

SYDE 556/750

Simulating Neurobiological Systems

Lecture 4: Temporal Representations

Chris Eliasmith

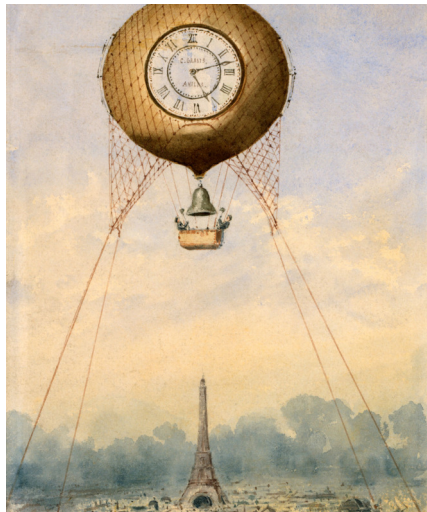
Sept 18 & 23, 2024

- ▶ Slide design: Andreas Stöckel
- ▶ Content: Terry Stewart, Andreas Stöckel, Chris Eliasmith

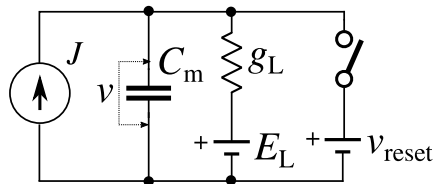
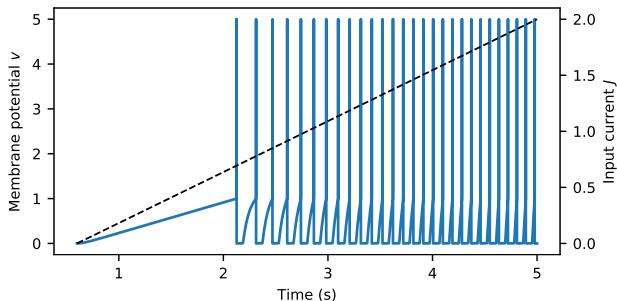


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Reminder: The LIF Neuron



$$\frac{d}{dt}v(t) = -\frac{1}{\tau_{RC}}(v(t) - J),$$

$$v(t) \leftarrow \delta(t - t_{\text{th}}),$$

$$v(t) \leftarrow 0,$$

$$\text{if } v(t) < 1,$$

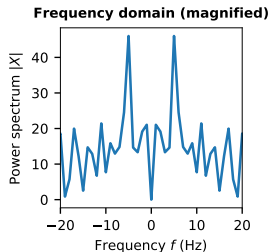
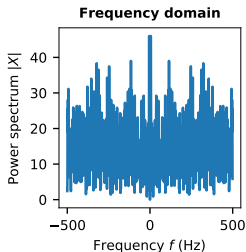
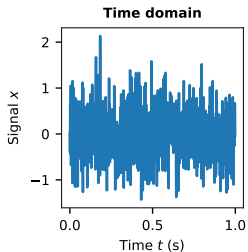
$$\text{if } t = t_{\text{th}},$$

$$\text{if } t > t_{\text{th}} \text{ and } t \geq t_{\text{th}} + \tau_{\text{ref}},$$

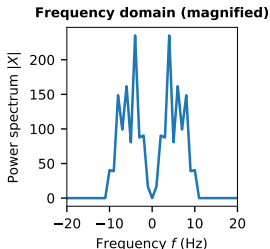
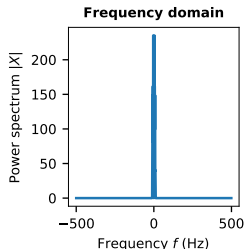
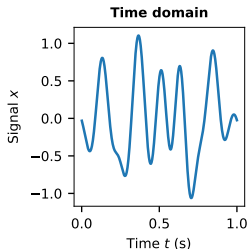
Temporal Decoding

- ▶ For population decoders, we needed to integrate their responses, $\mathbf{a}(\mathbf{x})$, over the represented variable, \mathbf{x} .
- ▶ For temporal decoders, we will likely want to integrate their responses, $\mathbf{a}(t)$, over the represented variable, $\mathbf{x}(t)$.
- ▶ What space do we want to sample to estimate the integrals?

