Spatial Semantic Pointers (SSPs)

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SYDE 556/750

Spatial Semantic Pointers

- Semantic pointers represent standard discrete structures (lists, trees, etc.)
- SSPs allow recurrent convolutions to have fractional powers

$$B^k = \underbrace{B \circledast B \circledast \ldots \circledast B}_{B \text{ appears } k \text{ times}}$$

- Compute fractional *k* in Fourier space

$$B^k = \mathcal{F}^{-1}\left\{\mathcal{F}\left\{B\right\}^k\right\}, \quad k \in \mathbb{R}$$

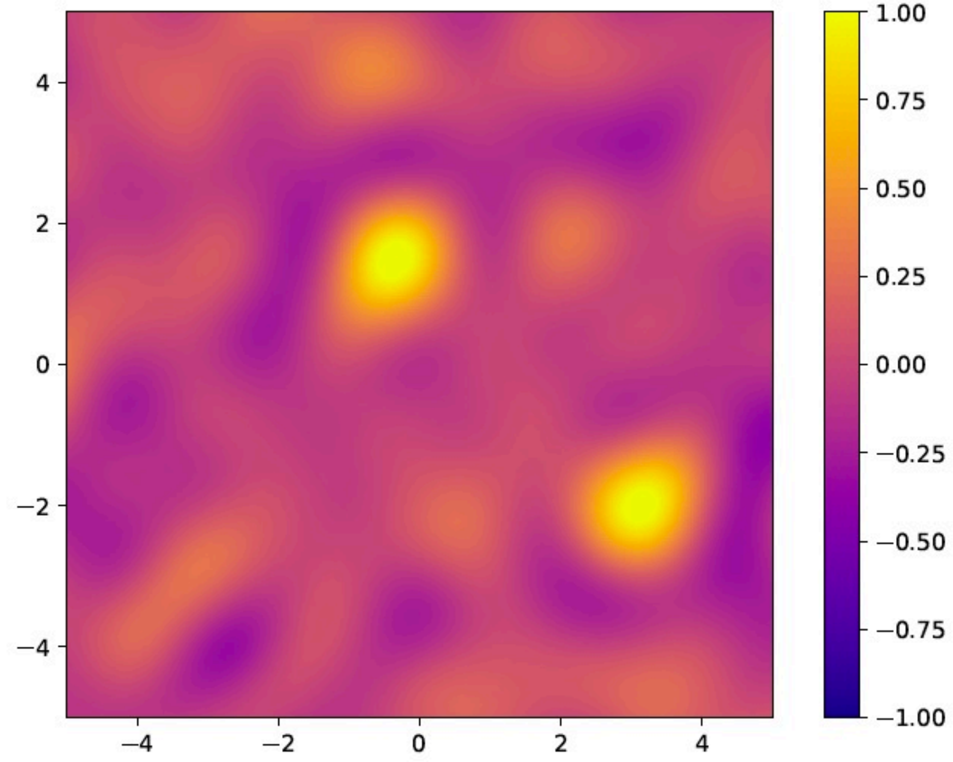
$$S(x,y) = X^x \circledast Y^y = \mathcal{F}^{-1}\{\mathcal{F}\{X\}^x \odot \mathcal{F}\{Y\}^y\}$$

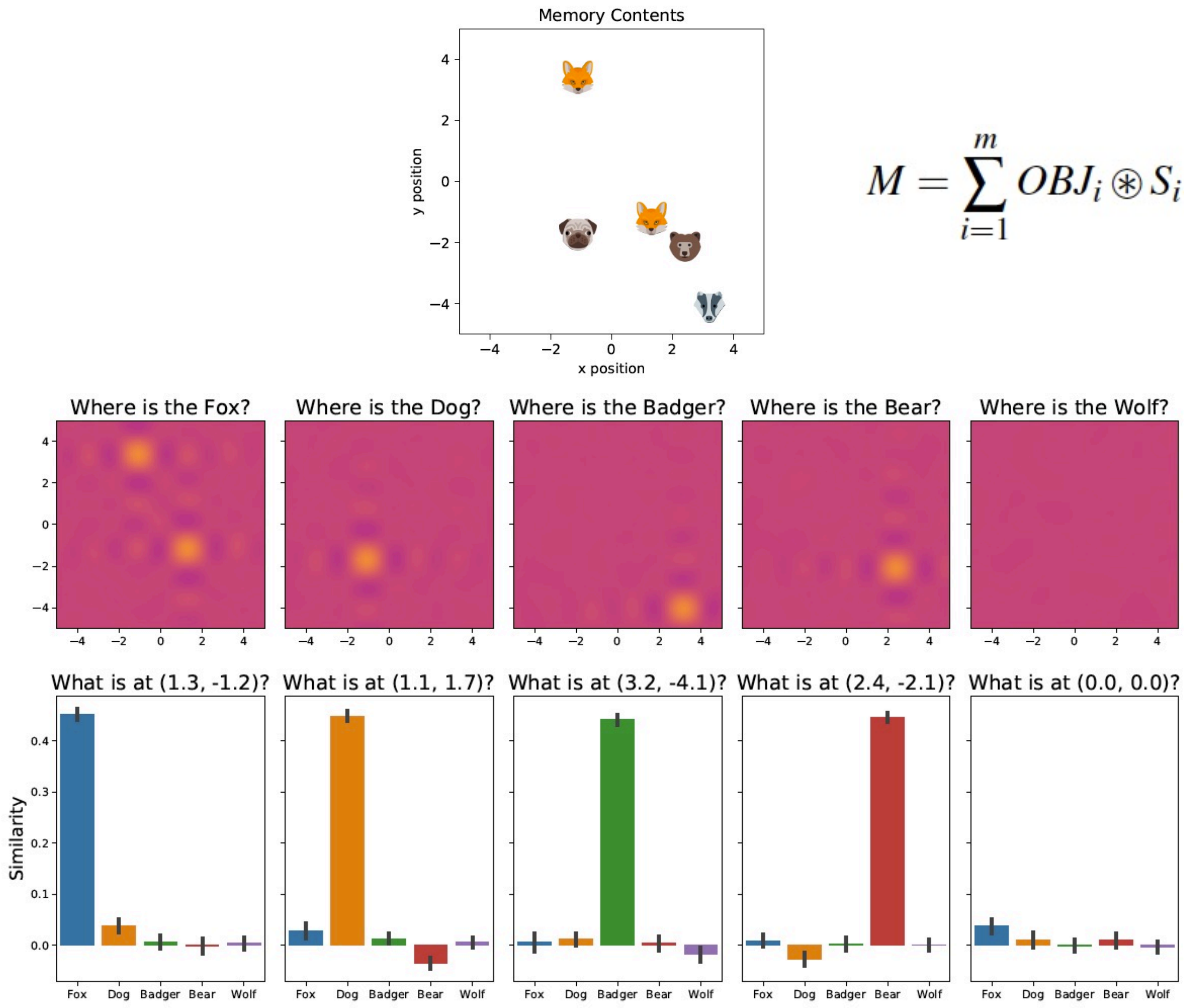
Spatial Semantic Pointers

- Represent continuous space (Clifford torus)

$$S(x, y) = X^x \circledast Y^y$$

- Heat map to visualize vector contents
- Dot product between SSP at every possible position and S




Memory Contents
y position
x position
$$M = \sum_{i=1}^{m} OBJ_i \circledast S_i$$
Where is the Fox?
Where is the Dog?
Where is the Badger?
Where is the Bear?
Where is the Wolf?
What is at (1.3, -1.2)?
What is at (1.1, 1.7)?
What is at (3.2, -4.1)?
What is at (2.4, -2.1)?
What is at (0.0, 0.0)?
Similarity
Fox
Dog
Badger
Bear
Wolf

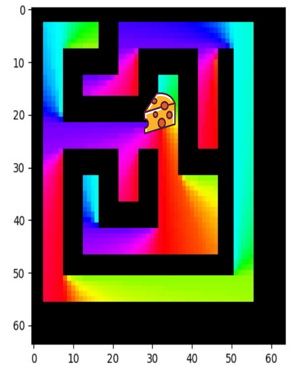
Spatial Manipulations

Desiderata	Accuracy	
	Non-Neural	Neural
Query single object	99.1%	95.7%
Query missing object	99.4%	96.7%
Query location	97.3%	94.7%
Query duplicate object	97.4%	95.3%
Query Region	90.4%	73.5%
Slide single object in group (all objects)	75.7%	67.3%
Slide single object in group (moved object)	100.0%	100.0%
Slide whole group	97.8%	96.7%
Readout x-y location from SSP	95.7%	94.1%
Construct SSP from x-y location	100.0%	99.0%

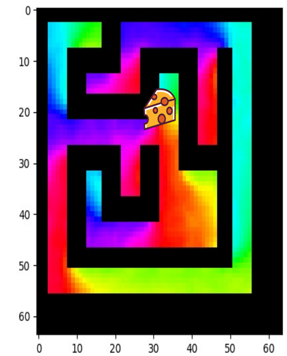
Navigating to Goal

Single layer MLP, same for each encoding (tried many more).

