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A STOCHASTIC VERSION OF A STACKELBERG-NASH-COURNOT EQUILIBRIUM MODEL

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A Stochastic Version of a Stackelberg-Nash-Cournot Equilibrium Model.

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ABSTRACT

We consider a stochastic version of the Stackelberg-Nash-Cournot model proposed by Murphy et al. (1983). In the first stage, the leader chooses and announces his production level taking into account the reaction of the followers. The decision of the leader is taken when market demand is uncertain. In the second stage, the followers, knowing the leader's output, react to this level according to the Cournot assumption. At this stage, demand is known. We study the extension of the Murphy et al. model and give a numerical illustration of this model using the European gas market.

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1 Introduction

The *supply side* of an oligopolistic market is modeled as follows. There are $N + 1$ firms which supply a homogeneous product in a noncooperative manner. One of the firms, hereafter named the leader, has to decide now what its future output will be. We can imagine that this firm has not yet installed production capacities. We suppose that this firm chooses its future output level taking into account the reaction of the other producers. The others firms, hereafter named the followers, then react after knowing the leader's output level. We assume that the followers arrive at a Nash Equilibrium, i.e. each firm chooses its output level maximizing its profit under the assumption that the other firms will hold their production at the existing level.

The *demand side* is characterized by uncertainty for the leader, i.e. demand is uncertain when the leader has to decide its future output level. We assume that this uncertainty can be modeled by M demand scenarios. The followers are assumed to choose their output when demand is known.

This model will be applied to the European gas market which provides the motivation for this work. The good supplied on this oligopolistic market can be considered as a homogeneous product. The main suppliers are the CEI, Norway, the Netherlands, Algeria and the United Kingdom. The leader is assumed to be Norway which has, for technical reasons, a longer development time for its fields. The particular situation of Norway is the following. Norway has the opportunity to develop a new very important field. The level of development of this new field has to be decided now to became effective in 10 years only. The demand curve for natural gas in the year 2000 is not known. In contrast, other producers have enough spare capacities to adjust their production to demand when demand will be known. Norway is the "leader" in the sense that it must decide now for the year 2000 in the most intelligent way, i.e. by taking into account the reaction of the other producers in 2000.

This paper is organized as follows. In the second section, we shall present our definition of a Stochastic Stackelberg-Nash-Cournot Equilibrium and its main properties. In the third section, we examine how the equilibrium search algorithm presented by Murphy et al. (1983) has to be adapted to deal with our problem. In a section 4, the empirical assumptions for the modeling of the European gas market will be presented. Section 5 will be devoted to a discussion of the most significant results of our simulations for the year 2000. We conclude in section 6.

2 Mathematical developments

2.1 Mathematical description of the market

The *supply side* of the market consists of $N+1$ producers, each of them described by a total cost function $f_i(q_i)$, $i = 0, ...N$. Index 0 refers to the leader and positive indices to the followers. Each function $f_i(q_i)$, $i = 0, 1...N$ is assumed to be convex and twice differenciable (Assumption 1).

The *demand side* of the market is characterized by uncertainty at the time the leader takes his output decision. This uncertainty is modelled through a set of M possible demand scenarios, $p_j(Q)$, $j = 1, ...M$ and their associated probabilities, π_j, $j = 1, ...M$. The (inverse) demand function $p_j(Q)$ gives the price at which consumers will demand a quantity Q in scenario j. Each of these functions is assumed to be strictly decreasing, twice differenciable and concave (Assumption 2).

We also suppose (Assumption 3) that there is a quantity that suppliers do not want to exceed. Mathematically this can be written as:

$$\exists \quad qu \quad s.t. \quad f_i'(q) \geq p_j(q) \quad \forall q \geq qu \quad \forall j = 1, ...M \quad \forall i = 0, ...N \tag{1}$$

This can be interpreted as follows: for $q \geq qu$, the marginal cost exceeds the price for every producer and none of thems wants to produce above this level.

We can now present the definition of what we call a Stochastic Stackelberg-Nash-Cournot Equilibrium.

