Credible Persuasion

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Motivation: Bayesian Persuasion

Persuasion: One party uses information to influence another party's decision.

Motivation: Bayesian Persuasion

Sender

Receiver

Motivation: Bayesian Persuasion

$$\begin{tikzcd}
\text{Sender} \arrow[d, "\text{commits to}"] \\
\text{Information Structure} \arrow[d, "\text{messages}"] \\
\text{Receiver}
\end{tikzcd}$$

Motivation: Bayesian Persuasion

$$\begin{tikzcd}
\text{Sender} \arrow[d, "\text{commits to}"] & & \text{School} \\
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\text{Receiver} & & \text{Employer}
\end{tikzcd}$$

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In practice, information structures are not observed.

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- Grading policy: grade distribution
- Rating policy: distribution of ratings

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This paper: introduce a notion of credibility for persuasion.

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Sender's information structure is **credible** if no other information structure

- gives the Sender a strictly higher payoff, and
- generates the same distribution of messages.

Example: Used Car Rating

$\times 3$

$\times 7$

Example: Used Car Rating

$\times 3$

$\times 7$

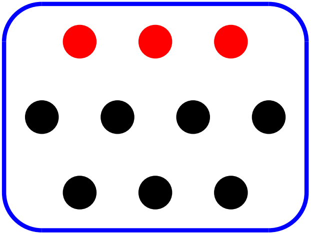
	Buy	Not Buy
Good	2	1
Bad	2	0

Seller Payoff

	Buy	Not Buy
Good	1	0
Bad	-1	0

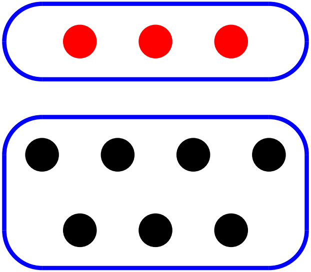
Buyer Payoff

Uninformative Rating Policy



$\longrightarrow$ Not Buy

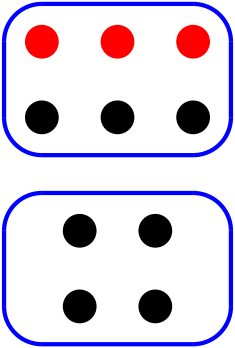
Fully-Revealing Rating Policy



“Good” $\longrightarrow$ Buy

“Bad” $\longrightarrow$ Not Buy

Optimal Rating Policy (Kamenica-Gentzkow)



“Pass” $\longrightarrow$ Buy

“Fail” $\longrightarrow$ Not Buy

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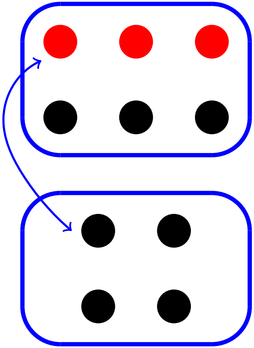

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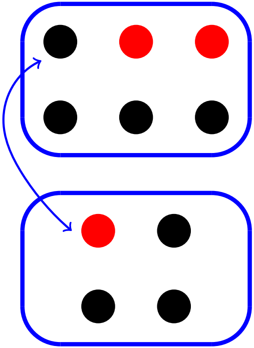

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Undetectable + Profitable $\Rightarrow$ Not Credible

Can Any Car Be Sold in a Credible Rating Policy?

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$\Rightarrow$ No cars can be sold in any credible rating policy.

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 $\times 3$

 $\times 7$

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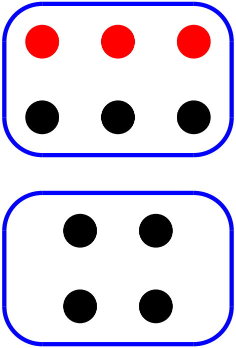
Employer Payoff

Optimal Grading under Full Commitment



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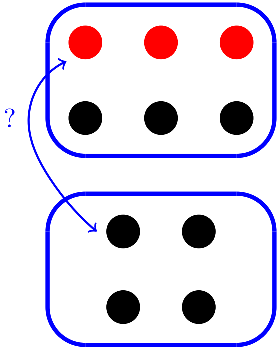

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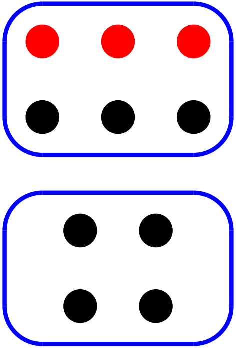

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The optimal full-commitment information structure is credible.

