

SYDE 556/750

Simulating Neurobiological Systems
Lecture 7: Temporal Basis Functions

Chris Eliasmith

October 21, 2024

- ▶ Slide design: Andreas Stöckel
- ▶ Content: Terry Stewart, Andreas Stöckel, Chris Eliasmith

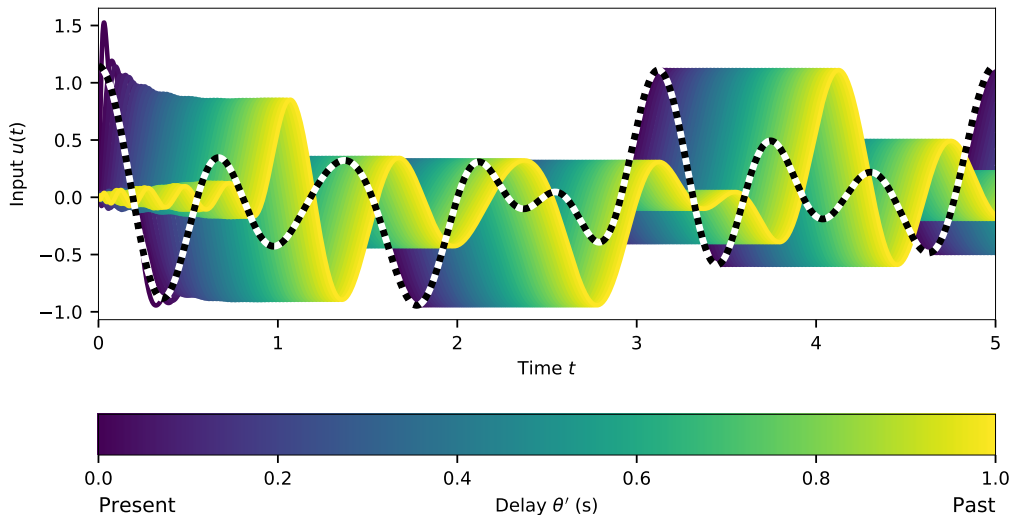


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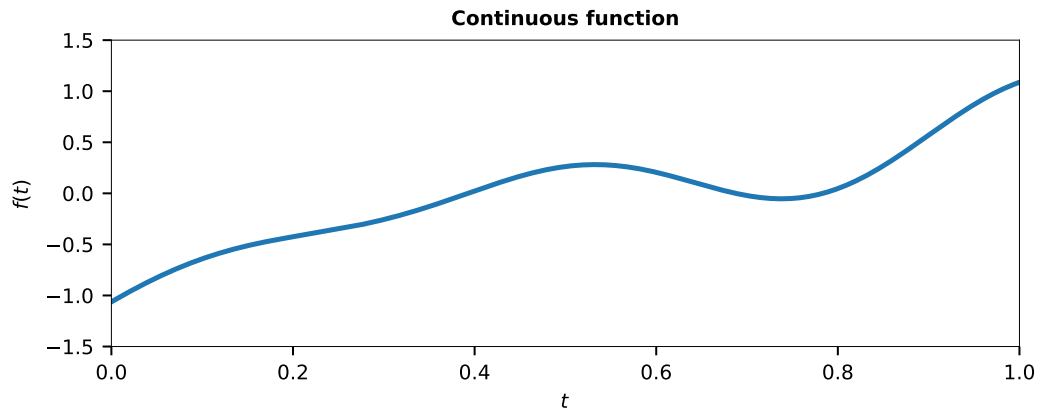
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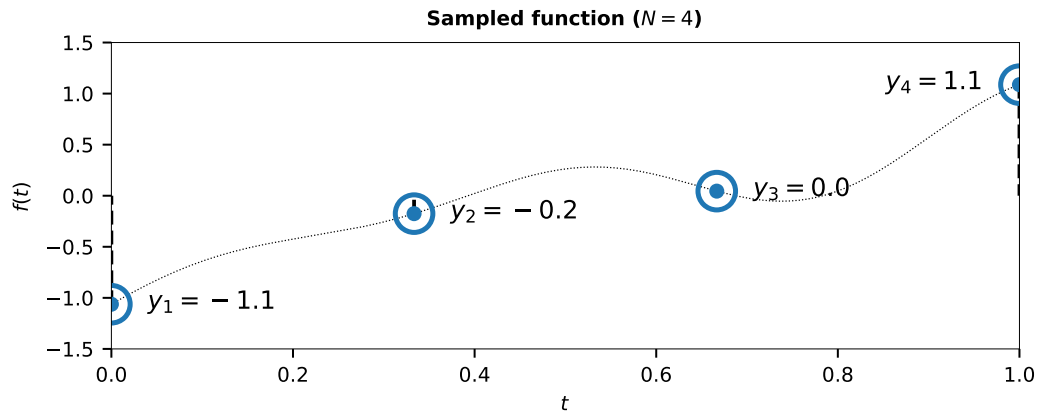
Representing Stimulus Histories