2.2 Definition of a stochastic SNC equilibrium

Definition 1 *The quantities $(x^*, (q^*_{ij})_{i=1,..N \quad j=1,..M})$ form a Stochastic Stackelberg-Nash-Cournot (SSNC) equilibrium if*

i) x^* solves the STACKELBERG PROBLEM:

$$SP: \quad \max_{x \geq 0} \left\{ x \sum_{j=1}^{j=M} \pi_j p_j \left[x + \sum_{i=1}^{i=N} q_{ij}(x) \right] \quad - \quad f_0(x) \right\} \tag{2}$$

ii) $q^*_{ij} \quad = \quad q_{ij}(x)$ solve the COURNOT PROBLEM:

$$CP_{ij} \quad : \quad \max_{q_{ij} \geq 0} \left\{ q_{ij} p_j \left(q_{ij} + x^* + \sum_{k \neq i} q^*_{kj} \right) - f_i(q_{ij}) \right\} \tag{3}$$

Some comments will help to clarify the above definition. *First*, the leader has to fix his output level, noted by x, when demand is uncertain. His objective is to maximize his expected profit. *Secondly*, the functions $q_{ij}(x)$, $i = 1, 2, ...N$, are the simultaneaous reaction functions of the followers to the leader's output of x units in scenario j. When the followers take their output decisions, the state of demand is known. And we must thus consider as many Cournot Problems as there are demand scenarios. Only the leader maximizes expected profit.

We also define the aggregate reaction function of the followers in scenario j, noted $Q_j(x)$, as the sum of the simultaneaous reaction functions of the followers:

$$Q_j(x) = \sum_{i=1}^{N} q_{ij}(x), \quad j = 1, \ldots M \tag{4}$$

2.3 Existence and uniqueness of joint reaction curves

By adapting the proof by Murphy et al. (1983), one can prove uniqueness:

Theorem 1 *For each fixed $x \geq 0$, in demand scenario j, there exists a unique set of quantities $[q_{1j}(x), ..., q_{Nj}(x)]$ satisfying the Cournot Conditions $CP_{ij}(x)$.*

Before studying the leader problem and the existence of a SSNC equilibrium, we recall an important property of the aggregate reaction function.

2.4 Properties of the aggregate reaction curve

Theorem 2 *For every scenario j, the aggregate reaction curve $Q_j(x) = \sum_{i=1}^{N} q_{ij}(x)$*
i) is a continuous function of x.
ii) verifies the following inequalities:

$$\begin{array}{rcccccccccc} -1 & < & Q_j^+(x) & < & 0 & if & Q_j(x) & > & 0 & & \\ & & Q_j^+(x) & = & 0 & if & Q_j(x) & = & 0 & and \quad x & > \quad 0 \end{array}$$

where $Q_j^+(x)$ denotes the right hand derivative of $Q_j(x)$ with respect to x.

Part two of this theorem shows that, if the leader increases his production by one unit, the followers will globally decrease their production, but not by more than one unit.

We have similar results for the joint reaction functions:

Corollary 1 *For $x > 0$, the joint reaction curves verify the following inequalities:*

$$q_{ij}^{+}(x) \leq 0, \qquad \forall i = 1, ...N, \ \forall j = 1, ...M$$

This tells us that when the leader increases his production, each of the followers decreases his production.

2.5 Existence of a stochastic SNC equilibrium

We can now state the fundamental existence result. We also give a sufficient condition for uniqueness of a SSNC equilibrium.

Theorem 3 *i) Under the above 3 assumptions, there exists a SSNC equilibrium.*
ii) Moreover, if the aggregate reaction curves $Q_j(x)$, $j = 1, ...M$, are convex, then this equilibrium is unique.

Proof. i) The proof is based on the above results. The leader faces a maximization problem SP involving a continuous function of x (the function $p_j(x + Q_j(x))$ is a continuous function of x since $Q_j(x)$ is continuous) over a compact non empty set $[0, qu]$. And thus an optimal solution x^* exists for this problem.

Futhermore, for such x^* fixed, Theorem 1 tells us that there exists a set of quantities $[q_{1j}(x^*), ..., q_{Nj}(x^*)]$ which solve the Cournot problems $CP_{ij}(x^*)$, $i = 1, ...N$. Thus $[x^*, (q_{ij}(x^*))_{i=1,...N \quad j=1,...M}]$ is a SSNC solution.

ii) To complete the proof, it must be shown that, since $Q_j(x)$, $j = 1, \ldots M$, are convex, the objective function of the Stackeleberg Problem SP is strictly concave on $(0,\infty)$. This can be done by an immediate extension of a similar result given in Murphy et al. (1983).

3 An algorithm to solve the leader problem

In this section, we shall review the basic points which lead to the algorithm presented by Murphy et al. (1983). We shall also see how to implement it in an efficient way.

