

SYDE 556/750

Simulating Neurobiological Systems
Lecture 9: Analysing Representations

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- ▶ Slide design: Andreas Stöckel
- ▶ Content: Terry Stewart, Andreas Stöckel, Chris Eliasmith



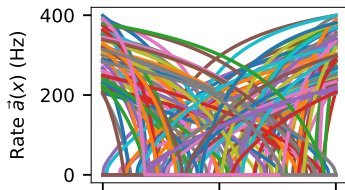
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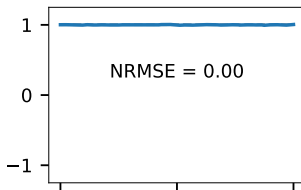


Decoding Polynomials

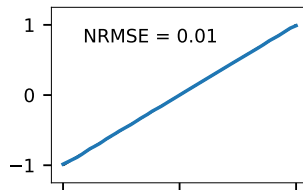
Tuning curves ($n = 128$)



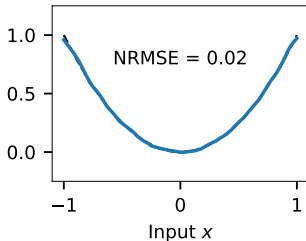
$f(x) = 1$



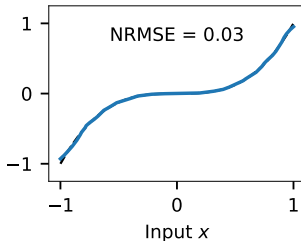
$f(x) = x$



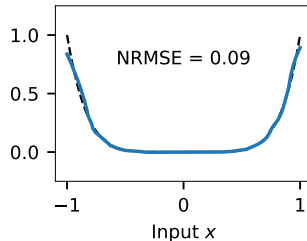
$f(x) = x^2$



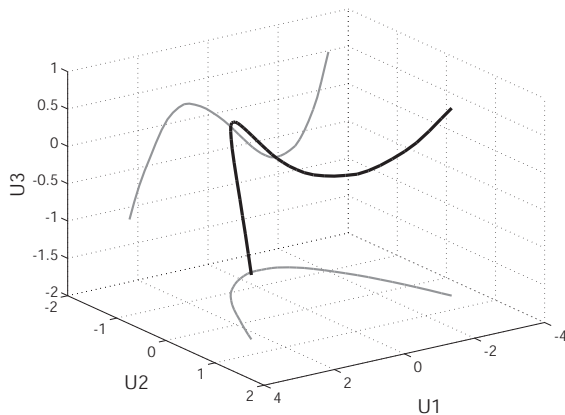
$f(x) = x^3$



$f(x) = x^6$



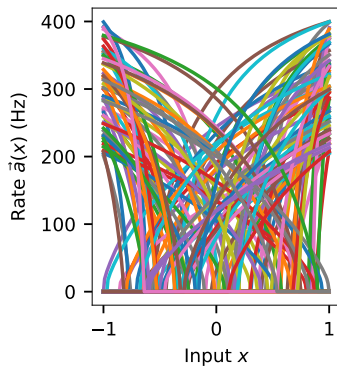
Projection of the Neuron Space to PCA



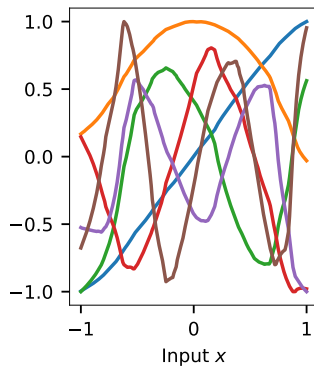
A subspace of neuron activity being projected onto the first few principle component planes. Notice that the axis scales are different, capturing the size of the singular value.

