Spatial Semantic Pointers (SSPs)

Chris Eliasmith
SYDE 556/750

Spatial Semantic Pointers

- Semantic pointers represent standard discrete structures (lists, trees, etc.)
- SSPs allow recurrent convolutions to have fractional powers

$$B^k = \underbrace{B \circledast B \circledast \ldots \circledast B}_{B \text{ appears } k \text{ times}}$$

- Compute fractional *k* in Fourier space

$$B^k = \mathcal{F}^{-1}\left\{\mathcal{F}\left\{B\right\}^k\right\}, \quad k \in \mathbb{R}$$

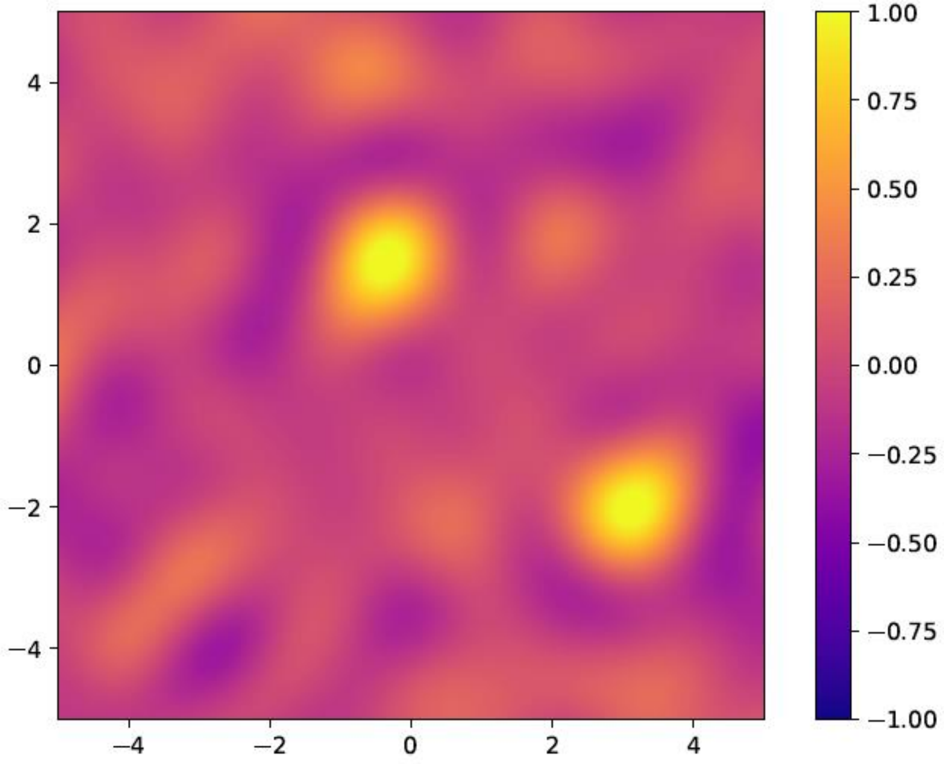
$$S(x,y) = X^x \circledast Y^y = \mathcal{F}^{-1}\{\mathcal{F}\{X\}^x \odot \mathcal{F}\{Y\}^y\}$$

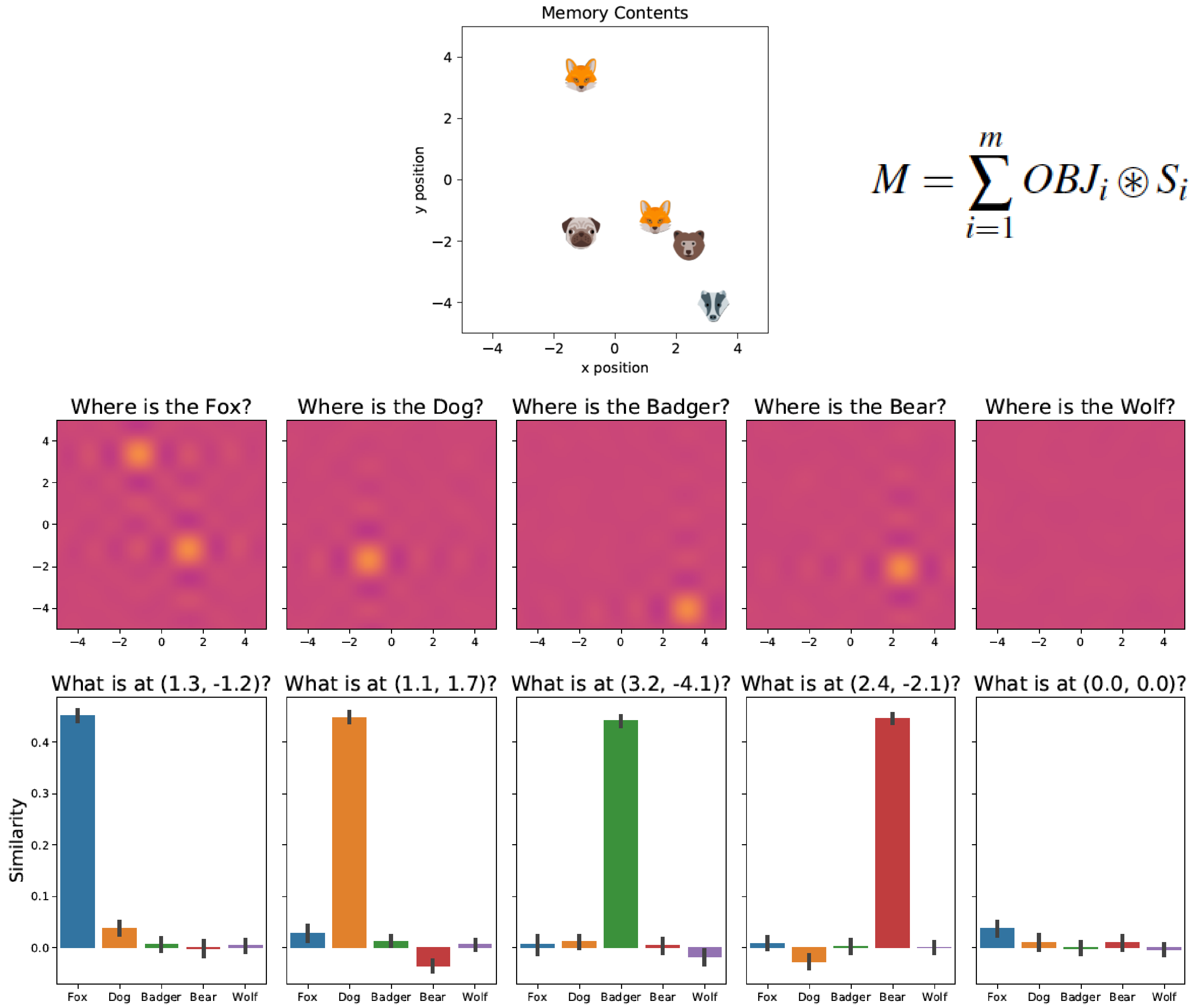
Spatial Semantic Pointers

- Represent continuous space (Clifford torus)

$$S(x,y) = X^x \circledast Y^y$$

- Heat map to visualize vector contents
- Dot product between SSP at every possible position and S




Memory Contents
y position
x position
$M = \sum_{i=1}^{m} OBJ_i \circledast S_i$
Where is the Fox?
Where is the Dog?
Where is the Badger?
Where is the Bear?
Where is the Wolf?
What is at (1.3, -1.2)?
What is at (1.1, 1.7)?
What is at (3.2, -4.1)?
What is at (2.4, -2.1)?
What is at (0.0, 0.0)?
Similarity
Fox
Dog
Badger
Bear
Wolf

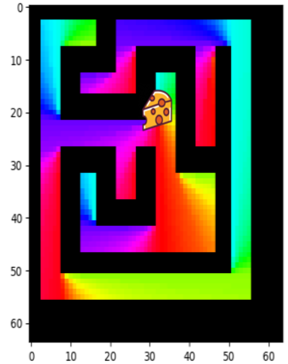
Spatial Manipulations

Desiderata	Accuracy	
	Non-Neural	Neural
Query single object	99.1%	95.7%
Query missing object	99.4%	96.7%
Query location	97.3%	94.7%
Query duplicate object	97.4%	95.3%
Query Region	90.4%	73.5%
Slide single object in group (all objects)	75.7%	67.3%
Slide single object in group (moved object)	100.0%	100.0%
Slide whole group	97.8%	96.7%
Readout x-y location from SSP	95.7%	94.1%
Construct SSP from x-y location	100.0%	99.0%

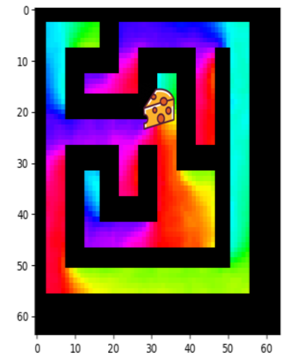
Navigating to Goal

Single layer MLP, same for each encoding (tried many more).

