SYDE 556/750

Simulating Neurobiological Systems
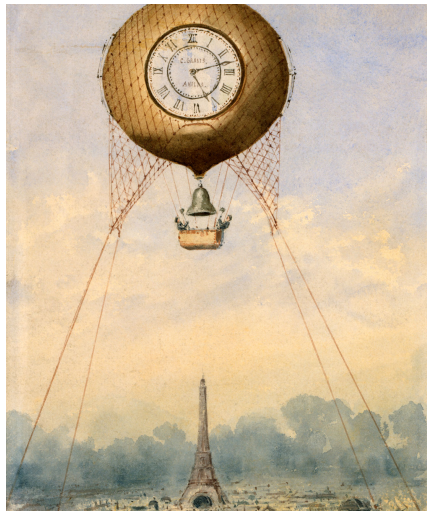**Lecture 4: Temporal Representations**

Chris Eliasmith

Sept 18 & 23, 2024

- ▶ Slide design: Andreas Stöckel
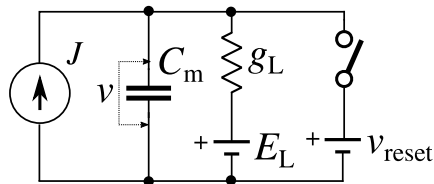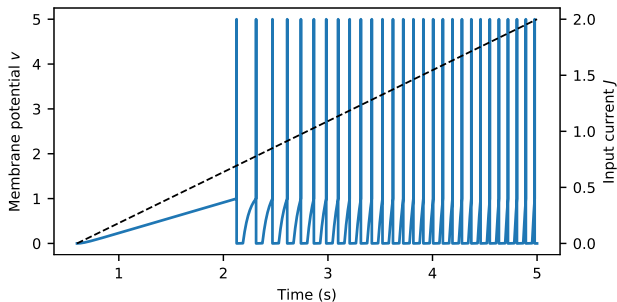- ▶ Content: Terry Stewart, Andreas Stöckel, Chris Eliasmith

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Reminder: The LIF Neuron

$$\frac{\mathrm{d}}{\mathrm{d}t}v(t) = -\frac{1}{\tau_{\mathrm{RC}}}\big(v(t) - J\big)\,, \qquad\qquad \text{if } v(t) < 1\,,$$

$$v(t) \leftarrow \delta(t - t_{\mathrm{th}})\,, \qquad\qquad\qquad \text{if } t = t_{\mathrm{th}}\,,$$

$$v(t) \leftarrow 0\,, \qquad\qquad\qquad\qquad \text{if } t > t_{\mathrm{th}} \text{ and } t \geq t_{\mathrm{th}} + \tau_{\mathrm{ref}}\,,$$
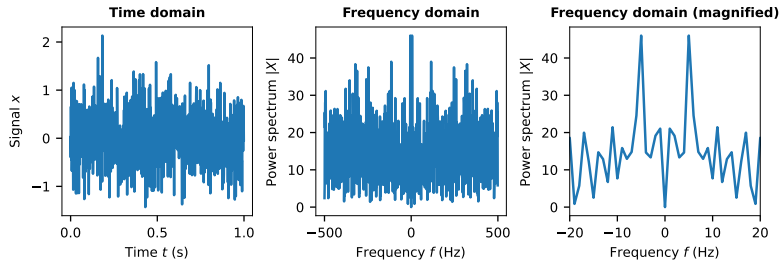
Temporal Decoding

▶ For population decoders, we needed to integrate their responses, $\mathbf{a}(\mathbf{x})$, over the represented variable, $\mathbf{x}$.
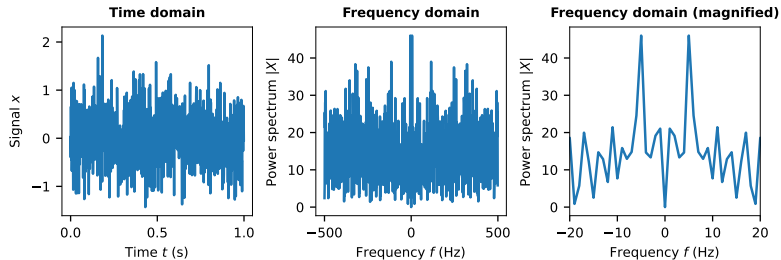
Temporal Decoding

▶ For population decoders, we needed to integrate their responses, $\mathbf{a}(\mathbf{x})$, over the represented variable, $\mathbf{x}$.

▶ For temporal decoders, we will likely want to integrate their responses, $\mathbf{a}(t)$, over the represented variable, $\mathbf{x}(t)$.

Temporal Decoding

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▶ For temporal decoders, we will likely want to integrate their responses, $\mathbf{a}(t)$, over the represented variable, $\mathbf{x}(t)$.

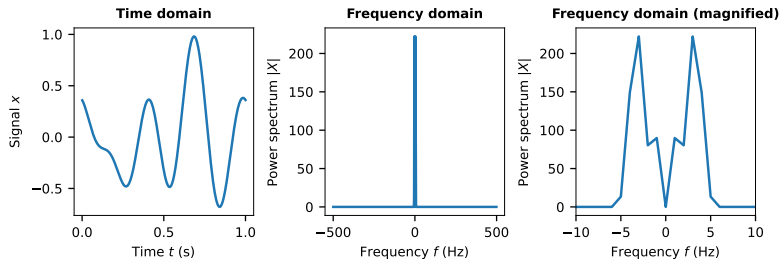▶ What space do we want to sample to estimate the integrals?

