

**SYDE 556/750**

**Simulating Neurobiological Systems**  
**Lecture 8: Learning**

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February 25 & 27, 2020

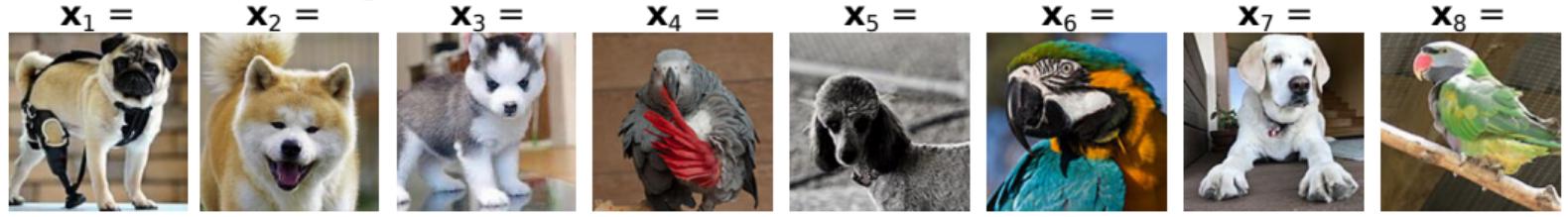


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**WATERLOO**

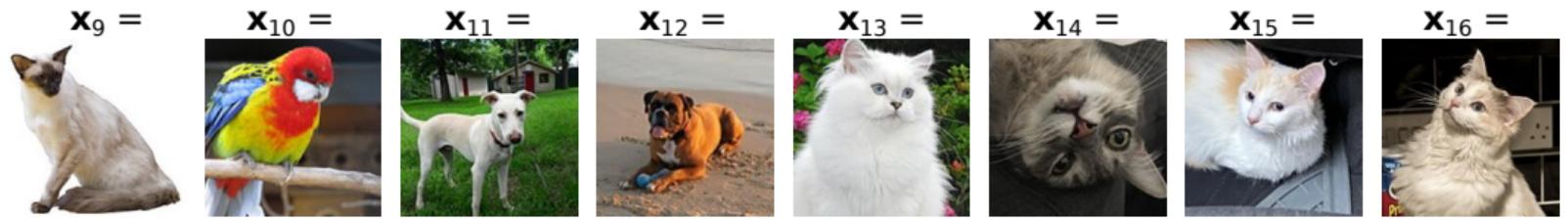
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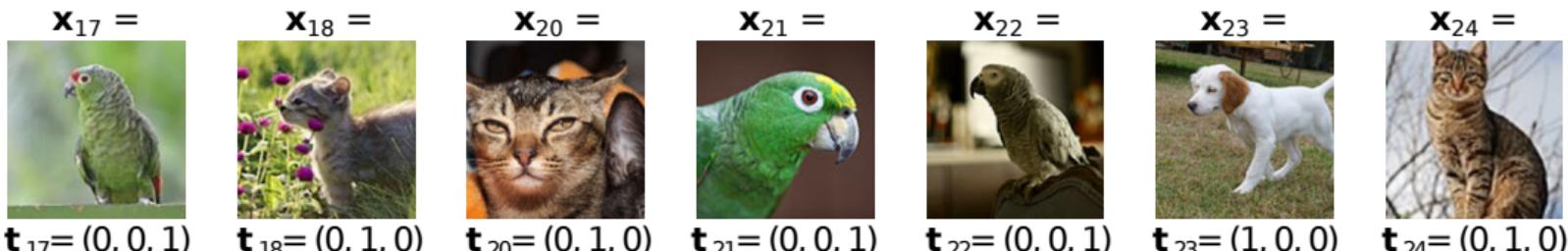
# Supervised Learning



$$\mathbf{t}_1 = (1, 0, 0) \quad \mathbf{t}_2 = (1, 0, 0) \quad \mathbf{t}_3 = (1, 0, 0) \quad \mathbf{t}_4 = (0, 0, 1) \quad \mathbf{t}_5 = (1, 0, 0) \quad \mathbf{t}_6 = (0, 0, 1) \quad \mathbf{t}_7 = (1, 0, 0) \quad \mathbf{t}_8 = (0, 0, 1)$$

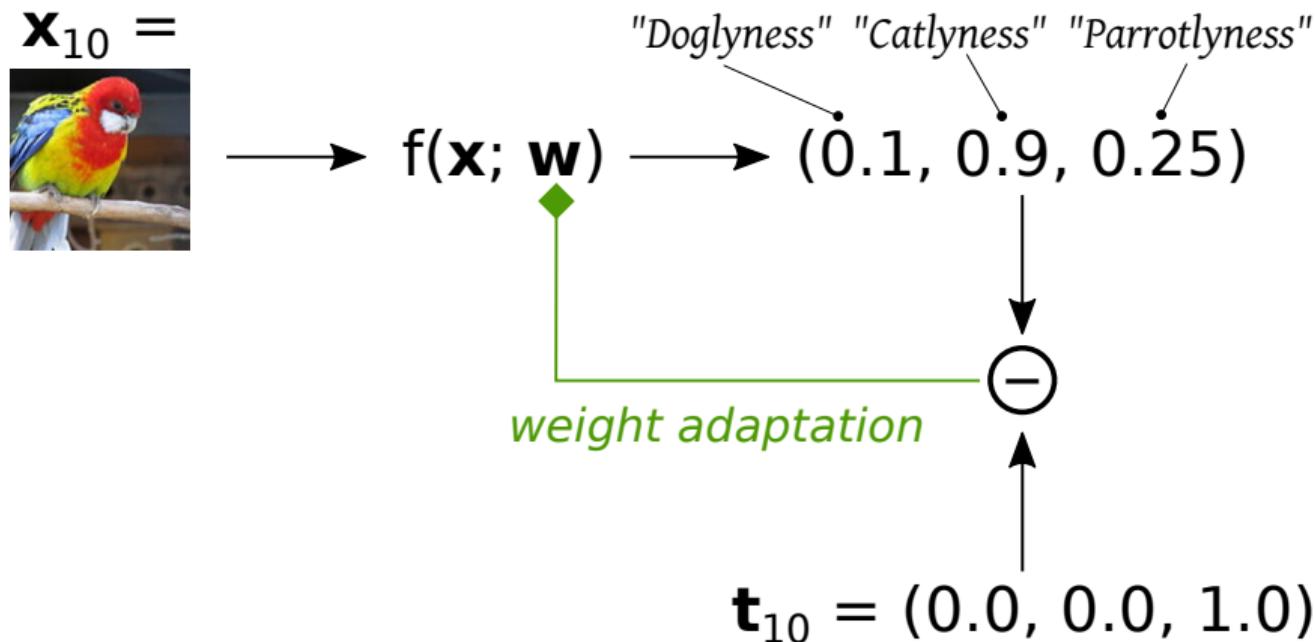


$$\mathbf{t}_9 = (0, 1, 0) \quad \mathbf{t}_{10} = (0, 0, 1) \quad \mathbf{t}_{11} = (1, 0, 0) \quad \mathbf{t}_{12} = (1, 0, 0) \quad \mathbf{t}_{13} = (0, 1, 0) \quad \mathbf{t}_{14} = (0, 1, 0) \quad \mathbf{t}_{15} = (0, 1, 0) \quad \mathbf{t}_{16} = (0, 1, 0)$$

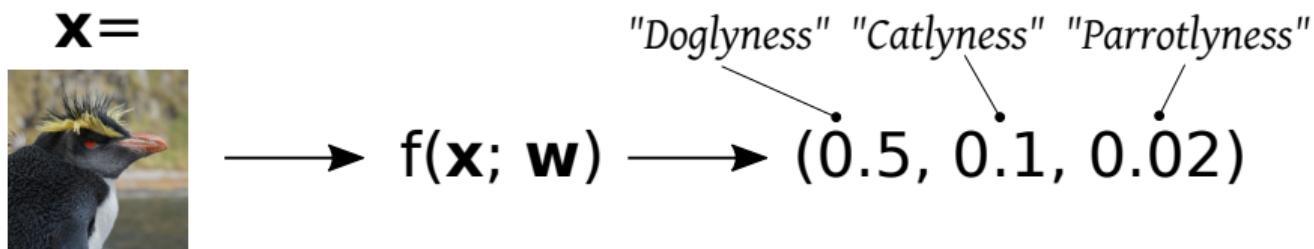


$$\mathbf{t}_{17} = (0, 0, 1) \quad \mathbf{t}_{18} = (0, 1, 0) \quad \mathbf{t}_{20} = (0, 1, 0) \quad \mathbf{t}_{21} = (0, 0, 1) \quad \mathbf{t}_{22} = (0, 0, 1) \quad \mathbf{t}_{23} = (1, 0, 0) \quad \mathbf{t}_{24} = (0, 1, 0)$$