Examples: Takeaway

Used Car:

When the car quality is higher, the buyer has a stronger incentive to trade, but the seller has a weaker incentive to trade.

School & Employer:

When the student ability is higher, both the school and the employer have a stronger incentive to have the student hired.

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$\Rightarrow$ Marginal incentives play a crucial role in the scope of credible persuasion.

Comparison with Canonical Models

Bayesian Persuasion: Sender's choice of information structure is unconstrained.

Cheap Talk: Sender has no profitable deviation to any other information structure.

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$$\text{Cheap Talk} \subset \text{Credible Persuasion} \subset \text{Bayesian Persuasion}$$

Comparison with Canonical Models

	Used Car	School–Employer
Bayesian Persuasion	KG-optimal	KG-optimal

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What We Do

Propose a notion of credibility for persuasion problems

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Characterization: credibility $\Leftrightarrow$ cyclical monotonicity

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Characterization: credibility $\Leftrightarrow$ cyclical monotonicity

Information transmission depends on the "alignment" of preferences

Application: market for lemons

Literature

Optimal Information Disclosure

- Ostrovsky and Schwarz (2010); Rayo and Segal (2010); Kamenica and Gentzkow (2011)

Limited Commitment

- Lipnowski, Ravid, and Shishkin (2021); Min (2021); Nguyen and Tan (2021); Perez-Richet and Skreta (2022)

Repeated Communication

- Renault, Solan, and Vieille (2013); Best and Quigley (2020); Kuvalekar, Lipnowski, and Ramos (2022); Mathevet, Pearce, and Stacchetti (2022);

Quota Mechanism and Multi-Issue Cheap Talk

- Jackson and Sonnenschein (2007); Chakraborty and Harbaugh (2007); Rahman (2010); Frankel (2014)

Examples

Model

Characterization

When is Credibility Restrictive

Market for Lemons

Summary

Model

One Sender (S) and one Receiver (R).

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State: $\theta \in \Theta$ with prior μ_0

Action: $a \in A$

Message: $m \in M$

All spaces are finite

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Payoff functions: $u_S(\theta, a)$ and $u_R(\theta, a)$

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Credibility and R-IC

Definition. A profile (T^*, σ^*) is credible if

$$T^* \in \underset{T \in D(T^*)}{\arg\max}\ U_S(T, \sigma^*)$$

$D(T^*)$: The set of information structures with the same message distribution as T^*

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A credible and R-IC profile always exists

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Summary

Outcome Distributions

An outcome distribution $\pi \in \Delta(\Theta \times A)$ is induced by a profile (T, σ) if

$$\pi(\theta, a) = \sum_{m \in \sigma^{-1}(a)} T(m|\theta)\mu_0(\theta)$$

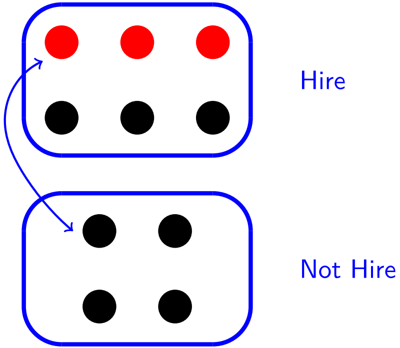
Cyclical Monotonicity

An outcome distribution $\pi \in \Delta(\Theta \times A)$ is u_S–cyclically monotone if for any n and $(\theta_1, a_1), \ldots, (\theta_n, a_n) \in \mathsf{supp}(\pi)$

$$\sum_{i=1}^{n} u_S(\theta_i, a_i) \geq \sum_{i=1}^{n} u_S(\theta_i, a_{i+1})$$

where $a_{n+1} \equiv a_1$.