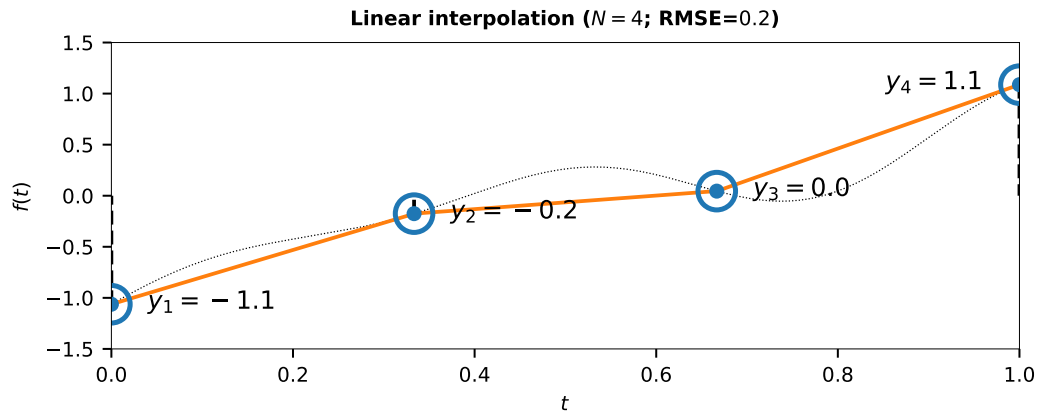
Representing Functions: Sampling



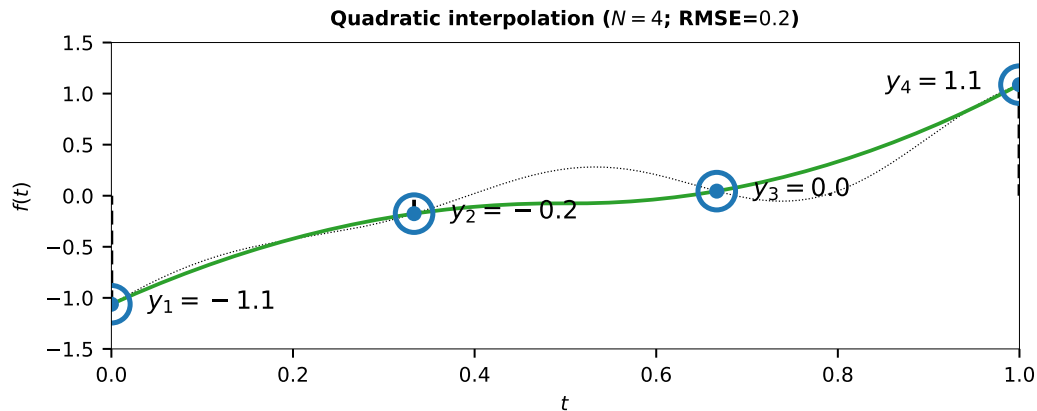
Representing Functions: Sampling



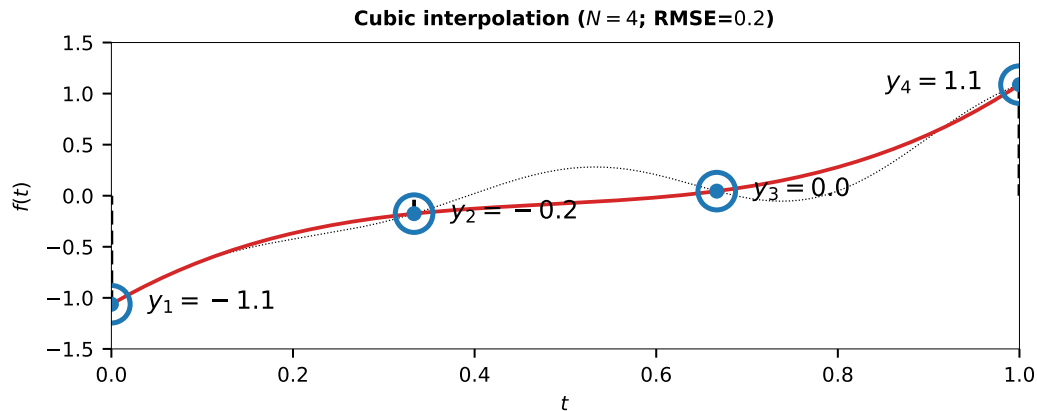
Representing Functions: Sampling



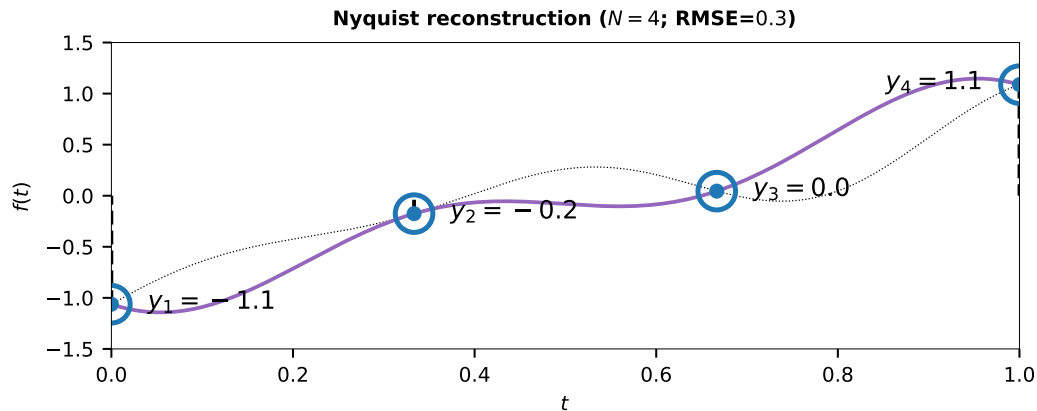
Representing Functions: Sampling



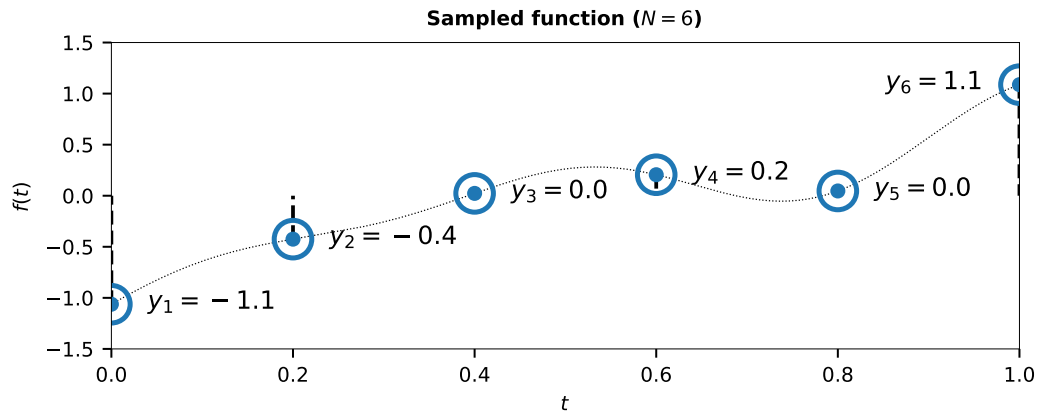
Representing Functions: Sampling



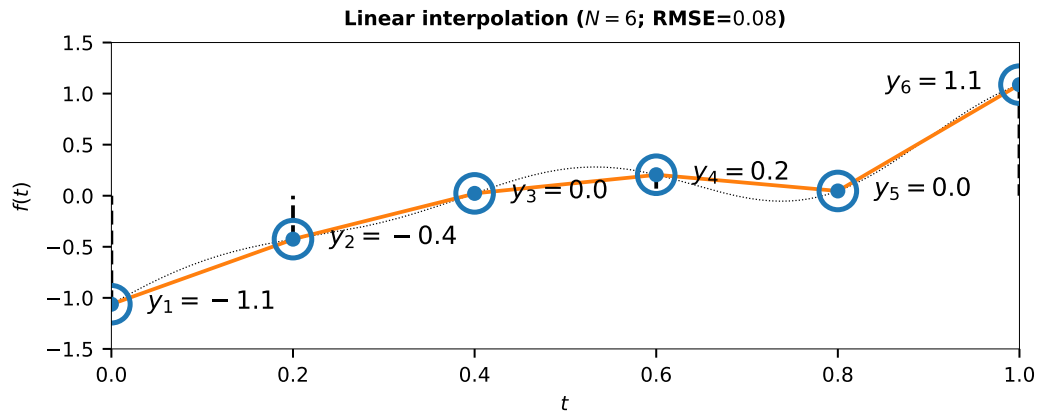
Representing Functions: Sampling



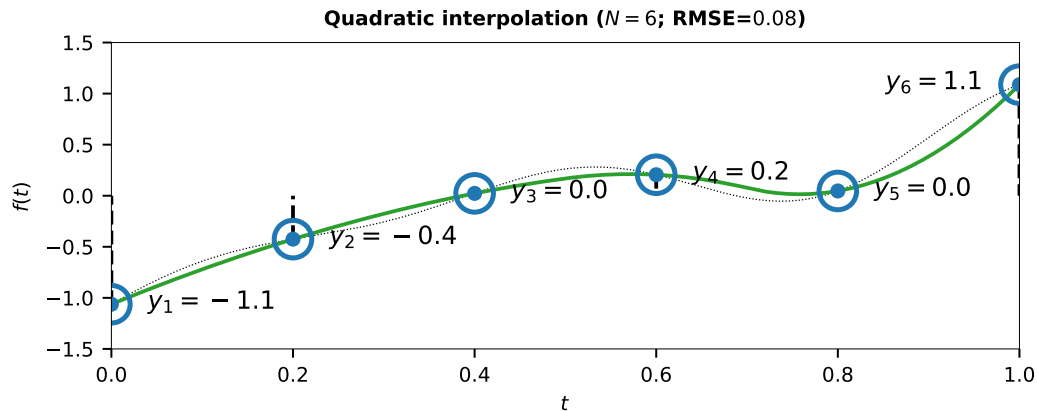
Representing Functions: Sampling



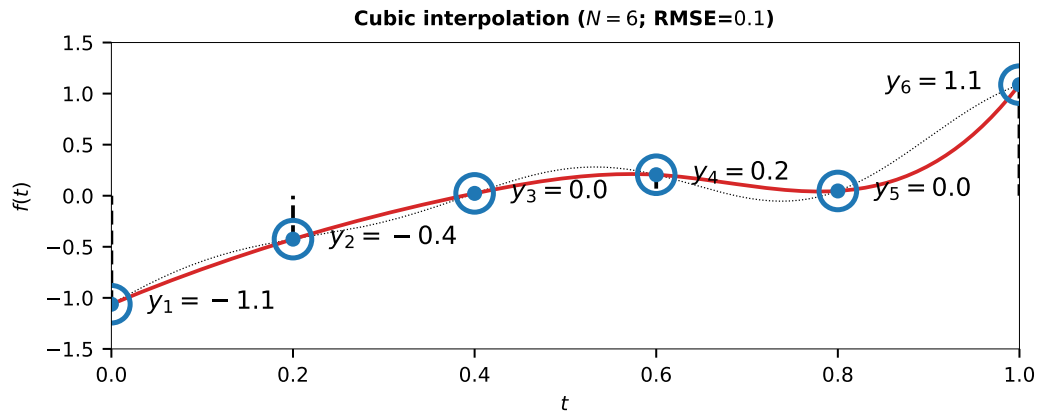
Representing Functions: Sampling



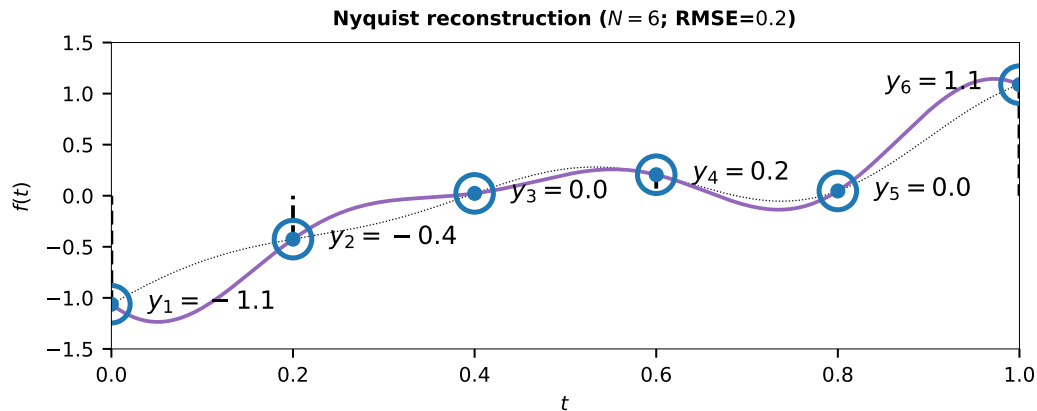
Representing Functions: Sampling



Representing Functions: Sampling



Representing Functions: Sampling

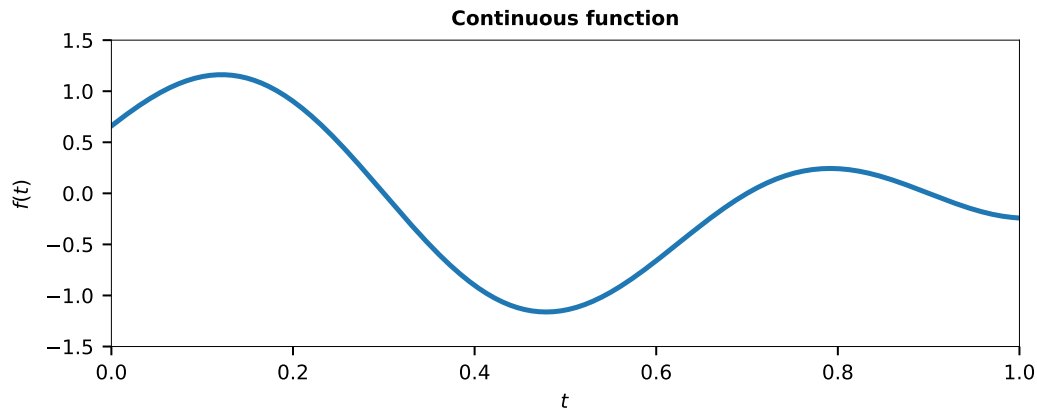


Nyquist Sampling

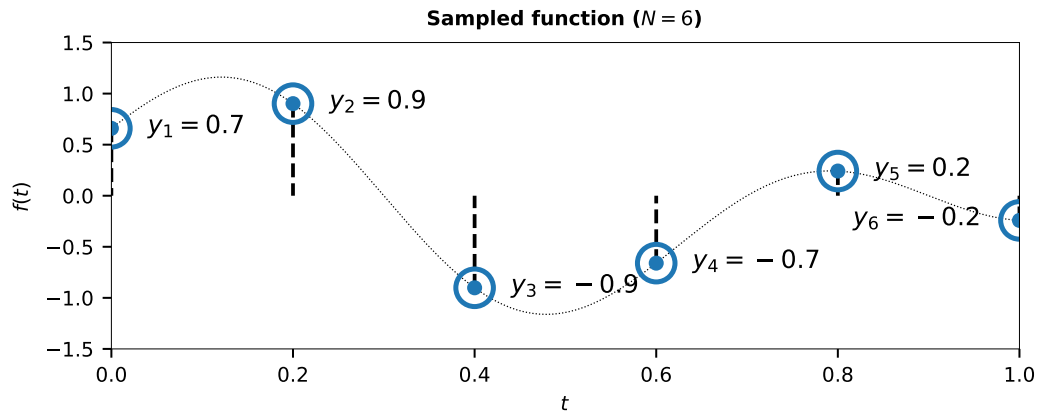
The Nyquist-Shannon Sampling Theorem

If $f(t)$ contains no frequencies greater than B then it is *completely* determined by samples spaced $\Delta t = \frac{1}{2B}$ apart ($N = 2BT$ equally spaced samples for a time-slice $[0, T)$). There is a *one-to-one mapping* between the samples x and the function $f(t)$.