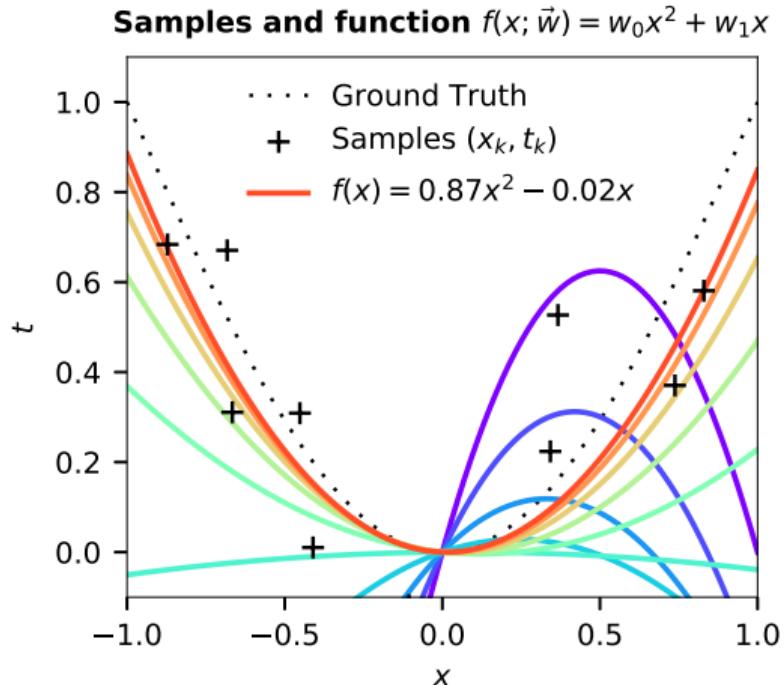
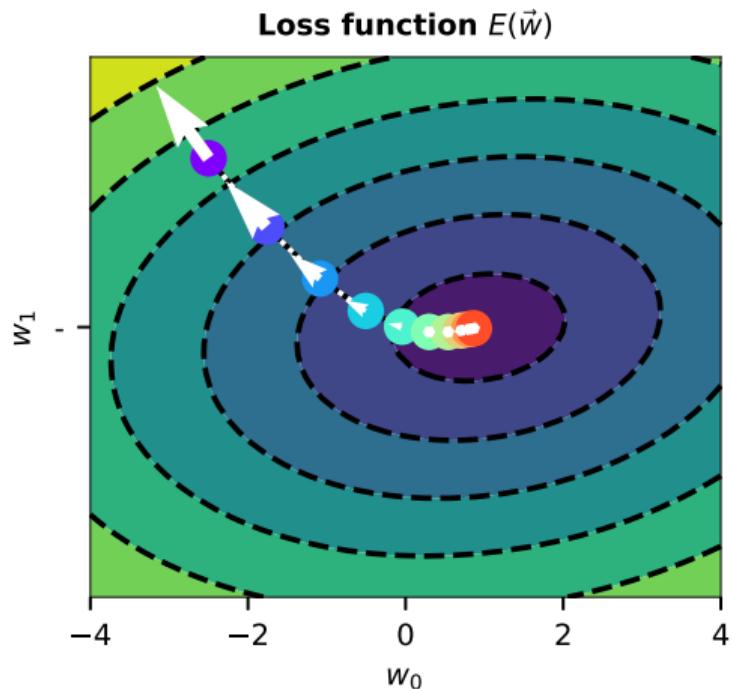
## Supervised Learning – Training



## Supervised Learning – Inference

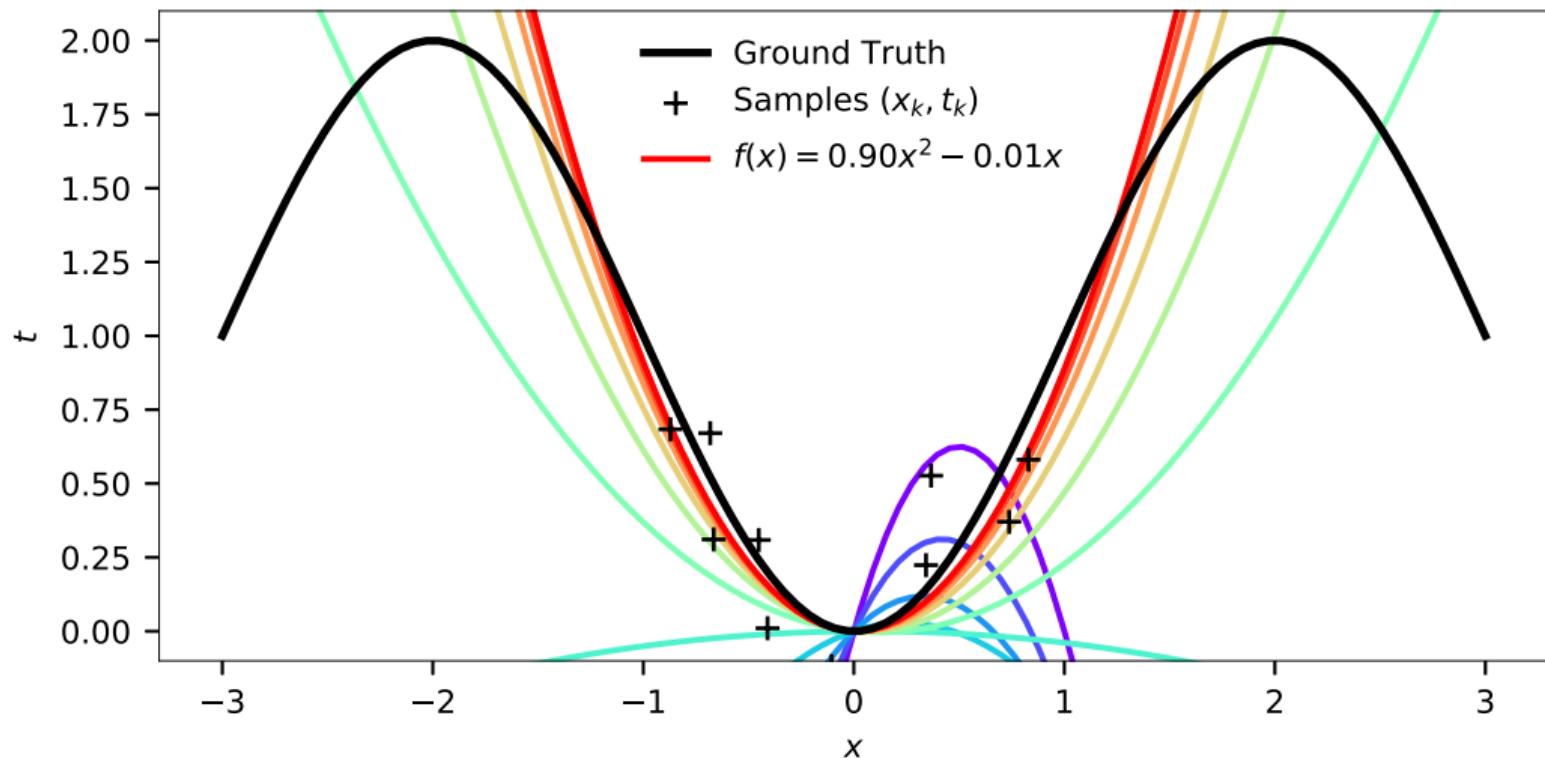


# Gradient Descent – Example



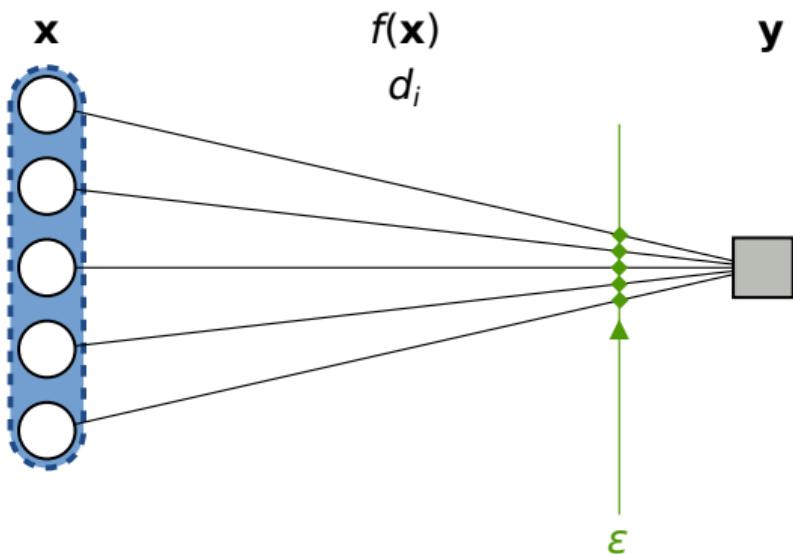
# Supervised Learning – Generalisation

**Samples and function**  $f(x; \vec{w}) = w_0x^2 + w_1x$



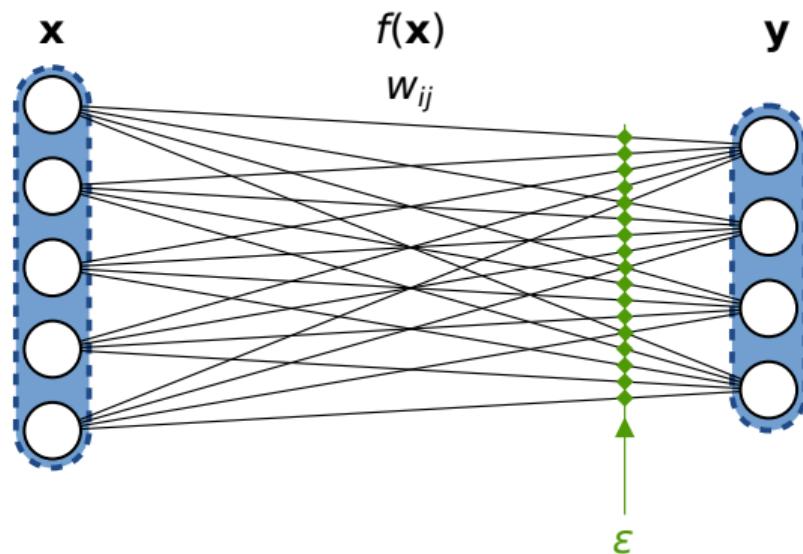
# Learning Decoders and Learning Weights

