Air cargo load and route planning in pickup and delivery operations

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1. The mathematical modeling

Given the assumptions, scenarios and parameters described in the previous section, we are ready to present the mathematical modeling of ACLP+RPDP, which is one of the contributions of our work.

The flight plan will be carried out by a **single** aircraft, which has predefined values of W_{max}, $limit^{CG}_{long}$, $limit^{CG}_{lat}$, d^{CG}_{pallet}, c_d and c_g. This aircraft has a set $M = \{p_1, p_2, \ldots, p_m\}$ of m pallets, where each pallet p_i, $1 \leq i \leq m$, has weight capacity wp_i, volume capacity vp_i, and distance dp_i to the CG of aircraft. Table ?? shows the data of the aircraft considered in this work.

Let $L = \{l_0, l_1, \ldots, l_K\}$ be the set of $K+1$ nodes (or destinations), where l_0 is the origin and the end of a tour. Let $d(l_i, l_j)$ be the distance from l_i to l_j, where $0 \leq i, j \leq K$. By definition, $d(l_i, l_i) = 0$. Let $C = \{c_{ij}\}$ be the cost matrix of the aircraft, where $c_{ij} = c_d * d(l_i, l_j), 0 \leq i, j \leq K$.

Let $S_K = \{s : \{1, \ldots, K\} \rightarrow \{1, \ldots, K\}\}$ be the set of the $K!$ permutations of the nodes, which correspond to all possible tours (or itineraries) that have l_0 as origin and end, passing through the others K nodes.

The objective of ACLP+RPDP is to find the permutation $\pi \in S_K$, with the corresponding allocation of items on the pallets at each node, that maximizes the function $\tilde{s}_\pi / \tilde{c}_\pi$ 1, where $\tilde{s}_\pi$ is the total score of transported items, and $\tilde{c}_\pi$ is the total cost of fuel consumed. In this way, the tour plan will be $l_0, l_{\pi(1)}, \ldots, l_{\pi(K)}, l_0$. Later, we will describe the calculations of $\tilde{s}_\pi$ and $\tilde{c}_\pi$.

$$\max_{\pi \in S_K} \tilde{s}_\pi / \tilde{c}_\pi \tag{1}$$

Let us now describe the input and output data at each node l_k of the flight plan, where $0 \leq k \leq K$. Let L_k 2 be the set of remaining nodes when the aircraft is in l_k, $0 \leq k \leq K$.

$$L_0 = L; L_k = L_{k-1} - \{l_{\pi(k)}\};\ k \in \{1, 2, \ldots, K\} \tag{2}$$

Let $N_k = \{t^k_1, t^k_2, \ldots, t^k_{n_k}\}$ be the input set of n_k items to be loaded in node l_k, $0 \leq k \leq K$. Each item t^k_j, $1 \leq j \leq n_k$, has score st^k_j, weight wt^k_j, volume vt^k_j, and destination $lt^k_j \in L_k$. Let $N = \bigcup_{0 \leq k \leq K} N_k$ be the input set of items of all nodes along a tour.

Let X^k_{ij} be output binary variables, where $0 \leq k \leq K$, $1 \leq i \leq m$ and $1 \leq j \leq n_k$. $X^k_{ij} = 1$ if t^k_j is assigned to p_i in node l_k, and 0 otherwise 3.

$$X^k_{ij} = 0;\ i \in \{1, 2, \ldots, m\};\ j \in \{1, 2, \ldots, n_k\};\ k \in \{1, 2, \ldots, K\};\ lt^k_j \notin L_k \tag{3}$$

In this way, we can consider $\tilde{s}_\pi$ as the sum of the scores of the items loaded on the aircraft throughout the flight plan 4.

$$\tilde{s}_\pi = \sum_{k=0}^{K} \sum_{i=1}^{m} \sum_{j=1}^{n_k} X^k_{ij} \times st^k_j \tag{4}$$

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Let $Q_k = \{a_1^k, a_2^k, \ldots, a_{m_k}^k\}$ be the input set of $m_k \leq m$ consolidated items at node l_k, $0 \leq k \leq K$. a_i^k, $1 \leq i \leq m_k$, is a package of items allocated in some of the previous nodes, with total weight wa_i^k, total volume va_i^k, and destination $la_i^k \in L_k$. By definition, $m_0 = 0$ 5. Consolidated items that were destined to node l_k are unloaded, that is, they are not considered in Q_k.

$$m_0 = 0 \tag{5}$$

Let Y_{iq}^k be output binary variables, where $0 \leq k \leq K$, $1 \leq i \leq m$ and $1 \leq q \leq m_k$. $Y_{iq}^k = 1$ if a_q^k is assigned to p_i in node l_k, and 0 otherwise 6.

$$Y_{iq}^k = 0;\ i \in \{1, 2, \ldots, m\};\ q \in \{1, 2, \ldots, m_k\};\ k \in \{1, 2, \ldots, K\};\ la_i^k \notin L_k \tag{6}$$

Therefore, allocations of items and consolidated items to pallets in node l_k can be seen as a bipartite graph $G_k(V_k, E_k)$, where $V_k = M \cup N_k \cup Q_k$, $E_k = E_k^N \cup E_k^Q$, $(p_i, t_j^k) \in E_k^N$ if $X_{ij}^k = 1$, and $(p_i, a_q^k) \in E_k^Q$ if $Y_{iq}^k = 1$.