Random Signals

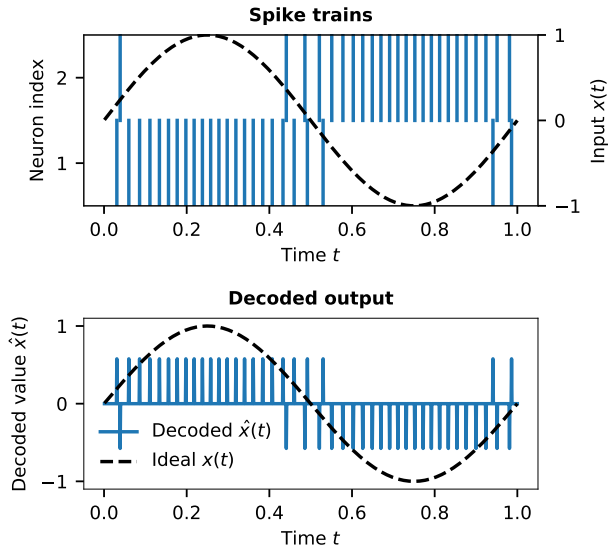
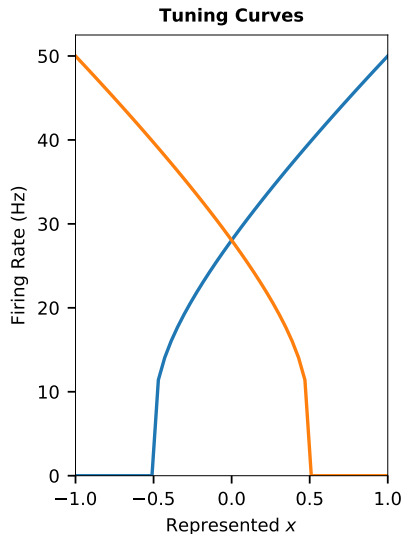


White Noise
(zero mean)

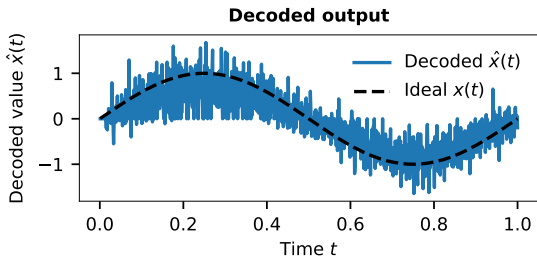
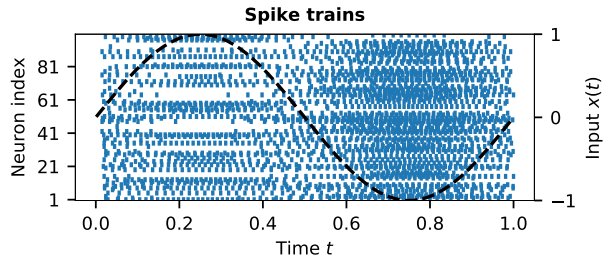
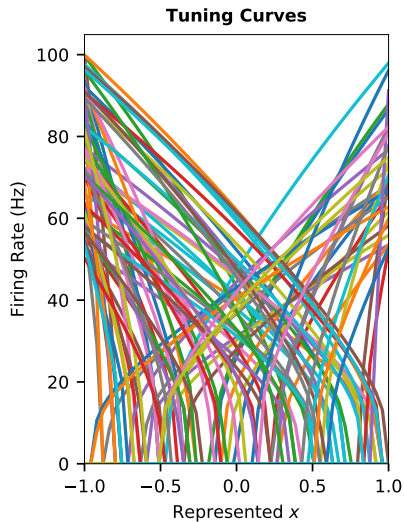