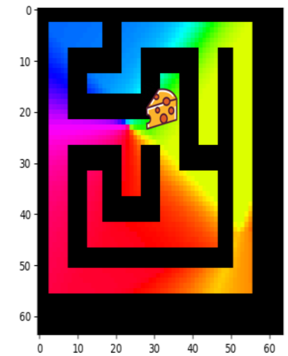
Ground Truth
RMSE: 0



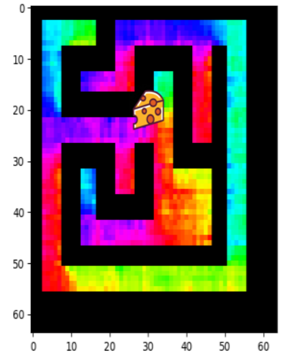
SSPs
RMSE: 0.0529



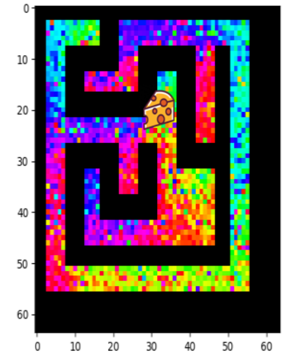
Random Projection
RMSE: 0.5351



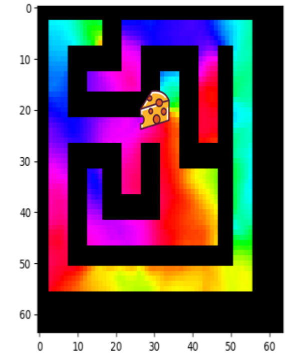
Trig Functions
RMSE: 0.1350



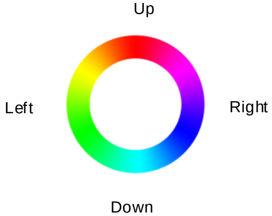
Random Mapping
RMSE: 0.2580



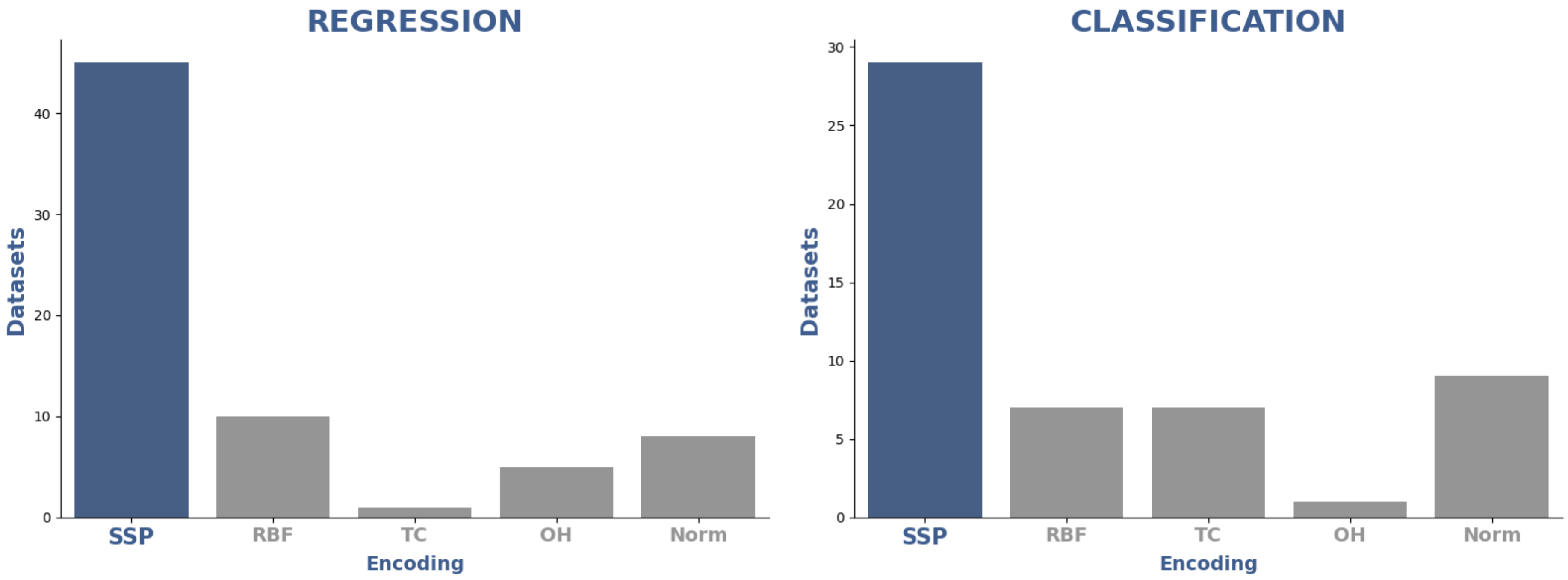
2D Coordinates
RMSE 0.1984



Legend

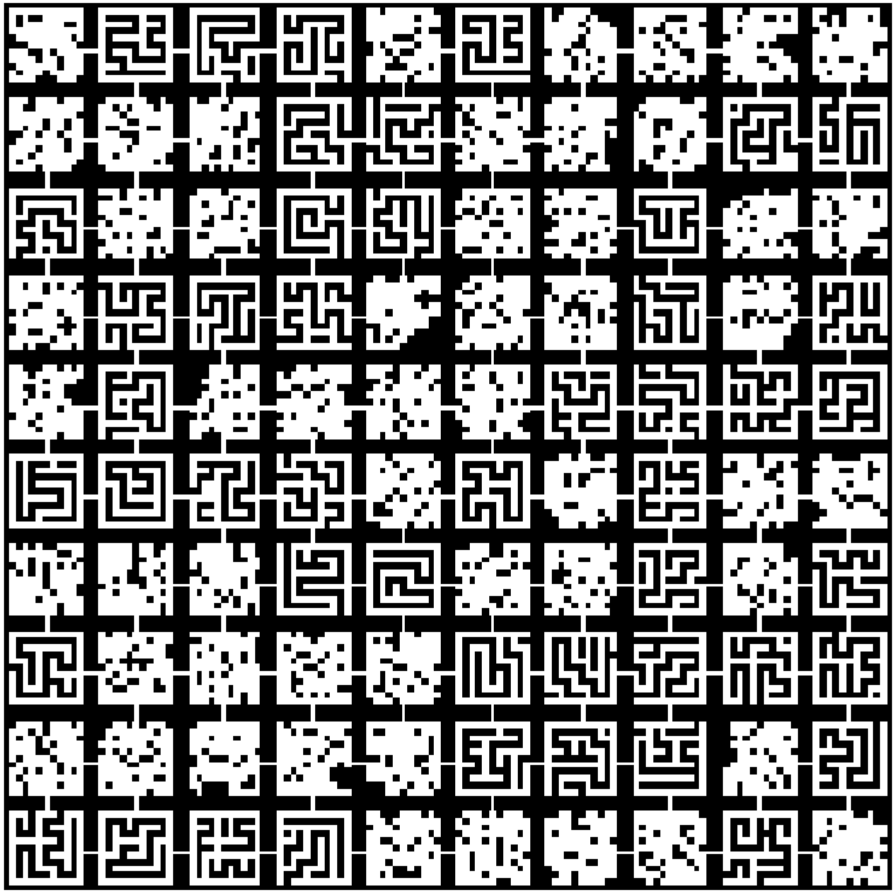


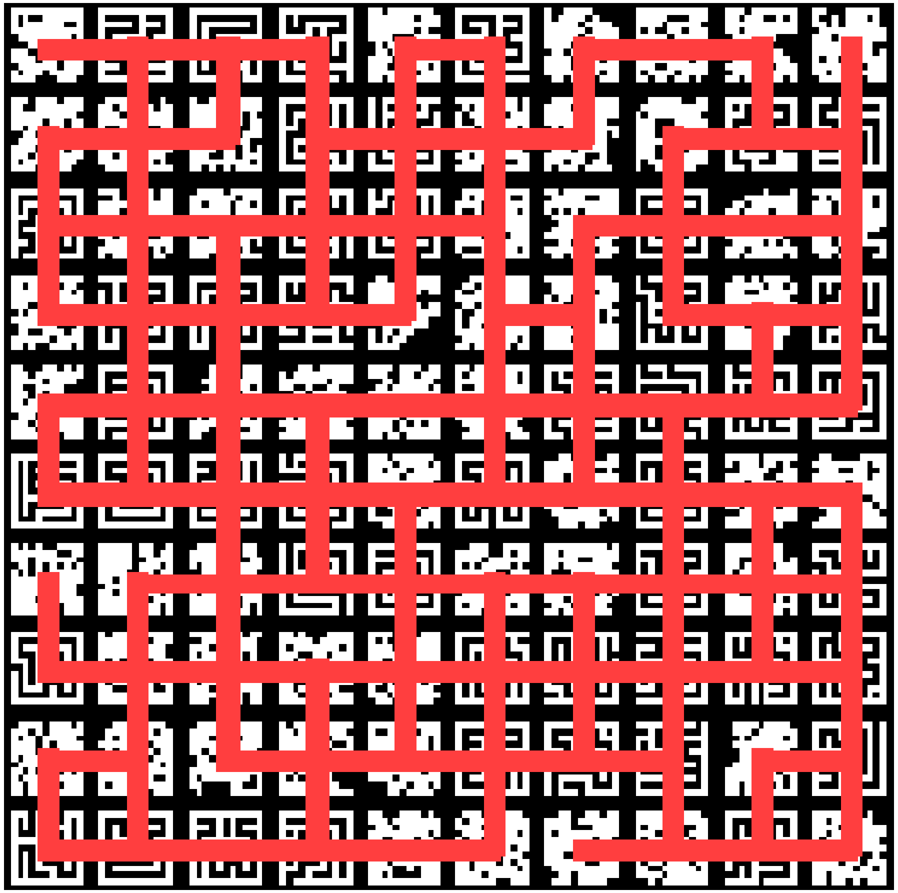
Good for General ML



SSPs are more accurate on a large majority of 122 standard ML benchmarks.

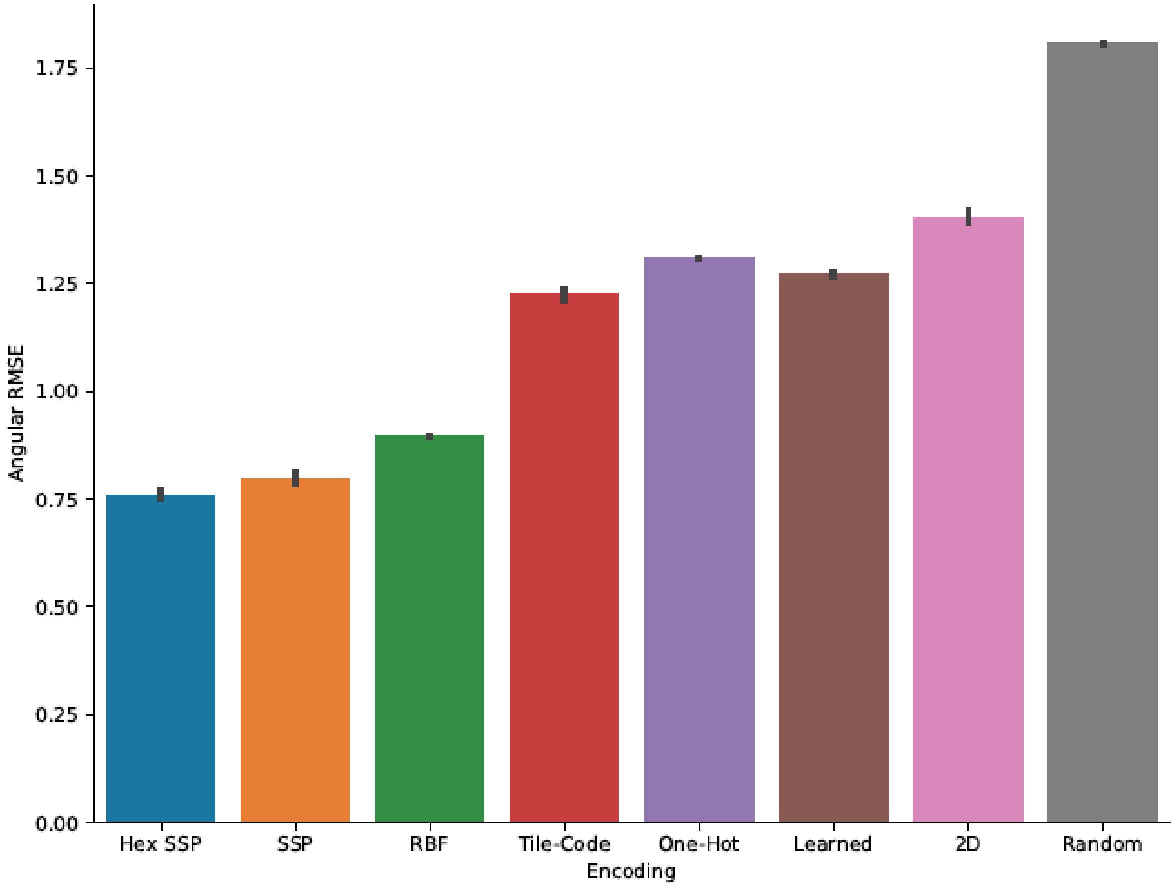
Scaling





Example 10x10 joined, hierarchical
environment

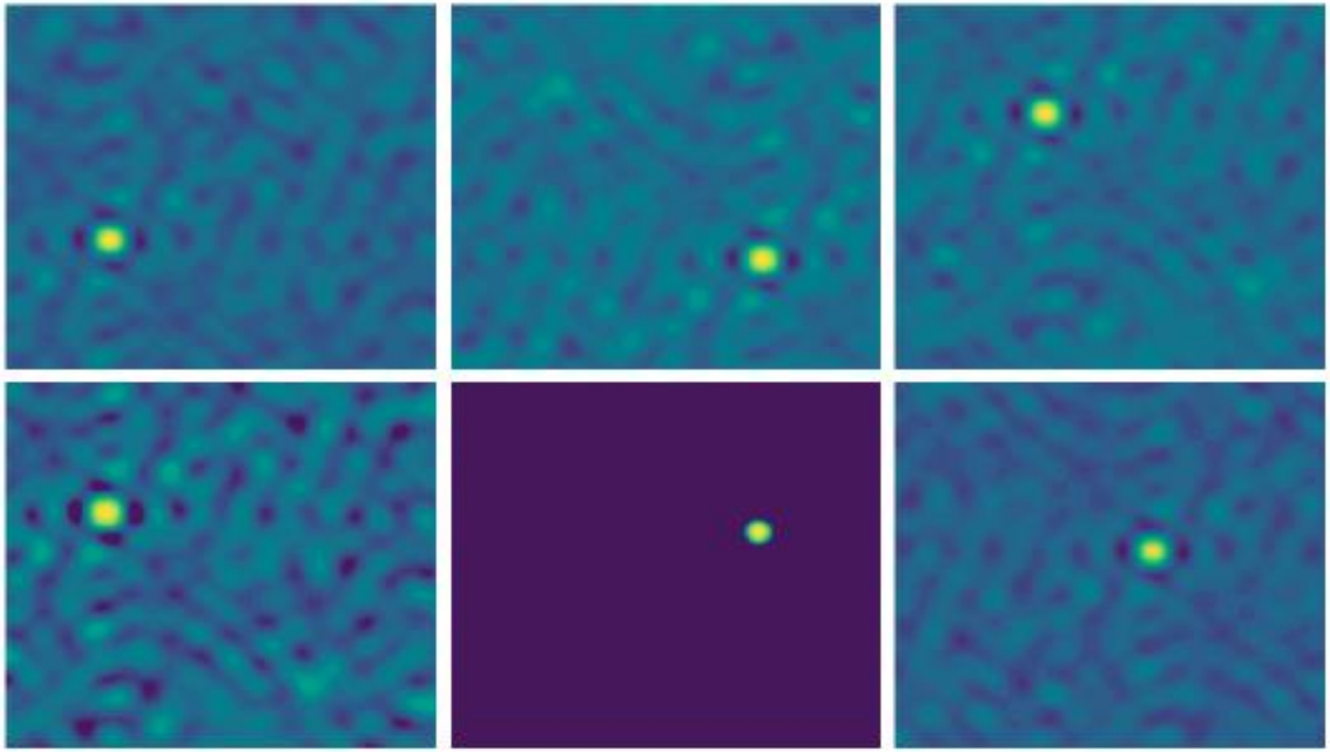
Scaling



Performance of different encoders on 10x10 large maze from any point to any other

