SYDE 556/750

Simulating Neurobiological Systems
Lecture 8: Learning

Chris Eliasmith

October 23, 28, & 30

- Slide design: Andreas Stöckel
- Content: Terry Stewart, Andreas Stöckel, Chris Eliasmith

UNIVERSITY OF WATERLOO | FACULTY OF ENGINEERING

Learning

Definition

Learning is a directed change in synaptic weights $\mathbf{W}$ while the network is active.

Learning is important because:

1. We might not know the function we want to compute at the beginning of a task.
2. The desired function might change over time.
3. The "optimal weights" we are solving for are not optimal.
4. We want to answer scientific questions about learning in nervous systems.

Supervised Learning

$\mathbf{x}_1 =$

$\mathbf{t}_1 = (1, 0, 0)$

$\mathbf{x}_2 =$

$\mathbf{t}_2 = (1, 0, 0)$

$\mathbf{x}_3 =$

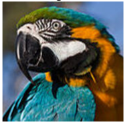
$\mathbf{t}_3 = (1, 0, 0)$

$\mathbf{x}_4 =$

$\mathbf{t}_4 = (0, 0, 1)$

$\mathbf{x}_5 =$

$\mathbf{t}_5 = (1, 0, 0)$

$\mathbf{x}_6 =$

$\mathbf{t}_6 = (0, 0, 1)$

$\mathbf{x}_7 =$

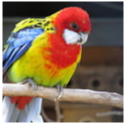
$\mathbf{t}_7 = (1, 0, 0)$

$\mathbf{x}_8 =$

$\mathbf{t}_8 = (0, 0, 1)$

$\mathbf{x}_9 =$

$\mathbf{t}_9 = (0, 1, 0)$

$\mathbf{x}_{10} =$

$\mathbf{t}_{10} = (0, 0, 1)$

$\mathbf{x}_{11} =$

$\mathbf{t}_{11} = (1, 0, 0)$

$\mathbf{x}_{12} =$

$\mathbf{t}_{12} = (1, 0, 0)$

$\mathbf{x}_{13} =$

$\mathbf{t}_{13} = (0, 1, 0)$

$\mathbf{x}_{14} =$

$\mathbf{t}_{14} = (0, 1, 0)$

$\mathbf{x}_{15} =$

$\mathbf{t}_{15} = (0, 1, 0)$

$\mathbf{x}_{16} =$

$\mathbf{t}_{16} = (0, 1, 0)$

$\mathbf{x}_{17} =$

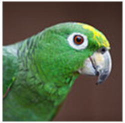
$\mathbf{t}_{17} = (0, 0, 1)$

$\mathbf{x}_{18} =$

$\mathbf{t}_{18} = (0, 1, 0)$

$\mathbf{x}_{20} =$

$\mathbf{t}_{20} = (0, 1, 0)$

$\mathbf{x}_{21} =$

$\mathbf{t}_{21} = (0, 0, 1)$

$\mathbf{x}_{22} =$

$\mathbf{t}_{22} = (0, 0, 1)$

$\mathbf{x}_{23} =$

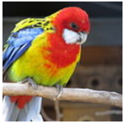
$\mathbf{t}_{23} = (1, 0, 0)$

$\mathbf{x}_{24} =$

$\mathbf{t}_{24} = (0, 1, 0)$

Supervised Learning – Training

$\mathbf{x}_{10} =$



$\longrightarrow f(\mathbf{x};\, \mathbf{w}) \longrightarrow (0.1,\ 0.9,\ 0.25)$

"Doglyness" *"Catlyness"* *"Parrotlyness"*

$\ominus$

weight adaptation

$\mathbf{t}_{10} = (0.0,\ 0.0,\ 1.0)$

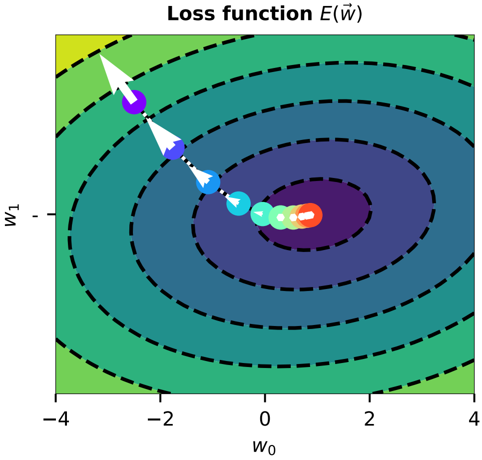
Supervised Learning – Inference

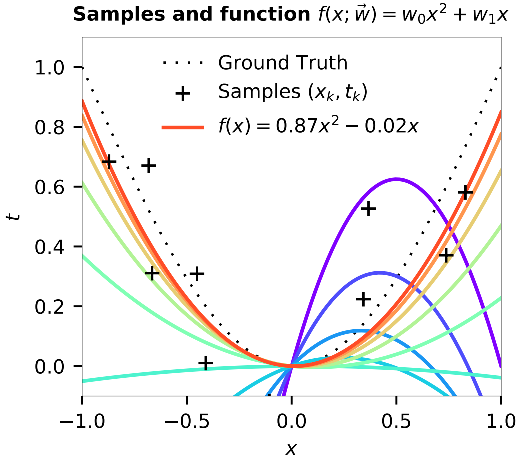
$\mathbf{x} =$

$$\longrightarrow f(\mathbf{x};\, \mathbf{w}) \longrightarrow (0.5,\ 0.1,\ 0.02)$$