Random Signals
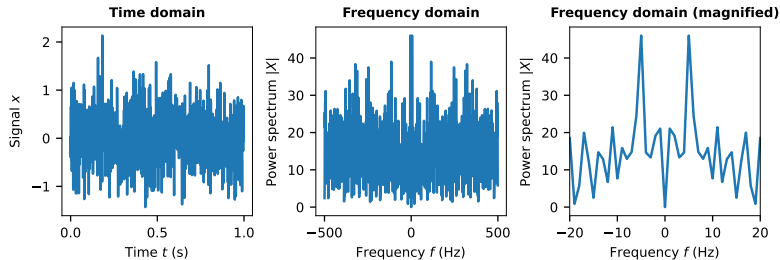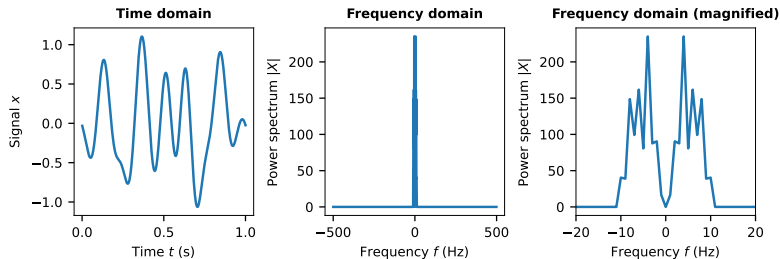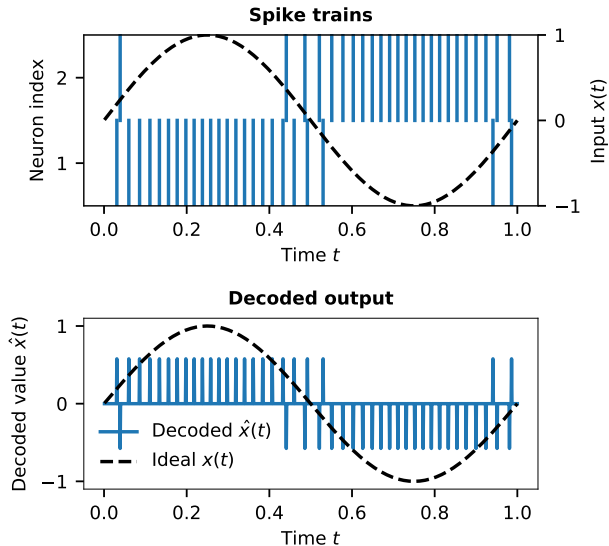
Random Signals

White Noise
(zero mean)

Bandlimited
White Noise
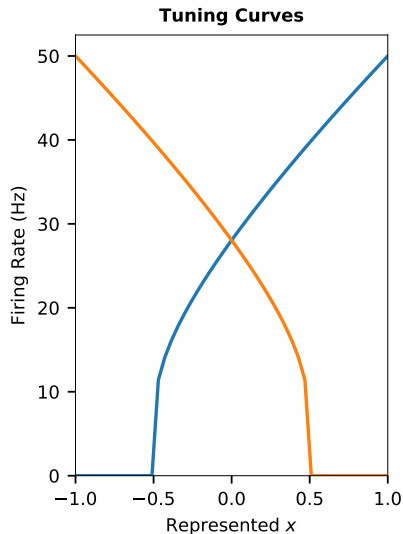(zero mean,
$5\,\mathrm{Hz}$ bandwidth)

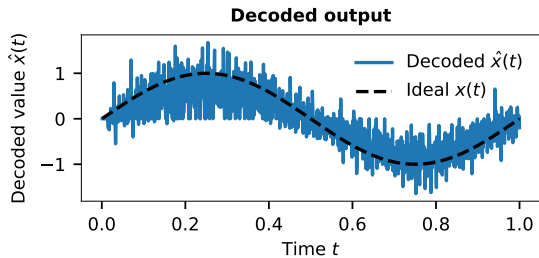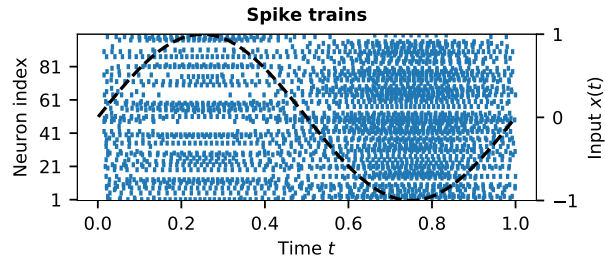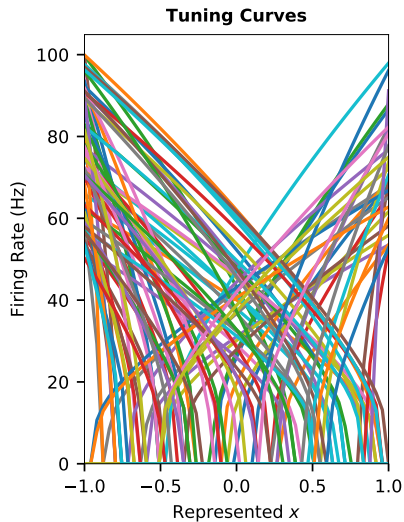# Random Signals

White Noise
(zero mean)

