SYDE 556/750

Simulating Neurobiological Systems
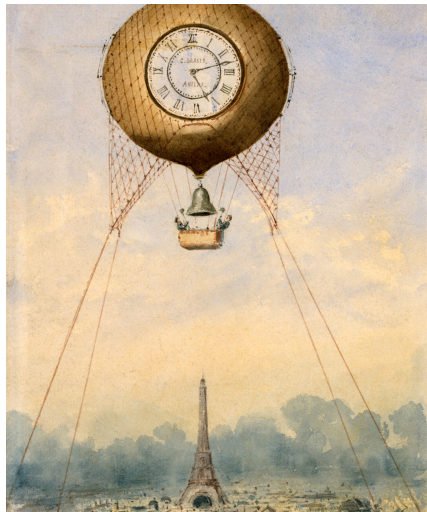**Lecture 4: Temporal Representations**

Chris Eliasmith

Sept 18 & 23, 2024

- ▶ Slide design: Andreas Stöckel
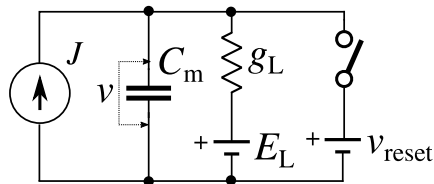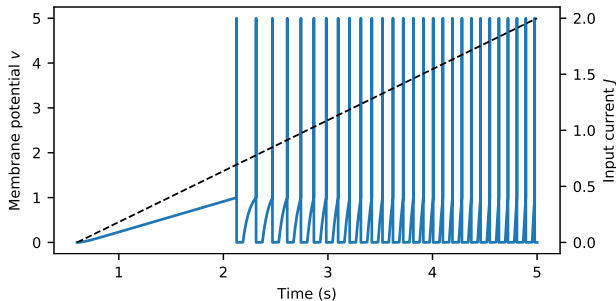- ▶ Content: Terry Stewart, Andreas Stöckel, Chris Eliasmith

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Reminder: The LIF Neuron

$$\frac{\mathrm{d}}{\mathrm{d}t}v(t) = -\frac{1}{\tau_{\mathrm{RC}}}\big(v(t) - J\big)\,, \qquad \text{if } v(t) < 1\,,$$
$$v(t) \leftarrow \delta(t - t_{\mathrm{th}})\,, \qquad\qquad \text{if } t = t_{\mathrm{th}}\,,$$
$$v(t) \leftarrow 0\,, \qquad\qquad \text{if } t > t_{\mathrm{th}} \text{ and } t \geq t_{\mathrm{th}} + \tau_{\mathrm{ref}}\,,$$

Temporal Decoding

▶ Our decoders to this point have ignored time, as we used a rate response function to calculate them.
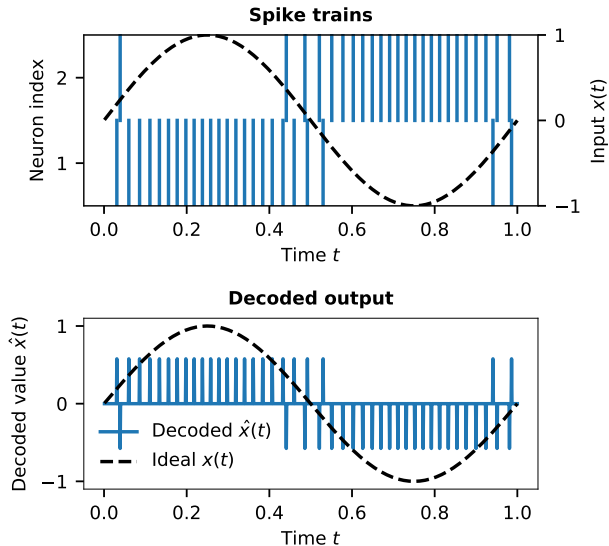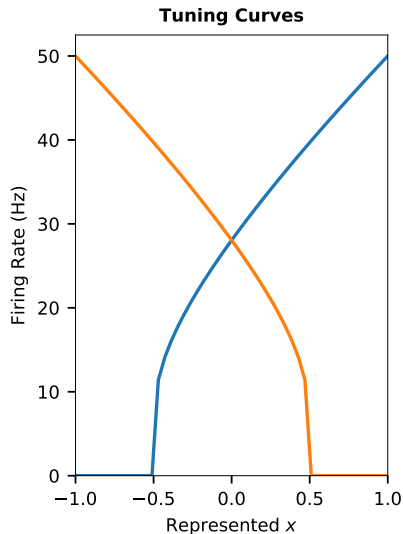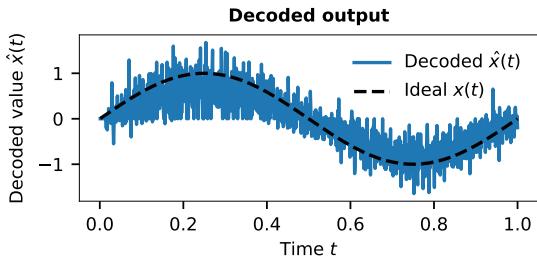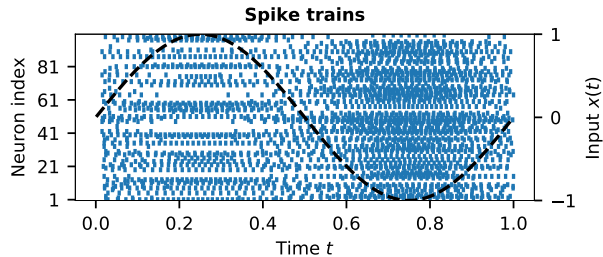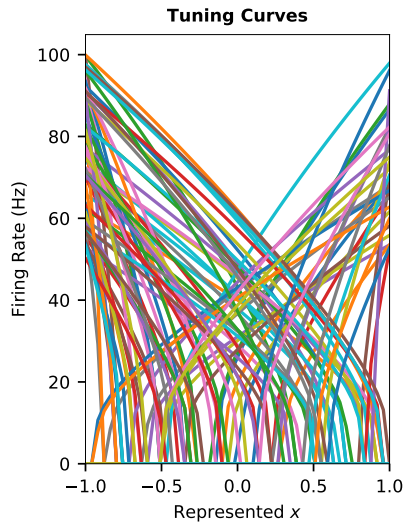
Temporal Decoding