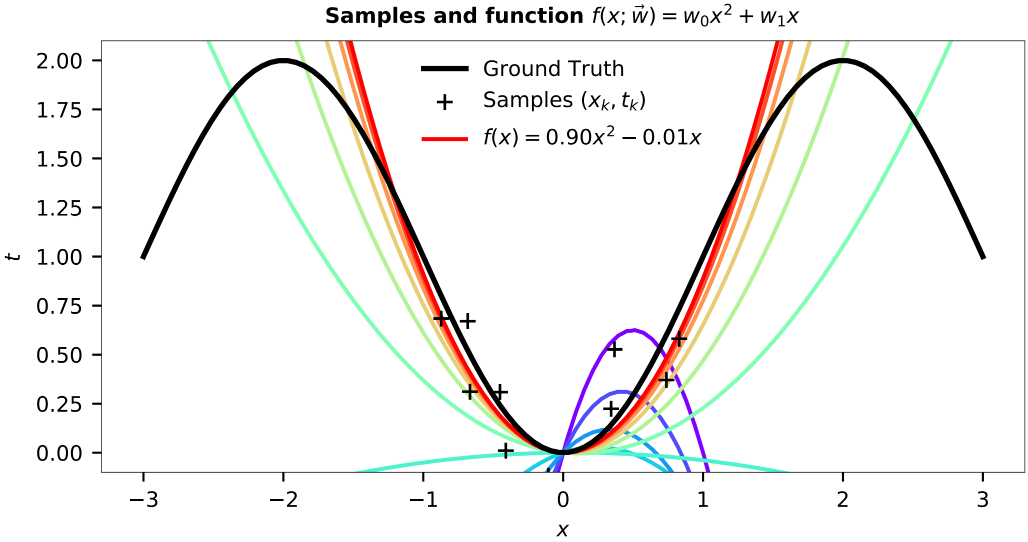
"Doglyness" *"Catlyness"* *"Parrotlyness"*

Gradient Descent – Example



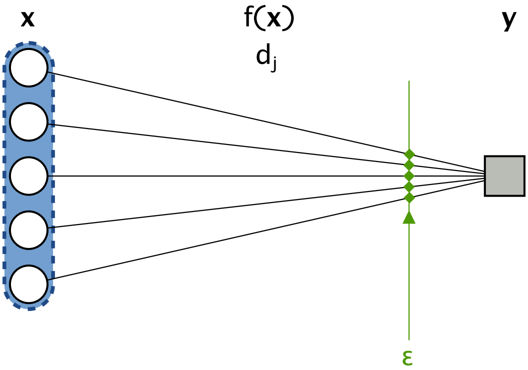


Supervised Learning – Generalisation



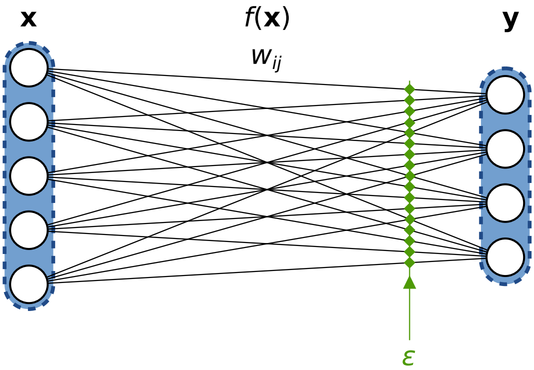
Learning Decoders and Learning Weights

Learning Decoders (Delta Rule)



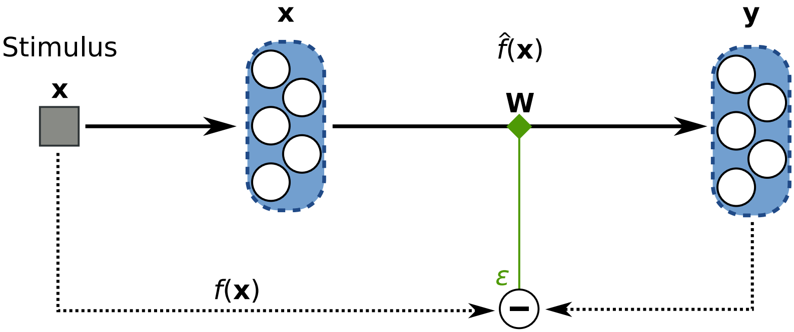
$$\Delta d_i = -\eta a_i(\mathbf{x}) \underbrace{\left(y(t) - y^{\mathrm{d}}(t)\right)}_{\varepsilon(t)}$$