Bandlimited
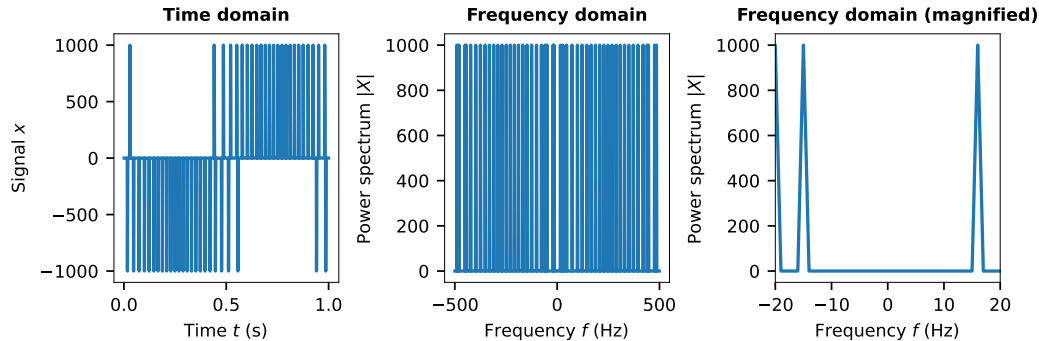White Noise
(zero mean,
$10\,\text{Hz}$ bandwidth)
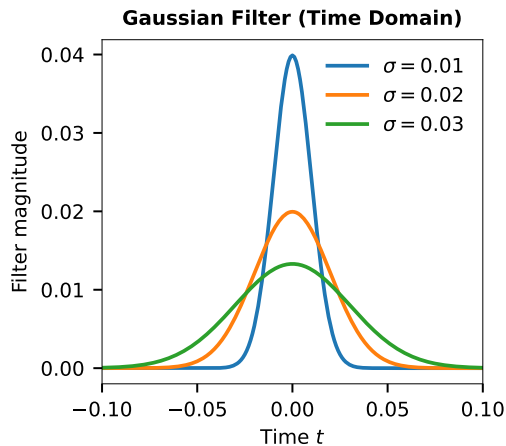
Temporal Decoding of Two Neurons - Weighted Spikes

Temporal Decoding of One Hundred Neurons - Weighted Spikes

Frequency Response of Two Neurons
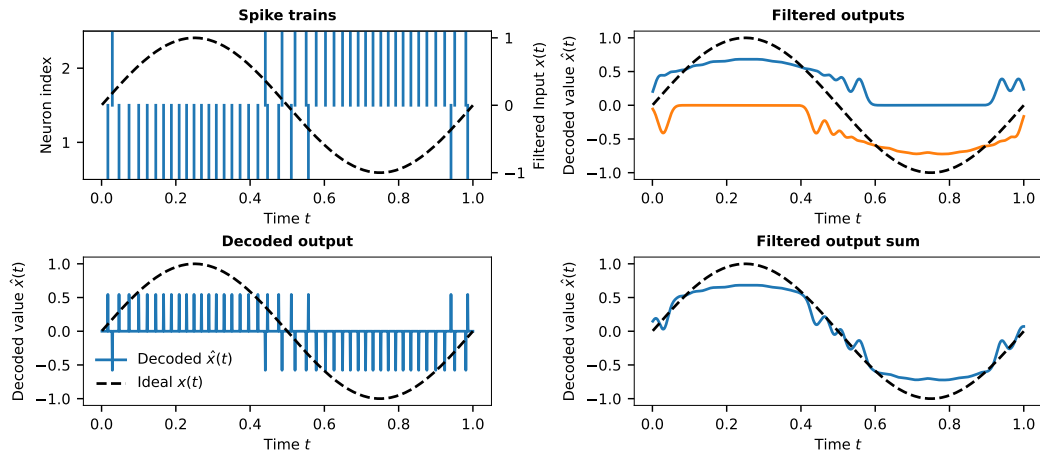
Filtering by Convolution

Gaussian Filter (Time Domain)

Gaussian Filter

$$h(t) = c \exp\left(\frac{-t^2}{\sigma^2}\right)$$

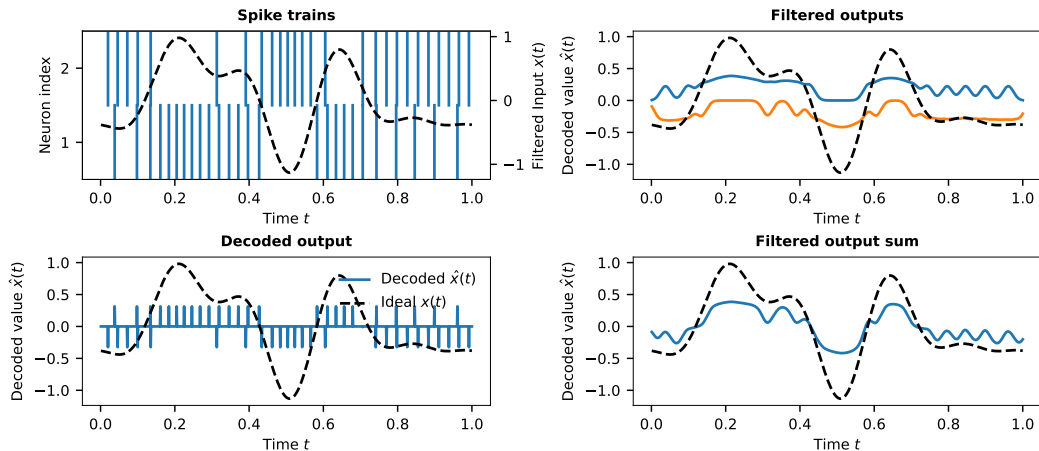where c chosen s.t. $\int_{-\infty}^{\infty} h(t)\,\mathrm{d}t = 1$

