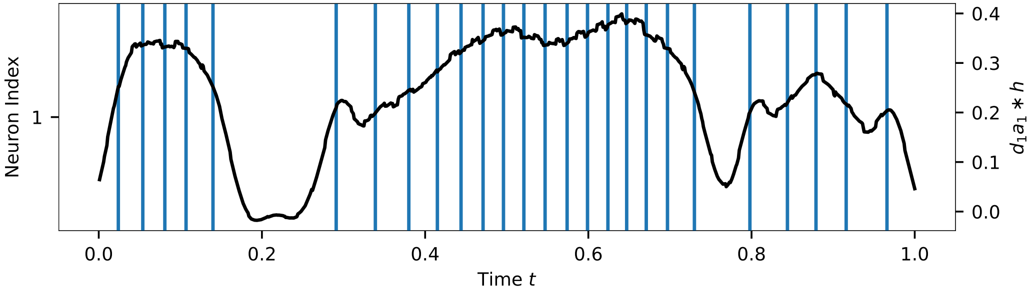
Filtering a Spike Train for a Random Signal (Improved Optimal Filter)



Filtering a Spike Train for a Random Signal (Improved Optimal Filter)



Pros and Cons of the Optimal Filter



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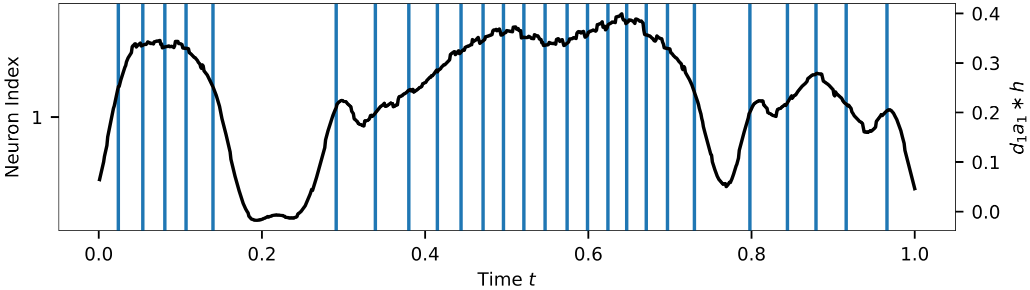
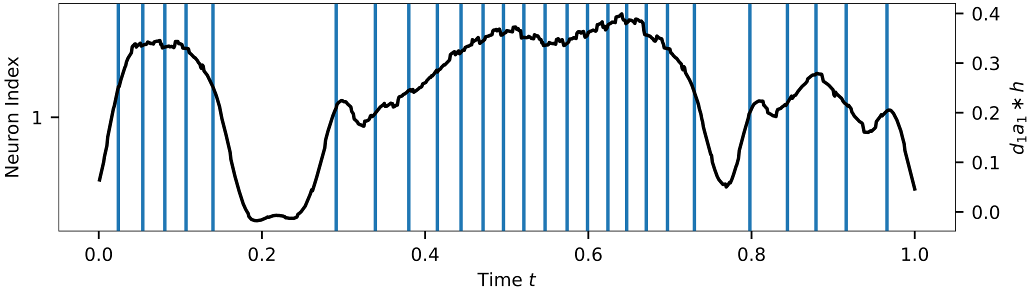