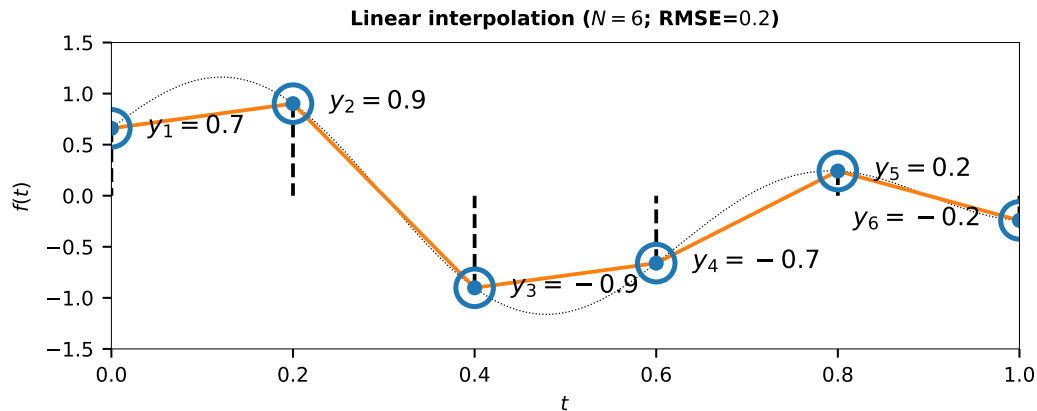
Representing Functions: Sampling $< 3\text{Hz}$



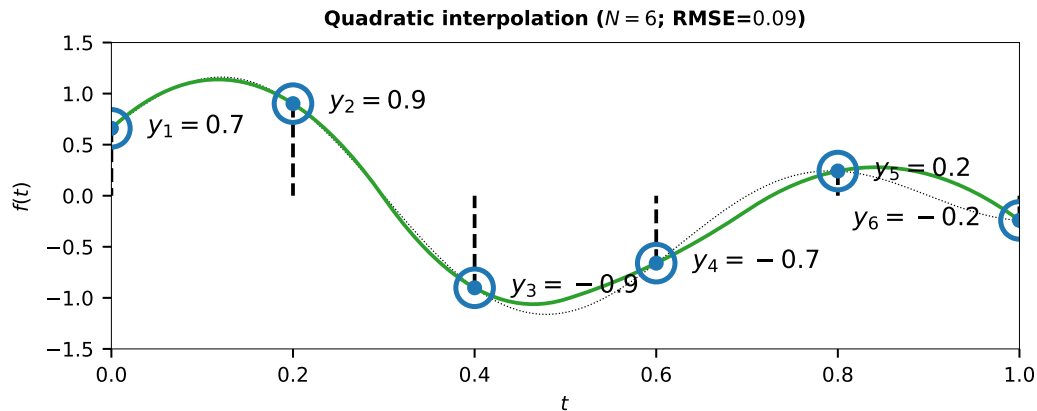
Representing Functions: Sampling $< 3\text{Hz}$



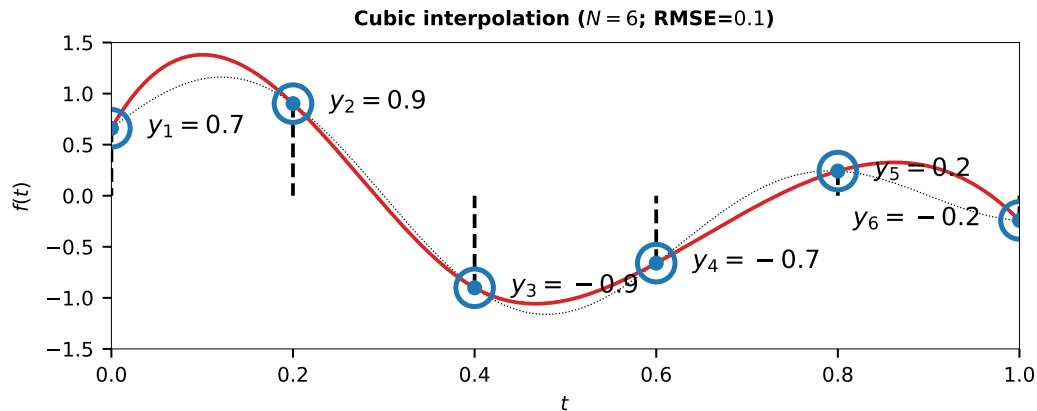
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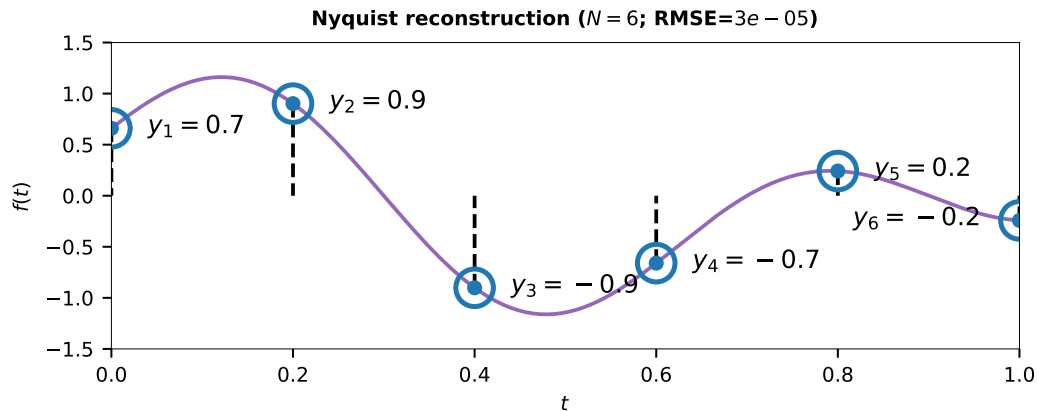
Representing Functions: Sampling $< 3\text{Hz}$



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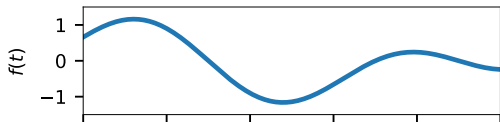


Representing Functions: Sampling $< 3\text{Hz}$

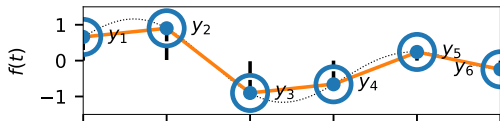


Representing Functions: Sampling $< 3\text{Hz}$

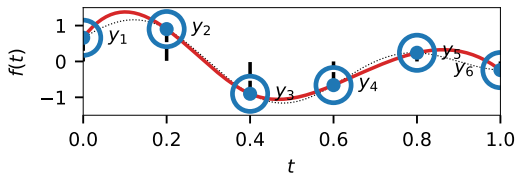
Continuous function



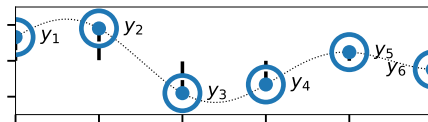
Linear interpolation ($N = 6$; RMSE=0.2)



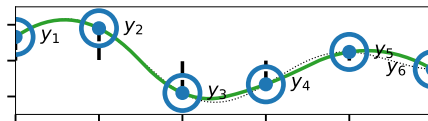
Cubic interpolation ($N = 6$; RMSE=0.1)



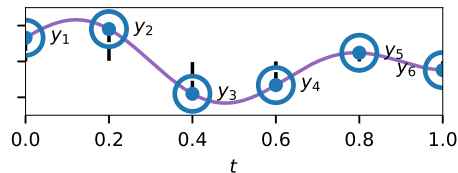
Sampled function ($N = 6$)



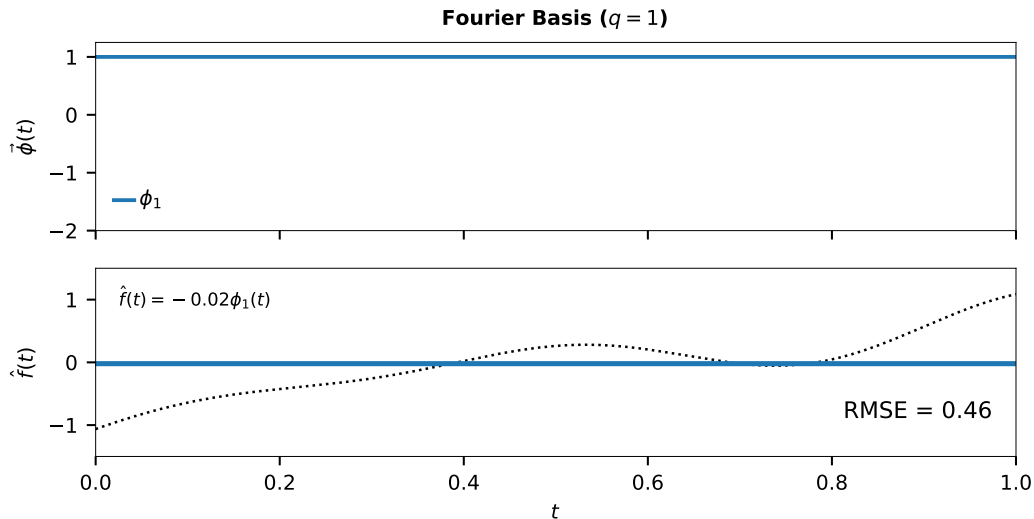
Quadratic interpolation ($N = 6$; RMSE=0.09)



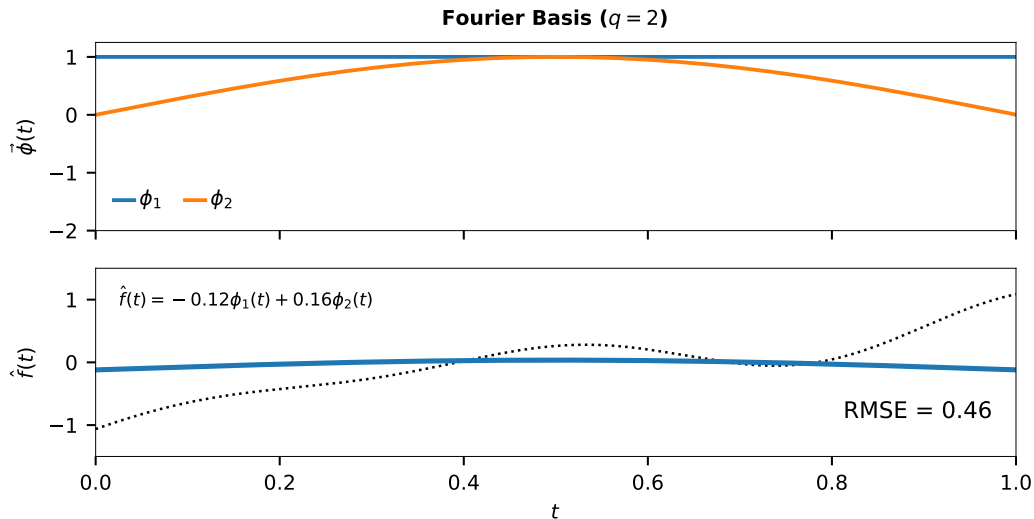
Nyquist reconstruction ($N = 6$; RMSE=3e-05)