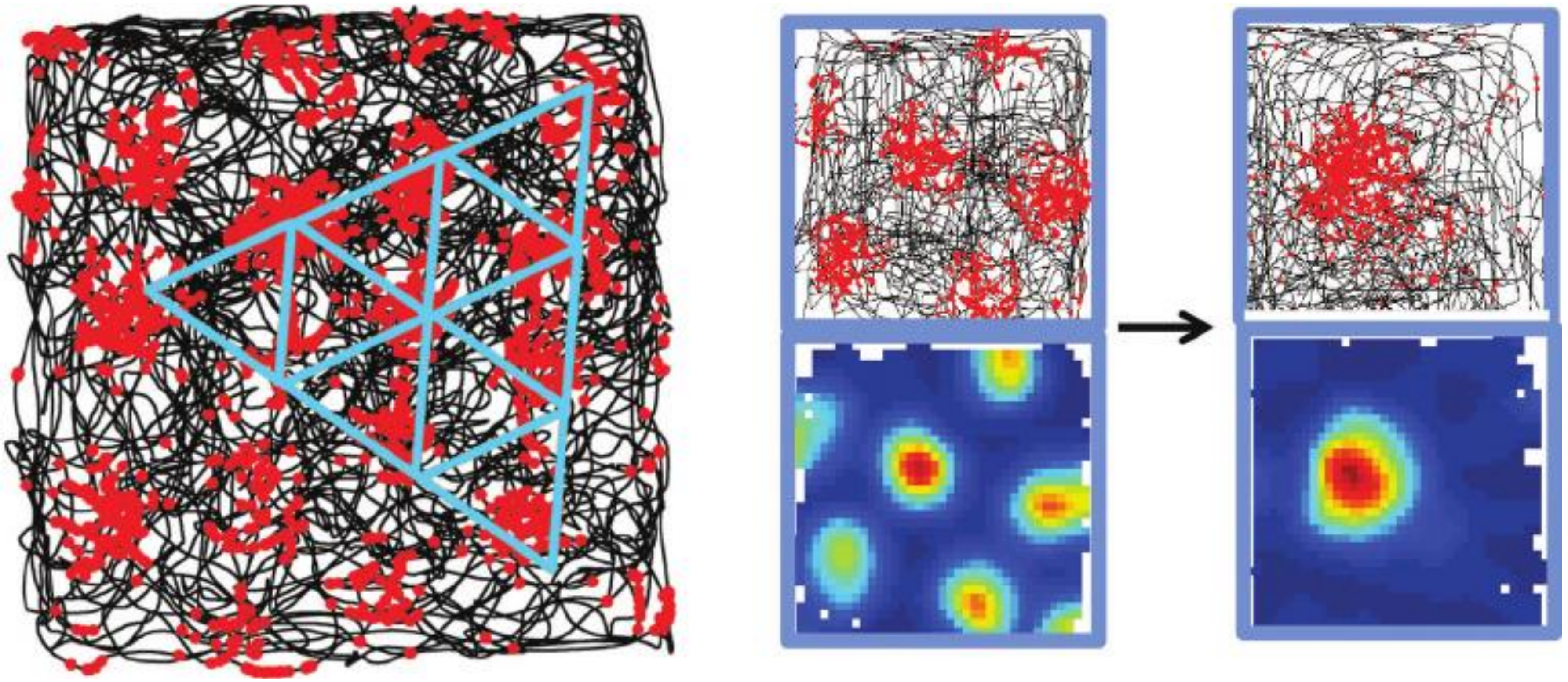
How to Choose Axis Vectors

- Randomly (examples til now)



Tuning curves of neurons with random axis vectors and evenly tiled SSPs as encoders

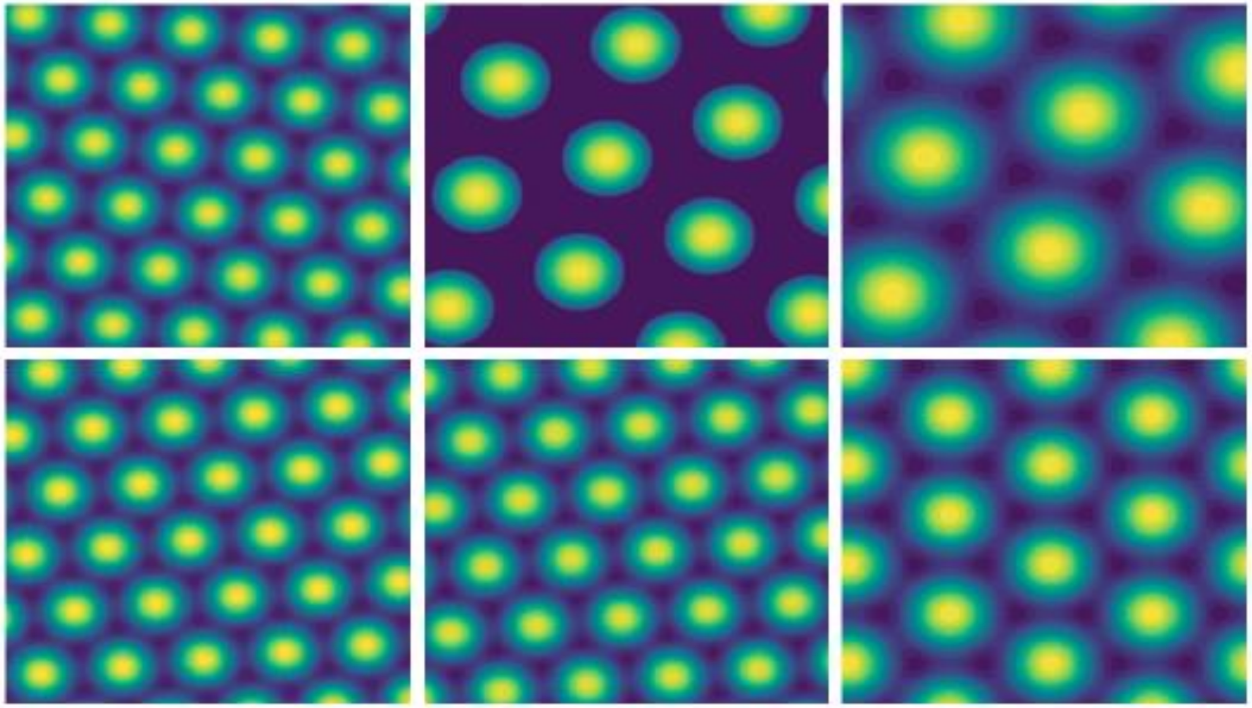
Empirical Grid Cells



Grid cells in rat entorhinal cortex (Moser et al., 2015)

How to Choose Axis Vectors

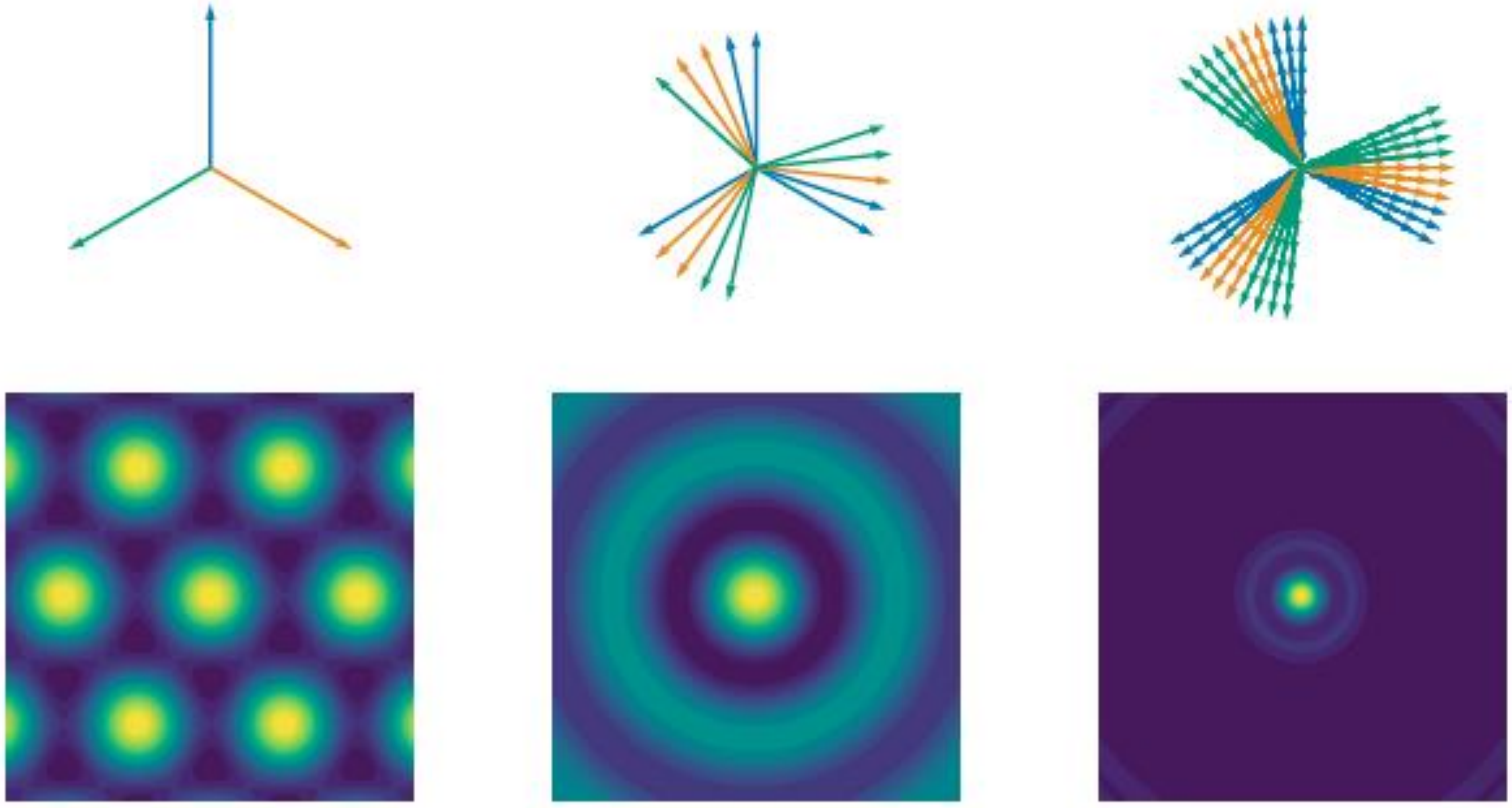
- With plane wave structure



Tuning curves of neurons with structured axis vectors and encoders picking out plane waves