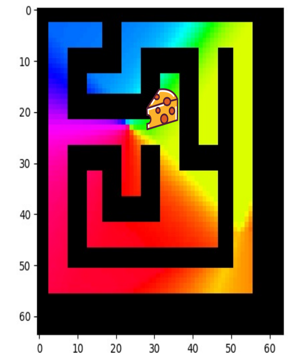
Ground Truth
RMSE: 0



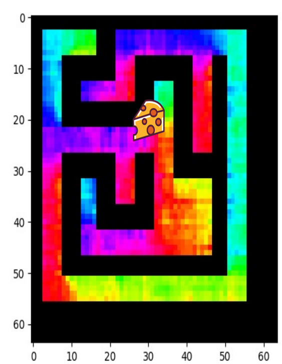
SSPs
RMSE: 0.0529



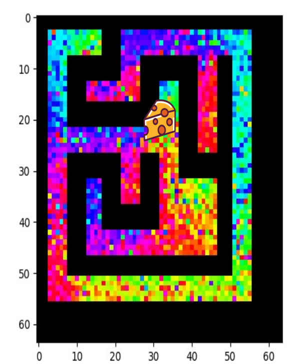
Random Projection
RMSE: 0.5351



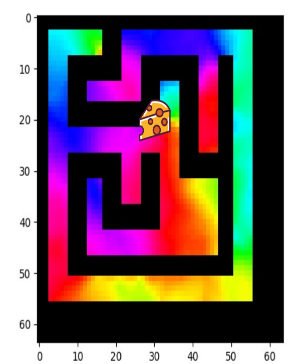
Trig Functions
RMSE: 0.1350



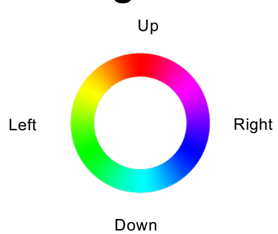
Random Mapping
RMSE: 0.2580



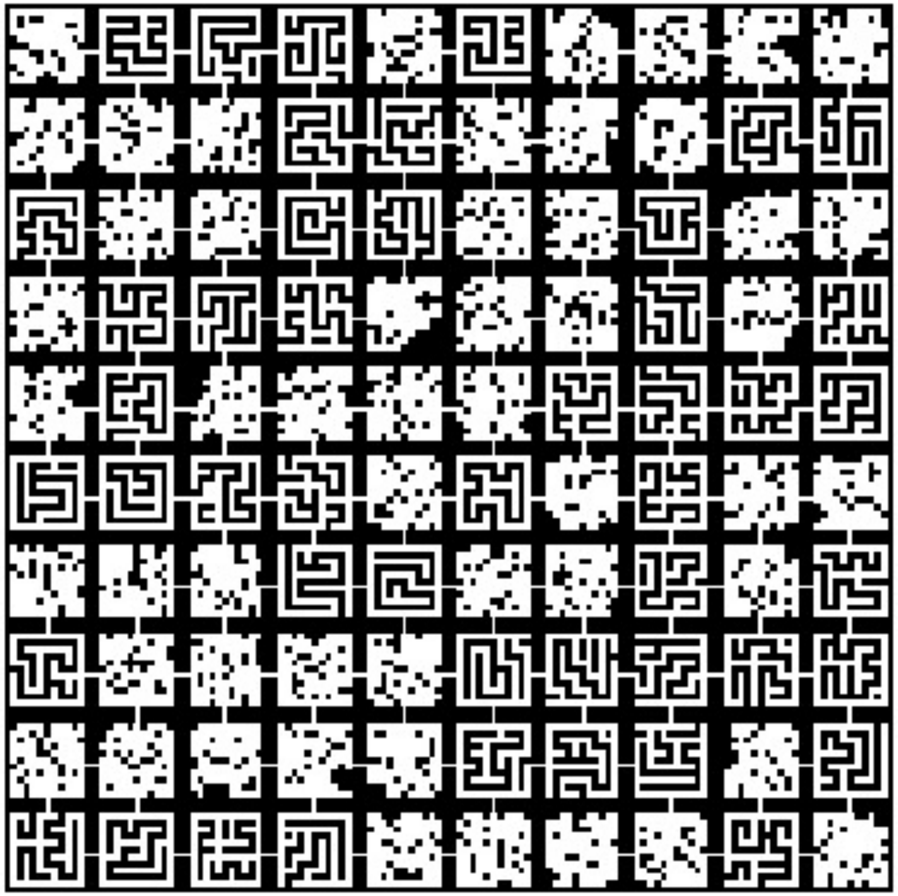
2D Coordinates
RMSE 0.1984

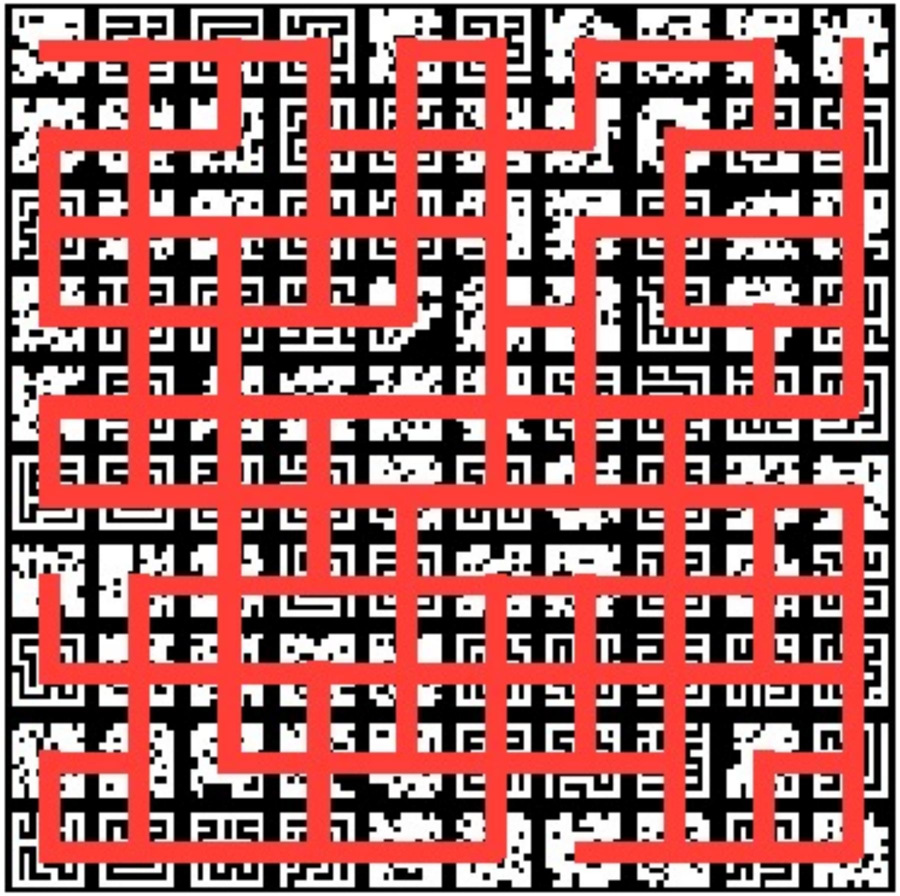


Legend



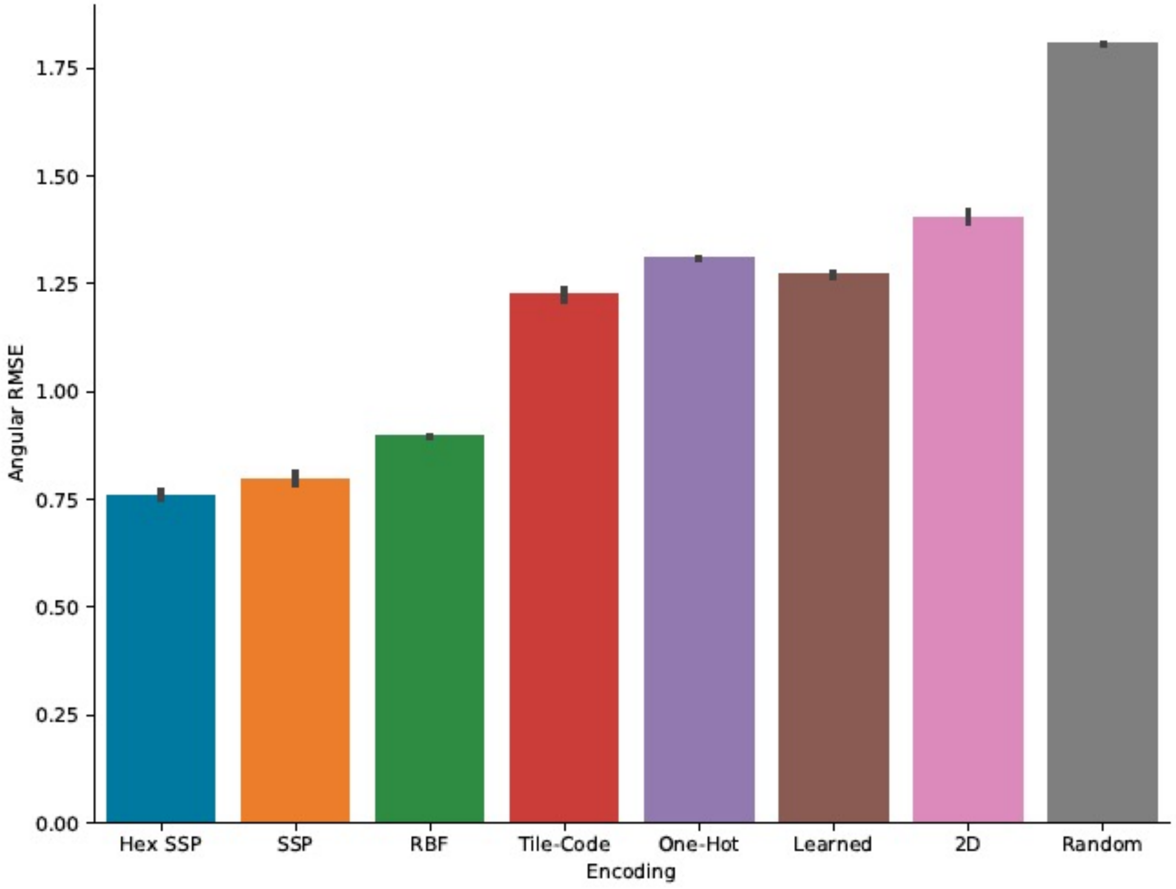