School Example

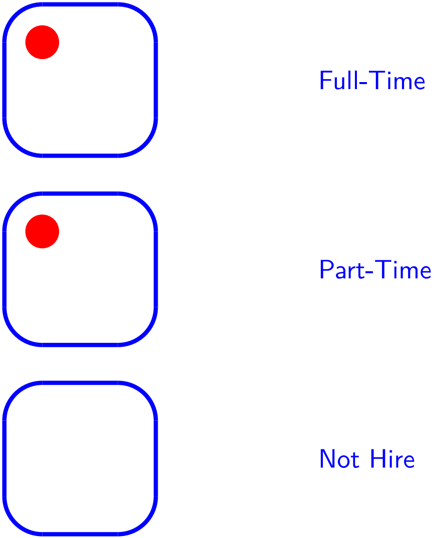


pairwise swappings = cycles of length 2

Full-Time

Part-Time

Not Hire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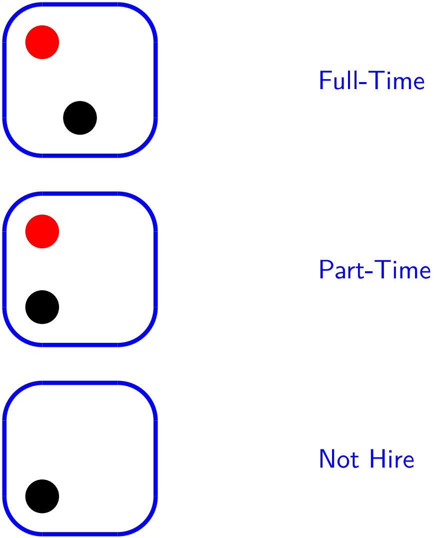
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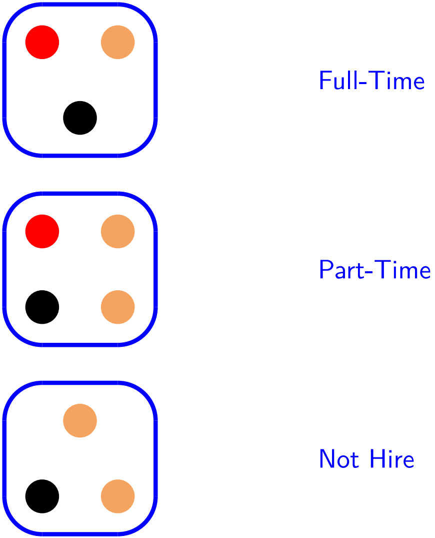
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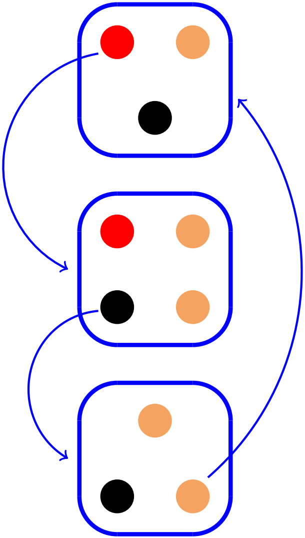

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Characterization

Theorem. An outcome distribution $\pi \in \Delta(\Theta \times A)$ can be induced by a credible and R-IC profile if and only if:

1. π is u_R-obedient: for any a in the support of π_A,

$$\sum_{\Theta} \pi(\theta|a)\, u_R(\theta, a) \geq \sum_{\Theta} \pi(\theta|a)\, u_R(\theta, a') \text{ for all } a' \in A.$$

2. π is u_S-cyclically monotone: for any $(\theta_1, a_1), \ldots, (\theta_n, a_n) \in \text{supp}(\pi)$,

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technical details

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$$\sum_{\theta,a} \pi(\theta,a)u_S(\theta,a) < \sum_{\theta,a} \pi'(\theta,a)u_S(\theta,a)$$

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In matrix form $\Rightarrow$

$$\pi \cdot u_S < \pi' \cdot u_S$$

where π and π' with the same row- and column- sums.

Sufficiency

Rescaling $+$ Splitting $\Rightarrow$

$$\underbrace{\mathbf{\Pi}}_{\text{bi-stochastic}} \cdot \mathbf{U_S} \quad < \quad \underbrace{\mathbf{\Pi'}}_{\text{bi-stochastic}} \cdot \mathbf{U_S}$$

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$\Rightarrow$ Profitable cyclical deviation.