**Learning Decoders** (Delta Rule)



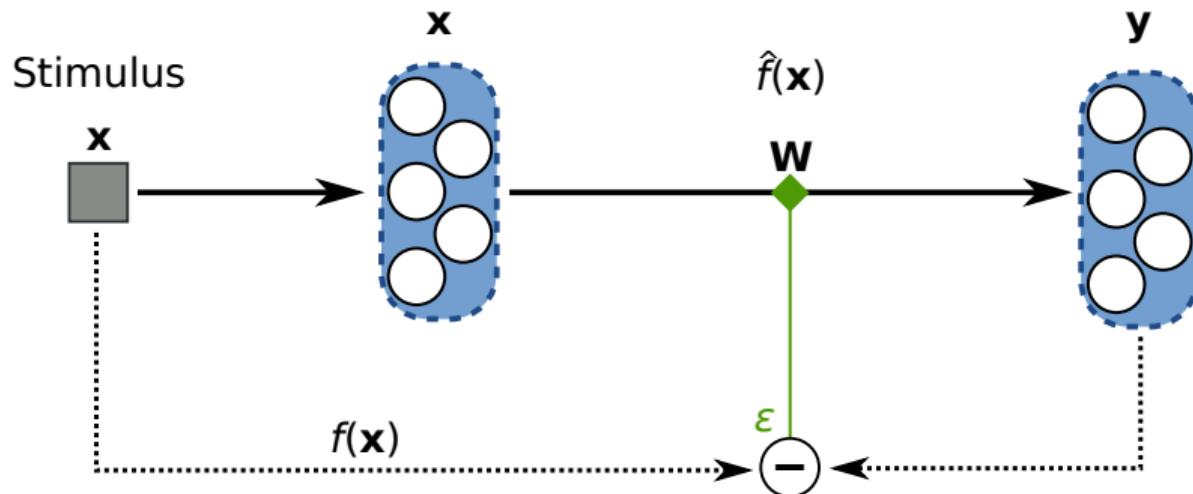
$$\Delta d_i = -\eta a_i(\mathbf{x}) \underbrace{(y(t) - y^d(t))}_{\varepsilon(t)}$$

**Learning Weights** (PES Rule)



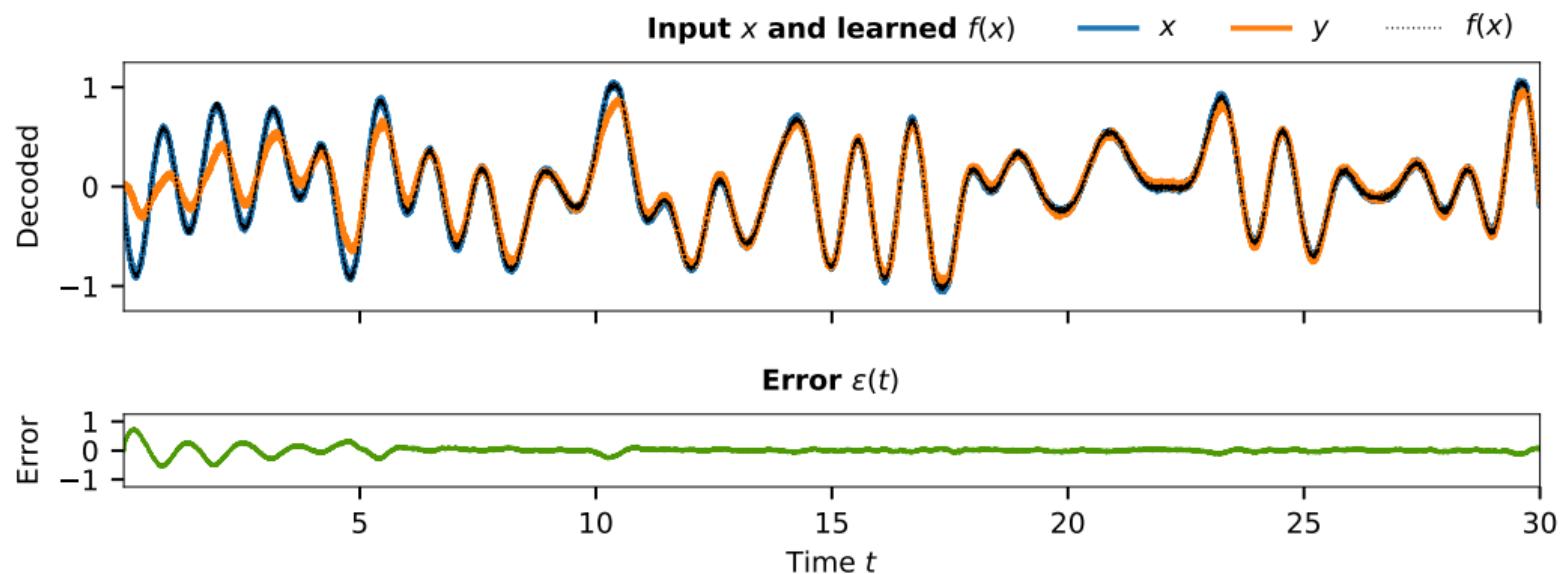
$$\Delta w_{ij} = -\eta a_i(\mathbf{x}) \left( \alpha_j \langle \mathbf{e}_j, \varepsilon(t) \rangle \right)$$

## Example: Learning Functions (I)



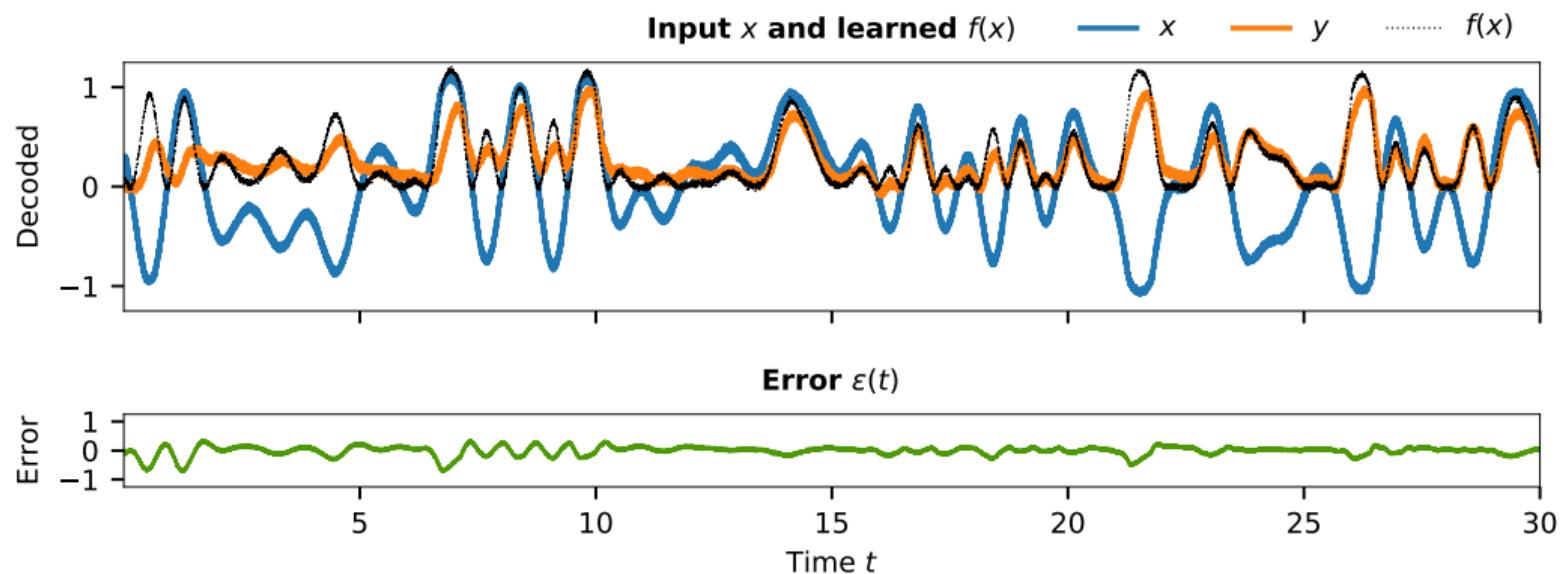
## Example: Learning Functions (II)

Communication Channel  $f(x) = x$



## Example: Learning Functions (III)

Square  $f(x) = x^2$

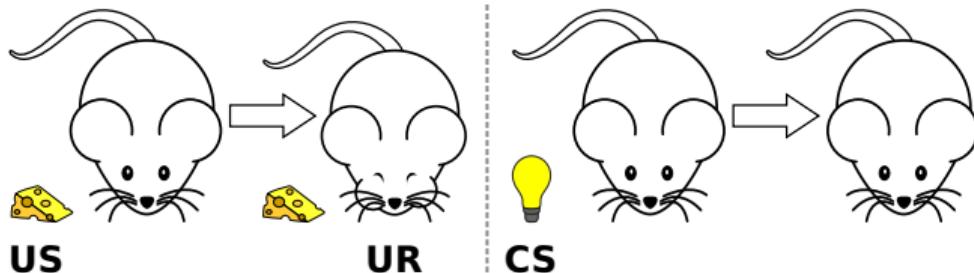


Works, but learns more slowly!