LIF Tuning Curve Principal Components

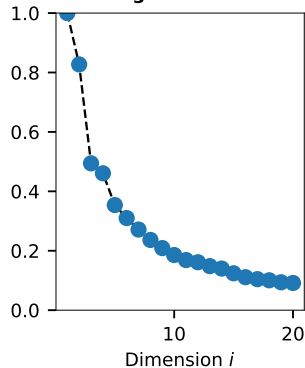
Tuning curves ($n = 128$)



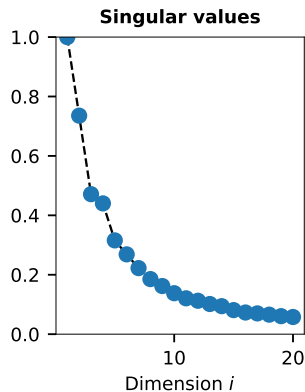
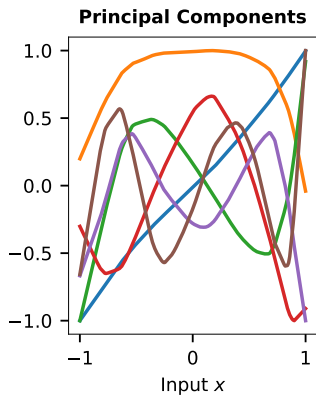
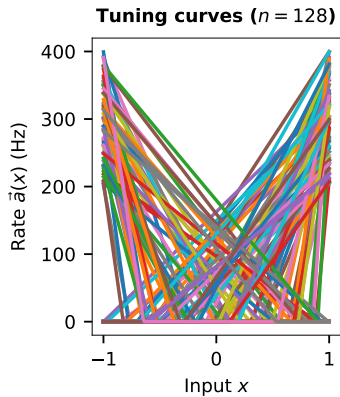
Principal Components



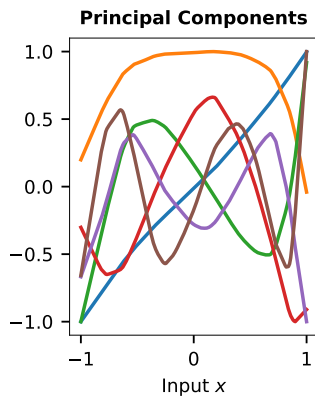
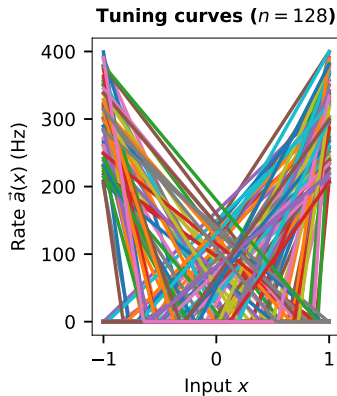
Singular values