- ▶ Our decoders to this point have ignored time, as we used a rate response function to calculate them.

- ▶ What happens if we use those decoders with the spike trains generated by spiking LIF neurons?

Temporal Decoding of Two Neurons - Weighted Spikes

Temporal Decoding of One Hundred Neurons - Weighted Spikes
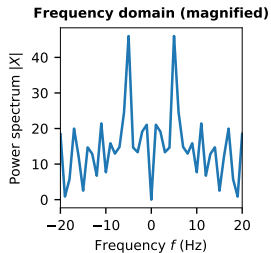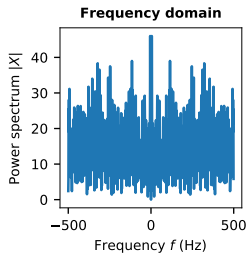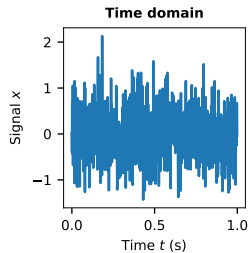
Temporal Decoding

▶ For population decoders, we needed to integrate their responses over the represented variable, x

Temporal Decoding

- ▶ For population decoders, we needed to integrate their responses over the represented variable, x

- ▶ What is the equivalent integration for time? That is, what space do we want to sample to estimate the integral?
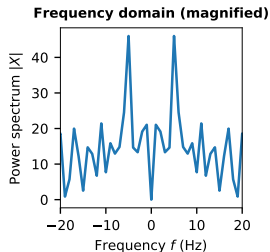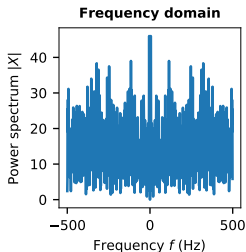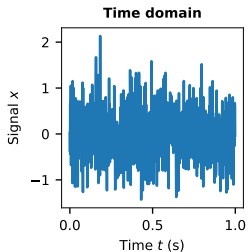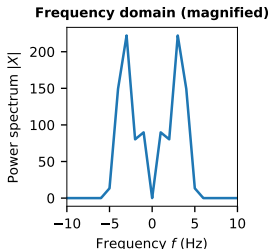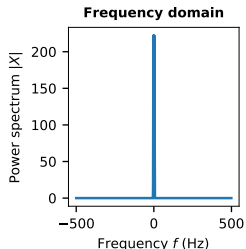
Random Signals

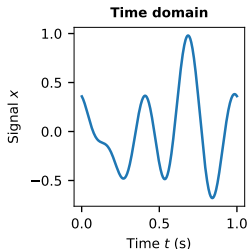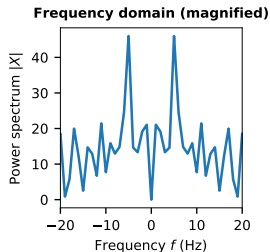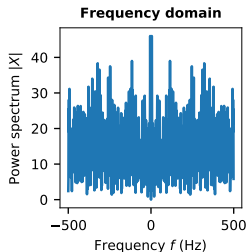
White Noise
(zero mean)

Random Signals

White Noise
(zero mean)

Bandlimited
White Noise
(zero mean,
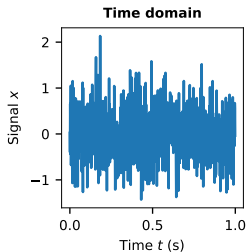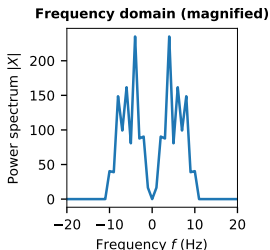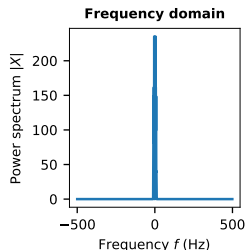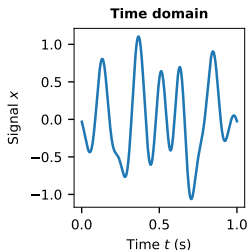$5\,\mathrm{Hz}$ bandwidth)