Bandlimited
White Noise
(zero mean,
10 Hz bandwidth)

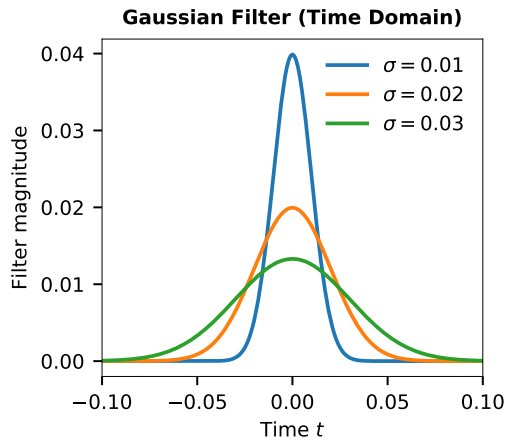
Temporal Decoding of Two Neurons - Weighted Spikes



Temporal Decoding of One Hundred Neurons - Weighted Spikes



Filtering by Convolution



Gaussian Filter

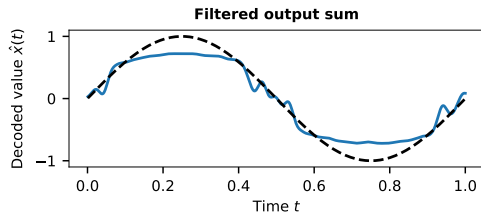
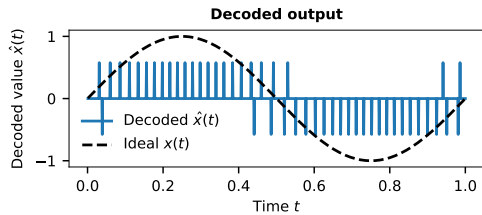
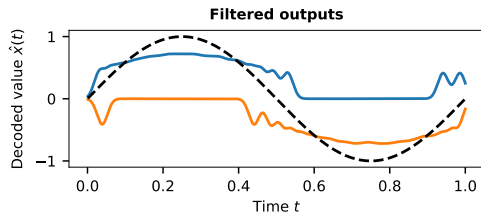
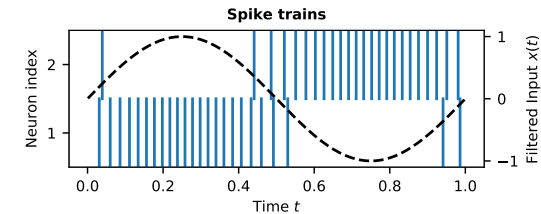
$$h(t) = c \exp\left(\frac{-t^2}{\sigma^2}\right)$$