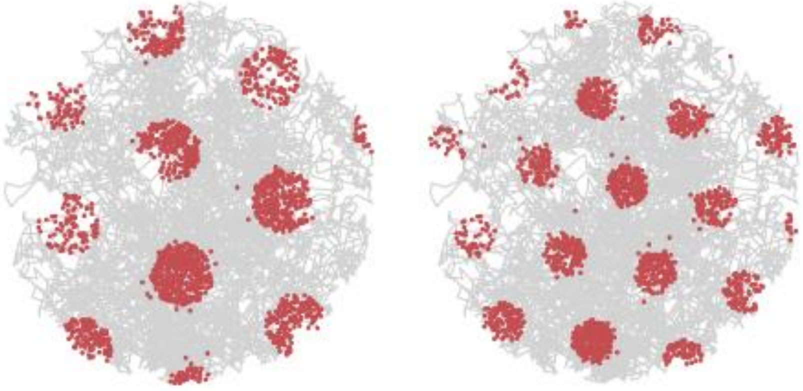
How to Choose Axis Vectors

- Sums of planar waves



Grid Cells

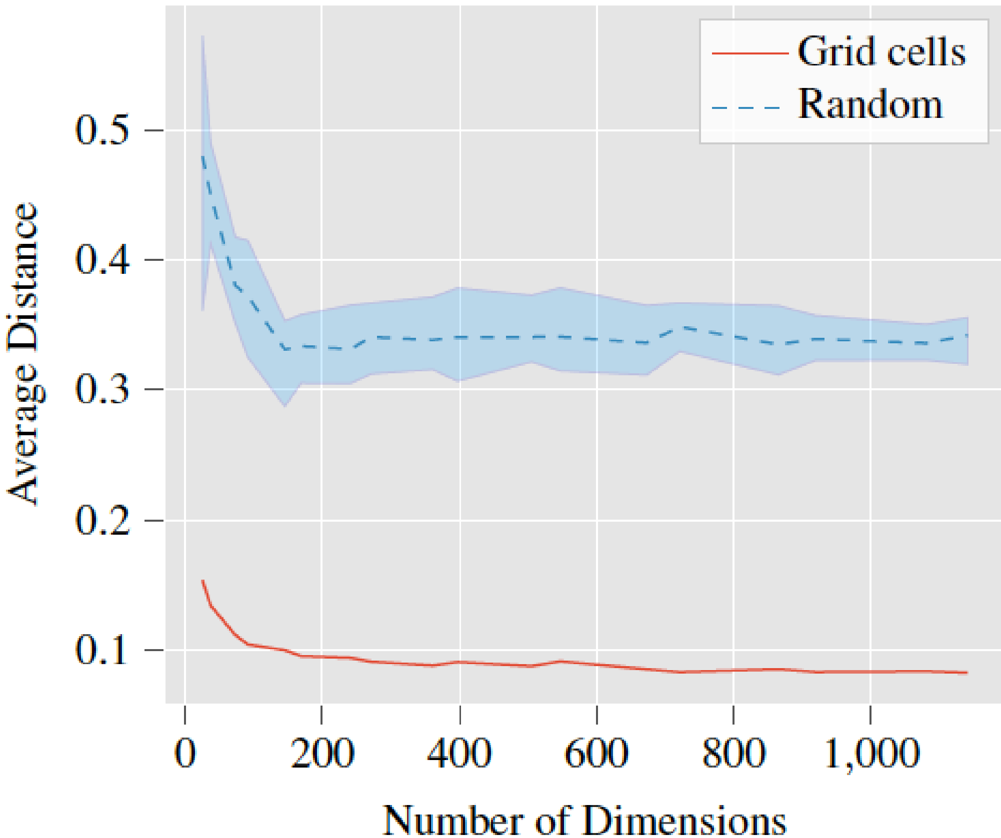
- With plane wave structure, spiking neurons give grid cell responses
- We can combine them to get place cells (with standard NEF decoders)

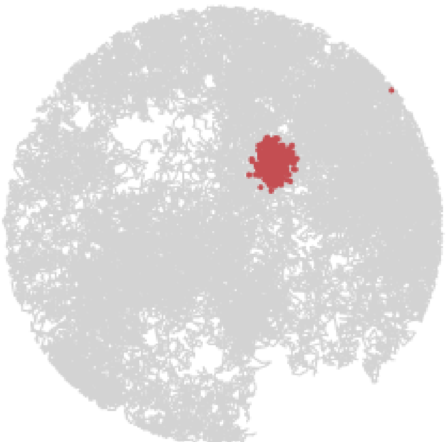


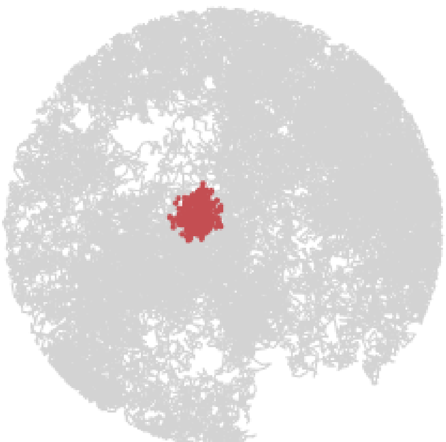
SSP Grid cells

Place Cells

Distance between ideal and represented place cells



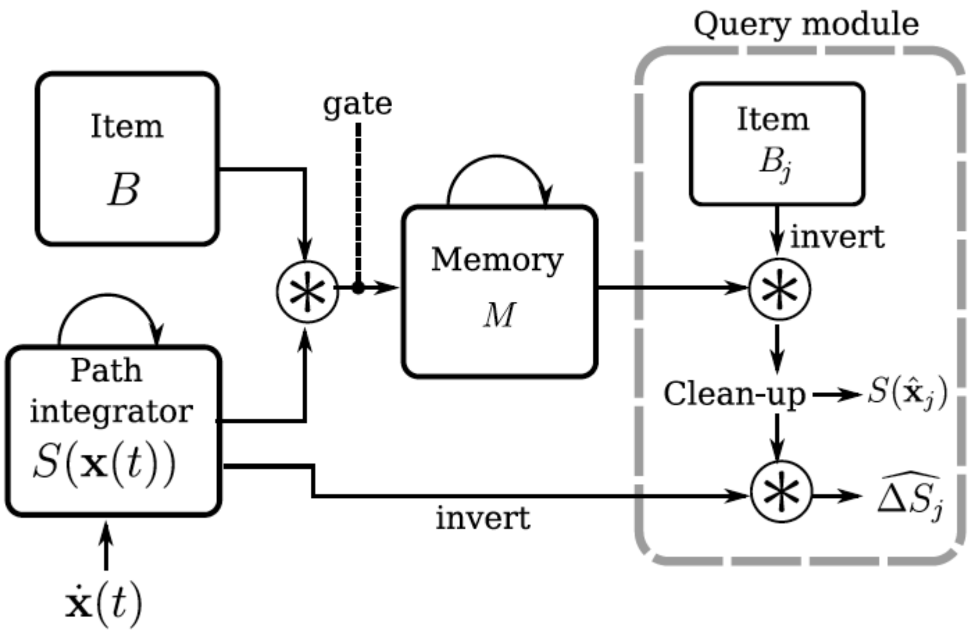




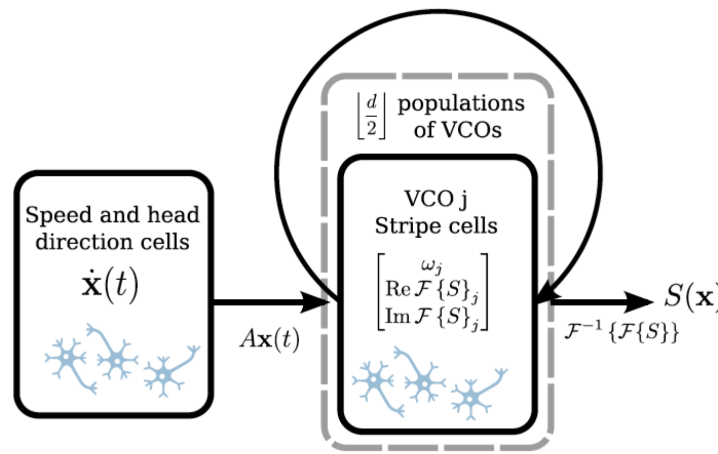
Cognitive SLAM

- SLAM with complex features at certain spatial locations
- Start with no knowledge, bind vector descriptions to particular location in space
- Combines spatial and ‘symbol’ repn in neural network
- Semantic map as opposed to standard ‘image registration’ map

Cognitive SLAM Model



Full Model

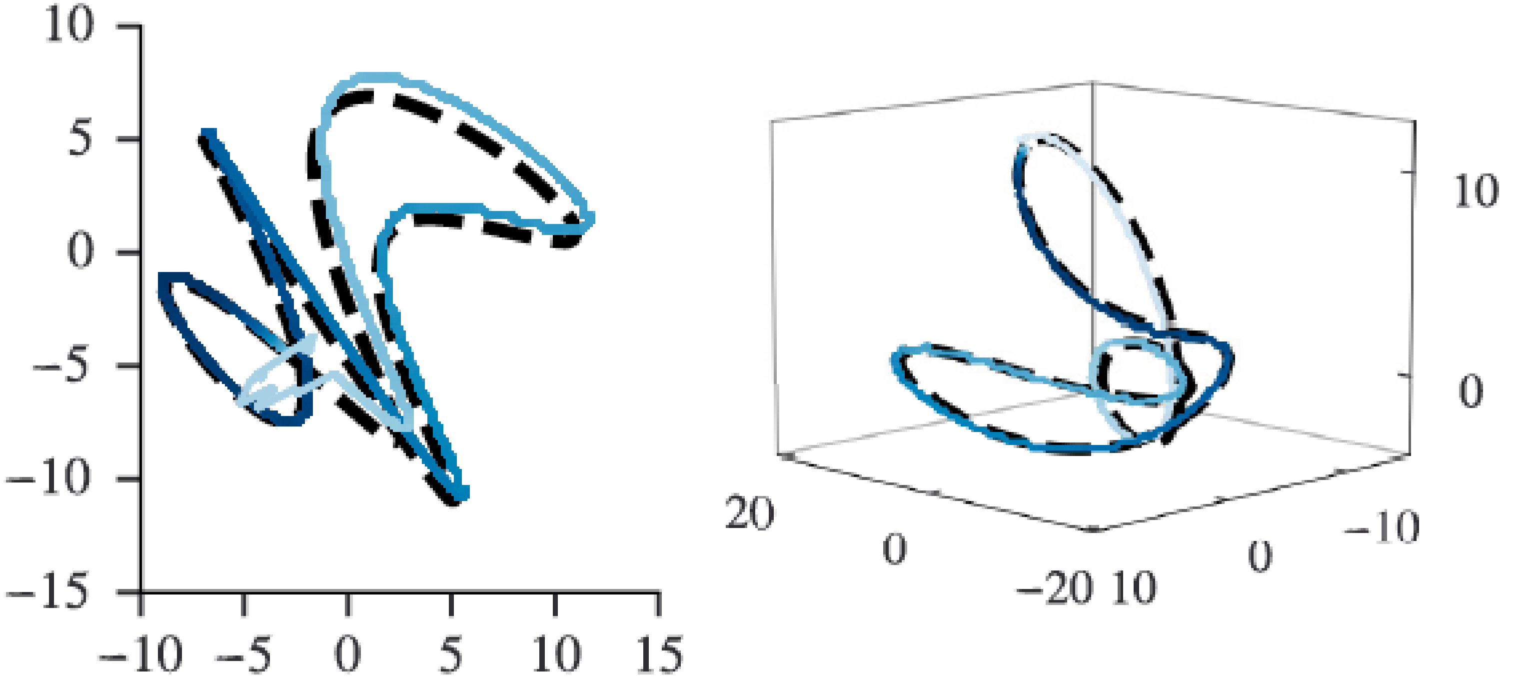


Path Integrator

- Path integrator tracks ego position from velocity
- SLAM model learns env map, outputs allo- and ego-centric position