Random Signals

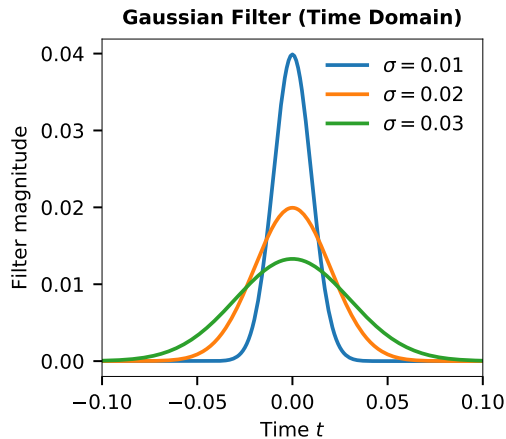**White Noise**
(zero mean)

Bandlimited
White Noise
(zero mean,
10 Hz bandwidth)

Filtering by Convolution

Gaussian Filter (Time Domain)
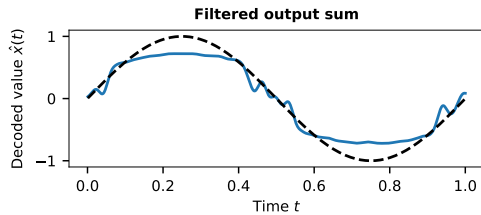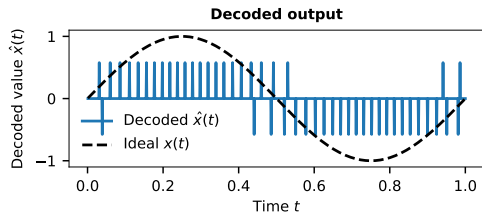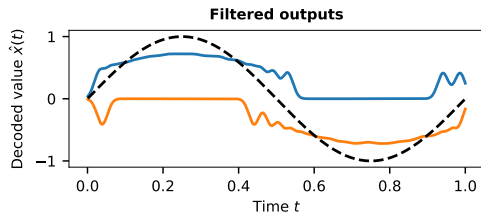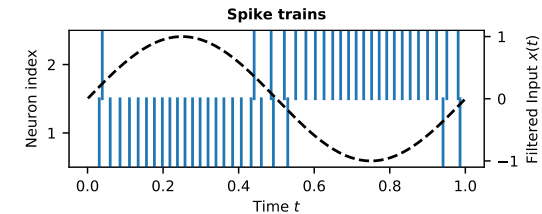
Gaussian Filter

$$h(t) = c \exp\left(\frac{-t^2}{\sigma^2}\right)$$

where c chosen s.t. $\int_{-\infty}^{\infty} h(t)\,\mathrm{d}t = 1$

Convolution

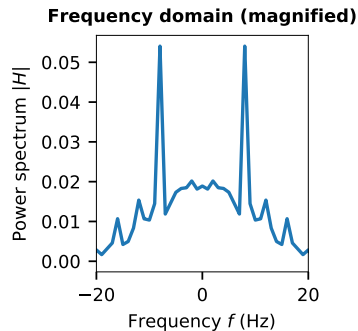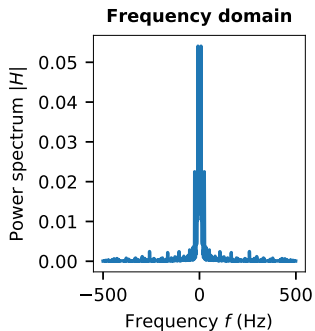$$\bigl(f * h\bigr)(t) = \int_{-\infty}^{\infty} f(t - t')h(t')\,\mathrm{d}t'$$
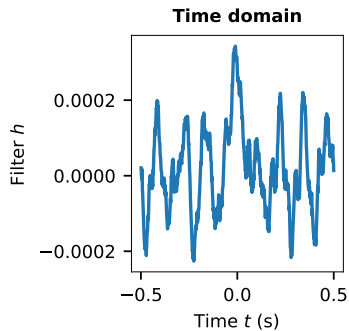
Filtering a Spike Train