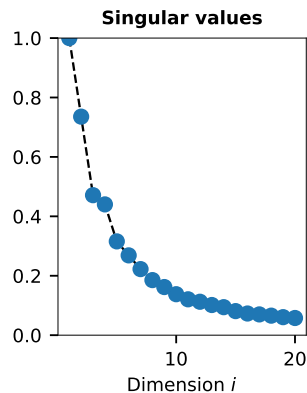
ReLU Tuning Curve Principal Components



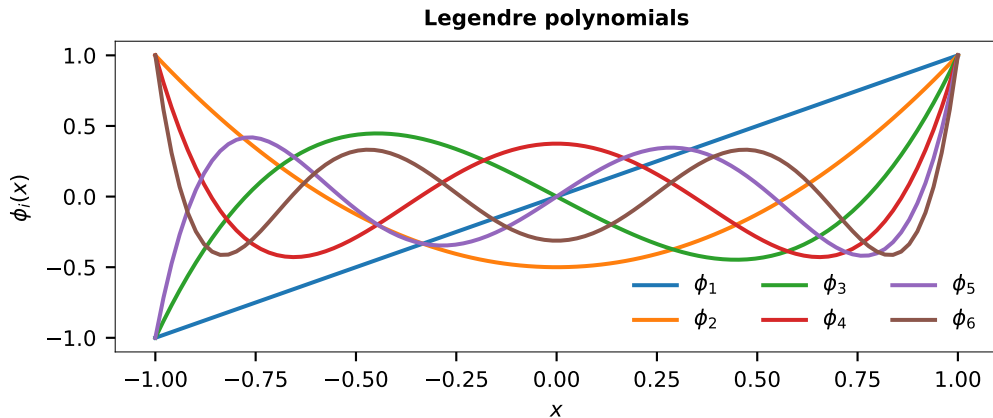
ReLU Tuning Curve Principal Components



\approx Legendre Basis



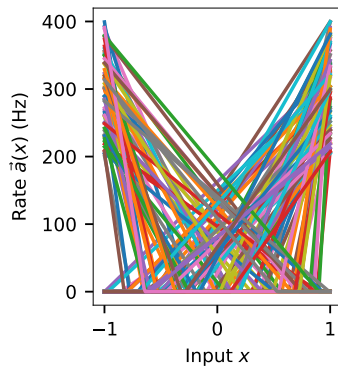
Reminder: Legendre Polynomials



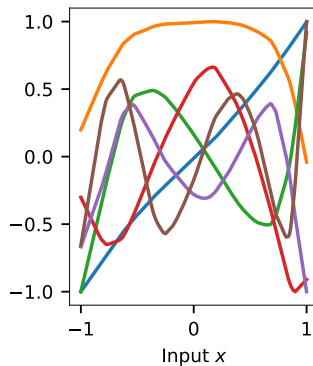
$$\varphi_i(x) = \frac{1}{2^i} \sum_{k=0}^i \binom{i}{k}^2 (x-1)^{i-k} (x+1)^k$$

Modifying the Basis – Same Maximum Rate (I)

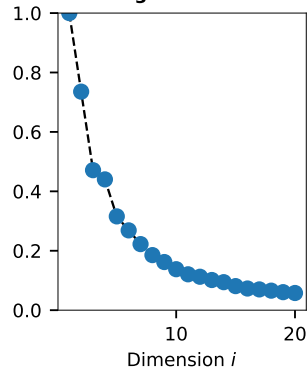
Tuning curves ($n = 128$)



Principal Components

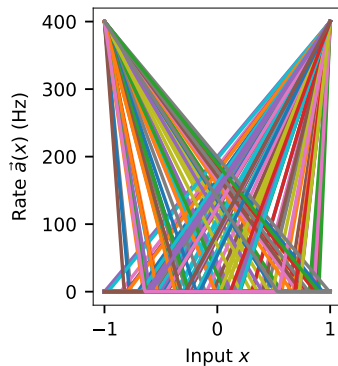


Singular values

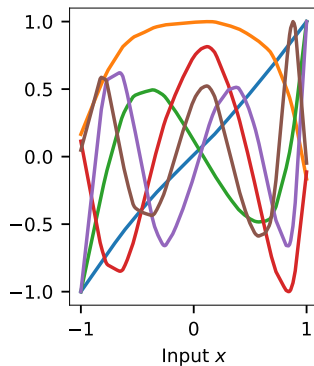


Modifying the Basis – Same Maximum Rate (I)

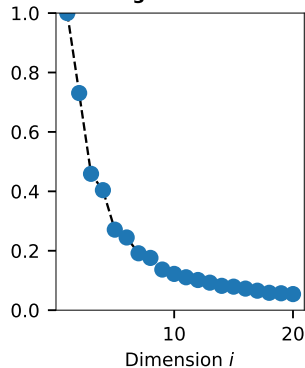
Tuning curves ($n = 128$)



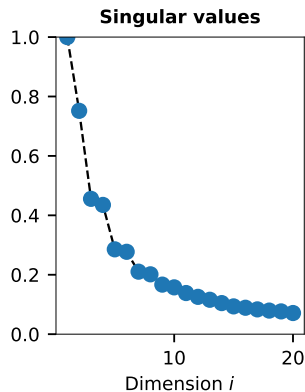
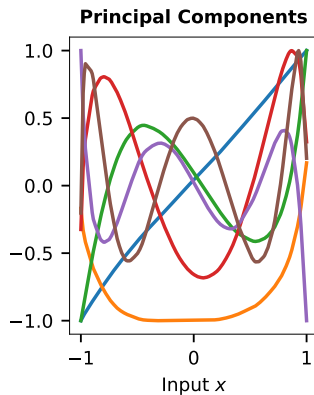
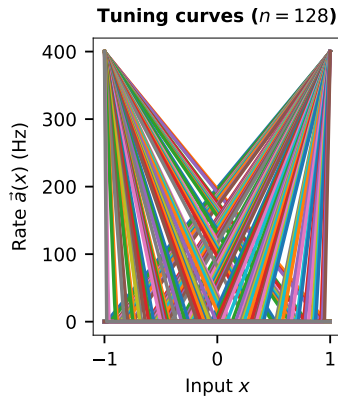
Principal Components