Scaling





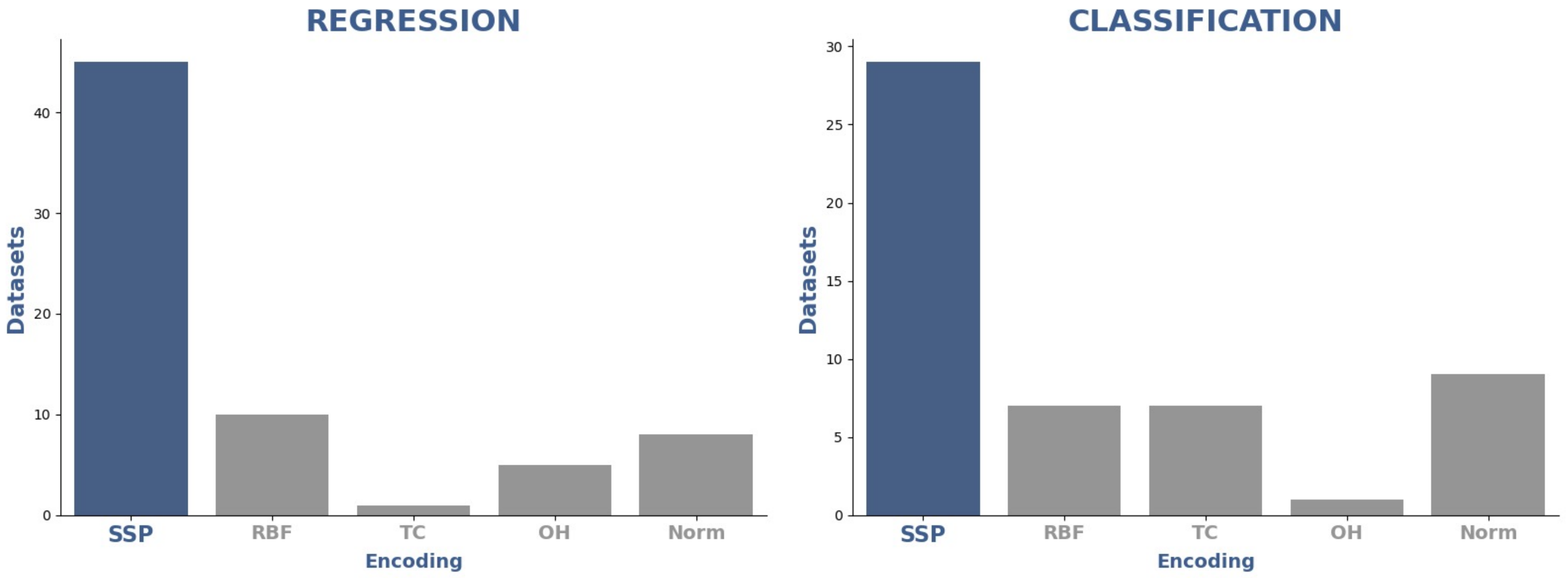
Example 10x10 joined, hierarchical environment

Scaling



Performance of different encoders on 10x10 large maze from any point to any other

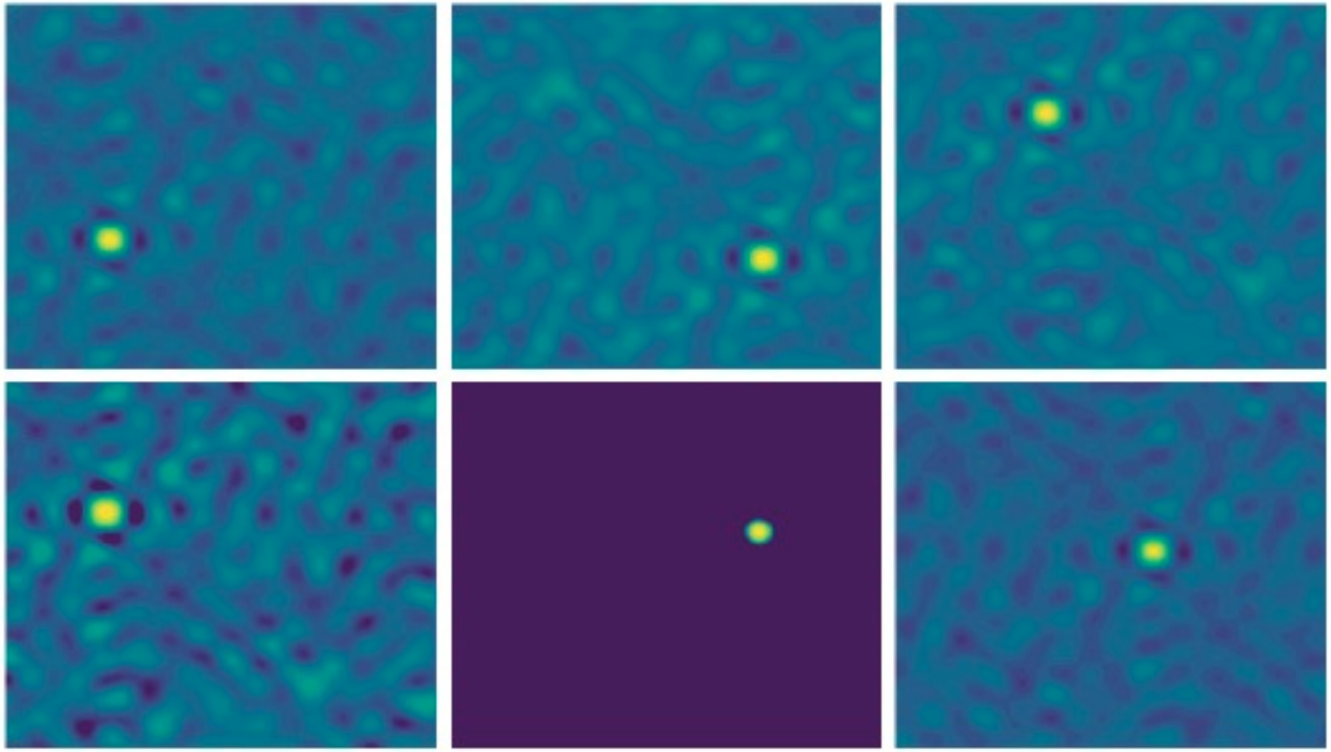
Good for General ML



SSPs are more accurate on a large majority of 122 standard ML benchmarks.

How to Choose Axis Vectors

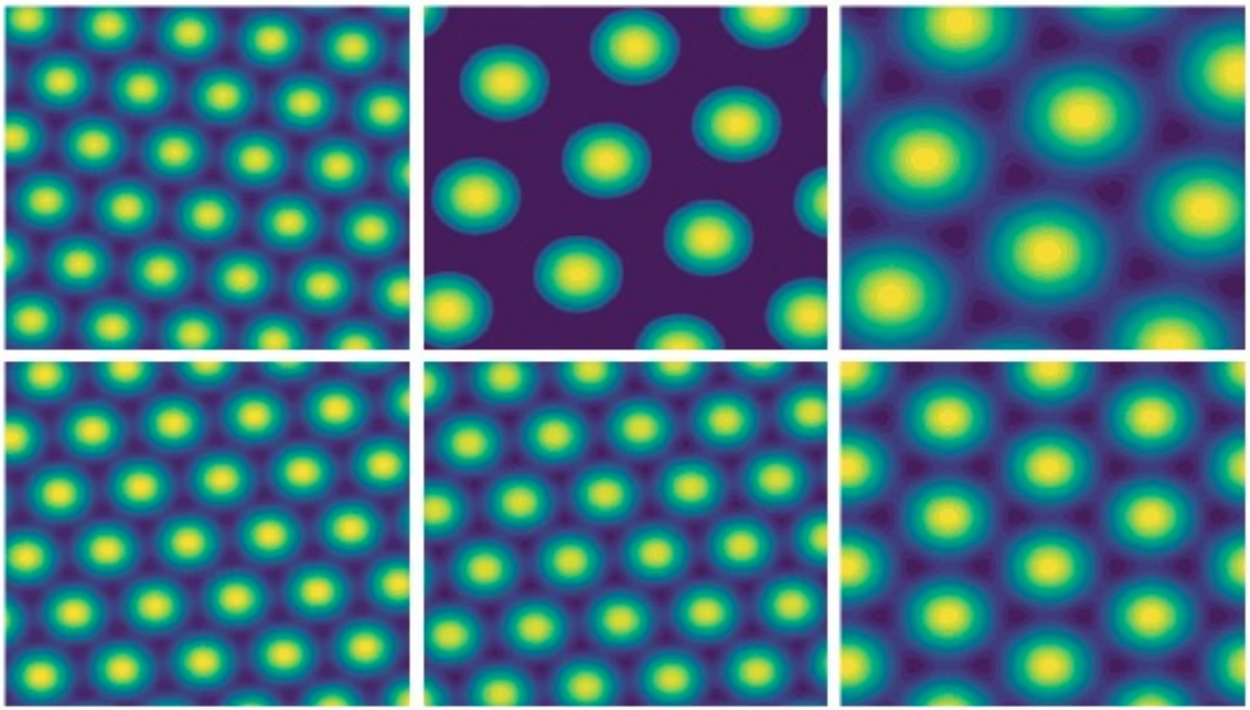
- Randomly (examples til now)



Tuning curves of neurons with random axis vectors and evenly tiled SSPs as encoders