Convolution

$$\left(f * h\right)(t) = \int_{-\infty}^{\infty} f(t - t')h(t')\,\mathrm{d}t'$$
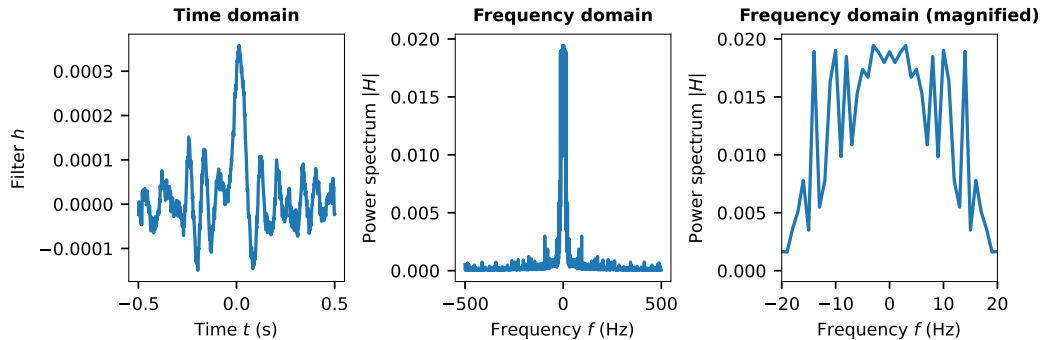
Filtering a Spike Train
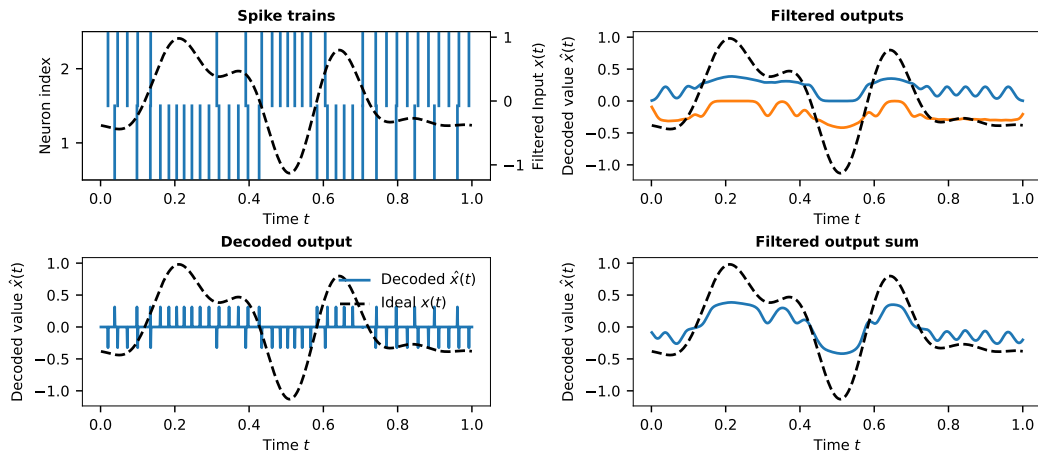
Filtering a Spike Train for a Random Signal
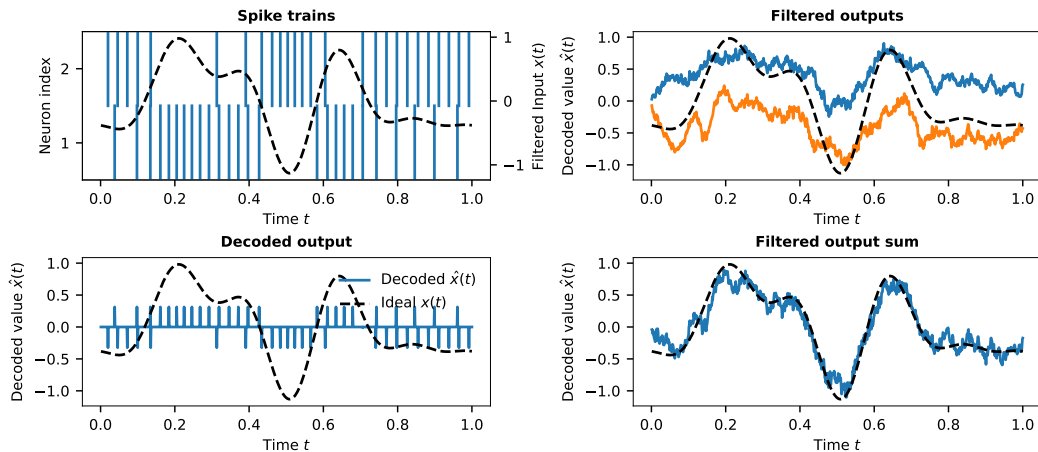
Optimal Filter

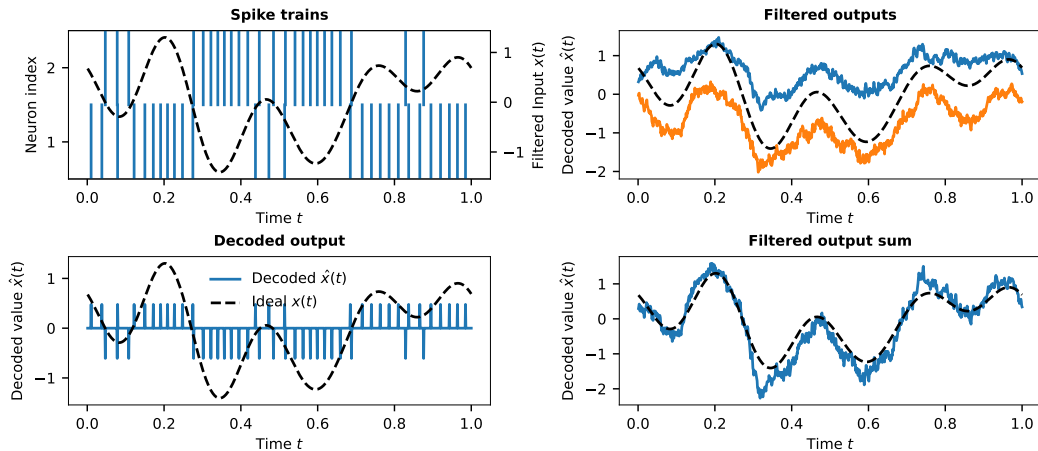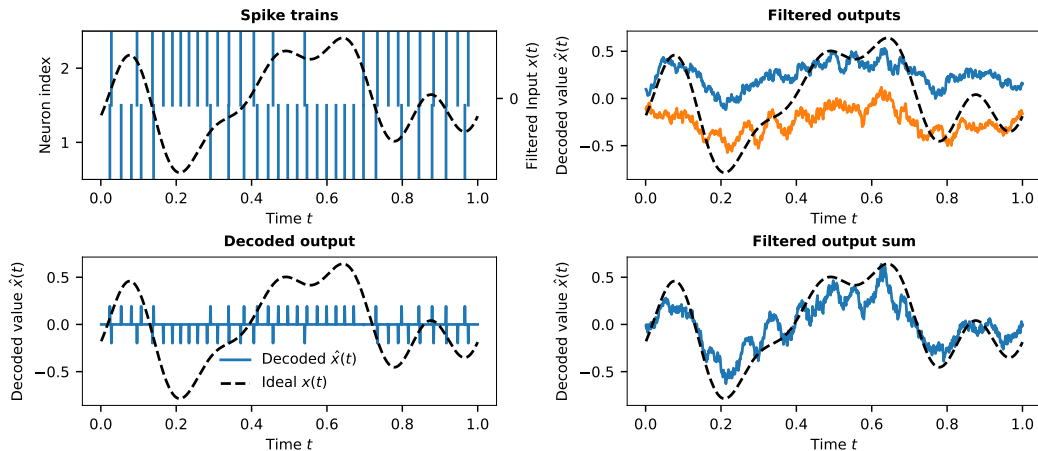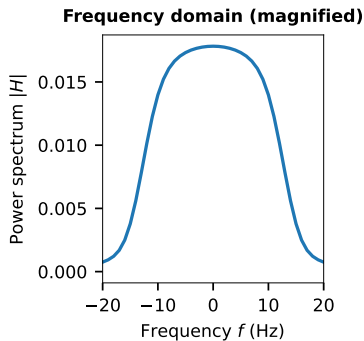$$H(\omega) = \frac{X(\omega)\overline{R}(\omega)}{|R(\omega)|^2}$$