Singular values

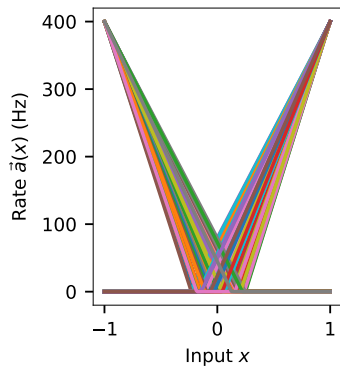


Modifying the Basis – Equidistant x -Intercepts (II)

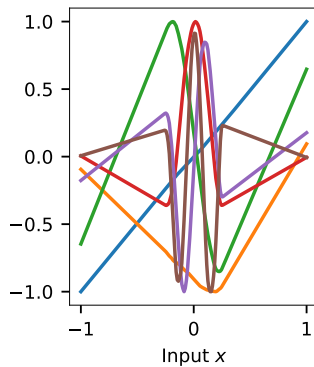


Modifying the Basis – Limited x-Intercepts (III)

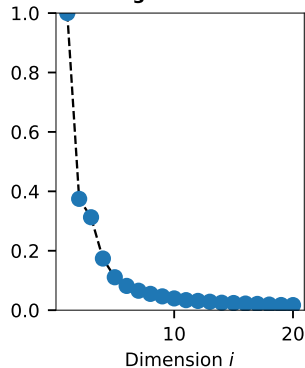
Tuning curves ($n = 128$)



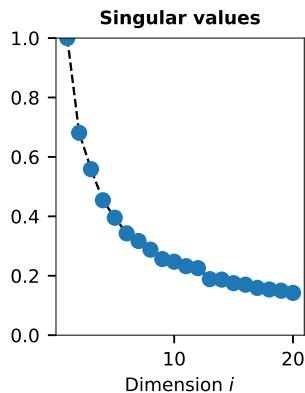
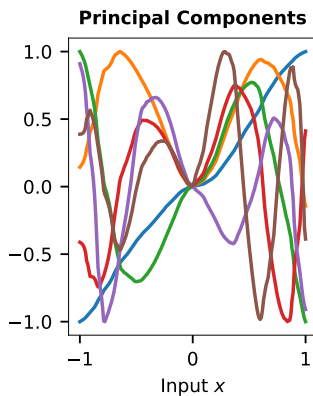
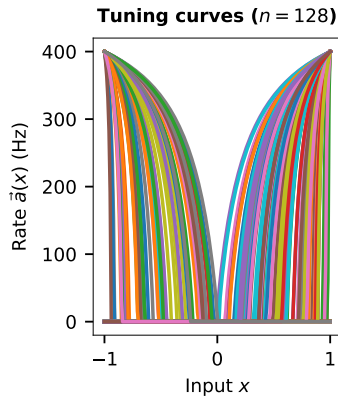
Principal Components



Singular values

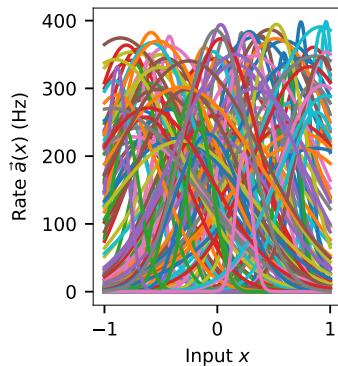


Modifying the Basis – Symmetric Tuning Curves (IV)

