Ver. 1.0

Notes on Task 1.7

1 Introduction

Task 1.7 considers a cross-section of decision regions (in a D-dimensional vector space) with a 2D-plane, where the plane is specified with the first two principal components and a position vector.

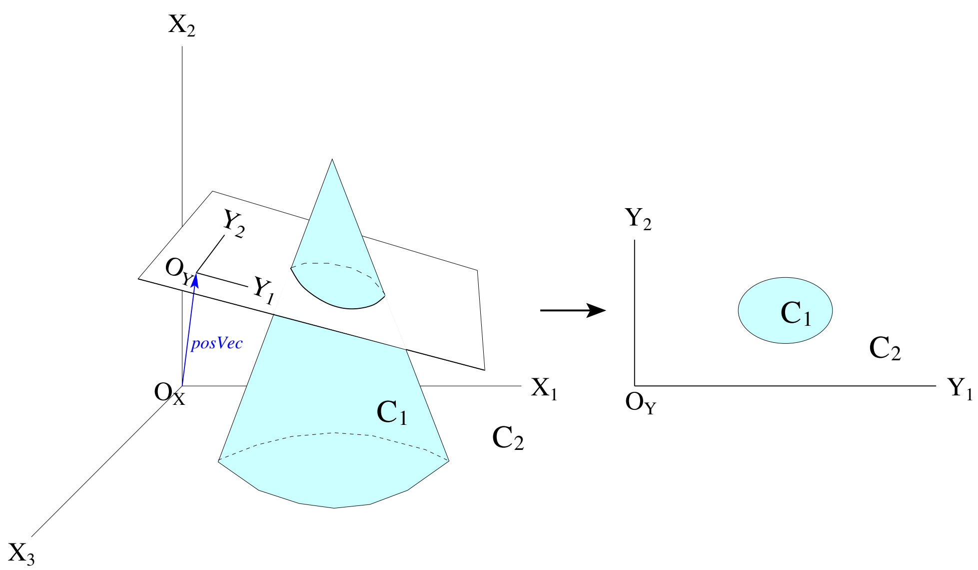

Figure 1: An example cross-section of a 3D-cone with a plane

Figure 1 shows an example of cross-section of a 3D cone and a plane, where we assume that the cone depicts the region of Class 1, and outside of the cone Class 2. The right figure shows the cross-section. Depending on the angle (orientation) and position of the plane, we obtain different cross-section images. In this task, we use the first two principal components to define the orientation of the plane, and `posVec` to define the position of the coordinate origin of the plane.

2 Displaying cluster regions on the plane

We can employ the same technique used in Lab 4, in which we display decisions boundaries/regions for k-NN (see section 2.1). The basic idea is that we consider a N-by-N grid (`Dmap`) that corresponds to the area to display on the plane. For each grid point (i,j), we run classification with k-NN to associate the point with a class number, so that `Dmap(i,j)` holds the class number for that point.

To display the result, we can use contour()/contourf() as is used in the in Lab 4, or other image visualisation functions such as image().

When classifying each grid point, we need to know the coordinates of the grid point at first. Let $(y_1, y_2)^T$ be the coordinates of a grid point on the 2D-plane. Since classification is done for feature vectors in the original coordinate system with D dimensions, we need to convert the coordinates and represent them as $\mathbf{x} = (x_1, \ldots, x_D)^T$ in the original coordinate system.

See Figure 2, where we consider a 3D coordinate system (X) in which the original data set resides and another coordinate system (Y) that are used for cross section. For simplicity, we assume that $Y_1 - Y_2$ plane is used for cross section. The figure shows the relationship among $\mathbf{y}$, $\mathbf{x}$, and $\mathbf{p}$ (position vector). If $\mathbf{p}$ is equal to the mean vector of data, it means that we move the origin of the coordinate system Y to the centre of the data.

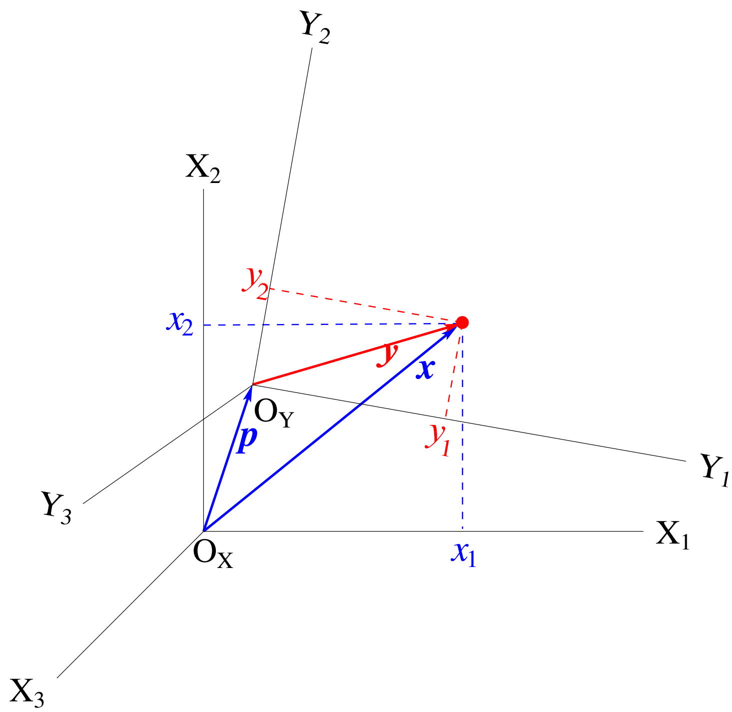

Figure 2: Relationship between the original coordinate system (X) and the one (Y) for cross-section

3 Details of coordinate conversion

In Figure 2, $(y_1, y_2)^T$ is a point on the $Y_1 - Y_2$ plane, but we represent is as $(y_1, y_2, y_3)^T$ by considering the 3rd axis Y_3, It is clear that $y_3 = 0$.

In the original 3D coordinate system (X), the following relationship holds:

$$\mathbf{y} = \mathbf{x} - \mathbf{p} \tag{1}$$

We use the first two principal components of the data set for the $Y_1 - Y_2$ plane in this task.

Let $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be the eigenvectors obtained with PCA, where $\mathbf{v}_i, i = 1, 2, 3$, is a unit vector of i-th principal component. The coordinates of $\mathbf{y}$ in the coordinate system Y are given as follows.

$$y_1 = \mathbf{v}_1^T(\mathbf{x} - \mathbf{p}) \tag{2}$$
$$y_2 = \mathbf{v}_2^T(\mathbf{x} - \mathbf{p}) \tag{3}$$
$$y_3 = \mathbf{v}_3^T(\mathbf{x} - \mathbf{p}) \tag{4}$$

or in a vector representation,

$$\mathbf{y}^{(Y)} = (\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)^T (\mathbf{x} - \mathbf{p}) \tag{5}$$

where $\mathbf{y}^{(Y)}$ holds the coordinates of the point in the coordinate system Y.

Given $\mathbf{y}^{(Y)}$, we can obtain $\mathbf{x}$ with a simple calculation.

We can extend the above example to a D-dimensional vector space, so that

$$\mathbf{y}^{(Y)} = V^T(\mathbf{x} - \mathbf{p}) \tag{6}$$

where $V = (\mathbf{v}_1, \ldots, \mathbf{v}_D)$.

Please note that all the vectors here are column vectors, whereas some vectors in the coursework data are row vectors, e.g. those in `Xtrn` and `Xtst`.