3.1 Idea of the method

The leader problem defined in (2) can be non convex. Denote by $g(x)$ the leader's objective function. We only know that $g(x)$ is a continuous function of x. Futhermore, the proof of theorem 3 tells us that if $Q_j(x)$ is convex, for $j = 1, ...M$, then $g(x)$ is concave.

The basic idea of the method is to discretise the range of x values. At the grid points, the aggregate reaction curves $Q_j(x)$ are then computed in each scenario. A linear interpolation is made between two consecutive grid points. Then the leader problem is solved on each of these intervals with $Q_j(x)$ replaced in (2) by its linear approximation. Recall that only the leader's problem is stochastic. And thus we solve the followers equilibrium problem scenario by scenario since we consider the mathematical expectation for the leader problem.

Let us explain the solution method in more detail.

3.2 Approximation of the leader's objective function

Denote by $(x_k)_{k=1,...T}$ the grid points: $0 \leq x_1 < x_2 < x_3 < ... < x_T \leq qu$ and by $Q_{kj}(x)$ the linear approximation of $Q_j(x)$ on $[x_k, x_{k+1}]$. This approximation is defined as follows:

$$Q_{kj}(x) \quad = \quad Q_j(x_k) \quad + \quad \gamma_{kj}(x - x_k) \quad for \quad x_k \leq x \leq x_{k+1} \tag{5}$$

$$where \quad \gamma_{kj} \quad = \quad \frac{Q_j(x_{k+1}) - Q_j(x_k)}{x_{k+1} - x_k} \tag{6}$$

In the interval $[x_k, x_{k+1}]$, the aggregate reaction curve $Q_j(x)$ is then replaced by its linear approximation $Q_{kj}(x)$. On this interval, the leader problem SP becomes :

$$SP_k : \quad \max_{x_k \leq x \leq x_{k+1}} \left\{ x \sum_{j=1}^{j=M} \pi_j p_j \left[x + Q_{kj}(x) \right] - f_0(x) \right\} \tag{7}$$

The solution of this convex program gives x_k^*, an approximation of the optimal leader output on $[x_k, x_{k+1}]$. This point is introduced in the set of grid points and the procedure can be repeated. When terminating with the best of the grid points $x_1, ...x_M, M > T$, (a grid point is now either an original point or an interior maximum) as an estimated optimal leader output, an error is made in comparison with the true optimal leader output.

This error can be estimated following the same lines as in Murphy et al. (1983).

3.3 Computation of $Q_j(x)$, x fixed.

One problem remains in order to completely solve the leader problem: the computation of the aggregate reaction curves $Q_j(x)$ for x fixed. This can be done by defining of a new Equilibrium Problem:

$$
\begin{array}{lll}
EP_j(x,Q) & : & \\
\max & p_j(Q+x)\sum_{i=1}^{i=N} q_{ij} + \frac{1}{2}p_j'(Q+x)\sum_{i=1}^{i=N} q_{ij}^2 - \sum_{i=1}^{i=N} f_i(q_{ij}) & \\
\text{subject to} & \sum_{i=1}^{i=N} q_{ij} = Q & (8)\\
 & q_{ij} \geq 0 \qquad i=1,...N &
\end{array}
$$

The two fundamental results which constitute the basis of the method for computing $Q_j(x)$ are summarized in the following theorem:

Theorem 4 *For each j fixed,*
i) An optimal solution $\left[q_{1j}^*, ...q_{Nj}^*\right]$ for the problem $EP_j(x,Q^*)$ satisfies the Cournot conditions CP_{ij}, $i=1,...N$ if and only if the Lagrange multiplier, $\lambda_j^*(x,Q)$, associated to the first constraint of problem (8) is equal to 0.

ii) This multiplier is a continuous, strictly decreasing function of Q. Moreover, there exist quantities Q_{lj} and Q_{uj} such that:

$$\lambda_j^*(x,Q_{lj}) \geq 0 \qquad \textit{and} \qquad \lambda_j^*(x,Q_{uj}) \leq 0$$

Proof: See Murphy et al. (1983).

This theorem tells us that finding $Q_j(x)$ is equivalent to finding the unique solution of

$$\lambda_j(x,Q) = 0.$$

Futhermore λ_j is a strictly decreasing function of Q. The method, illustrated in figure 1, is thus simple: starting from upper and lower bounds Q_l and Q_u, apply a line search on $\lambda_j(x,.)$ in order to find its root.