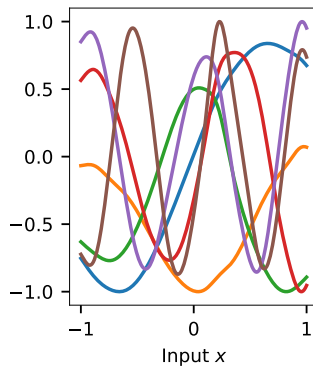


Gaussian Tuning Curve Principal Components

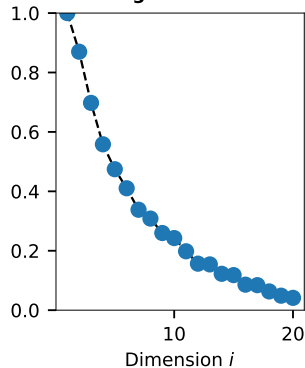
Tuning curves ($n = 128$)



Principal Components

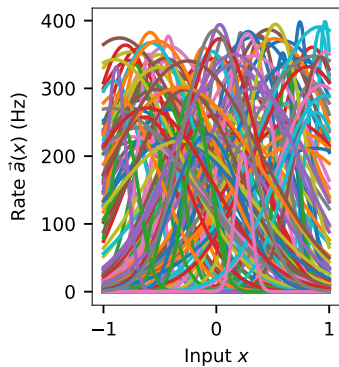


Singular values

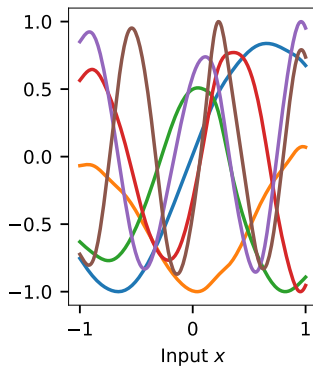


Gaussian Tuning Curve Principal Components

Tuning curves ($n = 128$)