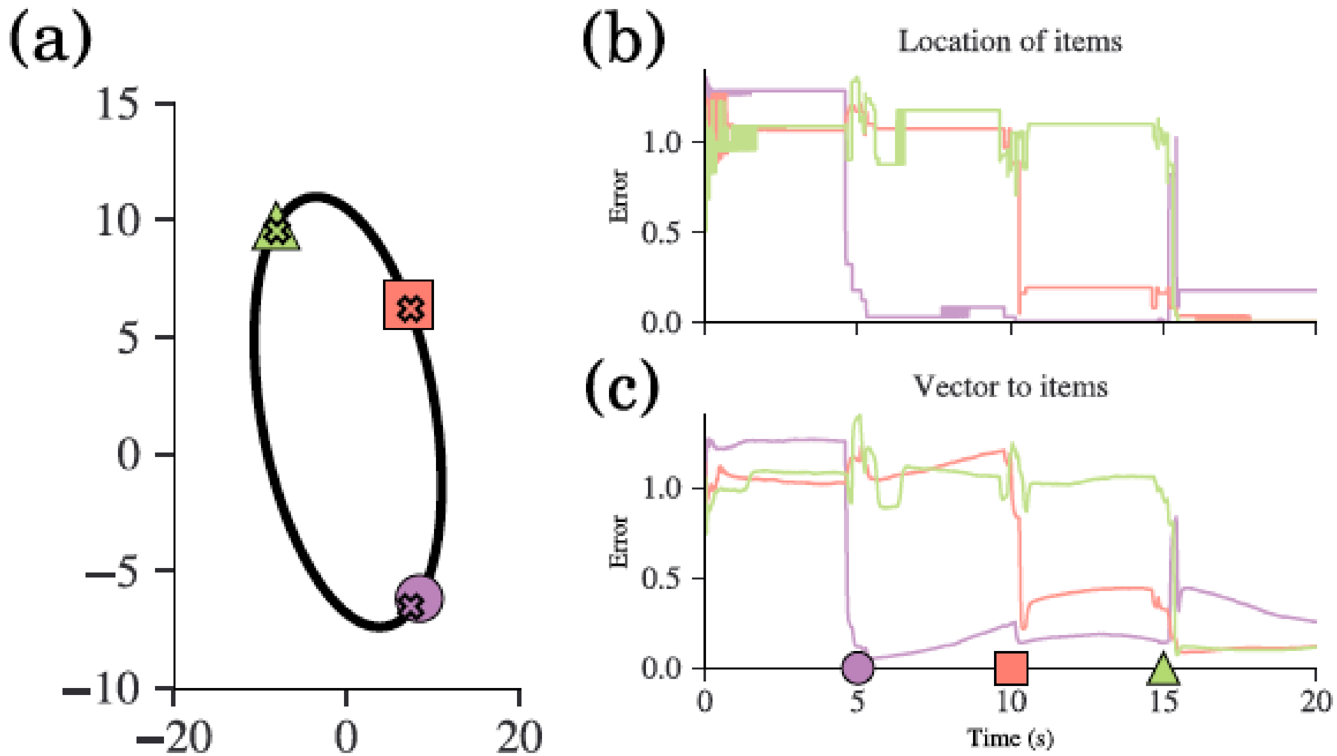
Path Integration

- One-minute long paths in 2D and 3D, spiking network



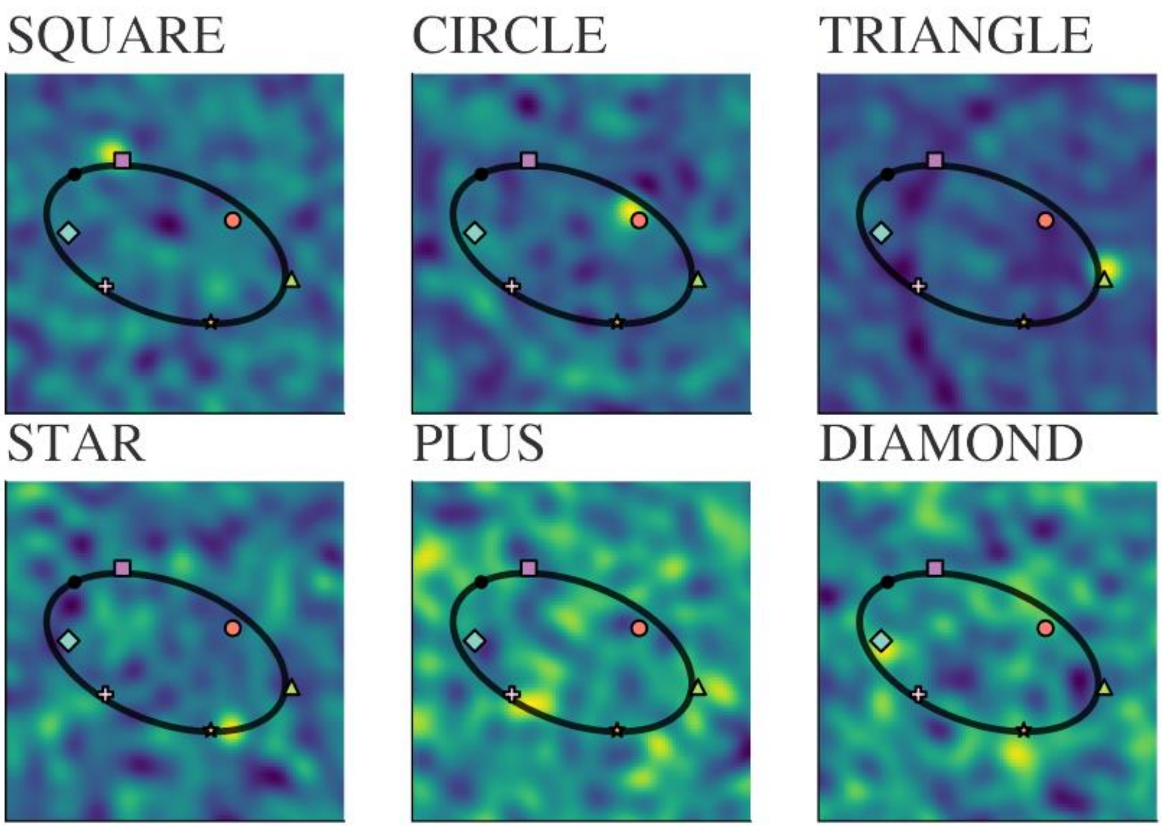
Cognitive SLAM

- Learns env by end of 20s path, full spiking
- Scaling up; symbols 'over' space



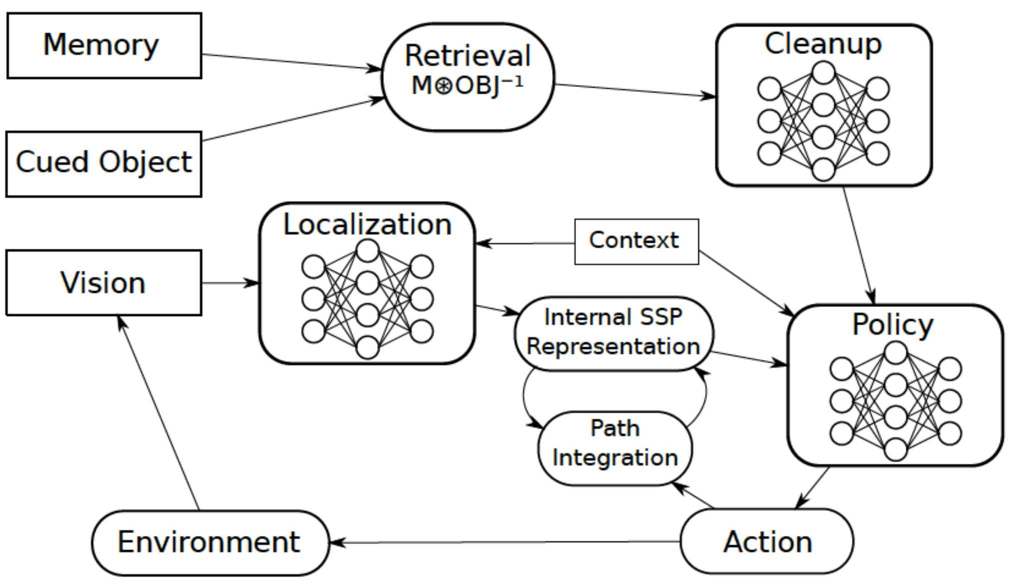
Cognitive SLAM

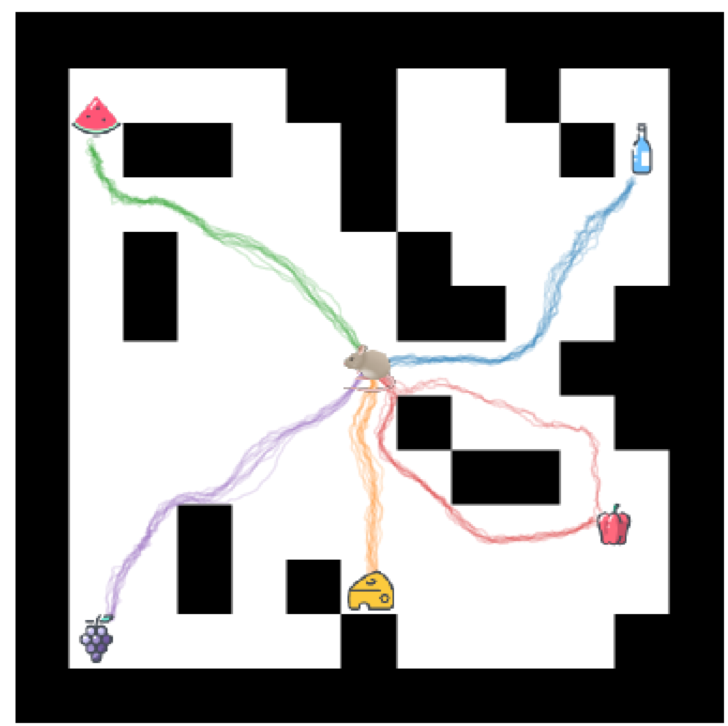
- Learns maps in LTM bound to position



Navigation network

Combined the above into a network to recall location and navigate to arbitrary objects in a maze.





Mathematical properties

- Euclidean space is preserved on a high-d (Clifford) torus

$$S(x_1, y_1) \circledast S(x_2, y_2) = S(x_1 + x_2, y_1 + y_2)$$

- E.g., $S(x, y) \circledast S(\Delta x, \Delta y) = X^{x+\Delta x} \circledast Y^{y+\Delta y}$

- We get the benefits of high-d representation, with accurate low-d Euclidean representation

SSPs as Probabilities

- SSPs can be a method for encoding and processing probabilities
- Method directly connects neural networks to probabilistic reasoning
- SSP based methods are efficient

SSPs as Probabilities

Background

- Kernel Density Estimators

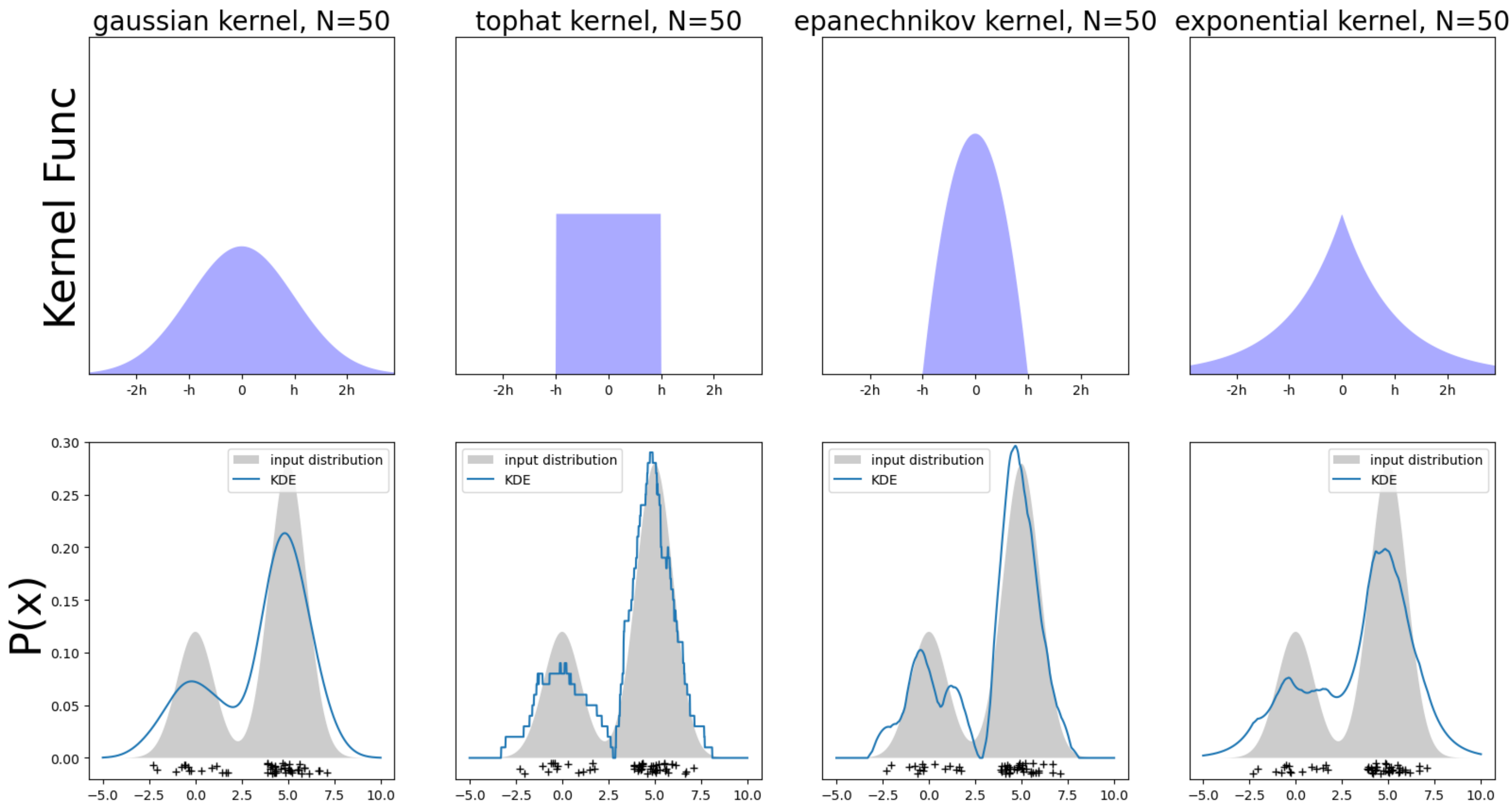
From a dataset $$\mathcal{D} = (x_1, x_2, \ldots, x_n)$$