where c chosen s.t. $\int_{-\infty}^{\infty} h(t) dt = 1$

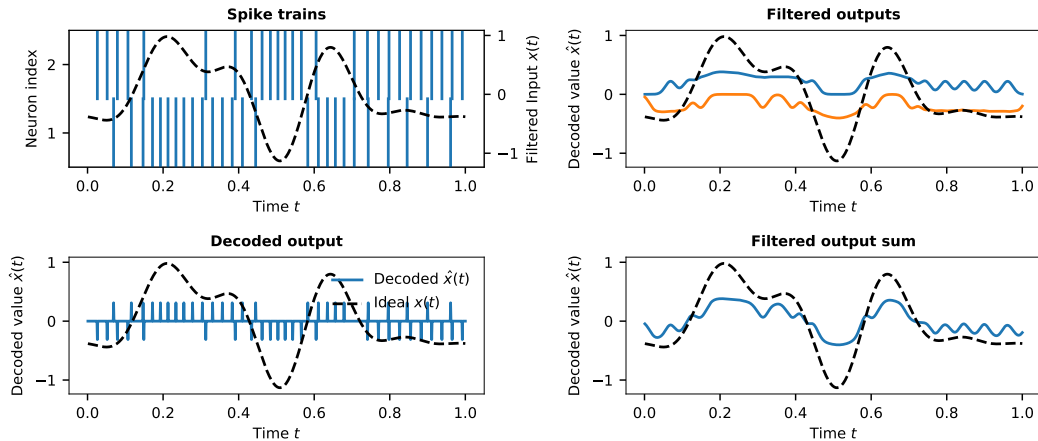
Convolution

$$(f * h)(t) = \int_{-\infty}^{\infty} f(t - t') h(t') dt'$$

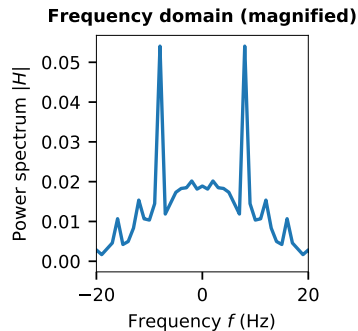
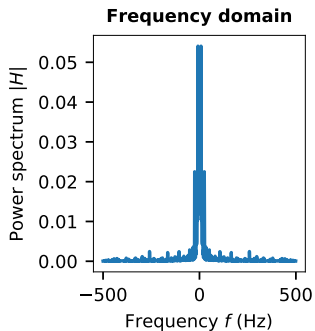
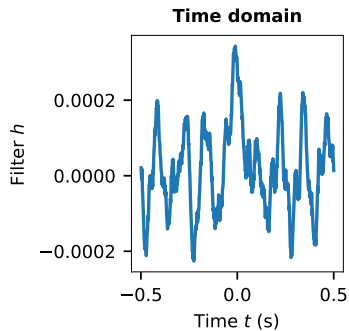
Filtering a Spike Train