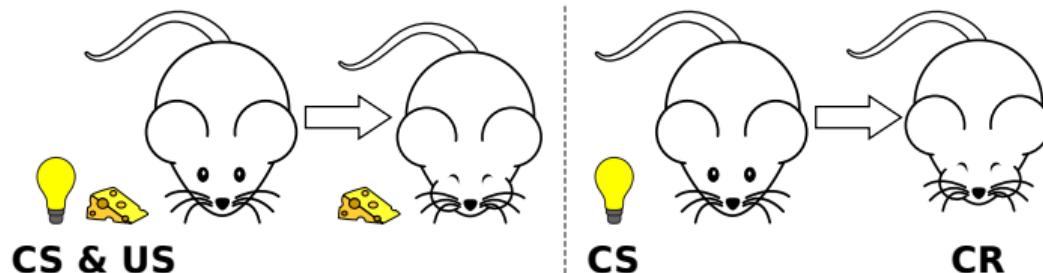
Where is the error signal  $\varepsilon(t)$  coming from?

## Example: Classical Conditioning (I)

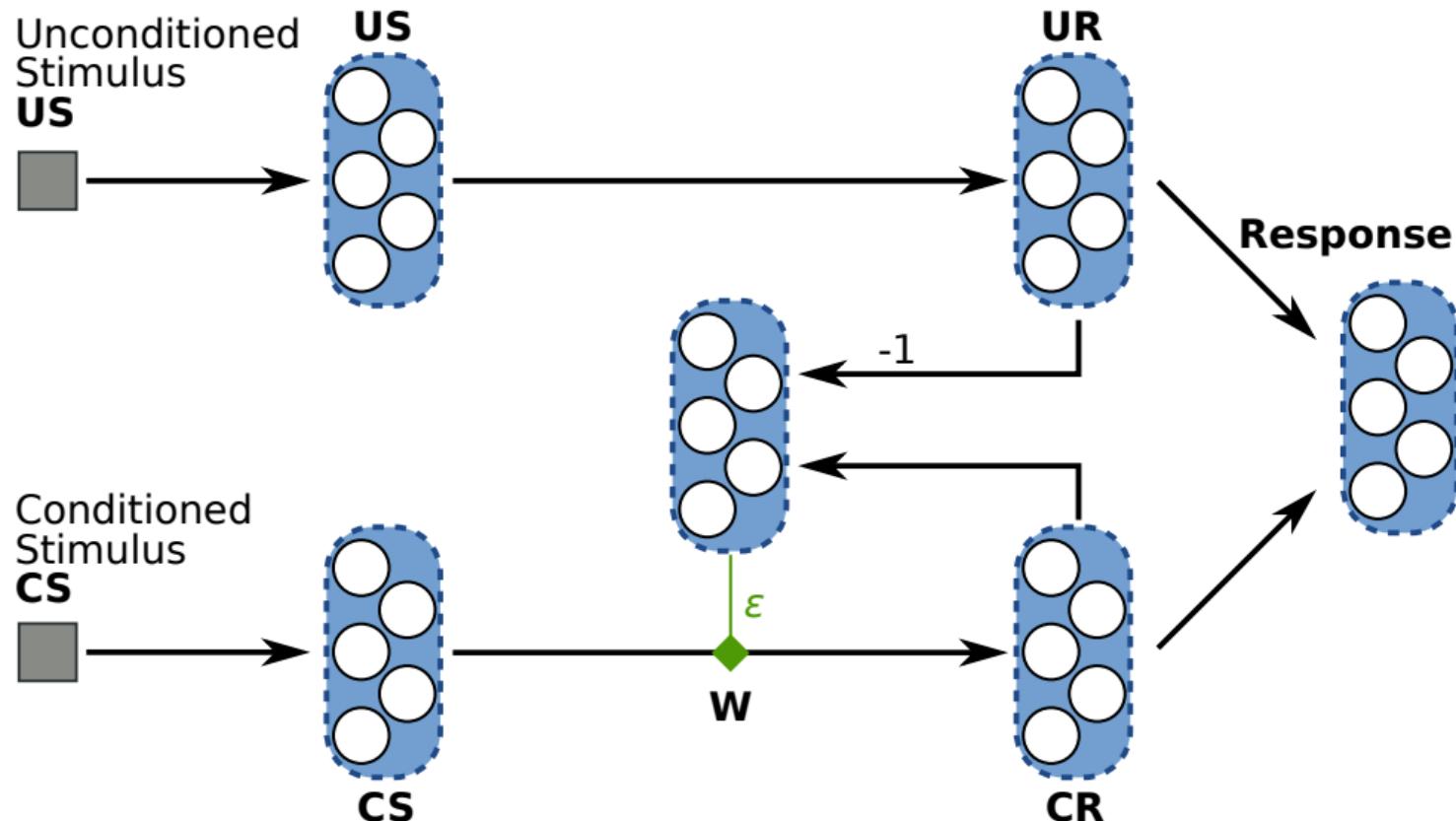
Before conditioning:



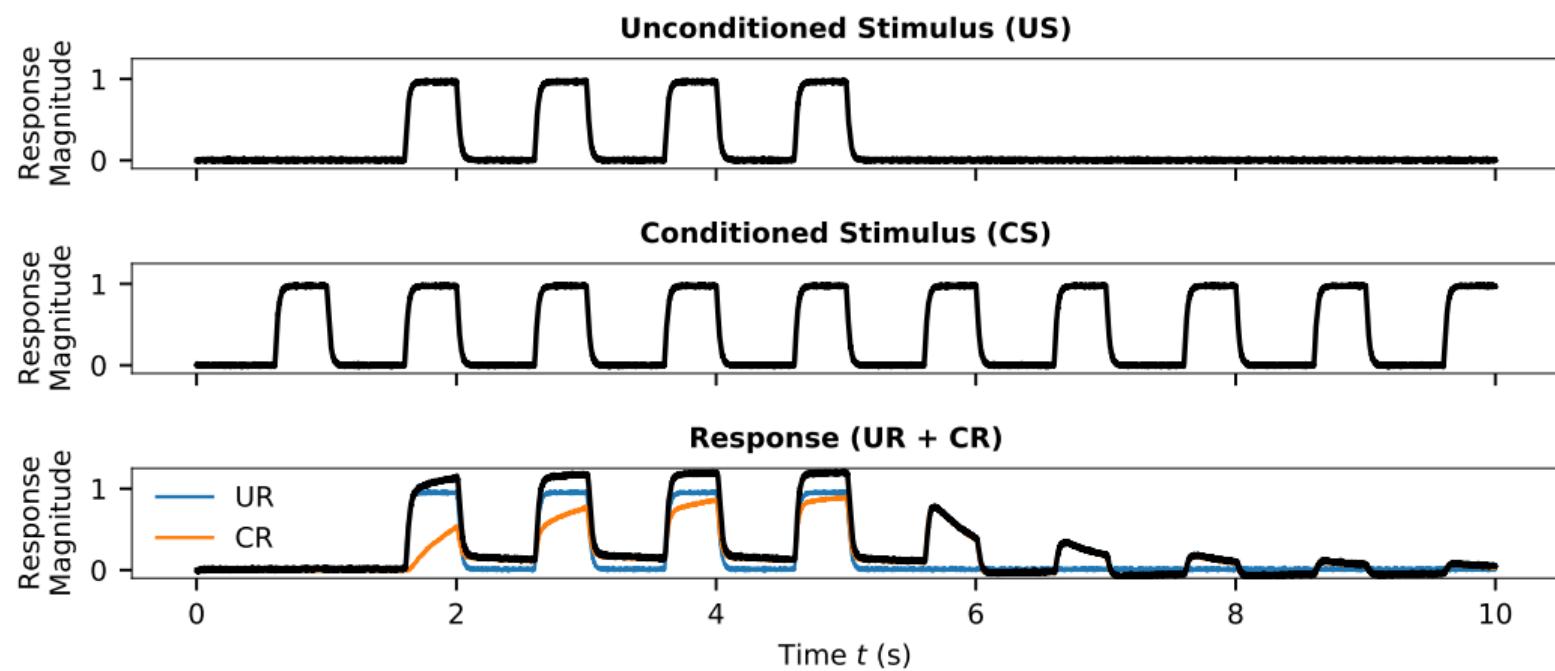
After conditioning:



## Example: Classical Conditioning (II)

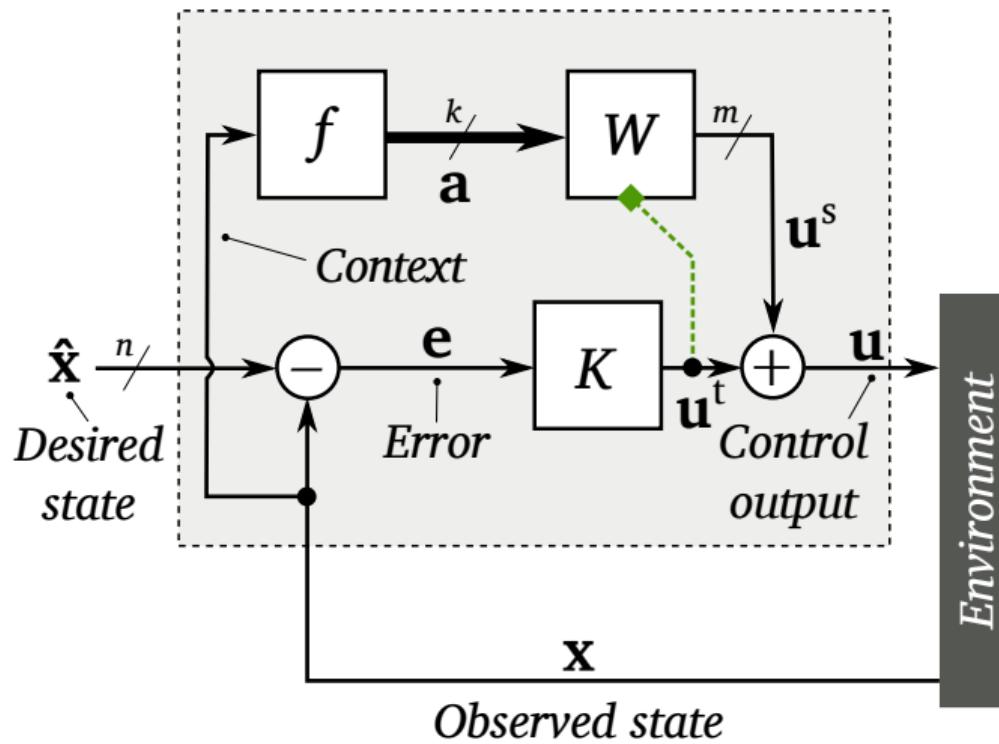


## Example: Classical Conditioning (III)

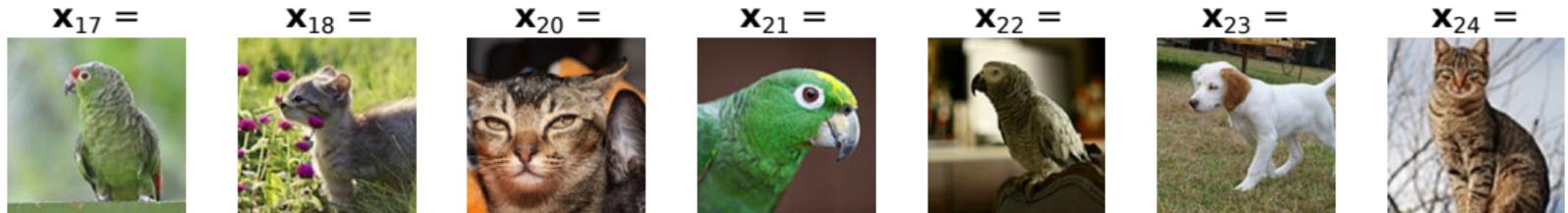
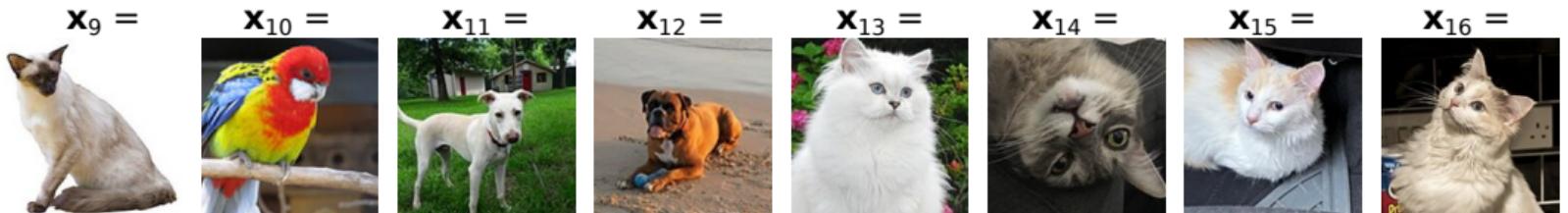
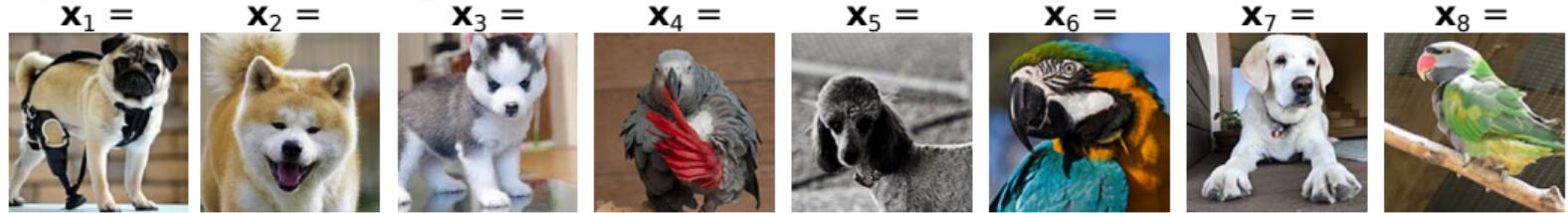