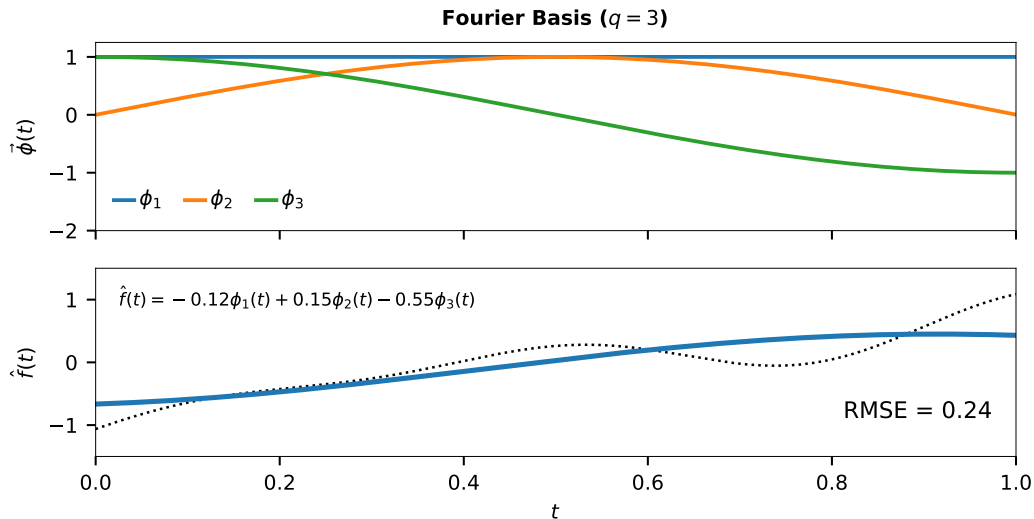
Representing Functions: Fourier Basis



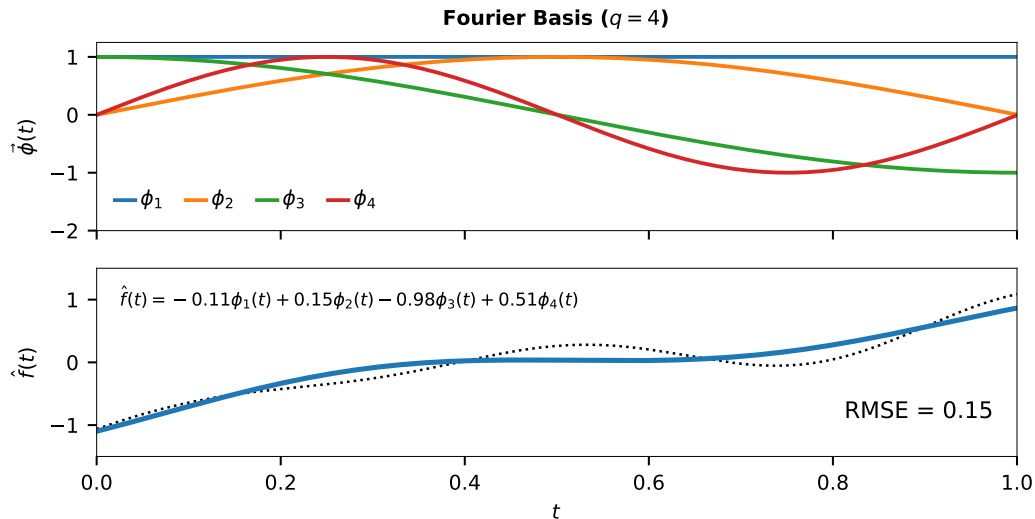
Representing Functions: Fourier Basis



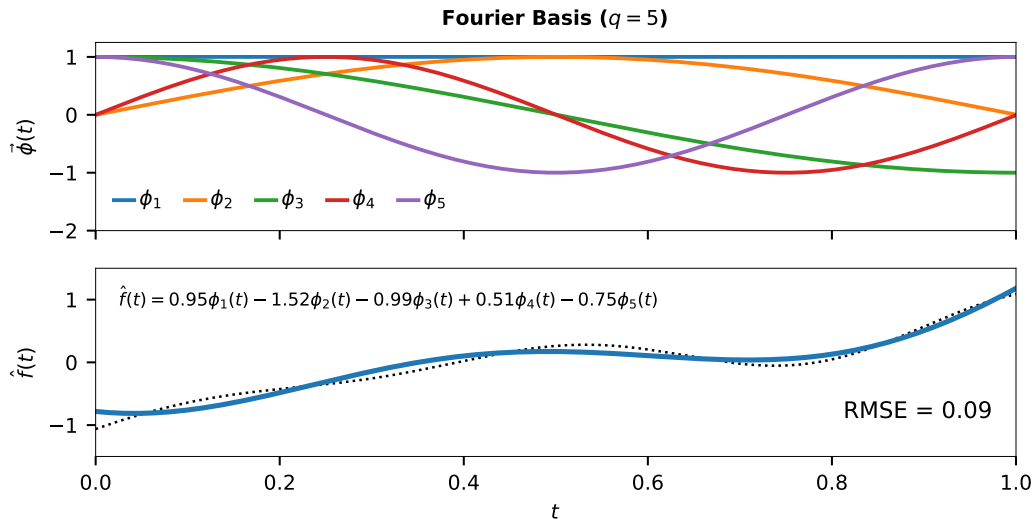
Representing Functions: Fourier Basis



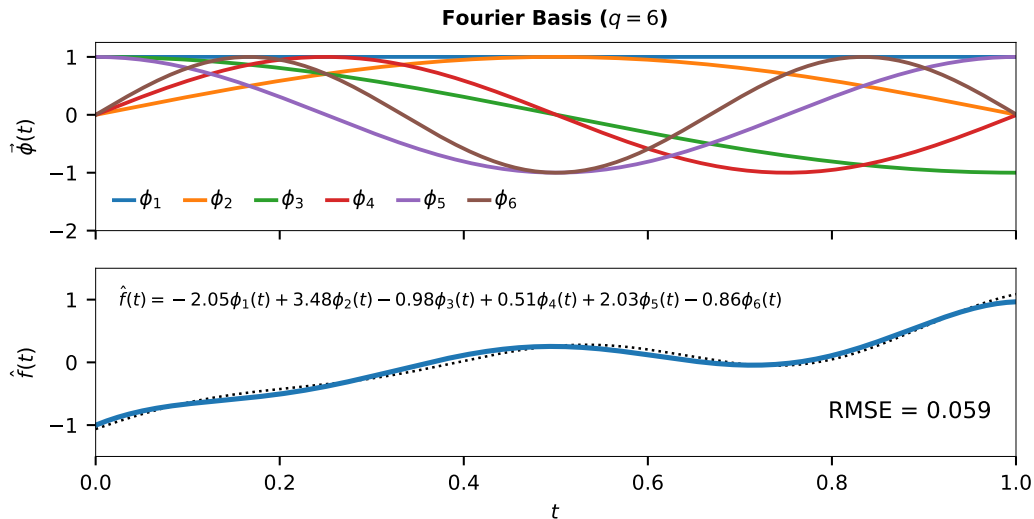
Representing Functions: Fourier Basis



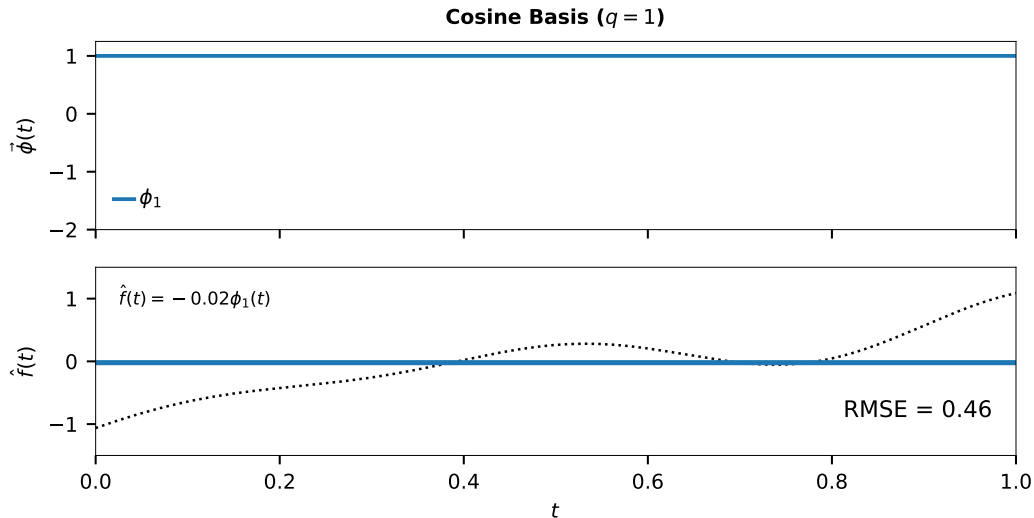
Representing Functions: Fourier Basis



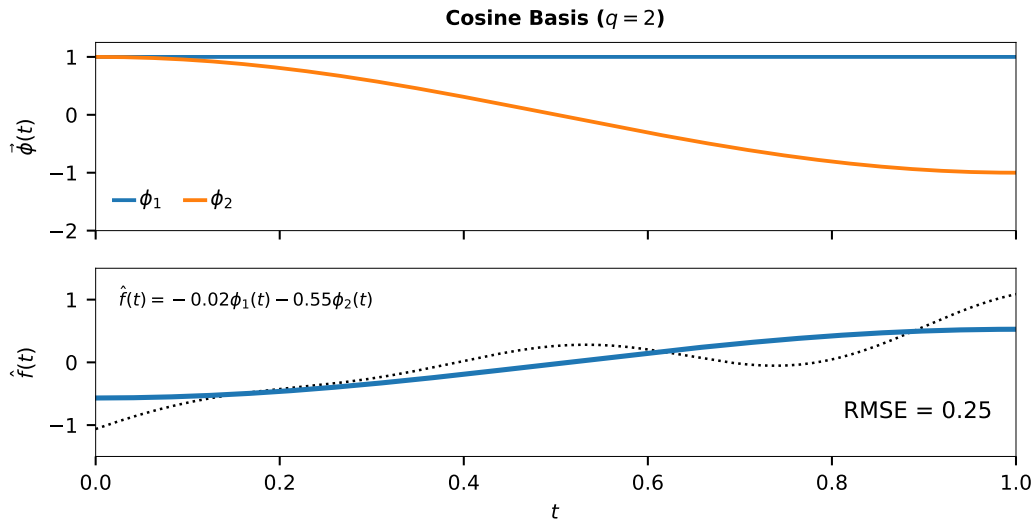
Representing Functions: Fourier Basis



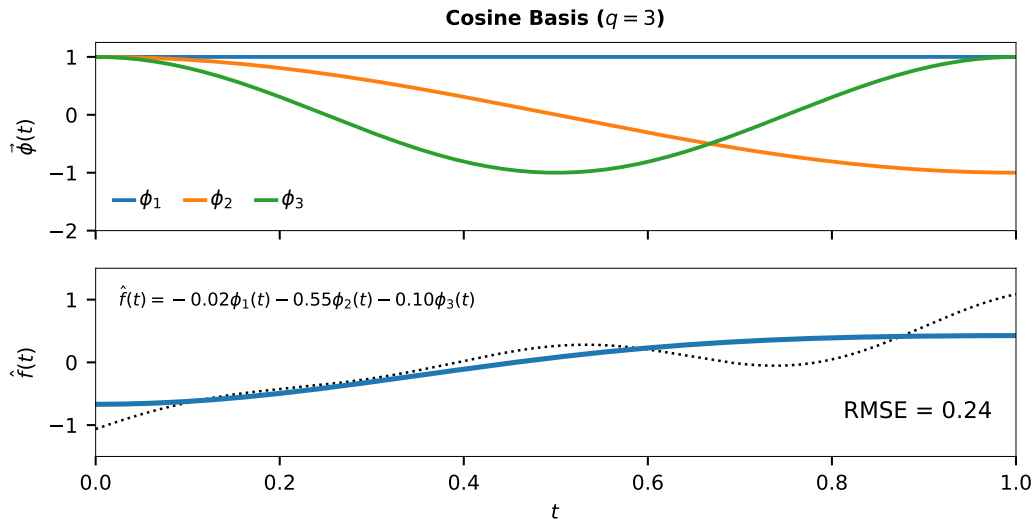
Representing Functions: Cosine Basis