Filtering a Spike Train for a Random Signal

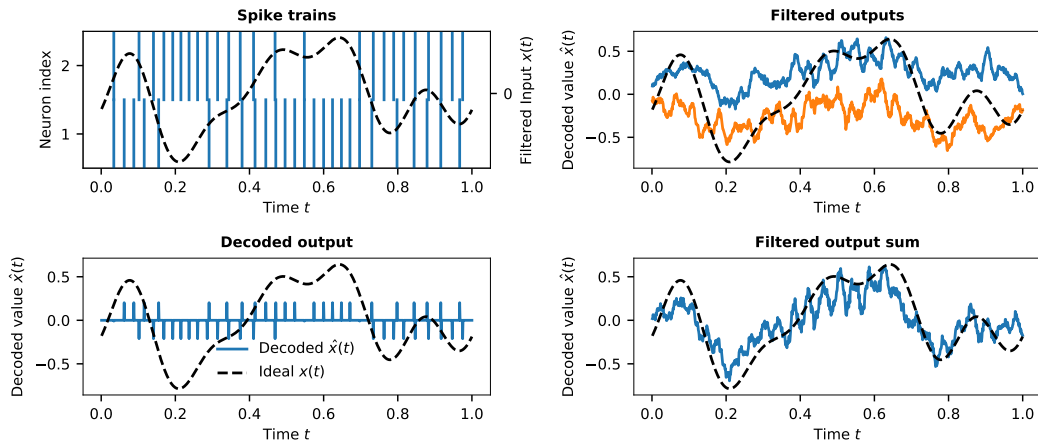


Optimal Filter

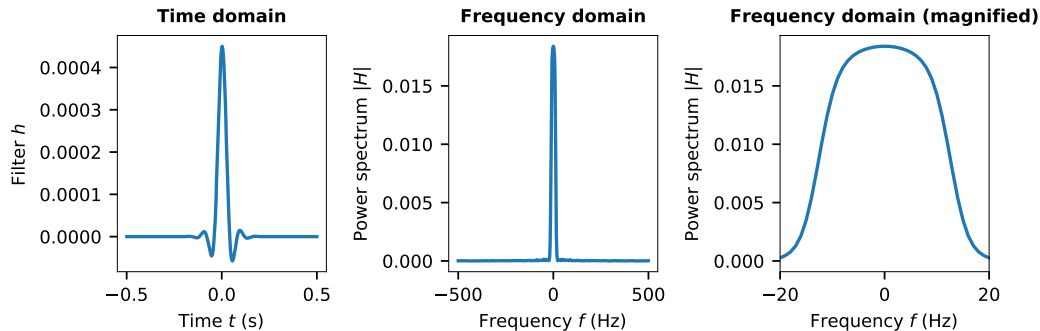


$$H(\omega) = \frac{X(\omega)\overline{R}(\omega)}{|R(\omega)|^2}$$

Filtering a Spike Train for a Random Signal (Optimal Filter)

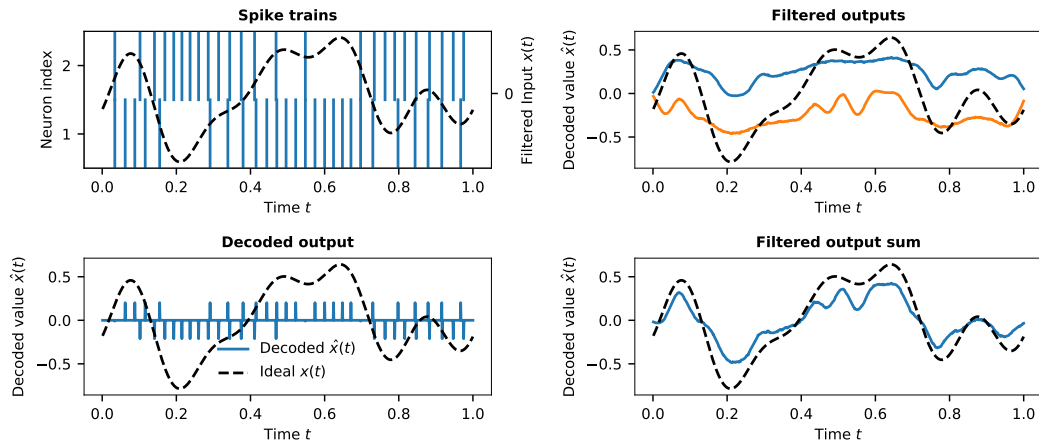


Optimal Filter (Improved)

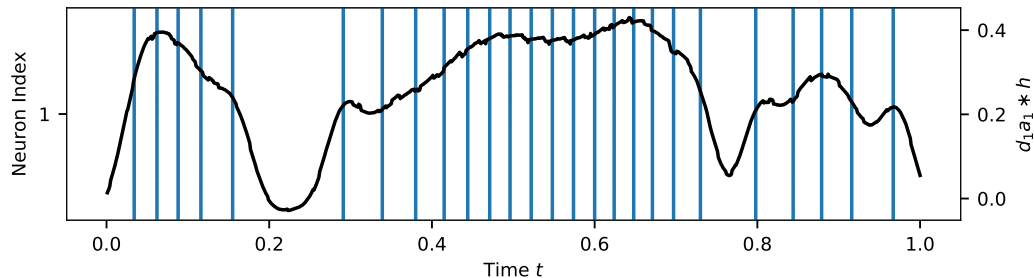


$$H(\omega) = \frac{X(\omega)\overline{R}(\omega) * W(\omega)}{|R(\omega)|^2 * W(\omega)}$$

Filtering a Spike Train for a Random Signal (Improved Optimal Filter)