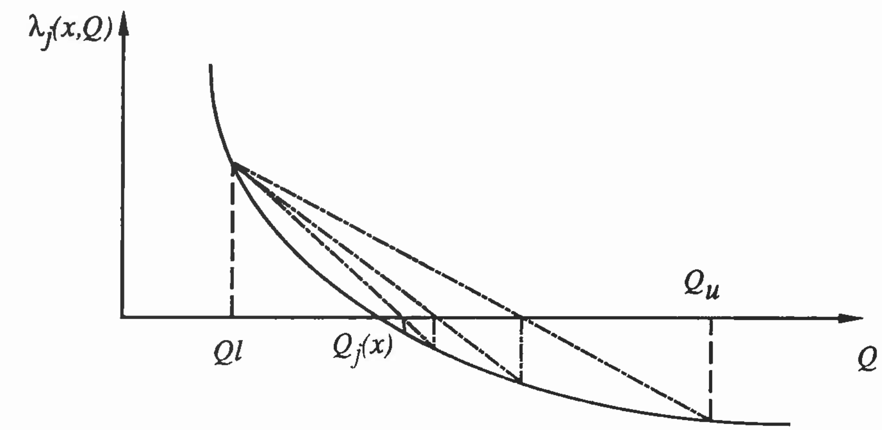

Figure 1: Method for finding $Q_j(x)$

3.4 Complete SSNC algorithm

The *data* to be provided are the following:

1. $f_i(q)$, $i=0,1,...N$, convex, twice differentiable;
2. $p_j(q)$, $j=1,2,...M$, strictly decreasing,
   twice differentiable, concave;
3. qu s.t. $f_i'(q) > p_j(q) \quad \forall q \geq qu \quad \forall j=1,...M \quad \forall i=1,...N$;
4. T, number of grid points for x;
5. ϵ, an error tolerance on the leader's optimal output.

The *algorithm* then consists of the four following steps:

i) Compute $Q_j(x_k)$, $k=1,...T$, $j=1,...M$.

ii) For $k=1,...T-1$:

-solve SP_k over $[x_k, x_{k+1}]$: Solution x_k^*.

-compute $g(x_k^*)$: Solution g_k^*

iii) Find the maximum of g_k^*

iv) Evaluate a bound on the error made on the leader's output.

3.5 Implementation

In order to implement efficiently the SSNC Algorithm, we must adequatly choose the method to solve the Equilibrium Problem $EP_j(x, Q)$ which is at the heart of the algorithm. With this implementation we shall be able to compute $Q_j(x)$ and solve the Stackelberg problem by discretisation.

Problem (8) is an optimization problem with a non linear objective and one linear constraint. The *reduced gradient method* which is very simple to implement in case there is only one constraint is very efficient to solve this problem.

4 Application to the European gas market.

4.1 Demand uncertainty

The modeling of demand uncertainty is similar to the one presented by Boucher, Hefting and Smeers (1985). We recall here the basic points. Demand for natural gas is a function of both the total demand of energy and of the relative price of gas with respect to oil. Assuming that all energy prices roughly move in line with oil, the overall demand for energy can be expressed as a function of the oil price only. The second effect (the relative price of natural gas with respect to oil) can then be considered as having a minor impact on the overall demand for energy: in fact the demand for natural gas is only a fraction of the total demand for energy and the modification of the relative price of gas with respect to oil has a minor influence on energy prices and hence on the demand for energy.

Each state of the world is characterized by an oil price level $OP(j)$ and by its associated probability $PROB(j)$. The oil price levels considered here range between a possible 50 % fall with respect to a given 2000 reference value and a possible 50 % increase with respect to this reference. Table 1 gives the oil prices and associated probabilities considered. Prices are given in indices; the value of 100 corresponds to a price of 34.24 USD per barrel in 2000.

Defining in each scenario a reference gas price that moves along with the corresponding oil price, we have

$$\frac{P(j)}{PR} = \frac{OP(j)}{OPR} \qquad J = 1, \ldots 9 \tag{9}$$

Scenario	$OP(j)$	$PROB(j)$
1	50	0.11
2	63	0.11
3	75	0.11
4	88	0.11
5	100	0.12
6	113	0.11
7	125	0.11
8	138	0.11
9	150	0.11

Table 1: Oil prices and associated probabilities

where $P(j)$ = reference gas price in state j;
$OP(j)$ = oil price in state j;
PR = reference gas price in the median scenario;
OPR = oil price in the median scenario.