Filtering a Spike Train for a Random Signal (Optimal Filter)

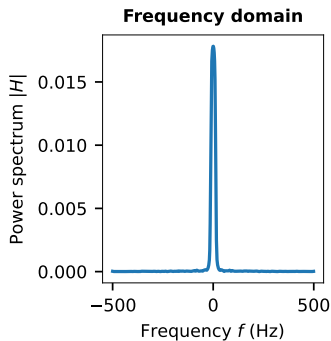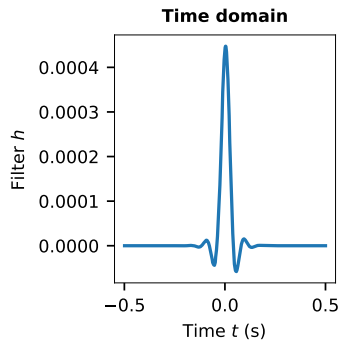# Filtering a Spike Train for a Random Signal (Optimal Filter)

Filtering a Spike Train for a Random Signal (Optimal Filter)

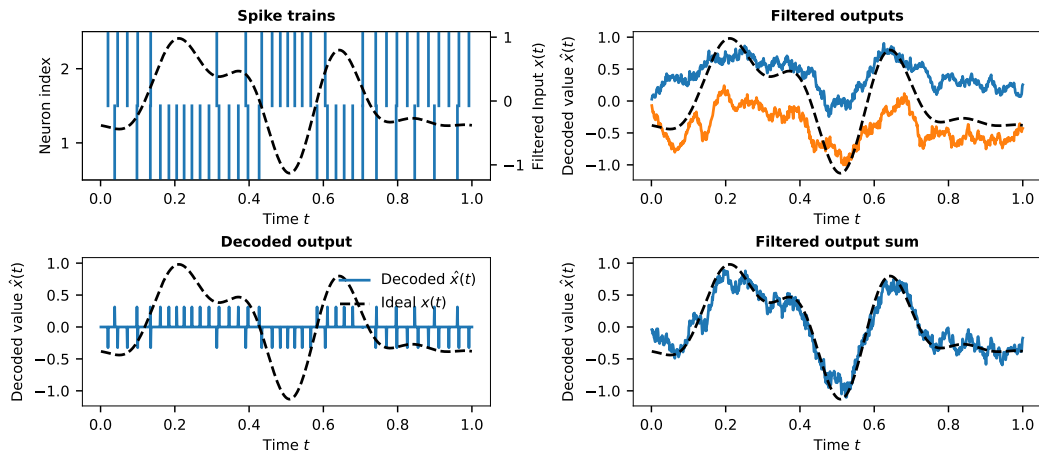# Filtering a Spike Train for a Random Signal (Optimal Filter)
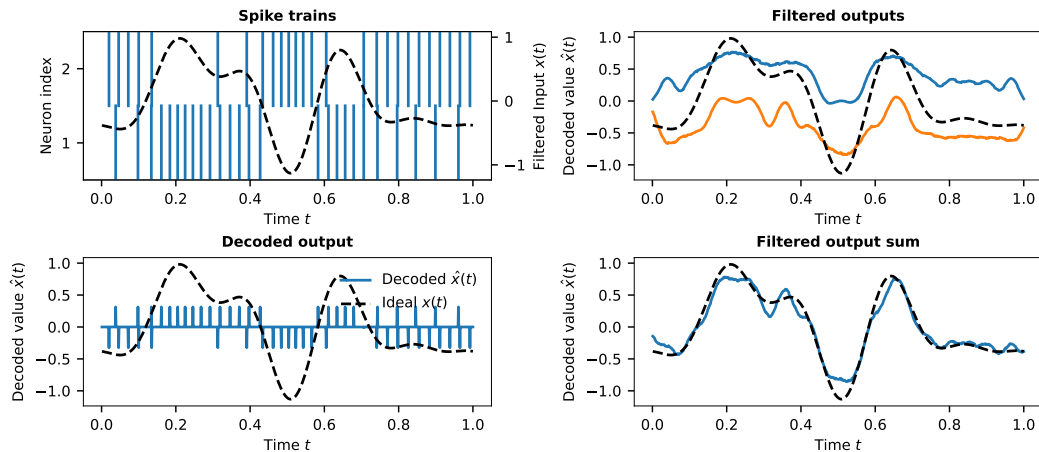
Optimal Filter (Improved)

