SYDE 556/750

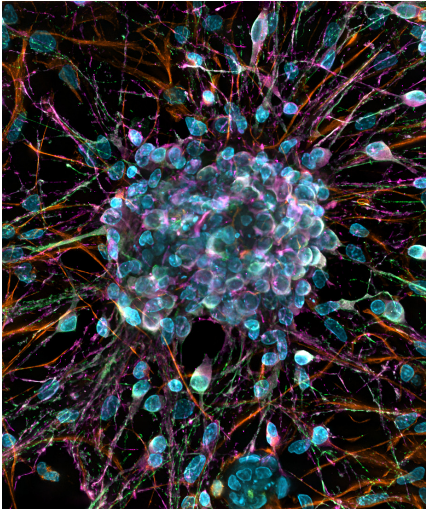
Simulating Neurobiological Systems
Lecture 2: Neurons

Chris Eliasmith

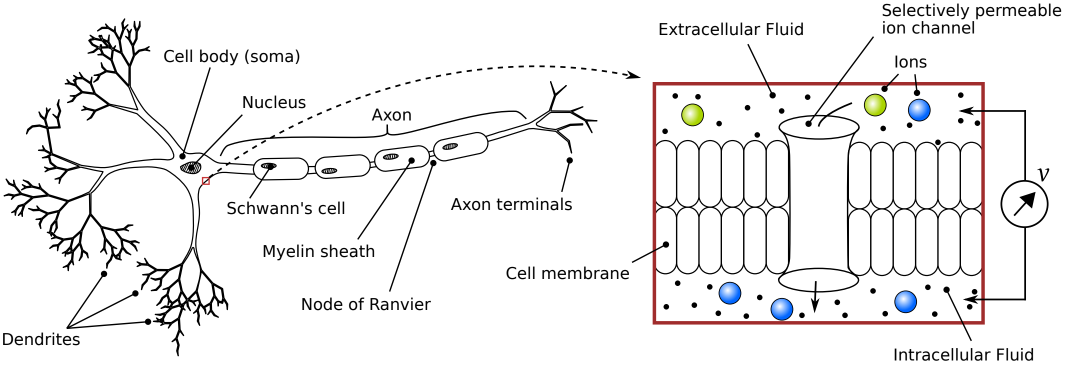
September 9, 2024

- Slide design: Andreas Stöckel
- Content: Terry Stewart, Andreas Stöckel, Chris Eliasmith

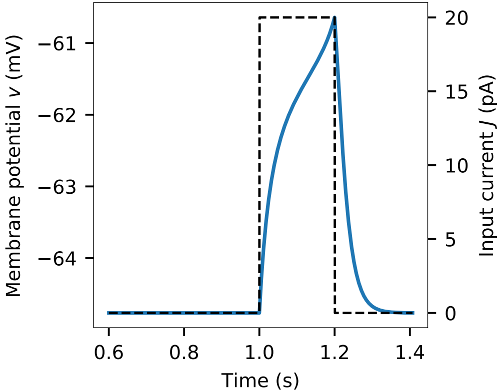
UNIVERSITY OF WATERLOO | FACULTY OF ENGINEERING



Textbook Neuron and Cell Membrane

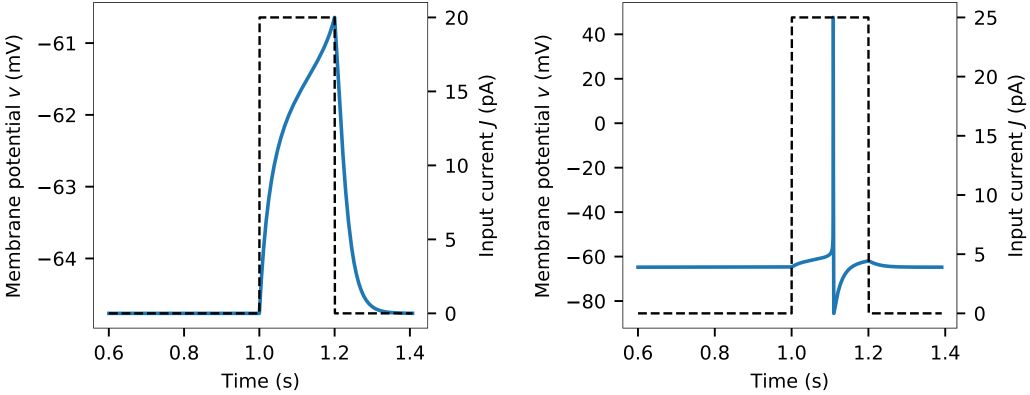


Injecting a Current Into a Detailed Neuron Model



Computer simulation of an Hodgkin-Huxley type neuron with Traub kinematics (Roger D. Traub and Richard Miles, *Neuronal Networks of the Hippocampus*, Cambridge University Press, 1991)