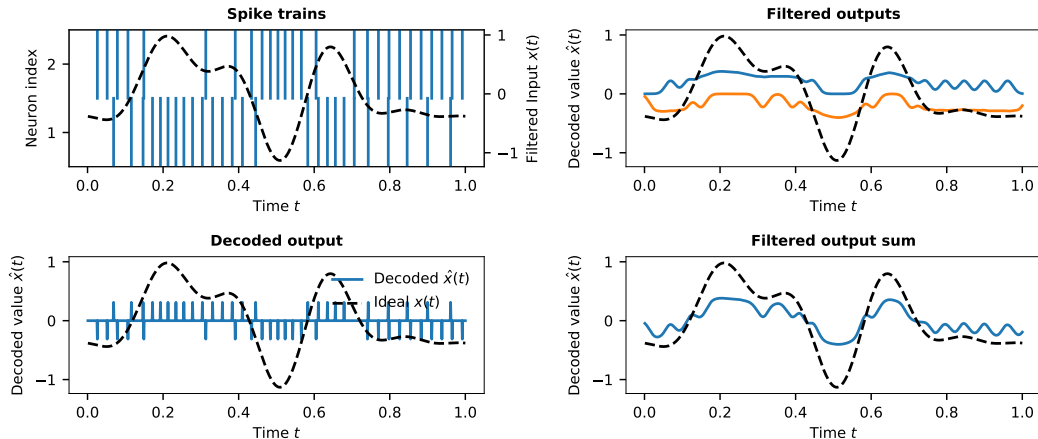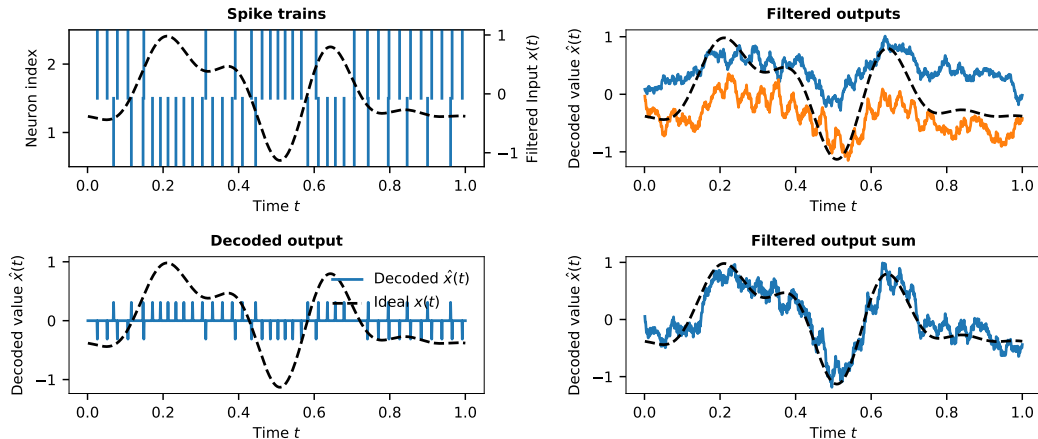# Filtering a Spike Train for a Random Signal
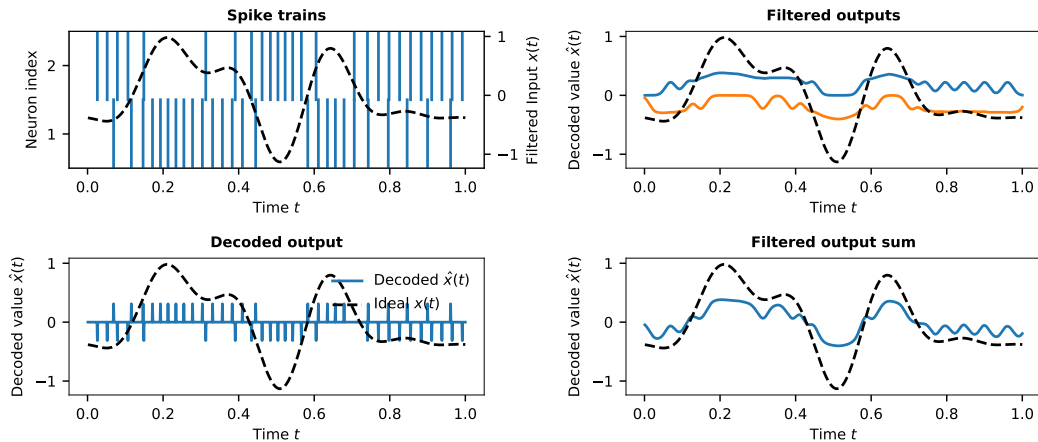
Optimal Filter

$$H(\omega) = \frac{X(\omega)\overline{R}(\omega)}{|R(\omega)|^2}$$

Filtering a Spike Train for a Random Signal (Optimal Filter)

Filtering a Spike Train for a Random Signal (Optimal Filter)

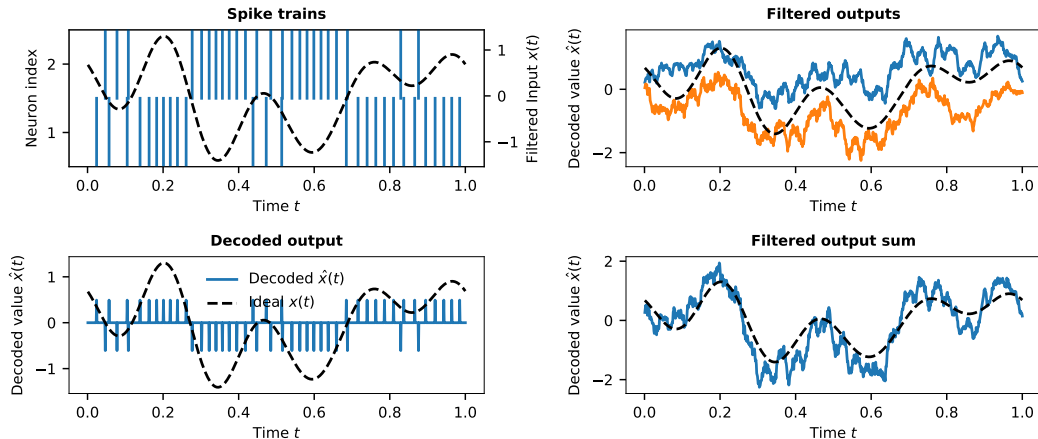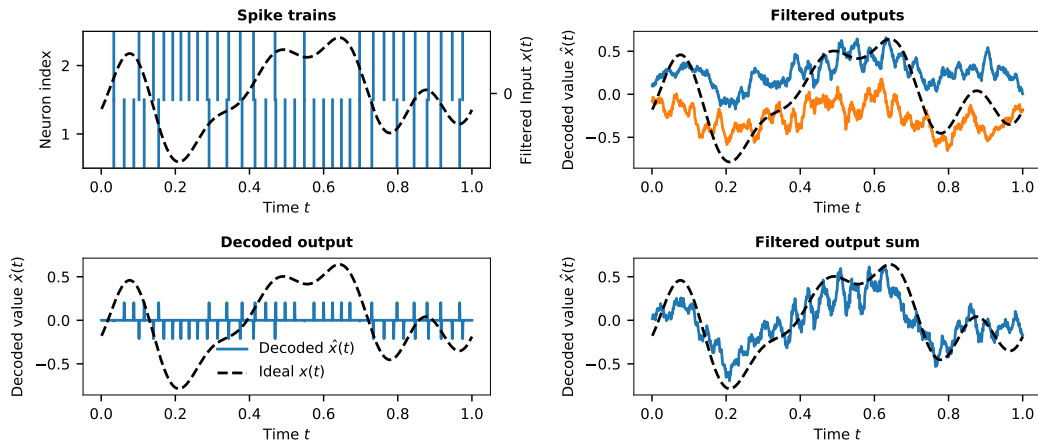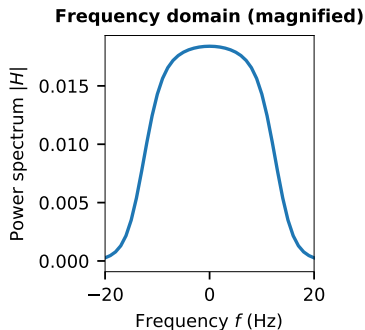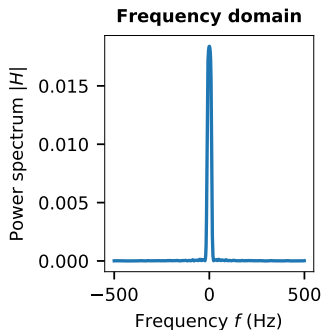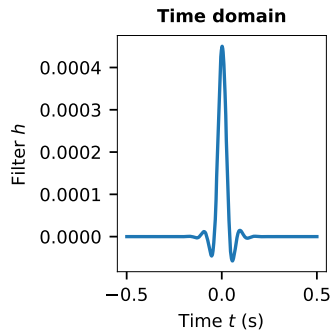# Filtering a Spike Train for a Random Signal (Optimal Filter)