The total demand for energy being a function of the oil price only, the reference demand $Q(j)$ in state j is given by

$$Q(j) \quad = \quad QR\left[\frac{OP(j)}{OPR}\right]^{-\eta} \tag{10}$$

where η = energy demand elasticity;
QR = gas demand in the median scenario.

In each state of the world, formulas (9) and (10) allow us to compute a reference price $P(j)$ and a reference demand for natural gas $Q(j)$. Starting from the reference demand $Q(j)$, we can now compute the demand of natural gas in state j when the price P of natural gas differs from P(j). We assume the demand for natural gas to be represented as is figure 2.

In each scenario, we assume that the demand curve is piecewise linear (like in Figure 2). The slopes of the two pieces, noted η_A and η_B, are identical in all scenarios (of course, $\eta_A \neq \eta_B$). Their values and the gas reference price in year 2000 are taken from Purvin and Gertz (1984).

4.2 The supply side

The European gas market is supplied by producers located inside and outside the European community. Domestic E.C. gas is produced mainly in the *Netherlands* and in the *United Kingdom*. Old fields also exist in France, Italy

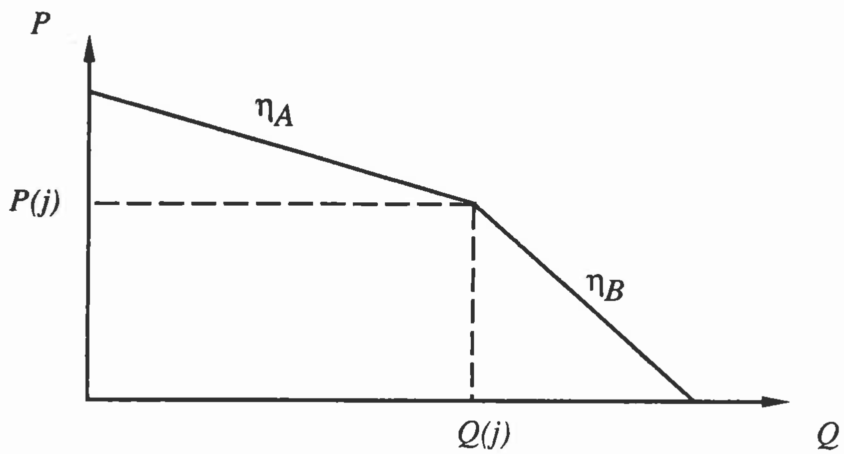

Figure 2: Demand curve in each scenario

and in Germany. The production of the latter countries is well determined and can be taken as given. The non Community gas comes from Algeria, Norway and the CEI. *Algeria*, with its 4 % share of the community market in 1982 has underutilized production capacities. *Norway* which, in 1982, provided 13 % of the gas supplies of the Community is affected by high production costs. The *CEI*, which supplied 11 % of the Community gas market in 1982 has large reserves.

After this brief description of the players present on the market, we turn to the form adopted for the marginal cost, Cm, (see Mathiesen et al. (1986)):

$$Cm(q) = a + \frac{b}{c - q} \tag{11}$$

As can be seen from figure 3, these curves present a discontinuity in their first derivative. These discontinuities can be explained by the decision of investment in a new field or in new transportation capacities. For exemple, the discontinuity for Norway corresponds to the decision to develop the new Troll field and the discontinuity for the CEI corresponds to the decision to build new pipe-line to Western countries.

Needless to say, contracts still in force in the year 2000 must also be taken into account. We present the latter through fixed volumes. Table 2 gives these quantities.