Pros and Cons of the Optimal Filter



+ Precise

Good for analysing data after the fact

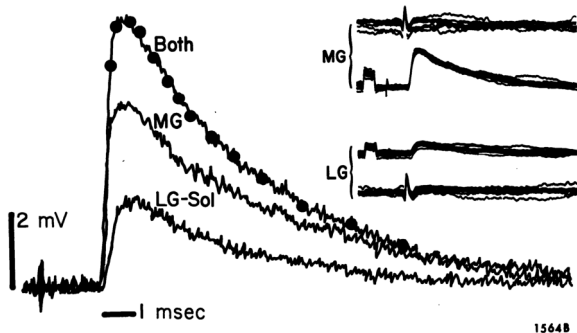
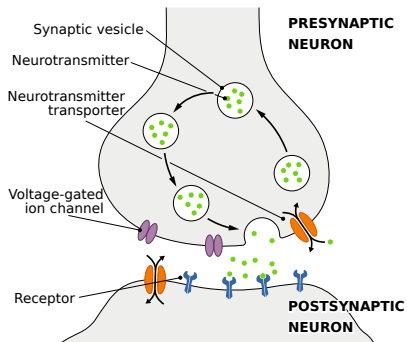
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Non-causal

Does not describe a biological process

We need to find a mechanism that low-pass filters spikes over time!

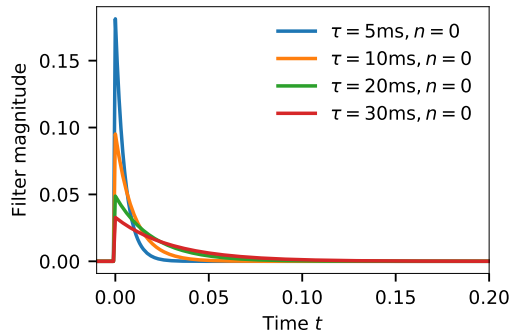
Synapses as Filters



Post-synaptic currents (EPSCs, IPSCs) are low-pass filtered spike trains!

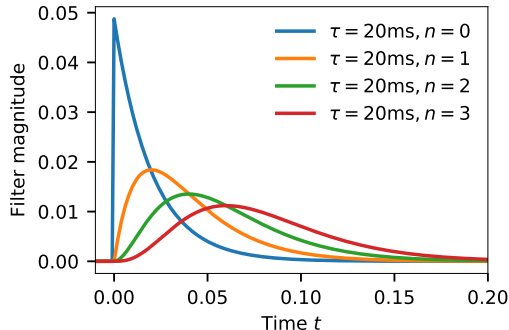
Exponential Low-Pass Filter (I)

Synaptic Filter (Time Domain)



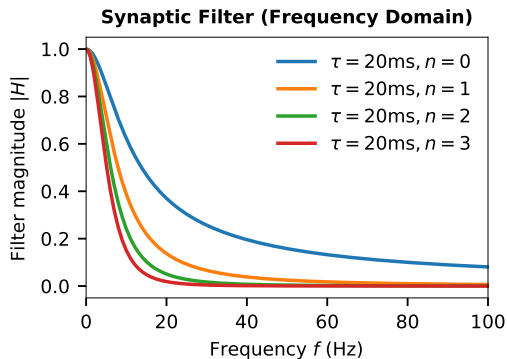
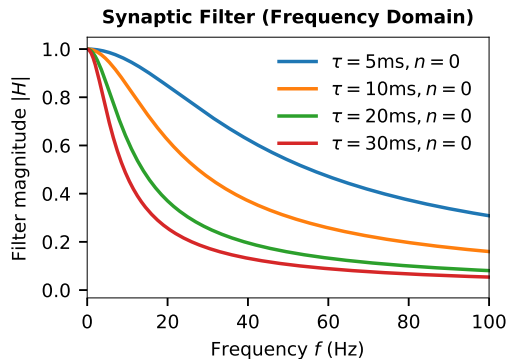
$$h(t) = \begin{cases} c^{-1} t^n \exp^{-t/\tau} & \text{if } t \geq 0, \\ 0 & \text{otherwise,} \end{cases}$$