Filtering a Spike Train for a Random Signal (Optimal Filter)

Optimal Filter (Improved)

Time domain   **Frequency domain**   **Frequency domain (magnified)**
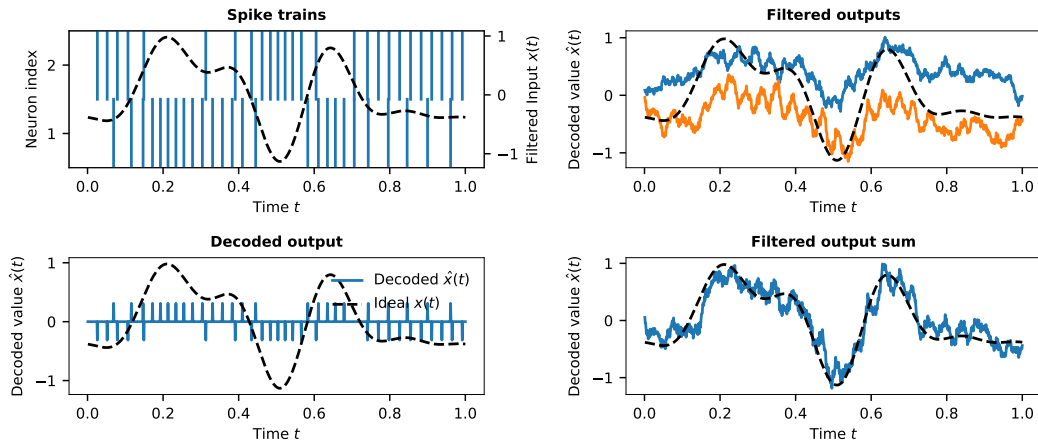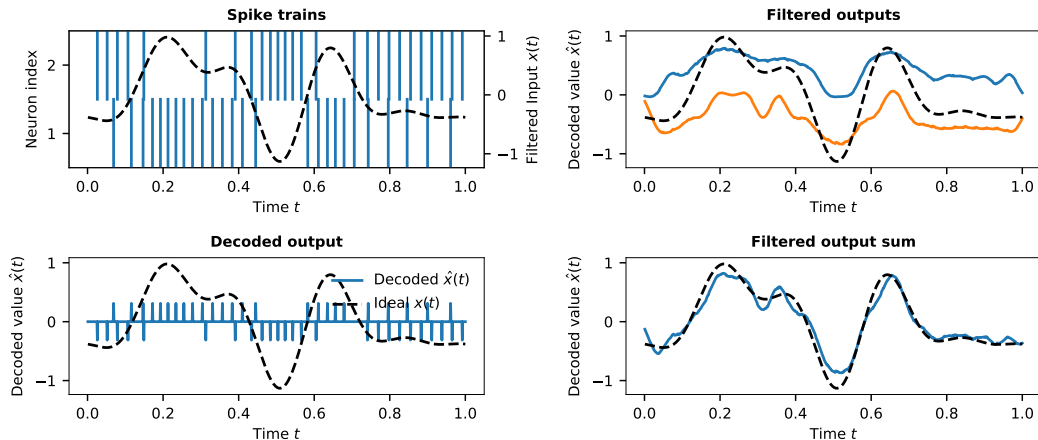
$$H(\omega) = \frac{X(\omega)\overline{R}(\omega) * W(\omega)}{|R(\omega)|^2 * W(\omega)}$$
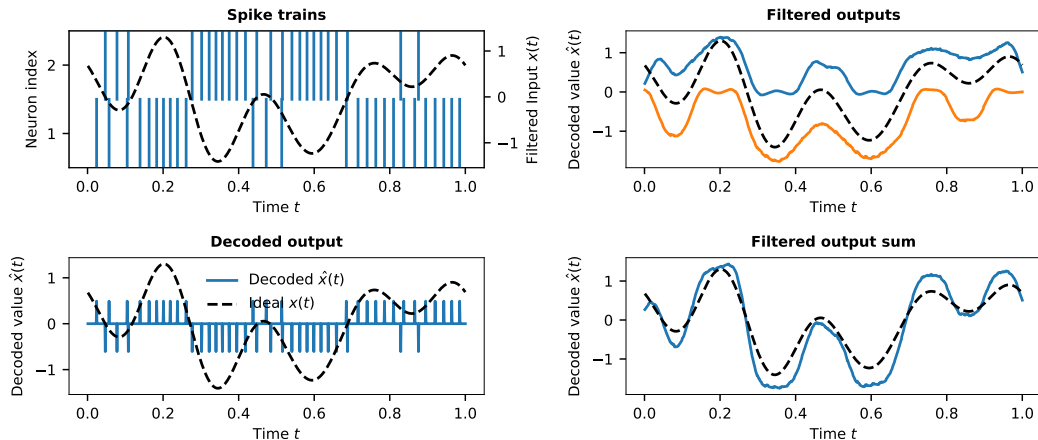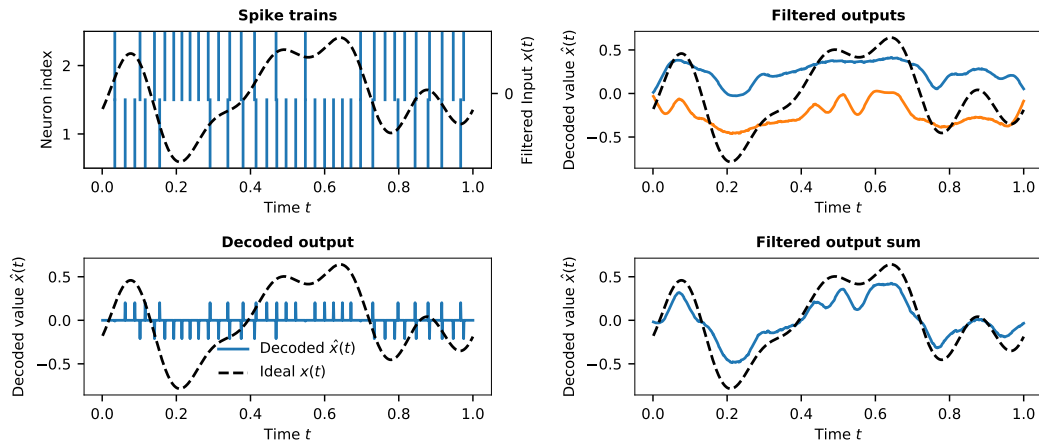
Filtering a Spike Train for a Random Signal (Improved Optimal Filter)