| Time domain | Frequency domain | Frequency domain (magnified) |

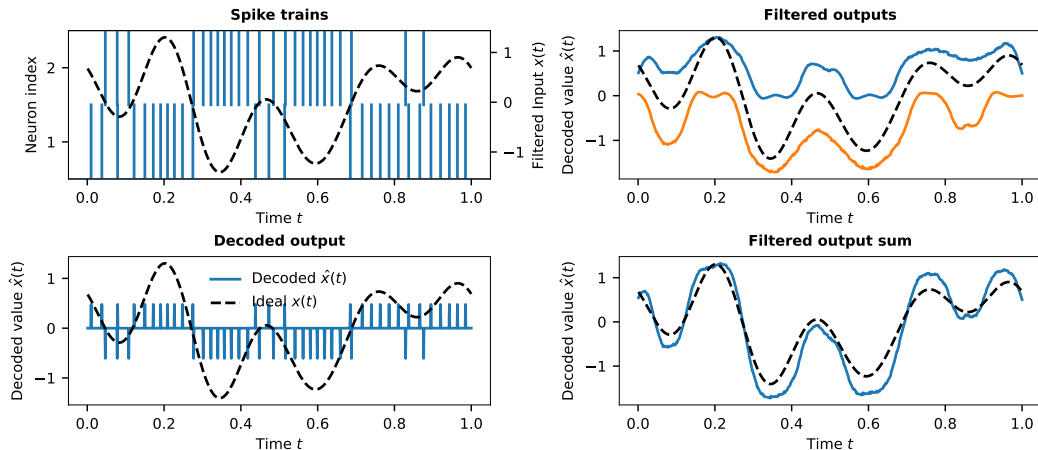$$H(\omega) = \frac{X(\omega)\overline{R}(\omega) * W(\omega)}{|R(\omega)|^2 * W(\omega)}$$
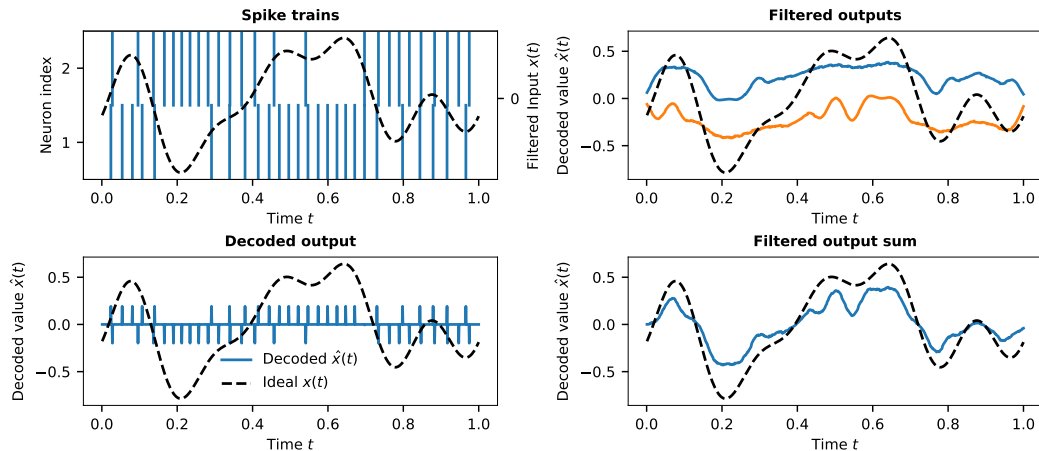
Filtering a Spike Train for a Random Signal (Improved Optimal Filter)

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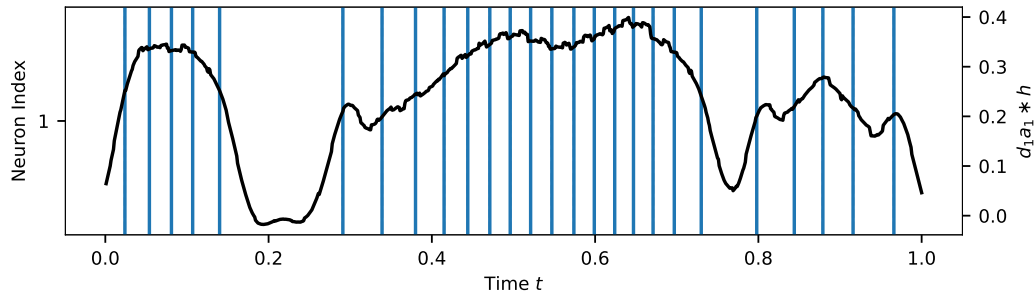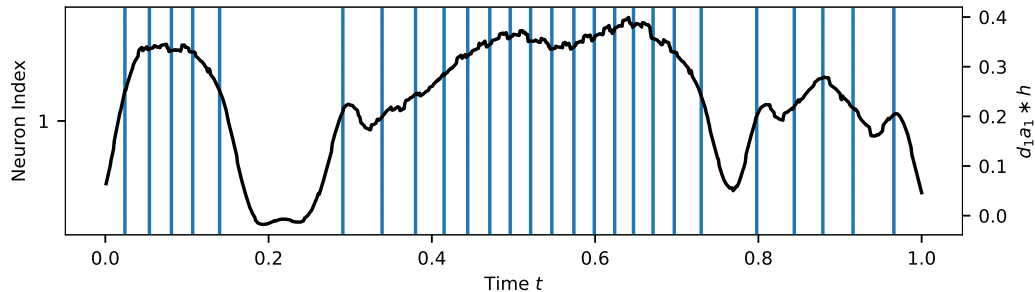# Filtering a Spike Train for a Random Signal (Improved Optimal Filter)