Through the output binary variables X_{ij}^k and Y_{iq}^k, we can calculate the lateral 78 and the longitudinal 9 torques, and the CG deviation 10, caused by shipped items at node l_k.

$$LatIt_k = \sum_{i=1}^{m} \sum_{j=1}^{n_k} (X_{ij}^k \times wt_j^k \times (i\%2) - X_{ij}^k \times wt_j^k \times (i+1)\%2);\ k \in \{0, 1, \ldots, K\} \tag{7}$$

$$LatCons_k = \sum_{i=1}^{m} \sum_{q=1}^{m_k} (Y_{iq}^k \times wa_q^k \times (i\%2) - Y_{iq}^k \times wa_q^k \times (i+1)\%2);\ k \in \{0, 1, \ldots, K\} \tag{8}$$

$$\tau_k = \sum_{i=1}^{m} [dp_i \times (\sum_{j=1}^{n_k} X_{ij}^k \times wt_j^k + \sum_{q=1}^{m} Y_{iq}^k \times wa_q^k)];\ k \in \{0, 1, \ldots, K\} \tag{9}$$

$$\epsilon_k = \frac{\tau_k}{W_{max} \times limit_{long}^{CG}};\ k \in \{0, 1, \ldots, K\} \tag{10}$$

With the CG deviations, the distances covered and the cost matrix of the aircraft, we calculate the total fuel consumption $\tilde{c}_\pi$ 11 along the flight plan.

$$\tilde{c}_\pi = c_{0,\pi(1)} \times (1 + c_g \times |\epsilon_0|) + \sum_{k=1}^{K-1} [c_{\pi(k),\pi(k+1)} \times (1 + c_g \times |\epsilon_{\pi(k)}|)] + c_{\pi(K),0} \times (1 + c_g \times |\epsilon_{\pi(K)}|) \tag{11}$$

Finally, we can consider the constraints on each node l_k:

- The latitudinal 12 and the longitudinal 13 torques must be within the limits of the aircraft;
- The items allocated to each pallet cannot exceed its weight 14 and volume 15 limits;
- At most, each item is associated with a single pallet 16;
- Consolidated items that have not yet reached their destination must remain on board 17;
- Items allocated on the same pallet must have the same destinations. In this case, we need two constraints: 18 and 19, where B is a big value. If $X_{ij}^k = X_{iq}^k = 1$, both constraints require that $lt_j^k = lt_q^k$; otherwise these constraints have no effect.
- If there is a consolidated item on the pallet, it must also have the same destination as the other items. Similarly, we use two constraints: 20 and 21.

$$s.t.: d_{pallet}^{CG} \times |LatIt_k + LatCons_k| \leq W_{max} \times limit_{lat}^{CG};\ k \in \{0, 1, \ldots, K\} \tag{12}$$

$$s.t.: |\tau_k| \leq W_{max} \times limit_{long}^{CG};\ k \in \{0, 1, \ldots, K\} \tag{13}$$

$$s.t.:\sum_{j=1}^{n_k} X_{ij}^k \times wt_j^k + \sum_{q=1}^{m_k} Y_{iq}^k \times wa_q^k \leq wp_i;\ i \in \{1,2,\ldots,m_k\};\ k \in \{0,1,\ldots,K\} \tag{14}$$

$$s.t.:\sum_{j=1}^{n_k} X_{ij}^k \times vt_j^k + \sum_{q=1}^{m_k} Y_{iq}^k \times va_q^k \leq\ vp_i;\ i \in \{1,2,\ldots,m_k\};\ k \in \{0,1,\ldots,K\} \tag{15}$$

$$s.t.:\sum_{i=1}^{m} X_{ij}^k \leq 1;\ j \in \{1,2,\ldots,n_k\};\ k \in \{0,1,\ldots,K\} \tag{16}$$

$$s.t.: Y_{iq}^k = 1;\ q \in \{1,2,\ldots,m_k\};\ k \in \{0,1,\ldots,K\};\ la_q^k \in L_k \tag{17}$$

$$s.t.: lt_j^k - lt_q^k \geq -B \times (1 - X_{ij}^k \times X_{iq}^k);\ j,q \in \{1,2,\ldots,n_k\};\ k \in \{0,1,\ldots,K\} \tag{18}$$

$$s.t.: lt_j^k - lt_q^k \leq B \times (1 - X_{ij}^k \times X_{iq}^k);\ j,q \in \{1,2,\ldots,n_k\};\ k \in \{0,1,\ldots,K\} \tag{19}$$

$$s.t.: lt_j^k - la_q^k \geq -B \times (1 - X_{ij}^k \times Y_{iq}^k);\ j \in \{1,2,\ldots,n_k\};\ q \in \{1,2,\ldots,m_k\};\ k \in \{0,1,\ldots,K\} \tag{20}$$

$$s.t.: lt_j^k - la_q^k \leq B \times (1 - X_{ij}^k \times Y_{iq}^k);\ j \in \{1,2,\ldots,n_k\};\ q \in \{1,2,\ldots,m_k\};\ k \in \{0,1,\ldots,K\} \tag{21}$$

Thus, ACLP+RPDP is defined through the equations 1 to 11, subject to constraints 12 to 21.