Sufficiency: $n \leq \min\{|\Theta|, |A|\}$

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Additively Separable Preference

Observation. If $u_S(\theta, a) = \phi(\theta) + \psi(a)$, every outcome distribution is cyclically monotone.

$\Rightarrow$ Credibility does not restrict Sender's ability to persuade.

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A special case: state-independent preference $u_S(\theta, a) = \psi(a)$

(e.g. Chakraborty and Harbaugh, 2010; Alonso and Câmara, 2016; Lipnowski and Ravid, 2020)

Supermodular Preference

Assume both Θ and A are subsets of $\mathbb{R}$.

Sender's payoff $u_S(\theta, a)$ is supermodular.

Definition. A payoff function $u : \Theta \times A \to \mathbb{R}$ is supermodular if for any $\theta_H > \theta_L$ and $a_H > a_L$,

$$u(\theta_H, a_H) - u(\theta_H, a_L) \geq u(\theta_L, a_H) - u(\theta_L, a_L).$$

Comonotonicity

Lemma. If $u_S(\theta, a)$ is strictly supermodular,

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Comonotonicity: for any $(\theta, a), (\theta', a') \in \mathsf{supp}(\pi)$,

$$\theta > \theta' \quad \Rightarrow \quad a \geq a'$$

Theorem. When $\Theta, A \subseteq \mathbb{R}$ and $u_S(\theta, a)$ is strictly supermodular, an outcome distribution $\pi \in \Delta(\Theta \times A)$ can be induced by a credible and R-IC profile if and only if:

1. π is u_R-obedient;
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Monotone Persuasion

Dworczak and Martini (2019); Goldstein and Leitner (2018); Mensch (2021); Ivanov (2020); Kolotilin (2018); and Kolotilin and Li (2020)

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An additional rationale

Information Transmission under Credibility

Information Transmission under Credibility

Proposition 1. Supermodular $+$ Submodular $\Rightarrow$ No information

full statement

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Proposition 2. Supermodular $+$ Supermodular $\Rightarrow$ Benefit from persuasion?

Information Transmission under Credibility

Proposition 1. Supermodular + Submodular $\Rightarrow$ No information

full statement

Proposition 2. Supermodular + Supermodular $\Rightarrow$ Benefit from persuasion?

counter example

Information Transmission under Credibility

Proposition 1. Supermodular + Submodular ⇒ No information

full statement

Proposition 2. Supermodular + Supermodular ⇒ Benefit from persuasion
+ Additional condition

full conditions

Information Transmission under Credibility

Proposition 1. Supermodular $+$ Submodular $\Rightarrow$ No information

full statement

Proposition 2. Supermodular $+$ Supermodular $\Rightarrow$ Benefit from persuasion

$+$ Additional condition

full conditions

Proposition 3. Supermodular $+$ Supermodular and $|A| = 2$

$\Rightarrow$ Optimal full commitment solution is credible

full statement

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Market for Lemons: Base Game

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The base game G:

- Seller observes θ, chooses an ask price $a \in [0, v(1)]$;
- Buyers submit bids $b_1, b_2 \in [0, v(1)]$;
- Asset traded at winning bid if higher than ask; no trade otherwise.

Market for Lemons: Information

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$\langle G, T \rangle$ forms a Bayesian game:

- Message from T observed publicly;
- Buyers update posteriors; seller observes θ;
- Seller and buyers simultaneously submit ask and bids.

$\Rightarrow$ Prices determined endogenously by beliefs (ratings).

Full Commitment

If the seller can commit:

fully reveal $\theta \Rightarrow$ seller captures all gains from trade.

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If the seller can commit:

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Is this credible?