Learning Weights (PES Rule)



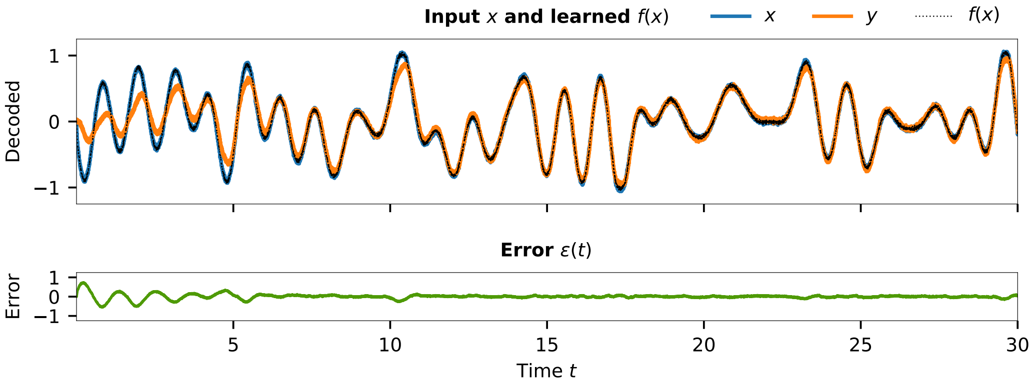
$$\Delta w_{ij} = -\eta a_i(\mathbf{x}) \Big(\alpha_j \langle \mathbf{e}_j, \varepsilon(t) \rangle\Big)$$

Example: Learning Functions (I)



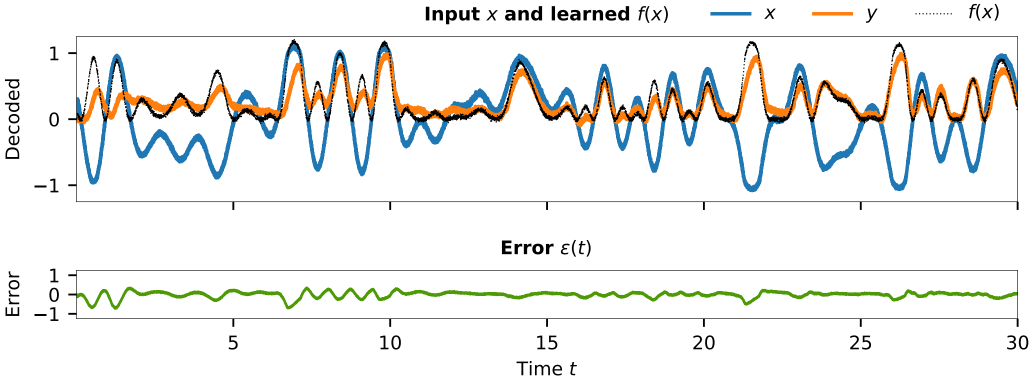
Example: Learning Functions (II)

Communication Channel $f(x) = x$



Example: Learning Functions (III)

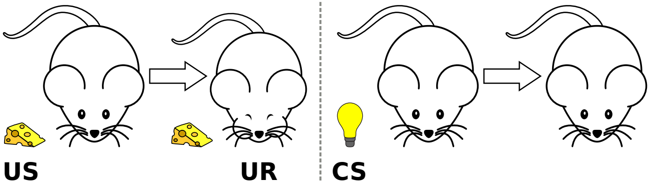
Square $f(x) = x^2$



Works, but learns more slowly! **Where is the error signal $\varepsilon(t)$ coming from?**

Example: Classical Conditioning (I)

Before conditioning:



After conditioning:

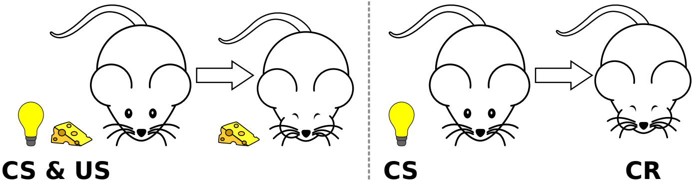
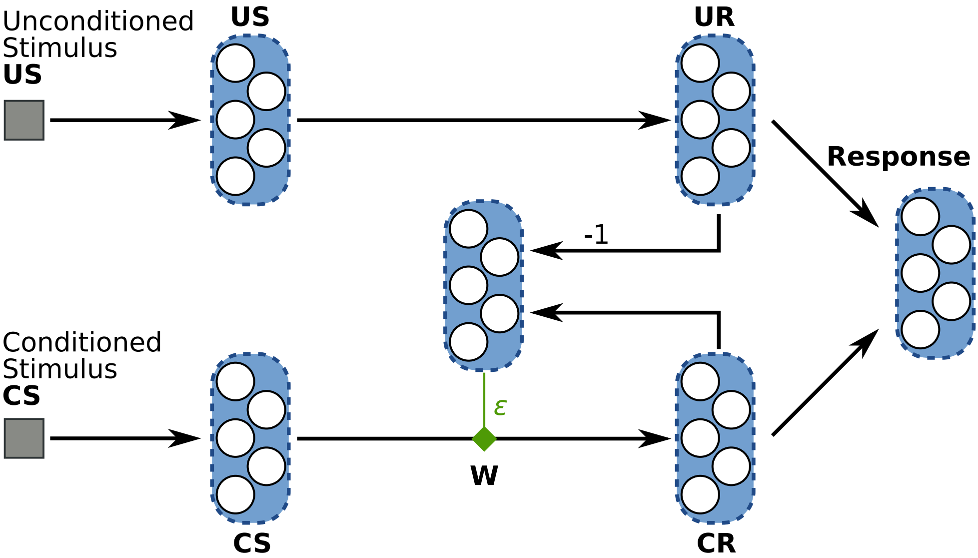
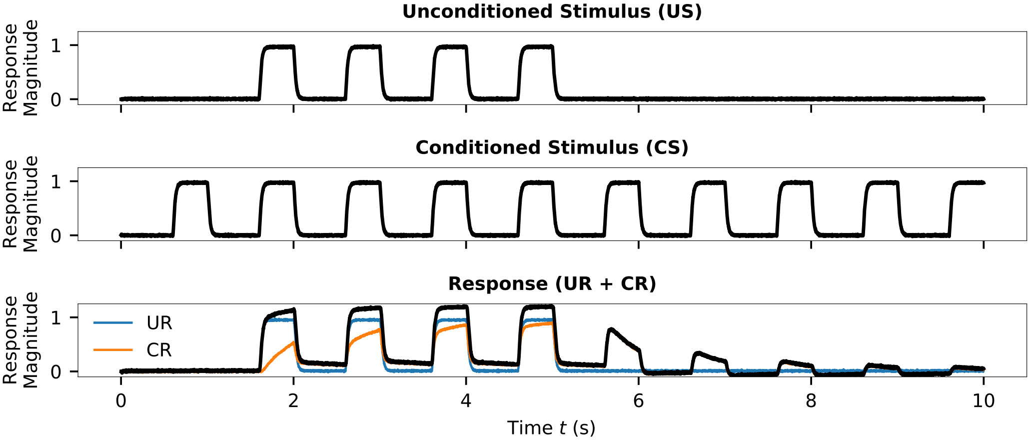


Image Sources. Images adapted from Wikimedia.

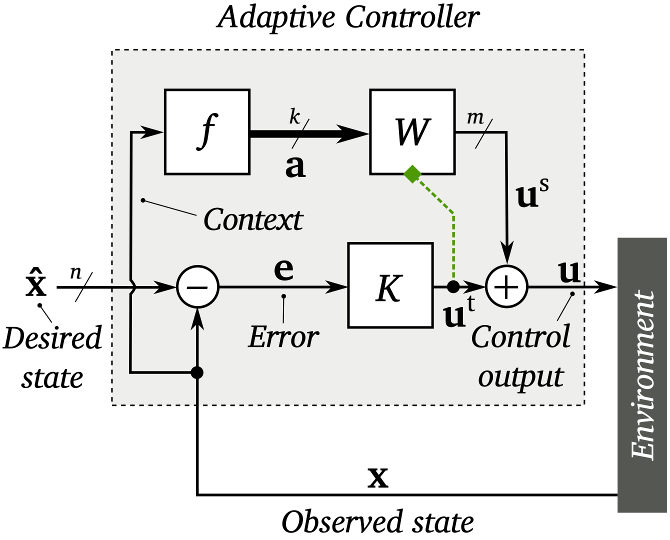
Example: Classical Conditioning (II)



Example: Classical Conditioning (III)



Example: Adaptive Controller



Unsupervised Learning

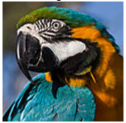
$\mathbf{x}_1 =$


$\mathbf{x}_2 =$


$\mathbf{x}_3 =$


$\mathbf{x}_4 =$

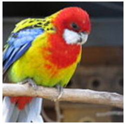

$\mathbf{x}_5 =$


$\mathbf{x}_6 =$


$\mathbf{x}_7 =$


$\mathbf{x}_8 =$


$\mathbf{x}_9 =$


$\mathbf{x}_{10} =$


$\mathbf{x}_{11} =$


$\mathbf{x}_{12} =$


$\mathbf{x}_{13} =$


$\mathbf{x}_{14} =$

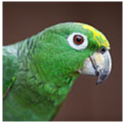

$\mathbf{x}_{15} =$


$\mathbf{x}_{16} =$


$\mathbf{x}_{17} =$


$\mathbf{x}_{18} =$


$\mathbf{x}_{20} =$


$\mathbf{x}_{21} =$


$\mathbf{x}_{22} =$


$\mathbf{x}_{23} =$


$\mathbf{x}_{24} =$


Unsupervised Learning – Training

Unsupervised Learning – Inference

Dimensionality Reduction Example: Faces

Face Database

- 84 images of 12 women with 7 different expressions
- Normalised eye location
- 45×60 pixels (2700 dimensions)
- Greyscale