Filtering a Spike Train for a Random Signal (Improved Optimal Filter)

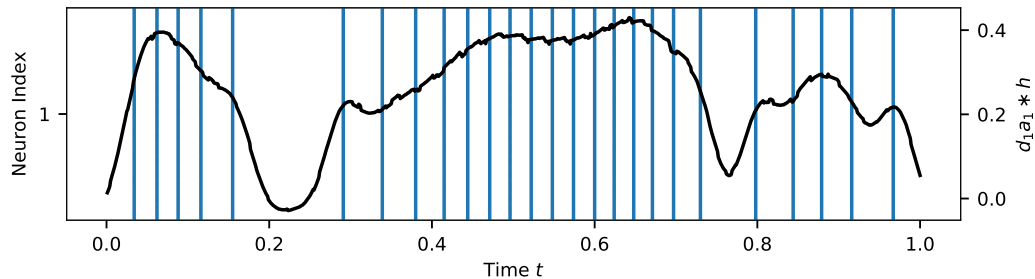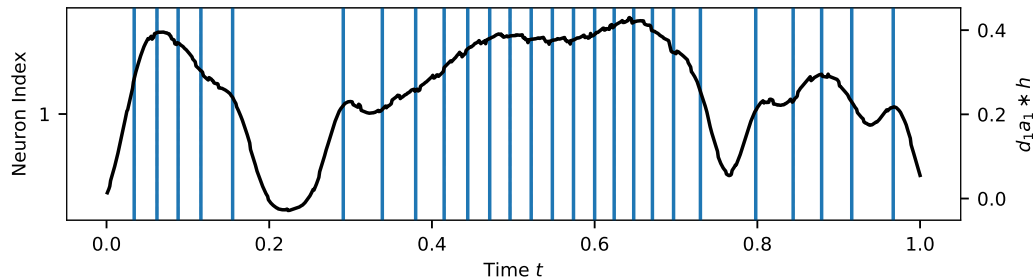# Filtering a Spike Train for a Random Signal (Improved Optimal Filter)

Filtering a Spike Train for a Random Signal (Improved Optimal Filter)