Market for Lemons: Credibility and IC

Definition. A profile of information structure and players' strategies (T^*, σ^*) is credible and IC if

- σ^* is a Bayesian Nash equilibrium in $\mathcal{G} = \langle G, T^* \rangle$
- T^* is optimal for the Sender among all information structures that generate the same message distribution

Market for Lemons: Credible Information Disclosure

Proposition. Under any credible and IC profile (T^*, σ^*),

$$\text{Seller's payoff} \leq \text{No information payoff}$$

Market for Lemons: Proof Idea

Step 1: $\exists$ a common trading threshold τ across all ratings

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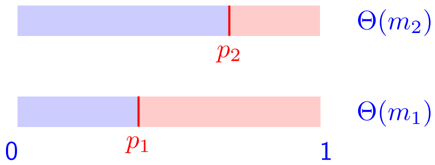
Suppose two different prices are induced:



Market for Lemons: Proof Idea

Step 1: $\exists$ a common trading threshold τ across all ratings

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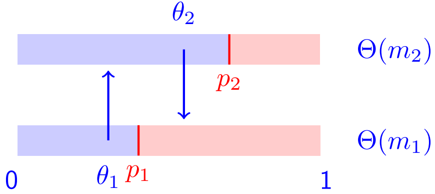

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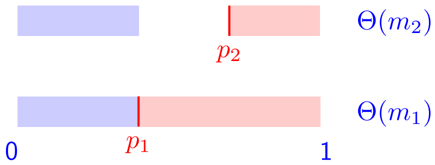

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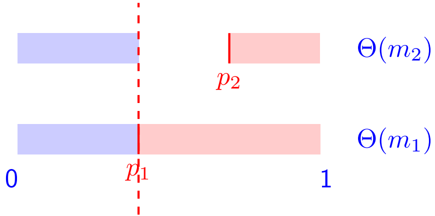

There should be no cars with quality $\theta \in (p_1, p_2)$ under message m_2.

A car is traded if and only if its quality $\theta \leq \tau = p_1$.

Market for Lemons: Proof Idea (Cont'd)

Step 2: Under the prior μ_0, $\exists$ equilibrium threshold $\tau^* \geq \tau$

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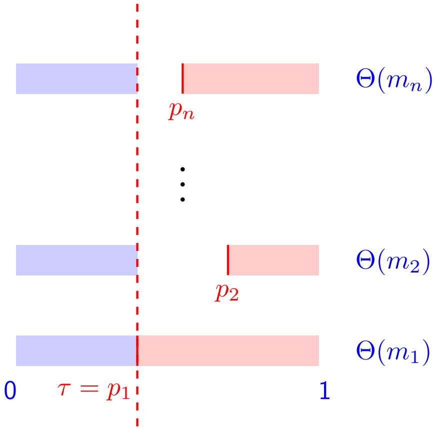


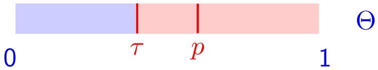
Market for Lemons: Proof Idea (Cont'd)

Step 2: Under the prior μ_0, $\exists$ equilibrium threshold $\tau^* \geq \tau$

Taking expectation over m gives

$$\tau \leq E_{\mu_0}[v(\theta)|\theta \leq \tau] = p$$

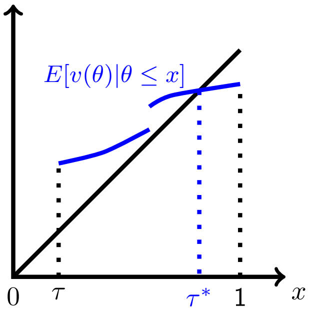
So with no information:



Market for Lemons: Proof Idea (Cont'd)

Tarski's fixed point theorem $\Rightarrow \quad \exists\, \tau^* \geq \tau$ such that

$$E_{\mu_0}[v(\theta)|\theta \leq \tau^*] = \tau^*.$$



Market for Lemons: Proof Idea (Cont'd)

There exists an equilibrium (under no information) where assets are traded at a higher threshold $\tau^* \geq \tau$.

Market for Lemons: Proof Idea (Cont'd)

There exists an equilibrium (under no information) where assets are traded at a higher threshold $\tau^* \geq \tau$.

$\Rightarrow$ The seller receives a higher payoff under no information.