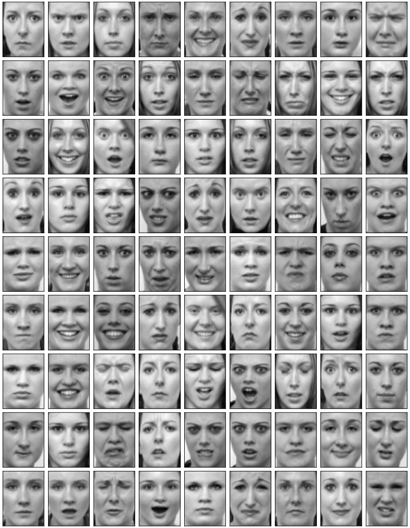
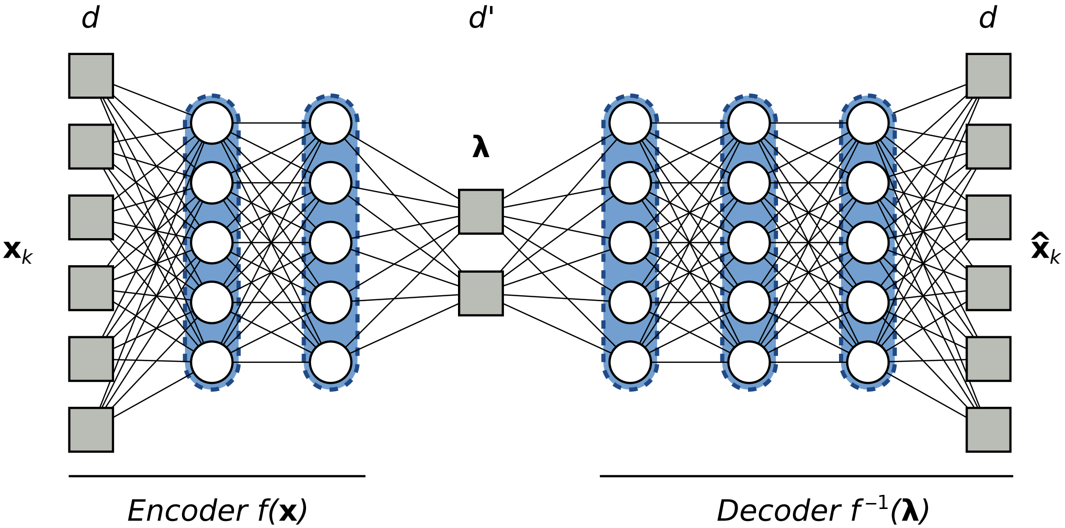
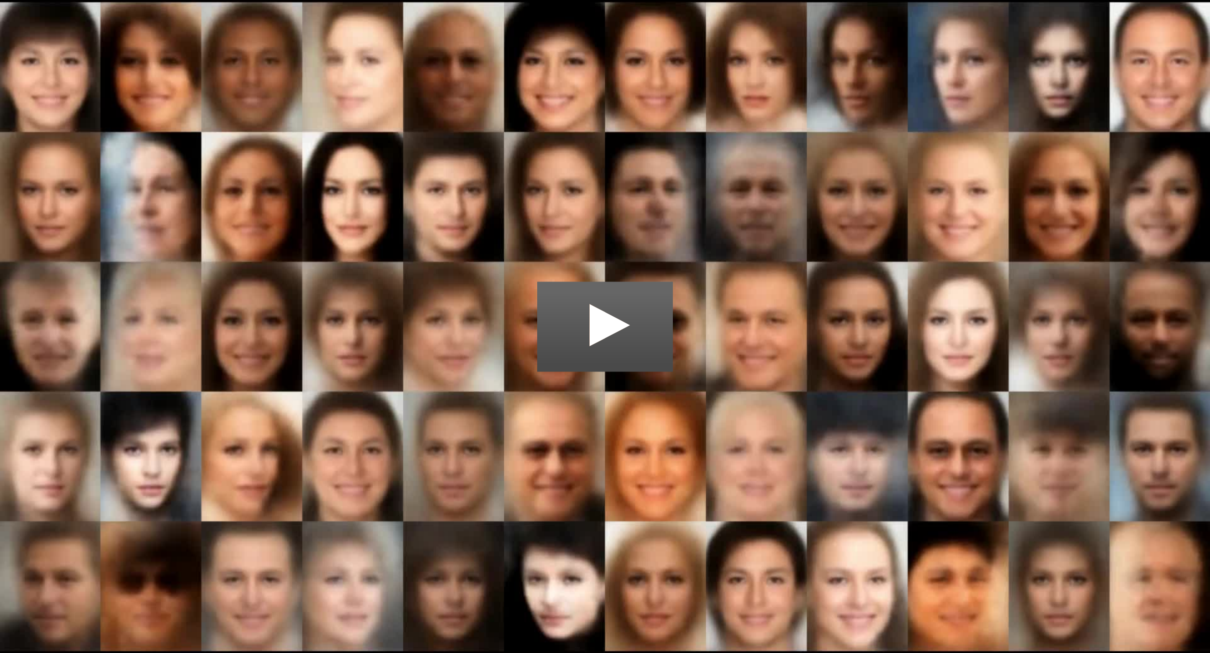