Pros and Cons of the Optimal Filter
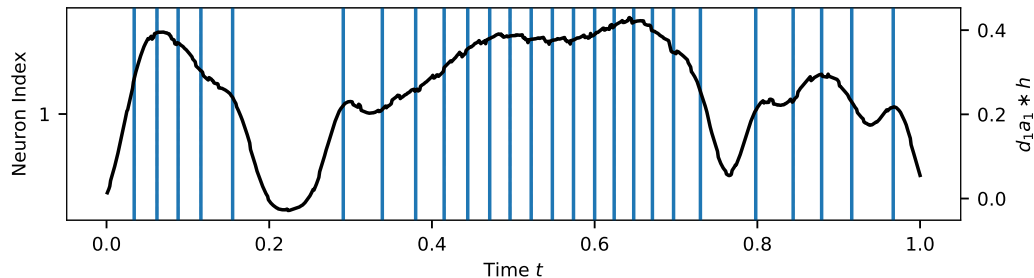
Pros and Cons of the Optimal Filter

➕ **Precise**
Good for analysing data after the fact
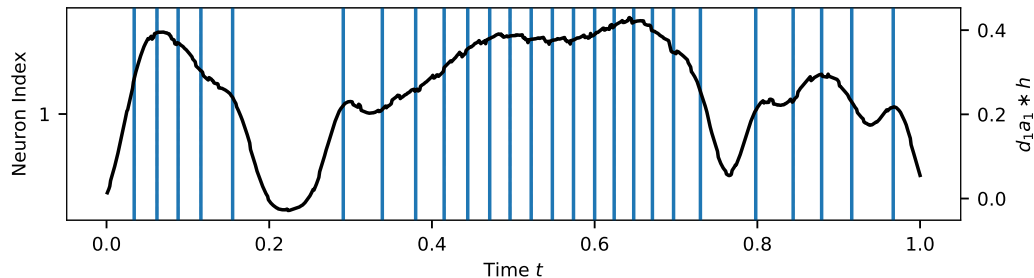
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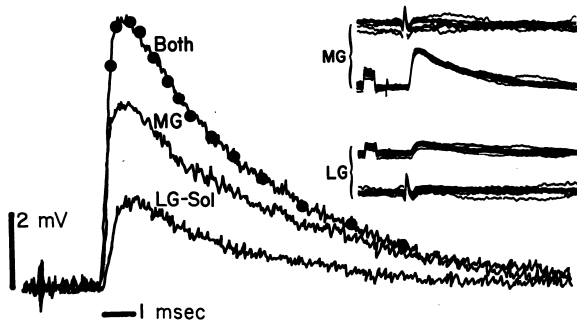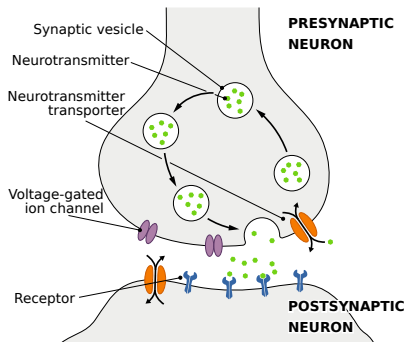Good for analysing data after the fact
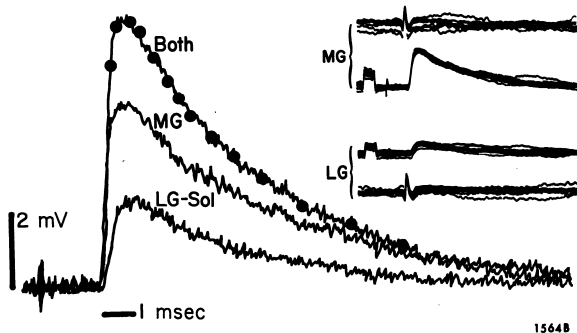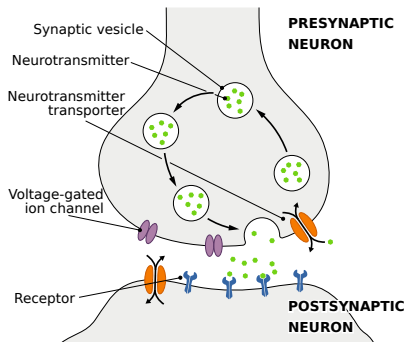
➖ **Non-causal**
Does not describe a biological process

We need to find a mechanism that low-pass filters spikes over time!