How to Choose Axis Vectors

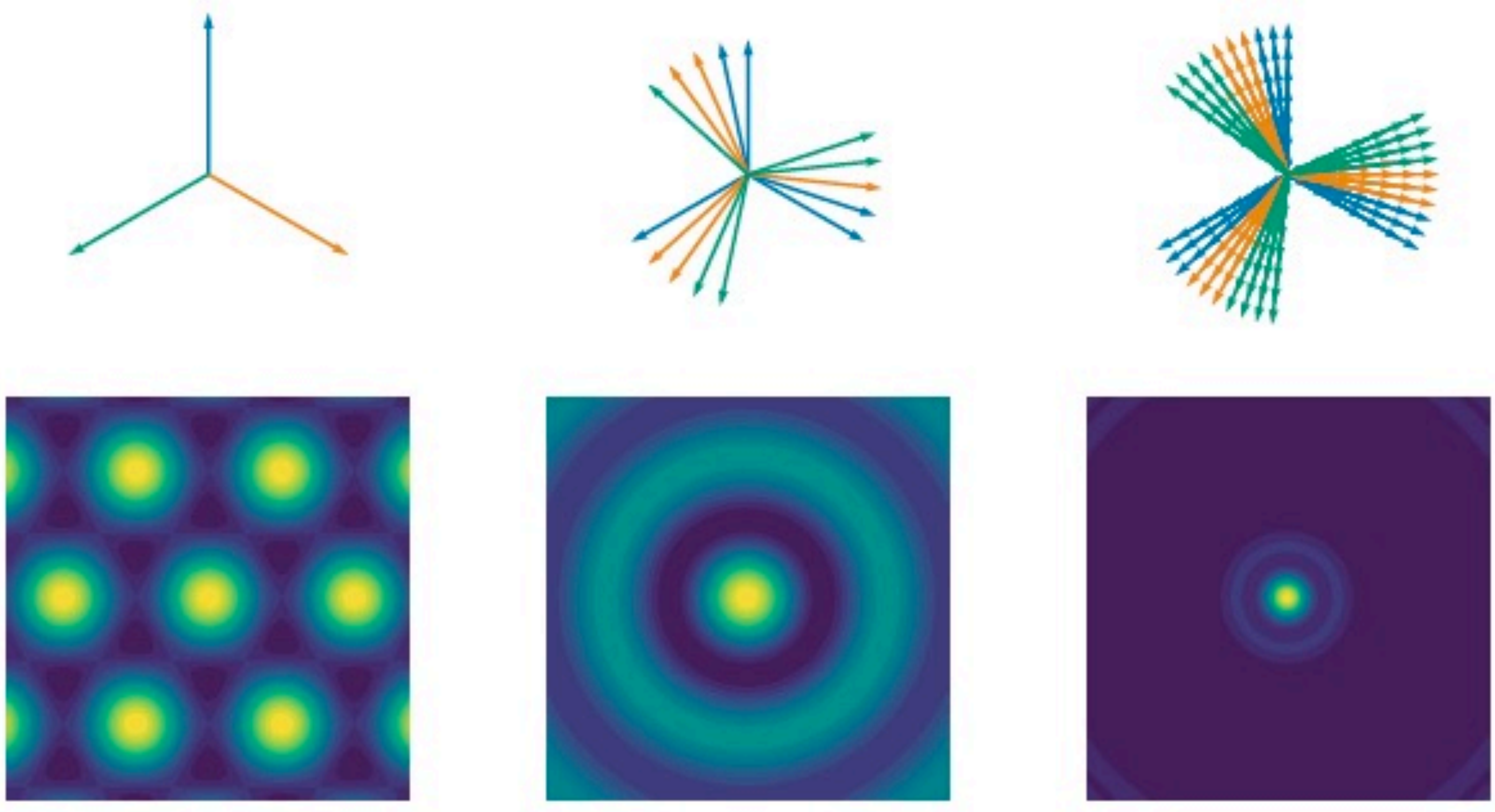
- With plane wave structure



Tuning curves of neurons with structured axis vectors and encoders picking out plane waves

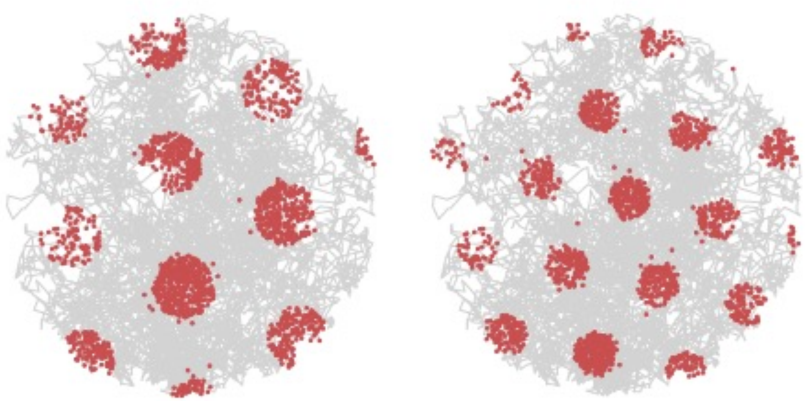
How to Choose Axis Vectors

- Sums of planar waves



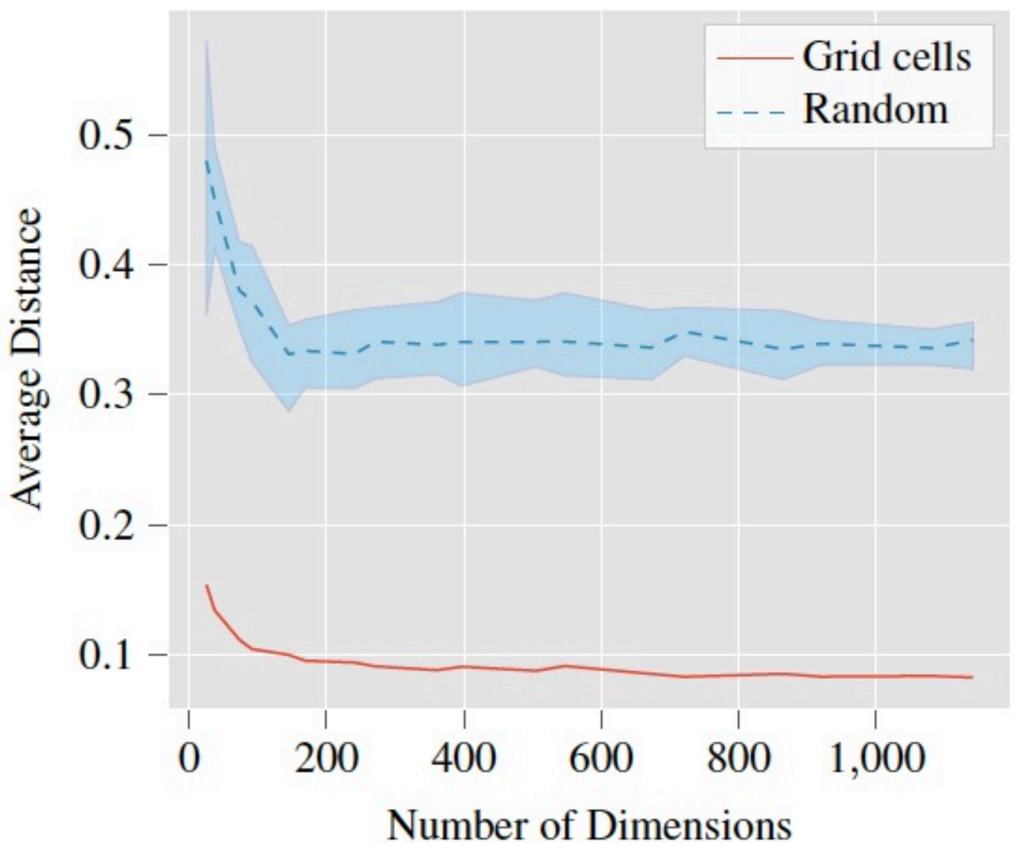
Grid Cells

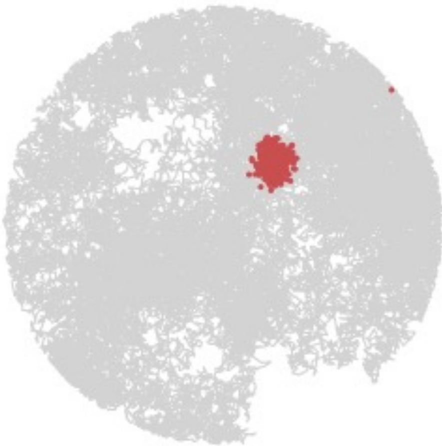
- With plane wave structure, spiking neurons give grid cell responses
- We can combine them to get place cells (with standard NEF decoders)

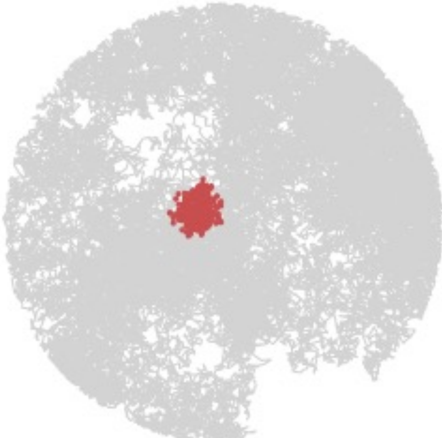


SSP Grid cells

Place Cells


Grid cells
Random
Average Distance
0.5
0.4
0.3
0.2
0.1
0
200
400
600
800
1,000
Number of Dimensions





Mathematical properties

- Euclidean space is preserved on a high-d torus

$$S(x_1, y_1) \circledast S(x_2, y_2) = S(x_1 + x_2, y_1 + y_2)$$

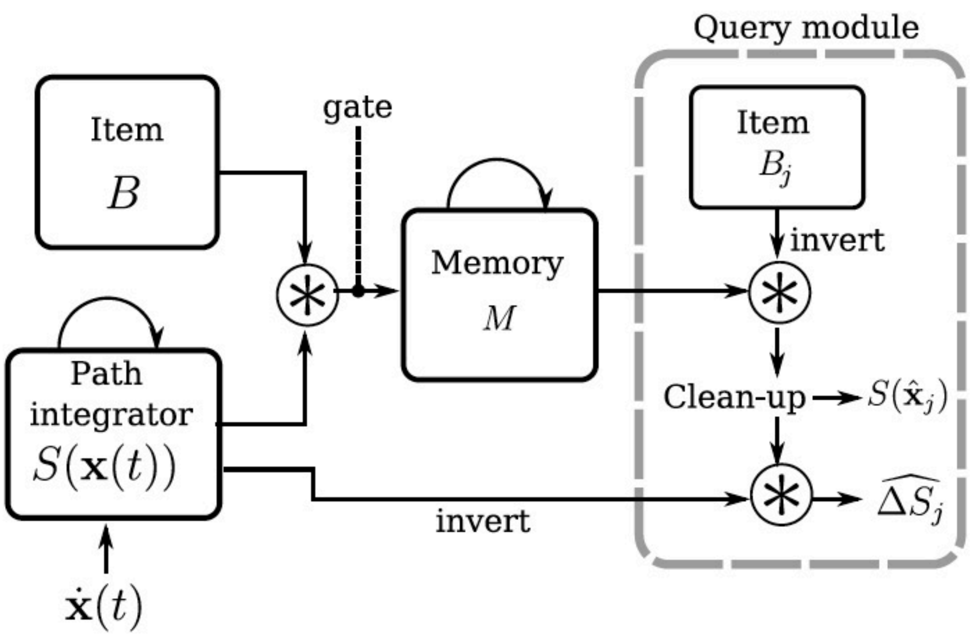
- E.g., $S(x, y) \circledast S(\Delta x, \Delta y) = X^{x+\Delta x} \circledast Y^{y+\Delta y}$

- We get the benefits of high-d representation, with accurate low-d Euclidean representation