Injecting a Current Into a Detailed Neuron Model

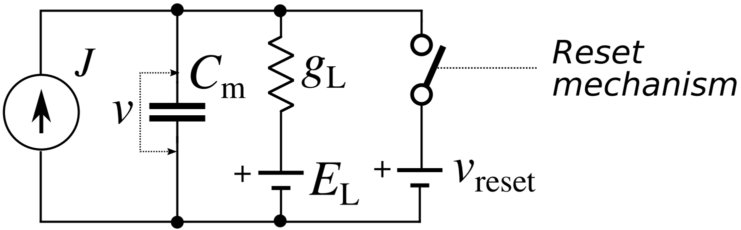


Computer simulation of an Hodgkin-Huxley type neuron with Traub kinematics (Roger D. Traub and Richard Miles, *Neuronal Networks of the Hippocampus*, Cambridge University Press, 1991)

Basic High-Level Details (Lapicque, 1907)

1. The cell acts like a *capacitor*, i.e., the voltage increases while we're injecting a current.
2. The capacitor is *leaky*. As soon as we stop injecting a current, the voltage collapses back to the resting potential E_{L}.
3. As soon as the voltage surpasses a certain value, the *threshold potential* v_{th}, the cell will generate a spike.
4. Shortly after the spike has been produced, the voltage drops below the resting potential. During this period, the *refractory period* of length τ_{ref}, we cannot get the neuron to spike again, even if we apply relatively large input currents J.

The Leaky Integrate-and-Fire Equivalent Circuit

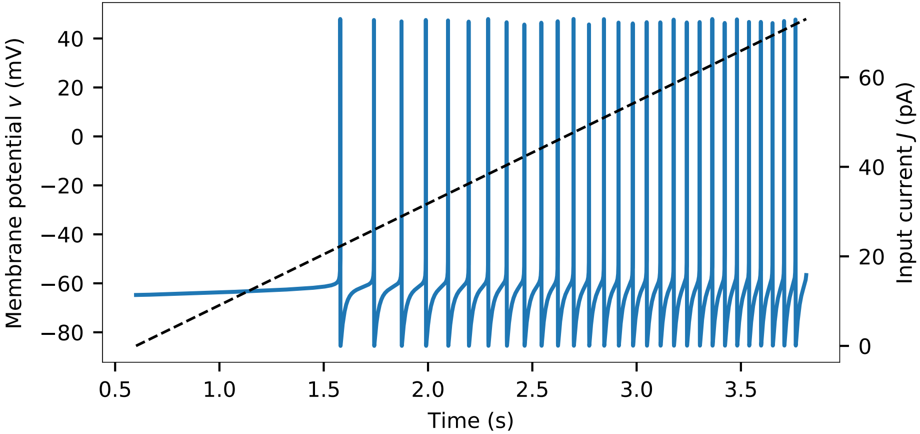


$$\frac{\mathrm{d}}{\mathrm{d}t}v(t) = \frac{1}{C_\mathrm{m}}\Big(g_\mathrm{L}(E_\mathrm{L} - v(t)) + J\Big)\,, \quad \text{if } v(t) < v_\mathrm{th}\,.$$

if $v(t) = v_\mathrm{th}$ at $t = t_\mathrm{th}$, output a spike $\big(\delta(t - t_\mathrm{th})\big)$ and:

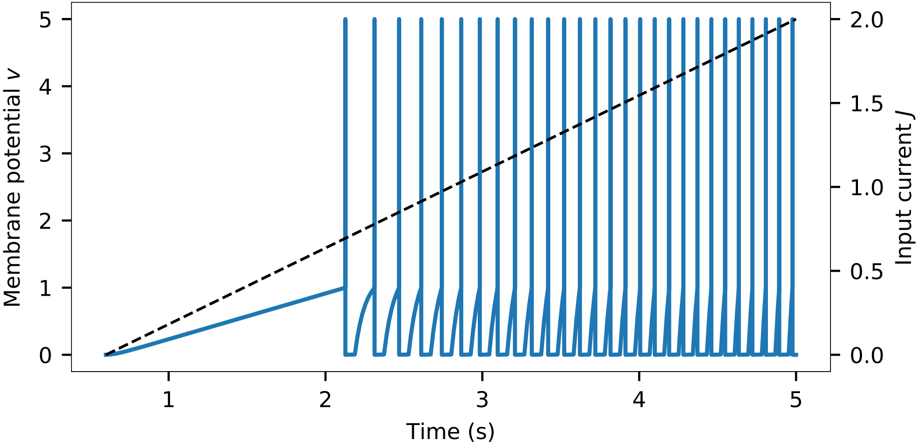
$$v(t) \leftarrow v_\mathrm{reset}\,, \quad \text{if } t_\mathrm{th} < t \leq t_\mathrm{th} + \tau_\mathrm{ref}\,,$$

Injecting a Current Ramp into a Detailed Neuron Model



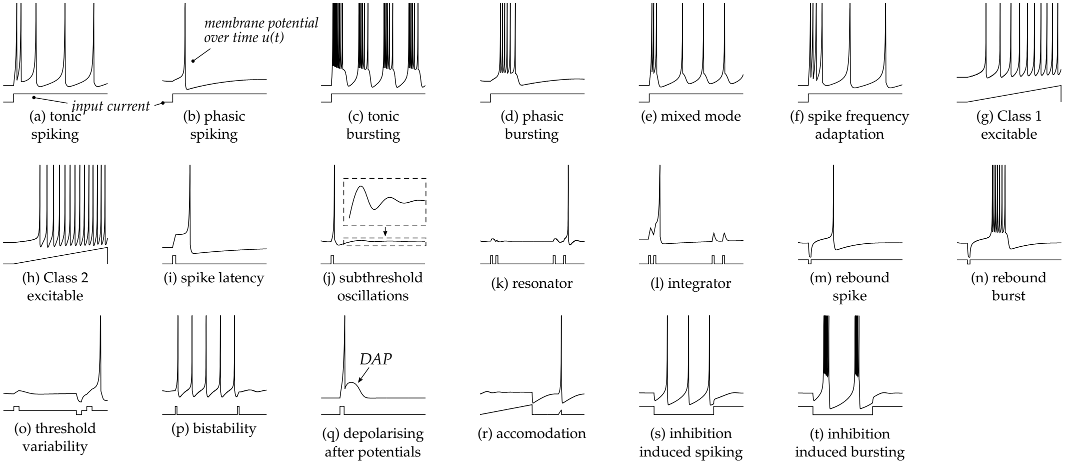
Computer simulation of an Hodgkin-Huxley type neuron with Traub kinematics (Roger D. Traub and Richard Miles, *Neuronal Networks of the Hippocampus*, Cambridge University Press, 1991)

Injecting a Current Ramp into a LIF Neuron Model

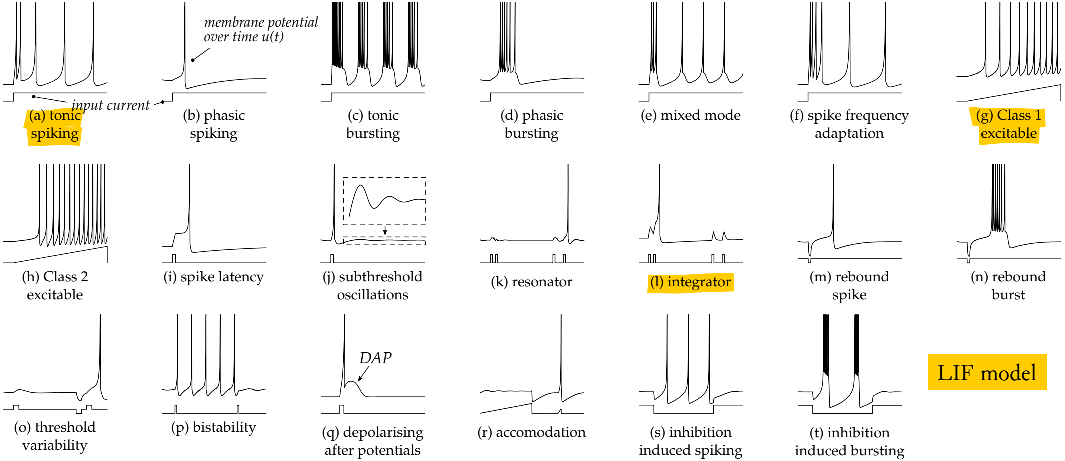


(note normalization to $v_{th} = 1$, $v_{reset} = E_L = 0$)