Examples

Model

Characterization

When is Credibility Restrictive

Market for Lemons

Summary

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We propose a new credibility notion for persuasion

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- These conditions depend on the "alignment" of preferences

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In general: credibility $\Leftrightarrow$ cyclical monotonicity

Provide conditions on how credibility affects communication

- These conditions depend on the "alignment" of preferences

Apply our approach to markets for lemons

- Information disclosure cannot credibly prevent inefficiency

Takeaways

A way to evaluate the commitment assumption in Bayesian Persuasion

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A rationale to explain why certain industries can effectively disclose information by utilizing their own rating system

- School designs grading rule
- Hospital designs treatment guideline

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A way to evaluate the commitment assumption in Bayesian Persuasion

A rationale to explain why certain industries can effectively disclose information by utilizing their own rating system

- School designs grading rule
- Hospital designs treatment guideline

but some other industries have to reply on other means

- Used car dealers
- Antique dealers

Literature

Optimal Information Disclosure

- Ostrovsky and Schwarz (2010); Rayo and Segal (2010); Kamenica and Gentzkow (2011)

Limited Commitment

- Lipnowski, Ravid, and Shishkin (2021); Min (2021); Nguyen and Tan (2021); Perez-Richet and Skreta (2022)

Repeated Communication

- Renault, Solan, and Vieille (2013); Best and Quigley (2020); Kuvalekar, Lipnowski, and Ramos (2022); Mathevet, Pearce, and Stacchetti (2022);

Quota Mechanism and Multi-Issue Cheap Talk

- Jackson and Sonnenschein (2007); Chakraborty and Harbaugh (2007); Rahman (2010); Frankel (2014)

Proposition. Suppose u_S and u_R are both supermodular.

1. If $u_S(\theta, a) > u_S(\theta, a')$ for any θ and $a > a'$, the Sender benefits from credible persuasion for generic prior as long as she benefits from persuasion;
2. If $u_S(\overline{\theta}, \overline{a}) > u_S(\overline{\theta}, a)$ for all $a \neq \overline{a}$ and $u_S(\underline{\theta}, \underline{a}) > u_S(\underline{\theta}, a)$ for all $a \neq \underline{a}$, she benefits from credible persuasion;
3. If the Sender is strictly better off from fully revealing outcome than from no information outcome, then the Sender benefits from credible persuasion.

Back

Extensive-Form Foundation

Timing:

1. Sender chooses an information structure T;
2. Receiver observes the distribution of messages $P_m(\cdot) = \sum_\theta \mu_0(\theta) T(\cdot|\theta)$;
3. Receiver chooses an action for each message $\sigma : M \to A$

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Proposition.

$$(T, \sigma) \text{ is a SPE outcome}$$

$$\Updownarrow$$

$$(T, \sigma) \text{ is Credible and R-IC} \quad \& \quad U_S(T, \sigma) \geq \text{No Information Value}$$

Back

Kantorovich Duality. Suppose X and Y are both finite sets, and $u : X \times Y \to \mathbb{R}$ is a real-valued function. Let $\mu \in \Delta(X)$, $\nu \in \Delta(Y)$, and $\Pi(\mu, \nu) = \{P \in \Delta(X \times Y) | P_X = \mu, P_Y = \nu\}$.

For any $\pi^* \in \Pi(\mu, \nu)$, the following three statements are equivalent:

1. $\pi^* \in \arg\max_{\pi \in \Pi(\mu,\nu)} \sum_{x,y} \pi(x, y) u(x, y)$;
2. π^* is u-cyclically monotone.
3. There exists $\psi : Y \to \mathbb{R}$ such that for any $(x, y) \in \text{supp}(\pi^*)$ and any $y' \in Y$,
$$u(x, y) - \psi(y) \geq u(x, y') - \psi(y').$$

Back

Proposition.

$$\text{Cheap Talk} + \text{Transfers} \quad \Leftrightarrow \quad \text{Credibility}$$

Approximation

Suppose

$$\sum_{\theta,a} \pi(\theta,a) u_S(\theta,a) < \sum_{\theta,a} \pi'(\theta,a) u_S(\theta,a)$$

Approximation

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By continuity, $\exists$ rational $\widetilde{\pi}$ and $\widetilde{\pi}'$ with the same support as π and π' such that

$$\sum_{\theta,a} \widetilde{\pi}(\theta,a) u_S(\theta,a) < \sum_{\theta,a} \widetilde{\pi}'(\theta,a) u_S(\theta,a)$$

Back

When Credibility Shuts Down Communication

Proposition. If u_S is strictly supermodular and u_R is submodular, then any outcome distribution that can be induced by a credible and R-IC profile is a no information outcome.