## Example: Adaptive Controller

*Adaptive Controller*



# Unsupervised Learning



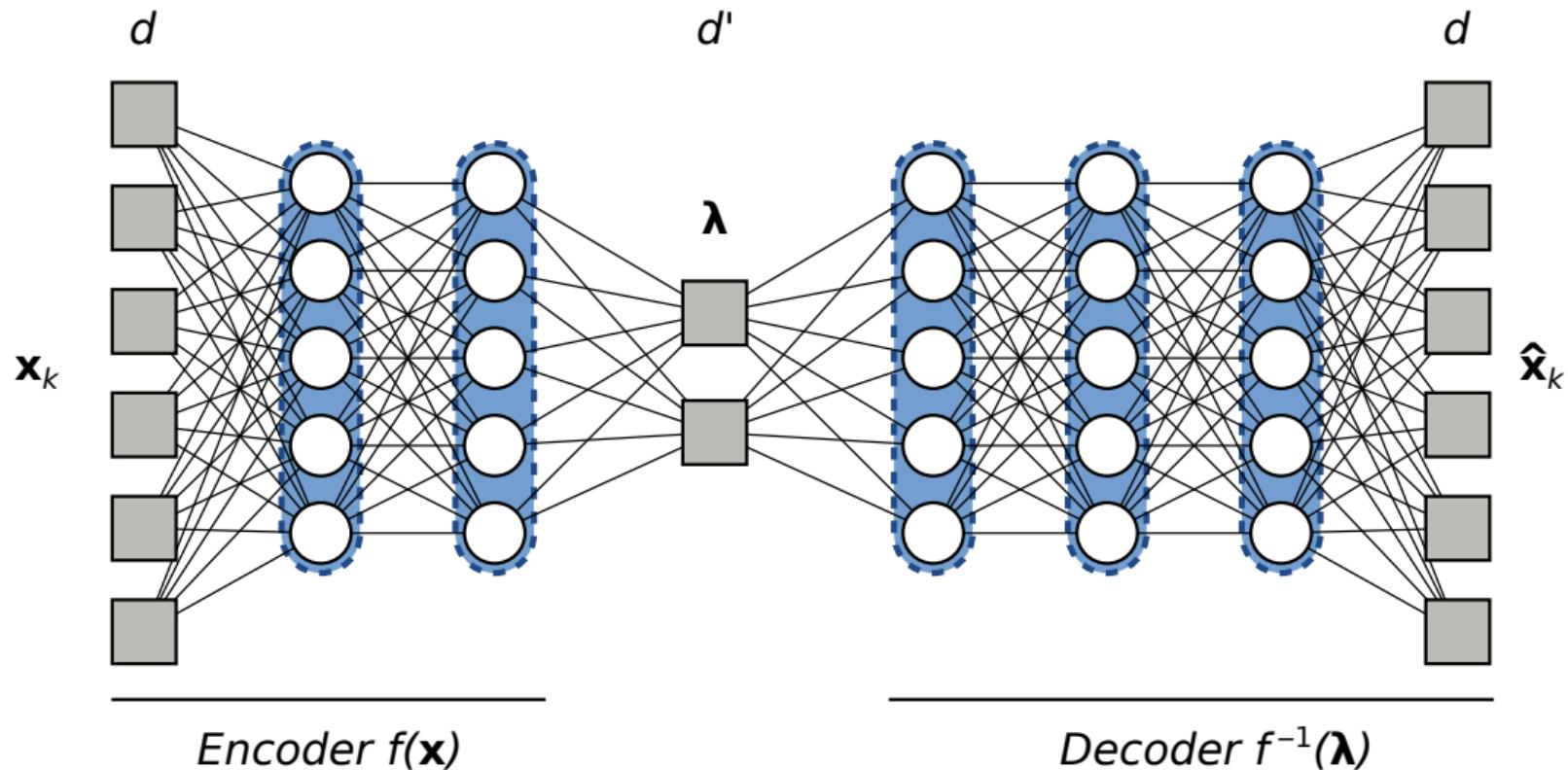
# Unsupervised Learning – Training

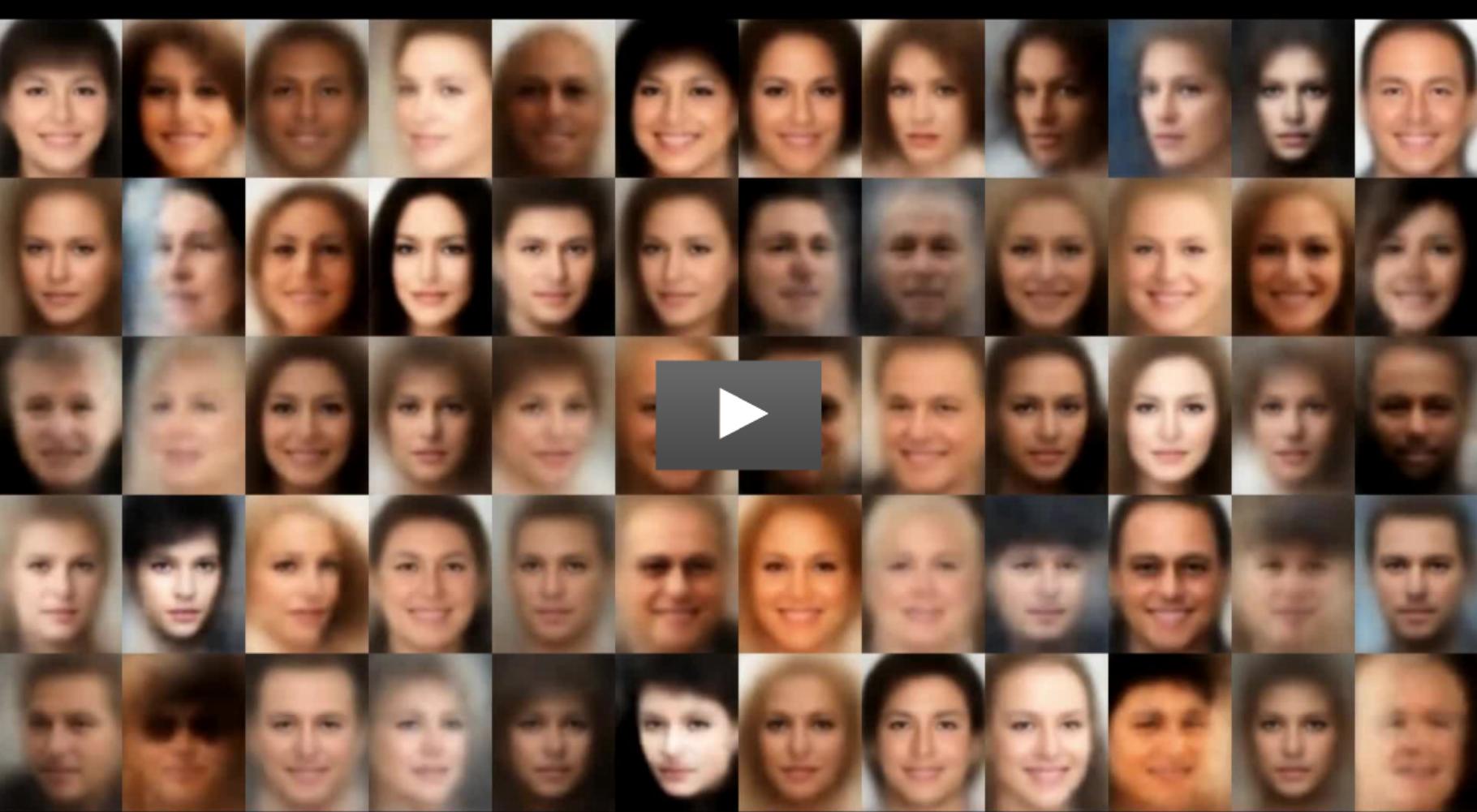


# Unsupervised Learning – Inference



# Autoencoder





# PCA in Python

```
🐍 def PCA(X): # X: N x d matrix
    N, d = X.shape
    X_cen = X - np.mean(X, axis=0)
    C = (X_cen.T @ X_cen) / (N - 1)
    L, V = np.linalg.eigh(C) # "eigh" faster than "eig" for symmetric matrices
    return V.T[::-1, :] # d x d matrix
```

```
🐍 def PCA_SVD(X): # X: N x d matrix
    return np.linalg.svd(X - np.mean(X, axis=0))[2]
```

# PCA Example: Source Images

## Face Database

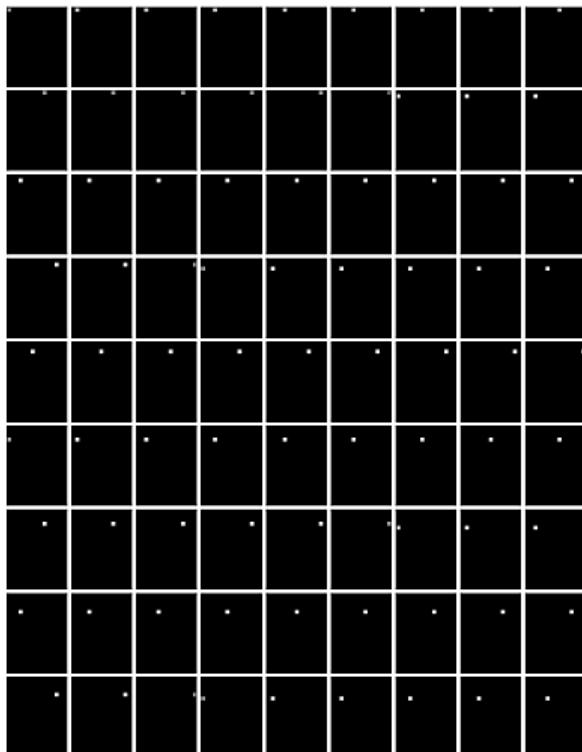
- ▶ 84 images of 12 women with 7 different expressions
- ▶ Normalised eye location
- ▶  $45 \times 60$  pixels (2700 dimensions)
- ▶ Greyscale



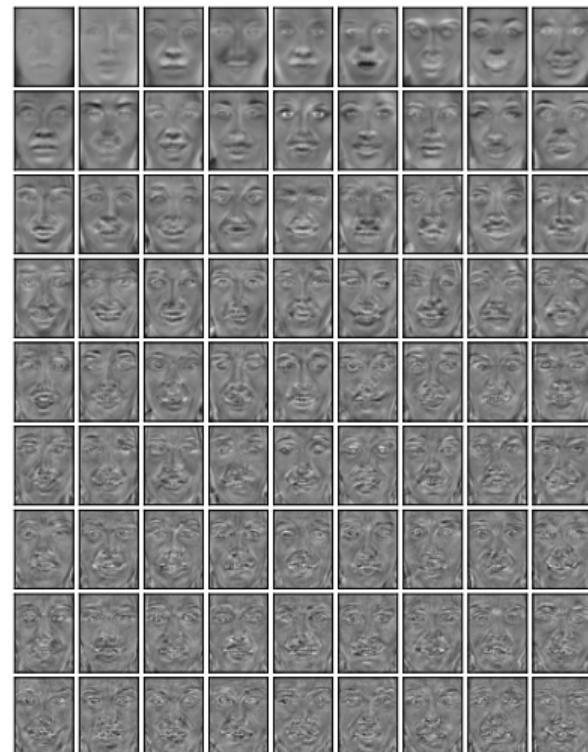
Image Sources. "Pain Expression Subset;" [http://pics.stir.ac.uk/2D\\_face\\_sets.htm](http://pics.stir.ac.uk/2D_face_sets.htm)