Image Sources. "Pain Expression Subset;" `http://pics.stir.ac.uk/2D_face_sets.htm`

Autoencoder





PCA in Python

```python
def PCA(X): # X: N x d matrix
    N, d = X.shape
    X_cen = X - np.mean(X, axis=0)
    C = (X_cen.T @ X_cen) / (N - 1)
    L, V = np.linalg.eigh(C) # "eigh" faster than "eig" for symmetric matrices
    return V.T[::-1, :] # d x d matrix
```

```python
def PCA_SVD(X): # X: N x d matrix
    return np.linalg.svd(X - np.mean(X, axis=0))[2]
```

PCA Example: Source Images

Face Database

- 84 images of 12 women with 7 different expressions
- Normalised eye location
- 45×60 pixels (2700 dimensions)
- Greyscale

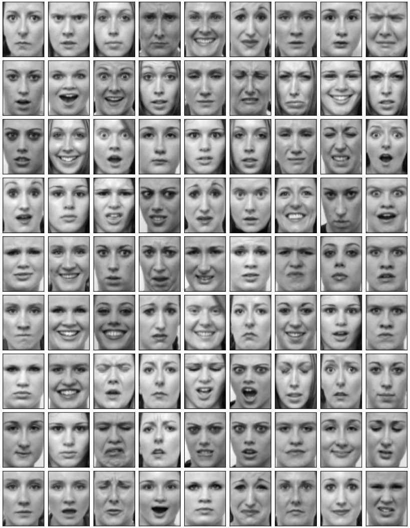
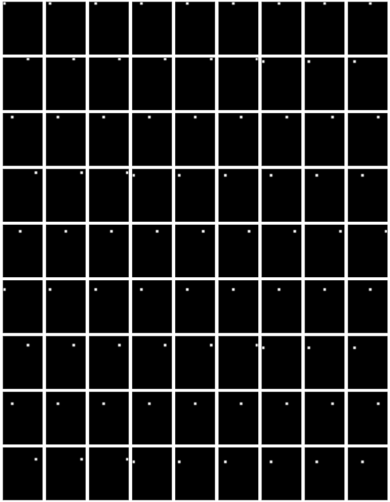


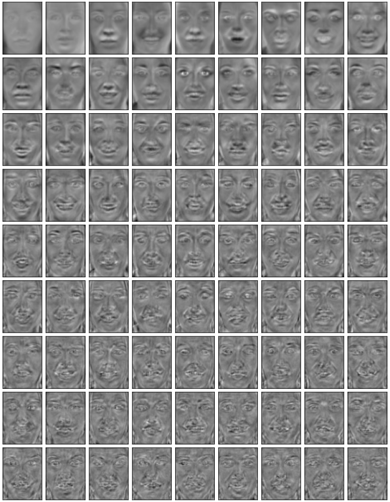
Image Sources. "Pain Expression Subset;" `http://pics.stir.ac.uk/2D_face_sets.htm`