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Submodularity + Obedience $\Rightarrow$ Higher states match with lower actions

Only one action can be induced

Back

Example

$\Theta = \{\theta_L, \theta_H\}$, $A = \{a_1, a_2, a_3, a_4\}$.

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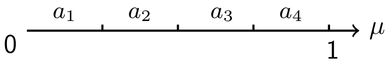

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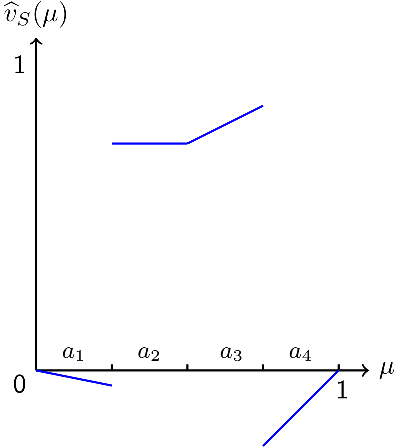

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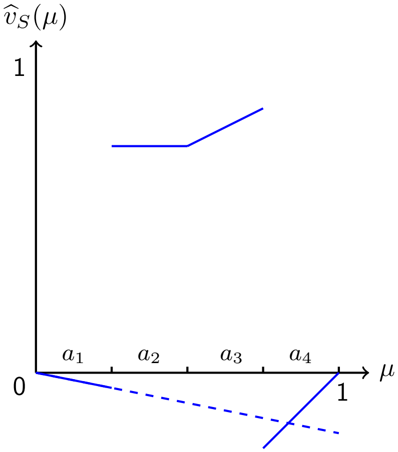

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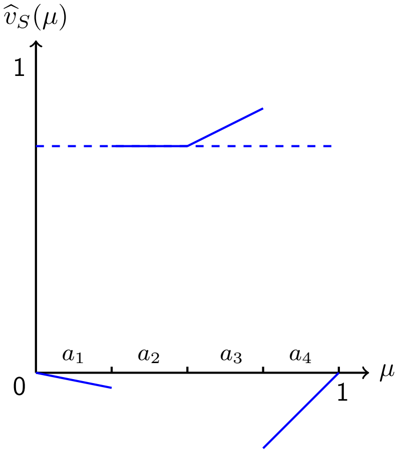

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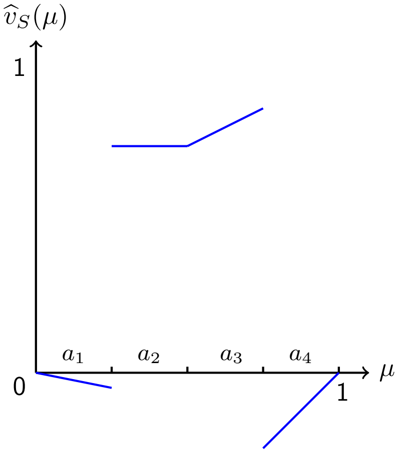

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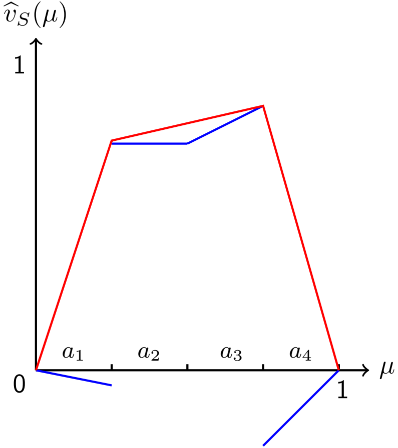