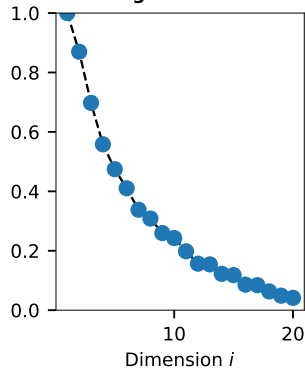


Principal Components

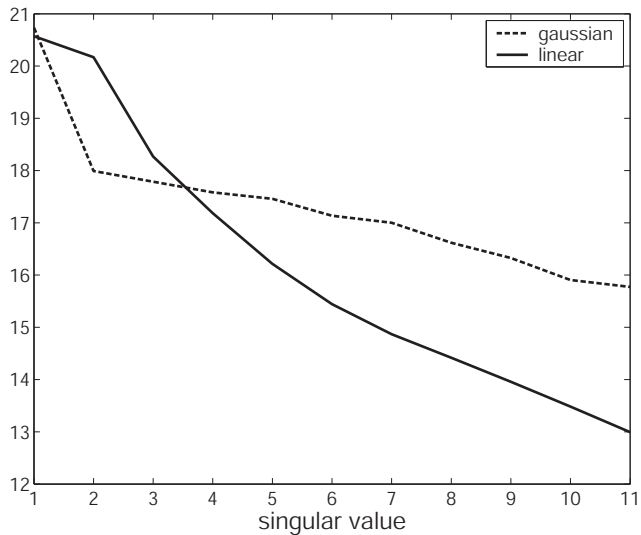


\approx Fourier Basis

Singular values

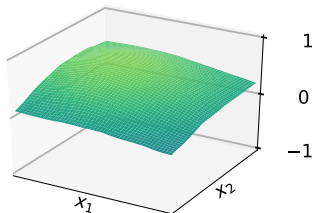


Gaussian vs LIF Singular Values

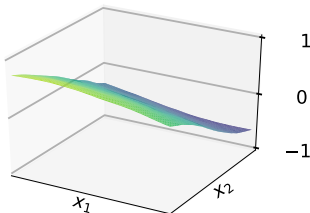


PCA of 2D Tuning Curves

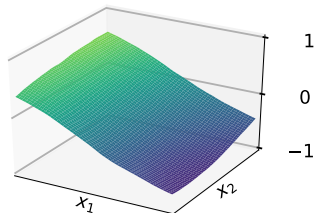
Principal component 1



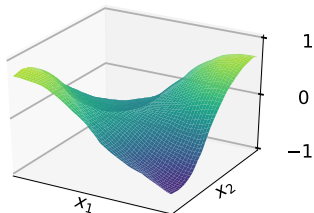
Principal component 2



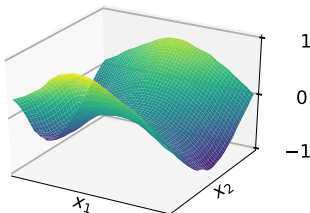
Principal component 3



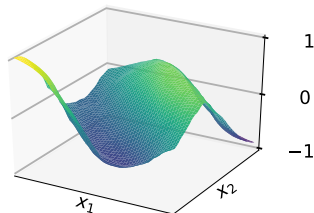
Principal component 4



Principal component 5

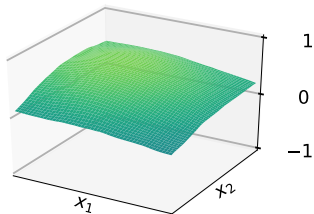


Principal component 6

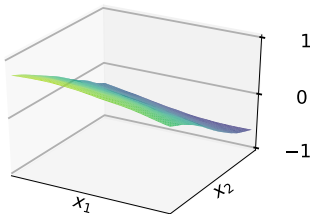


PCA of 2D Tuning Curves

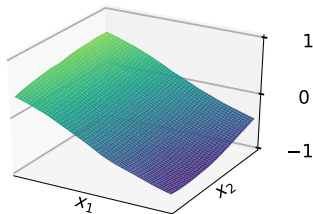
Principal component 1



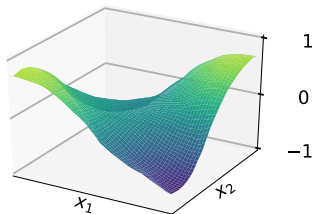
Principal component 2



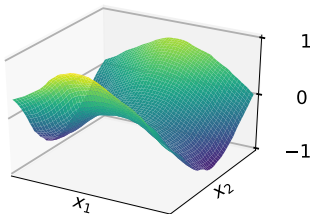
Principal component 3



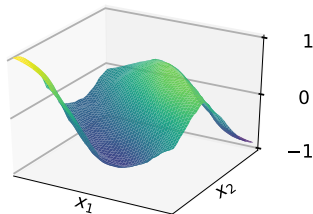
Principal component 4



Principal component 5

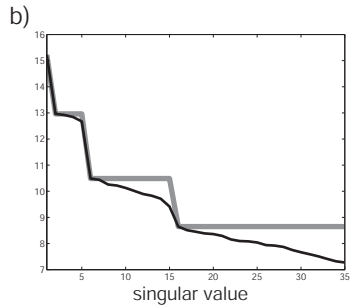
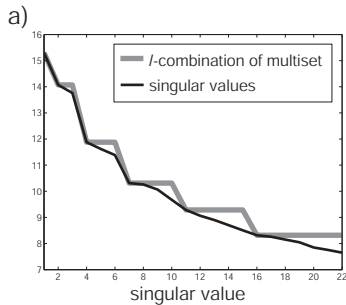


Principal component 6



Combination of 2D Polynomials

2D Singular Values



2D and 4D singular values compared to the prediction of the multiset (i.e., number of cross terms).

Conclusions

- ▶ Can use **PCA** to find the basis functions underlying neural representations
- ▶ **Singular values** inversely proportional to noise sensitivity
- ▶ **Basis function shape** depends on
 - ▶ max firing rate distribution (a bit)
 - ▶ x-intercept distribution
 - ▶ neuron response curve $G[J]$
- ▶ Finding optimal tuning curves for computing particular functions
⇒ Full network optimization (must use gradient descent)

Image sources

Title slide

Maurice Denis: Homage to Cézanne, 1900
From Wikimedia.