Synapses as Filters

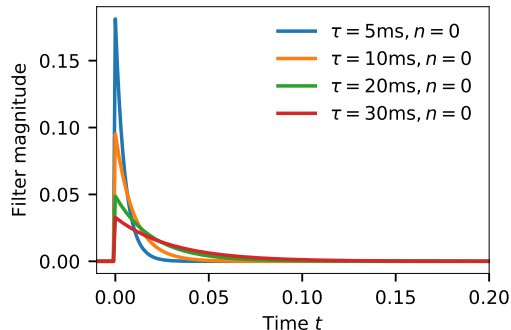## Synapses as Filters

Post-synaptic currents (EPSCs, IPSCs) are low-pass filtered spike trains!
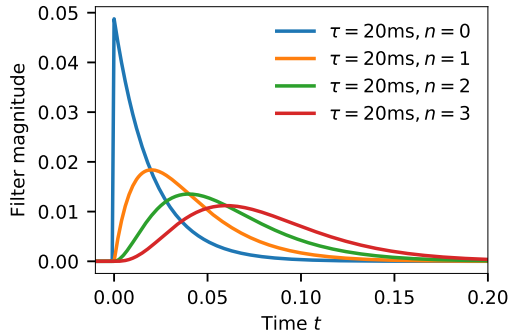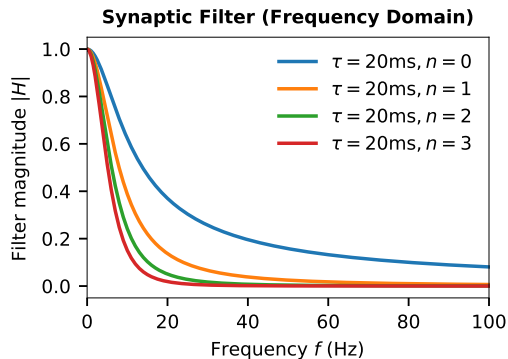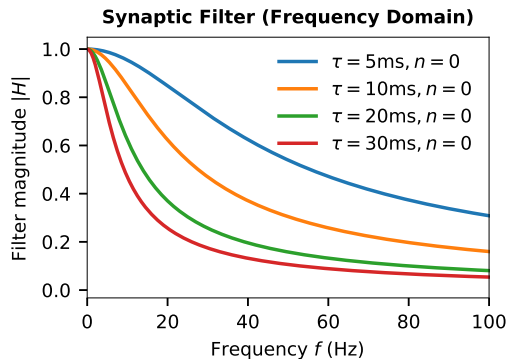
Exponential Low-Pass Filter (I)

$$h(t) = \begin{cases} c^{-1} t^n \exp^{-t/\tau} & \text{if } t \geq 0, \\ 0 & \text{otherwise}, \end{cases} \qquad \text{where } c = \int_0^\infty t^n \exp^{-t/\tau} \, \mathrm{d}t.$$
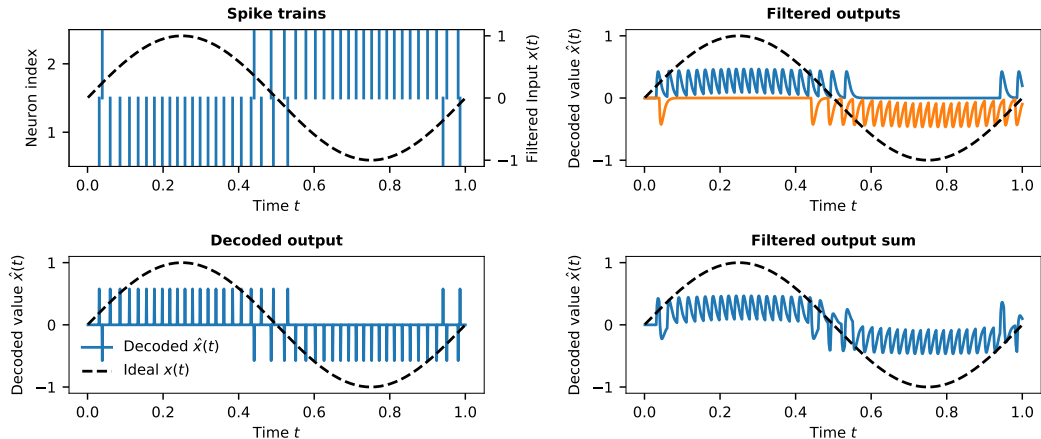
Exponential Low-Pass Filter (II)

$$h(t) = \begin{cases} c^{-1} t^n \exp^{-t/\tau} & \text{if } t \geq 0 \,, \\ 0 & \text{otherwise} \,, \end{cases} \qquad \text{where } c = \int_0^\infty t^n \exp^{-t/\tau} \, \mathrm{d}t \,.$$
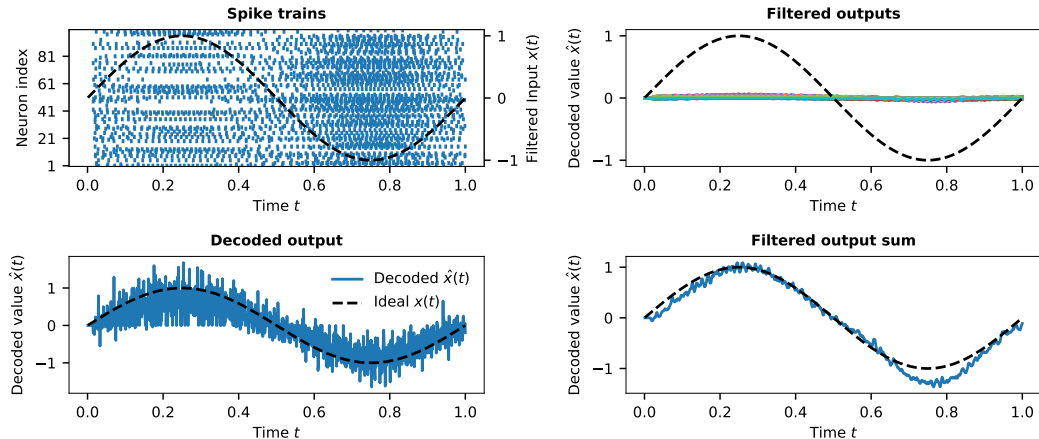
Example: Synaptic Filter for Two Neurons

$\tau = 5\,\text{ms}, n = 1$

Example: Synaptic Filter for One Hundred Neurons

$$\tau = 5\,\mathrm{ms}, n = 1$$

Image sources

Title slide
"Captive balloon with clock face and bell, floating above the Eiffel Tower, Paris, France."
Author: Camille Grávis, between 1889 and 1900.
From Wikimedia.