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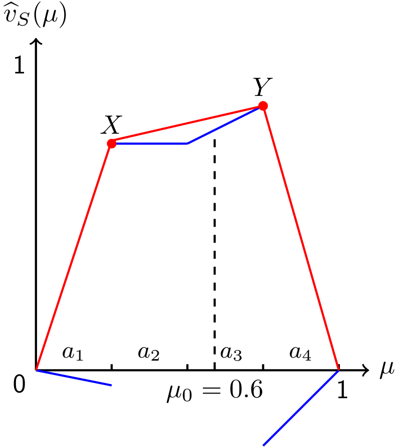
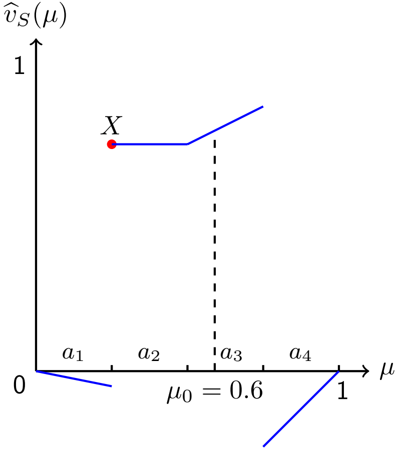
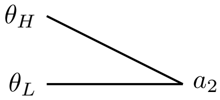
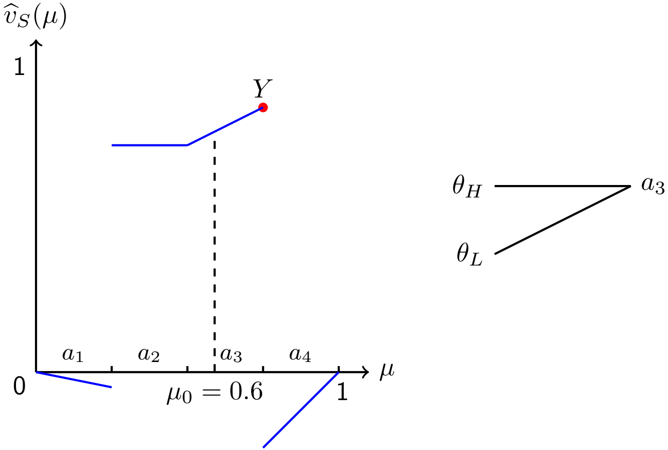



$\widehat{v}_S(\mu)$
1
X
a_1
a_2
a_3
a_4
μ
0
$\mu_0 = 0.6$
1


θ_H
θ_L
a_2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1
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0
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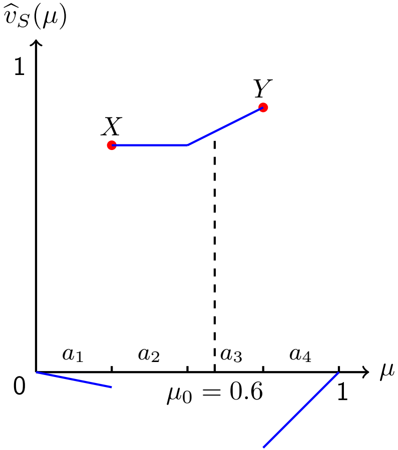
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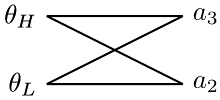
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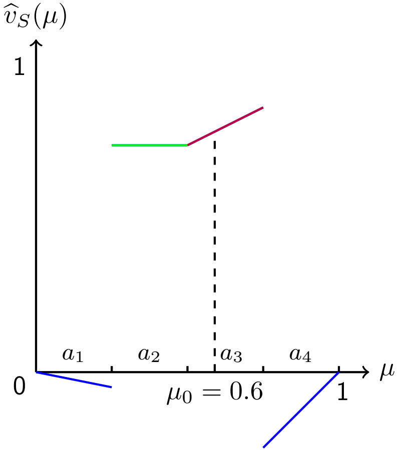

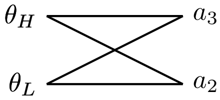

back

Additional conditions:

- Sender prefers a higher action regardless of the state;
- Sender and Receiver agree under extreme states;
- Fully revealing gives the Sender higher payoff than no information.

Back

When Credibility Imposes No Cost to the Sender

Proposition. Suppose u_S and u_R are both supermodular and $|A| = 2$, at least one optimal full-commitment outcome distribution can be induced by a credible and R-IC profile.

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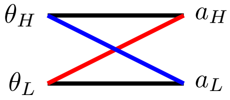
Intuition (Mensch 2021):

suppose an outcome distribution is obedient but not comonotone



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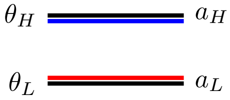
swapping non-comonotone pairs



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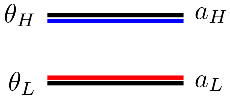
Intuition (Mensch 2021):

Sender's payoff weakly improves + the obedient constraints still hold



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