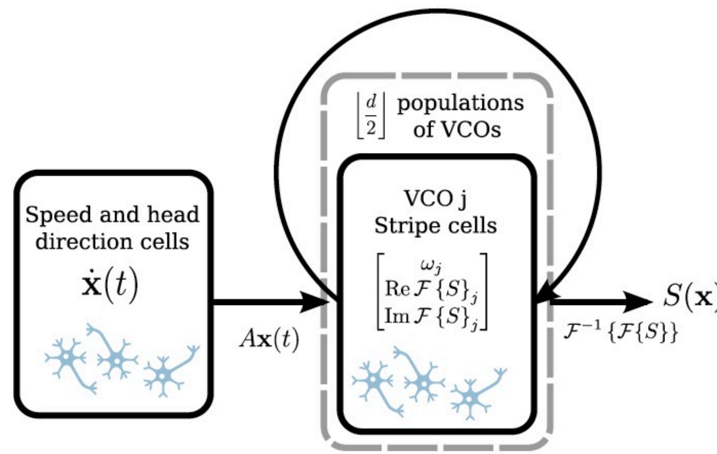
Cognitive SLAM

- SLAM with complex features at certain spatial locations
- Start with no knowledge, bind vector descriptions to particular location in space
- Combines spatial and ‘symbol’ repn in neural network
- Semantic map as opposed to standard ‘image registration’ map

Cognitive SLAM Model



Full Model

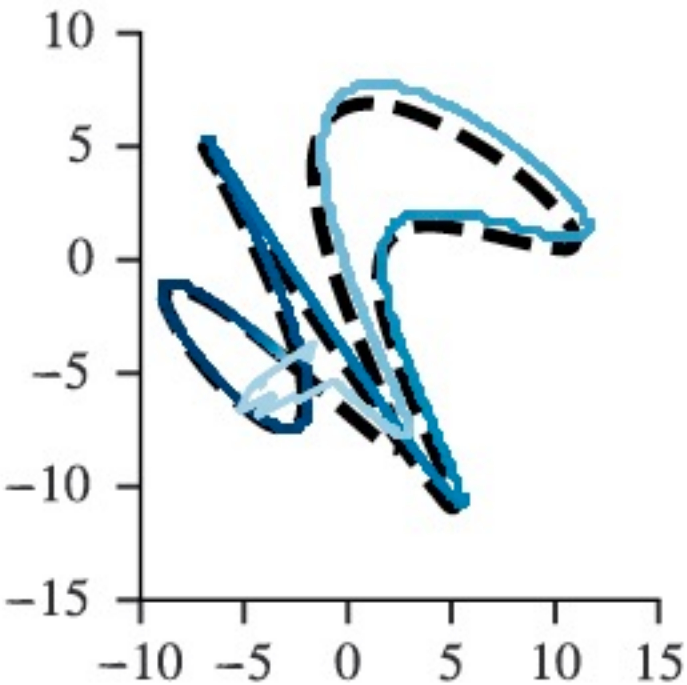


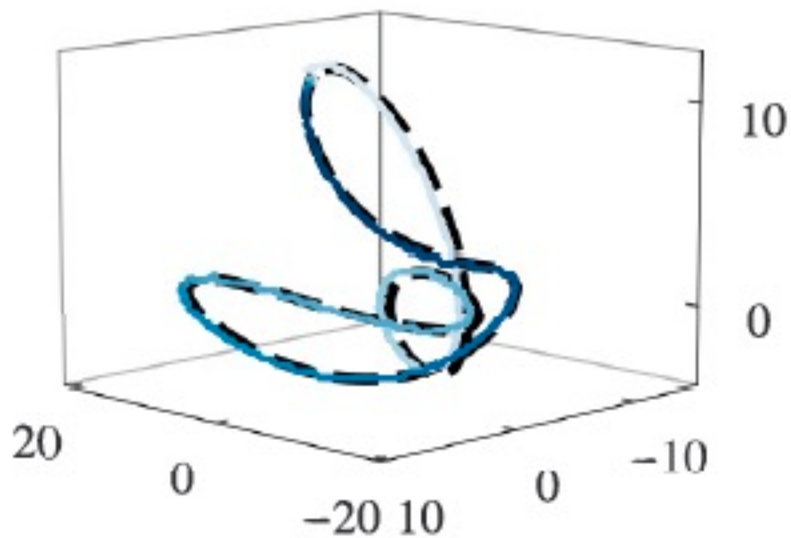
Path Integrator

- Path integrator tracks ego position from velocity
- SLAM model learns env map, outputs allo- and ego-centric position

Path Integration

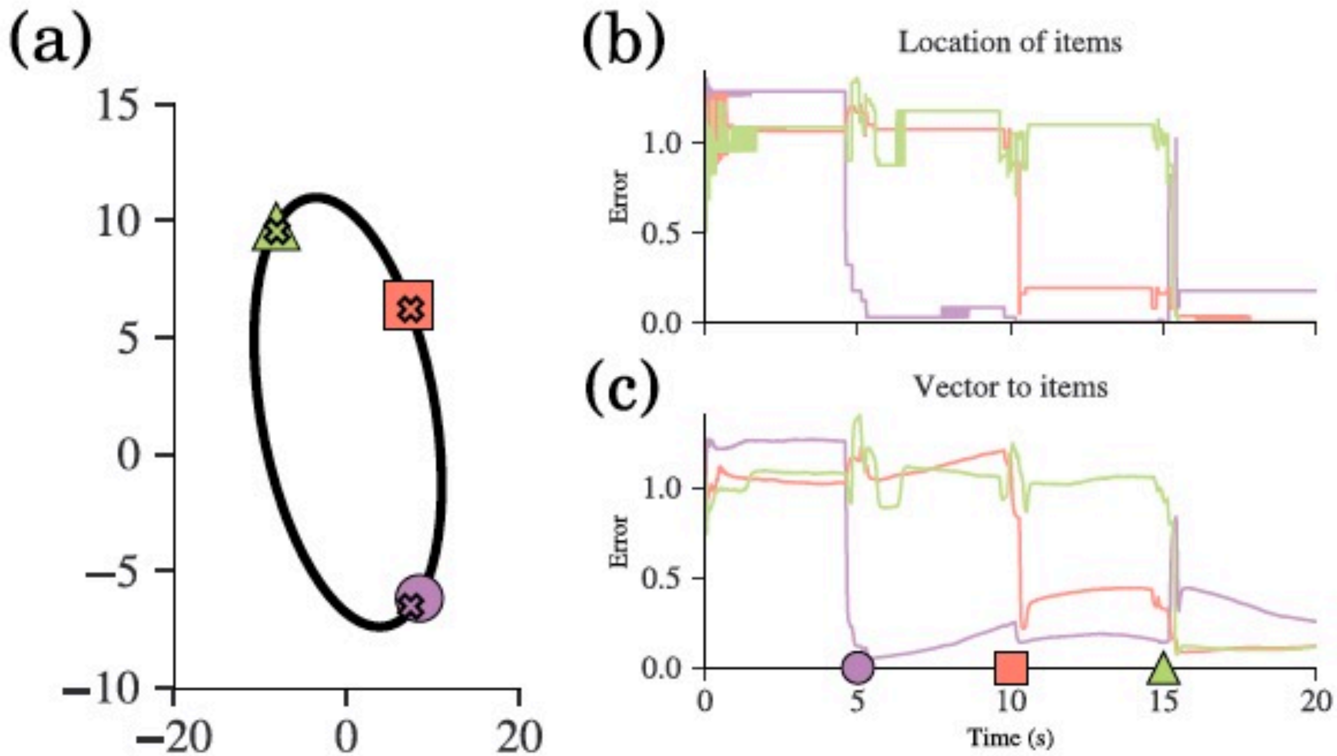
- One-minute long paths in 2D and 3D, spiking network