# PCA Example: Eigenfaces

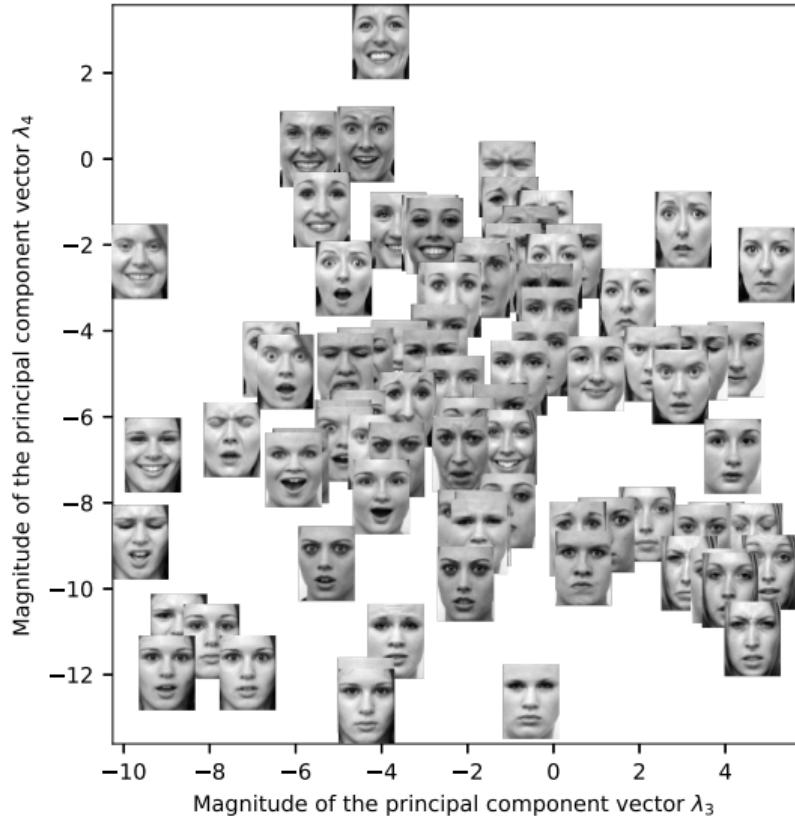
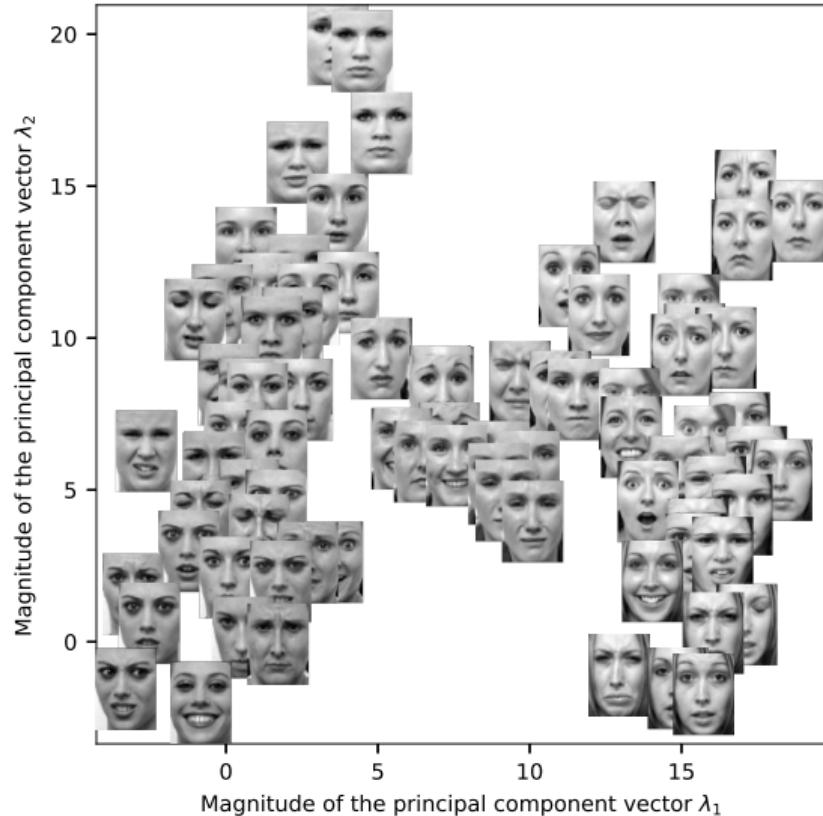
**Identity Basis**