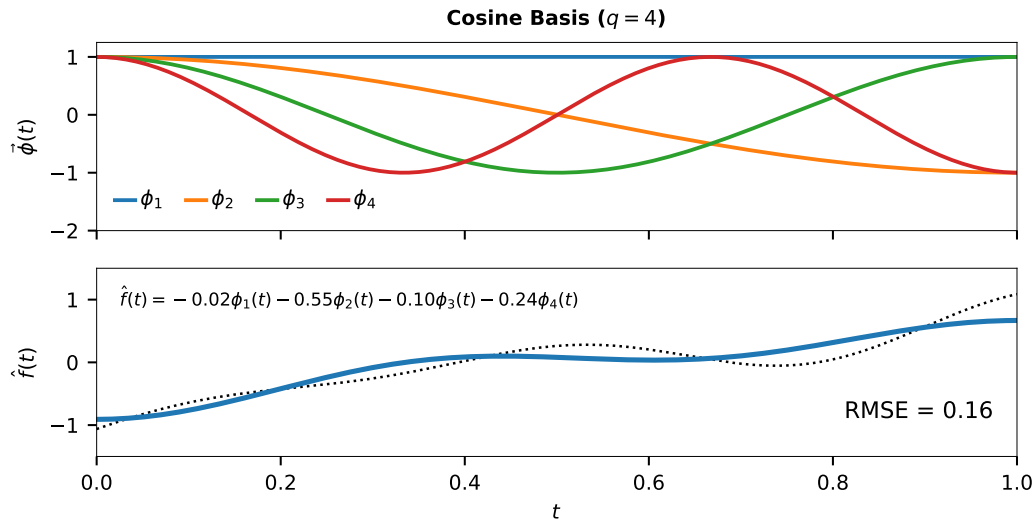
Representing Functions: Cosine Basis



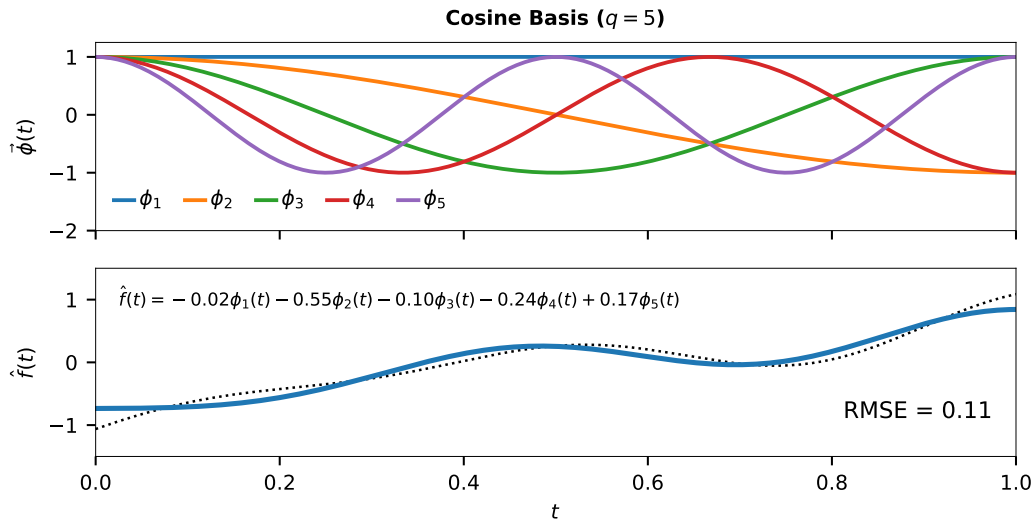
Representing Functions: Cosine Basis



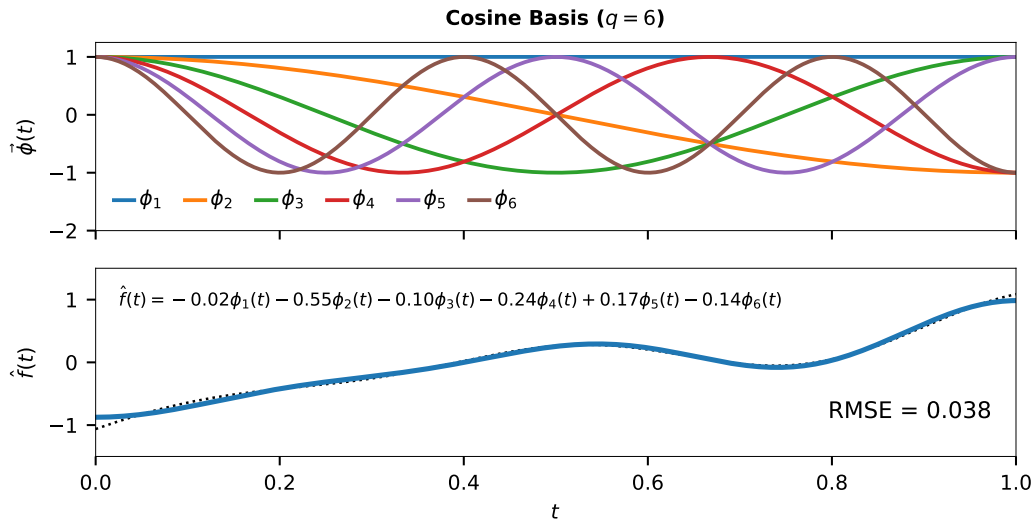
Representing Functions: Cosine Basis



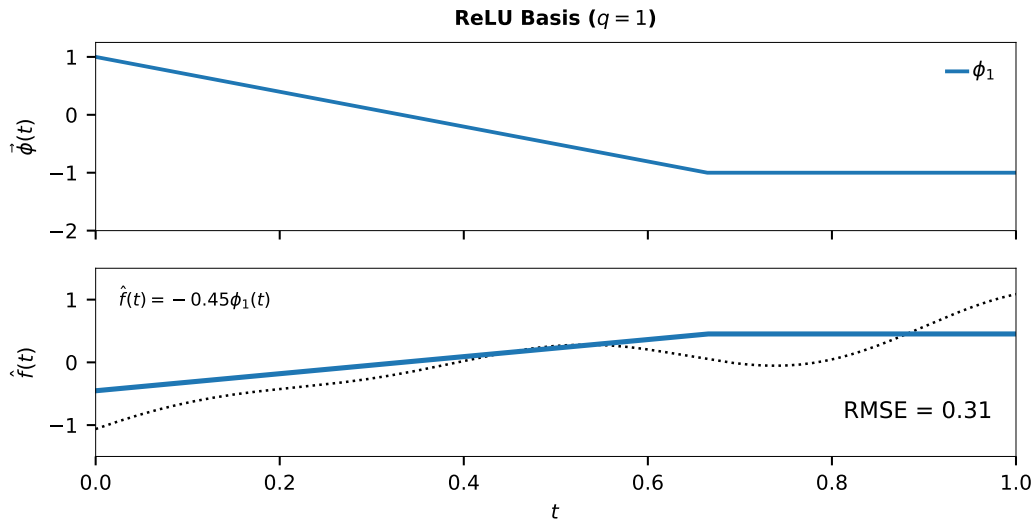
Representing Functions: Cosine Basis



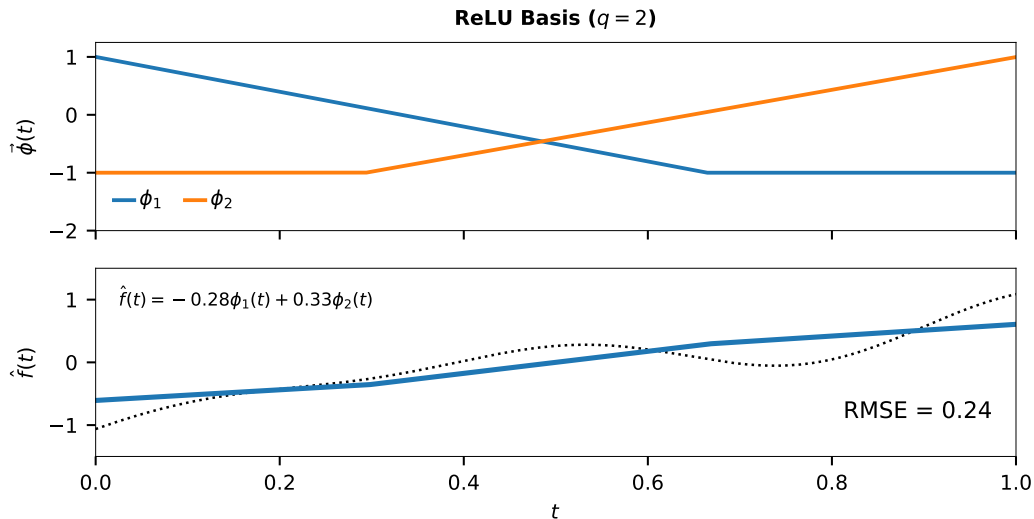
Representing Functions: Cosine Basis



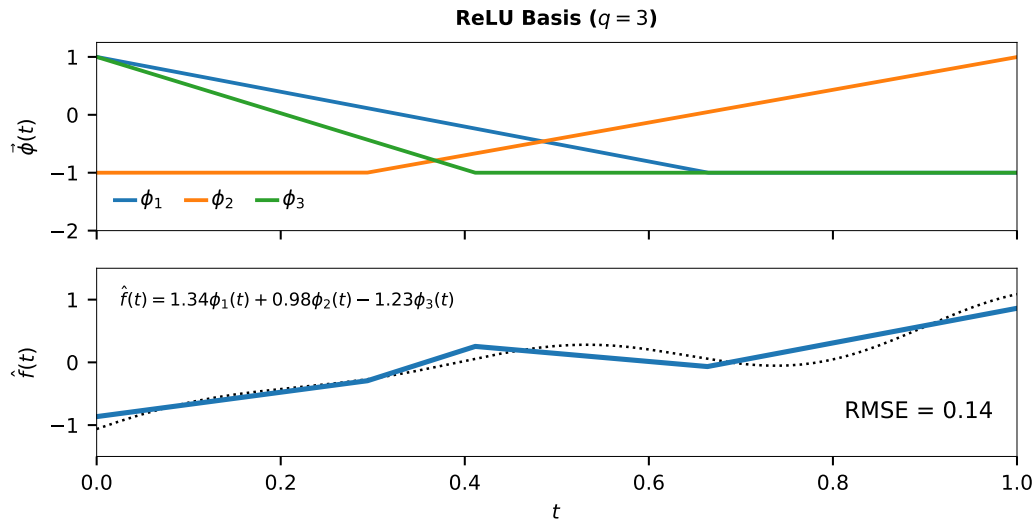
Representing Functions: ReLU Basis



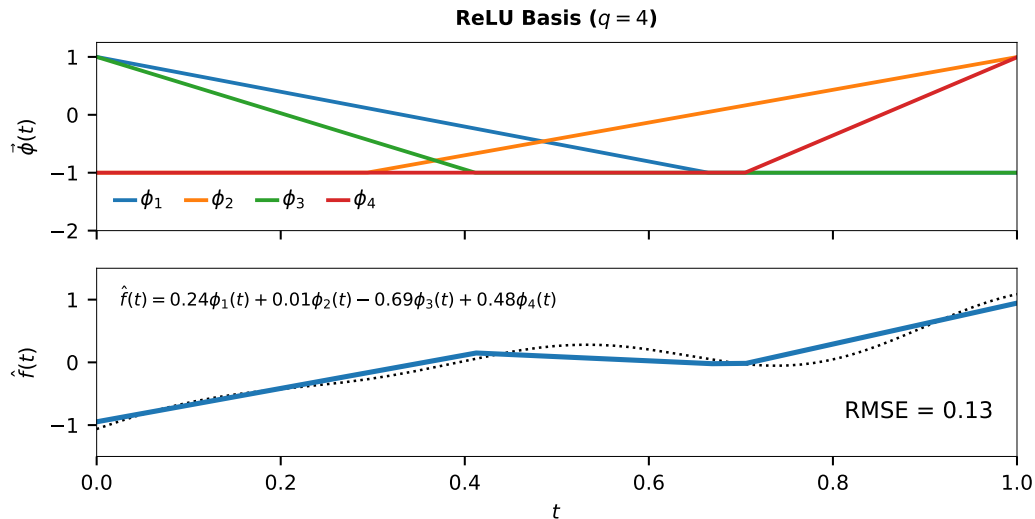
Representing Functions: ReLU Basis



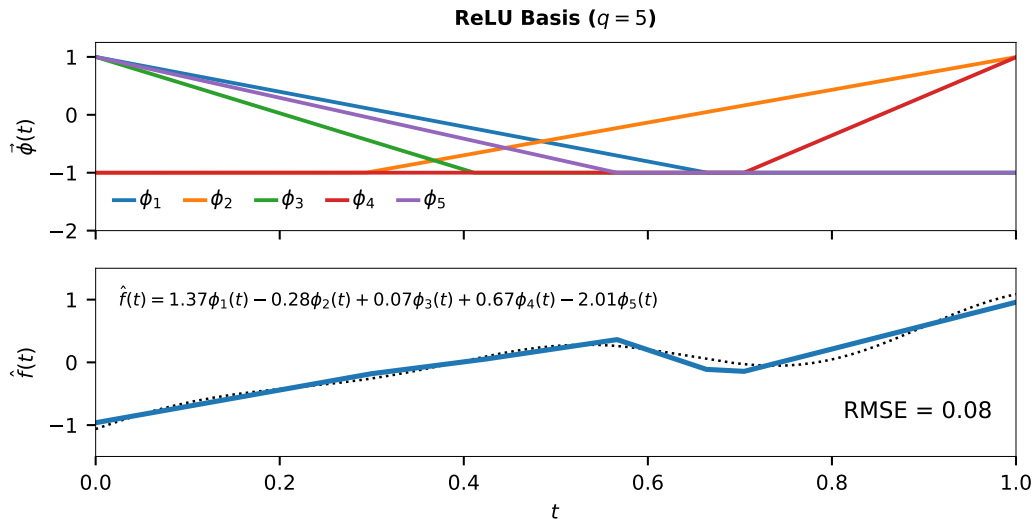