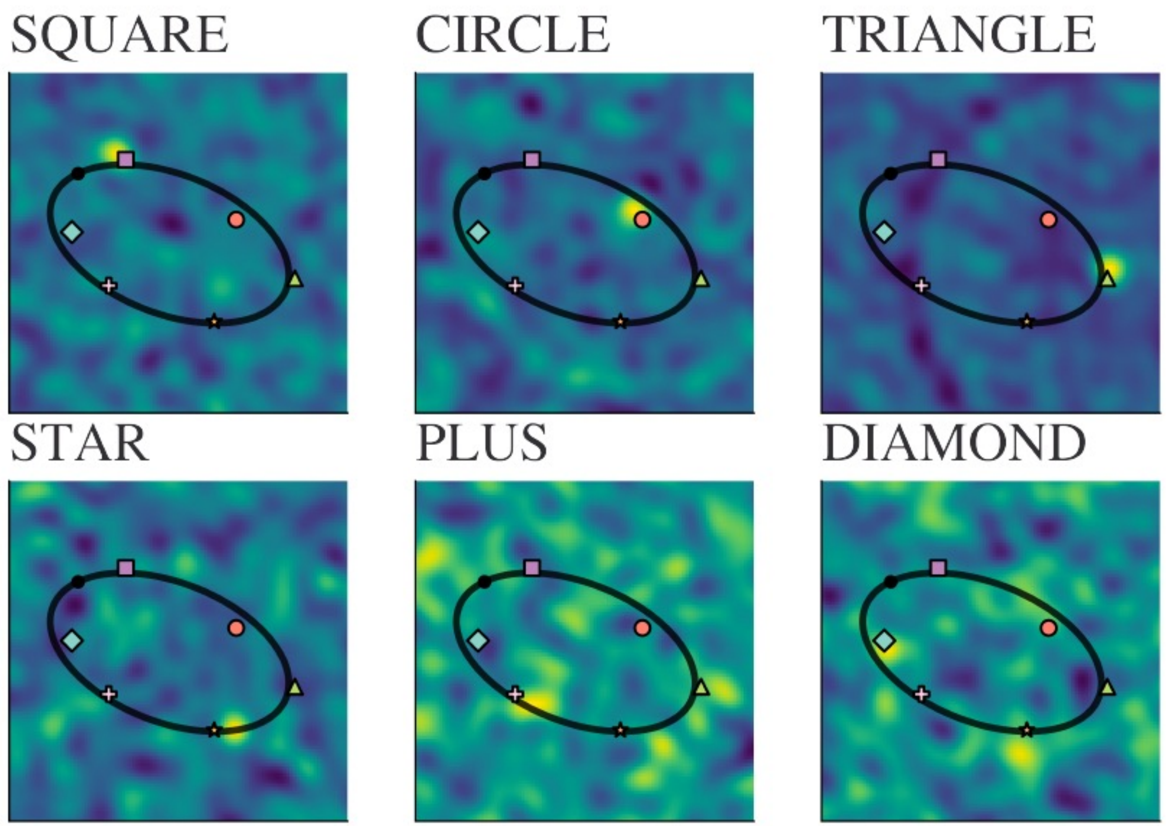
Cognitive SLAM

- Learns env by end of 20s path, full spiking
- Scaling up; symbols 'over' space



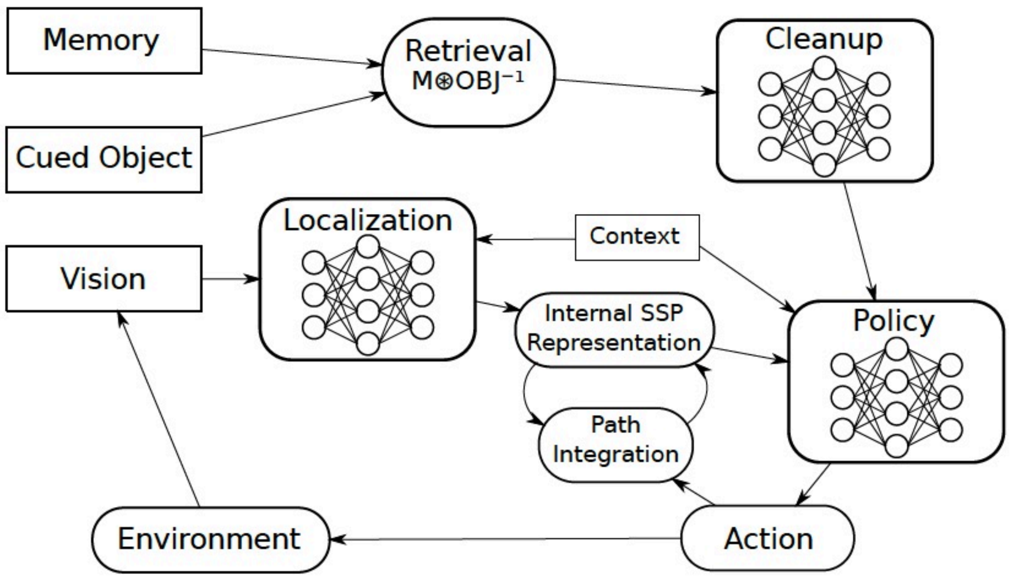
Cognitive SLAM

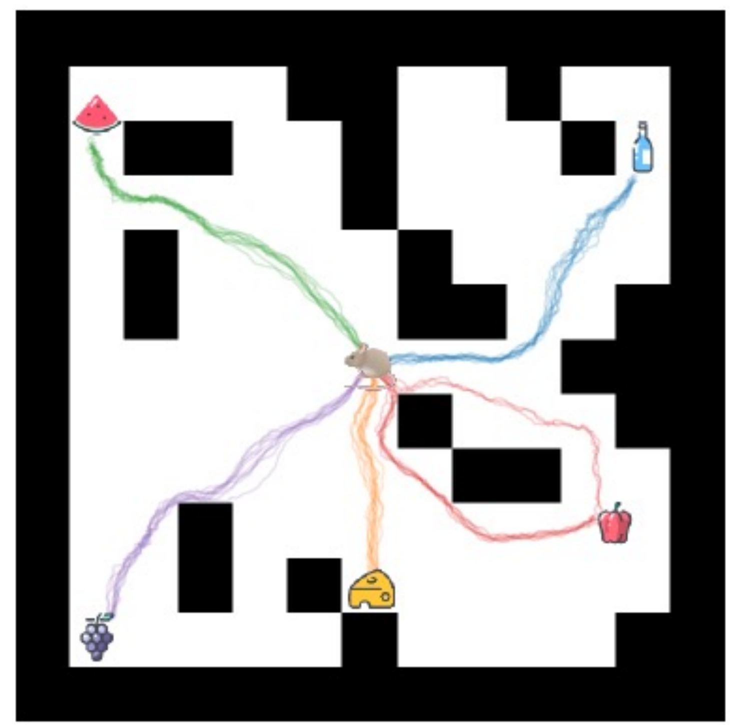
- Learns maps in LTM bound to position



Navigation network

Combined all of the above into a network to recall location and navigate to arbitrary objects in a maze.





SSPs as Probabilities

- SSPs can be a method for encoding and processing probabilities
- Method directly connects neural networks to probabilistic reasoning
- SSP based methods are efficient

SSPs as Probabilities

Background

- Kernel Density Estimators

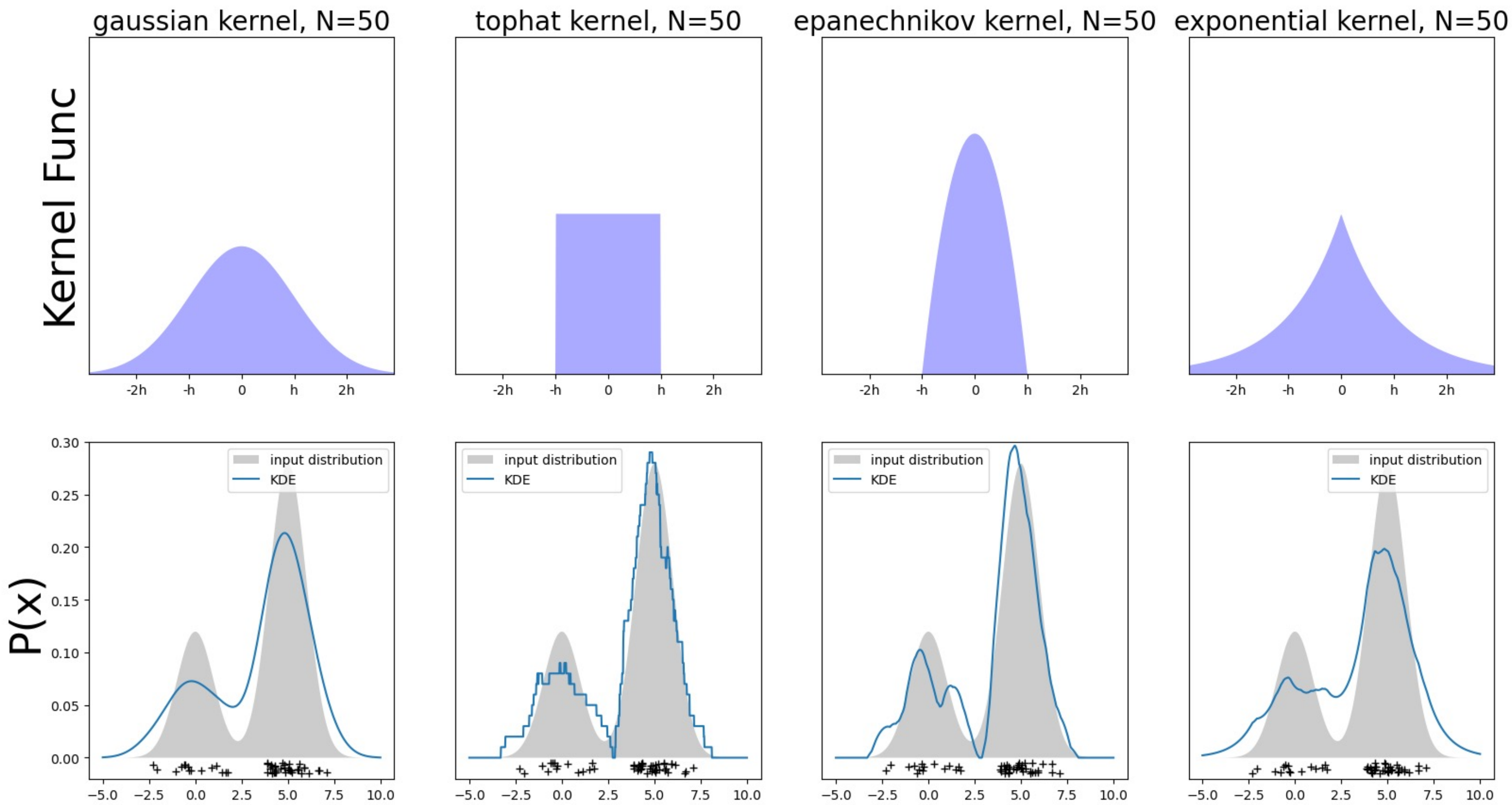
From a dataset $$\mathcal{D} = (x_1, x_2, \ldots, x_n)$$

With a kernel function $$k_h(x, x') = k\left(\frac{\|x - x'\|}{h}\right)$$

We can estimate the probability of x $$P_{\mathcal{D}}(X = x) = \frac{1}{nh}\sum_{x_i \in \mathcal{D}} k_h(x, x_i)$$

SSPs as Probabilities

- Kernel Density Estimators Examples



SSPs as Probabilities

Problems

- KDE memory grows linearly with the number of observations
- KDE time to compute a probability grow linearly with # of observations

But…

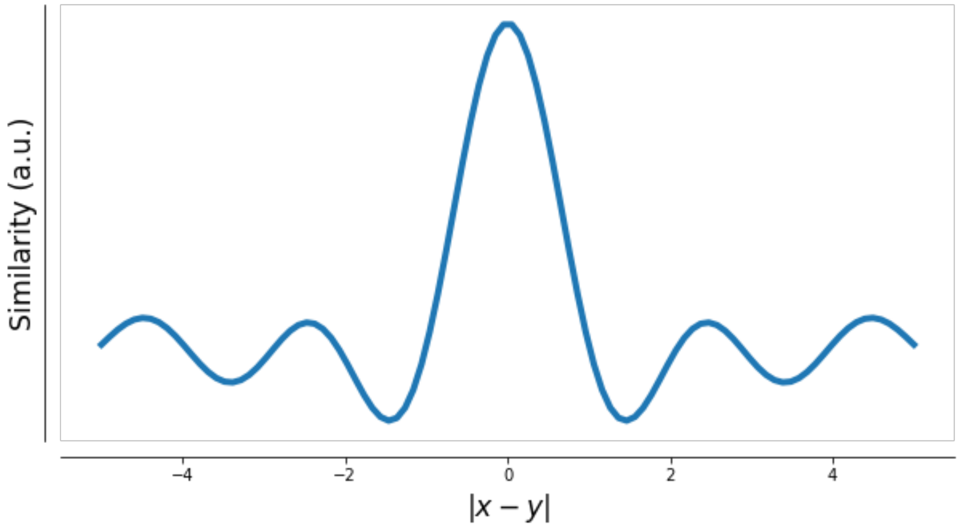
- Not if your kernel is a dot product

$$k_h(\mathbf{x}, \mathbf{x}') \approx \phi_h(\mathbf{x}) \cdot \phi_h(\mathbf{x}') \Longrightarrow P(\mathrm{X} = \mathbf{x}) = \frac{1}{nh} \sum_{\mathbf{x}_i \in \mathcal{D}} \phi_h(\mathbf{x}) \cdot \phi_h(\mathbf{x}_i)$$