Synaptic Filter (Time Domain)



$$\text{where } c = \int_0^{\infty} t^n \exp^{-t/\tau} dt.$$

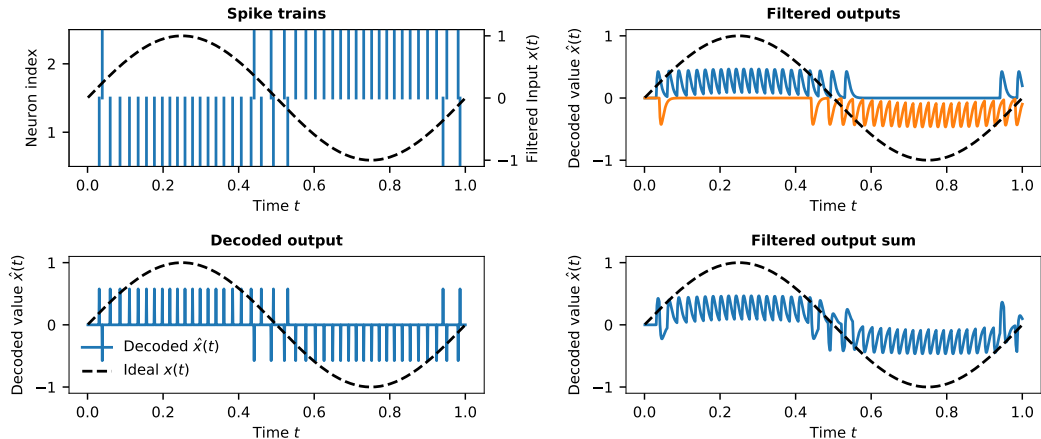
Exponential Low-Pass Filter (II)



$$h(t) = \begin{cases} c^{-1} t^n \exp^{-t/\tau} & \text{if } t \geq 0, \\ 0 & \text{otherwise,} \end{cases}$$

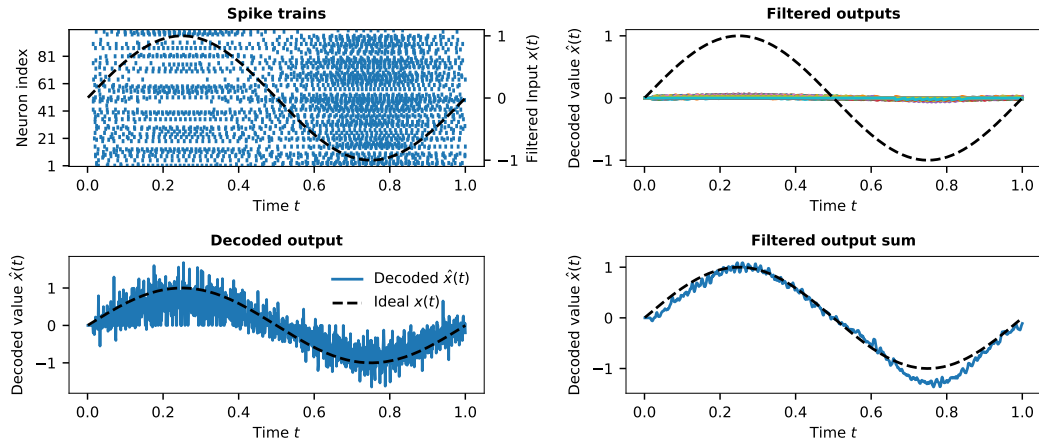
$$\text{where } c = \int_0^{\infty} t^n \exp^{-t/\tau} dt.$$

Example: Synaptic Filter for Two Neurons



$$\tau = 5 \text{ ms}, n = 1$$

Example: Synaptic Filter for One Hundred Neurons



$$\tau = 5 \text{ ms}, n = 1$$

Image sources

Title slide

“Captive balloon with clock face and bell, floating above the Eiffel Tower, Paris, France.”

Author: Camille Grávis, between 1889 and 1900.

From Wikimedia.