PCA Example: Eigenfaces

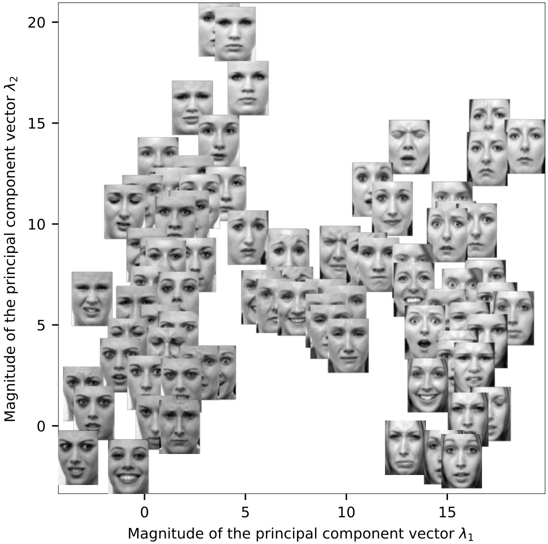
Identity Basis

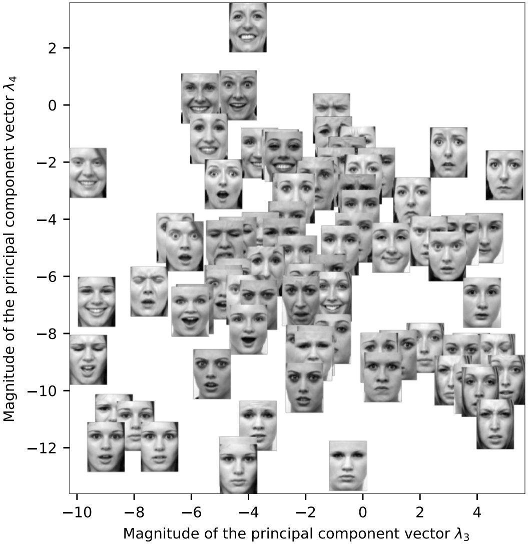


Principal Components

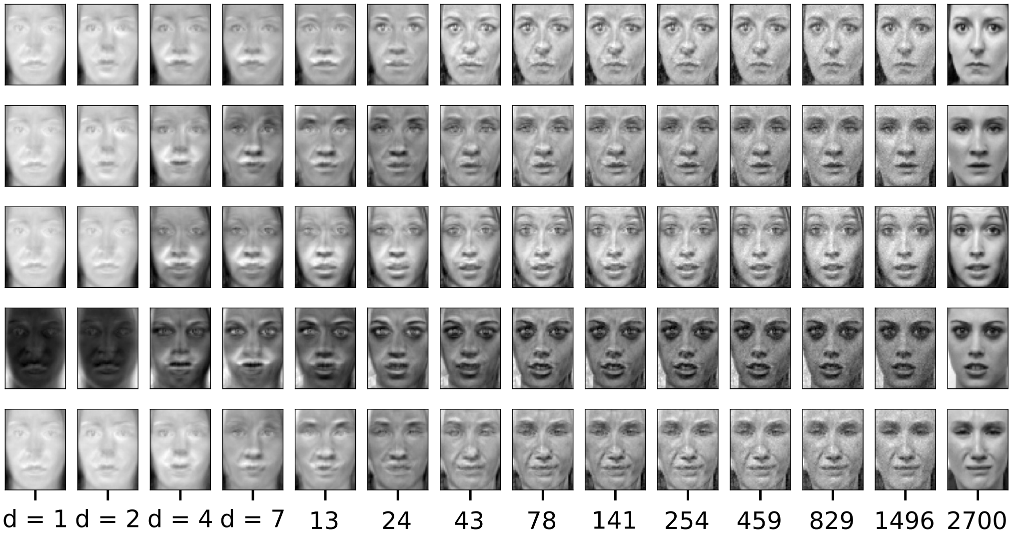


PCA Example: Face Spaces

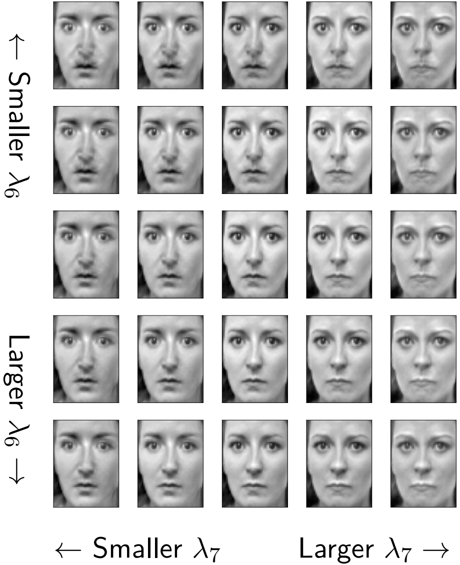




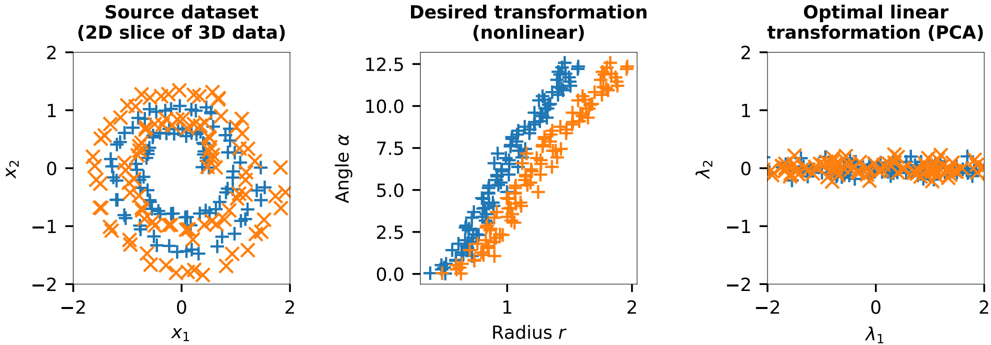
PCA Example: Sparse Vectors



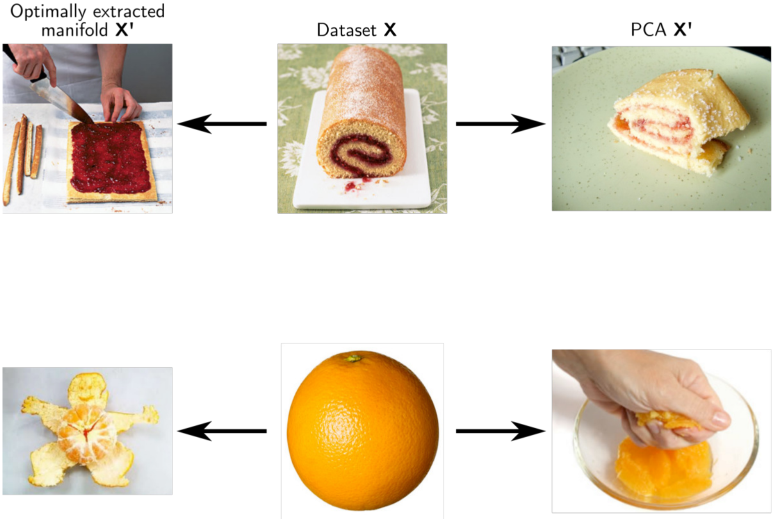
PCA Example: Modifying the Latent Space



Limitations of PCA: Classifying Two Groups



Limitations of PCA: Metaphorical Illustration



PCA

Projects ("squashes") data onto a hyperplane

Image Sources. Barbara Hammer, "Maschinelles Lernen im Web", Universität Bielefeld, WS 2014/15

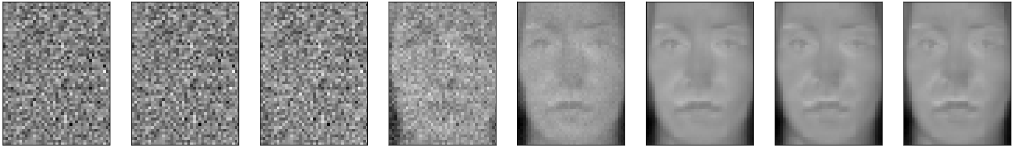
Hebbian Learning

> When an axon of cell A is near enough to excite a cell B and repeatedly or persistently takes part in firing it, some growth process or metabolic change takes place in one or both cells such that A's efficiency, as one of the cells firing B, is increased.
>
> — Donald O. Hebb, "The Organization of Behaviour", 1949

Hebbian Learning

$\Delta w_{ij} = \eta a_i a_j$