Precise

Good for analysing data after the fact

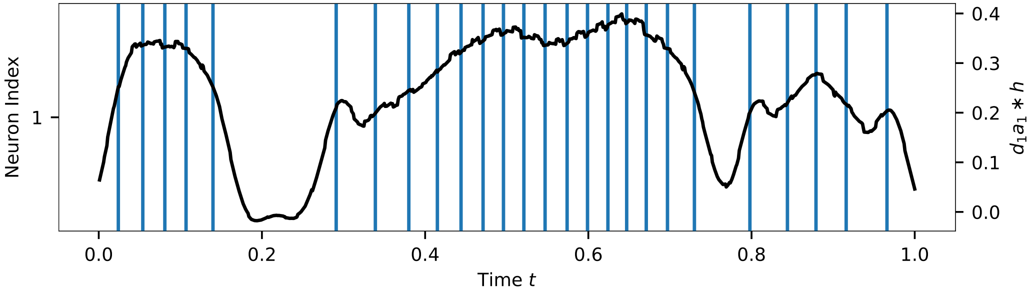
Pros and Cons of the Optimal Filter



Precise
Good for analysing data after the fact

Non-causal
Does not describe a biological process

Pros and Cons of the Optimal Filter



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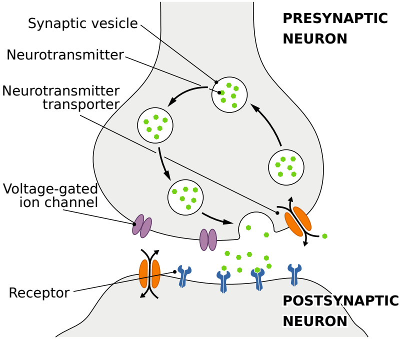
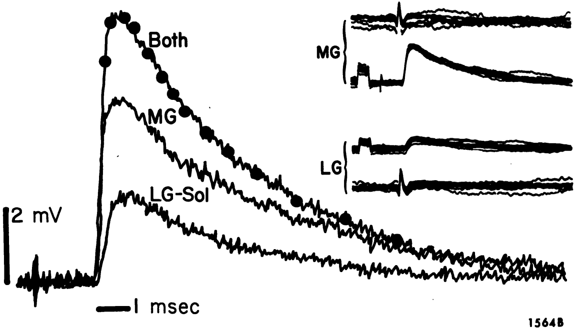
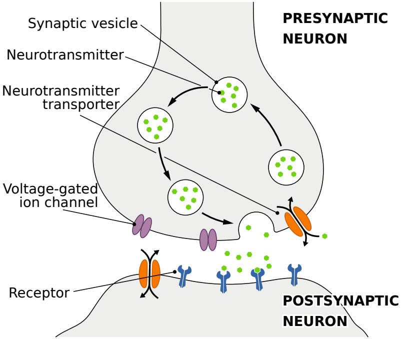
We need to find a mechanism that low-pass filters spikes over time!

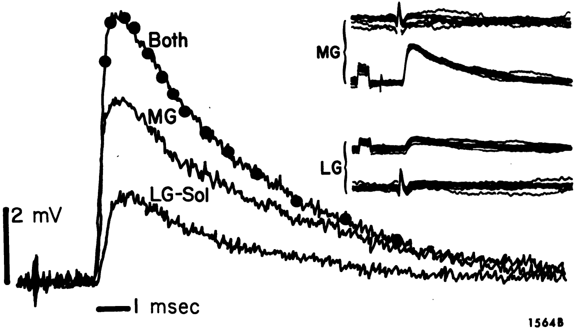
Synapses as Filters





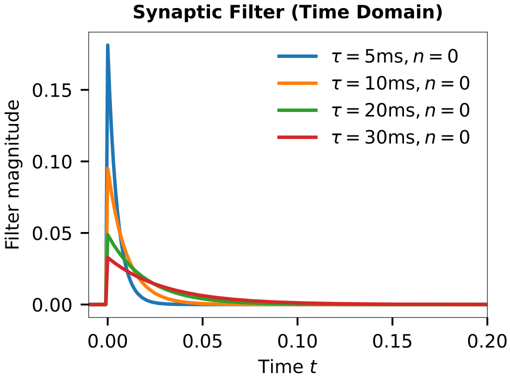
Synapses as Filters

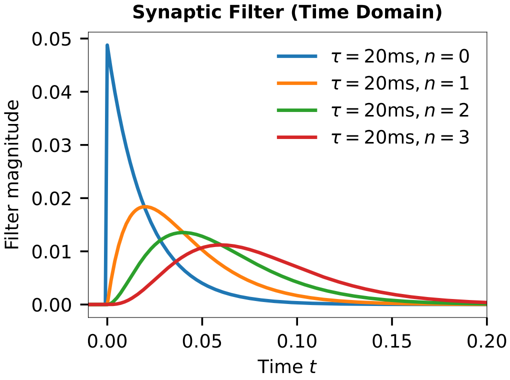




Post-synaptic currents (EPSCs, IPSCs) are low-pass filtered spike trains!

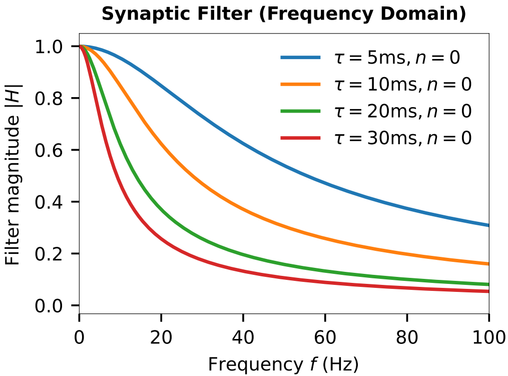
Exponential Low-Pass Filter (I)