SSPs as Probabilities

SSPs induce a quasi-kernel

- ‘quasi’ because there are negatives



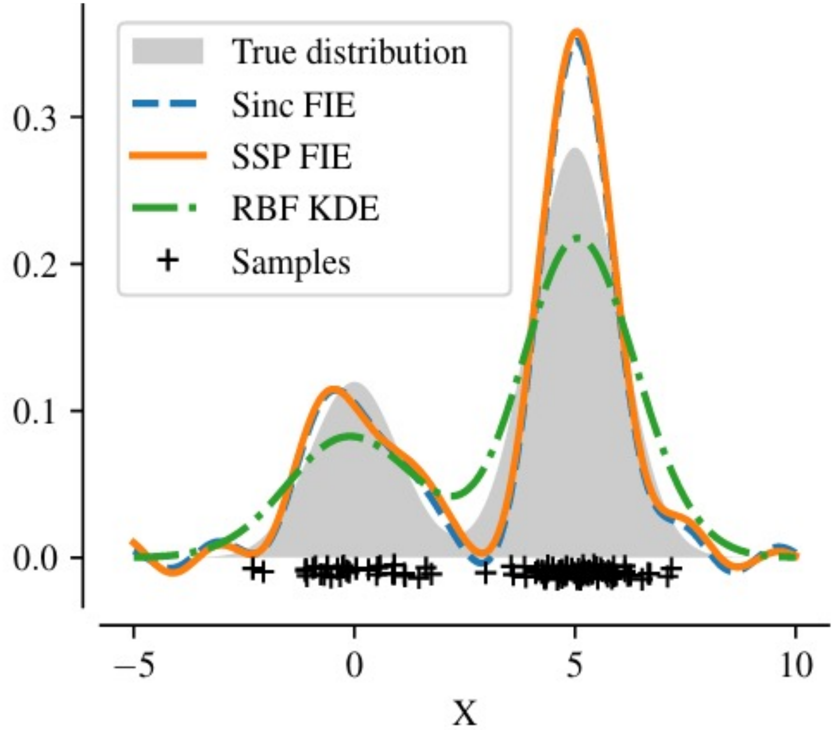
$$\phi(\mathbf{x}) \cdot \phi(\mathbf{y}) \approx \prod_{k=1}^{m} \mathrm{sinc}(|x_k - y_k|)$$

SSPs as Probabilities

Can convert to probability estimator

$$P_{\mathcal{D}}(X = x) = \max\left\{0, \phi(x) \cdot M_{\mathcal{D},h} - \xi\right\}$$

Glad et al, 2003



$$\boxed{\mathrm{ReLU}(\mathbf{w} \cdot \mathbf{z} + b)}$$

SSPs as Probabilities

So SSP memory is a latent probability distribution

- The distribution is stored in bundles of vector symbols.
- We can apply manipulations to bundles to produce probabilistic statements

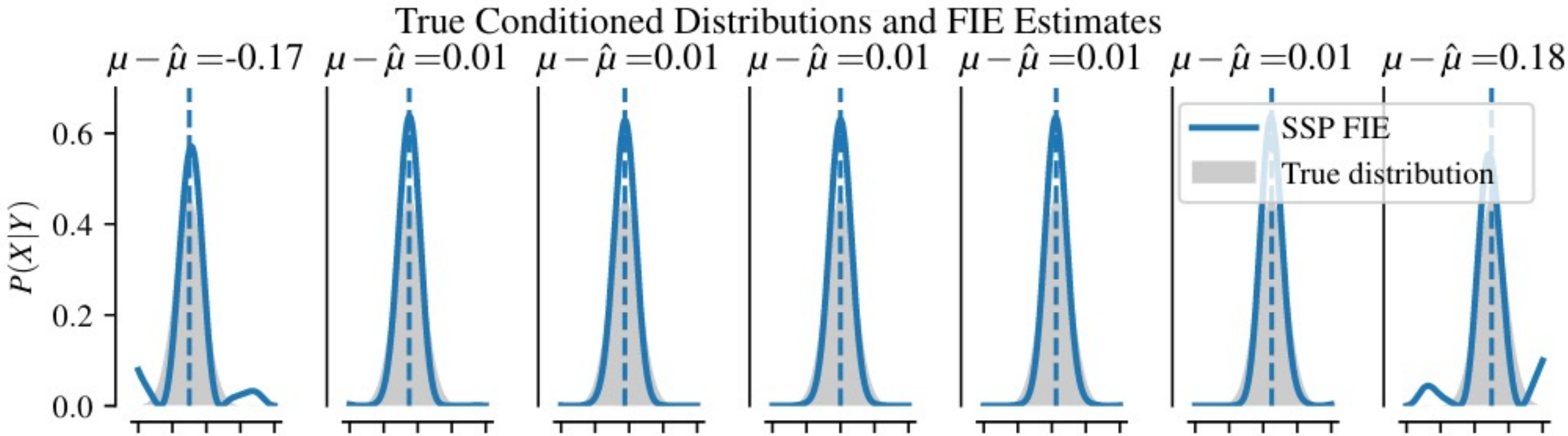
$$M_{\mathcal{D}} = \frac{1}{n} \sum_{\mathbf{x}_i \in \mathcal{D}} \phi(\mathbf{x}_i)$$

$$P(\mathrm{X} = \mathbf{x}) = \phi_h(\mathbf{x}) \cdot M_{\mathcal{D}}$$

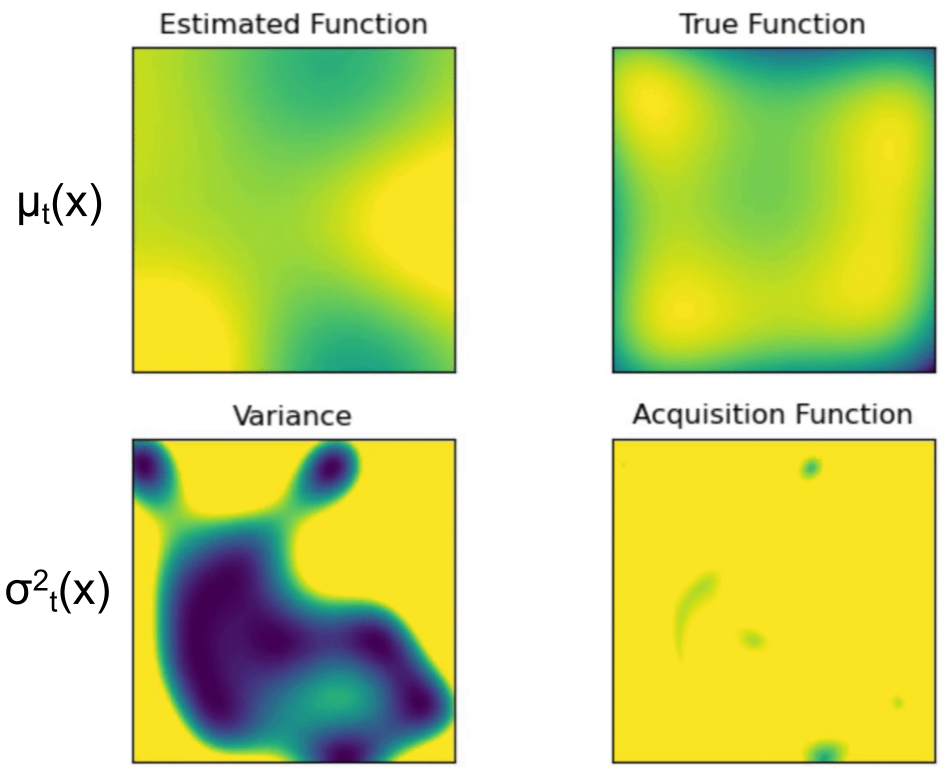
SSPs as Probabilities

Conditioning

$$P(\mathrm{X} = x \mid \mathrm{Y} = y) \approx \phi_{\mathrm{X}}(x) \cdot \left[M_{\mathcal{D}} \circledast \phi^{-1}(y)\right]$$



SSPs as Probabilities

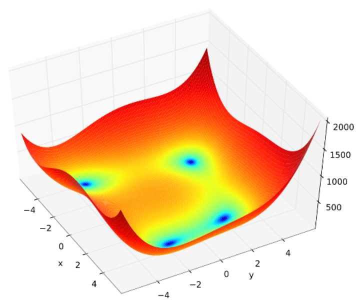


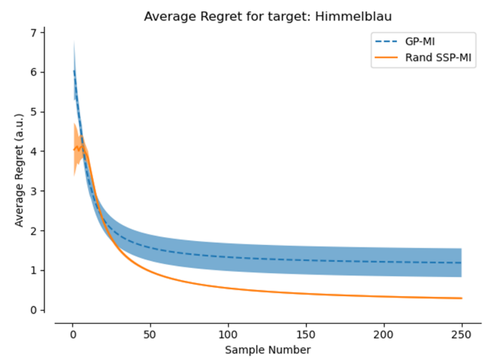
- Mutual Information (MI) is a common objective function used in exploration
- Gaussian Processes (GPs) are a convenient, but computationally intensive tool for computing MI
- We use Spatial Semantic Pointers and Bayesian linear regression to approximate a GP while improving in memory and time complexity