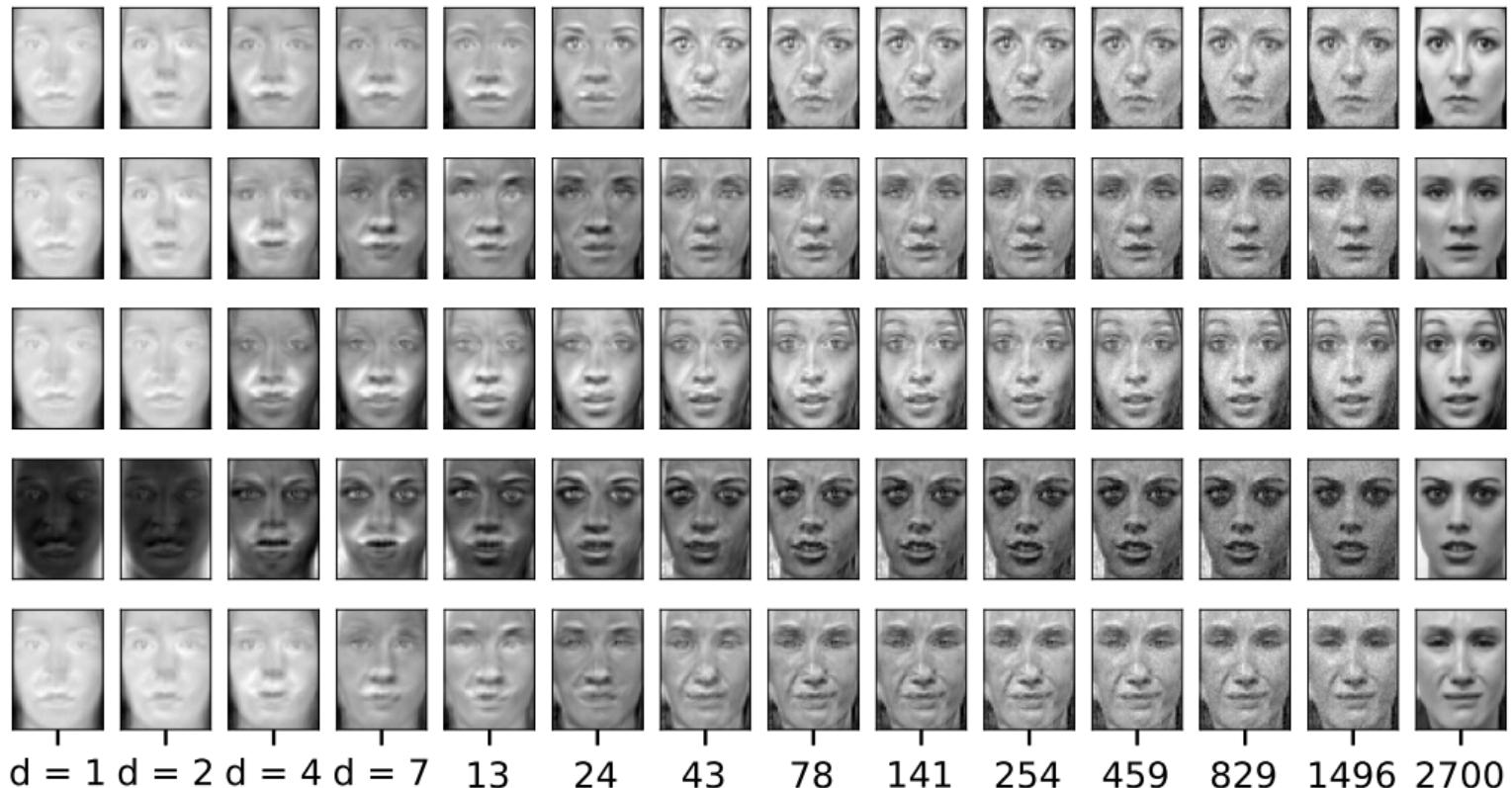
**Principal Components**



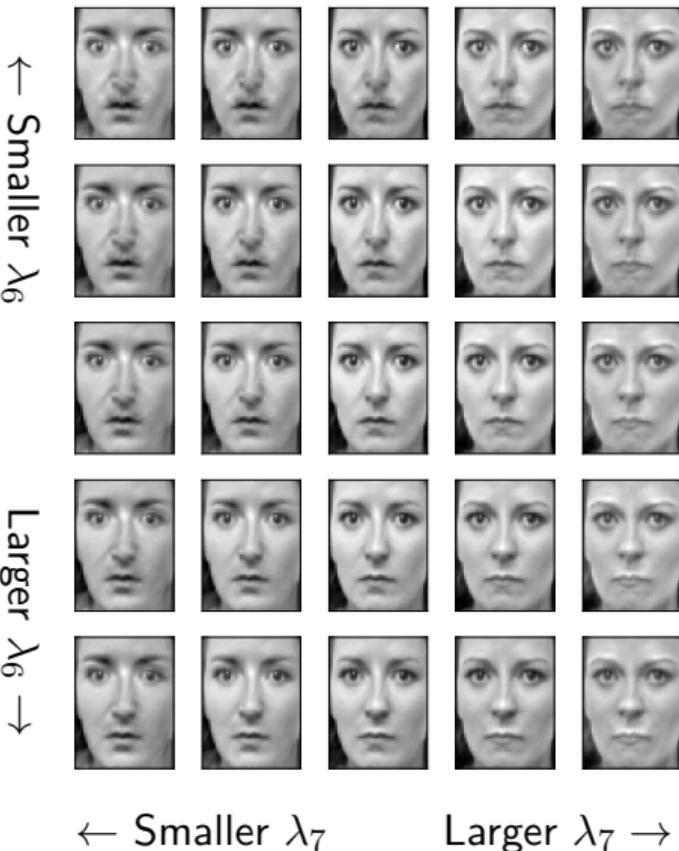
# PCA Example: Face Spaces



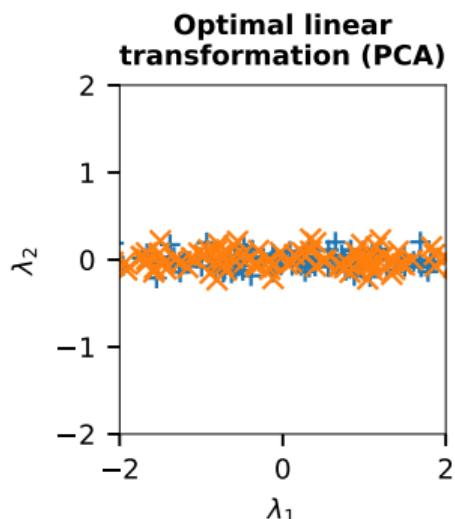
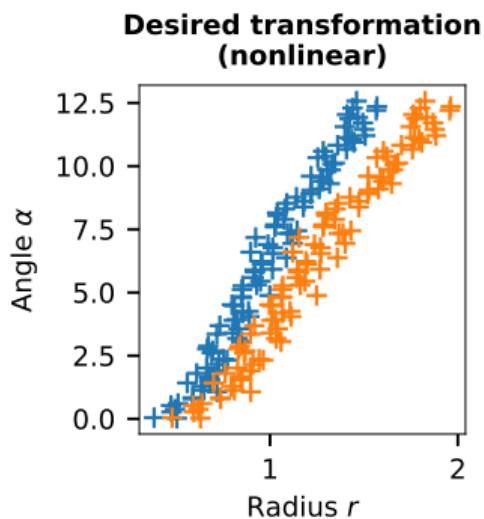
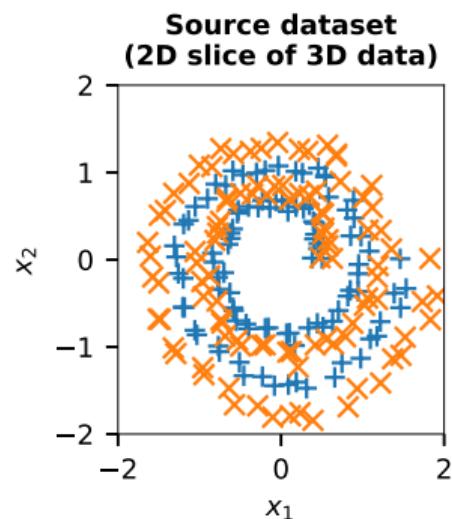
## PCA Example: Sparse Vectors



## PCA Example: Modifying the Latent Space



# Limitations of PCA: Classifying Two Groups



# Limitations of PCA: Metaphorical Illustration

Optimally extracted  
manifold  $X'$



Dataset  $X$

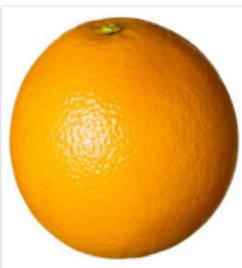


PCA  $X'$



PCA

Projects ("squashes") data  
onto a hyperplane

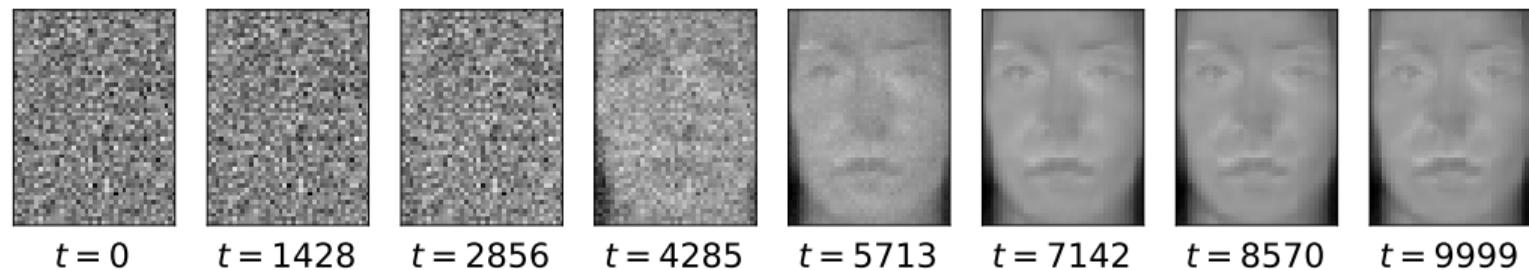


## Hebbian Learning

When an axon of cell  $A$  is near enough to excite a cell  $B$  and repeatedly or persistently takes part in firing it, some growth process or metabolic change takes place in one or both cells such that  $A$ 's efficiency, as one of the cells firing  $B$ , is increased.

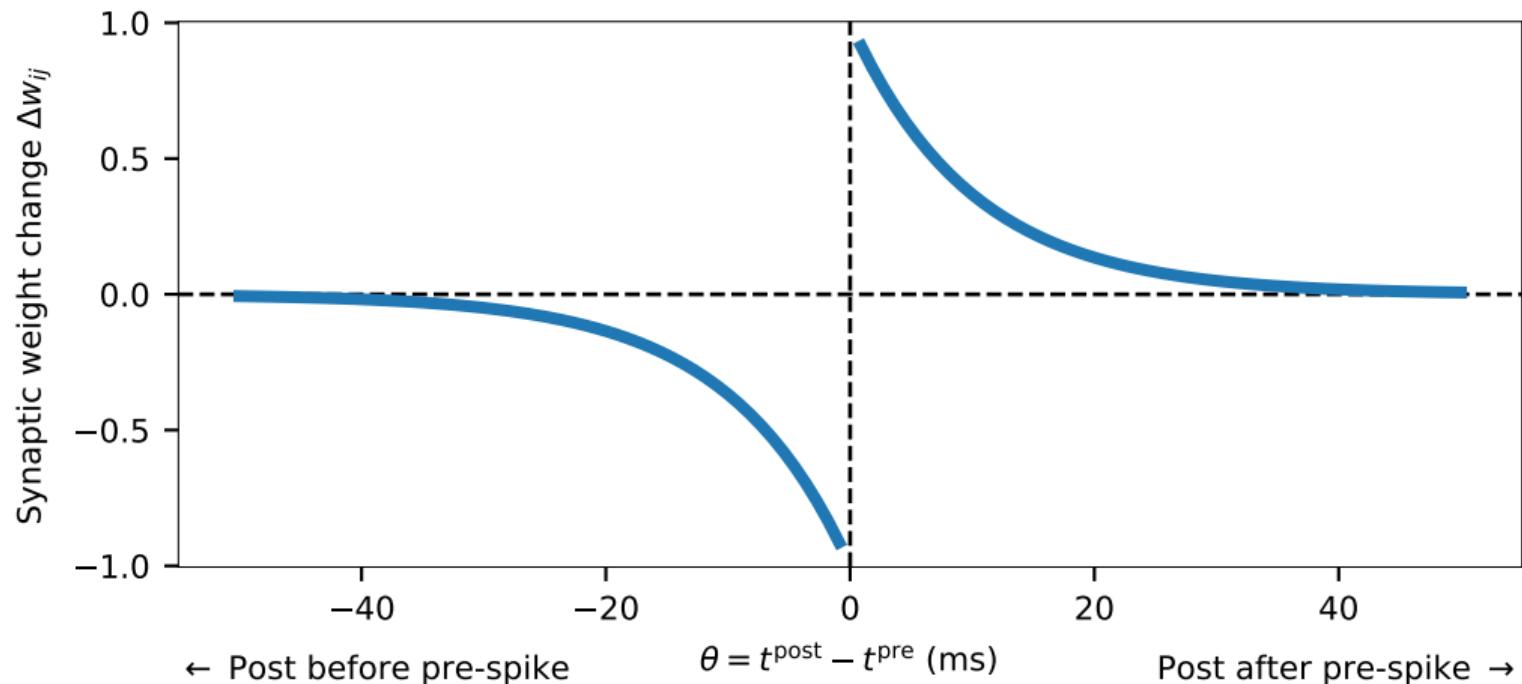
— Donald O. Hebb, “The Organization of Behaviour”, 1949

## Example: Normalised Hebbian Learning



Learning an encoder  $\mathbf{e}$ , with  $\|\mathbf{e}\| = 1$ , 10000 steps,  $\eta = 0.2 \times 10^{-4}$ ,  $\Delta\mathbf{e} = \eta(\mathbf{x} \circ \mathbf{e})$

# Spike-Time Dependent Plasticity



# Conclusion

## Supervised Learning

- ▶ Find  $\mathbf{w}$  such that  $f(\mathbf{x}_k; \mathbf{w}) \approx \mathbf{t}_k$
- ▶ *Hope:*  $f(\mathbf{x}_k; \mathbf{w}) \approx f_{\text{GT}}(\mathbf{x}_k)$
- ▶ Use gradient descent to find  $\mathbf{w}$
- ▶ Delta, PES learning rules
- ▶ Modulatory synapses in the brain

## Unsupervised Learning

- ▶ Dimensionality reduction  $f(\mathbf{x}_k) = \lambda_k$
- ▶ *Hope:* latent dimensions  $\lambda$  are “meaningful”
- ▶ Autoencoders (nonlinear), PCA (linear)
- ▶ Hebbian learning  $\Rightarrow$  learns PCA

# Image sources

## Title slide

Page from “Liber ethicorum des Henricus de Alemannia”. Title: “Henricus de Alemannia con i suoi studenti” (Henricus of Germany with his students), second half of 14th century.

From Wikimedia.