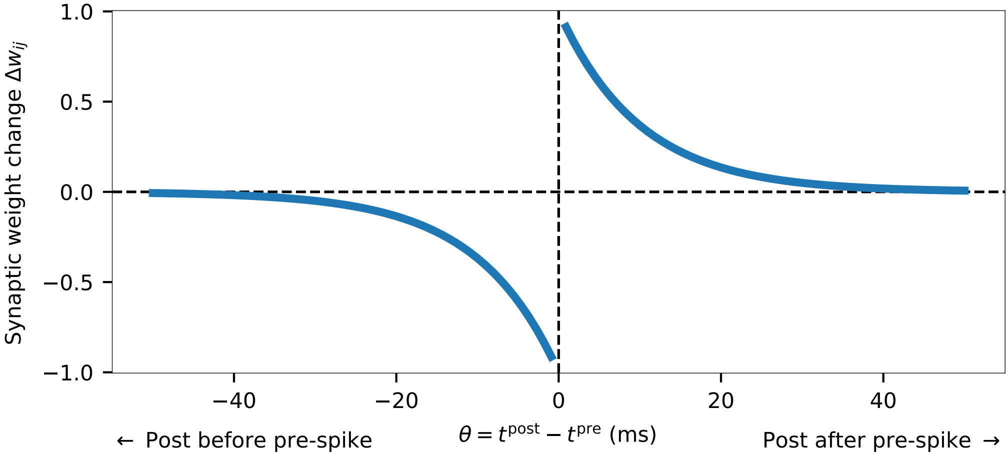
Example: Normalised Hebbian Learning



$t = 0$	$t = 1428$	$t = 2856$	$t = 4285$	$t = 5713$	$t = 7142$	$t = 8570$	$t = 9999$

Learning an encoder $\mathbf{e}$, with $\|\mathbf{e}\| = 1$, 10000 steps, $\eta = 0.2 \times 10^{-4}$, $\Delta\mathbf{e} = \eta(\mathbf{x} \circ \mathbf{e})$

Spike-Time Dependent Plasticity



Conclusion

Supervised Learning

- Find $\mathbf{w}$ such that $f(\mathbf{x}_k; \mathbf{w}) \approx \mathbf{t}_k$
- *Hope:* $f(\mathbf{x}_k; \mathbf{w}) \approx f_{\mathrm{GT}}(\mathbf{x}_k)$
- Use gradient descent to find $\mathbf{w}$
- Delta, PES learning rules
- Modulatory synapses in the brain

Unsupervised Learning

- Dimensionality reduction $f(\mathbf{x}_k) = \lambda_k$
- *Hope:* latent dimensions λ are "meaningful"
- Autoencoders (nonlinear), PCA (linear)
- Hebbian learning $\Rightarrow$ learns PCA

Image sources

Title slide
Page from “Liber ethicorum des Henricus de Alemannia”. Title: “Henricus de Alemannia con i suoi studenti” (Henricus of Germany with his students), second half of 14th century.
From Wikimedia.