Representing Functions: ReLU Basis



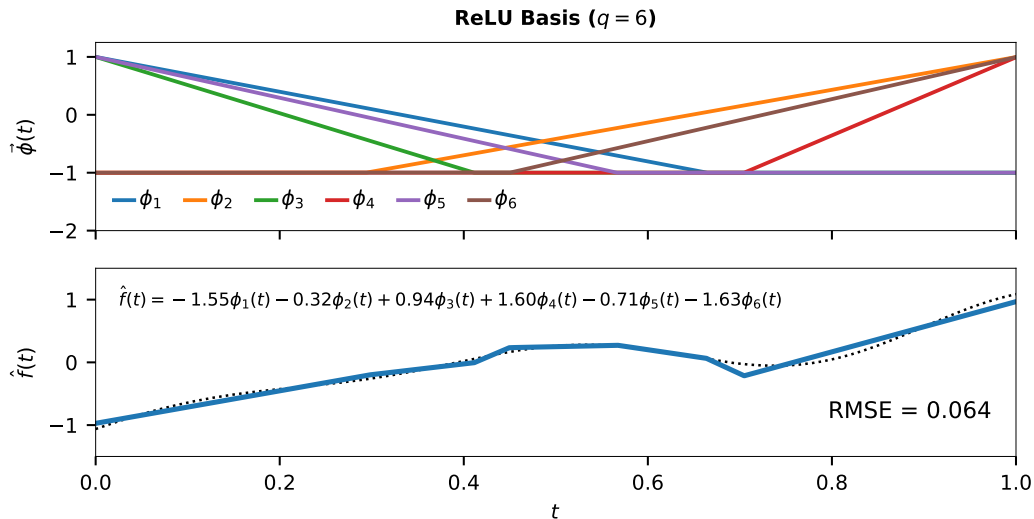
Representing Functions: ReLU Basis



Representing Functions: ReLU Basis

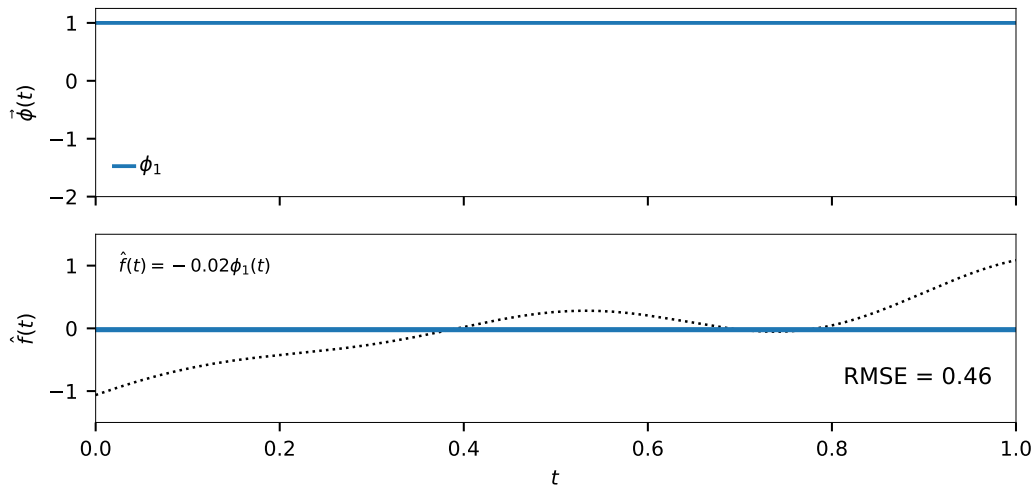


Representing Functions: ReLU Basis



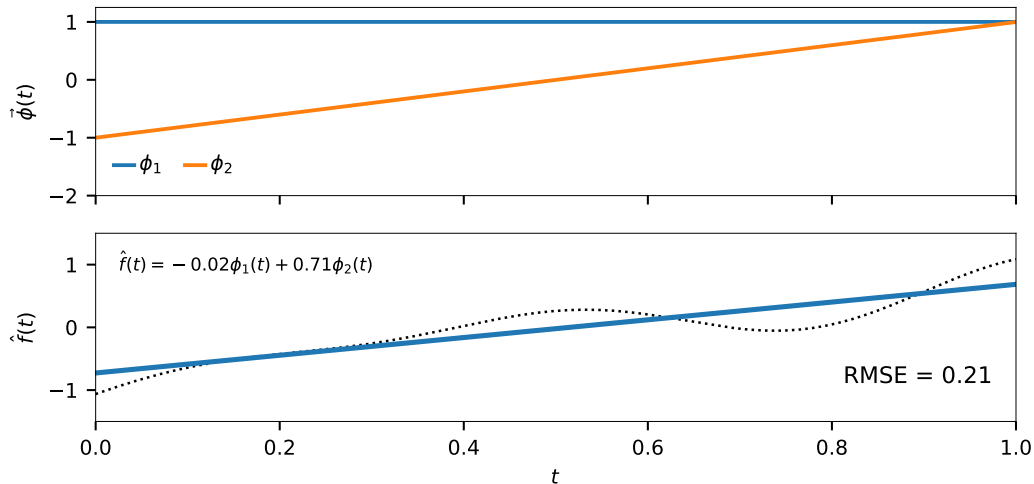
Representing Functions: Legendre Basis

Legendre Polynomials ($q = 1$)



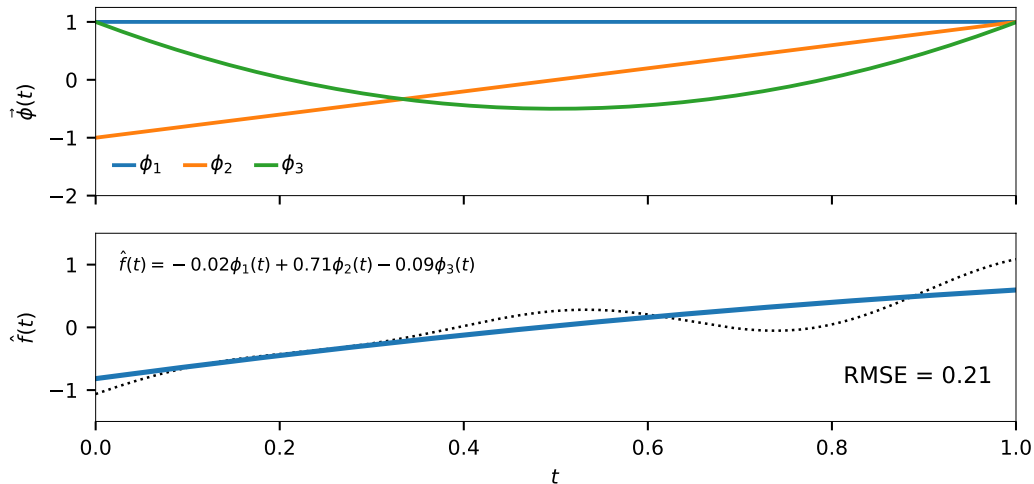
Representing Functions: Legendre Basis

Legendre Polynomials ($q = 2$)



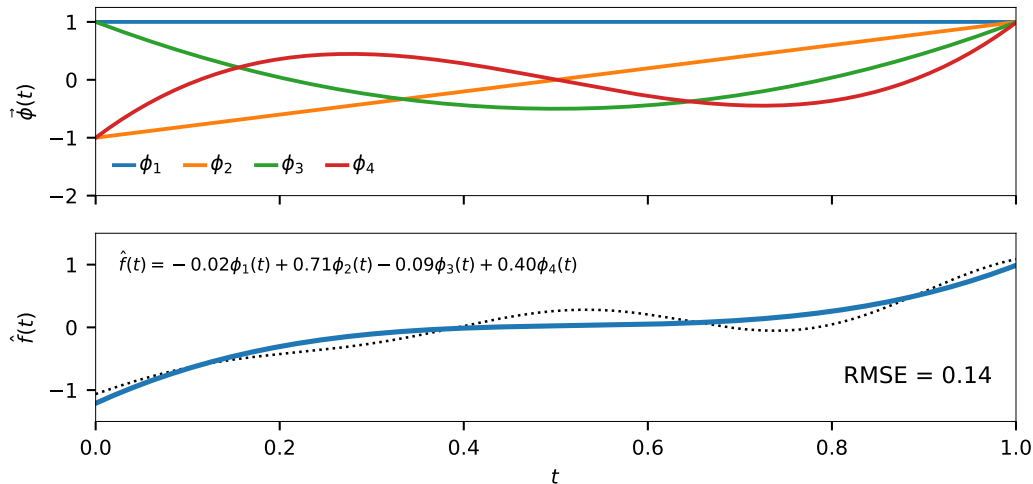
Representing Functions: Legendre Basis

Legendre Polynomials ($q = 3$)