Filtering a Spike Train for a Random Signal (Improved Optimal Filter)
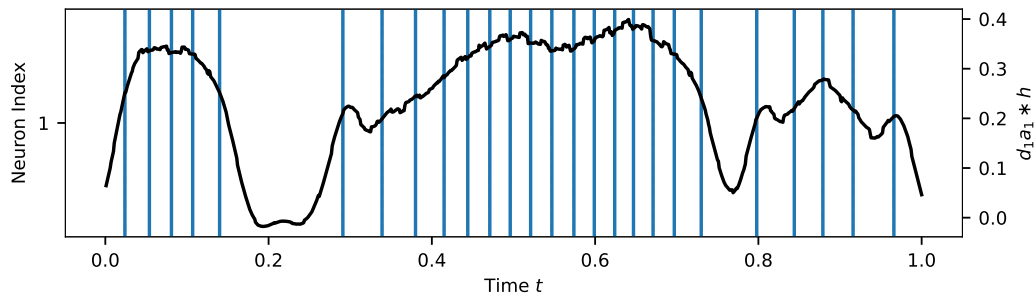
Pros and Cons of the Optimal Filter

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➕ **Precise**
Good for analysing data after the fact

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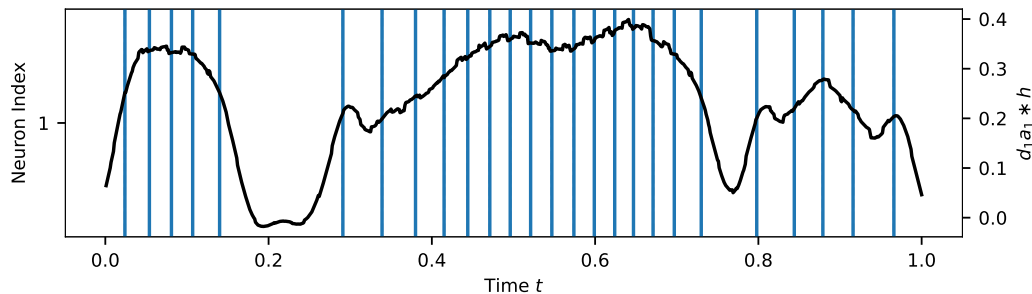

➕ **Precise**
Good for analysing data after the fact

➖ **Non-causal**
Does not describe a biological process

Pros and Cons of the Optimal Filter

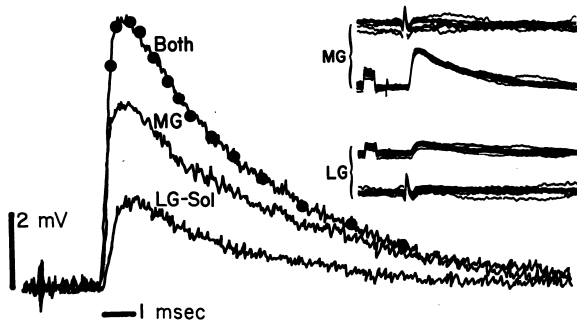
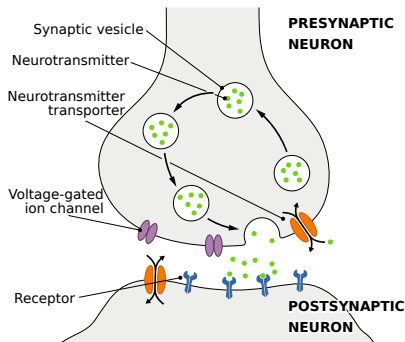➕ **Precise**
Good for analysing data after the fact
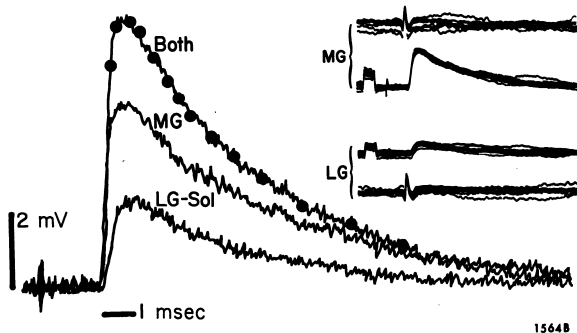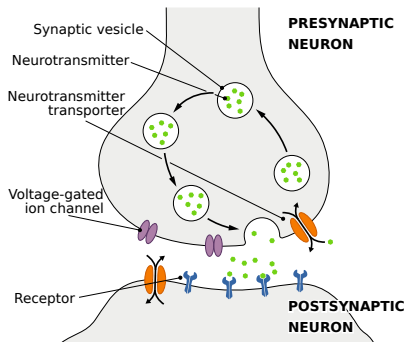
➖ **Non-causal**
Does not describe a biological process

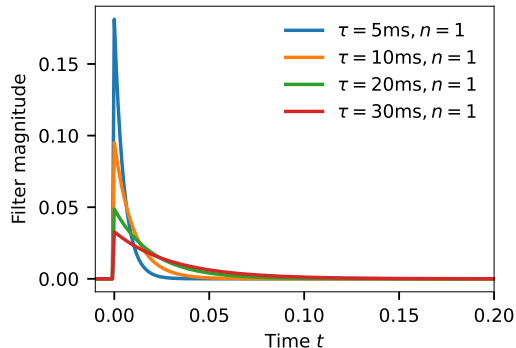We need to find a mechanism that low-pass filters spikes over time!
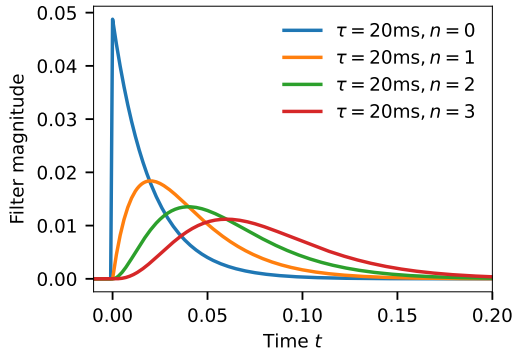
Synapses as Filters

Synapses as Filters

Post-synaptic currents (EPSCs, IPSCs) are low-pass filtered spike trains!