Limitations of the LIF Neuron Model



Limitations of the LIF Neuron Model



LIF Rate Approximation

- need to compute t_{th} (the time $v(t_{\mathrm{th}}) = v_{th}$)
- assume: J is constant and $v(0) = 0$.
- also: $g_L = 1/R$ and $\tau_{RC} = RC_m$

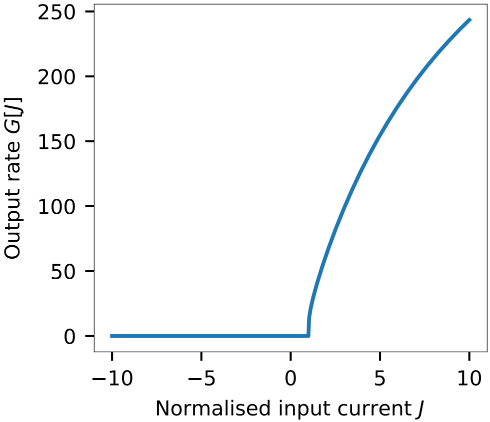
$$v(t) = -\int_0^t \frac{1}{\tau_{\mathrm{RC}}}\big(v(t') - RJ\big)\,\mathrm{d}t' = RJ\left(1 - e^{-\frac{t}{\tau_{\mathrm{RC}}}}\right).$$

$$v_{\mathrm{th}} = RJ\left(1 - e^{-\frac{t_{\mathrm{th}}}{\tau_{\mathrm{RC}}}}\right) \Leftrightarrow 1 - \frac{v_{\mathrm{th}}}{RJ} = e^{-\frac{t_{\mathrm{th}}}{\tau_{\mathrm{RC}}}},$$

$$t_{\mathrm{th}} = -\tau_{\mathrm{RC}} \log\left(1 - \frac{v_{\mathrm{th}}}{RJ}\right)$$

$$G[J] = \begin{cases} \frac{1}{\tau_{\mathrm{ref}} - \tau_{\mathrm{RC}} \log\left(1 - \frac{v_{\mathrm{th}}}{RJ}\right)} & \text{if } 1 - \frac{v_{\mathrm{th}}}{RJ} > 0, \\ 0 & \text{otherwise}. \end{cases}$$

Artifical Rate Neurons: LIF

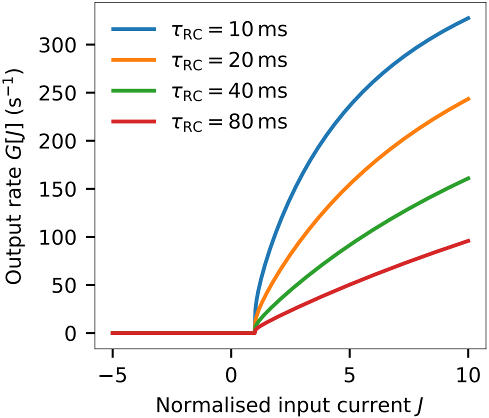


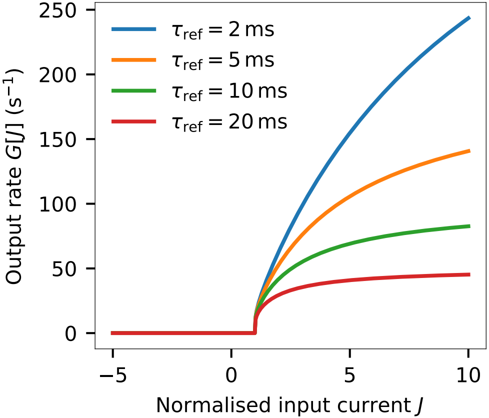
$$G[J] = \frac{1}{\tau_{\mathrm{ref}} - \tau_{\mathrm{RC}} \log\left(1 - \frac{1}{J}\right)}$$

Usefulness to neurobiological systems modellers:

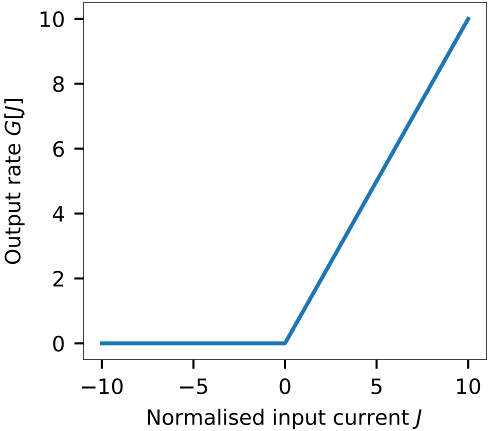
- ⊕ Biologically motivated
- ⊕ Captures saturation effects
- ● Relatively slow to evaluate numerically (for machine-learning people)
- ⊖ Spike onset is smooth in noisy systems

Exploring the LIF Rate Approximation





Artifical Rate Neurons: ReLU

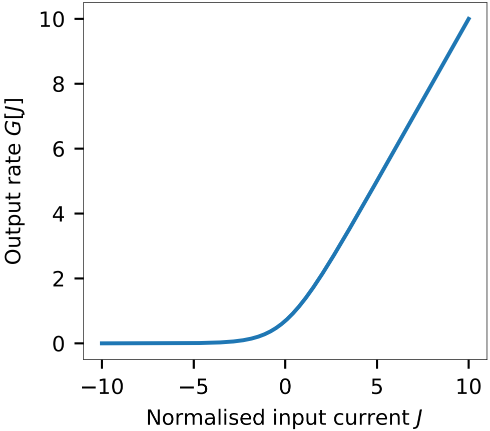


$$G[J] = \max\{0, J\}$$

Usefulness to neurobiological systems modellers:

- ⊕ Fast to evaluate
- ● Rough approximation of the LIF response curve
- ⊖ Does not capture saturation effects
- ⊖ Spike onset is smooth in noisy systems

Artifical Rate Neurons: Smooth ReLU (Softplus)

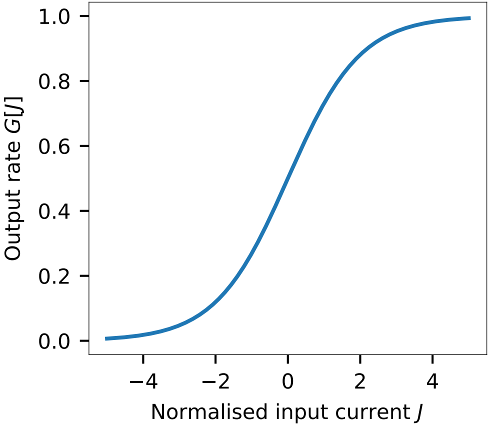


$$G[J] = \log(1 + \exp(J))$$

Usefulness to neurobiological systems modellers:

- ⊕ Models smooth spike onset
- ● Rough approximation of the LIF response curve
- ⊖ Does not capture saturation effects

Artifical Rate Neurons: Logistic Function

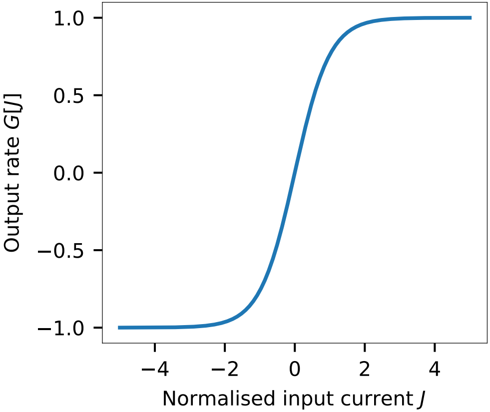


$$G[J] = \frac{1}{1 + e^{-J}}$$

Usefulness to neurobiological systems modellers:

- Models smooth spike onset and saturation (?)

Artifical Rate Neurons: Hyperbolic Tangent



$$G[J] = \tanh(J) = \frac{e^J - e^{-J}}{e^J + e^{-J}}$$

Usefulness to neurobiological systems modellers:

- Models smooth spike onset and saturation (?)
- Negative rates

Image sources

Title slide
Image of rat primary cortical neurons in culture.
Author: ZEISS Microscopy, `http://www.zeiss.com/celldiscoverer`.
From Wikimedia.