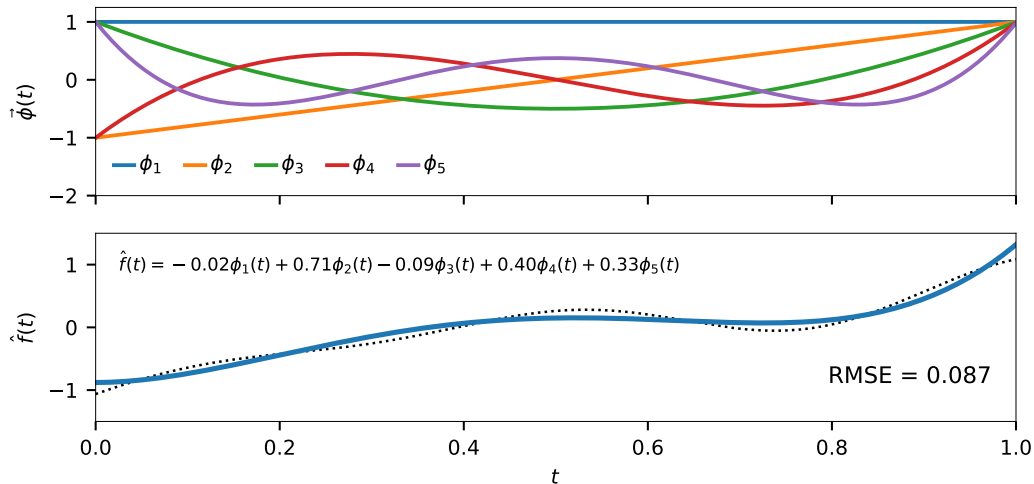
Representing Functions: Legendre Basis

Legendre Polynomials ($q = 4$)



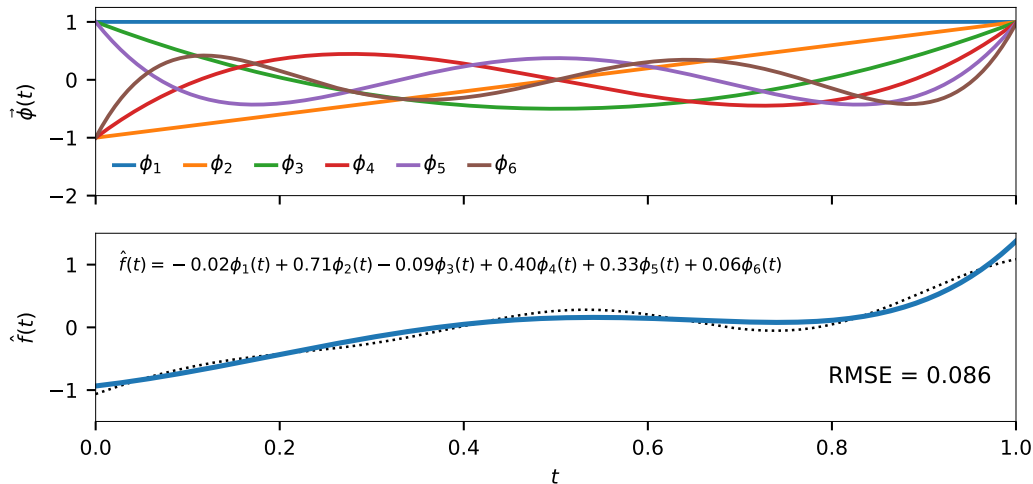
Representing Functions: Legendre Basis

Legendre Polynomials ($q = 5$)

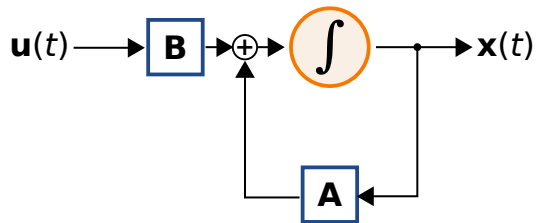


Representing Functions: Legendre Basis

Legendre Polynomials ($q = 6$)



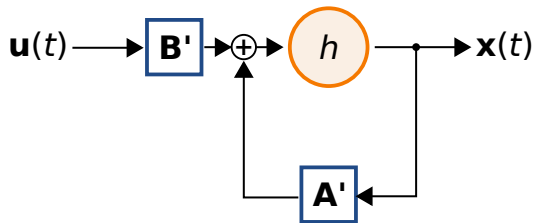
Implementing the Delay Network



$$\theta \mathbf{A} = a_{ij} \in \mathbb{R}^{q \times q},$$

$$\theta \mathbf{B} = b_i \in \mathbb{R}^q,$$

$$\mathbf{A}' = \tau \mathbf{A} + \mathbf{I}$$

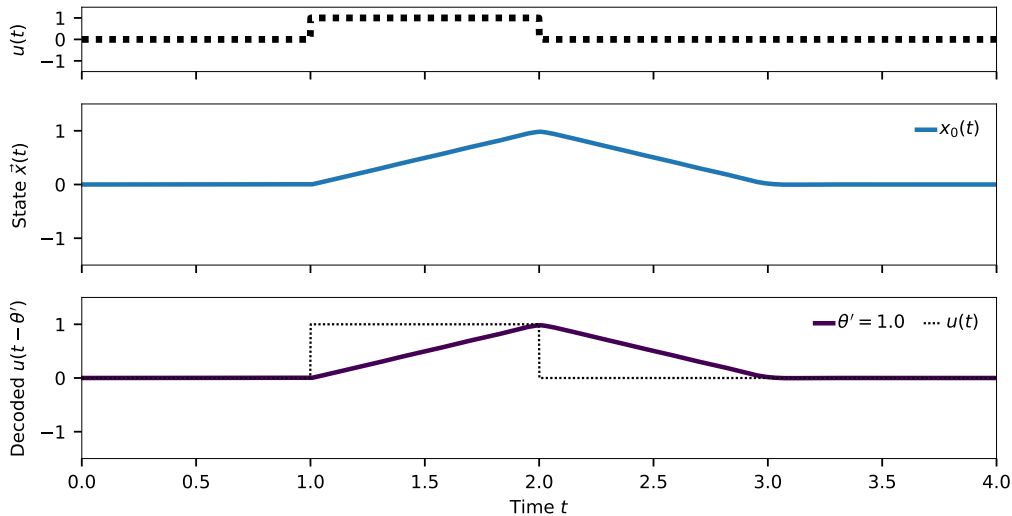


$$a_{ij} = \begin{cases} (2i+1)(-1) & i < j, \\ (2i+1)(-1)^{i-j+1} & i \geq j \end{cases}$$