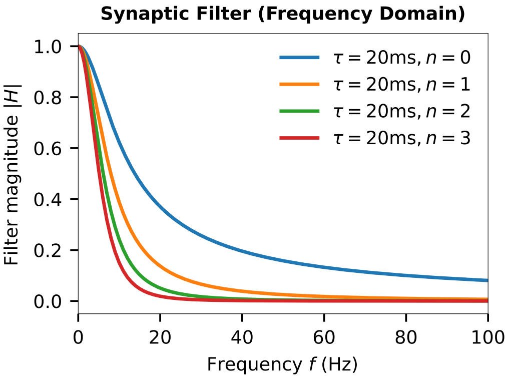




$$h(t) = \begin{cases} c^{-1} t^n \exp^{-t/\tau} & \text{if } t \geq 0\,, \\ 0 & \text{otherwise}\,, \end{cases} \qquad \text{where } c = \int_0^\infty t^n \exp^{-t/\tau} \, \mathrm{d}t\,.$$

Exponential Low-Pass Filter (II)





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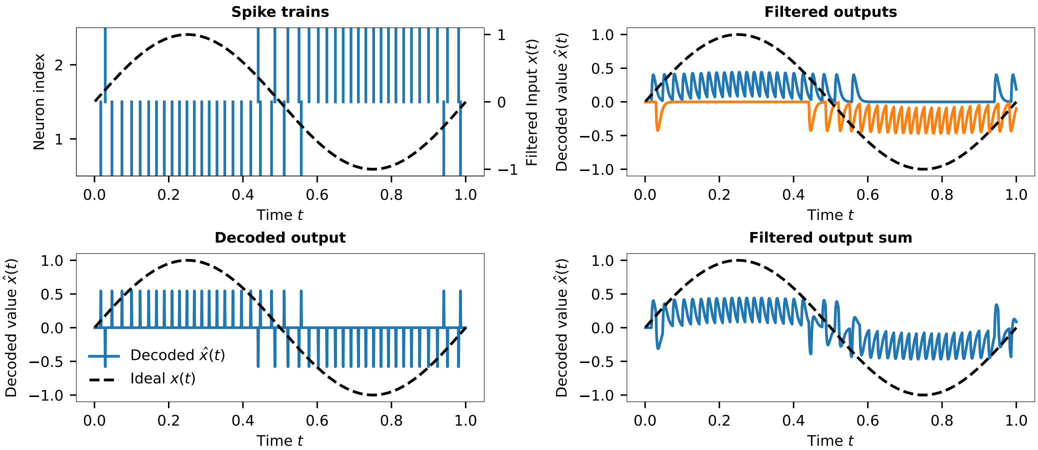
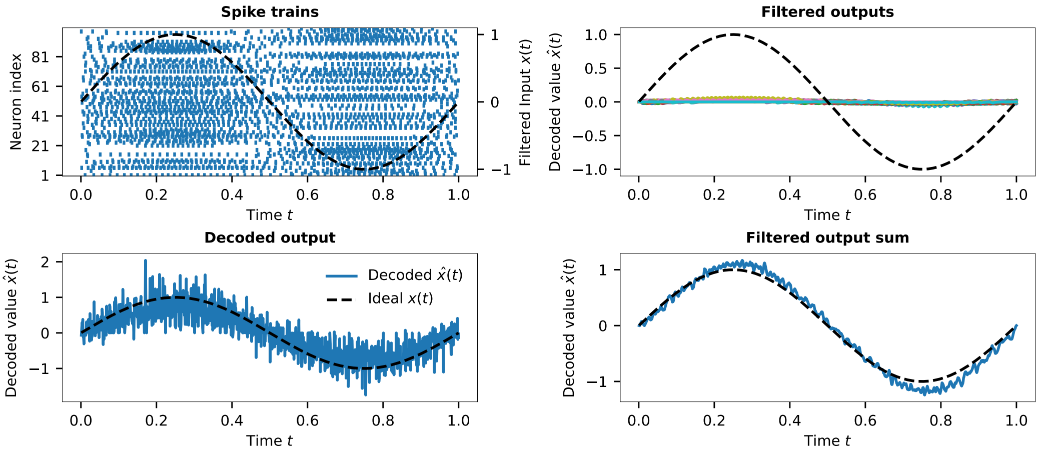
Example: Synaptic Filter for Two Neurons



$$\tau = 5\,\text{ms}, n = 1$$

Example: Synaptic Filter for One Hundred Neurons



$$\tau = 5\,\mathrm{ms},\, n = 1$$

Image sources

Title slide
"Captive balloon with clock face and bell, floating above the Eiffel Tower, Paris, France."
Author: Camille Grávis, between 1889 and 1900.
From Wikimedia.