With a kernel function $$k_h(x, x') = k\left(\frac{\|x - x'\|}{h}\right)$$

We can estimate the probability of x $$P_{\mathcal{D}}(X = x) = \frac{1}{nh} \sum_{x_i \in \mathcal{D}} k_h(x, x_i)$$

SSPs as Probabilities

- Kernel Density Estimators Examples



SSPs as Probabilities

Problems

- KDE memory grows linearly with the number of observations
- KDE time to compute a probability grow linearly with # of observations

But…

- Not if your kernel is a dot product

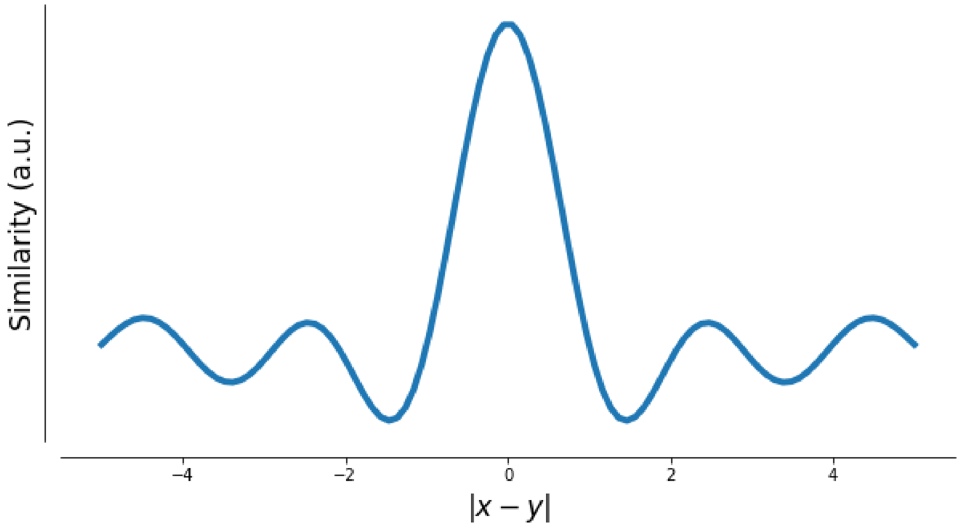
$$k_h(\mathbf{x}, \mathbf{x}') \approx \phi_h(\mathbf{x}) \cdot \phi_h(\mathbf{x}')$$

$$\Longrightarrow \quad P(X = \mathbf{x}) \approx \phi_h(\mathbf{x}) \cdot \sum_{\mathbf{x}_i \in \mathcal{D}} \phi_h(\mathbf{x}_i)$$

SSPs as Probabilities

SSPs induce a quasi-kernel

- 'quasi' because there are negatives



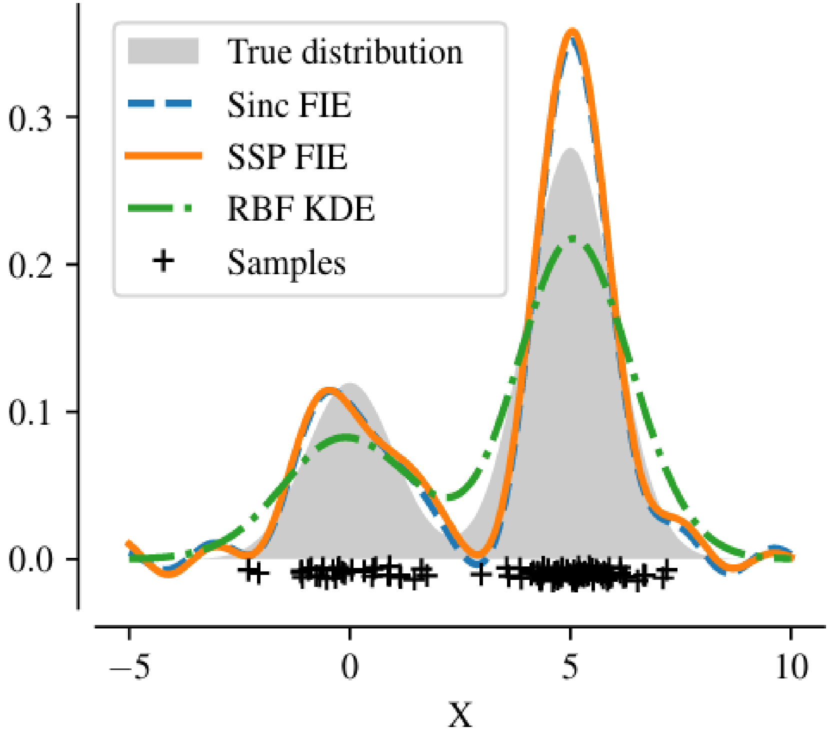
$$\phi(\mathbf{x}) \cdot \phi(\mathbf{y}) \approx \prod_{k=1}^{m} \mathrm{sinc}(|x_k - y_k|)$$

SSPs as Probabilities

Can use as a probability estimator

$$P_{\mathcal{D}}(X = x) = \max\left\{0, \phi(x) \cdot M_{\mathcal{D},h} - \xi\right\}$$

Glad et al, 2003



$$\boxed{\mathrm{ReLU}(\mathbf{w} \cdot \mathbf{z} + b)}$$

SSPs as Probabilities

So SSP memory is a latent probability distribution

- The distribution is stored in bundles of vector symbols.
- We can apply manipulations to bundles to produce probabilistic statements

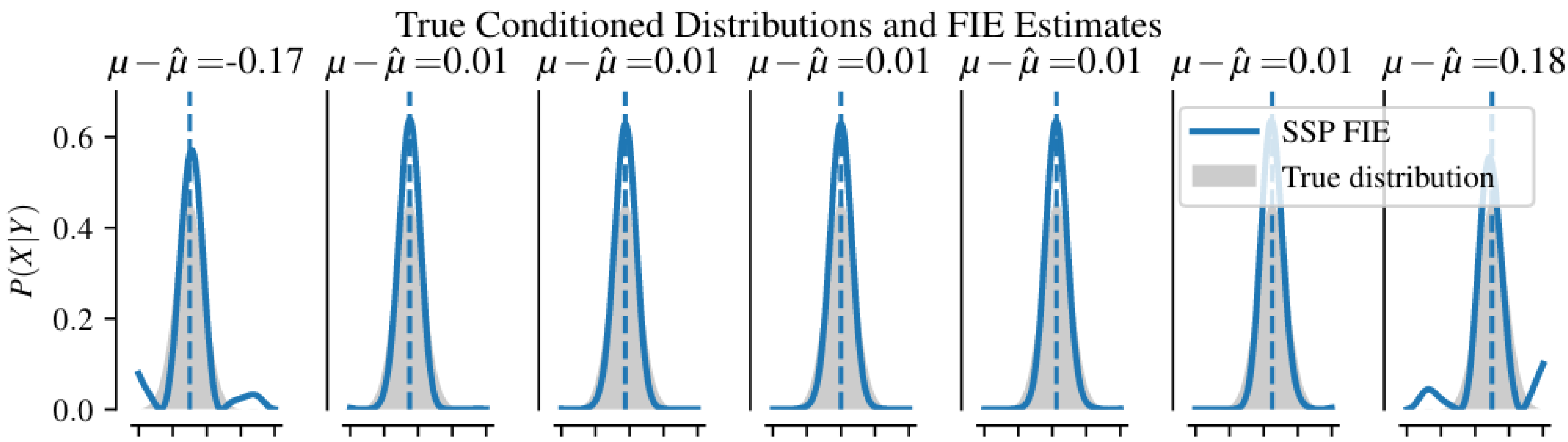
$$M_{\mathcal{D}} = \frac{1}{n} \sum_{\mathbf{x}_i \in \mathcal{D}} \phi(\mathbf{x}_i)$$

$$P(\mathrm{X} = \mathbf{x}) = \phi_h(\mathbf{x}) \cdot M_{\mathcal{D}}$$

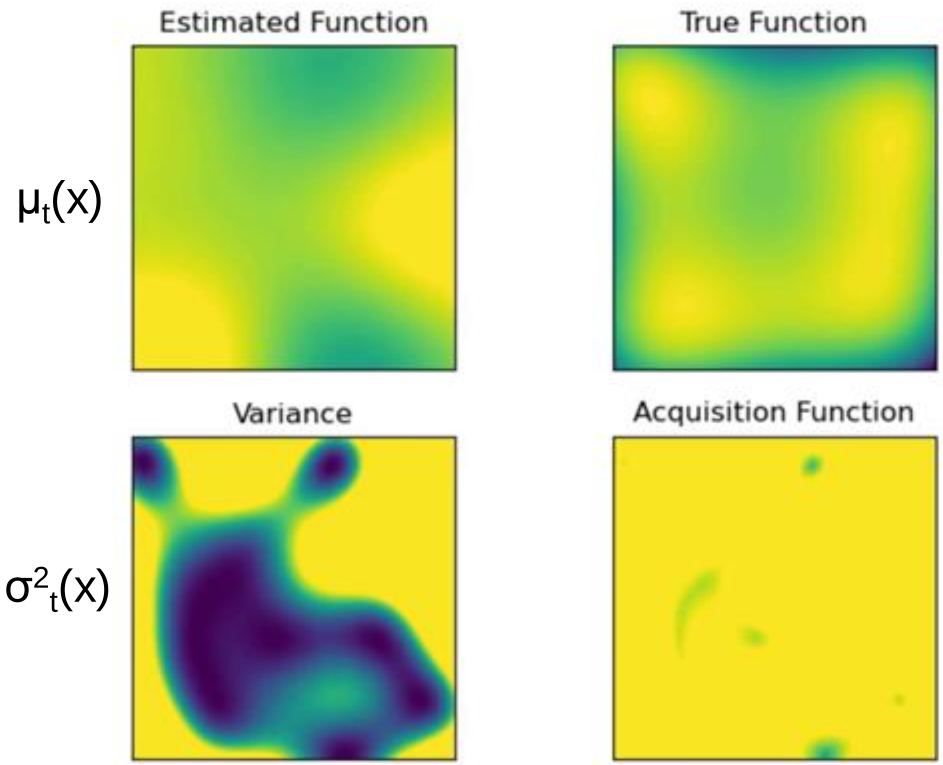
SSPs as Probabilities

Conditioning

$$P(\mathrm{X}=x \mid \mathrm{Y}=y) \approx \phi_{\mathrm{X}}(x) \cdot \left[M_{\mathcal{D}} \circledast \phi^{-1}(y)\right]$$