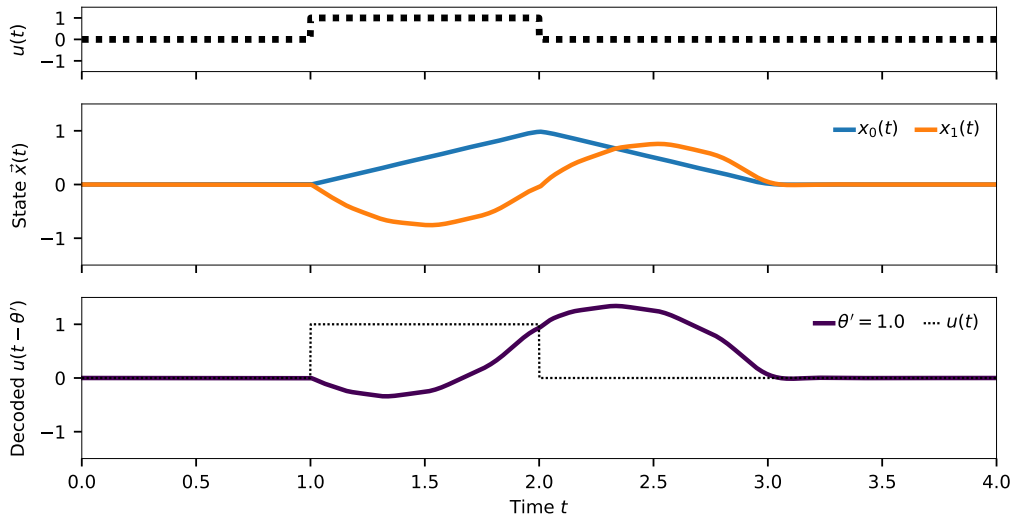
$$b_i = (2i+1)(-1)^i$$

$$\mathbf{B}' = \tau \mathbf{B}$$

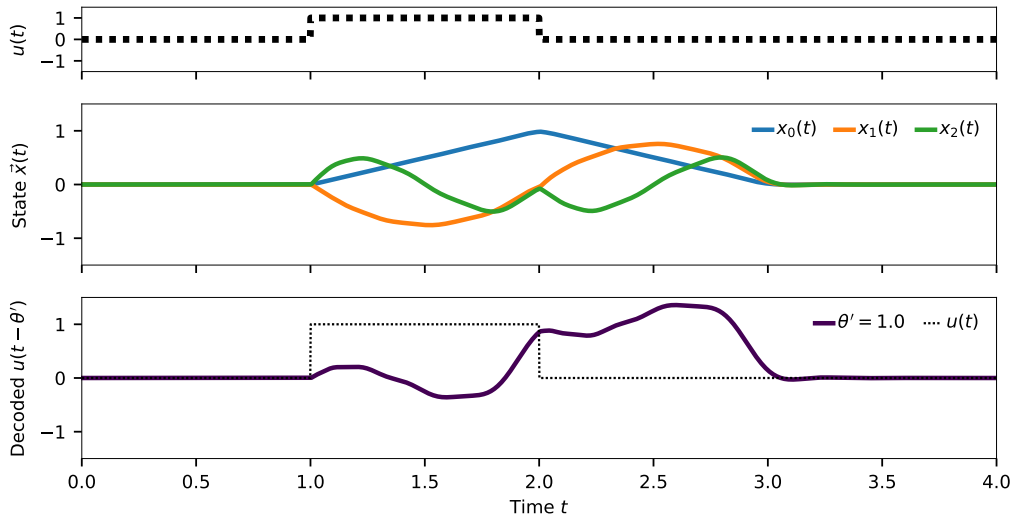
Delay Network: Step Function



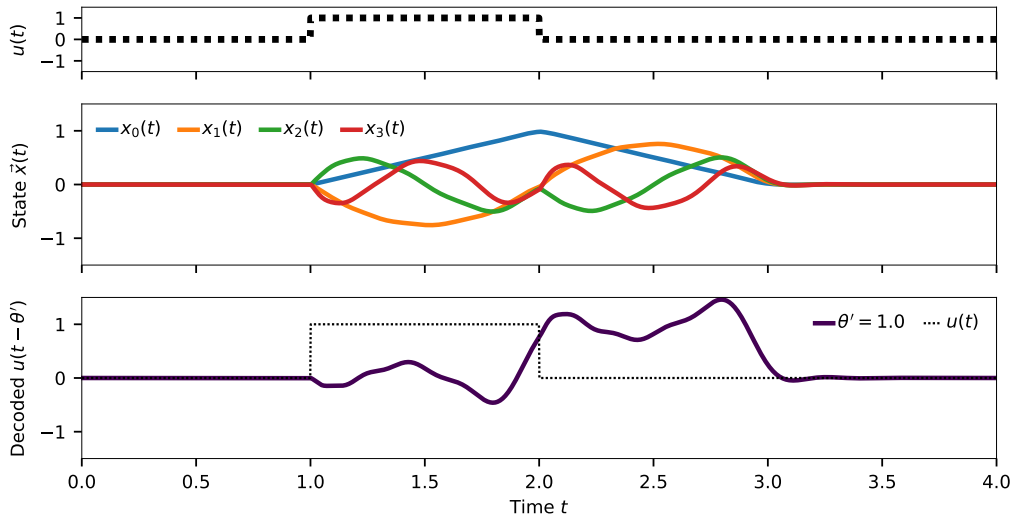
Delay Network: Step Function



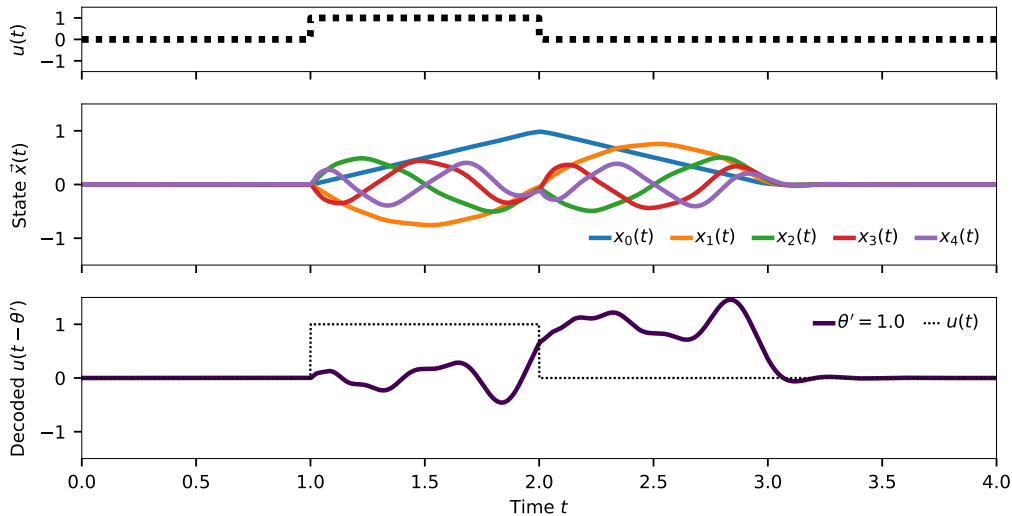
Delay Network: Step Function



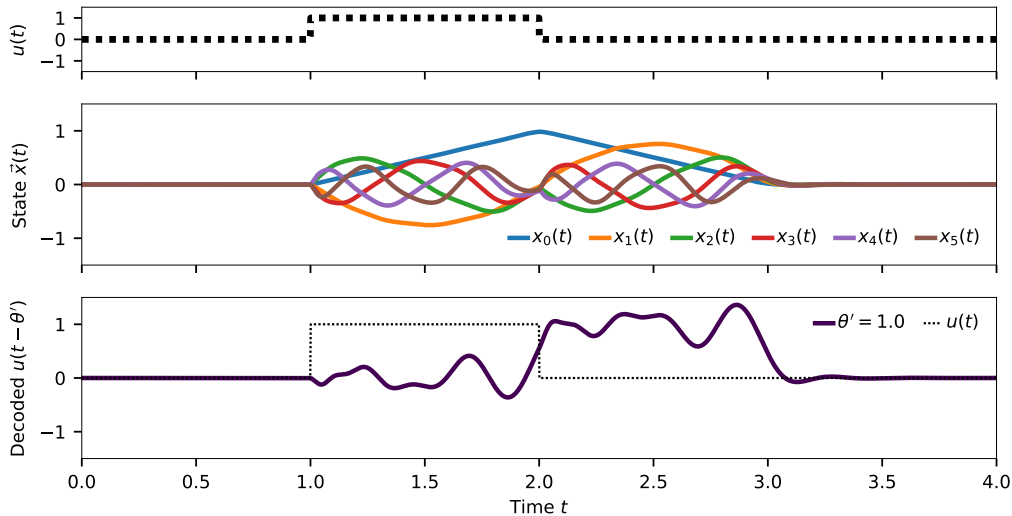
Delay Network: Step Function



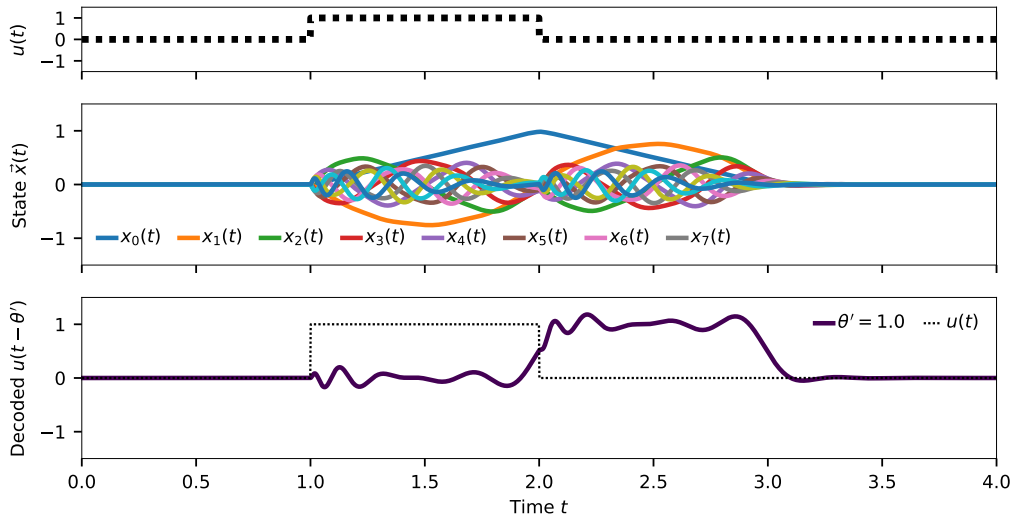
Delay Network: Step Function



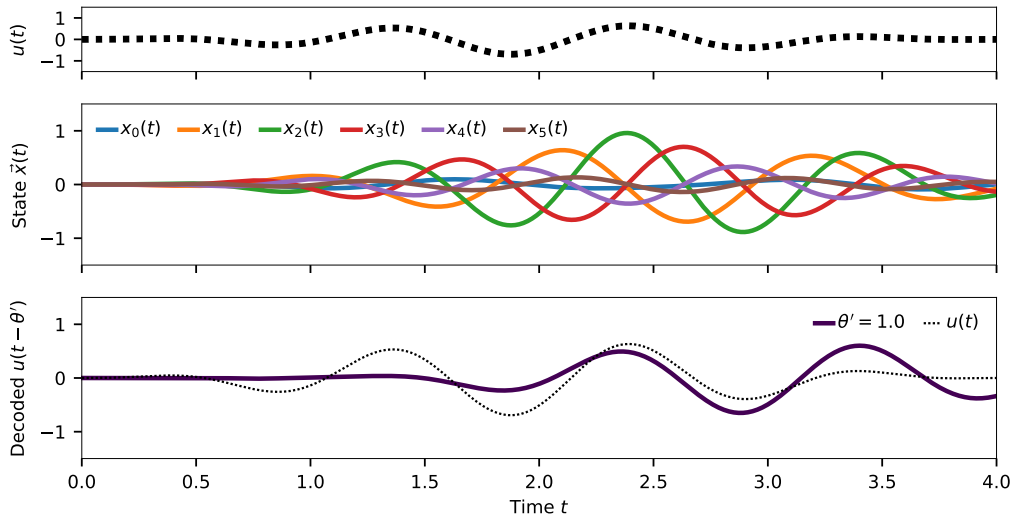
Delay Network: Step Function



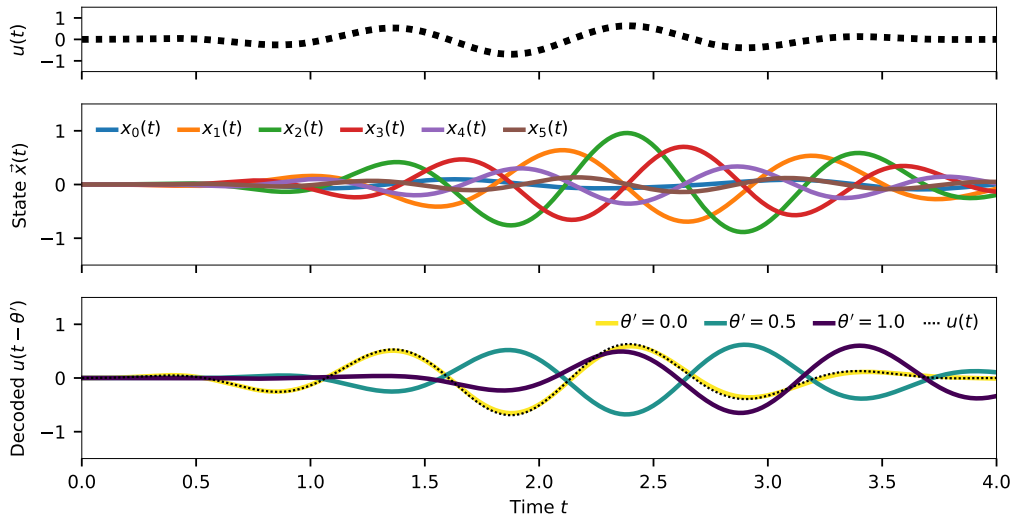
Delay Network: Step Function



Delay Network: Windowed Sine Function



Delay Network: Windowed Sine Function



Delay Network: Windowed Sine Function

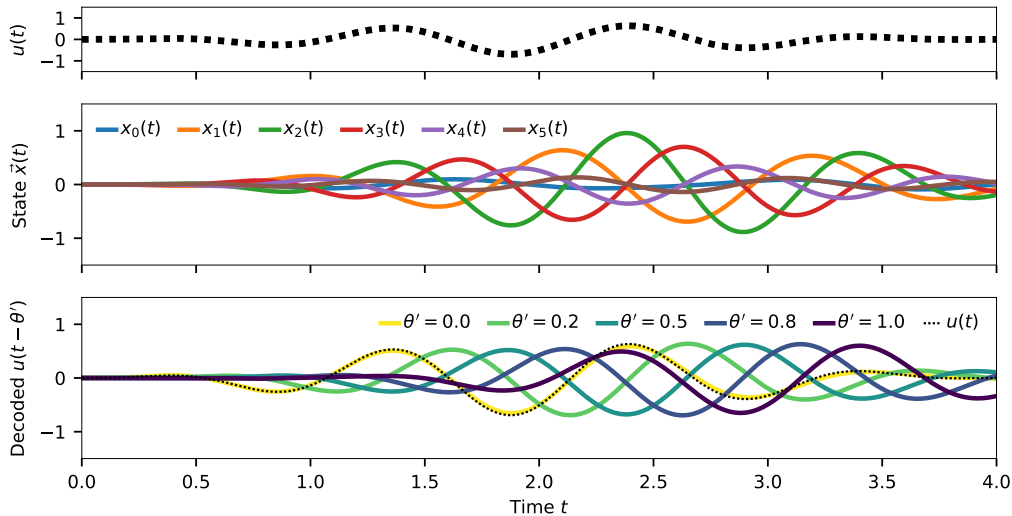


Image sources

Title slide

Infrared Photograph of a Sundial Near the Einstein Tower in Potsdam, Germany

Author: DrNRNowaczyk, 2007.

From Wikimedia.