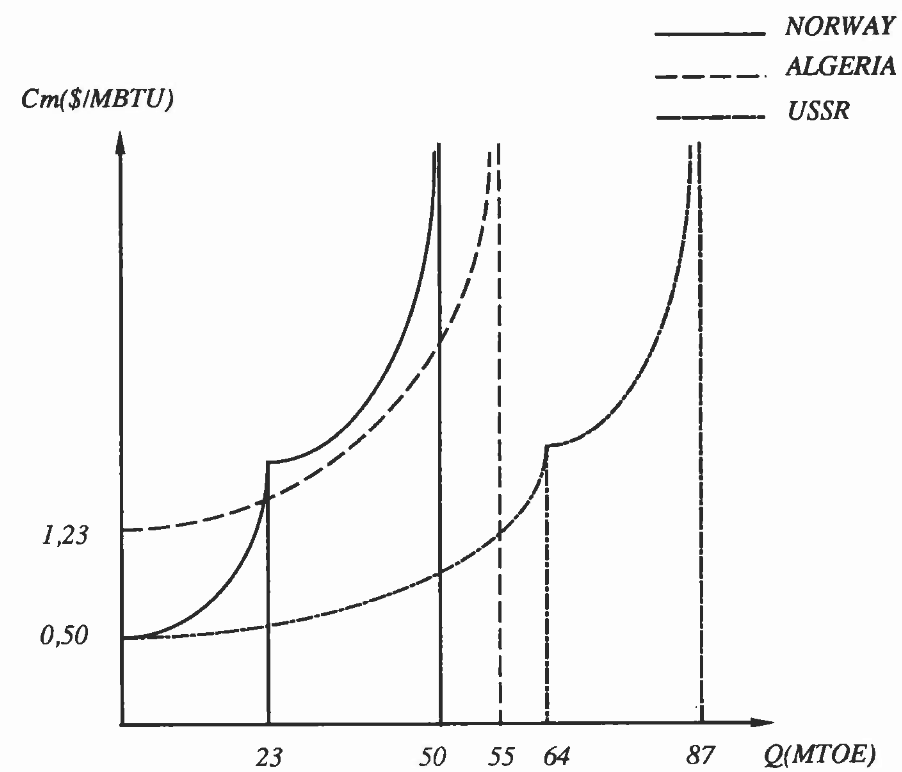

Figure 3: Marginal cost curves in the year 2000

Producer	Contracted quantity
Norway	6 (MMTOE)
CEI	41 (MMTOE)
Algeria	28 (MMTOE)
Netherlands	55 (MMTOE)
United Kingdom	37 (MMTOE)
France, Italy,FRG	34 (MMTOE)

Table 2: Contracted quantities for the year 2000

5 Numerical results

Three basic results are presented: the total production and market price in each scenario; the market share of the leader and of the followers; and the percentage of capacity utilised by each producer.

Table 3 presents for each scenario the two following ratios :

$Q/Q(j)$ which represents the ratio between the model production and the reference quantity for scenario j;

$P/P(j)$ which represents the ratio between the model price and the reference price for scenario j;

We recall here that scenario number 1 corresponds to the lowest reference price and highest reference demand. In contrast, scenario number 9 corresponds to the highest reference price and lowest reference demand.

SCENARIO	$Q/Q(j)$	$P/P(j)$
1	0.838	1.068
2	0.994	1.005
3	1.038	0.950
4	1.120	0.836
5	1.195	0.734
6	1.261	0.642
7	1.309	0.575
8	1.353	0.514
9	1.390	0.464

Table 3: Quantity and Price relative to reference.

Except for the two first scenarios, the **quantity supplied** is larger than the reference demand. Consequently the **price** falls and the decrease can reach as much as 50 % (scenario 9).

Table 4 presents the **market shares** of the producers in the two extreme scenarios and in the median scenario. As a reference point, the value of the market shares in the year 1985 is also given. As can be seen, the Norwegian share increases by about 50 %. The optimal strategy for Norway is to develop the Troll field to reach high production level.

Table 5 shows the **percentage of capacity used** for the differents producers and for the three same scenarios as those of table 4. The most striking feature is that half of the Algerian capacity remains idle.

SCENARIO	NORWAY	CEI	ALGER.	U.K.	N.L.	OTHERS
1	0.16	0.30	0.13	0.19	0.13	0.10
5	0.17	0.22	0.14	0.20	0.14	0.13
9	0.19	0.19	0.11	0.22	0.15	0.14
1985	0.11	0.14	0.08	0.17	0.33	0.17

Table 4: Market shares of the producers.

SCENARIO	NORWAY	CEI	ALGERIA
1	0.86	0.89	0.69
5	0.86	0.70	0.68
9	0.86	0.54	0.51

Table 5: Proportion of capacities used in 2000

6 Possible extensions

The formulation of the model and of the definition of a SSNC Equilibrium are easy to extend to the **multimarket** case. The possible extension of the mathematical properties of the model remains however an open question. In particular, the question of the sign of the derivative of the followers reaction curve on one market if the leader increases his production on an other market appears difficult to tackle.

Gas producers do not maximize their profit on an annual basis but over some time horizon. The model should thus be extended to a **multiperiod** environment. The same remarks as for the multimarket model would apply to these extensions.

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