SSPs as Probabilities

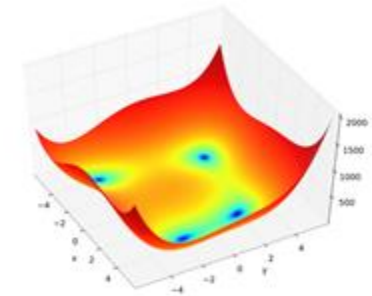


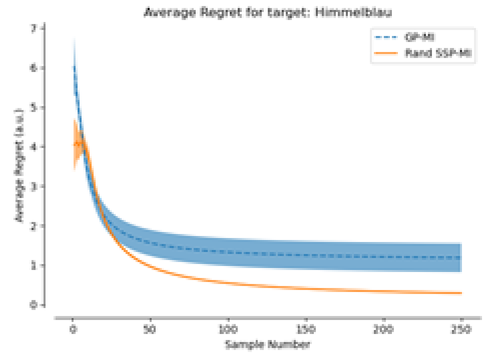
- Mutual Information (MI) is a common objective function used in exploration
- Gaussian Processes (GPs) are a convenient, but computationally intensive tool for computing MI
- Can use SSPs and Bayesian linear regression to approximate a GP while improving in memory and time complexity

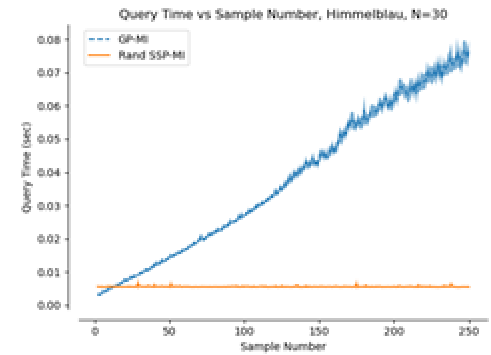
Results

Accuracy | Runtime

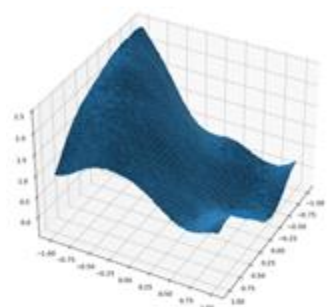
Himmelblau Function

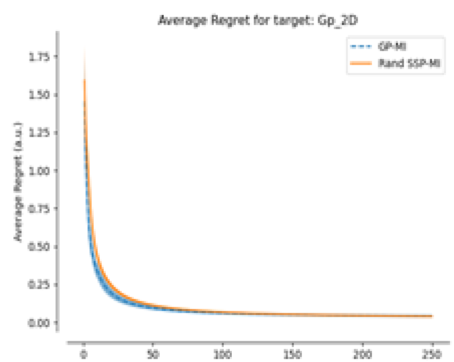


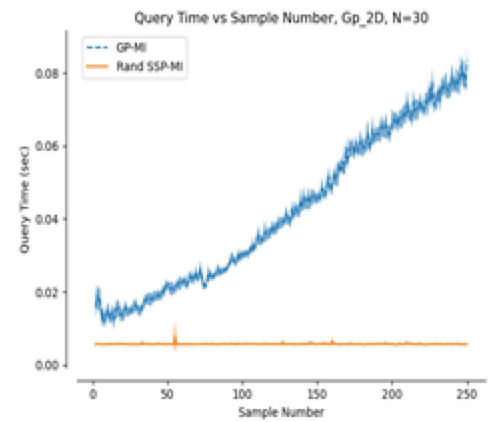




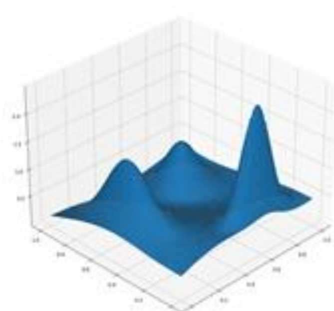
Sampled from GP with Matern Kernel + 1% noise

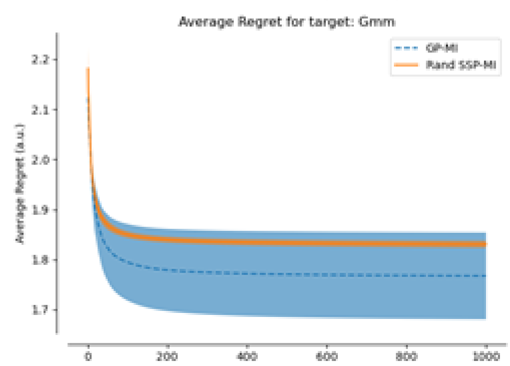


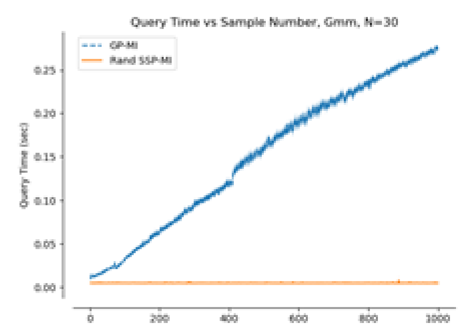




GMM with noise ~ (Matern Kernel + 1% noise)







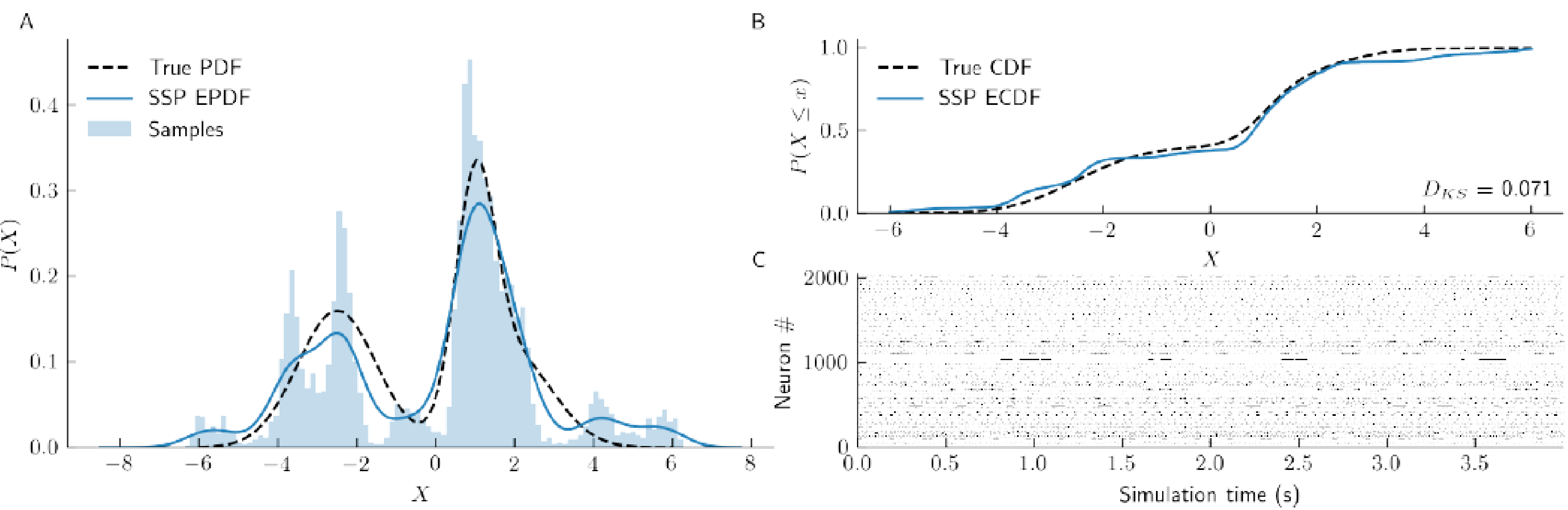
SSPs for Sampling

- Current methods require knowledge not accessible to a neural system (e.g., the encoding, gradients)
- Can use Langevin dynamics to do MC sampling with SSPs
- Supports conditioning easily as well
- Turns latent repns into samples in an encoding agnostic manner

SSPs for Sampling

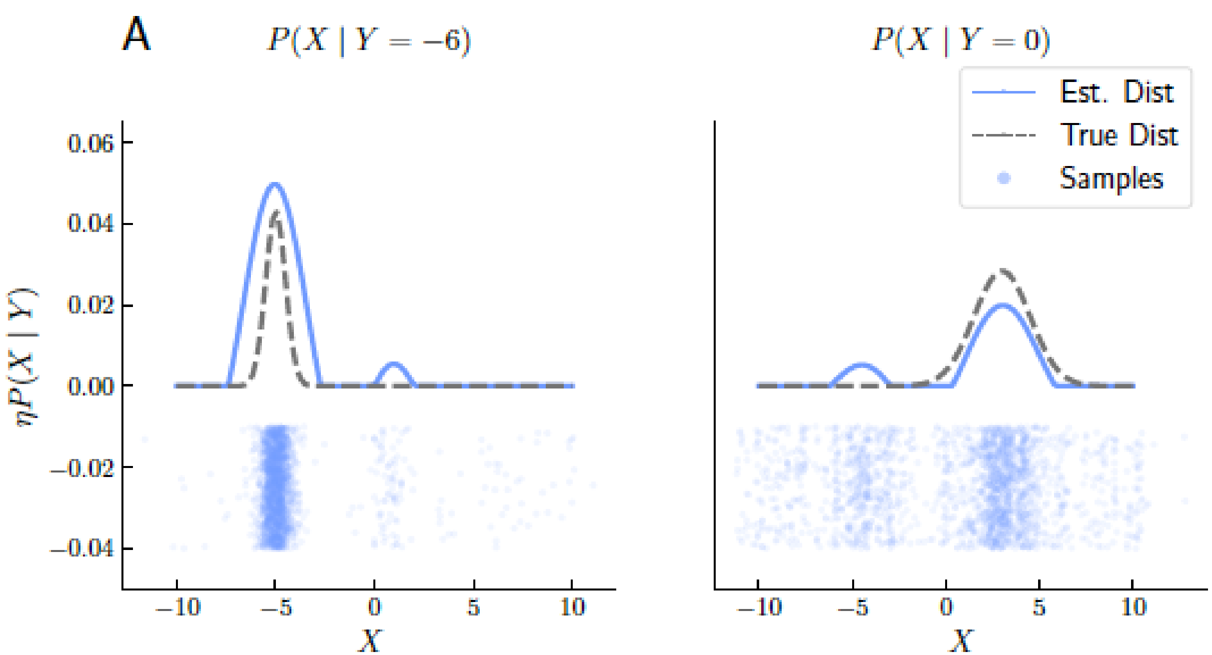
Implementing the dynamics in a SNN generates effective samples