Exponential Low-Pass Filter (I)

$$h(t) = \begin{cases} c^{-1} t^n \exp^{-t/\tau} & \text{if } t \geq 0, \\ 0 & \text{otherwise}, \end{cases} \qquad \text{where } c = \int_0^\infty t^n \exp^{-t/\tau} \, \mathrm{d}t.$$

Exponential Low-Pass Filter (II)
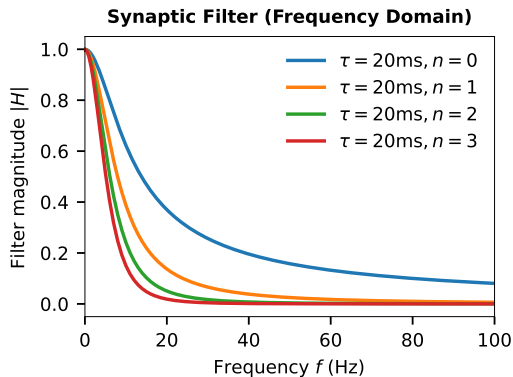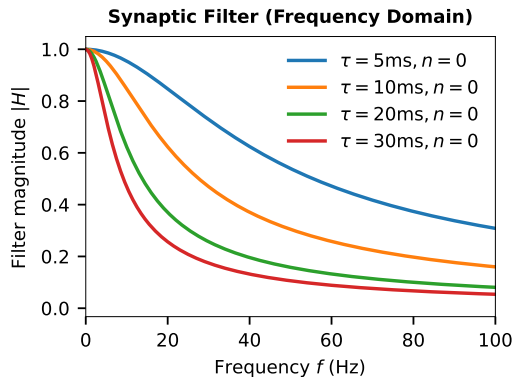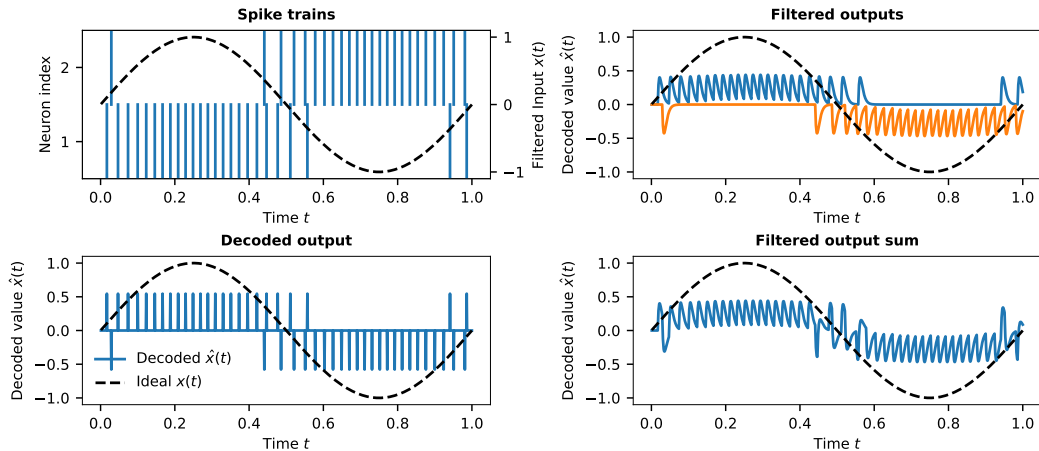

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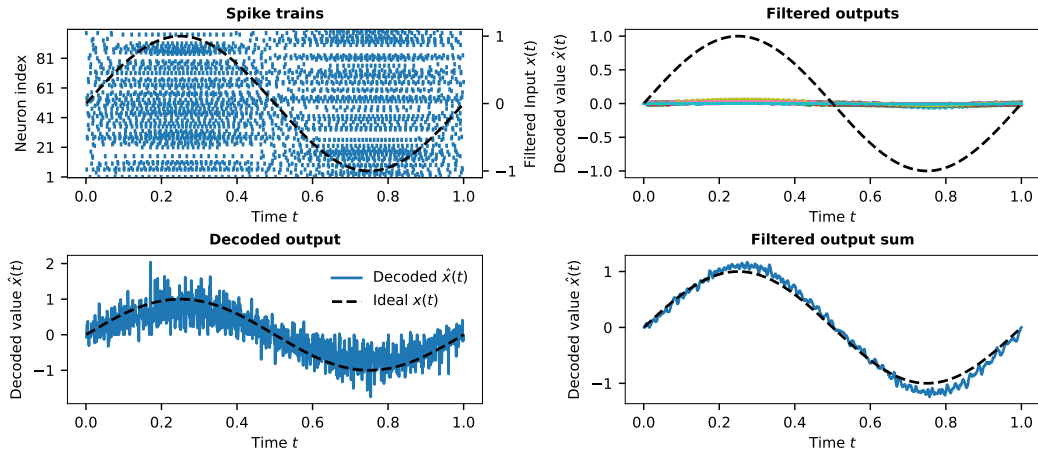
Example: Synaptic Filter for Two Neurons

$$\tau = 5\,\mathrm{ms}, n = 1$$

Example: Synaptic Filter for One Hundred Neurons

$$\tau = 5\,\mathrm{ms}, n = 1$$

Image sources

Title slide
"Captive balloon with clock face and bell, floating above the Eiffel Tower, Paris, France."
Author: Camille Grávis, between 1889 and 1900.
From Wikimedia.