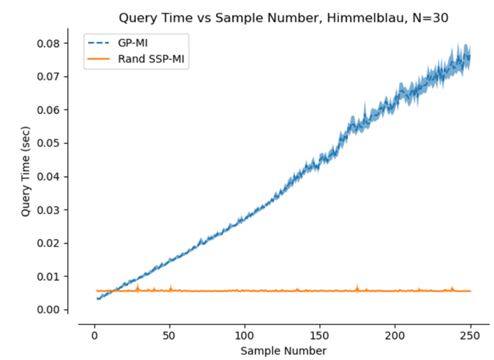
$$MI(X;Y) = H(Y) - H(Y|X) \text{ where } H(Y) = -\sum p(y)log(p(y))$$

Results

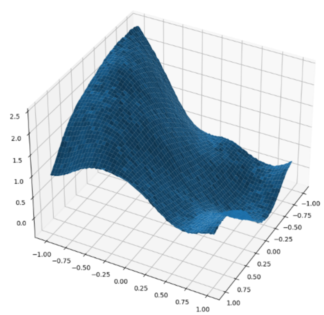
Himmelblau Function

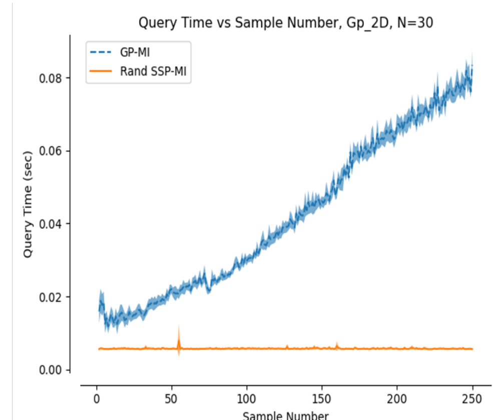




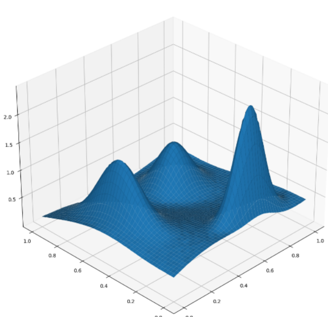


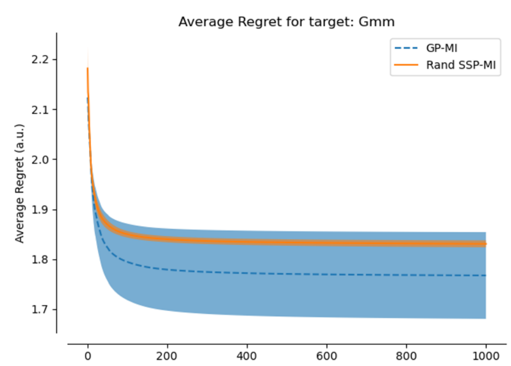
Sampled from GP with Matern Kernel + 1% noise

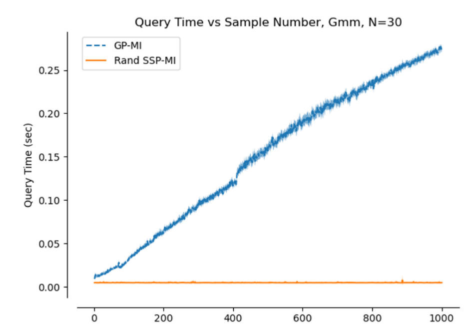




GMM with noise ~ (Matern Kernel + 1% noise)







SSPs as Probabilities

Where we differ:

- Provide a general and abstract framework for modelling probabilities
- Draw a direct connection between cognitive models and probability statements
- Provide network architectures for conditioning, marginalization, entropy, and mutual information

Conclusion

- SSPs support a variety of types of inference for cognitive models
  - Binding spatial and ‘symbolic’ representations
  - Representations of sampled data that can be used for probabilistic inference
- Improves
  - Interpretability
  - Efficiency (SWaP critical)
- Best engineered systems continue to learn from understanding biological solutions

List of Publications/Awards Attributed to the Grant

- *Best cognitive modeling prize - Language* Peter Blouw and Chris Eliasmith. Inferential role semantics for natural language. In Thora Tenbrink, Glenn Gunzelmann, Andrew Howes and Eddy Davelaar, editors, *Proceedings of the 39th Annual Conference of the Cognitive Science Society, 142–147*. Philadelphia, Pennsylvania, 2017. Cognitive Science Society.
- Peter Duggins, Terrence C. Stewart, Xuan Choo, and Chris Eliasmith. Effects of guanfacine and phenylephrine on a spiking neuron model of working memory. *Topics in Cognitive Science*, 2017.
- Jan Gosmann, Aaron R. Voelker, and Chris Eliasmith. A spiking independent accumulator model for winner-take-all computation. In *Proceedings of the 39th Annual Conference of the Cognitive Science Society*. London, UK, 2017. Cognitive Science Society.
- Jan Gosmann and Chris Eliasmith. Automatic optimization of the computation graph in the nengo neural network simulator. *Frontiers in Neuroinformatics*, 11:33, 2017.
- Ivana Kajić, Jan Gosmann, Brent Komer, Ryan W. Orr, Terrence C. Stewart, and Chris Eliasmith. A biologically constrained model of semantic memory search. In *Proceedings of the 39th Annual Conference of the Cognitive Science Society*. London, UK, 2017. Cognitive Science Society.

List of Publications/Awards Attributed to the Grant

- Ivana Kajić, Jan Gosmann, Terrence C. Stewart, Thomas Wennekers, and Chris Eliasmith. A spiking neuron model of word associations for the remote associates test. *Frontiers in Psychology*, 8:99, 2017.
- Daniel Rasmussen, Aaron R. Voelker, and Chris Eliasmith. A neural model of hierarchical reinforcement learning. *PLoS ONE*, 12(7):1–39, 2017.
- Peter Blouw and Chris Eliasmith (2018) Using Neural Networks to Generate Inferential Roles for Natural Language. *Frontiers in Psychology*.
- Terrence C. Stewart, Sverrir Thorgeirsson, and Chris Eliasmith. Supervised learning of action selection in cognitive spiking neuron models. In 40th Annual Conference of the Cognitive Science Society, 1086–1091. Cognitive Science Society, 2018.
- Sverrir Thorgeirsson, Terrence C. Stewart, and Chris Eliasmith. Analysis of learning action selection parameters in a neural cognitive model. In International Conference on Cognitive Modelling. 2018.
- Sverrir Thorgeirsson, Brent Komer, and Chris Eliasmith. Incorporating an adaptive learning rate in a neural model of action selection. In Cognitive Computational Neuroscience. Philadelphia, USA, 2018.
- Jan Gosmann and Chris Eliasmith. Vector-derived transformation binding: an improved binding operation for deep symbol-like processing in neural networks. Neural Computation, 31(5):849-869, 05 2019.

List of Publications/Awards Attributed to the Grant

- Aaron R. Voelker and Chris Eliasmith. Improving spiking dynamical networks: accurate delays, higher-order synapses, and time cells. Neural Computation, 30(3):569-609, 03 2018
- Brent Komer, Terrence C. Stewart, Aaron R. Voelker, and Chris Eliasmith. A neural representation of continuous space using fractional binding. In 41st Annual Meeting of the Cognitive Science Society. Montreal, QC, 2019. Cognitive Science Society.
- Joost de Jong, Aaron R. Voelker, Hedderik van Rijn, Terrence C. Stewart, and Chris Eliasmith. Flexible timing with delay networks – the scalar property and neural scaling. In 17th Annual Meeting of the International Conference on Cognitive Modelling (ICCM). 2019.
- Peter Duggins and Chris Eliasmith. A spiking neuron model of pharmacologically-biased fear conditioning in the amygdala. In SfN Abstracts. Chicago USA, 2019.
- Aaron R. Voelker, Ivana Kajić, and Chris Eliasmith. Legendre memory units: continuous-time representation in recurrent neural networks. In Advances in Neural Information Processing Systems, 15544–15553. 2019.
- Thomas Lu, Aaron R. Voelker, Brent Komer, and Chris Eliasmith. Representing spatial relations with fractional binding. In 41st Annual Meeting of the Cognitive Science Society. Montreal, QC, 2019. Cognitive Science Society.

List of Publications/Awards Attributed to the Grant

- Andreas Stöckel, Terrence C. Stewart, and Chris Eliasmith. Connecting biological detail with neural computation: application to the cerebellar granule-golgi microcircuit. In 18th Annual Meeting of the International Conference on Cognitive Modelling. Toronto, ON, 2020. Society for Mathematical Psychology. ***Best paper award
- Andreas Stöckel, Terrence C. Stewart, and Chris Eliasmith. A biologically plausible spiking neural model of eyeblink conditioning in the cerebellum. In 42nd Annual Meeting of the Cognitive Science Society, 1614–1620. Toronto, ON, 2020. Cognitive Science Society.
- Brent Komer and Chris Eliasmith. Efficient navigation using a scalable, biologically inspired spatial representation. In 42nd Annual Meeting of the Cognitive Science Society. Toronto, ON, 2020. Cognitive Science Society.
- Nicole Sandra-Yaffa Dumont and Chris Eliasmith. Accurate representation for spatial cognition using grid cells. In 42nd Annual Meeting of the Cognitive Science Society, 2367–2373. Toronto, ON, 2020. Cognitive Science Society.

List of Publications/Awards Attributed to the Grant

- Jan Gosmann and Chris Eliasmith. CUE: a unified spiking neuron model of short-term and long-term memory. Psychological Review, 2020.
- Narsimha Reddy Chilkuri and Chris Eliasmith. Parallelizing Legendre Memory Unit training. In Marina Meila and Tong Zhang, editors, Proceedings of the 38th ICML, volume 139 of Proceedings of Machine Learning Research, 1898–1907. PMLR, 18–24 Jul 2021.
- Andreas Stöckel, T C. Stewart, and C Eliasmith. Connecting biological detail with neural computation: application to the cerebellar granule-golgi microcircuit. Topics in Cognitive Science, 13(3):515-533, 2021.
- Joost de Jong, Aaron Voelker, Terry Stewart, Chris Eliasmith, and Heddrick van Rijn. A neurocomputational model of prospective and retrospective timing. In ICCM. Toronto, ON, 2021. Society for Mathematical Psychology.
- Andreas Stöckel and Chris Eliasmith. Passive nonlinear dendritic interactions as a computational resource in spiking neural networks. Neural Computation, 33(1):96-128, 2021. PMID: 33080158.
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