$$f(\phi(t)) = \tau\gamma\nabla_{\phi(t)} \log P(x) + \phi(t),$$

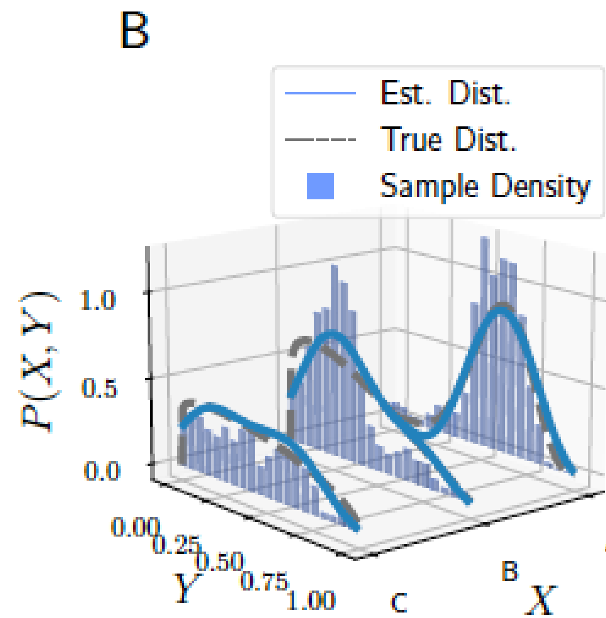


SSPs for Sampling

Sampling using dynamics for both continuous and discrete PDFs



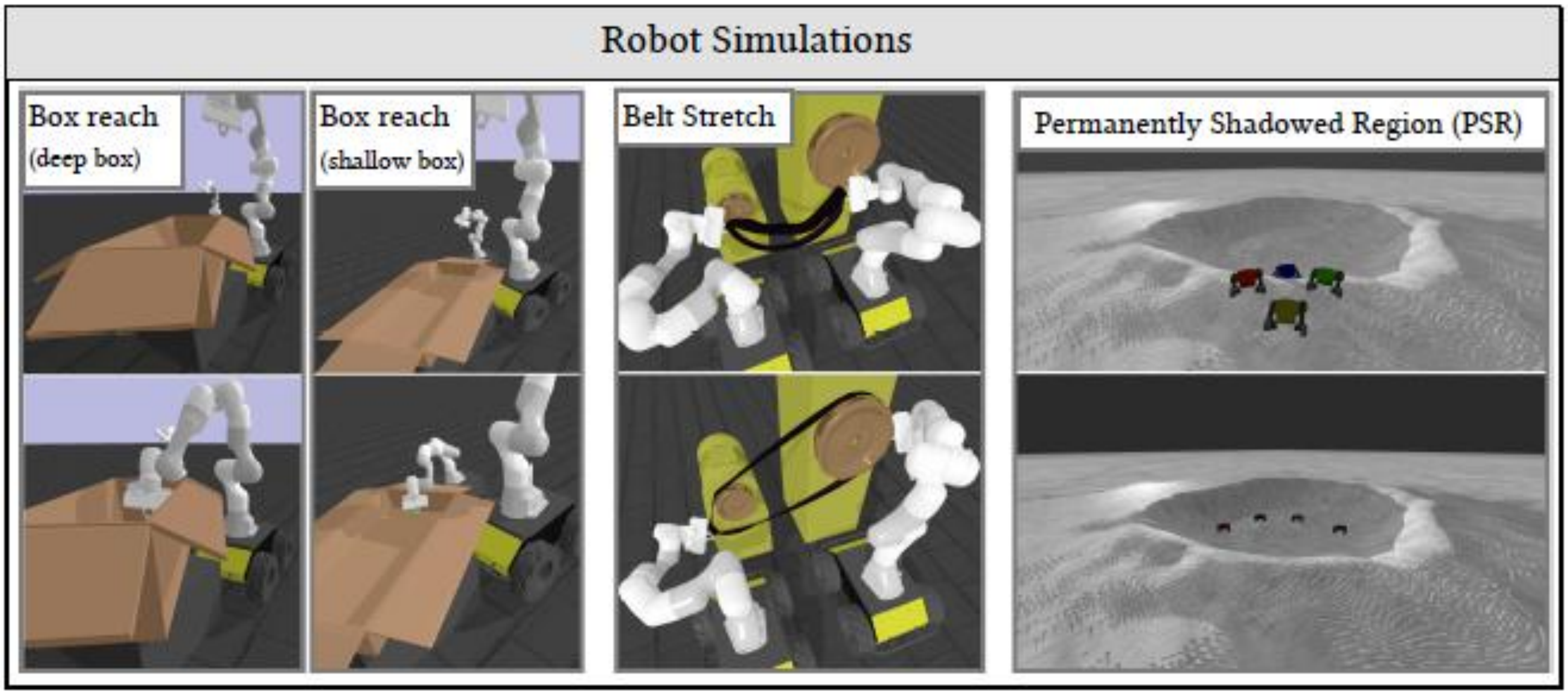
2D Gaussian Mixture Model



Hybrid Discrete/Continuous

SSPs for Optimization

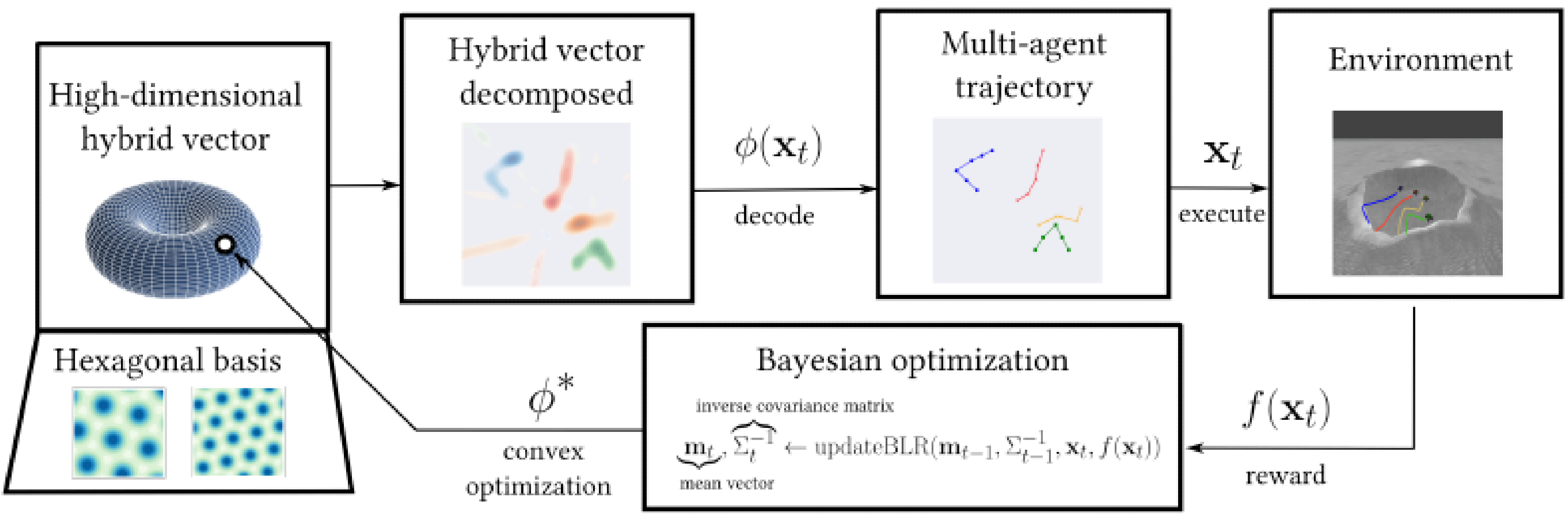
Three difficult optimization problems



SSPs for Optimization

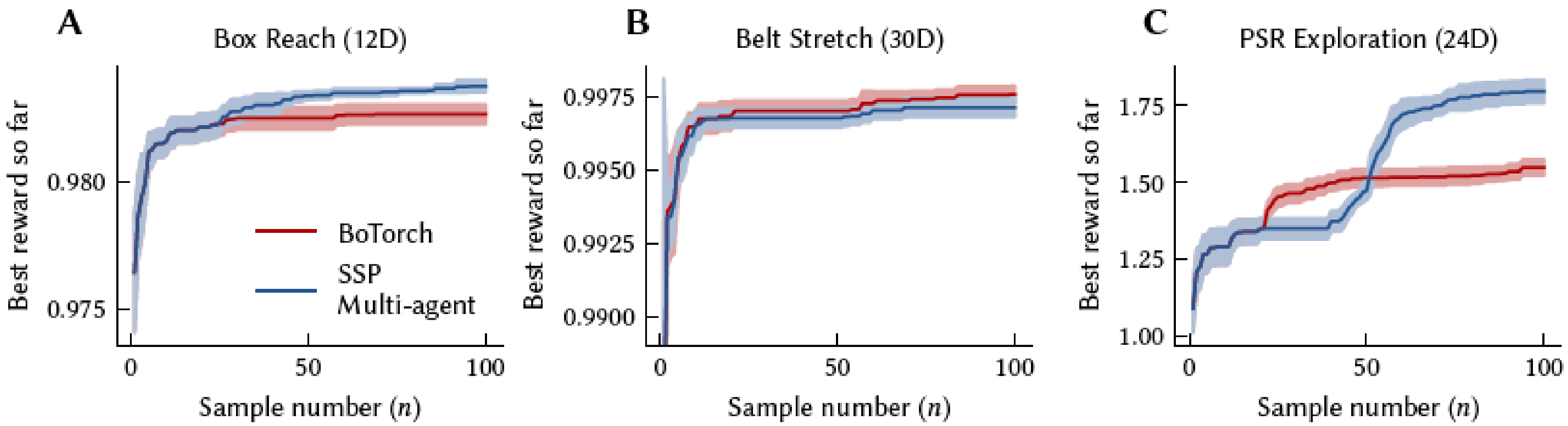
We use SSPs to represent possible trajectories

- Hybrid repn: sum(agent*trajectory)



SSPs for Optimization

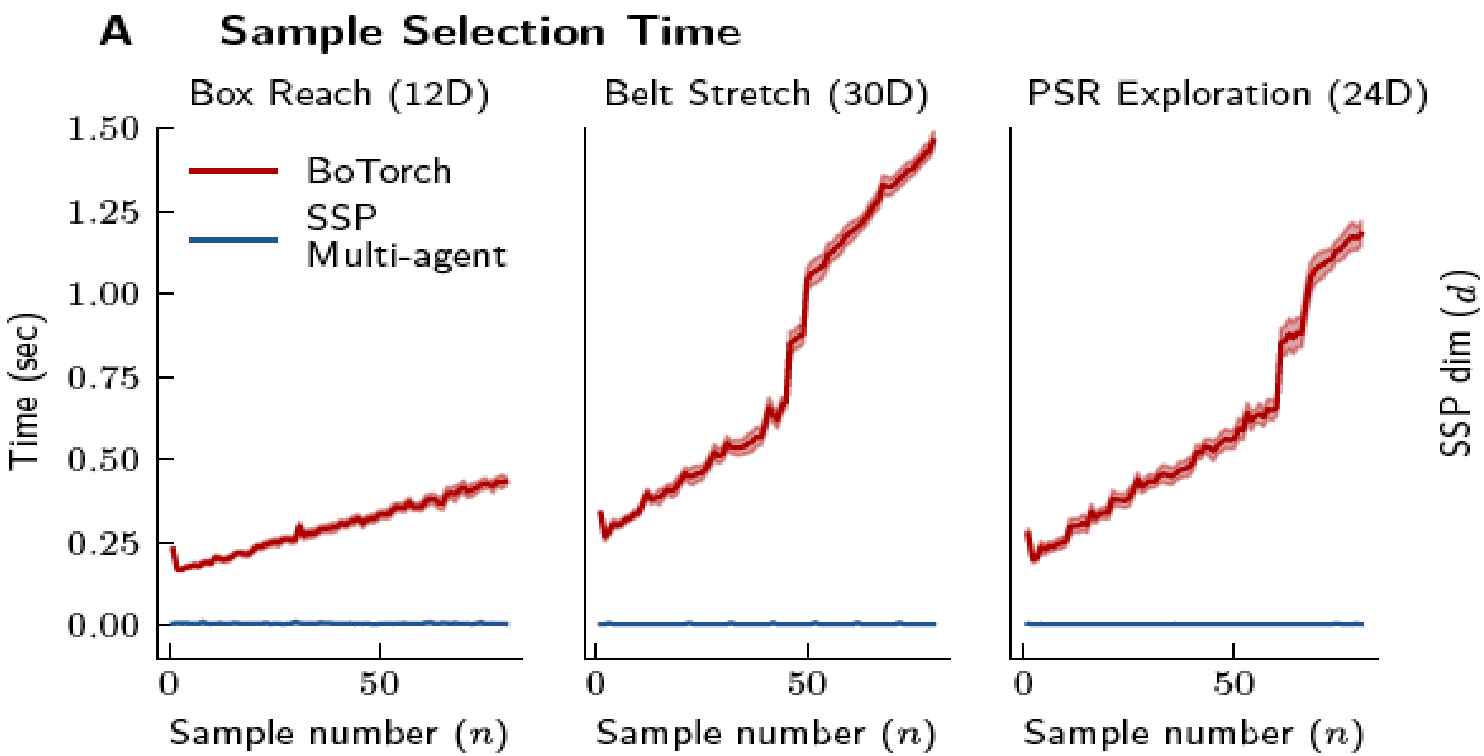
As good or better performance than SoA



SSPs for Optimization

Much faster (53x-175x)

- From $O(N^3)$ to $O(1)$ in number of samples



SSPs as Probabilities

Useful properties:

- Provide a general and abstract framework for modelling probabilities
- Draw a direct connection between cognitive models and probability statements
- Provide network architectures for conditioning, marginalization, entropy, sampling, and mutual information

Conclusion

- SSPs support a variety of types of inference for cognitive models
  - Hybrid spatial and ‘symbolic’ representations
  - Representations of sampled data that can be used for probabilistic inference
- Improves
  - Interpretability
  - Efficiency